Sustainable Economic Growth in an Economy with Exhaustible Resources and a Declining Population under the Balance-of-Payments Constraint

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Abstract

This study builds a growth model and theoretically investigated the effects of the depletion of resources, as well as an increase or decrease in population, on the growth rate of per capita consumption in an open economy that trades with the rest of the world. We specifically consider an open economy where final goods are produced with capital, labor, exhaustible resources, and imported intermediate goods. We examine two cases. In one case, the input ratio of exhaustible resources is fixed while in the other case, it is endogenously determined. In both cases, as long as the combinations of the parameters are confined within a specific range, the long-term growth rate of per capita consumption is positive, irrespective of whether the population growth rate is positive or negative. Comparing the case where the input ratio of exhaustible resources is fixed with the case where it is endogenized, in the latter case, the long-term growth rate of per capita consumption is more likely to be positive..

Keywords: exhaustible resources; population decline; international trade; balance-of-payments constraint; sustainable growth

JEL Classification: J11; O13; O41; Q32

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1 Introduction

Various studies have been conducted to shed light on sustainable and long-term economic growth. For this study, we built an economic growth model and investigated how the depletion of natural resources and population growth/decline affect the long-term growth of per capita real consumption in an open economy. Our study simultaneously deals with the relationship between population and growth, that between resources and growth, and that between international trade and growth.

Solow's (1956) neoclassical growth model, which is the basis for many economic growth models, indicates that the long-term growth rate of per capita income is ultimately determined by technological progress. However, in the neoclassical growth model, the technological progress rate is given exogenously. Hence, decisive factors for technological progress are not considered. Afterward, one of the important contributions of endogenous growth models, Romer (1990), invented a research and development (R&D) model and explained how technological innovation is born through the profit motives of firms and is diffused, leading to perpetual economic expansion. In the Romer model, an increase in the employment share of the R&D sector increases technological progress, thereby expanding per capita income growth. By contrast, Jones (1995) examined evidence that in the U.S., although the number of researchers increased, the growth rate of per capita income did not. He pointed out that the Romer model does not appropriately describe the reality of innovation. Regarding this, the Romer model has a shortcoming such that an increase in the scale of population leads to an increase in the per capita income growth rate, which seems to contradict reality. Jones (1995) improved the specifications of the technological progress function and presented a semi-endogenous growth model with no scale effects. In the Jones model, a rise in the growth rate of population (not the level of population) increases the long-term growth rate of per capita income. Since then, many theoretical and empirical studies have been conducted as to whether the Romer (1990) or Jones (1995) model is more realistic; that is, whether scale effects exist or not.[1]

Meadows et al. (1972) rang the alarm bell that if the population grew at its then-current rate in 1972 and if environmental pollution progressed, the depletion of natural resources and the deterioration of the environment would limit growth within 100 years. On the other hand, Stiglitz (1974) built a growth model to show that sustainable development is possible with the depletion of natural resources and population growth because technological progress alleviates the resource constraint; as such, Stiglitz disagreed with the limits of growth proposed by Meadows et al. (1972). In Stiglitz's (1974) model, final goods production requires

[1] For scale effects of economic growth models, see Jones (1999). For an empirical study of scale effects, see Ziesmer (2020).

capital, labor, and exhaustible resources, and the engine of growth is technological progress. Groth and Schou (2002), like Stiglitz (1974), built a growth model that describes a situation where final goods production requires capital, labor, and exhaustible resources. They also investigated the long-term consequences of economic growth. A major difference between their study and that of Stiglitz (1974) lies in the specifications of the production function. While Stiglitz (1974) assumed a constant returns to scale production function, Groth and Schou (2002) assumed an increasing returns to scale production function. They proved that even if the economy faces a resource constraint, its long-term growth rate of per capita output becomes positive, provided that the population growth rate is positive and the production function exhibits increasing returns with regard to capital and labor. Note that the latter condition concerning the production function is stronger than the assumption of increasing returns to scale.

Nowadays, globalization has progressed, and as part of it, international trade has flourished due to the advancements of transportation technology. Bardhan and Lewis (1970), in their classical work, explored how the economic growth rate and terms of trade are determined in a situation where an open economy is subject to the balance-of-payments constraint. They built a growth model in which final goods production requires capital, labor, and imported intermediate goods, and the firm's production function exhibits constant returns to scale. An important characteristic of their study is that the exports of the domestic country are constrained by the growth rate of the rest of the world.[2] Their model does not assume technological progress. In contrast, Christiaans (2003) introduced technological progress due to learning by elaborating on Bardhan and Lewis's model and scrutinizing the relationship between the growth rate of per capita output and that of population. Christiaans's model is a semi-endogenous growth model that considers international trade. He revealed that in some cases, population growth has a positive effect on per capita income growth, but in other cases has a negative effect.

Japan began to experience a population decline in 2005 that has only continued to fall. Some might think that few counties are experiencing a population decline. However, if we use the rate of "natural increase" as a measure of population change, surprisingly, many countries have been witnessing a population decrease. The rate of natural increase is the difference between the number of births and the number of deaths over a period of time, which removes the effect of immigration. According to *World Population Prospects* published by the United Nations, many countries and regions are expected to experience population declines in the future.

Nevertheless, few studies have considered population decline in terms of economic

[2]For growth models with the export constraint, see Ziesmer (1995) and Ziesmer and Hallonsten (2019).

growth.[3] Christiaans (2011) is one of the few exceptions and showed that in the Solow growth model, with an increasing returns to scale production function due to the positive externality of capital accumulation, the long-term growth rate of per capita output can be positive, even if the population growth rate is negative.[4]

Sasaki (2015) built a small open economy growth model with agricultural and manufacturing sectors with negative population growth and explored the relationship between trade patterns and economic development. Sasaki and Hoshida (2017) introduced negative population growth into Jones' (1995) semi-endogenous growth model and investigated the long-term growth rate of per capita output. These previous studies indicate that even in a situation of negative population growth, the long-term growth rate of per capita output can be positive.

Jones (2020) also incorporated negative population growth into an endogenous growth model. He built a growth model whose growth engine is the technological progress of firms' R&D activity, and examined two cases: In one case, an endogenously determined population growth rate is positive and in the other, it is negative. In the former steady state, population, knowledge, and living standards continue to rise exponentially, whereas in the latter, population continues to decline and knowledge and living standards stagnate. Jones (2020) emphasized technological progress due to knowledge production and did not consider capital accumulation. On the contrary, as we mentioned above, Christiaans (2011), Sasaki (2015), and Sasaki and Hoshida (2017) stressed capital accumulation.

As stated above, since there are few studies on population decline, there are also few studies that have considered population decline and exhaustible resources simultaneously. Sasaki (2021) extended the work of Stiglitz (1974) and Groth and Schou (2002), and investigated whether the long-term growth rate of per capita output can be positive when population continues to fall at a constant rate in an economy where final goods production requires capital, labor, and exhaustible resources. He revealed that even in a population declining economy, per capita output can attain sustainable growth as long as the population declining rate and input ratio of exhaustible resources satisfy certain conditions. Sasaki and Mino (2021), using Hotelling's (1931) rule, elaborated on the work of Sasaki (2021) and

[3]Ritschl (1985) indicated that in the standard Solow model, a negative savings rate is necessary for the steady state to exist.

[4]Sasaki (2019) found that in the Solow growth model with a CES production function if the elasticity of substitution between capital and labor is less than unity and the population growth rate is negative, exogenous technological progress is necessary for the long-term growth rate of per capita output to be positive. When the production function is the Cobb-Douglas type and the population growth rate is negative, the long-term growth rate of per capita output can be positive without technological progress, because the capital deepening effect strongly works. In contrast, when the elasticity of substitution is less than unity, this capital deepening effect is weak; hence, technological progress is required for the sustainable growth of per capita output. Christiaans (2017) considered negative population growth in a two-sector growth model.

endogenized the input ratio of exhaustible resources that is fixed in the research of Sasaki (2021). Also, in this case, the growth rate of per capita output can be positive even when the population growth rate is negative.[5]

The present study is the first to simultaneously consider the depletion of natural resources and population decline in an open economy. In the world of globalization, an economy becomes more connected with the rest of the planet. Hence, it is important to investigate the effects of the depletion of natural resources and population decline on economic growth in an open economy. We explored an economy in which final goods are produced by capital, labor, exhaustible resources, and imported intermediate goods, and the production function exhibits constant returns to scale. We introduced the positive externality effect due to capital accumulation, which leads to increasing returns to scale. Population is assumed to continue to expand at a positive or negative rate. Our model introduces endogenous technological progress and in addition, exhaustible resources into Bardhan and Lewis's (1970) model. Further, our model investigates how the depletion of natural resources and population growth/decline affect the long-term growth rate of per capita real consumption.

As long as the combination of the population growth rate and the input ratio of exhaustible resources is confined within some range, the long-term growth rate of per capita consumption is positive, irrespective of whether the population growth rate is positive or negative. In comparing a scenario where the input ratio of exhaustible resources is fixed with a situation where it is endogenized by the use of Hotelling's rule, we found that in the latter case, the long-term growth rate of per capita consumption is more likely to be positive.

The remainder of this paper is organized as follows. Section 2 presents our model. Section 3 outlines the long-term growth rate of per capita real consumption and establishes the conditions in which it is positive. Section 4 endogenizes the input ratio of exhaustible resources by using Hotelling's rule. Section 5 examines the conditions in which the long-term growth rate of per capita real consumption is positive under Hotelling's rule. Section 6 presents a numerical example based on the data for the Japanese economy because Japan experiences steady population decline since 2010. Section 7 concludes the paper.

[5] Mino and Sasaki (2021) built on the work of Sasaki (2021) and Sasaki and Mino (2021), and endogenized the household's savings rate by using dynamic optimization. Their results show that even when population growth is negative, balanced positive growth of per capita output and per capita consumption is possible, but for this situation to occur, we need to impose an unrealistically strong increasing returns to scale on the production function.

2 Model

Consider an economy in which final goods are produced with four production factors: capital, labor, imported intermediate goods, and exhaustible resources. A portion of produced goods is allocated to domestic consumption and investment, and the rest to foreign exports. A represented firm's production function takes the constant returns to scale Cobb-Douglas form, as follows:

$$X_t = A_t K_t^{\alpha} L_t^{\beta} M_t^{\gamma} R_t^{1-\alpha-\beta-\gamma}, \quad \alpha,\ \beta,\ \gamma \in (0,1),\ \alpha + \beta + \gamma < 1, \tag{1}$$

where X_t denotes total output, A_t signals total factor productivity, K_t is capital stock, L_t represents labor input, M_t indicates imported intermediate goods, and R_t refers to exhaustible resources. Time is denoted by t. All parameters are assumed to be larger than zero and smaller than unity.

Suppose that the total factor productivity increases along with the positive external effect due to capital accumulation.

$$A_t = K_t^{\phi}, \quad 0 < \phi < 1. \tag{2}$$

The restriction $0 < \phi < 1$ means that the external effect is not so large. The specification (2) is based on the work of Sasaki (2021) and other previous studies. Christiaans (2003) assumed that A_t is an increasing function of cumulative total output that leads to $\dot{A}_t = X_t$. In this study, by contrast, for ease of analysis, the level of A_t is an increasing function of K_t. Nevertheless, either specification produces a similar outcome.

Substituting (2) into (1), we can rewrite the production function as follows:

$$X_t = K_t^{\alpha+\phi} L_t^{\beta} M_t^{\gamma} R_t^{1-\alpha-\beta-\gamma}. \tag{3}$$

With regard to the parameters, we have $\alpha + \phi + \beta + \gamma + (1 - \alpha - \beta - \gamma) = 1 + \phi > 1$. Hence, the production function exhibits increasing returns to scale. In what follows, for population growth and semi-endogenous growth to be compatible, we assume the following restriction.

$$\alpha + \phi < 1. \tag{4}$$

This means that the capital elasticity of total output is less than unity. Instead, if we assume $\alpha + \phi \geq 1$, total output becomes infinity within finite time when the population growth rate is positive. Accordingly, this case has no economic meaning.

Suppose that labor is fully employed and that total labor force is equal to the total population. The growth rate of the total population is specified as follows:

$$\frac{\dot{L}_t}{L_t} = n \gtreqless 0. \tag{5}$$

The population growth rate is fixed over time and takes a positive or negative value.

Let S_t denote the stock of exhaustible resources. Then, S_t continues to decline over time because a part of S_t, which is represented by R_t, is used for final goods production. The dynamics of S_t is specified as follows:

$$\dot{S}_t = -R_t \quad \text{with} \int_0^\infty R_t\,dt \leq S_0 = \text{given} > 0, \tag{6}$$

where S_0 denotes the initial period endowment of exhaustible resources. Let $s_R \in (0, 1)$ refer to the input ratio of exhaustible resources at time t. Then, we have $R_t = s_R S_t$. As such, we obtain

$$\frac{\dot{S}_t}{S_t} = -\frac{R_t}{S_t} = -s_R = \frac{\dot{R}_t}{R_t}. \tag{7}$$

In this study, we investigate two cases: one in which s_R is fixed over time, and another in which s_R is an endogenous variable. When s_R is fixed, R_t continues to fall at a rate $-s_R < 0$. The assumption that s_R is fixed was also used by Jones and Vollrath (2003) and Sasaki (2021). For the endogenization of s_R, we employ the well-known method of Hotelling (1931); that is, Hotelling's rule.

We specify the export function as follows:

$$EX_t = p_t^{\eta} e^{\lambda t}, \quad \eta < 0,\ \lambda > 0, \tag{8}$$

where EX_t signals exports, p_t denotes the price of domestic goods in terms of imported intermediate goods, and λ indicates the growth rate of the rest of the world (ROW). The parameter $-\eta$ refers to the price elasticity of export demand. This export function is the same as that used by Bardhan and Lewis (1970) and Christiaans (2003).[6]

Following Bardhan and Lewis (1970) and Christiaans (2003), we assume that the trade balance will continue to be sustained over time. The trade balance condition is given by

$$p_t EX_t = M_t. \tag{9}$$

[6]Christiaans (2003) specifies the export demand function as $EX_t = p_t^{\eta} Y_{w,t}^{\varepsilon}$, where $Y_{w,t}$ denotes the income of the ROW and ε is the income elasticity of export demand. If $Y_{w,t}$ grows at a constant rate $\bar{\lambda} > 0$, then λ in equation (8) leads to $\lambda = \bar{\lambda}\varepsilon$.

Note that the price of imported intermediate goods is unity.

We investigate firms' profit maximization. Suppose the positive external effect due to capital accumulation is given for firms; that is, the Marshallian externality holds. Then, firms maximize their profits under perfect competition. From this, the value of marginal products of imported intermediate goods is equal to the price of imported intermediate products.[7]

$$p_t \frac{\partial X_t}{\partial M_t} = p_t \gamma \frac{X_t}{M_t} = 1. \tag{10}$$

From this, we obtain

$$M_t = p_t \gamma X_t. \tag{11}$$

This means that the ratio of the total amount of imported intermediate goods $1 \times M_t$ to total production $p_t X_t$ is γ and constant over time.

Since national income, in terms of imported intermediate goods, is equal to total output minus imported intermediate goods, we obtain

$$Y_t = p_t(1-\gamma)X_T. \tag{12}$$

This is nominal gross domestic product (GDP). From this, the ratio of the total amount of imported intermediate goods M_t to nominal GDP, Y_T is given by

$$\frac{M_t}{Y_t} = \frac{p_t \gamma X_t}{p_t(1-\gamma)X_t} = \frac{\gamma}{1-\gamma}, \tag{13}$$

which states that this ratio is constant over time.

We consider consumption and savings. Suppose that a fraction s of nominal GDP is spent on nominal savings and the rest $1-s$ on nominal consumption. Let C_t denote real consumption. Then, C_t is given by

$$C_t = \frac{(1-s)Y_t}{p_t} = (1-s)(1-\gamma)X_t. \tag{14}$$

[7]By using firms' profit maximization, we obtain each factor share of income as follows:

$$\begin{aligned} \text{labor share} &= \frac{\beta}{1-\gamma}, \\ \text{capital share} &= \frac{\alpha}{1-\gamma}, \\ \text{resource share} &= \frac{1-\alpha-\beta-\gamma}{1-\gamma}. \end{aligned}$$

These results indicate that all factor shares of income are constant over time.

Substituting $C_t = (1-s)(1-\gamma)X_t$ and $EX_t = M_t/p_t = \gamma X_t$ into the goods market equilibrium condition $p_tX_t = p_tC_t + p_tI_t + p_tEX_t$, we obtain gross investment $I_t = \dot{K}_t$, as follows:

$$\dot{K}_t = s(1-\gamma)X_t. \tag{15}$$

For ease of analysis, we abstract capital depreciation.[8]

Substituting equation (3) into equation (11) and solving the resultant expression for M_t, we obtain

$$M_t = \gamma^{\frac{1}{1-\gamma}} p_t^{\frac{1}{1-\gamma}} K_t^{\frac{\alpha+\phi}{1-\gamma}} L_t^{\frac{\beta}{1-\gamma}} R_t^{\frac{1-\alpha-\beta-\gamma}{1-\gamma}}. \tag{16}$$

Substituting equation (8) into equation (9), we obtain

$$M_t = p_t^{1+\eta} e^{\lambda t}. \tag{17}$$

Substituting equation (17) into equation and solving the resultant expression for p_t, we obtain

$$p_t = \gamma^{\frac{1}{\eta(1-\gamma)-\gamma}} K_t^{\frac{\alpha+\phi}{\eta(1-\gamma)-\gamma}} L_t^{\frac{\beta}{\eta(1-\gamma)-\gamma}} R_t^{\frac{1-\alpha-\beta-\gamma}{\eta(1-\gamma)-\gamma}} e^{-\frac{\lambda(1-\gamma)}{\eta(1-\gamma)-\gamma}t}. \tag{18}$$

Substituting equation (18) into equation (17), we obtain

$$M_t = \gamma^{\frac{1+\eta}{\eta(1-\gamma)-\gamma}} K_t^{\frac{(\alpha+\phi)(1+\eta)}{\eta(1-\gamma)-\gamma}} L_t^{\frac{\beta(1+\eta)}{\eta(1-\gamma)-\gamma}} R_t^{\frac{(1-\alpha-\beta-\gamma)(1+\eta)}{\eta(1-\gamma)-\gamma}} e^{-\frac{\lambda}{\eta(1-\gamma)-\gamma}t}. \tag{19}$$

Substituting equation (19) into equation (3), we obtain the following production function.

$$X_t = HK_t^a L_t^b R_t^c e^{dt}, \tag{20}$$

where the parameters of equation (20) are defined as follows:

$$H = \gamma^{\frac{(\eta+1)\gamma}{\eta(1-\gamma)-\gamma}} > 0, \tag{21}$$

$$a = (\alpha+\phi)\frac{\eta}{\eta(1-\gamma)-\gamma} > 0, \tag{22}$$

$$b = \beta\frac{\eta}{\eta(1-\gamma)-\gamma} > 0, \tag{23}$$

$$c = (1-\alpha-\beta-\gamma)\frac{\eta}{\eta(1-\gamma)-\gamma} > 0, \tag{24}$$

[8] Analytical results do not change so much even if we abstract capital depreciation. However, numerical results are affected by the introduction of the depreciation rate.

$$d = -\frac{\lambda\gamma}{\eta(1-\gamma)-\gamma} > 0. \tag{25}$$

The production function given by equation (20) can be regarded as the production function that endogenizes imports. The term e^{dt} corresponds to the exogenous progress rate, which is composed of the growth rate of ROW, the price elasticity of export demand $-\eta > 0$, and the share of imported intermediate goods to total output γ.

We examine the sizes of the parameters of equation (20). For b and c, we obtain $0 < b < 1$ and $0 < c < 1$. Next, for a, $a > 1$ is not likely to be $a > 1$. For $a > 1$ to hold, we need

$$(\alpha+\phi)\frac{\eta}{\eta(1-\gamma)-\gamma} > 1 \Longrightarrow (\alpha+\phi+\gamma-1)\eta < -\gamma. \tag{26}$$

Suppose that $\alpha + \phi + \gamma < 1$. Then, we have

$$\eta > -\frac{\gamma}{\alpha+\phi+\gamma-1} > 0. \tag{27}$$

However, this contradicts with $\eta < 0$. On the other hand, since α, ϕ, and γ are small, $\alpha + \phi + \gamma > 1$ is not likely to hold. In the following analysis, we assume the following restriction:

$$\alpha + \phi + \gamma < 1. \tag{28}$$

Under this restriction, we obtain $a < 1$. The restriction given by equation (28) is more stringent than that given by equation (4). This condition is satisfied under a realistically plausible parameter setting that will be given in Section 6.

3 Analysis of dynamics

This section analyzes the dynamics of our model. For this purpose, let the output capital ratio be $z_t = X_t/K_t$. The differential equation of z_t is given by

$$\dot{z}_t = z_t[s(1-\gamma)(a-1)z_t + bn - cs_R + d]. \tag{29}$$

This can be rewritten as

$$\dot{z}_t = z_t(Az_t + B), \tag{30}$$

where A and B are respectively defined as follows:

$$A = s(1-\gamma)(a-1) < 0, \tag{31}$$
$$B = bn - cs_R + d. \tag{32}$$

By investigating this differential equation of z_t, we obtain the dynamics of our model. Since $a < 1$, we obtain $A < 0$. For the sign of B, there are two possible cases:

$$n > \frac{cs_R - d}{b} \Longrightarrow B > 0, \tag{33}$$
$$n < \frac{cs_R - d}{b} \Longrightarrow B < 0. \tag{34}$$

These conditions suggest that it is likely to be $B > 0$ when the population growth is high, whereas $B < 0$ when it is low.[9]

Therefore, we consider the following two cases.

Case 1 $A < 0$ and $B > 0$.

Case 2 $A < 0$ and $B < 0$.

The results of the above two cases are displayed in Figure 1. The left figure corresponds to Case 1, that is, $B > 0$, while the right figure corresponds to Case 2, that is, $B < 0$. In either case, the economy stably converges to the respective steady state.

Let us consider economic welfare. Per capita GDP, in terms of consumption goods, is given by $y_t = Y_t/(p_t L_t)$, which we define as per capita real GDP. The growth rate of per

[9]This footnote briefly explains the meaning of the condition $B \gtrless 0$. According to the explanation of the export demand function by Christiaans (2003), the parameter λ is rewritten and

$$\lambda = \bar{\lambda}\varepsilon,$$

where $\bar{\lambda} > 0$ denotes the growth rate of the ROW and $\varepsilon > 0$ is the income elasticity of export demand. Using the above relationship, the condition $B \gtrless 0$ is rewritten as

$$\frac{\beta}{1-\gamma} \cdot n - \frac{1-\alpha-\beta-\gamma}{1-\gamma} \cdot s_R + \frac{\gamma}{1-\gamma} \cdot \frac{\varepsilon}{\mu} \cdot \bar{\lambda} \gtrless 0. \tag{35}$$

In words, the above condition can be written as follows:

$$\text{(Laboe share} \times \text{Population growth)} - \text{(Resource share} \times \text{Resource input ratio)}$$
$$+ \left(\text{Imported intermediates share} \times \frac{\text{Income elasticity of exports}}{\text{Price elasticity of exports}} \times \text{ROW's growth}\right) \gtrless 0.$$

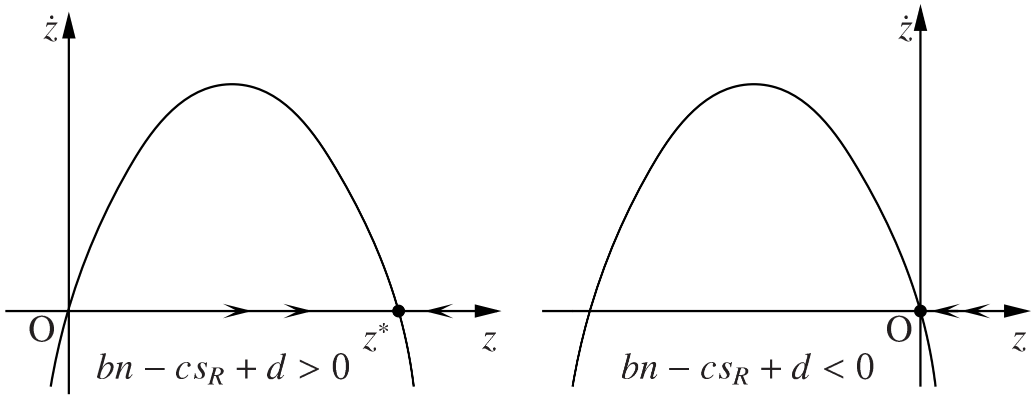

Figure 1: Convergence to the steady state

capita real GDP is given by

$$g_{y,t} = \frac{\dot{X}_t}{X_t} - n = as(1-\gamma)z_t + (b-1)n - cs_R + d. \tag{36}$$

The growth rate of the variable x_t is denoted by $g_{x,t}$. Next, we derive the growth rate of per capita real consumption $c_t = C_t/L_t$.

$$g_{c,t} \equiv \frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - n = \frac{\dot{X}_t}{X_t} - n = as(1-\gamma)z_t + (b-1)n - cs_R + d. \tag{37}$$

Therefore, the growth rate of per capita real consumption and that of per capita real GDP are equal. Substituting the corresponding steady state value into equation (37), we can obtain the long-term growth rate of per capita real consumption $g_{c,t}$.

3.1 Case 1: high population growth

In Case 1, z_t converges to the following value.

$$z^* = -\frac{B}{A} = \frac{bn - cs_R + d}{s(1-\gamma)(1-a)} > 0. \tag{38}$$

Substituting equation (38) into equation (37), the long-term growth rate of per capita real consumption in Case 1 is as follows:

$$g_c = \underbrace{\frac{1}{1-a}(bn - cs_R + d)}_{+} - n = \frac{1}{1-a}[(a+b-1)n - cs_R + d]. \tag{39}$$

Equation (39) means that the long-term growth rate of per capita real consumption is an increasing function of the population growth rate when $a + b > 1$; that is, the production function that endogenizes imports exhibits increasing returns as to both capital and labor. In contrast, it is a decreasing function when $a + b < 1$; that is, the production function displays decreasing returns. Moreover, equation (39) states that the long-term growth rate of per capita real consumption is a decreasing function of the input ratio of exhaustible resources, and that it is an increasing function of d.

In this study, we regard the population growth rate and the input ratio of exhaustible resources as important parameters. Then, in what follows, on the (n, s_R) plane, we examine the set such that $g_c > 0$ holds subject to $bn - cs_R + d > 0$.

The case of $bn - cs_R + d > 0$ is divided into two subcases: the case of $a + b > 1$ (Case 1-1) and the case of $a + b < 1$ (Case 1-2). Figure 2 corresponds to Case 1-1 where both $B > 0$ and $a + b > 1$ hold, while Figure 3 corresponds to Case 1-2 where both $B > 0$ and $a + b < 1$ hold. In these Figures, the line $g_p = 0$ corresponds to the border line at which the price of domestic goods does not change in the long run. At the domain above $g_p = 0$, we have $g_p > 0$ while at the domain below $g_p = 0$, we have $g_p < 0$ in the long run.[10]

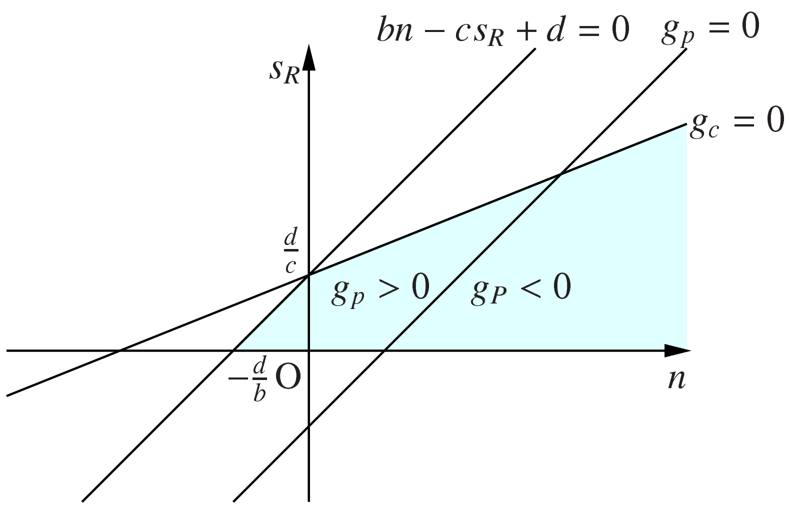

Figure 2: Set of (n, sR) such that $g_c > 0$ holds in Case 1-1

When $a + b > 1$, that is, the production function exhibits increasing returns as to both capital and labor, we can obtain $g_c > 0$ even if s_R is large as long as n is large. Moreover, we can obtain $g_c > 0$ even if n is negative as long as the absolute value of it is small. In Case 1-1, the larger n is and the smaller s_R is, the higher g_c is.

On the contrary, when $a + b < 1$, that is, the production function exhibits decreasing

[10]For details of the price change, see the Appendix.

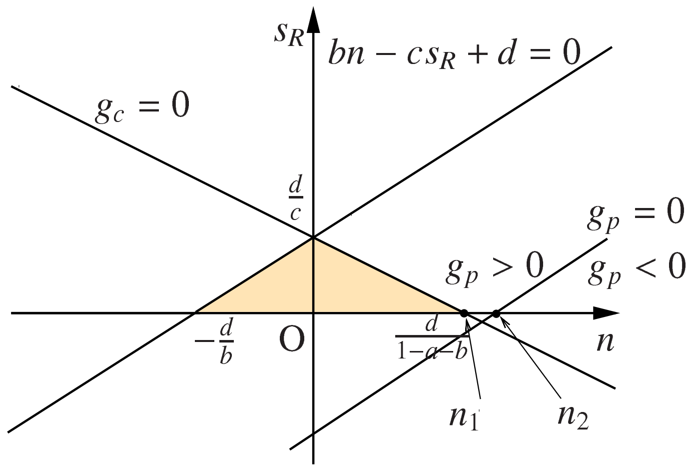

Figure 3: Set of (n, s_R) such that $g_c > 0$ holds in Case 1-2

returns as to both capital and labor, the set of (n, s_r) such that $g_c > 0$ holds, is narrow. Irrespective of whether n is positive or negative, we can obtain $g_c > 0$ but the set is narrow. In Case 1-2, the smaller n and s_R are, the higher g_c is.

3.2 Case 2: low population growth

In Case 2, z_t converges to zero in the long run.

$$z^{**} = 0. \tag{40}$$

Substituting $z^{**} = 0$ into equation (37), we obtain the long-term growth rate of per capita real consumption, as follows:

$$g_c = (b-1)n - cs_R + d = \underbrace{bn - cs_R + d}_{-} - n. \tag{41}$$

Equation (41) suggests that the long-term growth rate of per capita real consumption is a decreasing function of the population growth rate since $b < 1$. Accordingly, when $n < 0$ and its absolute value is large, $g_c > 0$ holds. Moreover, equation (41) indicates that the long-term growth rate of per capita real consumption is a decreasing function of the input ratio of exhaustible resources and an increasing function of d.

We examine the set of (n, s_R) such that $g_c > 0$ holds under the restriction of $bn - cs_R + d < 0$. Under this restriction, we obtain Figure 4. In this case, for $g_c > 0$ to hold, the population growth rate must be negative. Further, the smaller n and s_R are, the higher g_c is.

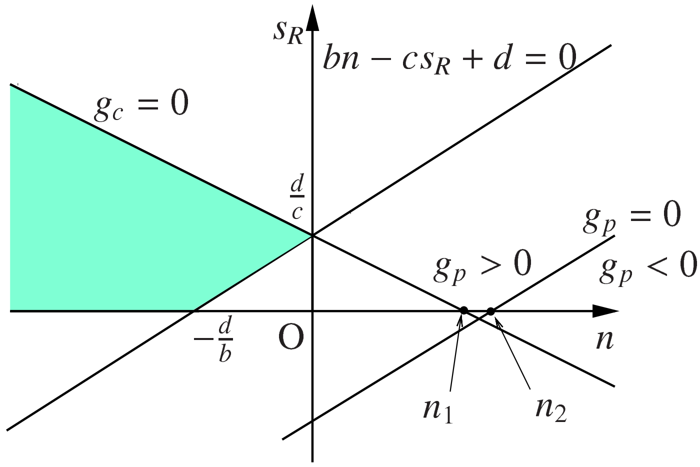

Figure 4: Set of (n, s_R) such that $g_c > 0$ holds in Case 2

From the above analysis, we develop the following proposition:

Proposition 1. *Consider an open economy in which final goods production requires capital, labor, imported intermediate goods, and exhaustible resources. Suppose that the production function exhibits increasing returns to scale. Moreover, suppose that the population growth rate and the input ratio of exhaustible resources are fixed over time. Then, irrespective of whether the population growth rate is positive or negative, the long-term growth rate of per capita real consumption is positive as long as the combination of the population growth rate and the input ratio of exhaustible resources is located within the appropriate domain.*

As seen above, when $B > 0$, we obtained different outcomes according to whether $a+b > 1$, that is, $\alpha + \beta + \gamma + \phi > 1$ (Case 1-1) or $a + b < 1$, that is, $\alpha + \beta + \gamma + \phi < 1$ (Case 1-2).

From equation (3), $\alpha + \beta + \gamma + \phi > 1$ or $\alpha + \beta + \gamma + \phi < 1$ corresponds to whether the production function exhibits increasing returns as to all capital, labor, and imported intermediate goods or decreasing returns. From equation (20), $a+b > 1$ or $a+b < 1$ corresponds to whether the production function that endogenizes imports manifests increasing returns as to both capital and labor or decreasing returns.

In the work of Groth and Schou (2002) and Sasaki (2021), for the long-term growth rate of per capita real consumption to be positive, the population growth rate must be positive when the production function exhibits increasing returns as to both capital and labor. In contrast, in our model, the long-term growth rate of per capita real consumption can be positive, even if the population growth rate is negative, as long as its absolute value is small.

In Sasaki (2021), who considered negative population growth in addition to positive population growth, for the long-term growth rate of per capita real consumption to be positive when the production function exhibits decreasing returns as to both capital and labor, the population growth rate must be negative. In contrast, in our model, the long-term growth rate of per capita real consumption can be positive, even if the population growth rate is positive, as long as its absolute value is small.

The differences between our study and prior ones are that our model takes international trade into account and hence, the parameter d has a similar effect to the exogenous technological progress. First, if $d = 0$, from equation (39), we have $g_c < 0$ under $n < 0$ when $a + b > 1$. In contrast, if $d > 0$, we have $g_c > 0$ under $n < 0$. Second, when $a + b < 1$, we have $g_c < 0$ under $n > 0$ if $n > 0$. However, if $d > 0$, we have $g_c > 0$ under $n > 0$. Thus, the parameter d, which captures the effect of international trade, has a similar effect to the exogenous technological progress; as such, per capita real consumption can continue to increase.

4 Endogenization of input ratio of exhaustible resources

Thus far, we have assumed that the input ratio of exhaustible resources is fixed over time. This corresponds to the analysis such that we regard s_R as a policy variable and determine the appropriate input ratio of exhaustible resources as a policy. Since the input ratio is fixed, it is not affected by other variables in the model. Therefore, the effect of declining population on the input ratio is not considered. However, if households own exhaustible resources as assets, they determine the amount of holding of exhaustible resources by comparing the return from resources with the return from another asset; that is, physical capital. This procedure is specified by Hotelling's (1931) rule. This section endogenously derives the input ratio s_R by using Hotelling's rule.

Let $p_{R,t}$ denote the price of exhaustible resources in terms of imported intermediate goods. Then, from the profit maximization of firms, we obtain the equalization between the value of the marginal product of exhaustible resources and their price.

$$p_t(1 - \alpha - \beta - \gamma)\frac{X_t}{R_t} = p_{R,t}. \tag{42}$$

Let $R_{K,t}$ denote the rental price of capital in terms of imported intermediate goods. Then, from the profit maximization of firms, we obtain the equalization between the value of the marginal product to capital and the rental price of capital.

$$p_t\alpha\frac{X_t}{K_t} = R_{K,t}. \tag{43}$$

Consider households' asset holdings. If capital stock and exhaustible resources are assets of households, then for both assets to exist at equilibrium, the holding of capital stock as an asset and the holding of exhaustible resources as assets must be indifferent. Accordingly, at equilibrium, from the no-arbitrage condition between the two assets, we obtain Hotelling's rule such that the rate of change in the real price of exhaustible resources is equal to the real rental price of capital stock, which is given by

$$\frac{d\log(p_{R,t}/p_t)}{dt} = \frac{R_{K,t}}{p_t} \Longrightarrow \frac{\dot{X}_t}{X_t} - \frac{\dot{R}_t}{R_t} = \alpha\,\frac{X_t}{K_t}. \tag{44}$$

From equation (44), we obtain the rate of change in R_t as follows:

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{X}_t}{X_t} - \alpha\,\frac{X_t}{K_t}. \tag{45}$$

Using equation (45), we obtain a system of differential equations as to z_t and $s_{R,t}$.

$$\dot{z}_t = \frac{s(1-\gamma)[\alpha-(1-c)]-\alpha c}{1-c}\,z_t^2 + \frac{bn+d}{1-c}\,z_t = \mathcal{A}z_t^2 + \mathcal{B}z_t, \tag{46}$$

$$\dot{s}_{R,t} = s_{R,t}^2 + \left\{-\frac{\alpha[1-s(1-\gamma)]}{1-c}\,z_t + \frac{bn+d}{1-c}\right\} s_{R,t} = s_{R,t}^2 + (\mathcal{C}z_t + \mathcal{B})s_{R_t}, \tag{47}$$

where the parameters b, c, and d are the same as those used in the model with fixed s_R. Equation (46) states that the differential equation of z_t depends on z_t itself and not on $s_{R,t}$. On the other hand, equation (47) states that the differential equation of $s_{R,t}$ depends on both $s_{R,t}$ and z_t.

We examine the coefficients of the differential equations. First, as to $\mathcal{A}$, we have $\mathcal{A} < 0$ if $\alpha < 1-c$. For realistic values of the parameters, c is rather small; hence, $\alpha < 1-c$ holds, which leads to $\mathcal{A} < 0$. We assume this condition, as outlined below.

$$\alpha < 1-c. \tag{48}$$

Second, since $0 < c < 1$, we have $\mathcal{C} < 0$. For $\mathcal{B}$, we have $\mathcal{B} > 0$ if $bn+d > 0$ while $\mathcal{B} < 0$ if $bn+d < 0$. Accordingly, depending on the size of the population growth rate, we can obtain either case.[11] Summarizing above discussions, we derive the following two cases:

Case A If $bn+d > 0$, we have $\mathcal{A} < 0$, $\mathcal{B} > 0$, and $\mathcal{C} < 0$.

[11]The meaning of condition $\mathcal{B} \gtrless 0$ can be explained as follows:

$$\frac{\beta}{1-\gamma}\cdot n + \frac{\gamma}{1-\gamma}\cdot\frac{\varepsilon}{\mu}\cdot\bar{\lambda} \gtrless 0, \tag{49}$$

Case B If $bn + d < 0$, we have $\mathcal{A} < 0$, $\mathcal{B} < 0$, and $C < 0$.

4.1 Analysis of dynamics in Case A

When $\mathcal{A} < 0$, $\mathcal{B} > 0$, and $C < 0$, we obtain the left phase diagram in Figure 5. If a unique steady state exists, it is a saddle point. Since z_t is a pre-determined state variable while $s_{R,t}$ is a jump variable, for given z_0, we can choose the initial value $s_{R,0}$ such that the economy is located on the saddle path that converges toward the steady state. Therefore, the steady state is saddle-path stable.

From the analysis of the phase diagram, the condition in which the unique steady state exists is given by $\mathcal{A} > C$, which leads to as follows:[12]

$$\alpha > s(1-\gamma). \tag{51}$$

Then, the steady state values of output capital ratio and the input ratio of exhaustible resources are respectively given by

$$z^* = -\frac{\mathcal{B}}{\mathcal{A}} > 0, \tag{52}$$

$$s_R^* = \mathcal{B}\left(\frac{C-\mathcal{A}}{\mathcal{A}}\right) > 0. \tag{53}$$

As the phase diagram shows, when the economy starts from z_0 that is higher than z^*, $s_{R,t}$ continues to declines and reaches the steady state value. On the other hand, when the economy starts from z_0 that is lower than z^*, $s_{R,t}$ continues to increase and reaches the steady state value.

4.2 Analysis of the dynamics in Case B

When $\mathcal{A} < 0$, $\mathcal{B} < 0$, and $C < 0$, the locus of $\dot{z}_t = 0$ coincides with the vertical axis. In this case, the steady state is a corner solution and saddle point. Like Case A, for given z_0, we can choose the unique initial value $s_{R,0}$, which is located on the saddle path that converges

In words, the above condition can be written as follows:

$$\text{(Labor share} \times \text{Population growth)} + \left(\text{Imported intermediates share} \times \frac{\text{Price elasticity of exports}}{\text{Income elasticity of exports}} \times \text{ROW's growth}\right) \gtrless 0. \tag{50}$$

[12]In the work of Sasaki and Mino (2021), who did not consider international trade, this condition is given by $\alpha > s$.

toward the steady state. Thus, the steady state is saddle-path stable. Then, the steady state values of the output capital ratio and the input ratio of exhaustible resources are respectively given by

$$z^{**} = 0, \tag{54}$$

$$s_R^{**} = -\mathcal{B} > 0. \tag{55}$$

As the phase diagram shows, $s_{R,t}$ continues to decline and reaches the steady state value. Hence, along the optimal path, the input ratio of exhaustible resources continues to fall and approaches a constant value.

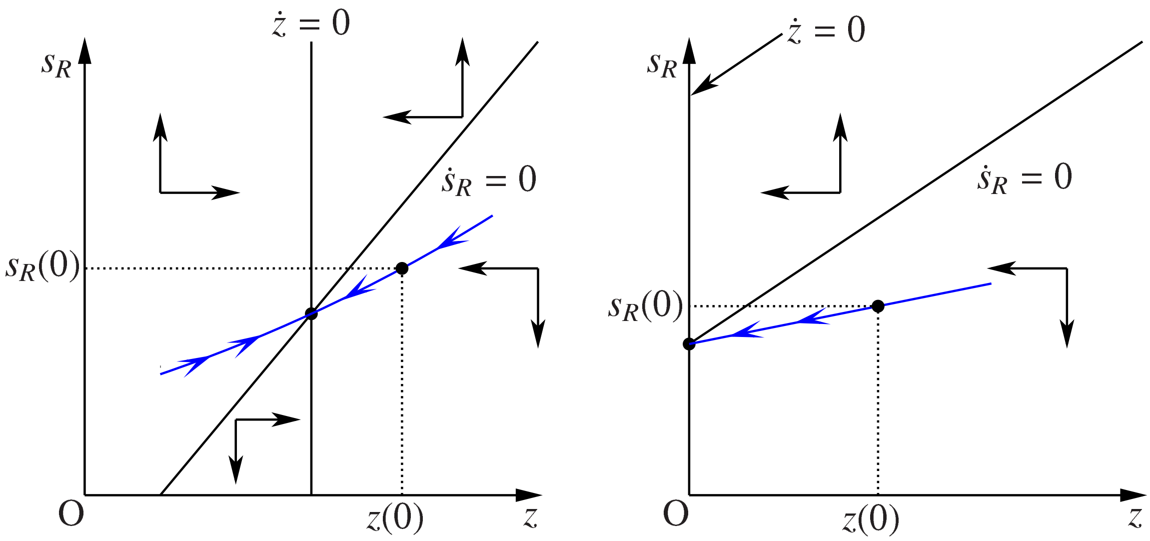

Figure 5: Phase diagram on (z, s_R)-plane

5 Per capita real consumption growth under Hotteling's rule

The growth rate of per capita real consumption under Hotteling's rule is given by

$$g_{c,t} = \frac{as(1-\gamma) + c\alpha}{1+c} z_t + \frac{b-1-c}{1+c} n + \frac{d}{1+c}. \tag{56}$$

We examine the condition in which $g_c > 0$ in Cases A and B in order.

5.1 Per capita real consumption growth in Case A

In Case A, we have $bn + d > 0$. Substituting $z^* = -\mathcal{B}/\mathcal{A} > 0$ into equation (56) and deriving the condition in which $g_c > 0$ holds, we obtain

$$g_c = \frac{[as(1-\gamma) + c\alpha](bn + d)}{s(1-\gamma)[(1-c) - \alpha] + \alpha c} - (1 - b + c)n + d > 0. \tag{57}$$

In rearranging this condition, we obtain

$$\begin{aligned}&\underbrace{\Big\{[as(1-\gamma) + c\alpha] + (b - c - 1)\Big\{s(1-\gamma)[(1-c) - \alpha] + \alpha c\Big\}\Big\}}_{\Omega} n \\ &\qquad + \underbrace{\Big\{as(1-\gamma) + c\alpha + s(1-\gamma)[(1-c) - \alpha] + \alpha c\Big\}}_{\Lambda} d > 0. \end{aligned} \tag{58}$$

$$\Longrightarrow \underset{-/+}{\Omega} n + \underset{+}{\Lambda} > 0. \tag{59}$$

Under the assumptions as to the sizes of the parameters, Λ is positive, and Ω is either positive or negative. Accordingly, we divide Case A into two subcases: Case A-1 with $\Omega < 0$, and Case A-2 with $\Omega > 0$.

5.1.1 Case A-1

When $\Omega < 0$, equation (58) can be rewritten as

$$n < \underbrace{-\frac{\Lambda}{\Omega}}_{+}. \tag{60}$$

The right-hand side of this condition is positive. In rearranging the condition $bn + d > 0$, we obtain

$$n > \underbrace{-\frac{d}{b}}_{-}. \tag{61}$$

The right-hand side of this condition is negative.

Accordingly, the growth rate of per capita real consumption is positive as long as the population growth rate is within the following interval.

$$\underbrace{-\frac{d}{b}}_{-} < n < \underbrace{-\frac{\Lambda}{\Omega}}_{+}. \tag{62}$$

This condition suggests that for the growth rate of per capita real consumption to be positive, the population growth rate must not be too high or too low.

5.1.2 Case A-2

When $\Omega > 0$, equation (58) leads to

$$n > \underbrace{-\frac{\Lambda}{\Omega}}_{-}. \tag{63}$$

The right-hand side of this condition is negative. We need to examine the size relationship between the right-hand side of equation (63) and that of $n > -d/b$. However, the combination of the parameters is very complicated; as such, analytical analysis is difficult. For example, if $\alpha = 0.27, \beta = 0.54$, $\gamma = 0.1$, $\phi = 0.3$, $\eta = -2$, $\lambda = 0.05$, and $s = 0.25$, then we have $\Omega 0$, $n > -d/b = -0.0046$, and $n > -\Lambda/\Omega = -0.135$. Therefore, if $n > -d/b = -0.0046$, the long-run growth rate of per capita consumption is positive.

5.2 Per capita real consumption growth in Case B

In Case B, $bn + d < 0$ holds. Substituting $z^{**} = 0$ into equation (56), we obtain

$$g_c = \frac{b - 1 - c}{1 + c} n + \frac{d}{1 + c} > 0. \tag{64}$$

In rearranging the above condition, we obtain

$$n < \frac{d}{1 - b + c}. \tag{65}$$

Comparing the right-hand side of $n < -(d/b)$ with that of equation (65), we find that the former is less than the latter. Accordingly, if the population growth rate contains the following inequality, the long-term growth rate of per capita real consumption is positive.

$$n < -\frac{d}{b} < 0. \tag{66}$$

In summarizing the above discussions, we obtain the following proposition.

Proposition 2. *Consider an open economy in which final goods production requires capital, labor, imported intermediate goods, and exhaustible resources. Suppose that the production function exhibits increasing returns to scale. Moreover, assume that the population growth rate is fixed over time, and that the input ratio of exhaustible resources is endogenously* *determined by Hotelling's rule. Then, irrespective of whether the population growth rate is positive or negative, the long-term growth rate of per capita real consumption is positive as long as the combination of the population growth rate is located within appropriate intervals.*

5.3 Summary

First, let us compare Case A with Case B. Suppose that $\alpha > s(1-\gamma)$ holds in both cases. The condition in which Case A is obtained is given by $\alpha > s(1-\gamma)$. In Case A, the lower limit of n, such that the growth rate of per capita real consumption is positive, is given by $-d/b$. In Case B, the upper limit of n, such that the growth rate of per capita real consumption is positive, is given by $-d/b$. In sum, as long as $\alpha > s(1-\gamma)$ holds, we can obtain $g_c > 0$ if $n < -\Lambda/\Omega$ when $\Omega < 0$, and we can obtain $g_c > 0$ no matter the value that n takes when $\Omega < 0$.

Second, let us compare the case where s_R is fixed and that where s_R is endogenized. In the endogenous s_R case, for a broader size of the population growth rate, the long-term growth rate of per capita real consumption is likely to be positive.

6 Numerical examples

This section presents numerical examples for the Japanese economy because Japan experiences steady population decline since 2010. Then, we calculate the long-run growth rate of per capita consumption and the input ratio of exhaustible resources. For this purpose, we need to set the parameters values based on actual data and existing studies.

For λ, we proceed as follows. The annual average growth rate of world GDP during 2000 and 2020 is 3.8%. According to Christiaans (2003), we have $\lambda = \bar{\lambda}\varepsilon$, where $\bar{\lambda}$ is the growth rate of world GDP and ε is the income elasticity of export demand. Then, we set $\bar{\lambda} = 0.038$. For ε, we use the estimate of Thorbecke and Salike (2018). They estimated the export demand function of the Japanese manufacturing by using the panel data. They state that the price elasticity of export demand is -0.31 and the income elasticity of export demand is 2.07. Accordingly, we set $\varepsilon = 2.07$ and $\eta = -0.31$. From this, we obtain $\lambda = \bar{\lambda}\varepsilon = 0.038 \times 2.07 = 0.07866$.

For the saving rate of households, we employ the empirical analysis of Unayama and Yoneda (2018). They calculated adjusted saving rates, and according to their results, the saving rate of households in Japan averages out around 10%. Therefore, we use $s = 0.1$.

For the population growth rate, we use the long-run economic statistics of Annual Report on the Japanese Economy and Public Finance 2021. The annual average rate of population decline in 2010–2020 is 0.19%, and hence, we have $n = -0.0019$.

For the labor share of income, we use the data for the System of National Accounts. The average value during 2000 and 2020 is around 0.6. For the capital share of income, we use 0.3.

For the input ratio of exhaustible resources, Nordhaus (1992) presents the value of 0.005. Table 12.1 of Hess (2016, p. 488) shows that the contribution of natural resources to GDP, that is, the rent share of total natural resources to GDP is 4.9%. Based on these studies, we use 0.005.

For the share of imported intermediate goods, we proceed as follows. Japanese Economy 2018-2019 published by the Cabinet Office states that the ratio of the value of imported intermediate goods to the value of imported final goods is around 1.5, and thus, the former is larger than the latter. Since the value of imported final goods is obtained by the System of National Accounts, we can calculate the value of imported intermediate goods as the value of imported intermediate goods = 1.5 × the value of imported final goods. Accordingly, the average value of the share if imported intermediate goods during 2000 and 2020 is 0.22, and therefore, we obtain $\gamma = 0.18$.

For the externality of capital accumulation, we use $\phi = 0.1$. Graham and Temple (2006) suggest $\phi = 0.3$, but it seems too high.

From the above statements, for the income shares, we obtain

$$\text{Imported intermediate goods share} = \frac{M}{Y} = \frac{\gamma}{1-\gamma} = 0.22, \tag{67}$$

$$\text{Labor share} = \frac{\beta}{1-\gamma} = 0.6, \tag{68}$$

$$\text{Capita share} = \frac{\alpha}{1-\gamma} = 0.3, \tag{69}$$

$$\text{Resource share} = \frac{1-\alpha-\beta-\gamma}{1-\gamma} = 0.098. \tag{70}$$

Existing studies report that resource share is around 0.03 and 0.05, and thus, the value of 0.098 may be too large. However, as an example, we proceed with 0.098.

Summarizing the above discussion, we use the following values of the parameters for the case where the input ratio of exhaustible resources is fixed.

$$\alpha = 0.25, \beta = 0.49, \gamma = 0.18, \phi = 0.1, n = -0.0019,$$
$$\eta = -0.31, \lambda = 0.079, s = 0.1, s_R = 0.005.$$

This corresponds to Case 1-2, in which the steady state value of the output capital ratio and the long-run growth rate of per capita consumption are as follows:

$$z^* = 0.52, \tag{71}$$

$$g_c = 0.044. \tag{72}$$

In the case where the input ratio of exhaustible resources is endogenously determined, the above parameter set corresponds to Case A-1, and we obtain

$$z^* = 0.45, \tag{73}$$

$$s_R^* = 0.076, \tag{74}$$

$$g_c = 0.05 \tag{75}$$

In both cases, g_c seems too high compared to the actual growth rate of per capita consumption. In Japan, it is calculated from the System of National Accounts and Annual Report on the Japanese Economy and Public Finance, and we obtain $g_c = -0.00031$ during the period of 2010 and 2020, and in additon, $g_c = 0.007$ during the period of 2000 and 2019 which excludes 2020 since the year is affected by COVID-2019.

7 Conclusion

We investigated whether sustainable growth of per capita consumption is possible in an economy that requires exhaustible resources and imported intermediate goods for final goods production when population growth is positive or negative. Our results show that irrespective of whether population growth is positive or negative, sustainable growth of per capita consumption is possible depending on the condition.

In the analysis, we considered two cases: one where the input ratio of exhaustible resources is a policy variable and hence fixed over time, and one where it is endogenously determined by Hotelling's rule, which considers households' asset choice. In either case, irrespective of whether population growth is positive or negative, sustainable growth of per capita consumption is possible depending on the condition, but in the endogenously determined input ratio case, the growth rate of per capita real consumption is likely to be positive for a broader population growth rate.

We endogenized the input ratio of exhaustible resources in the latter model, but from the viewpoint of analysis of the optimal growth path, we also need to endogenize the household savings rate. Introducing Ramsey-Cass-Koopmans' type of dynamic optimization into our model will be left for future research.

Appendix: Rate of change in price of domestic goods

In our model, the price of domestic goods continues to change in the long run. Specifically, depending on conditions, it continues to rise or fall at a constant rate. This outcome is similar to the results obtained by Bardhan and Lewis (1970) and Christiaans (2003).[13]

When trade is balanced, we have $p_t EX_t = p_t^{1-(-\eta)} e^{\lambda t} = M_t$. Thus, according to whether the price elasticity of export demand $(-\eta)$ is more than or less than unity, the effect of a change in the price of domestic goods on exports/imports is different. From the production function, we see that a rise in M_t has a positive effect on total output, while a decrease in M_t has a negative effect. When the price elasticity of export demand is more than unity, other things being equal, an increase in the price of domestic goods decreases imports and has a negative effect on total output, whereas a decline in the price of domestic goods increases imports and has a positive effect. On the other hand, if the price elasticity of export demand is less than unity, other things being equal, a rise in the price of domestic goods increases imports and has a positive effect on total output, whereas a drop in the price of domestic goods has a negative effect. Thus, a decline in the price of domestic goods is favorable for economic growth when the price elasticity is large, whereas an increase in the price is favorable when the price elasticity is small.

We investigate the rate of change in the price of domestic goods when the long-term growth rate of per capita real consumption is positive. In differentiating equation (18) with respect to time, we obtain the rate of change in p_t as follows:

$$g_{p,t} = \frac{(\alpha + \phi)s(1-\gamma)z_t + \beta n - (1 - \alpha - \beta - \gamma)s_R - \lambda(1-\gamma)]}{\eta(1-\gamma) - \gamma}. \tag{76}$$

Substituting the steady state value of z_t into equation (76), we obtain $g_{p,t}$ in the long run.

First, in Case 1, that is $B > 0$, the condition in which the price of domestic goods is

[13] The models presented by Wong and Yip (1999) and Sasaki (2008) have a similar characteristic: along the balanced growth path, the relative price continues to change at a constant rate. These models capture a small open economy; thus, the relative price is determined by the world market and exogenously given for a domestic country. By contrast, in our model, the relative price is determined by both domestic and foreign factors. If the price elasticity of export demand is infinity, our model is reduced to a small open economy model.

constant, that is, $g_p = 0$, is given by

$$s_R = \frac{b}{c}n + \underbrace{\frac{\frac{\lambda[\gamma-\eta(1-\gamma)](\alpha+\phi+\gamma-1)}{\eta(\alpha+\phi+\gamma-1)+\gamma}}{(\alpha+\phi)\frac{\eta(1-\alpha-\beta-\gamma)}{-\eta(\alpha+\phi+\gamma-1)-\gamma} + (1-\alpha-\beta-\gamma)}}_{-}. \tag{77}$$

This is a straight line whose slope is b/c and the intercept is negative on (n, s_R)-plane. This slope is the same as that of the straight line given by $B = 0$.

In Case 1-1 where $B > 0$ and $a+b > 1$ hold, we have $g_p > 0$ or $g_p < 0$, which is displayed in Figure 2. At the domain above $g_p = 0$, we have $g_p > 0$, while at the domain below $g_p = 0$, we have $g_p < 0$. Hence, in the long run, the price of domestic goods continues to decline when the population growth rate is high, while it continues to rise when the population growth rate is low.

In Case 1-2 where $B > 0$ and $a + b < 1$ hold, we always have $g_p > 0$, which is portrayed in Figure 3.[14] Therefore, irrespective of the size of the population growth rate, the price of domestic goods continues to rise in the long term.

Next, in Case 2 where $B < 0$ holds, the condition under which $g_p = 0$ is given by

$$s_R > \frac{b}{c}n - \underbrace{\frac{\lambda(1-\gamma)}{1-\alpha-\beta-\gamma}}_{-}. \tag{78}$$

This is a straight line whose slope is b/c and the intercept is negative. Hence, from Figure 4, we always have $g_p > 0$. Therefore, in the case of population decline, the price of domestic goods continues to rise when the long-term growth rate of per capita real consumption is positive.

Let us consider the relationship between the size of price elasticity of export demand and the growth rate of per capita real consumption. In Case 1-1, as Figure 2 indicates, we have $g_p < 0$ or $g_p > 0$. When $g_p < 0$, the price of domestic goods continues to decline, and

[14]In figures 3 and 4, n_1 denotes the value of n such that $g_c = 0$ line crosses the horizontal axis, and n_2 denotes the value of n such that $g_p = 0$ line crosses the horizontal axis. The size relationship between n_1 and n_2 is as follows. In Case 1-2 where $B > 0$ and $a + b < 1$ ($\alpha + \beta + \gamma + \phi - 1 < 0$) hold, we have

$$n_1 - n_2 = \lambda \frac{(\alpha+\beta+\gamma+\phi-1)[\eta(\alpha+\phi+\gamma-1)+\gamma]}{\beta[\eta(\alpha+\beta+\gamma+\phi-1)+\gamma]} < 0.$$

In Case 2 where $B < 0$ holds, we have

$$n_1 - n_2 = -\frac{\lambda(1-\beta-\gamma)[\eta(1-\gamma)-\gamma]}{\beta[\eta(1-\beta-\gamma)-\gamma]} < 0.$$

As such, in either case, n_2 is more than n_1.

the exports of the domestic country become larger as the price elasticity of export demand gets larger, which has a positive effect on the growth rate of per capita real consumption. In contrast, when $g_p > 0$, the price of domestic goods continues to rise, and the exports of the domestic country become smaller as the price elasticity of export demand grows larger, which has a negative effect on the growth rate of per capita real consumption. In Case 1-2 and Case 2, we have $g_p > 0$, which is the same as Case 1-1 with $g_p > 0$.

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