Finance and Growth Cycles

Kunieda, Takuma

Ryukoku University

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Takuma Kunieda*†
Department of Economics, Ryukoku University

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Abstract

This research examines the effect of financial development on volatility in economic growth. It demonstrates theoretically that financial development has a hump-shaped effect on volatility in economic growth. In early stages of the development of a financial sector, growth rates evolve monotonically. At the intermediate level of financial development, as the degree of credit market imperfections diminishes and as asymmetric information between borrowers and lenders is less pronounced, an economy exhibits endogenous growth cycles. However, as the financial sector matures, the volatility in the growth process dissipates and the growth rates evolve once again monotonically.

Keywords: Endogenous Growth Cycles; Financial Deepening; Credit market imperfections; Heterogeneous agents.

JEL Classification Numbers: E22, E23, O41.

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*Corresponding author. Takuma Kunieda, Department of Economics, Ryukoku University, 67 Fushimi-ku Fukakusa Tsukamoto-Cho, Kyoto, 612-8577, JAPAN, Phone: +81-75-642-1111, E-mail: kunieda@econ.ryukoku.ac.jp

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1 Introduction

A long tradition in economics has stressed the importance of finance for economic development (e.g. Schumpeter (1934)). Authors in this tradition (e.g. Hicks (1969)) give historical evidence such as the industrial revolution in England. They argue that the technology used for the first stage of the industrial revolution in England had been invented far earlier and that it was the development of the capital market in England that made sustained growth possible. Meanwhile, writers such as Keynes (1936) maintain that business fluctuations originate in the financial system of an economy. This hypothesis is also supported by historical evidence such as the Great Depression before World War II and the lost decade in Japan after the boom in the 1980s.

More recently a large number of economists have emphasized the importance of financial deepening in understanding macroeconomic phenomena such as economic growth and business cycles. The empirical literature has produced evidence for the positive effect of financial development on economic growth. (See for instance King and Levine (1993a,b), Levine (1997), Levine, et al. (2000), and Aghion, et al. (2005).) Also, the positive relationship between financial development and economic growth has been demonstrated theoretically in an extensive literature. (See for instance Greenwood and Jovanovic (1990), Galor and Zeira (1993), Greenwood and Smith (1997), and Aghion, et al. (2005).) Meanwhile, the relationship between the volatility of growth rates and financial development has been empirically investigated by several researchers (Easterly, et al (2000), Denizer, et al. (2002), Raddatz (2006), and Beck, et al. (2006)). Almost all of these empirical articles show that economies with well developed financial sectors experience low volatility of growth rates.[1] However, the relationship between financial deepening and the volatility of growth rates is still an open question theoretically.

In this paper, we investigate the relationship between financial development and the dynamic properties of growth rates with a dynamic general equilibrium model.

Since small exogenous technological shocks cannot lead to persistent fluctuations of growth rates with a perfect credit market (Aghion and Banerjee (2005)), credit market imperfections are a possible mechanism to derive such persistent fluctuations. Indeed, Kiyotaki and Moore (1997) construct a dynamic general equilibrium model in which persistent fluctuations arise due to credit constraints associated with the price of collateral. However, they do not study

[1]Beck, et al. (2006) do not discover a robust relationship between financial development and growth volatility. In particular, they give empirical evidence that in economies with less developed capital markets, financial intermediaries magnify the impact of inflation volatility on the volatility of growth rates.

under what degree of credit market imperfections, or equivalently, under what degree of financial deepening, persistent fluctuations are most likely to occur.

A few researchers have investigated this—but only a few. For example, Aghion, et al. (1999), Aghion, et al. (2004), and Pintus (2007) investigate the relationship between financial deepening and endogenous business cycles; however, they do not study the fluctuations of growth rates but instead study the fluctuations of output with technological progress exogenously given. By contrast, in this paper, we study endogenous growth and endogenous business fluctuations, both of which are affected by the degree of financial deepening. Our question is, "*How does financial deepening affect the dynamic properties of growth rates?*" In order to answer this question, we develop a dynamic general equilibrium model following the Schumpeterian approach of Aghion and Howitt (1992).

By the degree of financial deepening (financial development), we mean the degree of credit market imperfections. Credit market imperfections result from asymmetric information between savers and investors or between a financial intermediary and investors. Savers and investors are different entities in an economy (Keynes (1936), Harrod (1939), and Aghion, et al. (1999)). However, how and where do they interact with each other? If savers could find investors endowed with high-quality projects and lend funds to them, this must be good not only for savers and investors in the current period but also for the entire economy through all periods. However, asymmetric information between savers and investors makes this impossible .

If a banking system is established in an economy, savers and investors make financial trades anonymously via the banking system. Savers do not have to collect information about investors because they just make a deposit in a financial intermediary. On the other hand, investors can borrow from the financial intermediary at a given interest rate, which is determined endogenously in a financial market. Nevertheless, the agency problem remains because investors are motivated to default after they borrow.

In our model, all financial trades are executed via an infinitely lived financial intermediary, which we call the "Bank" following Grandmont (1983).[2] The Bank imposes credit constraints on investors so as to avoid default. Credit constraints are incentive compatible with borrowers not defaulting and are derived from the optimal monitoring behavior of the Bank. The idea for the formulation is based on Aghion, et al. (1999). In order to collect the obligated repayments, the Bank monitors a borrower only when he defaults. The Bank incurs monitoring costs

[2]We assume that there is no equity market for direct finance. In our model, only small firms borrow in order to finance their investments. One can imagine that it is hard for small firms to issue equities because of agency problems.

due to the moral hazard problem. The more monitoring cost the Bank pays, the higher is the probability that it can collect repayments from a defaulting borrower. In turn, when a borrower defaults, he has to pay default costs. Under this situation, however, if a borrower faces credit constraints, he never defaults because the returns when not defaulting are greater than the returns when defaulting.

As a financial sector becomes well developed, the monitoring costs decrease and the default costs go up. Financial deepening must be due to well-functioning institutions that protect property rights. As the monitoring costs go down and/or the default costs increase, credit constraints will relax and the credit market in an economy gets closer to a perfect one.

Our main findings are as follows. In early stages of the development of a financial sector, growth rates evolve monotonically. At the intermediate level of financial development, as the degree of credit market imperfections diminishes and as asymmetric information between borrowers and lenders is less pronounced, an economy exhibits endogenous growth cycles. However, as the financial sector matures, the volatility in the growth process dissipates and the growth rates evolve once again monotonically.

Aghion, et al. (1999) and Aghion, et al. (2004) obtain similar results to ours with respect to fluctuations of output, not growth rates. However, the mechanisms that derive their results are completely different. In their models, there is no choice for each agent to be a saver or to be an investor, whereas in our economy, the choice for each agent to be a saver or an investor is crucial to the emergence of endogenous growth cycles. In our model, agents are heterogeneous in creating capital goods, which become input goods for a research and development (R&D) sector. More capable agents wish to establish their own firms, borrow from the Bank, start investment projects and create capital goods as long as the interest rate is less than their marginal products. On the other hand, less capable agents prefer to make a deposit in the Bank rather than to borrow, not creating capital goods, when their marginal products are less than the interest rate. The cut-off level of the agents' productivity which divides agents into savers and investors is determined endogenously.

We obtain a dynamical system for this cut-off point, which originates in the dynamic equation for net total assets in the private sector. The growth rate of the economy is a one-to-one function of the cut-off point. Given the degree of financial deepening, if the cut-off point in the current period is small (large), then the number of investors is large (small), and thus the growth rate is high (low). Therefore, the dynamics of the cut-off point induces endogenous growth cycles.

This paper proceeds as follows. Section 2 reviews the literature related to this paper. Section 3 provides our model, and in section 4 we study equilibrium, deriving a function for the growth rate with respect to the cut-off point of agents' productivity. In section 5, we investigate the dynamic properties of the economy. Section 6 discusses why endogenous fluctuations arise in equilibrium. In section 7, we give numerical examples and observe that it is when the degree of financial deepening is at the intermediate level that the growth rates endogenously fluctuate. Section 8 compares our results to empirical evidence. Section 9 gives concluding remarks.

2 Literature Review

Our paper belongs to the literature on macroeconomics for credit market imperfections. (See for instance, Bernanke and Gertler (1989), Greenwood and Jovanovic (1990), Galor and Zeira (1993), Greenwald and Stiglitz (1993), Greenwood and Smith (1997), and Kiyotaki and Moore (1997).) In particular, as we referred to in introduction, our paper is closely related to Aghion, et al. (1999) and Aghion, et al. (2004). Aghion, et al. (1999) study a closed economy like ours, whereas Aghion, et al. (2004) investigate a small open economy.[3] They also study the relationship between financial development and the dynamic properties of an economy; however, our model differs from theirs in several respects.

First, as was already mentioned, in our economy, whether each agent becomes a saver or an investor is very important to the emergence of endogenous growth cycles, whereas in their models, each agent cannot make a choice to be a saver or to be an investor. In their models, the number of savers and investors is exogenously given. The second point is that in Aghion, et al. (1999) the total savings in a period are predetermined by the stock of wealth in an economy. The total investments are determined by entrepreneurs' wealth multiplied by a constant credit multiplier. Therefore, the total investments are predetermined as well. As a result, when the savings are smaller (greater) than the investments, the interest rate increases (decreases). By contrast, in our model, we observe opposite causation. In our model, an agent makes a decision on whether he becomes a lender or a borrower, taking into account a prospective interest rate, a prospective price of capital goods, and/or his own ability. For this reason, the total investments are determined by future variables. Accordingly, if a prospective interest rate is high (low), a large (small) number of people tend to deposit in the Bank, whereas a small (large) number of people tend to invest.

There are other articles which deal with the relationship between the de-

[3]See also Caballé, et al. (2006). They show the existence of a complex equilibrium with the model of Aghion, et al. (2004).

gree of credit market imperfections and the dynamic properties of an economy. Matsuyama (2004, 2006) investigates the accumulation of capital stock when an economy faces credit market imperfections. Although Matsuyama's models are overlapping generations models like ours, our model differs from Matsuyama's models. The main difference is that agents in his models can borrow only from the other agents within a same generation as long as the economy is closed, i.e., there is no intergenerational financial trades in Matsuyama's models.[4] As a result, the net credit position of a generation is zero. By contrast, in our economy, individuals borrow or lend via the Bank. Accordingly, the net total asset a generation holds can be positive or negative.

Our model also belongs to a long tradition of overlapping generations modeling. That is to say, our model has hybrid aspects of Samuelson (1958) and Diamond (1965). The hybrid aspects originate in the heterogeneity of agents' productivity, due to which our model is tractable in studying credit market imperfections. In our model, less capable agents make a deposit in the Bank, not starting investment projects. They are comparable to the agents in Samuelson's consumption loan model. On the other hand, more capable agents start investment projects. Those can be regarded as the agents in Diamond's production economy. A large number of articles have studied the dynamic properties for overlapping generations economies (see for instance Benhabib and Day (1982), Grandmont (1985), Reichlin (1986), Farmer (1986), Benhabib and Laroque (1988), Galor and Ryder (1989), Galor (1992), and Goenka, et al. (1998)). However, none of these articles make clear under what degree of financial deepening endogenous growth cycles are most likely to arise nor do they deal with credit market imperfections.

3 Model

The economy consists of overlapping generations: young and old agents. Time goes from 0 to ∞. Each agent lives for two periods. Following Bernanke and Gertler (1989), we may think of a "period" as the length of a typical financial contract. Therefore, we may consider each individual as a small firm. The rate of population growth is assumed to be $n > -1$. If the population of young agents at time t is L_t, then $L_{t+1} = (1+n)L_t$ holds. Young agents can borrow up to a certain limit from an infinitely lived financial intermediary which we have called the Bank following Grandmont (1983). Namely, the Bank imposes credit constraints on young agents.

[4] Matsuyama (2004) considers a small open economy as well.

3.1 Governmental Agency: The Bank

The government establishes the Bank, by which each agent makes financial trades in the anonymous financial market. The role of the Bank is two-fold. One is to provide loans and the other is to monitor defaulting borrowers. While the Bank is engaged in the loan business, the government's credit position is divided into two cases. If the government is indebted to the private sector, we call this the government "debt" case. If the government has ownership of the economy's capital, we call this the government "credit" case.

3.1.1 Government Debt and Credit

Some of the young agents deposit a part of their income in the Bank. Those deposits become financial resources for the Bank to loan. Meanwhile, each young agent can borrow against his future income by selling bonds to the Bank. Young agents can consume, or invest in a project, more than their income in the first period by going into debt to the Bank.[5]

If the total stock of deposits made by young agents is greater than the total stock of loans lent to them, this is the case of government debt. In each period, the old agents redeem their government bonds. The government finances this redemption by borrowing from young agents at the market interest rate. On the other hand, if the total stock of deposits made by young agents is less than the total stock of loans lent to them, this is the case of government credit. In this case, the government owns a stock of loans to the private sector. In each period, the old agents pay back their loans and the government reinvests the proceeds by lending them to young agents at the market interest rate.

A loan made by the Bank is an exchange of real output in this period for real output in the next period. The government debt at time t, B_t, is equal to total deposits minus total loans in the economy at time t. So $B_t > 0$ in the government debt case and $B_t < 0$ in the government credit case.[6] We assume that in either case, the government does not conduct any fiscal policy, i.e., the government does not levy any tax on agents or purchase any goods, except at time zero. Under these circumstances, it follows that:

$$B_{t+1} = r_{t+1} B_t, \tag{1}$$

[5] Similar assumptions are frequently made in the literature. See for instance Azariadis and Smith (1993) and Rochon and Polemarchakis (2006).

[6] At first sight, the assumption of negative B_t does not seem reasonable because we observe that many governments in the real world run deficits. However, we should note that B_t is the net worth in the balance sheet of a government *including tangible assets*. The positive net worth of a government is sometimes reported even if the government runs deficits. For example, Eisner and Pieper (1984) give evidence for the positive net worth of the U.S. government from 1970 to 1980 (from 1946 to 1980 if the state and local governments are included).

where r_{t+1} is the (real) interest rate at time $t+1$. Whether there is government debt or credit is determined at time zero, when the government makes a lump-sum transfer of B_0 to the old (the debt case) or collects taxes equal to $-B_0$ from the old (the credit case).[7]

Note that Eq.(1) is equivalent to a goods-market clearing condition, because of Walras' law. To see this, we aggregate all variables at time t as follows:

$$C_t + K_t + B_t = Y_t + r_t B_{t-1},$$

where C_t is aggregate consumption, K_t is aggregate investment, and Y_t is aggregate output. From the goods-market clearing condition, we have $C_t + K_t = Y_t$. Therefore $B_t = r_t B_{t-1}$ holds.

3.1.2 The Bank's Monitoring

In this subsection, we study why the Bank imposes credit constraints on agents. The Bank's monitoring gives a microfoundation for credit constraints. In what follows, we will derive the inequality for credit constraints by modifying the model of Aghion, et al. (1999). Suppose that each agent prepares his own resources $\tilde{w}_t$ to invest. If he borrows $-b_t > 0$, his total resources are $k_t = \tilde{w}_t - b_t$. Let the return on one unit of investments be R_{t+1}.

The Bank monitors borrowers only when the borrowers default in order to collect debts as much as possible. When the Bank monitors a defaulting borrower, it pays costs $-r_{t+1}b_t C(p)$ to collect $-pr_{t+1}b_t$, where $p \in (0,1)$ is the probability with which the Bank can collect the obligated repayment and $C : [0,1) \to \Re_+$ is twice continuously differentiable. We assume $\frac{\partial C(.)}{\partial p} > 0$, $\frac{\partial^2 C(.)}{\partial p^2} > 0$, $C(0) = 0$, $\lim_{p \to 1} C(p) = \infty$, and $C'(0) < 1$. As the Bank takes on more costs, the probability to succeed in monitoring the borrower increases.

In turn, in order to default, borrowers have to pay default costs $\theta r_{t+1} k_t$, where we assume $0 \leq \theta < 1 - {C'}^{-1}(1) < 1$.[8] The default costs are some proportion of the returns which borrowers would obtain if they deposited k_t in the Bank. The default costs are considered as fines or social sanctions.

Under this loan contract, the incentive compatibility constraint so as for a borrower not to default is given by:

$$R_{t+1}[\tilde{w}_t - b_t] + r_{t+1}b_t \geq [R_{t+1} - \theta r_{t+1}][\tilde{w}_t - b_t] + pr_{t+1}b_t, \tag{2}$$

[7] One should note that even if $B_t > 0 (< 0)$, there are borrowers (savers) in the generation. B_t is a "total" of assets held by the generation.

[8] We impose this assumption so that every borrower faces credit constraints which are severer than the natural debt limit. In fact, the closer is θ to $1 - {C'}^{-1}(1)$, the more nearly the credit market approaches a perfect one.

which is rewritten as:

$$b_t \geq -\frac{\theta}{1-p}k_t. \tag{3}$$

The left-hand side of Eq.(2) is the profits when the borrower invests in a project, whereas the right-hand side of Eq.(2) is the gain when the borrower defaults. We note that Eq.(3) is independent of the return on one unit of investments R_{t+1}. In the next subsection, we shall assume that the values of R_{t+1} vary between agents. That is to say, the return on one unit of investments depends upon the agents' heterogeneous productivity. While the Bank has to know how much the agent invests (k_t), the Bank does not have to know the agent's productivity directly. As long as the Bank imposes a credit constraint given by Eq.(3) on each agent, the incentive compatibility constraint Eq.(2) holds for any unobserved the agents' productivity which is reflected in R_{t+1}.

In order to choose an optimal probability, the Bank solves its maximization problem as follows:

$$\max_p \quad -pr_{t+1}b_t + r_{t+1}b_tC(p).$$

Since $-r_{t+1}b_t > 0$, this maximization problem is rewritten as:

$$\max_p \; p - C(p).$$

From the first-order condition, we have:

$$p = {C'}^{-1}(1). \tag{4}$$

From Eq.(3) and Eq.(4), we obtain:

$$b_t \geq -\frac{\theta}{1-{C'}^{-1}(1)}k_t, \tag{5}$$

where $\frac{\theta}{1-{C'}^{-1}(1)}$ reflects financial deepening. We can say that the financial sector is well developed in an economy if the monitoring costs decrease, i.e., if the function $C(p)$ shifts down so that ${C'}^{-1}(1)$ increases. In addition, financial deepening must be related to the social sanctions when an agent defaults. That is to say, the financial sector in an economy is well developed if θ increases. Since $\theta < 1 - {C'}^{-1}(1)$, we can let $\mu := \frac{\theta}{1-{C'}^{-1}(1)} \in [0, 1)$ and thus:

$$b_t \geq -\mu k_t,$$

which is a credit constraint. μ is the measure of financial deepening.

3.2 Individuals

3.2.1 Saving Methods

Each agent is born with one unit of endowments which we call labor. In the first period of his lifetime, he supplies labor inelastically and earns a wage income. An agent born at time t consumes c_{1t} when young and c_{2t+1} when old. There are two saving methods for each agent. One is depositing his income in the Bank. If an agent deposits one unit of his income in the Bank at time t, he will gain a claim to r_{t+1} units of consumption goods at time $t+1$.

Alternatively, an agent can start an investment project, which will produce capital goods. The capital goods created by agents are thought of broadly as human capital or physical capital. The capital goods are used as input goods for an R&D sector.

3.2.2 A Consumer's Problem

In the first period, each agent consumes, starts an investment project, deposits his income in the Bank, and/or borrows from the Bank. In the second period, he consumes all his earnings from the investments and from the deposits, and he repays the Bank if he borrowed in the first period.

Each agent maximizes his lifetime utility:

$$u(c_{1t}, c_{2t+1}) \tag{6}$$

which is a function of consumption (c_{1t}, c_{2t+1}) in youth and old age, subject to:

$$c_{1t} + k_t + b_t \leq w_t \tag{7}$$

$$z_{t+1} = \phi k_t \tag{8}$$

$$c_{2t+1} \leq q_{t+1} z_{t+1} + r_{t+1} b_t, \tag{9}$$

$$b_t \geq -\mu k_t, \quad 0 \leq \mu < 1 \tag{10}$$

$$k_t \geq 0. \tag{11}$$

Eq.(7) is a budget constraint for the first period. k_t is the investment in a project and b_t is a deposit if positive and is a debt if negative. w_t is the wage at time t. Eq.(8) is a production function for capital goods. z_{t+1} is the capital goods produced by the agent, and $\phi > 0$ is the productivity of the production. Eq.(9) is a budget constraint for the second period. q_{t+1} is the (real) price of the capital goods in terms of the consumption goods at time $t+1$. Again, r_{t+1} is the (real) interest rate at time $t+1$. Eq.(10) is a credit constraint which the agent faces. While the agent can deposit his income in the Bank as much as he wants to, he can borrow from the Bank only up to some proportion of the investments. As studied in the previous subsection, Eq.(10) is incentive compatible with the agent's not defaulting. Again, μ measures the degree of financial deepening: if μ is large, the financial sector is well developed, whereas if μ is small, it is less developed. Eq.(11) is a non-negativity constraint for the investment project.

The lifetime utility function $u : \Re^2_+ \to \Re$ is continuously differentiable and strictly quasi-concave on the interior of the consumption set $\Re^2_+$. The utility function is increasing in consumption in both periods: $u_i(.,.) > 0$ $(i = 1, 2)$ for $(c_{1t}, c_{2t+1}) \gg 0$, where $u_i(.,.)$ is the derivative with respect to the ith variable. Starvation is avoided in both periods, i.e., $\lim_{c_{1t}\to 0} u_2(.,.) = \infty$ and $\lim_{c_{2t+1}\to 0} u_2(.,.) = \infty$. Specifically, we assume $u(c_{1t}, c_{2t+1}) = c_{1t}^{\gamma} c_{2t+1}^{1-\gamma}$ throughout the current model, where $0 < \gamma < 1$. By specifying the utility as a Cobb-Douglas function, we can highlight the effects of financial deepening on the dynamic properties of the economy. If the income effect of the return to savings is too strong relative to the substitution effect, equilibrium cycles will appear without credit market imperfections.[9] If we use a Cobb-Douglas utility function, however, the income and the substitution effects are canceled by each other. Accordingly, equilibrium cycles will not arise without credit market imperfections.

3.2.3 Heterogeneous Agents

Now the heterogeneity of agents is introduced in terms of their productivity in producing capital goods. When an agent is born, he receives a shock for his productivity level ϕ from a time-invariant distribution $G(\phi)$ whose support is $[0, a]$, where $a > 0$. We impose some assumptions on the distribution function $G(\phi)$.

Assumption 1

- $\int_0^a \phi dG(\phi) < \infty$.
- *$G(\phi)$ has a continuous density $g(\phi)$ on $[0, a]$.*

The productivity of each agent is private information. Each agent knows his own productivity at his birth, while other agents do not know his productivity.[10] In the Diamond overlapping generations model, agents are homogeneous, i.e., $\phi = 1$ for every agent. By contrast, in our model, the agents' saving technology depends upon their heterogeneous productivity. As we solve a consumer's

[9] See for instance Grandmont (1985).

[10] Hence, less capable agents cannot ask more capable agents to produce capital goods. For this situation, one could imagine that less capable agents face prohibitively high costs to identify more capable agents.

problem in what follows, we shall find that the heterogeneity of agents is crucial when savers and investors are endogenously determined.

Let us solve the utility maximization problem for a consumer. Let $s_t := k_t + b_t$. Since $k_t \geq 0$ and $b_t \geq -\mu k_t$, it follows that $s_t \geq (1-\mu)k_t \geq 0$. Hence, we can rewrite an agent's maximization problem as follows:

$$\max_{s_t \geq 0, 0 \leq k_t \leq \frac{s_t}{1-\mu}} \quad u(w_t - s_t, r_{t+1}s_t + (q_{t+1}\phi - r_{t+1})k_t). \tag{12}$$

In this maximization problem, $s_t > 0$ holds because $\lim_{c_{2t+1} \to 0} u_2(.,.) = \infty$. Since $u_2(.,.) > 0$, if $r_{t+1} > q_{t+1}\phi$, then it is optimal for the agent to choose $k_t = 0$ and $s_t = b_t$. Since we have assumed $u(c_{1t}, c_{2t+1}) = c_{1t}^{\gamma} c_{2t+1}^{1-\gamma}$, the first-order condition with respect to s_t is given by: $-\gamma \frac{u(c_{1t}, c_{2t+1})}{c_{1t}} + (1-\gamma) r_{t+1} \frac{u(c_{1t}, c_{2t+1})}{c_{2t+1}} = 0 \iff c_{2t+1} = \frac{(1-\gamma)r_{t+1}}{\gamma} c_{1t}$. From this equation, Eq.(7), and Eq.(9) we obtain $b_t = (1-\gamma)w_t$. On the other hand, if $r_{t+1} < q_{t+1}\phi$, then the agent chooses $k_t = \frac{s_t}{1-\mu}$. The first-order condition is then given by: $-\gamma \frac{u(c_{1t}, c_{2t+1})}{c_{1t}} + (1-\gamma) \frac{q_{t+1}\phi - r_{t+1}\mu}{1-\mu} \frac{u(c_{1t}, c_{2t+1})}{c_{2t+1}} = 0 \iff c_{2t+1} = \frac{1-\gamma}{\gamma} \frac{q_{t+1}\phi - r_{t+1}\mu}{1-\mu} c_{1t}$. Now from this equation, Eq.(7) and Eq.(9), we obtain $k_t = \frac{(1-\gamma)w_t}{1-\mu}$ and $b_t = -\frac{\mu(1-\gamma)w_t}{1-\mu}$.

ϕ is used for the index of the heterogeneity of agents. Therefore, we henceforth put it on each variable as $c_{1t}(\phi)$, etc. Lemma 1 summarizes the above results.

Lemma 1
Let $\phi_t := \frac{r_{t+1}}{q_{t+1}}$. Then, the following claims hold.

- *If $\phi_t > \phi$, then $k_t(\phi) = 0$ and $b_t(\phi) = (1-\gamma)w_t$.*
- *If $\phi_t < \phi$, then $k_t(\phi) = \frac{(1-\gamma)w_t}{1-\mu}$ and $b_t(\phi) = -\frac{\mu(1-\gamma)w_t}{1-\mu}$.*

Proof: The claims have been proven in the above discussion. □

As seen in lemma 1, ϕ_t is a cut-off point, i.e., if an agent's productivity is greater than ϕ_t, he starts an investment project, borrowing from the Bank. In this case, the returns to his savings are subject to his productivity. He obtains more returns from the investment in a project than from a deposit in the Bank. On the other hand, if an agent's productivity is less than ϕ_t, he only deposits a part of his income in the Bank. He prefers to deposit in the Bank rather than to invest in a project. We can ignore agents with $\phi = \phi_t$ because they have no impact on the economy. As the parameter for financial deepening μ gets greater, both investments and borrowings of the agents with $\phi > \phi_t$ become large.

3.3 Final Production Sector

The general goods are produced in a final production sector, which becomes consumption goods and investment goods. The general goods are produced from a continuum of intermediate goods, which is distributed uniformly in $[0,1]$. A CES production function is assumed for the production of the general goods, and is given by:

$$Y_t = \left[\int_{i \in [0,1]} (A_{it} x_{it})^{\alpha} di \right]^{\frac{1}{\alpha}}, \tag{13}$$

where A_{it} is the quality of the ith intermediate good and x_{it} is its quantity. We define $\epsilon := \frac{1}{1-\alpha}$ as the elasticity of substitution between input goods, which is greater than one since $\alpha \in (0,1)$.

The final production sector is competitive and the representative firm solves the maximization problem given by:

$$\max_{x_{it}} \; Y_t - \int_{i \in [0,1]} \tilde{p}_{it} x_{it} di, \tag{14}$$

where $\tilde{p}_{it}$ is the price of the intermediate good i. From the first-order condition, we have an inverse demand function for the intermediate good i:

$$\tilde{p}_{it} = A_{it}^{\alpha} x_{it}^{\alpha-1} Y_t^{1-\alpha}. \tag{15}$$

3.4 Intermediate Sector

The intermediate sector consists of a continuum of firms, which is distributed uniformly in $[0,1]$. This distribution is time-invariant because for the intermediate sector, new innovators come out into the market at each period. Due to the newly invented technologies, new innovators make monopolistic profits. The newly invented technologies may be protected by patents or it may take time for the technologies to be imitated. The monopolistic profits, however, will disappear in one period since the next newly invented technologies are introduced into the market by other innovators after one period goes by.[11]

The intermediate goods are produced from labor with a one-for-one technology, i.e., an intermediate firm needs one unit of labor to produce one unit of x_{it}. The maximization problem for an intermediate firm is given by:

$$\begin{aligned} & \max_{x_{it}} \; \tilde{p}_{it} x_{it} - w_t x_{it} \\ = & \max_{x_{it}} \; A_{it}^{\alpha} x_{it}^{\alpha} Y_t^{1-\alpha} - w_t x_{it}, \end{aligned} \tag{16}$$

[11] The profits gained by the newly invented technologies are greater than those by the old technologies. Therefore, the newly invented technologies are always adopted rather than the old ones.

where w_t is the wage rate. From the first-order condition, we obtain:

$$w_t = \alpha A_{it}^{\alpha} x_{it}^{\alpha-1} Y_t^{1-\alpha}, \tag{17}$$

and

$$\pi_{it} = (1-\alpha) A_{it}^{\alpha} x_{it}^{\alpha} Y_t^{1-\alpha}. \tag{18}$$

From Eq.(15) and Eq.(17), we note that $\tilde{p}_{it} = \frac{w_t}{\alpha}$. The prices of all the intermediate goods are the same in each period.

An intermediate firm supplies the intermediate good with up-to-date quality, which is developed in an R&D department within the firm.[12] The quality of the intermediate goods is improved by the R&D activities. Ha and Howitt (2007) empirically examine a production function for the quality improvement of intermediate goods. Using the data for the USA, the UK, France, Germany, and Japan, they study which model is most suitable to reality: the first generation endogenous models, the semi-endogenous models, or the fully-endogenous models with product proliferation.[13] They conclude that a functional form which is used in the fully-endogenous models with product proliferation such as Howitt's (1999) model is most plausible. Following them, we assume a functional form for the quality improvement as follows:

$$\frac{A_{it+1} - A_{it}}{A_{it}} = \eta \left(\frac{z_{it+1}}{A_{it} L_t} \right)^{\sigma}, \tag{19}$$

where we assume that $0 < \sigma \leq 1$.[14] This assumption guarantees that the demand function of input goods slopes downward. η is a productivity parameter of the R&D department, z_{it+1} is the capital goods for the R&D activities, and L_t is the population of young agents at time t. Following Jones' (1995) critique, the right-hand side is adjusted for scale effects in three respects. First, as A_{it} becomes big, it is difficult for the quality of the intermediate goods to be improved. We divide z_{it+1} by A_{it} in order to remove this kind of scale effect and obtain the quality-adjusted input goods. Second, we divide z_{it+1}/A_{it} by L_t in order to deal with a scale effect which comes from the increasing number of individuals. Finally, the production exhibits non-increasing returns to scale with respect to the input goods, i.e., $0 < \sigma \leq 1$.

[12] As discussed in Aghion and Howitt (1992), we may assume that separate firms are engaged in the R&D activities. In this case, the firms in an R&D sector sell their newly invented technologies to the firms in an intermediate sector. No results will change with this alternative assumption.

[13] For details, see Ha and Howitt (2007).

[14] Although the current economy is closed and thus all markets clear within the economy, we may allow technology transfers from abroad, by replacing Eq.(19) with an equation which expresses the rate of technology adoption as in Aghion, et al. (2005) and Howitt and Mayer-Foulkes (2005).

Since each R&D department is in the patent race, the demands for capital goods are determined by the research-arbitrage condition:

$$\pi_{it+1} = q_{t+1} z_{it+1},$$

where again q_{t+1} is the (real) price of capital goods. Using Eq.(18), we rewrite this equation as:

$$(1-\alpha) A_{it+1}^{\alpha} x_{it+1}^{\alpha} Y_{t+1}^{1-\alpha} = q_{t+1} z_{it+1}. \tag{20}$$

4 Equilibrium

4.1 Growth Rate

The heterogeneity of the intermediate firms is assumed away: all the intermediate firms are symmetrical. From the labor market clearing condition, we have $\int_{i\in[0,1]} x_{it} di = L_t$. Since due to the symmetry, the same amount of labor is used in each intermediate firm, $x_t = L_t$ holds. Each R&D division uses the same amount of input goods as well. Let this amount be $\tilde{z}_t$. Then, $\tilde{z}_t := \int_{i\in[0,1]} z_{it} di$ holds. Hereafter, all variables are independent of i. From the assumption of symmetry and Eqs.(13), (17), and (20), we have the following equations:

$$\begin{aligned} Y_t &= A_t L_t \\ w_t L_t &= \alpha A_t L_t = \alpha Y_t \\ q_t \tilde{z}_t &= (1-\alpha) A_t L_t = (1-\alpha) Y_t. \end{aligned}$$

We note that $Y_t = w_t L_t + q_t \tilde{z}_t$ holds, namely the final output is distributed to the wages and the returns to the investments.

From lemma 1 and $w_t = \alpha A_t$, the market clearing condition (of the R&D sector) is given by:

$$\begin{aligned} & \tilde{z}_{t+1} = \int_{\phi_t}^{a} \frac{(1-\gamma) w_t}{1-\mu} L_t \phi dG(\phi) \\ \Longleftrightarrow \quad & \tilde{z}_{t+1} = \frac{(1-\gamma)\alpha A_t L_t}{1-\mu} F(\phi_t), \end{aligned} \tag{21}$$

where $F(\phi_t) = \int_{\phi_t}^{a} \phi dG(\phi)$. Given ϕ_t, the right-hand side of Eq.(21) is a supply of input goods for the R&D sector.

Substituting Eq.(21) into Eq.(19), we can derive a growth rate (which is defined by the growth rate of per capita output) as follows:[15]

$$\begin{aligned} \Gamma(\phi_t) &:= \Big(\frac{A_{t+1}}{A_t}\Big) - 1 \\ &= \eta\Big(\frac{(1-\gamma)\alpha}{1-\mu} F(\phi_t)\Big)^{\sigma}. \end{aligned} \tag{22}$$

[15] Again, due to symmetry, Eq.(19) is independent of i.

We note that the first-order effect of μ on the growth rate is positive, i.e., for a given ϕ_t, as μ increases the growth rate goes up. We also note that the growth rate is a decreasing function with respect to ϕ_t. As ϕ_t increases, the number of investors decreases. Accordingly, for a given μ, the total investments go down and thus the growth rate decreases.

The above discussion about the effect of μ captures only the first-order effect on the growth rate. However, a change in μ will affect the value of ϕ_t indirectly, which is thought of as a general equilibrium effect. In particular, as we will see, μ has a positive effect on ϕ_t in the steady states. Therefore, at this point, if an economy is in a steady state, we are uncertain whether μ has a positive or negative effect on the growth rate because the growth rate is decreasing with ϕ_t. In the analysis of the steady state below, we will discuss the effect of μ on the growth rate, taking into account the indirect effect.

4.2 Dynamics

B_t is net total assets held by young agents at time t, i.e., B_t is the government debt at time t, which is equal to total deposits minus the total loans. From lemma 1 and $w_t = \alpha A_t$, we obtain:

$$\begin{aligned} B_t = & \int_0^{\phi_t} (1-\gamma)\alpha A_t L_t dG(\phi) \\ & - \int_{\phi_t}^{a} \frac{\mu(1-\gamma)\alpha A_t L_t}{1-\mu} dG(\phi) \\ = & \frac{(1-\gamma)\alpha Y_t}{1-\mu}[G(\phi_t)-\mu]. \end{aligned} \tag{23}$$

We note that the total loans $\int_{\phi_t}^{a} \frac{\mu(1-\gamma)\alpha A_t L_t}{1-\mu} dG(\phi)$ are affected by the degree of financial deepening μ. Since $q_{t+1}\tilde{z}_{t+1} = (1-\alpha)Y_{t+1}$, substituting Eq.(21) into this yields $q_{t+1} = \frac{(1-\alpha)(1-\mu)Y_{t+1}}{(1-\gamma)\alpha F(\phi_t)Y_t}$. Therefore, $r_{t+1} = q_{t+1}\phi_t = \frac{(1-\alpha)(1-\mu)\phi_t Y_{t+1}}{(1-\gamma)\alpha F(\phi_t)Y_t}$. Substituting the last and Eq.(23) into Eq.(1), we have:

$$\underbrace{\frac{(1-\gamma)\alpha Y_{t+1}}{1-\mu}[G(\phi_{t+1})-\mu]}_{B_{t+1}} = \underbrace{\frac{(1-\alpha)(1-\mu)\phi_t}{(1-\gamma)\alpha F(\phi_t)}\frac{Y_{t+1}}{Y_t}}_{r_{t+1}} \underbrace{\frac{(1-\gamma)\alpha Y_t}{1-\mu}[G(\phi_t)-\mu]}_{B_t}, \tag{24}$$

which reduces to a difference equation of the cut-off point, ϕ_t:

$$G(\phi_{t+1}) = \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi_t(G(\phi_t)-\mu)}{F(\phi_t)} + \mu. \tag{25}$$

Let us define a function as follows:

$$\Psi(\phi) := \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi(G(\phi)-\mu)}{F(\phi)} + \mu.$$

We note that the difference equation Eq.(25) is independent of η , σ, the population growth n, and the quality of input goods, A_t. The growth rate $\Gamma(\phi_t)$ is a one-to-one, continuous function of $\phi_t \in [0, a]$.[16] Therefore, the dynamic properties of the growth rates are deduced directly from those of the cut-off point. In what follows, we shift our focus to the study of the dynamic properties of the cut-off point for awhile.

4.3 Steady-State Analysis

4.3.1 Two Steady-State Equilibria

A steady-state equilibrium $\phi_t = \bar{\phi}$ solves the following equation:

$$G(\bar{\phi}) = \frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)} \frac{\bar{\phi}(G(\bar{\phi}) - \mu)}{F(\bar{\phi})} + \mu. \tag{26}$$

We note that there exist two steady-state equilibria, $\bar{\phi} = \phi^*$ and $\bar{\phi} = \phi^{**}$, (unless they are repeated values,) such that:

$$G(\phi^*) = \mu \tag{27}$$

and

$$\frac{\phi^{**}}{F(\phi^{**})} = \frac{\alpha(1-\gamma)}{(1-\alpha)(1-\mu)}, \tag{28}$$

respectively.

In the steady-state equilibrium with $\bar{\phi} = \phi^*$, a generation's net credit position is always zero, whereas in the steady-state equilibrium with $\bar{\phi} = \phi^{**}$, the net credit position is positive or negative. We call equilibria with $\bar{\phi} = \phi^*$ and with $\bar{\phi} = \phi^{**}$ a non-trade steady-state equilibrium and a trade steady-state equilibrium, respectively. Whether a steady-state equilibrium is called a non-trade or a trade steady-state equilibrium, agents within a generation make financial trades via the Bank. In the non-trade steady state, the credit market clears within a generation, whereas in the trade steady state, agents make financial trades intergenerationally and the credit market clears over two generations. In this sense, by "non-trade" we mean that agents in a generation do not trade with the agents in the other generation.

4.3.2 Comparative Statics with respect to μ

Now we investigate the effect of the change of μ on a growth rate, taking into account a general equilibrium effect.

[16] The "one-to-one" is due to assumption 1.

Proposition 1
As a financial sector is well developed, i.e., as μ increases, the following hold:

- *The growth rate goes up in the non-trade steady state, i.e.,* $\frac{\partial \Gamma(\phi^*)}{\partial \mu} > 0$.
- *The growth rate goes up in the trade steady state as well, i.e.,* $\frac{\partial \Gamma(\phi^{**})}{\partial \mu} > 0$.

Proof: See appendix.

King and Levine (1993a,b) and Levine, et al. (2000) give empirical evidence for the positive effect of financial development on economic growth. Our results for the steady states are consistent with their discoveries. From Eqs.(27) and (28), we note that both ϕ^* and ϕ^{**} increase as μ goes up. This means that the number of investors decreases, which negatively affects the growth rate. However, from lemma 1, although there are fewer investors, we note that each investor borrows and invests more now than before μ went up. This latter positive effect is stronger than the former negative effect in both steady states. Therefore, the growth rate always goes up in the steady states.

The model captures well the properties of financial deepening. As a financial sector is well developed, the mis-allocation of production factors is corrected. Namely, as μ goes up, less capable investors turn into savers and the economic resources concentrate on more capable investors. As a result, efficiency in the economy is promoted. Less capable agents can utilize the ability of more capable agents. This is an essential characteristic of financial deepening.

5 Dynamic Properties

In this section, we investigate the dynamic properties of the economy. As seen in Eq.(22), $\Gamma(\phi_t)$ is a one-to-one, continuous function of $\phi_t \in [0, a]$. This means that if the equilibrium sequence, $\{\phi_t\}_{t=0}^{\infty}$, exhibits cyclical behavior, then so does the equilibrium sequence of the growth rates, $\{\Gamma(\phi_t)\}_{t=0}^{\infty}$.

We define a compact interval in $\Re$ as $X = [0, \max\{\phi^*, \phi^{**}\}]$. We restrict the domain of the dynamical system of Eq.(25) to X so as to obtain economically meaningful equilibria. If $\{\phi_t\}_{t=0}^{\infty}$ starts with $\phi_0 \in (\max\{\phi^*, \phi^{**}\}, a]$, then $G(\phi_t)$ becomes greater than one in finite time. Such a sequence does not become an equilibrium. We assume that the minimum of $\Psi(\phi)$ (which is just the right-hand side of Eq.(25)) is no less than zero unless we explicitly state otherwise.[17]

[17] If the minimum of $\Psi(\phi)$ is less than zero, then for almost all the initial values $\phi_0 \in X$, the sequence $\{\phi_t\}_{t=0}^{\infty}$ escaping from X is not an equilibrium because the credit market clearing condition does not hold. The remaining subset of X is a Cantor set, whose Lebesgue measure is equal to zero. In fact, even if the minimum of $\Psi(\phi)$ is less than zero, the equilibria exist. On the remaining Cantor set, we easily observe two steady-state equilibria. There also exists a period-two equilibrium cycle. To the best of my knowledge, in the literature of economics, only Boldrin, et al. (2001) deal with equilibria on Cantor sets.

Having restricted the domain of the system to X, then the map, $\Psi : X \to X$, is continuous and maps X into itself. Henceforth, we use the pair (X, Ψ) to denote our dynamical system.

We linearize the difference equation Eq.(25) around a steady state:

$$\phi_{t+1} - \bar{\phi} = \Phi(\bar{\phi})(\phi_t - \bar{\phi}),$$

where $\Phi(\phi) = \frac{(1-\alpha)(1-\mu)\phi}{\alpha(1-\gamma)F(\phi)} \left[\left(\frac{1}{\phi g(\phi)} + \frac{\phi}{F(\phi)} \right) (G(\phi) - \mu) + 1 \right]$.

The local stability depends upon whether ϕ^{**} is greater than ϕ^* or not.

Proposition 2

- *If $\phi^{**} > \phi^*$, then the non-trade steady-state equilibrium ($\bar{\phi} = \phi^*$) is locally stable, whereas the trade steady-state equilibrium ($\bar{\phi} = \phi^{**}$) is locally unstable.*
- *If $\phi^{**} < \phi^*$, then the non-trade steady-state equilibrium ($\bar{\phi} = \phi^*$) is locally unstable, whereas the stability of the trade steady-state equilibrium ($\bar{\phi} = \phi^{**}$) is ambiguous.*

Proof: If $\phi^* < \phi^{**}$, then $|\Phi(\phi^*)| = |\frac{(1-\alpha)(1-\mu)\phi^*}{\alpha(1-\gamma)F(\phi^*)}| < |\frac{(1-\alpha)(1-\mu)\phi^{**}}{\alpha(1-\gamma)F(\phi^{**})}| = 1$ and $|\Phi(\phi^{**})| = \left| \left(\frac{1}{\phi^{**} g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})} \right) (G(\phi^{**}) - \mu) + 1 \right| > 1$. If $\phi^* > \phi^{**}$, then $|\Phi(\phi^*)| = |\frac{(1-\alpha)(1-\mu)\phi^*}{\alpha(1-\gamma)F(\phi^*)}| > |\frac{(1-\alpha)(1-\mu)\phi^{**}}{\alpha(1-\gamma)F(\phi^{**})}| = 1$. However, we cannot know whether $|\Phi(\phi^{**})| = \left| \left(\frac{1}{\phi^{**} g(\phi^{**})} + \frac{\phi^{**}}{F(\phi^{**})} \right) (G(\phi^{**}) - \mu) + 1 \right|$ is greater than one or not. $\square$

The phase diagrams for each case are given in Figures 1-3. As seen in Figure 3, if the trade equilibrium is locally unstable, it is possible for the dynamical system to exhibit cycles. At minimum, in this case we must have a period-two cycle.

[Figures 1-4 around here]

Proposition 3
Suppose that $\phi^{**} < \phi^*$. If the trade steady-state equilibrium is locally unstable, there exists a period-two cycle of $\{\phi_t\}_{t=0}^{\infty}$ in equilibrium.

Proof: Let $\tilde{\Psi}(\phi) = G^{-1}(\Psi(\phi))$. Then Eq.(25) is written as $\phi_{t+1} = \tilde{\Psi}(\phi_t)$. We can take ϕ_0 close to ϕ^* so that $\tilde{\Psi}^2(\phi_0) < \phi_0 < \phi^*$ because $\Phi(\phi^*) > 1$. If the trade steady-state equilibrium is locally unstable, then $\Phi(\phi^{**}) < -1$. So we can take ϕ_0' close to ϕ^{**} so that $\tilde{\Psi}(\phi_0') < \phi^{**} < \phi_0' < \tilde{\Psi}^2(\phi_0') < \tilde{\Psi}^2(\phi_0)$. Therefore, by continuity, there exists $\bar{\phi}_0$ such that $\tilde{\Psi}(\bar{\phi}_0) < \phi^{**} < \bar{\phi}_0 = \tilde{\Psi}^2(\bar{\phi}_0) < \phi^*$, which means that there exists a period-two cycle. □

A graphical analysis gives a proof of proposition 2 as well. Since $\phi^{**} < \phi^*$, if the trade steady-state equilibrium is locally unstable, $\Phi(\phi^{**}) < -1$ holds. The configuration of $\phi_{t+1} = G^{-1}(\Psi(\phi_t))$ in this case is drawn in Figure 4. Its mirror image relative to the 45 degree line is drawn as well. As seen in the figure, there exists at least one pair of $\bar{\phi}_0$ and $\bar{\phi}_1$, where $\bar{\phi}_1 \neq \bar{\phi}_0$, such that $\bar{\phi}_1 = G^{-1}(\Psi(\bar{\phi}_0))$ and $\bar{\phi}_0 = G^{-1}(\Psi(\bar{\phi}_1))$.

Since the initial value ϕ_0 is determined by the individuals' expectations, it can jump. Therefore, the equilibrium is indeterminate globally, i.e., for any initial value $\phi_0 \in X$, the sequence, $\{\phi_t\}_{t=0}^{\infty}$ is an equilibrium of this economy. There is an extensive literature on this topic in overlapping generations models. Therefore, we do not investigate indeterminacy here. Our interest is in how financial deepening affects the deterministic dynamic properties of the growth rates.

6 Financial Deepening and Cycles

In the previous section, we have examined the dynamical system analytically and we have seen the appearance of cycles in equilibrium. In this section, we investigate under what conditions are endogenous growth cycles more or less likely to arise.

Let us consider the case in which $\mu = 0$. If $\mu = 0$, then $\phi^* = 0$ holds from Eq.(27). Therefore, from the first part of proposition 2 and from its phase diagram, $\{\phi_t\}_{t=0}^{\infty}$ monotonically converges to $\bar{\phi} = \phi^* = 0$ whenever $\{\phi_t\}_{t=0}^{\infty}$ starts with $\phi_0 \in [0, \phi^{**})$. Therefore, cycles do not arise. By continuity, given other parameters and the distribution function for ϕ, there exists μ_0 such that for $\mu \in [0, \mu_0]$ almost all of the sequences $\{\phi_t\}_{t=0}^{\infty}$ which start with an arbitrary value of $\phi_0 \in X$ monotonically converge to $\bar{\phi} = \phi^* = 0$. Meanwhile, if μ is sufficiently close to one, the first term of Eq.(25) degenerates and thus wherever $\{\phi_t\}_{t=0}^{\infty}$ starts in X, it converges to an asymptotically stable steady state.[18] In this case, no cycles appear either. Again by continuity, we can claim that given other parameters and the distribution function for ϕ, there exists μ_1 such that for $\mu \in [\mu_1, 1)$ almost all of the sequences $\{\phi_t\}_{t=0}^{\infty}$ with $\phi_0 \in X$ converge to an asymptotically stable steady state. From these discussions, we note that if a financial sector is well developed or less developed, endogenous growth cycles do not appear. It is when financial development is at an intermediate level that endogenous growth cycles might arise. We summarize these discussions in

[18]However, in this case, we cannot specify to which steady state, $\bar{\phi} = \phi^*$ or $\bar{\phi} = \phi^{**}$, the economy converges.

proposition 4

Proposition 4
Suppose that $\mu_1 > \mu_0$. Then, the following hold.

- *If μ is in $[0, \mu_0]$ or $[\mu_1, 1)$, the economy converges to an asymptotically stable steady state.*
- *If μ is in (μ_0, μ_1), the economy might exhibit endogenous, deterministic growth cycles.*

From proposition 2, we note that it is in the government credit case that the economy oscillates. Essentially, the dynamic properties of $\{\phi_t\}_{t=0}^{\infty}$ depend upon the market clearing condition, i.e., $B_{t+1} = r_{t+1}B_t$ because Eq.(25) originates in this equation. In it, both r_{t+1} and B_t are functions of ϕ_t. Since $r_{t+1} = \phi_t q_{t+1} = \frac{(1-\alpha)(1-\mu)(1+n)\phi_t}{(1-\gamma)\alpha F(\phi_t)}(\Gamma(\phi_t)+1) = \frac{(1-\alpha)(1-\mu)(1+n)\phi_t}{(1-\gamma)\alpha}\Big[\frac{\eta}{F(\phi_t)^{1-\sigma}}\big(\frac{(1-\gamma)\alpha}{1-\mu}\big)^{\sigma} + \frac{1}{F(\phi_t)}\Big]$, r_{t+1} is increasing with ϕ_t. From Eq.(23), B_t is increasing with ϕ_t as well. Nevertheless, an increase in ϕ_t at the beginning of period t has an ambiguous effect on ϕ_{t+1}. This is because B_t could be negative. As we have seen in the previous section, if the trade steady-state equilibrium $\bar{\phi} = \phi^{**}$ is locally unstable, then the economy exhibits endogenous growth cycles in equilibrium. In what follows, we will see that it is possible that the economy oscillates when μ is the intermediate value.

[Figure 5 around here]

Suppose that the economy is in the trade steady-state equilibrium at the beginning of time t. In this case, since $\frac{(1-\alpha)(1-\mu)}{\alpha(1-\gamma)}\frac{\phi^{**}}{F(\phi^{**})} = 1$, we obtain the steady-state interest rate as follows:

$$r^{**} = (1+n)(\Gamma(\phi^{**})+1). \tag{29}$$

We note that if the growth rate is zero, i.e., $\Gamma(\phi^{**}) = 0$, then the steady-state interest rate r^{**} is equal to the "biological" interest rate. On the other hand, from Eq.(23) the government debt in the steady state is given by:

$$B^{**} = \Big[(1-\gamma)\alpha G(\phi^{**}) - \frac{\mu(1-\gamma)\alpha}{1-\mu}(1-G(\phi^{**}))\Big]Y_t, \tag{30}$$

where Y_t is given when we look at B^{**} at time t. Since we assume the government credit case now, $B^{**} < 0$ holds.

Now suppose that the economy faces a shock so that the prospective interest rate at time $t+1$, r_{t+1}, goes up. In this case, agents whose productivity is slightly greater than ϕ^{**} turn from investors into savers. Therefore, relative to the trade steady state, the number of savers increases, whereas the number of investors decreases. As a result, B_t goes up; however, it is still negative and thus the absolute value of B_t decreases. If μ is not close to one, i.e., μ is an intermediate value, then from Eq.(30) the increase in B_t is small relative to the increase in r_{t+1}. In this case, $r_{t+1}B_t$ decreases, which means that the financial resources of the Bank at time $t+1$ go up. Accordingly, the prospective interest rate at time $t+2$, r_{t+2}, goes down. As a result, ϕ_{t+1} becomes smaller than ϕ^{**} and thus the economy oscillates. If the sequence $\{\phi_t\}_{t=0}^{\infty}$ does not converge, equilibrium cycles emerge.

From the proof of the second part of proposition 2, it follows that if $\Big(\frac{1}{\phi^{**}g(\phi^{**})}+\frac{\phi^{**}}{F(\phi^{**})}\Big)(G(\phi^{**})-\mu) < -2$, then the trade steady-state equilibrium is locally unstable. In this case, we note from the phase diagram that endogenous growth cycles arise without fail, whether the minimum of $\Psi(\phi)$ is no less or less than zero.[19] From the assumption of the second part of proposition 2, we have $G(\phi^{**})-\mu < 0$. Therefore, the condition for cycles must hold if $\frac{1}{\phi^{**}g(\phi^{**})}+\frac{\phi^{**}}{F(\phi^{**})}$ is very big. For instance, whenever the density at ϕ^{**} is very thin, i.e., $g(\phi^{**})$ is very small, this value becomes large. Therefore, depending upon the distribution of ϕ, if financial development is at an intermediate level, it is always possible that endogenous growth cycles arise.

Figure 5 is a plot of the set of values (α, γ, μ) which give cycles in equilibrium when we assume a uniform distribution in $[0, 1]$ for ϕ (where again the minimum of $\Psi(\phi)$ is no less or less than zero). We also note that as α goes up, the set of μ, which results in cycles, becomes small. Therefore, in order to obtain cycles in equilibrium, the high complementarity between intermediate goods is needed. As α increases, the first-period income also increases. Accordingly, each agent does not have to borrow so much. As a result, the value of B_t increases and becomes positive and ϕ^{**} becomes greater than ϕ^* (see proposition 2.2). We also note that as γ becomes greater, the set of μ gets bigger. This is because as γ becomes greater, each agent puts more weight on first-period consumption and thus the aggregate amount of borrowings becomes greater. As a result, B_t becomes negative and its absolute value gets large. Then, a small change in the value of r_{t+1} (reflecting the change in ϕ_t) effects a large change in the value of B_{t+1}.

7 Numerical Analysis

A growth rate has a one-to-one relationship with a cut-off point. Therefore, we have investigated the dynamic properties of the cut-off point. We have made

[19]By studying the phase diagram for the case in which the minimum of $\Psi(\phi)$ is less than zero, we can note that there exists at minimum a period-two cycle in such a case.

clear under what value of μ endogenous growth cycles appear. However, we did not make clear to what kinds of cycles the economy converges in equilibrium. In this section, in order to investigate the dynamic properties of the sequences $\{\phi_t\}_{t=0}^{\infty}$ and $\{\Gamma(\phi_t)\}_{t=0}^{\infty}$ numerically, we create bifurcation diagrams. These diagrams are helpful for us to understand the asymptotic dynamic properties of the economy concretely. We again assume that $\phi \sim U(0,1)$. We would like to study the relationship between the degree of financial deepening and the dynamic properties of the growth rates. Therefore, we create bifurcation diagrams with respect to μ.

The dynamics of growth rates is given by the system of two equations:

$$\phi_{t+1} = \Psi(\phi_t) = \frac{2(1-\alpha)(1-\mu)}{\alpha(1-\gamma)} \frac{\phi_t(\phi_t - \mu)}{1-\phi_t^2} + \mu$$

and

$$\Gamma(\phi_t) = \eta\Big(\frac{(1-\gamma)\alpha}{1-\mu} F(\phi_t)\Big)^{\sigma}. \tag{31}$$

We create the bifurcation diagrams for the dynamical system iterating 10000 times. In this numerical analysis, it is hard to pin down the value of α. The parameter α captures three different things. The first is the labor share of output excluding human capital. As mentioned before z_{t+1} is broadly thought of as physical and human capital and w_t is paid to young agents who are probably unskilled. If we use the lower limit of the unskilled labor share estimated by Mankiw, et al. (1992), $\alpha = 0.29$. Second, $1/\alpha$ is the mark-up ratios of intermediate goods. The mark-up ratios of United States industries are estimated empirically (for example Hall (1988), Norrbin (1993) and Basu (1996)). Third, $1/\alpha$ captures complementarity between intermediate goods. The smaller is α, the greater is the complementarity. However, the complementarity between intermediate goods is difficult to measure. In this numerical analysis, we assume high complementarity between intermediate goods and we let $\alpha = 0.25$. We examine the various cases for γ: $\gamma = 0.55$, $\gamma = 0.56$, and $\gamma = 0.57$. If we suppose that the annual discount rate is 0.92, then $\gamma = 0.54$ and $\gamma = 0.57$ approximately correspond to the discount rates for 2 years and 3.5 years, respectively. This interval must be the intermediate terms of debt contracts between financial intermediaries and small firms. We choose the initial condition to be $\phi_0 = 0.01$.[20]

[20]We can verify that $\Psi : X \to X$ is a (upside-down) unimodal map: the critical point is $m := \frac{\mu}{1+\sqrt{1-\mu^2}}$. In this case, it is well known that for a given μ, if a Schwarzian derivative, $S(\Psi) := \frac{\Psi'''(\phi)}{\Psi'(\phi)} - \frac{3}{2}(\frac{\Psi''(\phi)}{\Psi'(\phi)})^2 < 0$ for all $\phi \in X - \{m\}$, then the asymptotic behavior of almost all sequences $\{\phi_t\}_{t=0}^{\infty}$ will be the same. Therefore, the choice of initial conditions does not matter. See for example Guckenheimer and Holmes (1983). Our Schwarzian derivative is too complicated to investigate its sign analytically. However, we plotted the values of our

[Figures 6-8 around here]

Figures 6-8 give bifurcation diagrams for ϕ_t with respect to the degree of financial deepening, i.e., μ. As predicted in the previous section, for all cases, if μ is small or large, $\{\phi_t\}_{t=0}^{\infty}$ converges to an asymptotically stable steady state and thus the growth rates converge to a stationary growth rate as well. For each case, however, if the value of μ is in the intermediate region, endogenous fluctuations appear. When $\gamma = 0.55$, as μ goes up from zero or goes down from one, the economy experiences period doubling bifurcations. At the intermediate values of μ, a globally stable period-eight cycle arises. When $\gamma = 0.56$ and $\gamma = 0.57$, the economy exhibits a complex dynamics in the intermediate region. For example, when $\gamma = 0.57$, as μ increases from zero, we observe the first period doubling bifurcation around $\mu = 0.298$ and the second period doubling bifurcation around $\mu = 0.393$. These bifurcations are repeated over and over again and eventually the economy enters a complex region (shaded regions in the diagrams). As seen in figure 2-7, when $\gamma = 0.56$ similar things happen.

[Figures 9-11 around here]

A new finding from these bifurcation diagrams is that the amplitudes of cycles increase as γ goes up. We note from lemma 1 that as γ increases, agents put more weight on the first-period consumption and thus the total borrowings in the economy go up. While the total borrowings go up, the investments by each investor go down (lemma 1), which leads the price of capital goods to climb up. As a result, the cut-off point goes down and the number of investors increases. This means that more agents are subject to the credit constraints when γ is large than when it is small. Hence, we may say that the enlarged amplitudes of cycles are due to the credit market imperfections agents are facing.

Figures 9-11 are the corresponding bifurcation diagrams for $\Gamma(\phi_t)$. As predicted in proposition 2.1, as μ increases, the growth rates go up for all cases. For each case, we observe endogenous growth cycles in the intermediate values for μ. The amplitudes of the growth rates are around 2-3%. Of course, we cannot compare the amplitudes in these numerical examples with those in a real economy because these analyses are not rigorous calibrations, although the amplitudes seem small. The amplitudes depend upon the distribution of ϕ, as well as upon the other parameters. In particular, the function $F(\phi_t)$ affects the amplitudes. If the average of ϕ is large, the amplitudes become big because

Schwarzian derivatives and numerically confirmed that the signs are negative. In addition, to make sure, we examined various initial values: we found that the asymptotic behavior of the dynamical system is invariant to the initial condition.

$F(0)$ becomes big in this case. Furthermore, we should note that these amplitudes are for the stationary states without any exogenous shocks. If we take into account exogenous shocks or noise, the amplitudes must be greater. Even in that case, endogenous growth cycles play an important role, complementing the role of exogenous shocks for business cycles.

8 Relation to Empirical Evidence

We have investigated the relationship between financial deepening and the dynamic properties of growth rates. We have discovered that (i) if a financial sector is very developed or poorly developed, an economy never exhibits endogenous growth cycles, and that (ii) if financial deepening is at an intermediate level, endogenous growth cycles may appear.

Easterly, at al. (2000) and Denizer, et al. (2002) give empirical evidence that exogenous macroeconomic shocks, such as productivity shocks and monetary shocks, are more repressed in an economy with a very developed financial sector than in an economy with a poorly developed financial sector. That is to say, a well developed financial sector plays a role in stabilizing and reducing growth volatility. Although they emphasize the reduction of growth volatility generated by exogenous shocks and they do not consider the structural non-linearity of an economy, their results are not inconsistent with the results of our model.

In our model, if the economy faces exogenous shocks, then persistent growth cycles occur if the development of a financial sector is at an intermediate level. Since their results are probably obtained with the data for countries with a middle-to-high degree of financial development (because it is hard to obtain data for countries with a lowest degree of financial development), their results are consistent with our predictions.

9 Concluding Remarks

Financial deepening has effects on macroeconomic phenomena such as economic growth and business cycles. Our main findings are as follows: (i) if the development of a financial sector is at an intermediate level, endogenous, deterministic growth cycles will arise and (ii) if a financial sector is very developed or poorly developed, the economy converges to an asymptotically stable steady state. Namely, it is when the development of a financial sector is at an intermediate level that growth rates are highly volatile.

We make a final remark for future research. If we rewrite our model with nominal variables, it can be applied to a study of the relationship between financial deepening and excess volatility in inflation rates. To the best of our knowledge, no article has studied such a relationship theoretically or empirically. However, it is very important to understand excess volatility in inflation rates in terms of financial deepening. In our model, $\frac{1}{r_{t+1}} - 1$ is equal to a net inflation rate if we use nominal variables. From $r_{t+1} = \frac{(1-\alpha)(1-\mu)(1+n)\phi_t}{(1-\gamma)\alpha F(\phi_t)}(\Gamma(\phi_t)+1)$, we guess that if the development of a financial sector is at the intermediate level, due to the $\frac{\phi_t}{F(\phi_t)}$, the inflation rates are far more volatile than the growth rates, which seems consistent with episodes in the real world. This investigation is left for future research.

Appendices

i) Proof of proposition 1

We take four steps to show proposition 1.

Step 1: If $\bar{\phi} \in [0, a)$, then $F(\bar{\phi}) > \bar{\phi}(1 - G(\bar{\phi}))$, since $F(\bar{\phi}) = \int_{\bar{\phi}}^{a} \phi dG(\phi) > \int_{\bar{\phi}}^{a} \bar{\phi} dG(\phi) = \bar{\phi}(1 - G(\bar{\phi}))$.

Step 2: $\frac{\partial \phi^*}{\partial \mu} = \frac{1}{g(\phi^*)}$, which follows from $G(\phi^*) = \mu$.

Step 3: $F(\phi^{**}) - (1-\mu)\phi^{**} g(\phi^{**}) \frac{\partial \phi^{**}}{\partial \mu} = (1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial \phi^{**}}{\partial \mu}$. This is because from $\frac{\phi^{**}}{F(\phi^{**})} = \frac{\alpha(1-\gamma)}{(1-\alpha)(1-\mu)}$, we have:

$$\log \phi^{**} - \log F(\phi^{**}) = \log\left[\frac{\alpha(1-\gamma)}{(1-\alpha)}\right] - \log(1-\mu).$$

Therefore, we have:

$$\begin{aligned} & \left[\frac{1}{\phi^{**}} + \frac{\phi^{**} g(\phi^{**})}{F(\phi^{**})}\right]\frac{\partial \phi^{**}}{\partial \mu} = \frac{1}{1-\mu} \qquad (32) \\ \iff \quad & F(\phi^{**}) - (1-\mu)\phi^{**} g(\phi^{**})\frac{\partial \phi^{**}}{\partial \mu} = (1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial \phi^{**}}{\partial \mu}. \end{aligned}$$

Step 4: Case 1: $\bar{\phi} = \phi^*$. From step 1, it holds that:

$$\frac{\phi^*}{F(\phi^*)} < \frac{1}{1 - G(\phi^*)} = \frac{1}{1-\mu}. \qquad (33)$$

From step 2 and Eq.(33), we have:

$$\begin{aligned} & \frac{-\phi^*(1-\mu) + F(\phi^*)}{(1-\mu)^2} > 0 \\ \iff \quad & \frac{-\phi^* g(\phi^*)(1-\mu)\frac{1}{g(\phi^*)} + F(\phi^*)}{(1-\mu)^2} > 0 \end{aligned}$$

$$\begin{aligned}
&\iff \quad \frac{-\phi^* g(\phi^*)(1-\mu)\frac{\partial \phi^*}{\partial \mu} + F(\phi^*)}{(1-\mu)^2} > 0 \\
&\iff \quad \frac{\partial}{\partial \mu}\Big[\frac{F(\phi^*)}{1-\mu}\Big] > 0 \\
&\iff \quad \frac{\partial \Gamma(\phi^*)}{\partial \mu} > 0.
\end{aligned}$$

Case 2: $\bar{\phi} = \phi^{**}$. From Eq.(32), $\frac{\partial \phi^{**}}{\partial \mu} > 0$ holds. Then, from step 3, we have:

$$\begin{aligned}
& \quad \frac{(1-\mu)\frac{F(\phi^{**})}{\phi^{**}}\frac{\partial \phi^{**}}{\partial \mu}}{(1-\mu)^2} > 0 \\
&\iff \quad \frac{-\phi^{**} g(\phi^{**})(1-\mu)\frac{\partial \phi^{**}}{\partial \mu} + F(\phi^{**})}{(1-\mu)^2} > 0 \\
&\iff \quad \frac{\partial}{\partial \mu}\Big[\frac{F(\phi^{**})}{1-\mu}\Big] > 0 \\
&\iff \quad \frac{\partial \Gamma(\phi^{**})}{\partial \mu} > 0. \qquad \square
\end{aligned}$$

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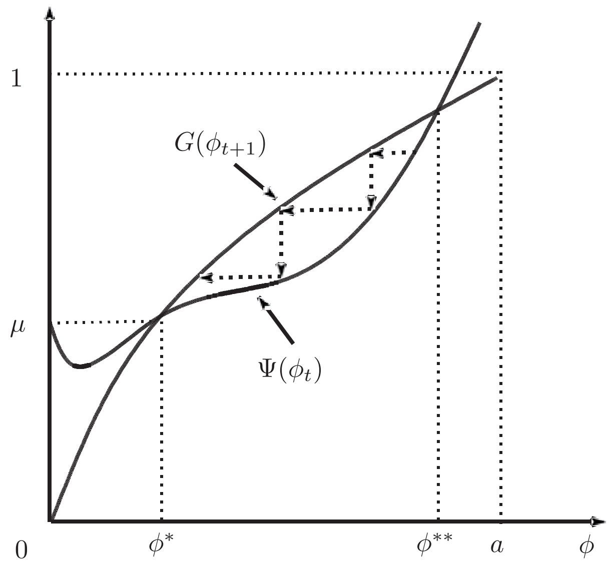

Figure 1: $\phi^{**} > \phi^{*}$

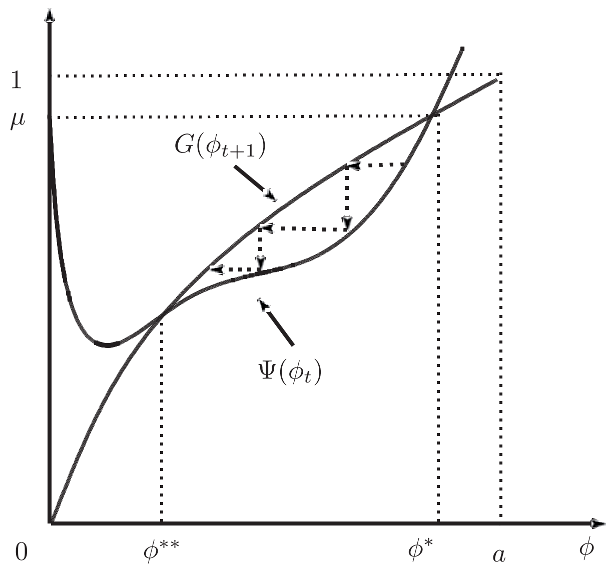

Figure 2: $\phi^{**} < \phi^{*}$, No Cycles

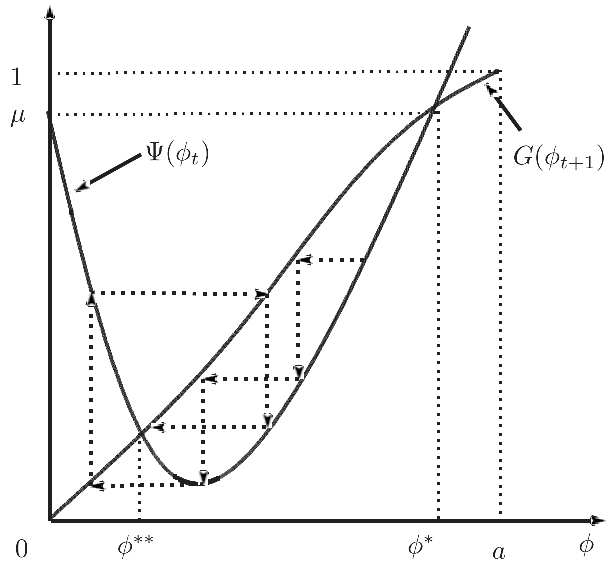

Figure 3: $\phi^{**} < \phi^{*}$, Cycles

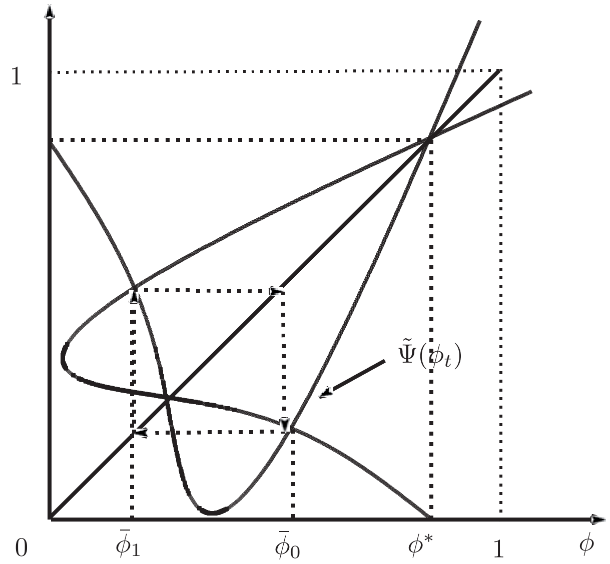

Figure 4: Period-Two Cycle

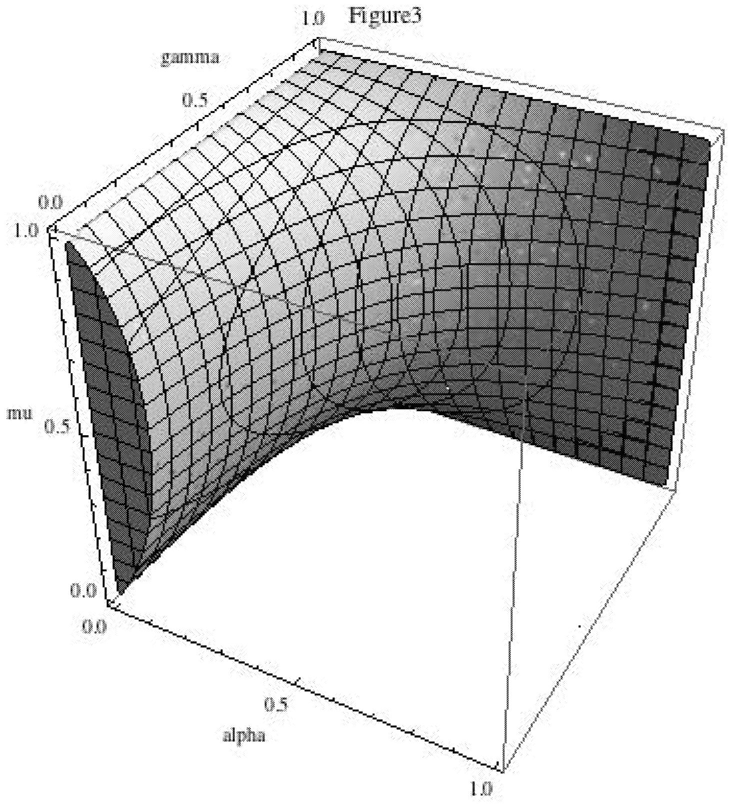

Figure 5: $\{\alpha, \mu, \gamma\}$ for cycles

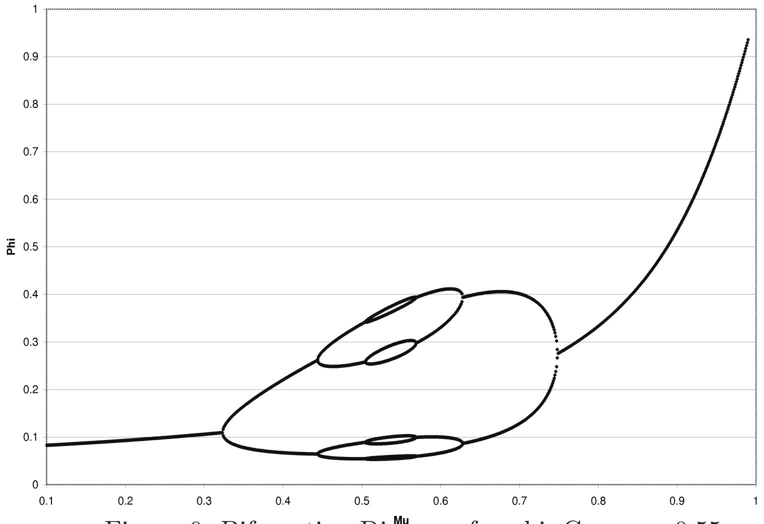

Figure 6: Bifurcation Diagram for phi, Gamma=0.55

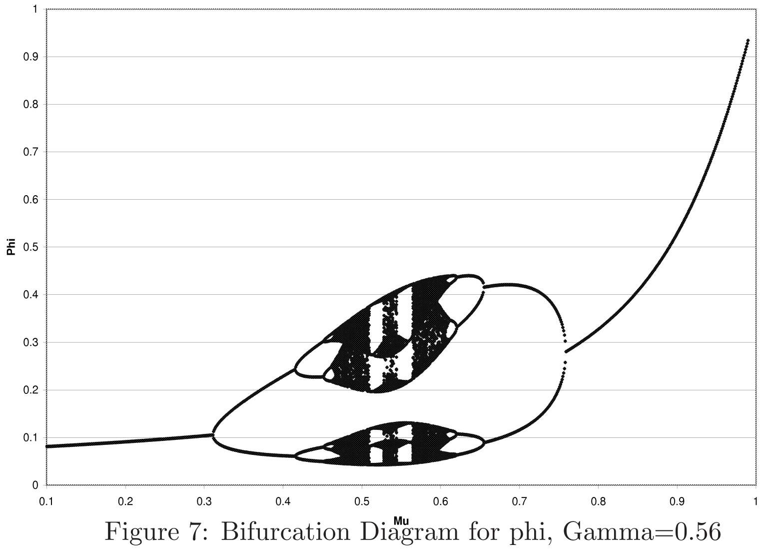

Figure 7: Bifurcation Diagram for phi, Gamma=0.56

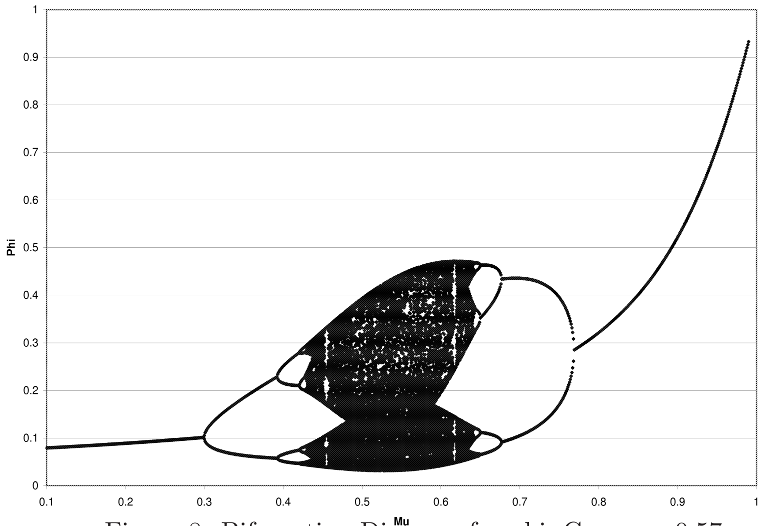

Figure 8: Bifurcation Diagram for phi, Gamma=0.57

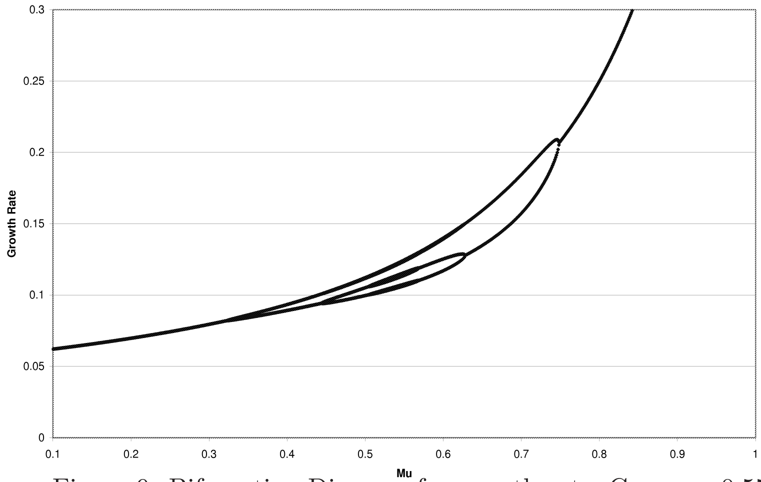

Figure 9: Bifurcation Diagram for growth rate, Gamma=0.55

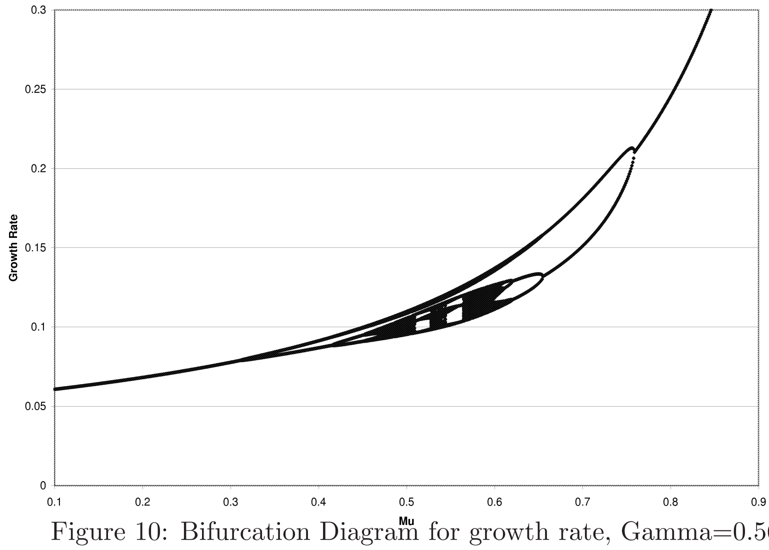

Figure 10: Bifurcation Diagram for growth rate, Gamma=0.56

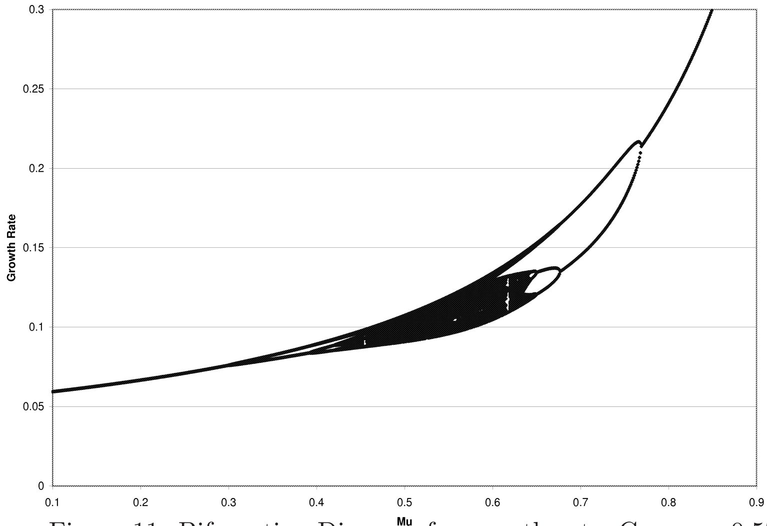

Figure 11: Bifurcation Diagram for growth rate, Gamma=0.57