Consumer Naivete and Competitive Add-on pricing on platforms

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Abstract

Two sellers trade vertically and horizontally differentiated goods on a platform which charges them a commission fee. Some consumers are naive and do not observe, or consider, add-on prices until after they commit to buying the base good from a seller. We address the following questions. First, how do consumer naïveté and costs asymmetries (arising from differences in fees) influence pricing strategies. Second, we examine the welfare loss arising from sub-optimal decisions made by naive consumers who buy the bundle, but fail to factor in its total price at the outset. Third, how does naïveté affect seller and platform payoffs.

Keywords: add-on pricing, consumer naïveté, cost asymmetry, horizontal differentiation, vertical differentiation, platform fee, cost pass-through

JEL classification codes: L11, L15, L14, D43

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1 Introduction

This paper is motivated by the observation that firms often shroud the prices of their add-ons - e.g. additional cover in insurance, printer toner cartridges, hotel mini bars, etc. - from their consumers till the last stages of a transaction. Such obfuscation, to the extent that it precludes a comparison of the aggregate price of a good (inclusive of the add-on) across sellers, may be to the detriment of consumers. Indeed, a market study by the Financial Conduct Authority (FCA) on general insurance add-ons in the UK concluded that the "... *practice of revealing add-on prices only at the point of sale of the primary product is a very powerful barrier to consumers looking for alternatives...*".[1] Similarly, a market investigation into private motor insurance by the Competition and Markets Authority (CMA) in the UK found that some consumers were "... *less likely to compare the price of add-ons from different providers once they had selected add-ons from their preferred provider of the basic [insurance] policy which [...] could lead to consumers paying higher prices for add-ons*".[2]

Alternatively, some consumers may be unable to foresee their demand for add-ons, until after they have committed to purchase the base good from a seller. This, again, results in purchase decisions that are entirely based on prices of the base good alone, as the CMA found in its market investigation into retail banking in the UK: "... *some overdraft customers may believe they do not need to shop around because they believe that they use their overdraft much less than they actually do or not at all. They may therefore not even think about searching for a product offering better overdraft charges and terms*".[3]

We examine the implications of such consumer naïveté - whereby consumers either do not observe, or consider, add-on prices until after they have committed to purchase the base good from a seller - in markets where sellers trade horizontally and vertically differentiated goods. An increasing volume of transactions occur on platforms (such as Amazon, price comparison websites) that match buyers and sellers and we model a situation where the sellers trade their goods on a platform. More specifically, we analyse the impact of consumer naïveté on consumer choice and welfare, sellers' pricing strategies and profits, and the platform's fee and revenues. In addition, this paper throws light on the role played by platforms

[1] Provisional Findings, General insurance add-ons market study, FCA, 2016, page 25.
[2] Final Report, Private Motor Insurance Market Investigation, CMA, 2014, para 7.45.
[3] Final Report, Retail Banking Market Investigation, CMA, 2016, para 6.66.

- via differences in the fee charged - in shaping the pricing strategies of sellers who appear in its listings and consumer welfare.

In our model, two sellers located at the ends of a Hotelling line sell a base good, and a bundle comprising of the base good and an add-on. The sellers list their goods on a platform such as a price comparison web site which charges them a flat commission fee each time a sale is made via the platform. Fees are typically determined by the platform which may potentially vary across sellers. Further, we assume that the marginal costs of production are identical for the two goods and across sellers.

Consumers, distributed uniformly along the Hotelling line, differ from one another in three dimensions. First, consumers are differentiated by an idiosyncratic taste parameter that captures how well the sellers' goods match their preferences. Second, they differ in their level of sophistication. We assume that naive consumers are, for the reasons mentioned above, entirely guided by the prices of base goods when choosing the seller to buy from, while sophisticated consumers take into account the *total* cost of the good (inclusive of the add-on) when choosing a seller. Finally, consumers differ in their marginal utility of income, i.e. their valuation of quality. Thus, consumers can either have a low valuation of quality (be of "low type"), or have a high valuation of quality (be of "high type").

The timing of the game is as follows. First, the platform decides the commission fee to be paid; fees are common knowledge. Next, sellers simultaneously choose the prices of their base goods and bundles. Finally, consumers decide whether to purchase the base good only or the bundle, if at all, and the seller to buy from.

We examine the following questions. First, how does consumer naïveté influence the price structure - the relative prices of base goods and add-ons - and the level of prices across sellers. Further, to the extent that differences in commission fees introduce cost asymmetries across sellers, how does this affect their pricing strategies. Second, naive consumers consider only base good prices when they make purchasing decisions and yet, some of them may buy additional add-ons at the point of sale. This failure to consider the total price of the bundle at the outset may lead to sub-optimal decisions, involving a wrong choice of a seller, by some naive consumers. We explore how the interplay of the forces described above influences the extent of this welfare loss. Third, what are the factors that determine a platform's fee and revenues, and sellers' profits? Are sellers and platforms benefitted by the presence of naive consumers who are captive at the point of sale and can be sold expensive add-ons? Or are their revenues increasing in the fraction of sophisticated, well informed consumers who may drive down prices, thereby leading to greater buyer traffic and revenues for platforms?

We find that, as expected, sellers charge a lower price for the base good than for the bundle, at the equilibrium, and consumers self-select, with the low type consumers buying only the base good and the high type consumers buying the add-on too. But, the more interesting findings of the paper are to do with the impact of naïveté and differing consumer valuations of quality on sellers' pricing strategies and profits, consumer welfare and platform fee and revenues.

When the platform charges sellers a commission fee, it raises their costs, and via subsequent cost pass-through by sellers to consumers, brings about changes in the relative prices of the base goods and the bundles. Even if sellers have uniform marginal costs of production, differences in platform fees introduce cost asymmetries that have a bearing on these prices. The first result of the paper characterises conditions under which the fees paid by the sellers are asymmetric. Thereafter, in the rest of the paper, we assume that seller 2 is charged a higher fee for clear exposition of the results.

We then go on to examine sellers' pricing strategies. We find that seller 1 sets a lower price for its base good, but conversely, its add-on is more expensive, and becomes increasingly so with a decrease in the proportion of sophisticated consumers in the population, β. The relative prices of the sellers' bundles depend on β. We find that as the latter decreases, so that consumers are increasingly either of low type (and purchase the base good only), or of high type, but naive, base food prices become pivotal to purchasing decisions. In response, both sellers reduce the price of their base good but the lower costs of seller 1 allow it to reduce the price of its base good to a greater extent. Initially, despite a concomitant increase in the price of its add-on (to extract the greater surplus of the naive consumers), seller 1's bundle remains cheaper but when β is sufficiently low, so that consumers are mostly naive, the price of seller 1's add-on increases to an extent that its bundle becomes more expensive than that of seller 2.

In effect, seller 1's low headline price draws a greater share of the naive consumers who, once locked-in, buy its expensive add-on. As sales of the bundle are more profitable owing to their greater margin, seller 2 attempts to pass on its higher cost primarily through the price of its base good. At the same time, it keeps the price of its add-on low, in an attempt to keep the total price of its bundle as competitive as possible.

That the naive high type consumers buy the bundle at the point of sale but do not factor in its total price at the outset when they choose the seller to buy from, suggests that there may be a mass of such consumers who, *ex post*, make sub-optimal decisions. More specifically, we find that some of these consumers are drawn to seller 1 due to its low headline price and buy its bundle, but would be better off buying it from seller 2 instead. The next proposition of the paper outlines how the mass of naive consumers who make poor decisions varies with the extent of naïveté in the population and the fee charged by the platform. In fact, we find that any movement in the parameters of the model that reduces the magnitude of the difference in add-on prices across sellers results in a shrinking of this mass. Thus, a decrease in fee or an increase in β results in fewer naive consumers suffering this decrease in their payoff.

The final result of this paper examines sellers' profits and the platform's fee and revenues. We find that the latter are higher when β is high. The intuition behind this result hinges on the changes that movements in β bring about in the relative prices of the base goods and bundles, and thereby in the relative demand faced by the sellers. Thus, an increase in β results in a shift in demand to seller 2 and leads to, *ceteris paribus*, greater revenues for the platform as the fee paid by the latter is higher. In response, the platform increases the fee paid by this seller. This allows it to earn higher fees revenues by leveraging the greater demand that seller 2 now faces.

Conversely, sellers' profits are found to decrease in β. Intuitively, the main force driving this result appears to be the drop in the margin on sales of the bundle - due to a lowering of the price of the bundle for both sellers, and in the case of seller 2, an increase in its fee as well - that accompanies an increase in β. This suggests that platforms and sellers may have conflicting incentives when it comes to reducing the extent of naïveté in the population, whether by disseminating information about aggregate prices of bundles or educating consumers about the potential need for add-ons that may arise later on.

Related Literature: This paper is at the interface of three streams of literature. The first examines add-on pricing strategies of firms; more specifically, the factors influencing a firm's decision to bundle add-ons, their pricing and the impact on profits. While early studies (Gabaix and Laibson (2006), Lal and Matutes (1994), Verboven (1999)) argued that any profits earned on add-ons would be competed away in the form of lower prices of the base good, Ellison (2005) was the first to show that add-ons can actually raise profits by creating an adverse selection problem that makes price-cutting unappealing. Other studies focus on factors that impinge on a firm's decision of whether to bundle add-ons, and their pricing. For instance, Geng et al. (2018) explores how a firm's add-on strategy is influenced by an online platform's distribution contract choice. Similarly, Choudhary and Zhang (2016) analyses the impact of competition on firms' decisions of whether to bundle add-ons or offer them à la carte, when firms have asymmetric qualities. Borenstein et al. (2000) looks at whether competition in the durable-equipment market necessarily suppresses the exercise of market power in the aftermarket.

The second stream of literature studies how consumer naïveté affects market outcomes. Shapiro (1995) identifies the presence of myopic/ poorly informed consumers as one of the causes of market failure in aftermarkets. Verboven (1999) studies firms' add-on pricing strategies in an environment with vertical and horizontal differentiation, where there is a group of myopic consumers in the market. Gabaix and Laibson (2006) examines firms' decisions of whether to advertise or shroud their add-on prices when some consumers are boundedly rational or myopic and derives conditions under which competitive price cutting and educational advertising does not occur in equilibrium.

Armstrong (2015) discusses when the presence of savvy consumers results in search externalities that improve the deals available to all consumers, and when non-savvy consumers give rise to ripoff externalities and fund generous deals for all consumers. Shulman and Geng (2013, 2016) examine the implications for consumer welfare and profits when firms are horizontally and vertically differentiated and there is a segment of naive consumers who are unaware of add-on fees at the outset. Johnen and Somogyi (2021) studies when a platform discloses additional fees - seller fees or its own additional fees - when some buyers naively ignore shrouded fees. Similarly, Inderst and Obradovits (2021) studies how under vertical differentiation and context-dependent preferences, competition may rather exacerbate consumer and societal harm.

More recently, an emerging stream of literature focuses on agency pricing and examines how involvement of trading platforms and the nature of the contract between them and firms influences pricing strategies. Hao and Fan (2014) studies pricing of e-books and e-readers and social welfare under wholesale and agency pricing models. Ronayne (2020) examines whether consumers, including those who do not use price comparison websites, are better off with the presence of such sites in the market place. Wang and Wright (2020) argues that despite the possibility of showrooming, insistence on price parity clauses such as MFNs often harm consumers. Other work in this area includes Tian et al. (2017) that examines profitability under wholesale pricing vs. agency pricing. Similarly, Hao et al. (2016) examines advertising revenue-sharing contracts under agency pricing for app sales.

This paper adds to the literature by examining how naïveté influences consumer behaviour and welfare, sellers' pricing strategies and profits, and platform fees and revenues when sellers are charged a commission fee. Further, it highlights the role that differences in platform fees can play in shaping market outcomes by conferring costs advantages on some sellers. The rest of the paper is structured as follows. The next section lays out the model while Section 3 describes the key findings of the paper. Section 4 concludes the paper. Proofs are collected in an Appendix.

2 Model

There are two sellers in the market, indexed by $i \in \{1, 2\}$ and located at the two ends of a Hotelling line, that sell two vertically differentiated goods. These include a base good of low quality, and a bundle of higher quality comprising of the base good with an additional add-on, at prices p_{il} and p_{ih} respectively. We assume that marginal costs of production, k, are constant and identical for the two goods, and across sellers.

Sellers list their goods on a platform such as a price comparison web site, which charges a flat commission fee each time a sale is made via the platform. As new sellers continually enter the market and negotiate with platforms to appear in their listings and contracts between platforms and sellers are typically long-term and sticky in nature, we model a situation where there is a seller, say, seller 1, with a pre-existing contract (involving a commission fee $\bar{c}$) while the fee, c, paid by seller 2 is determined by the platform in the model so as to maximise its revenues.

There is a measure one of consumers, distributed along the Hotelling line, who differ from one another in three dimensions. First, consumers are differentiated by an idiosyncratic taste parameter $\theta \sim U[0, 1]$ that characterises how well the sellers' goods match their preferences. Second, consumers differ in their marginal utility of income, α; for a proportion γ of consumers (of "low type"), $\alpha = \alpha_l$ while for the remaining $1 - \gamma$ consumers (of "high type"), $\alpha = \alpha_h$, where $0 < \alpha_h < \alpha_l$.

Finally, consumers differ in their level of sophistication. More specifically, some consumers are naive and either cannot observe, or do not consider, sellers' add-on prices - possibly because they do not foresee their demand for the add-on - until after they have committed to purchase the base good from a seller. Therefore, such consumers only consider the price of the base good when they decide which seller to buy from. However, once locked-in they may buy the add-on at the point of sale. In contrast, sophisticated consumers consider the aggregate price of a good, i.e. inclusive of the add-on, when choosing which product to buy and the seller to buy from. We assume that a proportion β of consumers are sophisticated while the remaining consumers are naive.

The utility of a consumer of type (θ, α) from consumption of the high quality bundle is $u - \theta - \alpha p_{1h}$ when he buys from seller 1 and $u - (1 - \theta) - \alpha p_{2h}$ if he buys from seller 2. His utility is lower if he buys the base good only; it equals $u - a - \theta - \alpha p_{1l}$ if he buys it from seller 1 and $u - a - (1 - \theta) - \alpha p_{2l}$ if he buys it from seller 2 where $a > 0$. Note that the high types are less sensitive to price differences between the two sellers owing to their high valuation of quality. Each consumer buys at most one unit of the good and derives zero utility if he does not make a purchase. We assume that u is large enough so that the market is covered.

The timing of the game is as follows. First, the platform decides the commission fee, c, to be paid by seller 2; fees are common knowledge. Second, sellers simultaneously choose the prices of their goods, p_{il} and p_{ih}, $i \in \{1, 2\}$. Finally, consumers decide whether to purchase the base good only or the high quality bundle, if at all, and the seller to buy from. Note that the add-on cannot be bought in isolation.

3 Equilibrium

We examine the equilibrium where low type consumers owing to their low valuation of quality buy the base good only while high type consumers buy the add-on too. For such purchasing behavior to hold, the following individual rationality and incentive compatibility constraints must be satisfied.

i. For high type, naive consumers who buy the bundle from, say, seller 1:

$$u - a - \theta - \alpha_h p_{1l} \geq \max(0, u - a - (1-\theta) - \alpha_h p_{2l}) \tag{1}$$

and

$$u - \theta - \alpha_h p_{1h} \geq u - a - \theta - \alpha_h p_{1l} \tag{2}$$

Naive consumers compare only base good prices when they decide which seller to buy from, and the first inequality implies that it is optimal for them to choose seller 1 if they initially decide to buy the base good only. However, they eventually buy the add-on too at the point of sale, and the second inequality implies that it is optimal for them to buy the bundle (and not the base good only).

ii. For high type, sophisticated consumers who buy the bundle from, say, seller 1:

$$\begin{aligned} u - \theta - \alpha_h p_{1h} \geq \max(0, u - a - \theta - \alpha_h p_{1l}, u - a - (1-\theta) - \alpha_h p_{2l}, \\ u - (1-\theta) - \alpha_h p_{2h}) \end{aligned} \tag{3}$$

The sophisticated, high type consumers take into consideration the total price of the good when they decide which good to buy, and who to buy from.

iii. For low type consumers who buy the base good from, say, seller 1:

$$\begin{aligned} u - a - \theta - \alpha_l p_{1l} \geq \max(0, u - \theta - \alpha_l p_{1h}, u - (1-\theta) - \alpha_l p_{2h}, \\ u - a - (1-\theta) - \alpha_l p_{2l}) \end{aligned} \tag{4}$$

Note that of the low type consumers, only those who are sophisticated take the price of the add-on into consideration (though they buy the base good only) while the naive consumers focus only on base good prices.

Similar constraints apply for consumers who purchase from the other seller.

Now, low type consumers purchase from seller 1 if

$$u - a - \theta - \alpha_l p_{1l} \geq u - a - (1-\theta) - \alpha_l p_{2l}$$

or,

$$\theta \leq \theta_l = \frac{1}{2} + \frac{\alpha_l(p_{2l} - p_{1l})}{2}$$

while the remaining consumers buy from the other seller. Seller 1's profit from such consumers is then given by

$$\gamma\left(\frac{1}{2} + \frac{\alpha_l(p_{2l} - p_{1l})}{2}\right)(p_{1l} - \bar{c} - k) \tag{5}$$

Similarly, the high type sophisticated consumers buy from seller 1 if

$$\theta \leq \theta_s = \frac{1}{2} + \frac{\alpha_h(p_{2h} - p_{1h})}{2}$$

while the remaining consumers buy from the other seller. Then, seller 1's profit from such consumers is given by

$$(1-\gamma)\beta\left(\frac{1}{2} + \frac{\alpha_h(p_{2h} - p_{1h})}{2}\right)(p_{1h} - \bar{c} - k) \tag{6}$$

Finally, we consider the demand from the high type naive consumers. They buy from seller 1 if

$$\theta \leq \theta_n = \frac{1}{2} + \frac{\alpha_h(p_{2l} - p_{1l})}{2}$$

But, as they eventually buy the add-on too, the seller's profit from such consumers is

$$(1-\gamma)(1-\beta)\left(\frac{1}{2} + \frac{\alpha_h(p_{2l} - p_{1l})}{2}\right)(p_{1h} - \bar{c} - k) \tag{7}$$

Then, from equations (5), (6) and (7), seller 1's optimisation problem is

$$\begin{aligned} \max_{p_{1l}, p_{1h}} \pi_1 = \gamma\left(\frac{1}{2} + \frac{\alpha_l(p_{2l} - p_{1l})}{2}\right)(p_{1l} - \bar{c} - k) + \\ (1-\gamma)\beta\left(\frac{1}{2} + \frac{\alpha_h(p_{2h} - p_{1h})}{2}\right)(p_{1h} - \bar{c} - k) + \\ (1-\gamma)(1-\beta)\left(\frac{1}{2} + \frac{\alpha_h(p_{2l} - p_{1l})}{2}\right)(p_{1h} - \bar{c} - k) \end{aligned} \tag{8}$$

Similarly, seller 2's optimisation problem is

$$\begin{aligned}\max_{p_{2l},p_{2h}} \pi_2 = \gamma\left(\frac{1}{2}-\frac{\alpha_l(p_{2l}-p_{1l})}{2}\right)(p_{2l}-c-k) + \\ (1-\gamma)\beta\left(\frac{1}{2}-\frac{\alpha_h(p_{2h}-p_{1h})}{2}\right)(p_{2h}-c-k) + \\ (1-\gamma)(1-\beta)\left(\frac{1}{2}-\frac{\alpha_h(p_{2l}-p_{1l})}{2}\right)(p_{2h}-c-k)\end{aligned} \tag{9}$$

Suppose the prices that maximise sellers' profits are $p^*_{1l}(c), p^*_{2l}(c), p^*_{1h}(c)$ and $p^*_{2h}(c)$. Then, the platform's payoff, comprising of the revenues from the commission fee it charges the sellers, is given by

$$\begin{aligned}\pi_P = \bar{c}\Bigg[\gamma\left(\frac{1}{2}+\frac{\alpha_l(p^*_{2l}-p^*_{1l})}{2}\right) + (1-\gamma)\beta\left(\frac{1}{2}+\frac{\alpha_h(p^*_{2h}-p^*_{1h})}{2}\right) + \\ (1-\gamma)(1-\beta)\left(\frac{1}{2}+\frac{\alpha_h(p^*_{2l}-p^*_{1l})}{2}\right)\Bigg] + c\Bigg[\gamma\left(\frac{1}{2}-\frac{\alpha_l(p^*_{2l}-p^*_{1l})}{2}\right) + \\ (1-\gamma)\beta\left(\frac{1}{2}-\frac{\alpha_h(p^*_{2h}-p^*_{1h})}{2}\right) + (1-\gamma)(1-\beta)\left(\frac{1}{2}-\frac{\alpha_h(p^*_{2l}-p^*_{1l})}{2}\right)\Bigg]\end{aligned} \tag{10}$$

and the platform maximises its payoff with respect to the commission fee, c.

We characterise consumer choice and the fee charged by the platform from seller 2 at the equilibrium.

Lemma 1 (Equilibrium)**.** *For $a \in [\underline{a}, \bar{a}]$ and $u \geq \tilde{u}$, where $\underline{a}, \bar{a}, \tilde{u} > 0$, there exists an equilibrium where*

i Low type consumers with a high marginal utility of income, α_l, buy only the base good while the high type consumers with a lower marginal utility of income, α_h, purchase the add-on too.

ii If $\frac{\alpha_l}{\alpha_h} \geq \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$, the platform charges a higher fee from seller 2, i.e. $c \geq \bar{c}$.

The intuition behind the last result is as follows. When the platform charges sellers a commission fee, it raises their costs, and via a subsequent cost pass-through by sellers to consumers, brings about changes in the relative prices of the base goods and the bundles. An increase in c, *ceteris paribus*, raises the platform's revenues. But it also has an impact on the demand faced by either seller, which in turn depends on the direction and the magnitude of change in the relative prices, $p_{2l} - p_{1l}$ and $p_{2h} - p_{1h}$, following the fee hike, as described above. We find that if $\frac{\alpha_l}{\alpha_h} \geq \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$, an increase in c raises the demand faced by seller 1. Therefore, at the equilibrium, c exceeds $\bar{c}$, so that there is a concomitant loss in revenues that exactly offsets the higher revenues arising from the fee hike. In the rest of the paper, we assume that $\frac{\alpha_l}{\alpha_h} > \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$, so that the platform charges a higher fee from seller 2. Note that as the marginal costs of production are uniform across the sellers, a difference in platform fees introduces cost asymmetries between them.

The following proposition characterises the impact of β, i.e. the proportion of sophisticated consumers in the population, on sellers' pricing strategies at the equilibrium, when α_l is large enough.

Proposition 1 (Dependence of pricing strategies on β)**.** *At the equilibrium, seller 1 charges a lower price for its base good than seller 2, with the gap between the prices, $p^*_{2l} - p^*_{1l}$, widening with a decrease in β, i.e. with an increase in the proportion of naive consumers in the population. Though add-on prices are equal at $\beta = 1$, seller 1's add-on becomes increasingly more expensive as β falls. Finally, for $\beta \in (\bar{\beta}, 1]$, its bundle is less expensive, with the gap in bundle prices, $p^*_{2h} - p^*_{1h}$, shortening, and eventually, reversing in sign for $\beta \in (0, \bar{\beta}]$.*

We find that, at the equilibrium, seller 1 sets a lower price for its base good, i.e. $p_{1l} < p_{2l}$. When β equals one, i.e. none of the consumers is naive, the sellers charge the same price for their add-on. However, as β decreases and the proportion of sophisticated consumers in the population falls, consumers, increasingly, either have a low valuation of quality and buy the base good only, or have a high valuation of quality and are naive. Either way, $p_{il}, i = \{1, 2\}$ becomes pivotal to buying decisions, both for the naive consumers who are lured to a seller by low base good prices and for the low type consumers whose purchase decision, too, hinges on these prices alone. In response, both sellers reduce the price of their base good but the lower costs of seller 1 (as $\bar{c} < c$) allow it to reduce the price of its base good to a greater extent and the gap in base good prices, $p^*_{2l} - p^*_{1l}$, widens with a decrease in β. At the same time, its add-on becomes increasingly more expensive as this allows it to extract the higher surplus of the high type naive consumers.

Initially (when β is high), despite this increase in the price of its add-on, seller 1's bundle remains cheaper. This is strategic as a high β necessitates that bundle prices be kept in check for the sophisticated consumers who take into account the total price of the bundle when they decide who to buy from. As β decreases and the population increasingly comprises of naive consumers, both bundle prices increase and we find that p_{1h} increases to a greater extent (due to a large rise in seller 1's add-on price), so much so that when β is sufficiently low, its bundle becomes more expensive than that of seller 2.

The figure below depicts how the prices of the base goods and the bundles respond to changes in the proportion of sophisticated consumers in the population.

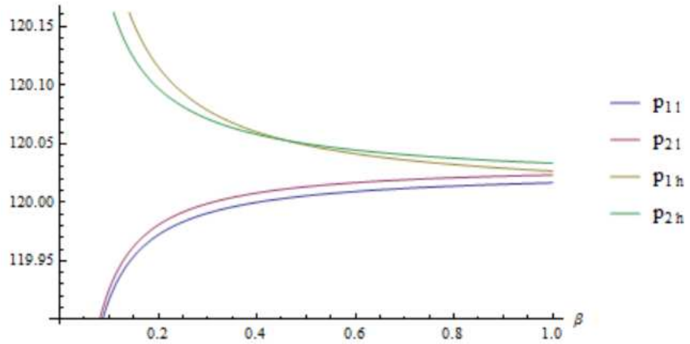

Figure 1: Sellers' pricing strategies as a function of the proportion of sophisticated consumers in the population. Parameters: $\gamma = \frac{1}{2}, \alpha_h = 50, \alpha_l = 100, \bar{c} = 100, k = 20$.

Thus, seller 1's base good is cheaper but its add-on is more expensive than that of seller 2; its bundle is cheaper when β is high enough, while the converse holds when β is low. Due to its low headline prices, it draws a greater share of the low type consumers and the high type naive consumers as they are entirely guided by the relative prices of base goods in their choice of seller. As sales of the bundle are more profitable owing to their greater margin, seller 2 attempts to pass on its higher cost primarily through the price of its base good. At the same time, it keeps the price of its add-on low, in an attempt to keep the total price of its bundle as competitive as possible.

That the naive high type consumers' choice of the seller to buy from is based entirely on a comparison of base good prices, but they are locked-in at the point of sale and eventually buy the add-on too, suggests that there may be a mass of such consumers who, *ex post*, make sub-optimal decisions. Indeed, we find that some naive consumers who are drawn to seller 1 due to its low headline price, but eventually buy its more expensive add-on as well, would be better off buying the bundle from seller 2 instead. Interestingly, the size of this mass is partly determined by the extent of consumer naïveté in the population and the commission fee charged by the platform. To see this, consider the lemma below that details the impact of an increase in the fee on sellers' base good and add-on prices.

Lemma 2. *As long as α_l is large enough, an increase in the fee, c, results in an increase in base good prices and the price of seller 1's add-on, and a decrease in the price of seller 2's add-on.*

An increase in the fee charged from seller 2 results in a widening of the gap, $p_{1h} - p_{1l} - (p_{2h} - p_{2l})$, between add-on prices for the following reason. An increase in the fee raises seller 2's costs, that it attempts to recover by raising its prices. As mentioned above, the higher margins on sales of the bundle imply that seller 2 passes on its higher costs mostly through the price of its base good. At the same time, it reduces the price of its add-on, so that the total price of its bundle remains as competitive as possible. In response to this price hike by seller 2, and given that the entire market is covered, seller 1 raises its prices too (including that of its add-on), though to a smaller extent. We find that this movement in add-on prices, resulting either from a change in fees or in β (Proposition 1), has implications for the welfare of a section of the naive consumers who make poor decisions.

Proposition 2 (Welfare)**.** *There is a mass of naive consumers, increasing in c and decreasing in β, who buy the high quality bundle from seller 1 but who would have been better off buying it from seller 2 instead.*

When the fee, c, charged from seller 2 decreases, the gap between the add-on prices shortens (Lemma 2), leading to a drop in the mass of such consumers. Similarly, a high β not only implies that there are fewer high type, naive consumers in the population, but also shortens the gap in add-on prices (Proposition 1), leading to a shrinking of this mass.

The last proposition outlines the impact of consumer naïveté on platform fees and revenues and sellers' profits.

Proposition 3 (Platform fees and revenues and sellers' profits)**.** *Platform fees and revenues rise with an increase in the proportion, β, of sophisticated consumers in the population. Conversely, sellers' profits decrease with an increase in β.*

From Proposition 1, when β increases, the gap, $p_{2l} - p_{1l}$, between base good prices shortens while that between bundle prices, $p_{2h} - p_{1h}$, widens. This, in turn, implies that, *ceteris* *paribus*, the demand for seller 1's product from the low type and high type naive consumers decreases, but that from high type sophisticated consumers, who are also greater in number now, increases. We find that the former effect dominates, so that the total demand faced by seller 1 decreases with an increase in β. As the market is covered, this translates into a shift in demand towards seller 2. As seller 2 is charged a higher fee, this results in an increase in the platform's revenues. In addition, an increase in β also results in an increase in the fee charged from seller 2, *per se*, leading to higher revenues - a higher fee allows the platform to earn greater revenues by leveraging the higher demand that seller 2 now faces. Hence, the platform's revenues rise.

We find that, conversely, sellers' profits are decreasing in β. Intuitively, the main force driving this result appears to be the drop in the margin on sales of the bundle - due to a lowering of the price of the bundle for both sellers, and in the case of seller 2, an increase in its fee as well - that accompanies an increase in β. In the case of seller 2, this is further compounded by the decrease in the number of sophisticated high type consumers who visit the seller (due to a widening of the gap between bundle prices, $p_{2h} - p_{1h}$). Though a greater number of low type and naive high type consumers visit the seller than before (due to a shortening in the gap between base good prices, $p_{2l} - p_{1l}$), the former two forces appear to be stronger, resulting in a drop in seller 2's profits. In the case of seller 1, in addition to the drop in the margin on sales of the bundle, fewer low type consumers and naive high type consumers visit as well. Though the margin on sales of the base good are higher and a greater number of sophisticated high type consumers visit than before, the former two forces appear to be stronger, resulting in a drop in seller 1's profits.

Thus, it appears that while an increase in β is in the interests of the naive high type consumers - in that it leads to a shrinking of the mass of consumers who make sub-optimal decisions - and also results in higher profits for the platform, it is to the detriment of the sellers. This is due to the resultant adverse changes in the fee charged by the platform and the concomitant changes in relative prices and the margin on sales of the bundles sold by the sellers. Thus, there may be a conflict of interest between platforms and sellers regarding incentives to bring about a decrease in the extent of consumer naïveté in the population, whether by disseminating greater information about the aggregate price of products and/ or prompting consumers about the need for add-ons at the outset, so that consumers can make more informed choices.

4 Conclusion

This paper explores the impact of consumer naïveté in markets where sellers trade a base good, and a bundle comprising of the base good and an add-on, on a platform and are charged a commission fee. More specifically, we examine the impact of naïveté on sellers' pricing strategies and profits, consumer welfare, and platform fees and revenues.

We find that differences in the fee charged by a platform can introduce cost asymmetries across sellers that interact with consumer naïveté and differing consumer valuations of quality to influence sellers' pricing strategies. In particular, these asymmetries can impart cost advantages to a seller that allow it to set lower headline and higher add-on prices than the other seller, while the relative prices of their bundles depend on the extent of naïveté in the population. Additionally, we find that the gap between base good and add-on prices charged by the two sellers widens as β decreases, i.e. the proportion of naive consumers in the population increases.

In turn, this has implications for the welfare of a section of naive consumers who either do not observe, or consider, add-on prices until after they have committed to purchase the base good from a seller. As these consumers' purchase decisions are based entirely on a comparison of base good prices but they buy the add-on too at the point of sale, there exists a mass of such consumers who make sub-optimal decisions. Specifically, they are drawn to a seller due to its low base good price but once locked in, buy its relatively expensive add-on too, in the process receiving a payoff that is lower than what they would have received had they purchased the bundle from the other seller. We find that a decrease in the fee charged by the platform and/ or an increase in the proportion of sophisticated consumers in the population leads to a shrinking of this mass by bringing about a favourable realignment in the add-on prices of the sellers.

Finally, we examine the impact of consumer naïveté on sellers' profits and the platform's fee and revenues. We find that while the sellers' profits are decreasing in the proportion of sophisticated consumers in the population, the converse holds for the platform's fee and revenues. This suggests that platforms and sellers may have conflicting incentives when it comes to disseminating information about aggregate prices of bundles or educating consumers about the potential need for add-ons.

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6 Appendix

Proof of Lemma 1

Differentiating equations (8) and (9), with respect to p_{1l}, p_{1h} and p_{2l}, p_{2h} respectively, and solving for the optimal prices yields $p^*_{1l}(c), p^*_{2l}(c), p^*_{1h}(c)$ and $p^*_{2h}(c)$. Substituting these in the platform's payoff, i.e. equation (10) and maximising with respect to c yields

$$c^* = \frac{3\beta\gamma\alpha_l(3+2\bar{c}\gamma\alpha_l) + 2(1-\gamma)\alpha_h((1-\beta)^2 + 3\bar{c}\beta\gamma\alpha_l)}{6\beta\gamma\alpha_l(\gamma\alpha_l + (1-\gamma)\alpha_h)} \tag{11}$$

Note that

$$c^* - \bar{c} = \frac{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h}{6\beta\gamma\alpha_l(\gamma\alpha_l + (1-\gamma)\alpha_h)}$$

so that as long as $\frac{\alpha_l}{\alpha_h} \geq \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$, $c^* > \bar{c}$. Finally, substituting for c^* in $p^*_{1l}(c), p^*_{2l}(c), p^*_{1h}(c)$ and $p^*_{2h}(c)$, we get the optimal prices:

$$p^*_{2h} = \frac{6\gamma^2\alpha_l^2 - 2(1-\gamma)\alpha_h^2((1-\beta)^2 - 3(\bar{c}+k)\beta\gamma\alpha_l) + \gamma\alpha_l\alpha_h(5+7\beta-6\gamma+6(\bar{c}+k)\beta\gamma\alpha_l)}{6\beta\gamma\alpha_l\alpha_h((1-\gamma)\alpha_h + \gamma\alpha_l)} \tag{12}$$

$$p^*_{2l} = \frac{6\gamma\alpha_l(2\beta - 1 + \gamma + (\bar{c}+k)\beta\gamma\alpha_l) + (1-\gamma)\alpha_h(-7+9\beta-2\beta^2+6\gamma+6(\bar{c}+k)\beta\gamma\alpha_l)}{6\beta\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} \tag{13}$$

$$p^*_{1l} = \frac{3\gamma\alpha_l(2(\bar{c}+k)\beta\gamma\alpha_l - 2(1-\gamma) + 3\beta) + (1-\gamma)\alpha_h(6\gamma - 7(1-\beta) + 6(\bar{c}+k)\beta\gamma\alpha_l)}{6\beta\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} \tag{14}$$

$$p^*_{1h} = \frac{6(\bar{c}+k)\beta(1-\gamma)\alpha_h^2 + 6\gamma\alpha_l + \alpha_h(7+2\beta-6\gamma+6(\bar{c}+k)\beta\gamma\alpha_l)}{6\beta\alpha_h((1-\gamma)\alpha_h + \gamma\alpha_l)} \tag{15}$$

Next, consider the individual rationality and incentive compatibility constraints. Substituting the value of θ for the indifferent consumer of low type, and the sophisticated and naive high type, i.e. θ_l, θ_s and θ_n, in (1), (2), (3) and (4) and the corresponding constraints for consumers buying from seller 2, the constraints reduce to $a \in [\alpha_h(p^*_{1h} - p^*_{1l}), \alpha_l(p^*_{2h} - p^*_{2l})]$, while

$$u \geq \max\left(\frac{1}{2} + a + \frac{\alpha_l(p^*_{2l} + p^*_{1l})}{2}, \frac{1}{2} + \frac{\alpha_h(2p^*_{1h} + p^*_{2l} - p^*_{1l})}{2}\right) \tag{16}$$

Let

$$
\begin{aligned}
\underline{a} &= \alpha_h(p^*_{1h} - p^*_{1l}) \\
\bar{a} &= \alpha_l(p^*_{2h} - p^*_{2l}) \\
\tilde{u} &= \max\left(\frac{1}{2} + a + \frac{\alpha_l(p^*_{2l} + p^*_{1l})}{2}, \frac{1}{2} + \frac{\alpha_h(2p^*_{1h} + p^*_{2l} - p^*_{1l})}{2}\right)
\end{aligned}
$$

■

Proof of Proposition 1

From equations (14), (15), (13) and (12),

$$
\frac{\delta p^*_{1l}}{\delta\beta} = \frac{(1-\gamma)((7-6\gamma)\alpha_h + 6\gamma\alpha_l)}{6\beta^2\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0
$$

$$
\frac{\delta p^*_{1h}}{\delta\beta} = -\frac{(7-6\gamma)\alpha_h + 6\gamma\alpha_l}{6\beta^2\alpha_h((1-\gamma)\alpha_h + \gamma\alpha_l)} < 0
$$

Further,

$$
\frac{\delta p^*_{2l}}{\delta\beta} = \frac{(1-\gamma)(6\gamma\alpha_l - (7-2\beta^2-6\gamma)\alpha_h)}{6\beta^2\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0
$$

as long as α_l is large enough, while

$$
\frac{\delta p^*_{2h}}{\delta\beta} = \frac{2(1-\beta^2)(1-\gamma)\alpha_h^2 + (6\gamma-5)\gamma\alpha_h\alpha_l - 6\gamma^2\alpha_l^2}{6\beta^2\gamma\alpha_l\alpha_h((1-\gamma)\alpha_h + \gamma\alpha_l)} < 0
$$

The denominator of the expression above is positive while the numerator is a quadratic in β with a negative leading coefficient, $-2\alpha_h^2(1-\gamma)$, i.e. it opens downwards. At $\beta = 0$, the numerator equals $2(1-\gamma)\alpha_h^2 - 5\gamma\alpha_h\alpha_l - 6\gamma^2\alpha_l(\alpha_l - \alpha_h)$, which is negative if α_l is large enough. As the maximum of the quadratic is attained at $\beta = 0$, where it takes a negative value, it is negative for all β. Next, note that

$$
p^*_{2l} - p^*_{1l} = \frac{2(1-\beta)(1-\gamma)\alpha_h + 3\gamma\alpha_l}{6\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0
$$

and

$$
\frac{\delta(p^*_{2l} - p^*_{1l})}{\delta\beta} = -\frac{(1-\gamma)\alpha_h}{3\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} < 0
$$

Similarly,

$$
p^*_{2h} - p^*_{1h} = -\frac{2(1-\beta)^2(1-\gamma)\alpha_h + (2-5\beta)\gamma\alpha_l}{6\beta\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)}
$$

The expression above is negative, i.e. $p^*_{2h} < p^*_{1h}$, when $\beta < \frac{2}{5}$. When $\beta > \frac{2}{5}$ and α_l is high enough, $p^*_{1h} < p^*_{2h}$. Further,

$$\frac{\delta(p^*_{2h} - p^*_{1h})}{\delta\beta} = \frac{(1-\beta^2)(1-\gamma)\alpha_h + \gamma\alpha_l}{3\beta^2\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0$$

Finally,

$$p^*_{1h} - p^*_{1l} - (p^*_{2h} - p^*_{2l}) = \frac{1-\beta}{3\beta\gamma\alpha_l} > 0$$

Note that the add-on prices are equal when $\beta = 1$. Also,

$$\frac{\delta(p^*_{1h} - p^*_{1l})}{\delta\beta} = -\frac{7\alpha_h + 6\gamma(\alpha_l - \alpha_h)}{6\beta^2\gamma\alpha_h\alpha_l} < 0$$

and

$$\frac{\delta(p^*_{2h} - p^*_{2l})}{\delta\beta} = -\frac{5\alpha_h + 6\gamma(\alpha_l - \alpha_h)}{6\beta^2\gamma\alpha_h\alpha_l} < 0$$

so that

$$\frac{\delta(p^*_{1h} - p^*_{1l} - (p^*_{2h} - p^*_{2l}))}{\delta\beta} = -\frac{1}{3\beta^2\gamma\alpha_l} < 0$$

■

Proof of Lemma 2

As long as $\frac{\alpha_l}{\alpha_h} \geq \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$,

$$\frac{\delta(p_{2h} - p_{2l})}{\delta c} = -\frac{(1-\beta)((1-\gamma)\alpha_h + \gamma\alpha_l)}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} < 0$$

and

$$\frac{\delta(p_{1h} - p_{1l})}{\delta c} = \frac{(1-\beta)((1-\gamma)\alpha_h + \gamma\alpha_l)}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} > 0$$

so that,

$$\frac{\delta(p_{1h} - p_{1l} - (p_{2h} - p_{2l}))}{\delta c} = \frac{2(1-\beta)(1-\gamma)\alpha_h + \gamma\alpha_l}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} > 0$$

Also, note that

$$\frac{\delta p_{1l}}{\delta c} = \frac{-(1-\beta)(1-\gamma)\alpha_h + 3\beta\gamma\alpha_l}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} > 0$$

and

$$\frac{\delta p_{2l}}{\delta c} = \frac{-(1-3\beta+2\beta^2)(1-\gamma)\alpha_h + 6\beta\gamma\alpha_l}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} > 0$$

as long as α_l is high enough. Finally,

$$\frac{\delta p_{1h}}{\delta c} = \frac{(1+2\beta)\gamma\alpha_l}{9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h} > 0$$

■

Proof of Proposition 2

Naive high type consumers decide to purchase from seller 1 when

$$u - a - \theta - \alpha_h p^*_{1l} \geq u - a - (1-\theta) - \alpha_h p^*_{2l} \tag{17}$$

$$\text{or, } \theta \leq \theta_n = \frac{1}{2} + \frac{\alpha_h(p^*_{2l} - p^*_{1l})}{2}$$

However, as these consumers buy the add-on too, this decision is optimal *ex post* only when

$$u - \theta - \alpha_h p^*_{1h} \geq u - (1-\theta) - \alpha_h p^*_{2h} \tag{18}$$

$$\text{or, } \theta \leq \theta_s = \frac{1}{2} + \frac{\alpha_h(p^*_{2h} - p^*_{1h})}{2}$$

As seller 1 sells a more expensive add-on,

$$p_{1h} - p_{1l} > p_{2h} - p_{2l}$$

or,

$$\frac{1}{2} + \frac{\alpha_h(p^*_{2h} - p^*_{1h})}{2} < \frac{1}{2} + \frac{\alpha_h(p^*_{2l} - p^*_{1l})}{2}$$

i.e. $\theta_n > \theta_s$. Thus, buying the bundle from seller 1 is optimal for a consumer if and only if $\theta < \theta_s$, but there is a mass of consumers located in the interval $[\theta_s, \theta_n]$ who also buy from seller 1. This mass is given by

$$(1-\gamma)(1-\beta)(\theta_n - \theta_s) = (1-\gamma)(1-\beta)\alpha_h(p_{1h} - p_{1l} - (p_{2h} - p_{2l})) = \frac{(1-\gamma)(1-\beta)^2\alpha_h}{3\beta\gamma\alpha_l}$$

This mass shrinks in size when the gap between the add-on prices shortens, as in the case where c decreases (from Lemma 2) or β increases (from Proposition 1).

■

Proof of Proposition 3

The platforms's revenue is

$$\pi_P = \frac{3\beta\gamma\alpha_l(3 + 8\bar{c}\gamma\alpha_l) - 2(1-\gamma)\alpha_h((1-\beta)^2 - 12\bar{c}\beta\gamma\alpha_l)}{24\beta\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)}$$

Differentiating the above with respect to β, we get

$$\frac{\delta\pi_P}{\delta\beta} = \frac{(1-\beta^2)(1-\gamma)\alpha_h}{12\beta^2\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0$$

Similarly, differentiating c in (11) with respect to β,

$$\frac{\delta c}{\delta\beta} = \frac{(1-\beta^2)(1-\gamma)\alpha_h}{3\beta^2\gamma\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)} > 0$$

Further, recall from Proposition 1 that

$$\frac{\delta(p_{2l} - p_{1l})}{\delta\beta} = -\frac{(1-\gamma)\alpha_h}{3\gamma\alpha_l((1-\gamma)\alpha_l + \gamma\alpha_l)} < 0$$

$$\frac{\delta(p_{2h} - p_{1h})}{\delta\beta} = \frac{(1-\beta^2)(1-\gamma)\alpha_h + \gamma\alpha_l}{3\beta^2\gamma\alpha_l((1-\gamma)\alpha_l + \gamma\alpha_l)} > 0$$

so that

$$\frac{\delta D_2}{\delta\beta} = \frac{3(1-\beta^2)(1-\gamma)\gamma(c-\bar{c})\alpha_l\alpha_h((1-\gamma)\alpha_h + \gamma\alpha_l)}{(9\beta\gamma\alpha_l - 2(1-\beta)^2(1-\gamma)\alpha_h)^2} > 0$$

as long as $\frac{\alpha_l}{\alpha_h} \geq \frac{2(1-\beta)^2(1-\gamma)}{9\beta\gamma}$. Finally,

$$\frac{\delta\pi_1}{\delta\beta} = \frac{(1-\gamma)((7 - 7\beta^2 + 15\gamma - 18\gamma^2)\alpha_h^2 + 3(12\gamma - 5)\gamma\alpha_h\alpha_l - 18\gamma^2\alpha_l^2)}{36\beta^2\gamma\alpha_h\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)}$$

The denominator of the expression above is always positive, while the numerator is a quadratic in β with a negative leading coefficient, $-7\alpha_h^2(1-\gamma)$, i.e. it opens downwards. One can show that the expression attains its maximum at $\beta = 0$, where it equals $(7\alpha_h + 6\gamma(\alpha_l - \alpha_h))(\alpha_h(1+3\gamma) - 3\gamma\alpha_l)$ which is negative as long as α_l is large enough. In this case, $\frac{\delta\pi_1}{\delta\beta} < 0$ for all β. Similarly,

$$\frac{\delta\pi_2}{\delta\beta} = \frac{(1-\gamma)(-5 + 5\beta^2 + 21\gamma - 18\gamma^2)\alpha_h^2 + 3\gamma(-7 + 12\gamma)\alpha_h\alpha_l - 18\gamma^2\alpha_l^2)}{36\beta^2\gamma\alpha_h\alpha_l((1-\gamma)\alpha_h + \gamma\alpha_l)}$$

The denominator of the expression above is always positive, while the numerator is a quadratic in β with a positive leading coefficient, $5\alpha_h^2(1-\gamma)$, i.e. it opens upwards. At $\beta = 0$, the value is $-\left(6\gamma(\alpha_l - \alpha_h) + 5\alpha_h\right)\left(\alpha_h + 3\gamma(\alpha_l - \alpha_h)\right) < 0$. Similarly, at $\beta = 1$, the value is $-3\gamma\left(\alpha_l - \alpha_h\right)\left(7\alpha_h + 6\gamma(\alpha_l - \alpha_h)\right) < 0$. Thus, $\frac{\delta\pi_2}{\delta\beta} < 0$ for all β.

■