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Abstract

This study develops an innovation-driven growth model with natural selection of heterogeneous households and endogenous takeoff. Families differ in their ability to accumulate human capital. In an early stage of development, households with lower education ability accumulate less human capital but choose to have more children and enjoy an evolutionary advantage. In a later stage of development, families with high education ability increase their number of children as their human capital rises over time. In the long run, high-ability households accumulate more human capital, and all families choose the same steady-state fertility rate. Therefore, households' population share and human capital converge to stationary distributions. Initially, the heterogeneity of households makes it more likely for an endogenous takeoff to occur; however, the temporary evolutionary disadvantage of high-ability families has a lasting negative impact on long-run growth. Finally, we provide evidence that heterogeneity in education indeed has adverse effects on education, innovation and economic growth in the long run.

JEL classification: O30, O40
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1 Introduction

Modern macroeconomic models often feature a representative household or a fixed composition of heterogeneous households. However, when heterogeneous households choose to have different fertility rates, their composition in the economy changes over time. This differential reproduction of individuals is famously known as natural selection. In this study, we explore how the heterogeneity of households and natural selection of heterogeneous households affect the macroeconomy. Family attitudes toward the education of their children last long, and the intra-family educational attitudes and human capital transmission abilities matter.[1] Unfortunately, not all households are equally endowed, so heterogeneity matters for human capital accumulation. How does this heterogeneity affect fertility? And how would the resulting natural selection influence technological progress and the takeoff of the economy? To explore these questions, we develop a novel growth model with endogenous fertility, an endogenous activation of innovation and natural selection of heterogeneous households, which persistently differ in their propensity to educate their children.

Following the seminal unified growth theory of Galor (2005, 2011, 2022), we assume that households differ in their ability to accumulate human capital. In this case, families that are more able to provide high-quality learning focus on child quality and have fewer children than less able families.[2] Naturally, this quality-quantity tradeoff magnifies the share of less able families in the economy, at least temporarily. Therefore, in an early stage of development, households that have a lower education ability accumulate less human capital but choose to have more children and enjoy an evolutionary advantage. In a later stage of development, households with a higher education ability choose to increase their number of children as their human capital rises over time because their higher level of human capital compensates for their lower fertility. In the long run, households with a higher education ability end up having a higher level of human capital, and all households choose the same steady-state fertility rate. Therefore, households' population share and human capital converge to stationary distributions.

Initially, the heterogeneity of households makes it more likely for an endogenous takeoff to occur. The presence of heterogeneous households implies that some households supply more human capital for production and innovation, whereas some households supply less. In our model, the former effect dominates the latter effect such that the initial amount of human capital available for production and innovation increases as a result of this heterogeneity. The intuition can be explained as follows. For an economy to activate its innovation process, the market size of the economy is critical. The core idea in Romer (1990) is that ideas are non-rival, so that a larger market size implies more profits for new ideas developed by innovative firms. Hence, paradoxically, the higher-fertility families being less willing to educate their children initially contribute more to the workforce and to a larger market size of the economy rewarding the innovation pioneers with more profits extracted from a larger mass of low-skilled workers.

However, the evolutionary disadvantage of high-ability households during the transitional dynamics implies that the population share of high-ability households decreases and the population share of low-ability households increases towards the steady state. The lower long-run

[1] For example, Alesina *et al.* (2021) find that differences in family attitudes toward education persist and rebound after even some of the most forceful attempts to eliminate differences in the population.

[2] This negative relationship between child quantity and quality is consistent with the empirical evidence in Klemp and Weisdorf (2019).

share of high-ability households is due to a well-known property that a temporary growth effect has a permanent level effect. Suppose two variables start at an equal level. Then, one of them grows at a slower rate temporarily before growing at the same rate as the other variable. In this case, the temporary disadvantage of the former will endure forever. So, despite population trends being similar in the long run, a temporarily lower population growth rate of the higher-ability households will never be compensated. The scale-invariant property of our model then implies that economic growth depends on the average level of human capital in the economy and that the lower share of high-ability households in the long run gives rise to a lower steady-state equilibrium growth rate as a result of natural selection of heterogeneous households. Finally, we provide evidence that heterogeneity in education indeed has adverse effects on education, innovation and economic growth in the long run.

This study relates to the literature on innovation and economic growth. The pioneering study by Romer (1990) develops the seminal innovation-driven growth model; see also Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) for other early studies. Some subsequent studies introduce endogenous fertility into variants of the innovation-driven growth model to explore the relationship between economic growth and endogenous population growth; see, for example, Jones (2001), Connolly and Peretto (2003), Chu *et al.* (2013), Peretto and Valente (2015) and Brunnschweiler *et al.* (2021). This study contributes to this literature by exploring the endogenous fertility decisions of heterogeneous households and their evolutionary differences in an innovation-driven growth model.

This study also relates to the literature on endogenous takeoff and economic growth. An early study by Galor and Weil (2000) develops the unified growth theory that explores the endogenous transition of an economy from pre-industrial stagnation to modern economic growth;[3] see Galor (2005) for a comprehensive review of unified growth theory and also Galor and Mountford (2008), Galor, Moav and Vollrath (2009) and Ashraf and Galor (2011) for subsequent studies and empirical evidence that supports unified growth theory. Galor and Moav (2002), Galor and Michalopoulos (2012) and Carillo *et al.* (2019) explore how natural selection of different traits, such as the quality preference of fertility, the degree of risk aversion and the level of family-specific human capital, affects the transition from stagnation to growth. This study complements these interesting studies by exploring how natural selection of heterogeneous households with different ability to accumulate human capital affects the transition of an economy from human capital accumulation to innovation-driven growth.

Therefore, this study also relates to a recent branch of this literature on the endogenous transition from pre-industrial stagnation to innovation-driven growth; for example, Peretto (2015) develops a Schumpeterian growth model with the endogenous activations of variety-expanding innovation and quality-improving innovation. Subsequent studies extend the model in Peretto (2015) to explore different mechanisms that trigger an endogenous takeoff; see for example, Chu, Fan and Wang (2020) on status-seeking culture, Chu, Kou and Wang (2020) on intellectual property rights, Iacopetta and Peretto (2021) on corporate governance, Chu, Furukawa and Wang (2022) on rent-seeking government, and Chu, Peretto and Wang (2022) on agricultural revolution. This study contributes to this branch of the literature by introducing natural selection of heterogeneous households to a tractable innovation-driven growth model

[3] Other early studies on endogenous takeoff and economic growth include Hansen and Prescott (2002), Jones (2001) and Kalemli-Ozcan (2002).

with endogenous takeoff.

The rest of this study is organized as follows. Section 2 sets up the model. Section 3 presents the two stages of economic development. Section 4 explores the implications of heterogeneous households and natural selection. Section 5 provides empirical evidence. Section 6 concludes.

2 An R&D-based growth model with natural selection

To model natural selection, we introduce heterogeneous households and endogenous fertility to the seminal Romer model. To keep the model tractable, we consider a simple structure of overlapping generations and human capital accumulation.[4] Each individual lives for three periods. In the young age, the individual accumulates human capital. In the working age, the individual allocates her time between work, fertility and education of the next generation. In the old age, the individual consumes her saving.

2.1 Heterogeneous households

There is a unit continuum of households indexed by $i \in [0, 1]$. Within household i, the utility of an individual who works at time t is given by

$$U^t(i) = u\left[n_t(i), h_{t+1}(i), c_{t+1}(i)\right] = \eta \ln n_t(i) + \gamma \ln h_{t+1}(i) + \ln c_{t+1}(i), \tag{1}$$

where $c_{t+1}(i)$ is the individual's consumption at time $t+1$, $n_t(i)$ denotes the number of children the individual has at time t, $\eta > 0$ is the fertility preference parameter, $h_{t+1}(i)$ denotes the level of human capital that the individual passes onto each child, and γ is the quality preference parameter. We assume that all individuals within the same household i have the same level of human capital at time 0. Then, they will also have the same level of human capital for all t as an endogenous outcome.

The individual allocates $e_t(i)$ units of time to her children's education. The accumulation equation of human capital is given by

$$h_{t+1}(i) = \phi(i)e_t(i) + (1-\delta)h_t(i), \tag{2}$$

where the ability parameter $\phi(i) > 0$ is heterogeneous across households $i \in [0, 1]$ and follows a general distribution with the following mean:[5]

$$\overline{\phi} \equiv \int_0^1 \phi(i)di.$$

The heterogeneity of households is captured by their differences in $\phi(i)$, which in turn give rise to an endogenous distribution of human capital. We focus on heterogeneity in $\phi(i)$ because it

[4]The formulation is based on Chu, Furukawa and Zhu (2016) and Chu, Kou and Wang (2022) with homogeneous households and exogenous fertilty.

[5]It is useful to note that $\overline{\phi}$ is the unweighted mean which is exogenous, whereas the weighted mean changes endogenously as the population share of households evolves over time.

allows for a stationary distribution of the population share of different households in the long run, whereas heterogeneity in other parameters, such as η or γ, imply that households with the largest η or smallest γ would dominate the population in the long run. As for the parameter $\delta \in (0,1)$, it is the depreciation rate of human capital that a generation passes onto the next.

An individual in household i allocates $1 - e_t(i) - \sigma n_t(i)$ units of time to work and earns $w_t\left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)$ as real wage income, where the parameter $\sigma \in (0,1)$ determines the time cost of fertility. For simplicity, we assume that there are economies of scale in educating children, and the cost of having more children is reflected in the time cost of childrearing.[6] The individual devotes her entire wage income to saving at time t and consumes the return at time $t+1$:[7]

$$c_{t+1}(i) = (1 + r_{t+1}) w_t \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i), \tag{3}$$

where r_{t+1} is the real interest rate. Substituting (2) and (3) into (1), the individual maximizes

$$\max_{e_t(i),\, n_t(i)} U^t(i) = \eta \ln n_t(i) + \gamma \ln\left[\phi(i) e_t(i) + (1-\delta) h_t(i)\right] + \ln\left\{(1 + r_{t+1}) w_t \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i)\right\},$$

taking $\{r_{t+1}, w_t, h_t(i)\}$ as given. The utility-maximizing level of fertility $n_t(i)$ is

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right], \tag{4}$$

which is decreasing in $\phi(i)$ but increasing in $h_t(i)$. In other words, households with a lower ability to accumulate human capital and a higher level of human capital choose to have more children. The utility-maximizing level of education $e_t(i)$ is

$$e_t(i) = \frac{1}{1+\eta+\gamma}\left[\gamma - (1+\eta)(1-\delta)\frac{h_t(i)}{\phi(i)}\right], \tag{5}$$

which is increasing in $\phi(i)$ but decreasing in $h_t(i)$. In summary, for a given $h_t(i)$, households with a larger $\phi(i)$ choose a higher level of education $e_t(i)$ but a smaller number $n_t(i)$ of children, reflecting the quality-quantity tradeoff.

Substituting (5) into (2) yields the autonomous and stable dynamics of human capital as

$$h_{t+1}(i) = \frac{\gamma}{1+\eta+\gamma}\left[\phi(i) + (1-\delta) h_t(i)\right], \tag{6}$$

where $h_{t+1}(i)$ is increasing in $\phi(i)$ and $h_t(i)$. The total amount of human capital in the economy at time t is

$$H_t = \int_0^1 h_t(i) L_t(i) di,$$

where $L_t(i)$ is the working-age population size of household i. The law of motion for $L_t(i)$ is

$$L_{t+1}(i) = n_t(i) L_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] L_t(i), \tag{7}$$

[6] This time cost is equivalent to a reduction in income of $\sigma w_t h_t(i)$ per child.

[7] Our results are robust to individuals consuming also in the working age; derivations available upon request.

and the size of the aggregate labor force in the economy at time t is

$$L_t = \int_0^1 L_t(i)di.$$

Let's define $s_t(i) \equiv L_t(i)/L_t$ as the working-age-population (i.e., labor) share of household i.

Lemma 1 *The labor share $s_t(i)$ of household i at time $t \geq 1$ is given by*

$$s_t(i) = \frac{\prod_{\tau=0}^{t-1} n_\tau(i) L_0(i)}{\int_0^1 \prod_{\tau=0}^{t-1} n_\tau(i) L_0(i) di},$$

where the fertility decision $n_t(i)$ of household i at time $t \geq 1$ is given by

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)} \left\{ \sum_{\tau=0}^{t-1} \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma} \right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma} \right]^t \left[1 + (1-\delta) \frac{h_0(i)}{\phi(i)} \right] \right\},$$

which is an increasing function of $h_0(i)/\phi(i)$.
Proof. See Appendix A. ■

Notice that changes to $n_\tau(i)$ in any one period will affect $s_t(i)$ in all future generations. The reason is general and does not depend on the specific assumptions of this model: a temporary growth effect has a permanent level effect. Therefore, if the fertility rate of an ability group drops temporarily, this group would ceteris paribus forever have a lower population share than it would otherwise have had. As we will later see, if the high-ability household experiences a temporary reproduction loss, the economy will have a lower share of high-ability people forever. We will also show that this loss will permanently lower per capita output, R&D investment, and productivity growth.

2.2 Final good

Perfectly competitive firms use the following production function to produce final good Y_t, which is chosen as the numeraire:

$$Y_t = H_{Y,t}^{1-\alpha} \int_0^{N_t} X_t^\alpha(j) dj, \tag{8}$$

where the parameter $\alpha \in (0,1)$ determines production labor intensity $1-\alpha$, and $H_{Y,t}$ denotes human-capital-embodied production labor. $X_t(j)$ denotes a continuum of differentiated intermediate goods indexed by $j \in [0, N_t]$. Firms maximize profit, and the conditional demand functions for $H_{Y,t}$ and $X_t(j)$ are given by

$$w_t = (1-\alpha) \frac{Y_t}{H_{Y,t}}, \tag{9}$$

$$p_t(j) = \alpha \left[\frac{H_{Y,t}}{X_t(j)} \right]^{1-\alpha}. \tag{10}$$

2.3 Intermediate goods

Each intermediate good j is produced by a monopolistic firm, which uses a one-to-one linear production function that transforms $X_t(j)$ units of final good into $X_t(j)$ units of intermediate good $j \in [0, N_t]$. The profit function is

$$\pi_t(i) = p_t(i)X_t(i) - X_t(i), \tag{11}$$

where the marginal cost of production is constant and equal to one (recall that final good is the numeraire). The monopolist maximizes (11) subject to (10) to derive the monopolistic price as

$$p_t(j) = \frac{1}{\alpha} > 1, \tag{12}$$

where $1/\alpha$ is the markup ratio. One can show that $X_t(j) = X_t$ for all $j \in [0, N_t]$ by substituting (12) into (10). Then, we substitute (10) and (12) into (11) to derive the equilibrium amount of monopolistic profit as

$$\pi_t = \left(\frac{1}{\alpha} - 1 \right) X_t = (1-\alpha)\alpha^{(1+\alpha)/(1-\alpha)} H_{Y,t}. \tag{13}$$

2.4 R&D

We denote v_t as the value of a newly invented intermediate good at the end of time t. The value of v_t is given by the present value of future profits from time $t+1$ onwards:

$$v_t = \sum_{s=t+1}^{\infty} \left[\pi_s / \prod_{\tau=t+1}^{s} (1 + r_\tau) \right]. \tag{14}$$

Competitive R&D entrepreneurs invent new products by employing $H_{R,t}$ units of human-capital-embodied labor. We specify the following innovation process:

$$\Delta N_t = \frac{\theta N_t H_{R,t}}{L_t}, \tag{15}$$

where $\Delta N_t \equiv N_{t+1} - N_t$. The parameter $\theta > 0$ determines R&D productivity $\theta N_t / L_t$, where N_t captures intertemporal knowledge spillovers as in Romer (1990) and $1/L_t$ captures a dilution effect that removes the scale effect.[8] If the following free-entry condition holds:

$$\Delta N_t v_t = w_t H_{R,t} \Leftrightarrow \frac{\theta N_t v_t}{L_t} = w_t, \tag{16}$$

then R&D $H_{R,t}$ would be positive at time t. If $\theta N_t v_t / L_t < w_t$, then R&D does not take place at time t (i.e., $H_{R,t} = 0$). Lemma 2 provides the condition for $H_{R,t} > 0$, which requires R&D productivity θ to be sufficiently high in order for innovation to take place.

[8] See Laincz and Peretto (2006) for a discussion of the scale effect.

Lemma 2 *R&D $H_{R,t}$ is positive at time t if and only if the following inequality holds:*

$$\int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i) s_t(i) di > \frac{1}{\theta}. \tag{17}$$

Proof. See Appendix A. ■

2.5 Aggregation

Imposing symmetry on (8) yields $Y_t = H_{Y,t}^{1-\alpha} N_t X_t^{\alpha}$. Then, we substitute (10) and (12) into this equation to derive the aggregate production function as

$$Y_t = \alpha^{2\alpha/(1-\alpha)} N_t H_{Y,t}. \tag{18}$$

Using $N_t X_t = \alpha^2 Y_t$, we obtain the following resource constraint on final good:

$$C_t = Y_t - N_t X_t = (1 - \alpha^2) Y_t, \tag{19}$$

where C_t denotes aggregate consumption. Finally, the resource constraint on human-capital-embodied labor is

$$\int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i) L_t(i) di = H_{Y,t} + H_{R,t}. \tag{20}$$

2.6 Equilibrium

The equilibrium is a sequence of allocations $\{X_t(j), Y_t, e_t(i), n_t(i), c_t(i), C_t, h_t(i), H_t, H_{Y,t}, H_{R,t}, L_t\}$ and prices $\{p_t(j), w_t, r_t, v_t\}$ that satisfy the following conditions:

- individuals choose $\{e_t(i), n_t(i), c_t(i)\}$ to maximize utility taking $\{r_{t+1}, w_t, h_t(i)\}$ as given;
- competitive firms produce Y_t to maximize profit taking $\{p_t(j), w_t\}$ as given;
- a monopolistic firm produces $X_t(j)$ and chooses $p_t(j)$ to maximize profit;
- competitive entrepreneurs perform R&D to maximize profit taking $\{w_t, v_t\}$ as given;
- the market-clearing condition for the final good holds such that $Y_t = N_t X_t + C_t$;
- the resource constraint on human-capital-embodied labor holds such that $H_{Y,t} + H_{R,t} = \int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i) L_t(i) di$;
- total saving equals asset value such that $w_t \int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i) L_t(i) di = N_{t+1} v_t$.

3 Stages of economic development

Our model features two stages of economic development. The first stage features only human capital accumulation. The second stage features both human capital accumulation and innovation.[9] The activation of innovation and the resulting transition from the first stage to the second stage are endogenous and do not always occur.

3.1 Stage 1: Human capital accumulation only

The initial level of human capital for each individual in household i is $h_0(i)$. Suppose the following inequality holds at time 0:

$$\int_0^1 [1 - e_0(i) - \sigma n_0(i)]\, h_0(i) s_0(i) di = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) s_0(i) di < \frac{1}{\theta}, \quad (21)$$

which uses (4) and (5). In (21), both the initial labor share $s_0(i) \equiv L_0(i)/L_0$ and initial human capital $h_0(i)$ are exogenously given. Then, Lemma 2 implies that $H_{R,0} = 0$ and

$$H_{Y,0} = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) L_0(i) di. \quad (22)$$

In this stage of development, the economy features only human capital accumulation. Human capital $h_t(i)$ accumulates according to the autonomous and stable dynamics in (6), and $s_t(i)$ evolves according to Lemma 1. However, so long as the following inequality holds at time t:

$$\int_0^1 [1 - e_t(i) - \sigma n_t(i)]\, h_t(i) s_t(i) di = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di < \frac{1}{\theta}, \quad (23)$$

we continue to have $H_{R,t} = 0$ and

$$H_{Y,t} = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) L_t(i) di. \quad (24)$$

Substituting (24) into (18) yields the level of output per worker as

$$y_t \equiv \frac{Y_t}{L_t} = \alpha^{2\alpha/(1-\alpha)} N_0 \frac{H_{Y,t}}{L_t} = \frac{\alpha^{2\alpha/(1-\alpha)} N_0}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di, \quad (25)$$

where N_0 remains at the initial level and output increases as human capital accumulates.

[9] See Iacopetta (2010), who considers a model in which innovation occurs before human capital accumulation.

3.2 Stage 2: Innovation and human capital accumulation

Equation (6) shows that human capital $h_t(i)$ converges to a steady state given by

$$h^*(i) = \frac{\gamma\phi(i)}{1+\eta+\gamma\delta}, \tag{26}$$

which is increasing in household i's ability $\phi(i)$. Substituting (26) into (4) and (5) yields the steady-state levels of education and fertility given by

$$e^*(i) = e^* = \frac{\gamma\delta}{1+\eta+\gamma\delta}, \tag{27}$$

$$n^*(i) = n^* = \frac{\eta}{\sigma(1+\eta+\gamma\delta)}, \tag{28}$$

where we assume positive population growth (i.e., $n^* > 1$) by imposing $\eta > (1+\gamma\delta)\sigma/(1-\sigma)$. Also, n^* is the same across all households because they are independent of $\phi(i)$. In other words, the negative effect of $\phi(i)$ and the positive effect of $h^*(i)$ on $n^*(i)$ cancel each other. As a result, the distribution of the population share of different households is stationary in the long run. In this case, Lemma 2 implies that if the following inequality holds:

$$(1-e^*-\sigma n^*)\int_0^1 h^*(i)s^*(i)di = \frac{\gamma}{(1+\eta+\gamma\delta)^2}\int_0^1 \phi(i)s^*(i)di > \frac{1}{\theta}, \tag{29}$$

then human capital accumulation eventually triggers the activation of innovation, under which the R&D condition in (16) holds and R&D $H_{R,t}$ becomes positive.

We now derive the equilibrium growth rate in the presence of innovation. Substituting (18) into (9) yields the equilibrium wage rate as

$$w_t = (1-\alpha)\alpha^{2\alpha/(1-\alpha)}N_t. \tag{30}$$

Then, substituting (30) into (16) yields the equilibrium invention value as

$$\frac{v_t}{L_t} = \frac{(1-\alpha)\alpha^{2\alpha/(1-\alpha)}}{\theta}. \tag{31}$$

The structure of overlapping generations implies that the value of assets at the end of time t must equal the amount of saving at time t given by wage income at time t:

$$N_{t+1}v_t = w_t\int_0^1 \left[1-e_t(i)-\sigma n_t(i)\right] h_t(i)L_t(i)di = w_t(H_{Y,t}+H_{R,t}), \tag{32}$$

where the second equality uses (20). Substituting (30) and (31) into (32) yields

$$N_{t+1} = \frac{\theta N_t}{L_t}(H_{Y,t}+H_{R,t}). \tag{33}$$

Combining (15) and (33) yields the equilibrium level of $H_{Y,t}$ as

$$\frac{H_{Y,t}}{L_t} = \frac{1}{\theta} \tag{34}$$

for all t. Substituting (4), (5) and (34) into (20) yields the equilibrium level of $H_{R,t}$ as

$$\frac{H_{R,t}}{L_t} = \int_0^1 \left[1 - e_t(i) - \sigma n_t(i)\right] h_t(i) s_t(i) di - \frac{H_{Y,t}}{L_t} = \frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di - \frac{1}{\theta}. \tag{35}$$

We can now substitute (35) into (15) to derive the equilibrium growth rate of N_t as

$$g_t \equiv \frac{\Delta N_t}{N_t} = \frac{\theta H_{R,t}}{L_t} = \frac{\theta}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_t(i)}{\phi(i)}\right] h_t(i) s_t(i) di - 1, \tag{36}$$

which is also the equilibrium growth rate of output per worker $y_t = \alpha^{2\alpha/(1-\alpha)} N_t/\theta$. Finally, the steady-state equilibrium growth rate of N_t and y_t is

$$g^* = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2} \int_0^1 \phi(i) s^*(i) di - 1. \tag{37}$$

In the steady state, $s^*(i)$ is also the population share of household i and still depends on the initial distribution of $h_0(i)$ and the exogenous distribution of $\phi(i)$ as shown in Lemma 1.

4 Heterogeneous households and evolutionary differences

Equation (21) shows that the activation of innovation-driven growth occurs at time 0 if and only if the following inequality holds:

$$\frac{1}{1+\eta+\gamma} \int_0^1 \left[1 + (1-\delta)\frac{h_0(i)}{\phi(i)}\right] h_0(i) s_0(i) di > \frac{1}{\theta}. \tag{38}$$

Suppose we consider a useful benchmark of an equal initial labor share $s_0(i) = 1$ and an equal initial level of human capital $h_0(i) = h_0$ for all $i \in [0,1]$. Then, the left-hand side of (38) simplifies to

$$\frac{h_0}{1+\eta+\gamma} \left[1 + (1-\delta) h_0 \int_0^1 \frac{1}{\phi(i)} di\right] > \frac{h_0}{1+\eta+\gamma} \left[1 + \frac{(1-\delta)h_0}{\overline{\phi}}\right], \tag{39}$$

where $\int_0^1 [1/\phi(i)] di > 1/\overline{\phi}$ due to Jensen's inequality. In other words, the presence of heterogeneity in $\phi(i)$ makes the activation of innovation-driven growth more likely to occur at time 0 than the absence of heterogeneity (i.e., $\phi(i) = \overline{\phi}$ for all $i \in [0,1]$) does. Due to heterogeneity, some households supply more human capital for production and innovation while others supply less. Equation (39) implies that the former effect dominates the latter effect such that the initial amount of human capital available for production and innovation increases as a result of heterogeneity. The intuition can be explained as follows.

Although some low-ability households may devote almost no time to education and most of their time to work (and fertility), high-ability households always spend some time to work, as the following shows:

$$1 - e_0(i) - \sigma n_0(i) = \frac{1}{1+\eta+\gamma} \left[1 + \frac{(1-\delta)h_0}{\phi(i)}\right] > \frac{1}{1+\eta+\gamma} > 0.$$

The convexity of $1/\phi(i)$ in $1 - e_0(i) - \sigma n_0(i)$ gives rise to the positive effect of heterogeneity on the amount of human capital available for production and innovation. To put it differently, the low-ability households being less willing to educate their children contribute to a larger workforce, which in turn rewards the innovation pioneers with more profits extracted from a larger market size of the economy. We summarize this result in the following proposition.

Proposition 1 *The heterogeneity of households makes it more likely for innovation to be activated at time 0.*

Proof. If the following inequality holds:

$$\frac{h_0}{1+\eta+\gamma}\left[1+(1-\delta)h_0\int_0^1 \frac{1}{\phi(i)}di\right] > \frac{1}{\theta} > \frac{h_0}{1+\eta+\gamma}\left[1+\frac{(1-\delta)h_0}{\overline{\phi}}\right], \tag{40}$$

which is a nonempty parameter space due to $\int_0^1 [1/\phi(i)]di > 1/\overline{\phi}$, then the takeoff of the economy occurs at time 0 under heterogeneous households but not under homogeneous households. ■

Next we examine how the labor share of households evolves over time. Given the benchmark of an equal initial labor share $s_0(i) = 1$ and an equal initial level of human capital $h_0(i) = h_0$ for all $i \in [0, 1]$, the fertility of household i at time 0 is

$$n_0(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1+(1-\delta)\frac{h_0}{\phi(i)}\right],$$

which is decreasing in $\phi(i)$. For households with $\phi(i) > \overline{\phi}$, their growth rate $n_0(i)$ would be lower than $n_0(\overline{\phi})$. However, they will have a higher level of human capital in the next period:

$$h_1(i) = \gamma\frac{\phi(i)+(1-\delta)h_0}{1+\eta+\gamma} > \gamma\frac{\overline{\phi}+(1-\delta)h_0}{1+\eta+\gamma}.$$

This higher level of human capital gives rise to a higher growth rate $n_1(i)$ and reduces the difference between $n_1(i)$ and $n_1(\overline{\phi})$. However, as shown in Lemma 1, $n_t(i)$ remains lower than $n_t(\overline{\phi})$ for $\phi(i) > \overline{\phi}$ until $h_t(i)$ converges to its steady-state level in (26) at which point the population growth rate of all households $i \in [0, 1]$ converges to n^* in (28). Therefore, the population growth rates of households with $\phi(i) > \overline{\phi}$ are lower than the population growth rates of households with $\phi(i) < \overline{\phi}$ until $h_t(i)$ converges to its steady-state level in (26). This temporary evolutionary disadvantage of high-ability households will never be compensated despite population trends being equal across households in the long run.

The above analysis implies that there exists a threshold for $\phi(i)$ above (below) which $s^*(i) < 1$ ($s^*(i) > 1$). This in turn implies that[10]

$$\int_0^1 \phi(i)s^*(i)di < \int_0^1 \phi(i)di = \overline{\phi}, \tag{41}$$

[10]See the proof of Proposition 2 in Appendix A.

because the households with larger $\phi(i)$ end up having a lower steady-state population share $s^*(i)$. Therefore, we also have the following inequality:

$$g^* = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\int_0^1 \phi(i)s^*(i)di - 1 < \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\overline{\phi} - 1, \tag{42}$$

where the right-hand side of the inequality is the steady-state equilibrium growth rate under homogeneous households (i.e., $\phi(i) = \overline{\phi}$ for all $i \in [0,1]$). In other words, the steady-state growth rate g^* becomes lower because the heterogeneity in households and the temporary evolutionary disadvantage of the high-ability households reduce the average level of human capital and consequently the level of innovation (recall that $g_t = \theta H_{R,t}/L_t$) in the long run. We summarize the above result in the following proposition.

Proposition 2 *The temporary evolutionary disadvantage of the high-ability households causes a lower steady-state equilibrium growth rate g^* than the case of homogeneous households.*

Proof. See Appendix A. ■

4.1 An example

In this section, we provide a simple parametric example to illustrate our results more clearly. We consider two types of households. Specifically, $\phi(i) = \overline{\phi} + \varsigma$ for $i \in [0, 0.5]$ and $\phi(j) = \overline{\phi} - \varsigma$ for $j \in [0.5, 1]$. As before, the households own the same initial amount of human capital (i.e., $h_0(i) = h_0$ for $i \in [0,1]$). Their initial population shares are also the same (i.e., $s_0(i) = 1$ for $i \in [0,1]$); in this case, the mean of $\phi(i)$ is simply $\overline{\phi}$ and the coefficient of variation in $\phi(i)$ is $\varsigma/\overline{\phi}$. Therefore, for a given $\overline{\phi}$, an increase in ς raises the coefficient of variation in $\phi(i)$.

From (42), the steady-state growth rate g^* is given by

$$g^* = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\left[(\overline{\phi}+\varsigma)s_H^* + (\overline{\phi}-\varsigma)s_L^*\right] - 1 = \frac{\theta\gamma}{(1+\eta+\gamma\delta)^2}\left\{\overline{\phi} - \varsigma\left[\underset{+}{s_L^*(\varsigma)} - \underset{-}{s_H^*(\varsigma)}\right]\right\} - 1, \tag{43}$$

where $s_L^* \equiv \int_{0.5}^1 s^*(j)dj = s^*(j)/2$ is the steady-state population share of household $j \in [0.5, 1]$ with low ability $\phi(j) = \overline{\phi} - \varsigma$ whereas $s_H^* \equiv \int_0^{0.5} s^*(i)di = s^*(i)/2$ is the steady-state population share of household $i \in [0, 0.5]$ with high ability $\phi(i) = \overline{\phi} + \varsigma$. We note that $s_H^* + s_L^* = 1$. Then, from Lemma 1, we have

$$\frac{s_L^*}{s_H^*} = \frac{\prod_{t=0}^{\infty} n_t(j)}{\prod_{t=0}^{\infty} n_t(i)} > 1, \tag{44}$$

where

$$\begin{aligned} n_t(j) &= \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{\sum_{\tau=0}^{t-1}\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t \left[1 + (1-\delta)\frac{h_0}{\overline{\phi}-\varsigma}\right]\right\}, \\ n_t(i) &= \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{\sum_{\tau=0}^{t-1}\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^\tau + \left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t \left[1 + (1-\delta)\frac{h_0}{\overline{\phi}+\varsigma}\right]\right\}. \end{aligned}$$

Therefore, s_L^*/s_H^* is increasing in ς, which together with $s_H^*+s_L^*=1$ implies that s_L^* is increasing in ς and s_H^* is decreasing in ς as stated in (43).

In summary, an increase in ς leads to an immediate increase in the coefficient of variation in $\phi(i)$ given by $\varsigma/\overline{\phi}$ and a subsequent decrease in the steady-state growth rate g^* given by (43) by reducing the average level of human capital and the level of innovation in the long run due to the temporary evolutionary disadvantage of the high-ability households.

5 Empirical evidence

In the previous section, we show that heterogeneity in the ability to accumulate human capital reduces the average level of education, innovation and economic growth in the long run. In this section, we use cross-country data to test this theoretical result. Specifically, we use the coefficient of variation in the level of education as a scale-invariant measure of heterogeneity in ability and estimate its effects on education, innovation and economic growth in the long run. The coefficient of variation in education is calculated from the Barro-Lee educational attainment dataset.[11]

The regression equation is specified as

$$y_{i,t+m} = \beta_0 + \beta_1 var_{i,t} + \beta_2 h_{i,t} + Z_{i,t} + \epsilon_{i,t},$$

where $y_{i,t+m}$ is the dependent variable (i.e., education, innovation or economic growth) in country i at time $t+m$, $var_{i,t}$ is the coefficient of variation in education and $h_{i,t}$ is the level of human capital in country i at time t. $Z_{i,t}$ is a vector of control variables including log population, log GDP per capita, trade as a share of GDP, gross capital formation as a share of GDP, and government expenditure as a share of GDP.[12] In order to capture the long run effect of variation in education, all explanatory variables are lagged 25 years (i.e., $m=25$).[13] Our theory predicts that $\beta_1 < 0$ and $\beta_2 > 0$. In other words, upon controlling for the level of human capital, heterogeneity in education (reflecting heterogeneity in the ability to accumulate human capital) has a negative effect on education, innovation and economic growth in the next period (i.e., 25 years later).

Table 1 reports our main empirical results. In the first two columns, the dependent variables are the share of the population with at least some primary education and the log of the average years of education, respectively. These two variables reflect the average level of education. In

[11] The Barro-Lee educational attainment dataset provides the fraction of each group completely or incompletely having attained primary, secondary and higher education. The duration for primary education and secondary education in each country is available from the UNESCO Statistical Yearbook. As in Barro and Lee (2013), we use a duration of four years for higher education and assign two years to persons who entered tertiary school but did not complete it. We compute the average years of education for each group and calculate their standard deviation in each country.

[12] Except for the coefficient of variation in education, the human-capital index, the number of researchers in R&D and the number of patent applications, all other variables are from the Penn World Table. The human-capital index, the number of researchers in R&D and patent applications are from the World Bank. We provide the summary statistics in Appendix B.

[13] Here, we choose a lag of 25 years since the age of most people graduated from college is between 20 and 25. In Appendix B, we also consider a lag of 20 years and a lag of 30 years.

Table 1: Effects of heterogeneity in education

	Education		Innovation		Growth
	(1)	(2)	(3)	(4)	(5)
Heterogeneity in education	-20.187***	-0.381***	-0.679**	-0.847***	-0.992***
	(1.875)	(0.039)	(0.284)	(0.158)	(0.360)
Human capital	3.952	0.174***	0.809***	1.884***	1.311**
	(2.469)	(0.053)	(0.255)	(0.268)	(0.518)
log population	0.453	0.016**	0.227***	1.336***	0.278***
	(0.365)	(0.008)	(0.058)	(0.077)	(0.087)
log GDP per capita	2.843***	0.147***	1.047***	1.111***	-0.702***
	(0.849)	(0.022)	(0.133)	(0.153)	(0.191)
Trade share to GDP	1.566	0.005	-3.121***	-0.100	-0.251
	(1.812)	(0.055)	(0.752)	(0.400)	(0.457)
Capital formation share	8.206	0.267	0.934	1.977	0.793
	(5.570)	(0.167)	(1.117)	(1.265)	(1.602)
Government expenditure share	3.950	0.320*	1.948*	1.903*	0.275
	(5.869)	(0.175)	(1.065)	(1.143)	(1.697)
Year fixed effect	Yes	Yes	Yes	Yes	Yes
R-squared	0.826	0.827	0.747	0.825	0.090
Observations	954	954	244	624	954

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. The dependent variables correspond to the share of population with schooling and log of average years of education respectively in the first two columns. The dependent variables correspond to log of the number of researchers in R&D (per million people) and log of the number of patent applications respectively in the columns 3-4. The dependent variable is the growth rate of GDP per capita in the last column. In all columns, we control year fixed effects. All independent variables are lagged 25 years.

columns (3) and (4), the dependent variables are the log of the number of researchers in R&D (per million people) and the log of the number of patent applications, respectively. These two variables reflect the level of innovation. Finally, in the last column, the dependent variable is the GDP per capita growth rate. From Table 1, we see that the coefficients of $var_{i,t}$ are all significantly negative, whereas the coefficients of $h_{i,t}$ are mostly significantly positive. This finding implies that upon controlling for the level of human capital, heterogeneity in education harms education, innovation and economic growth in the long run.

We also calculate the coefficients of variation in education for the male and female populations, respectively. Once again, the data is from the Barro-Lee educational attainment dataset. As shown in Table 3 in Appendix B, all main results still hold for both samples. The coefficients of variation in education are all significantly negative (except for the impact of heterogeneity in female education on the number of researchers in R&D). Comparing panel A and B, the negative impact of heterogeneity in the ability to accumulate human capital is more significant for the male population with a larger magnitude of coefficients. In the baseline results, the explanatory variables are lagged 25 years. If the explanatory variables are lagged 20 or 30 years instead, all the main results still hold (see Table 4 in Appendix B for more details).

6 Conclusion

In this study, we have developed a tractable innovation-driven growth model with endogenous takeoff and natural selection of heterogeneous households. Specifically, households differ in their ability to accumulate human capital. Within this growth-theoretic framework, we obtain the following results. Initially, the heterogeneity of households makes it more likely for an endogenous transition to innovation-driven growth to occur. However, our model features a rather surprising survival-of-the-weakest scenario in the short run. Only in the long run, the high-ability households would have accumulated enough human capital to overcome their temporary evolutionary disadvantage, which however has a lasting negative impact on long-run economic growth. We also examine cross-country data and find that heterogeneity in education indeed has adverse effects on education, innovation and economic growth in the long run.

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Appendix A: Proofs

Proof of Lemma 1. The labor share of household i is $s_t(i) \equiv L_t(i)/L_t$, where

$$L_t(i) = n_{t-1}(i)L_{t-1}(i) = n_{t-1}(i)n_{t-2}(i)L_{t-2}(i) = ... = \prod_{\tau=0}^{t-1} n_\tau(i)L_0(i). \tag{A1}$$

From (4), the fertility choice at time 0 is given by

$$n_0(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]. \tag{A2}$$

From (6), the level of human capital at time 1 is given by

$$h_1(i) = \frac{\gamma\phi(i)}{1+\eta+\gamma}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]. \tag{A3}$$

Substituting (A3) into (4) yields the fertility choice at time 1 as

$$n_1(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A4}$$

Substituting (A3) into (6) yields the level of human capital at time 2 as

$$h_2(i) = \frac{\gamma\phi(i)}{1+\eta+\gamma}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A5}$$

Substituting (A5) into (4) yields the fertility choice at time 2 as

$$n_2(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^2\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}. \tag{A6}$$

Then, we can continue the process to derive the fertility choice at time $t \geq 3$ as

$$n_t(i) = \frac{\eta}{\sigma(1+\eta+\gamma)}\left\{1+\frac{\gamma(1-\delta)}{1+\eta+\gamma}+...+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^{t-1}+\left[\frac{\gamma(1-\delta)}{1+\eta+\gamma}\right]^t\left[1+(1-\delta)\frac{h_0(i)}{\phi(i)}\right]\right\}, \tag{A7}$$

which can then be re-expressed using a summation sign as in Lemma 1. ■

Proof of Lemma 2. If (17) holds, then (35) shows that $H_{R,t} > 0$. Now, let's consider the case in which

$$\int_0^1 \left[1-e_t(i)-\sigma n_t(i)\right]h_t(i)\frac{L_t(i)}{L_t}di < \frac{1}{\theta}. \tag{A8}$$

Recall that the value of assets at the end of time t must equal the amount of saving at time t given by wage income at time t such that

$$N_{t+1}v_t = w_t\int_0^1 \left[1-e_t(i)-\sigma n_t(i)\right]h_t(i)L_t(i)di. \tag{A9}$$

Substituting (A9) into (A8) yields

$$w_t > \frac{\theta N_{t+1} v_t}{L_t} \geq \frac{\theta N_t v_t}{L_t}, \tag{A10}$$

where the second inequality uses $N_{t+1} \geq N_t$. Equation (A10) implies that $\Delta N_t v_t = w_t H_{R,t}$ in (16) cannot hold unless $H_{R,t} = 0$. ■

Proof of Proposition 2. From Lemma 1, the steady-state population share of household i is given by

$$s^*(i) = \frac{\prod_{t=0}^{\infty} n_t(i)}{\int_0^1 \prod_{t=0}^{\infty} n_t(i) di},$$

where we have used $L_0(i) = L_0$ for all i. Lemma 1 shows that $n_t(i)$ is monotonically decreasing in $\phi(i)$ before reaching the steady state n^* in (28), which then becomes independent of $\phi(i)$. Therefore, it must be the case that

$$s^*(i) < s^*(j) \Leftrightarrow \phi(i) > \phi(j).$$

Given that $\int_0^1 s^*(i)di = 1$, there must exist a threshold for $\phi(i)$ above (below) which $s^*(i) < 1$ ($s^*(i) > 1$). Let's define:

$$\Delta \equiv \int_0^1 \phi(i)s^*(i)di - \overline{\phi} = \int_0^1 \phi(i)s^*(i)di - \int_0^1 \phi(i)di = \int_0^1 \phi(i)[s^*(i) - 1]di.$$

We order the households such that $\phi(i) > \phi(j)$ for any $i < j$. In this case, $s^*(i) < 1$ for $i \in [0, \varepsilon]$ and $s^*(i) > 1$ for $i \in [\varepsilon, 1]$. Therefore, we can re-express Δ as

$$\Delta = \underbrace{\int_0^{\varepsilon} \phi(i)[s^*(i) - 1]di}_{<0} + \underbrace{\int_{\varepsilon}^1 \phi(i)[s^*(i) - 1]di}_{>0}.$$

If $\phi(i) = \phi(j) = \phi(\varepsilon)$ for all $i \in [0, \varepsilon]$ and $j \in [\varepsilon, 1]$, then $\Delta = 0$ because

$$\phi(\varepsilon) \int_0^{\varepsilon} [s^*(i) - 1]di + \phi(\varepsilon) \int_{\varepsilon}^1 [s^*(i) - 1]di = \phi(\varepsilon) \int_0^1 [s^*(i) - 1]di = 0.$$

Otherwise, $\Delta < 0$ because $\phi(i) > \phi(\varepsilon) > \phi(j)$ for any $i \in [0, \varepsilon)$ and $j \in (\varepsilon, 1]$ such that

$$\int_0^{\varepsilon} \phi(i)[s^*(i) - 1]di < \phi(\varepsilon) \int_0^{\varepsilon} [s^*(i) - 1]di < 0,$$

$$\phi(\varepsilon) \int_{\varepsilon}^1 [s^*(i) - 1]di > \int_{\varepsilon}^1 \phi(i)[s^*(i) - 1]di > 0,$$

implying $\Delta < \phi(\varepsilon) \int_0^1 [s^*(i) - 1]di = 0$. Therefore, (41) and (42) hold. ■

Appendix B: Data and robustness

Table 2: Summary statistics

Variable	Obs	Mean	S.D.	Min	Max
Share of population with schooling (%)	954	81.15	20.86	13.72	100
Years of education (log)	954	1.895	0.509	-0.191	2.586
Number of researchers (log)	244	6.728	1.665	2.003	8.952
Number of patent applications (log)	624	5.313	2.829	0	13.78
Growth of GDP per capita (%)	954	1.949	4.889	-50.23	35.26
Coefficient of variation in education	954	1.140	0.822	0.220	8.075
Variation in education (male)	954	0.991	0.638	0.228	5.984
Variation in education (female)	954	1.432	1.409	0.209	17.71
Human capital	954	1.808	0.613	1.009	3.463
log population	954	1.883	1.686	-2.212	7.067
log GDP per capita	954	8.587	1.175	5.683	12.38
Trade share to GDP (%)	954	-0.050	0.332	-8.188	0.860
Capital formation share (%)	954	0.211	0.133	0.002	2.000
Government expenditure share (%)	954	0.178	0.106	0.012	1.122

Note: The coefficient of variation in education is calculated from the Barro-Lee educational attainment dataset. The human-capital index, the number of researchers in R&D and the number of patent applications are from the World Bank. All other variables are from the Penn World Table.

Table 3: Effects of heterogeneity in education (male vs female)

	Education		Innovation		Growth
	(1)	(2)	(3)	(4)	(5)
Panel A: Male					
Heterogeneity in education	-20.872***	-0.372***	-1.072***	-1.048***	-1.148***
	(1.904)	(0.041)	(0.346)	(0.216)	(0.439)
Human capital	1.904	0.173***	0.704***	1.915***	1.415***
	(1.975)	(0.045)	(0.251)	(0.267)	(0.537)
R-squared	0.768	0.782	0.752	0.824	0.088
Observations	954	954	244	624	954
Panel B: Female					
Heterogeneity in education	-12.362***	-0.279***	-0.309	-0.423***	-0.534***
	(2.096)	(0.044)	(0.193)	(0.068)	(0.200)
Human capital	11.684***	0.252***	0.896***	1.901***	1.391***
	(3.372)	(0.073)	(0.246)	(0.268)	(0.417)
R-squared	0.798	0.836	0.745	0.826	0.091
Observations	954	954	244	624	954
Country-level controls	Yes	Yes	Yes	Yes	Yes
Year fixed effect	Yes	Yes	Yes	Yes	Yes

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. The dependent variables correspond to the share of population with schooling and log of average years of education respectively in the first two columns. The dependent variables correspond to log of the number of researchers in R&D (per million people) and log of the number of patent applications respectively in the columns 3-4. The dependent variable is log of GDP per capita in the last column. In all columns, we control year fixed effects. Country-level controls include log population, log GDP per capita, trade share to GDP, gross capital formation share to GDP and government expenditure share to GDP. All independent variables are lagged 25 years.

Table 4: Effects of heterogeneity in education (20 or 30 years lags)

	Education		Innovation		Growth
	(1)	(2)	(3)	(4)	(5)
Panel A: 20-year lag					
Heterogeneity in education	-22.536***	-0.442***	-0.477	-0.890***	-1.213***
	(1.860)	(0.037)	(0.330)	(0.166)	(0.350)
Human capital	5.993**	0.187***	1.028***	1.942***	0.921*
	(2.327)	(0.047)	(0.255)	(0.262)	(0.484)
R-squared	0.852	0.864	0.746	0.829	0.099
Observations	1091	1091	254	654	1091
Panel B: 30-year lag					
Heterogeneity in education	-17.946***	-0.333***	-0.797***	-0.787***	-0.723***
	(1.882)	(0.039)	(0.274)	(0.158)	(0.276)
Human capital	2.083	0.165***	0.864***	1.893***	1.092**
	(2.632)	(0.057)	(0.291)	(0.283)	(0.475)
R-squared	0.798	0.788	0.716	0.818	0.089
Observations	817	817	234	587	817
Country-level controls	Yes	Yes	Yes	Yes	Yes
Year fixed effect	Yes	Yes	Yes	Yes	Yes

Notes: *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Standard errors in parentheses are clustered by country. The dependent variables correspond to the share of population with schooling and log of average years of education respectively in the first two columns. The dependent variables correspond to log of the number of researchers in R&D (per million people) and log of the number of patent applications respectively in the columns 3-4. The dependent variable is log of GDP per capita in the last column. In all columns, we control year fixed effects. Country-level controls include log population, log GDP per capita, trade share to GDP, gross capital formation share to GDP and government expenditure share to GDP. All independent variables are lagged 20 years in panel A and lagged 30 years in panel B.