Generational Distribution of Fiscal Burdens: A Positive Analysis

Uchida, Yuki and Ono, Tetsuo

Seikei University, Osaka University

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Yuki Uchida[†]
Seikei University

Tetsuo Ono[‡]
Osaka University

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Abstract

This study presents a political economy model with overlapping generations to analyze the effects of population aging on fiscal policy formation and the resulting distribution of the fiscal burden across generations. We show that population aging incentivizes the government to raise the capital and labor income tax rates as well as the ratio of public debt to GDP; this result is consistent with the cross-country evidence of OECD countries. We then undertake a model-based simulation over the period 2000-2070 for Japan and the United States and show that Japan is anticipated to face higher labor income tax rates, a greater public debt-to-GDP ratio, and a lower government expenditure-to-GDP ratio than the United States throughout the entire period. Moreover, starting form 2040, Japan is predicted to surpass the United States in terms of the capital tax rate.

- Keywords: Generational burden, Overlapping generations, Political economy, Population aging, Public debt
- JEL Classification: D70, E24, E62, H60

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[†]Yuki Uchida: Faculty of Economics, Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo 180-8633, Japan. E-mail: yuchida@econ.seikei.ac.jp.

[‡]Tetsuo Ono: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: tono@econ.osaka-u.ac.jp.

1 Introduction

How is the burden of fiscal policy distributed across generations? How do demographic changes affect the pattern of generational burdens? To answer these questions, several studies have explored the political determinants of fiscal policy in overlapping generations models. Some examples are the works of Renström (1996), Beauchemin (1998), Boldrin and Rustichini (2000), Razin et al. (2004), and Razin and Sadka (2007), which are based on tractable models of the economy and voting process. Recently, Forni (2005), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Mateos-Planas (2008, 2010), Ono and Uchida (2016), and Bishnu and Wang (2017) studied taxation and public expenditure in a framework in which voting yields time-consistent policies. However, all of them assume a balanced government budget in each period, and thus ignore the possibility of a shift of fiscal burdens onto future generations via public debt issuance.

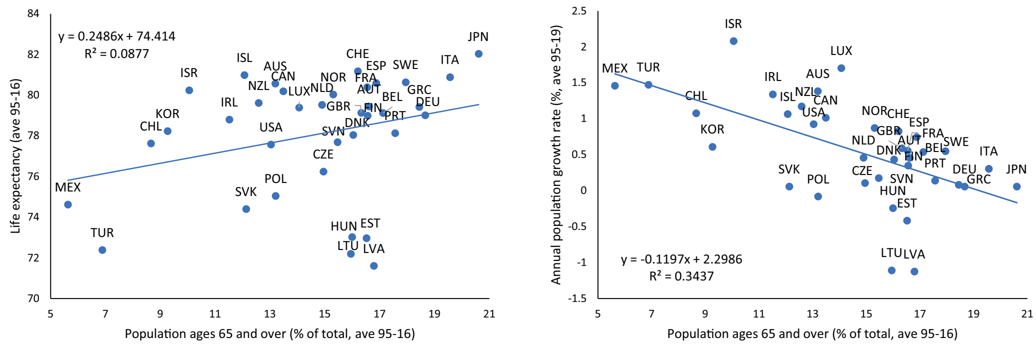
Several researchers, such as Cukierman and Meltzer (1989) as an early study and more recently Song et al. (2012), Müller et al. (2016), Röhrs (2016), Arawatari and Ono (2017), Arai et al. (2018), Ono and Uchida (2018), and Andersen (2019), have addressed the political economy of public debt. In these studies, labor income tax on the working generation is the only tax instrument; capital income tax on the retired, a possible additional instrument, is abstracted from the analyses. An exception is Arcalean (2018), who considered dynamic fiscal competition over public spending financed by labor and capital taxes and public debt and focused on the effects of fiscal cross-border externalities on welfare and growth.[1] In other words, these studies did not fully address the generational conflict over age-specific taxes and the resultant distribution of the fiscal burden across generations. However, as Mateos-Planas (2010) indicated, demographic changes, such as increasing longevity and declining birth rates, affect voters' interest in taxing different factors, and thus drive the change in the mix of capital and labor income taxes over time. Therefore, in considering the effects of demographic changes on the intergenerational distribution of fiscal burdens, a model that includes both labor and capital income taxes and public debt in a unified manner should be used.

1.1 Motivating Evidence

Figure 1 plots the cross-country average demographic data for Organization for Economic Cooperation and Development (OECD) member countries from 1995 to 2016. In each panel of the figure, the horizontal axis represents the share of the population aged 65 and over in total population, and the vertical axis represents life expectancy (Panel (a)) and annual population growth rates (Panel (b)). A positive association between life expectancy and the share of the population aged 65 and over in total population is observed in Panel (a) and a negative

[1] Uchida and Ono (2021) also considered this association, but limited the analysis to the case of inelastic labor supply and productive public expenditure that benefits only the young; therefore, they did not fully address the issue of the fiscal burden across generations. Instead, their analyses focused on the effects of debt ceilings on policy formation and economic growth.

association between annual population growth rates and the share in Panel (b). The evidence suggests that both an increase in life expectancy and a decrease in the population growth rate contribute to population aging although there are some exceptions to this trend, such as some Eastern European countries, as seen in Panel (a). Thus, we primarily employ the share of the population aged 65 and over in total population as an index of population aging, and consider the effects of each of the two aging factors through analysis.



(a) (b)

Figure 1: Each panel plots the data for OECD countries during 1995-2016. The horizontal axis represents the average share of the population aged 65 years and over in the total population. The vertical axis represents the average life expectancy (Panel (a)) and the average annual population growth rate (Panel (b)). Note: An equation in each panel shows the result of a simple linear regression analysis of the impact of x on y, where x and y represent the variables in horizontal and vertical axes, respectively, and R^2 shows the R-squared values of the linear regression. This also applies to Figure 2. Source: OECD.Stat (`https://stats.oecd.org/`) (accessed in April 6, 2021)

Figure 2 plots the cross-country average data for OECD member countries from 1995 to 2016 on the share of the population aged 65 and over and fiscal policy variables. In each panel, the horizontal axis represents the share of the population aged 65 and over in total population, and the vertical axis represents the labor income tax rate (Panel (a)), capital income tax rate (Panel (b)), public debt-GDP ratio (Panel (c)), and government expenditure-GDP ratio (panel (d)).[2] The numbers in parentheses in the estimation results are t-values. As the sample comprises 34 (Panels (a) and (c)), 33 (Panel (b)), and 35 (Panel (d)) countries, the t statistic follows a t distribution with 32, 31, and 33 degrees of freedom, and the critical value is 2.037, 2.040, and 2.035, respectively, at the 5% significance level.[3]

The results in Figure 2 show that the t-values of x are 2.100, 2.833, and 3.981 for the labor income tax rate, public debt-GDP ratio, and government expenditure-GDP ratio, respectively. The estimated coefficient values of 0.932, 4.713, and 0.645 are considered significant at the 5%

[2]The government expenditure is defined as the sum of government consumption expenditure and the general government gross-fixed capital formation. Appendix A.3 describes sources of data.

[3]The sample comprises 36 OECD member countries, excluding Colombia and Costa Rica that joined after 2020. Owing to missing data, the following countries are not included in the regression analysis: Chile and Iceland in Panel (a); Chile, Iceland, and Israel in Panel (b); Korea and New Zealand in Panel (c); and Iceland in Panel (d).

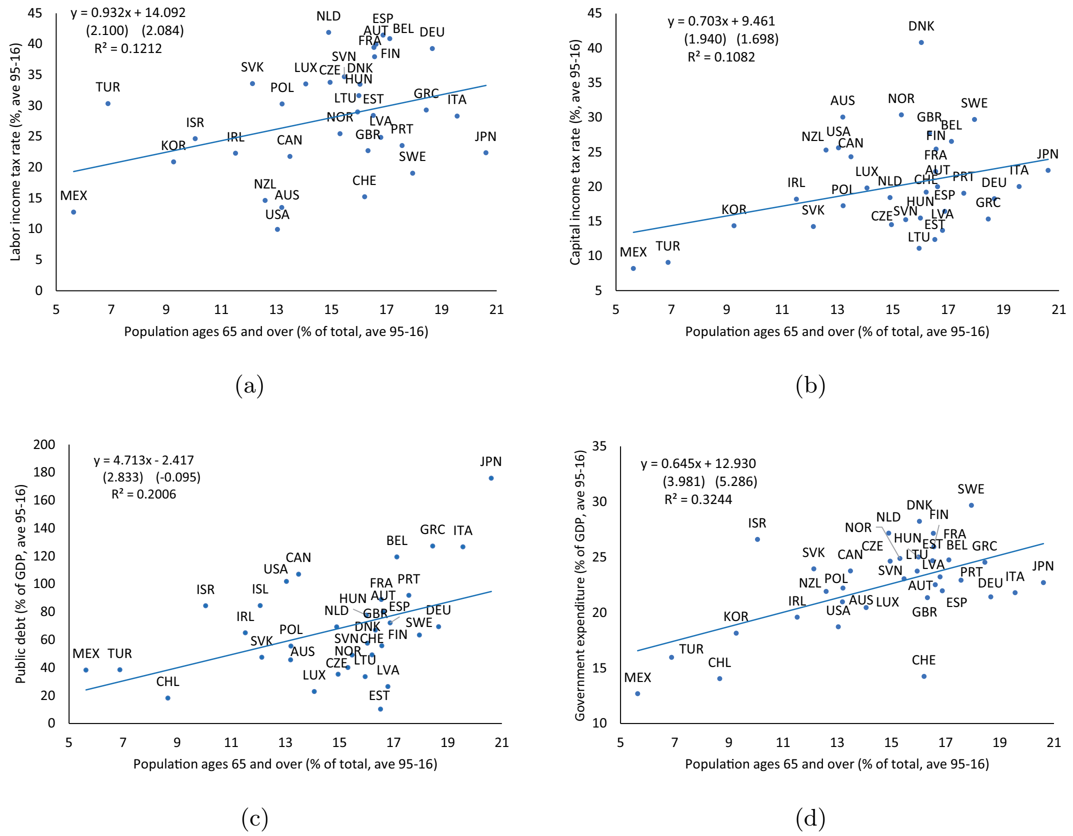


Figure 2: Each panel plots the data for OECD countries during 1995-2016. The horizontal axis represents the average share of the population aged 65 years and over in the total population. The vertical axis represents the average labor income tax rate (Panel (a)), the average capital income tax rate (Panel (b)), the average ratio of public debt to GDP (Panel (c)), and the average ratio of government expenditure to GDP (Panel (d)). In Panel (d), the government expenditure is defined as the sum of government consumption expenditure and the general government gross fixed capital formation. Each panel presents the OLS equation estimated results. The numbers in parentheses are t-values. Sources: OECD.Stat (`https://stats.oecd.org/`) (accessed on April 6, 2021); and Professor McDaniel's data archive (`https://www.caramcdaniel.com/research`) (accessed on April 6, 2021).

significance level. Regarding the capital income tax rate, the t-value is 1.940, suggesting that the results do not show significance at the 5% level, but the obtained t-value is close to the critical value at the 5% level and significant at the 10% level. Although there is a caveat to this finding, it can be generally concluded that population aging is associated with increases in the labor and capital income tax rates, public debt-GDP ratio, and government expenditure-GDP ratio in OECD member countries.[4]

The evidence from Figure 2 is contrary to intuition derived from a life-cycle model. Consider a two-period life-cycle model where workers pay labor income tax and the retired older adults

[4]A fresh econometric analysis would be necessary to determine the effects of population aging on the fiscal policy variables. For example, there could be omitted variables that correlated with old-age dependency ratios. As our main interest is in the theoretical and quantitative consideration of the impact of aging on fiscal policy through the political process, we do not concern ourselves much with econometric issues in this study.

pay capital income tax. Population aging strongly reflects the preferences of retired older adults in policy-making through an increase in their share of the vote. Thus, the natural prediction is that the labor income tax rate and public debt issuance increase while the capital income tax rate decreases with population aging. However, the evidence in Figure 2 indicates that countries with a higher share of the retired older adult in the population generally tend to have higher capital income tax rates, suggesting that the tax burden on all generations increases as the population ages. This counterintuitive evidence is the motivation for our study, and the objective is to present a model that explains this counterintuitive result.

1.2 Analysis and Result

We present an overlapping generation model in which individuals are identical within each generation and live in two periods, middle age and older adulthood. They work in middle age and retire in older adulthood but face uncertain lifetimes: they might die at the end of their middle age with a certain probability that represents their life expectancy. Public good expenditure, which benefits both the middle-aged and older adults, is financed by labor and capital taxes and public debt issuance. Following Song et al. (2012) and the subsequent literature, we assume probabilistic voting (e.g., Lindbeck and Weibull, 1987; Persson and Tabellini, 2002) in which fiscal policy in each period is determined to maximize the weighted sum of utility of the middle-aged and older adults. The weight of each generation depends on two demographic factors, population growth rate and life expectancy.

The aging demographic, marked by reduced population growth and increased longevity, significantly impacts forming fiscal policy toward older adults. Nonetheless, this policy also places a financial strain on middle-aged people, limiting their capacity to save. This leads to reduced physical capital formation and provision of public goods in the future. The reduction in physical capital then raises interest rates and intensifies the burden of debt repayment and capital income tax rates in the future. Previous research has utilized numerical approaches to analyze the intertemporal effects of aging on fiscal policy formation (e.g., Song et al., 2012; Lancia and Russo, 2016; Müller et al., 2016; Katagiri et al., 2020); however, this study provides closed-form solutions, allowing for an analytical examination of the impact of the two aging factors on policy formation.

Using the policy functions of the model, we investigate the effects of the two aging factors on fiscal policy formation and the resultant fiscal burdens across generations. We first consider the inelastic labor supply case and show that under certain conditions, the result is consistent with the evidence from Figure 2 for the labor income tax rate. We also show that the ratio of public debt to GDP increases as life expectancy increases: this result is also consistent with the evidence from Figure 2. However, the result shows that the ratio of public debt to GDP decreases as population growth rate declines, and that the capital income tax rate decreases as life expectancy increases and population growth rate declines. These findings are clearly inconsistent with the evidence in Figure 2.

To resolve some discrepancies between theory and evidence, we consider the elastic labor supply case based on a numerical approach. We calibrate the model to match the key statistics of the average OECD countries over the period 1995-2016. We investigate the effects of the two aging factors on fiscal policy and show that the ratio of public debt to GDP has a stronger tendency to increase in response to increased life expectancy than in the inelastic labor supply case. In addition, the negative effect of a declining population growth rate on the ratio, observed under inelastic labor supply, disappears if we assume elastic labor supply because the effect of reduced labor income tax burden from public debt issuance is strengthened by elastic labor supply. Overall, the two aging factors together work to increase the ratio of public debt to GDP more definitively in the elastic labor supply case than in the inelastic one.

Our study demonstrates that, under conditions of elastic labor supply, the rate of capital income tax follows a U-shaped pattern in relation to changes in population growth and life expectancy. As the population ages, the proportion of older adults in the voting population increases, resulting in a reduction of the capital income tax burden to align with their preferences. This accounts for one aspect of the U-shape. Conversely, elastic labor supply leads to a positive association between the two aging factors and the cost of debt repayment. As life expectancy increases, the cost of debt repayment increases because of a growth in public debt and an increase in interest rates caused by reduction in capital accumulation. Furthermore, a decline in the population growth rate also exacerbates the costs of debt repayment by elevating the labor income tax rate (which will be mentioned later), hindering capital accumulation, thereby raising interest rates. In response, the government may raise capital income tax rates to finance these increased costs. This constitutes the other side of the U-shape. The positive association observed on one side of the U-shape is unique to elastic labor supply and is supported by the data presented in Figure 2. Thus, elastic labor supply plays a crucial role in the relationship between aging and capital income tax rates, as shown in the findings in Figure 2.

For the labor income tax rate, we show that the tax rate increases as population growth rate declines while it exhibits an inverted U-shaped pattern against increased life expectancy. Combining the two effects yields a higher labor income tax rate associated with a higher proportion of the older adult population, which fits well with the OECD data observed. For the ratio of government expenditure to GDP, we find that both decline in population growth rate and increase in life expectancy lead to a decrease in the ratio. This result, which is contrary to the evidence observed in Figure 2, would be due to model specification and assumptions.[5] Despite this discrepancy between theory and evidence, the simulation results are generally consistent with the observations in Figure 2 regarding the impact of aging on the ratio of public debt to GDP and capital and labor income tax rates.

Based on the calibrated model, we undertake a model-based time series analysis predicting

[5]This point is further discussed in Section 4.

how fiscal policy changes over the period 2000-2070 as the population ages. We focus on two contrasting countries: Japan with the highest rate of population aging among OECD countries, and the United States with a higher population growth rate than other developed countries. Our results are threefold. First, capital and labor income tax rates are expected to rise in both countries in line with the projected future population aging. Second, the ratio of public debt to GDP is also expected to increase in both countries. Third, Japan is expected to experience a higher labor income tax rate, a higher ratio of public debt to GDP, and a lower ratio of government expenditure to GDP than the United States. Moreover, it is predicted that, compared with the United States, Japan will have a lower capital income tax rate for the initial few decades; however, it is projected that it will have a higher capital income tax rate than the United States in the long run. Thus, the model prediction suggests that Japan would pass on a higher tax burden to future generations than the United States.

1.3 Contribution to the Literature

Our study is closely related to Mateos-Planas (2010), who analyzed the impact of demographic changes on labor and capital income tax rates. Mateos-Planas (2010) reported that in the United States, the age of the median voter declined from 43.78 years to 40.74 years between 1965 and 1990, and that the labor income tax rate increased while the capital income tax rate declined accordingly. Based on the population projection that the age of the median voter increases from 40.74 years to 46.42 between 1995 and 2025, the author runs a model-based simulation and predicts a decrease in the labor income tax rate and an increase in the capital income tax rate between 1995 and 2025. The results suggest that population aging leads to a shift in the tax burden from the younger to the older generation.

Our study differs from Mateos-Planas (2010) in three aspects. First, the government can finance its expenditure through debt issues in addition to the labor and capital income tax rates. With this assumption, we can quantitatively measure the extent to which the aging of a society induces the postponement of the fiscal burden to future generations through the issuance of public debt. Second, we extend the scope of the projection to Japan and the United States and compare these countries with different stages of aging to examine the impact of aging on fiscal policy in detail. Third, in contrast to Mateos-Planas (2010), who limited the scope of his projections to the year 2025, our analysis extends the scope to 2070, which enables us to investigate the long-term impact of aging on fiscal policy.

This study is also related to recent theoretical contributions on fiscal politics. Razin et al. (2004), Razin and Sadka (2007), Bassetto (2008), Mateos-Planas (2008), and Lancia and Russo (2016) focused on the association between population aging and income taxation. They assumed a balanced government budget; therefore, they abstracted from public debt issuance. Song et al. (2012), Müller et al. (2016), and Katagiri et al. (2020) focused on public debt finance but they abstracted capital income taxation from their analysis. Thus, our study bridges the gap in the literature by comprehensively evaluating the effects of population aging on fiscal policy, including capital and labor income taxes as well as public debt issuance and the resulting fiscal burden on current and future generations.

Among the above-mentioned studies, our study is closely related to Müller et al. (2016) who took the small open economy model in Song et al. (2012) and extended it by introducing political shocks that shift voters' preference intensity for public goods provision. It follows Müller et al. (2016) in that we also assume disutility from labor effort following Greenwood et al. (1988). It differs from Müller et al. (2016) in that we assume a large economy in which general equilibrium effects through the interest rate are present, and we consider the effects of population aging, rather than political shocks, to fiscal policy formation. Within this extended framework, we derive a closed-form solution even when labor supply is elastic despite the Müller et al. (2016) argument that the model does not admit a closed-form solution in an elastic labor supply case. Thus, our work provides a technical contribution to the literature, and presents a possible method for addressing several points left unresolved in previous studies.

From another perspective, our study is closely related to Hassler et al. (2003, 2005, 2007) who derived closed-form solutions of policy functions consisting of multiple fiscal policy variables through majority voting. To obtain the solutions, they abstracted production and government deficit from their analysis and assumed a quadratic form of the utility function. In other words, they relied on a highly abstracted model to drive the closed-form solutions, limiting its application to quantitative evaluation. Conversely, our analysis uses standard utility and production functions commonly used in political economy analysis, albeit in a simplified two-period model. This makes it possible to apply existing methods of quantitative valuation, which is our contribution.

The study also contributes to the literature on time-consistent fiscal policy (Klein and Ríos-Rull, 2003; Klein et al., 2008; Martin, 2009, 2010; Ortigueira et al., 2012; Ortigueira and Pereira, 2022). In their frameworks with long-lived agents, the government chooses the Markov strategy in each period. Thus, current policies depend on payoff-relevant state variables. This study follows the policy strategy of these works, but differs from theirs by assuming a short-lived government representing only existing generations. Under this alternative assumption, we consider the conflict of interest among generations and its generational consequences.

The remainder of this paper is organized as follows. The next Section 2 presents the model. Section 3 presents the characterization of the political equilibrium and investigates the policy response to population aging under inelastic labor supply. Section 4 calibrates the model to match some key statistics from the OECD countries over the period 1995-2016, and investigates the effects of population aging on fiscal policy formation. It also undertakes a model-based simulation to run some quantitative experiments. Section 5 presents the concluding remarks. The Appendix provides the proofs of propositions and supplementary explanations of the analytical and numerical methods.

2 Model

The discrete time economy begins in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for two periods: middle age and older adulthood.[6] Individuals face uncertain lifetimes in the second period of life. Let $\pi \in [0,1]$ denote life expectancy (i.e., the probability of living in older adulthood). This is idiosyncratic for all individuals and constant across periods. Each middle-aged individual gives birth to $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1} = (1+n)N_t$.

Individuals

Individuals have the following economic behaviors over their life cycles. In middle age, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. In older adulthood, they retire, and receive and consume returns from savings.

Consider a middle-aged individual in period t. The individual is endowed with one unit of time. They supply it elastically in the labor market and obtain labor income $w_t l_t$, where w_t is wage rate per unit of labor and $l_t \in (0,1)$ is the amount of labor supply. After paying tax $\tau_t w_t l_t$, where τ_t is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings held as an annuity and invested in physical capital, s_t. Therefore, the period-t budget constraint for the middle-aged becomes

$$c_t + s_t \leq (1-\tau_t) w_t l_t. \tag{1}$$

The period $t+1$ budget constraint in older adulthood is

$$d_{t+1} \leq \left(1-\tau_{t+1}^K\right) \frac{R_{t+1}}{\pi} s_t, \tag{2}$$

where d_{t+1} is consumption, τ_{t+1}^K is the period-$t+1$ capital income tax rate, and R_{t+1} is the gross return from savings. If an individual dies at the end of the middle age, then their annuitized wealth is transferred to the individuals who live throughout old age via annuity markets. Therefore, the return from saving becomes R_{t+1}/π under the assumption of perfect annuity markets.

The preferences of a middle-aged individual in period t are specified by the following expected utility function:

$$\ln\left(c_t - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\pi\left(\mu \ln d_{t+1} + \theta \ln g_{t+1}\right),$$

where g_t is per-capita public goods in period t, $\beta \in (0,1)$ is a discount factor, $\theta(>0)$ is the degree of preferences for public goods, and $\mu\,(>0)$ is the preference weight for consumption relative to

[6] In conventional terminology, the first period of life is called youth. We refer to it as middle age instead of youth, because in Section 4, we introduce pre-employment youth into the model and extend it to a three-period version to apply the model for numerical analysis. This extension does not intrinsically affect the structure of the model because the young do not make any decisions; they depend on their parents.

public goods in old age.[7] Following Greenwood et al. (1988) and Müller et al. (2016), we assume that the disutility from labor effort is $(l_t)^{1+1/v}/(1+1/v)$, where $v(>0)$ parameterizes the Frisch elasticity of labor supply. This specification is called GHH preferences after Greenwood et al. (1988).

We substitute the budget constraints (1) and (2) into the utility function to form the unconstrained maximization problem:

$$\max_{\{s_t,l_t\}} \ln\left((1-\tau_t)w_t l_t - s - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\pi\left(\mu \ln\left(1-\tau^K_{t+1}\right)\frac{R_{t+1}}{\pi}s_t + \theta \ln g_{t+1}\right).$$

By solving this problem, we obtain the following labor supply and savings functions:

$$l_t = \left[(1-\tau_t)w_t\right]^v, \tag{3}$$

$$s_t = \frac{\beta\pi\mu}{1+\beta\pi\mu}\cdot\frac{1/v}{1+1/v}\left[(1-\tau_t)w_t\right]^{1+v}. \tag{4}$$

The labor supply and savings decrease as the labor income tax rate, τ_t, increases, but they increase as the wage rate, w_t, increases.

The three key assumptions about individuals are logarithmic utility, GHH preferences, and two-period life cycle. These assumptions follow Müller et al. (2016) who assumed a small open economy where the interest rate is provided. Conversely, this study assumes a large economy where the interest rate is endogenously determined through market clearing. Although the interest rate is endogenous to the model of this paper, the three assumptions allow us to derive closed-form solutions of the political equilibrium described later.

The role of the three assumptions in the analysis is as follows. First, the logarithmic utility allows the derivation of a savings function independent of the after-tax interest rate. In other words, the assumption enables us to obtain a savings function independent of the next-period capital income tax rate. Second, by assuming the GHH preferences, also employed in Battaglini and Coate (2008) and Azzimonti et al. (2016), we obtain a labor supply function independent of the after-tax interest rate.

Third, the two-period life cycle enables us to derive a labor supply function independent of asset holdings at the beginning of the first period. If an individual's lifetime lasts three or more periods, the labor supply in period $t(\geq 2)$ depends on the assets held at the beginning of period t. However, owing to the second assumption of the GHH preferences, effects of assets on labor supply are removed from the model. Thus, the second and third assumptions are closely connected to each other, and together with the first assumption, we can derive the labor supply and savings functions that enable us to derive closed-form solutions although labor supply and interest rate are both endogenous.

[7] Adding μ into the model helps in calibrating the model in Section 4. An alternative is to introduce $\lambda\,(>0)$ that represents a preference weight for public goods in older adulthood as in Song et al. (2012) and Müller et al. (2016). The alternative approach does not do as good a job of finding an empirically plausible set of parameters as the one adding μ on the utility of old-age consumption. Thus, we introduce the preference weight for old-age consumption rather than for old-age public goods into the model.

Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers and produce the final output Y_t with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A\,(K_t)^{\alpha}\,(L_t)^{1-\alpha}$. Here, $A(>0)$ is total factor productivity, which is constant across periods, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share in production.

In each period, a firm chooses capital and labor to maximize its profit, $A\,(K_t)^{\alpha}\,(L_t)^{1-\alpha} - R_tK_t - w_tL_t$, where R_t is the gross return on physical capital and w_t is the wage rate. The firm's profit maximization leads to

$$K_t : R_t = \alpha A\,(k_t)^{\alpha-1}\,(l_t)^{1-\alpha}\,, \tag{5}$$

$$L_t : w_t = (1-\alpha)A\,(k_t)^{\alpha}\,(l_t)^{-\alpha}\,, \tag{6}$$

where $k_t \equiv K_t/N_t$ is per-capita capital and $l_t \equiv L_t/N_t$ is per-capita labor. Capital fully depreciates in a single period.

Government budget constraint

Government expenditure is financed by both taxes on capital and labor income and public debt issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is $\tau_t w_t l_t N_t + \tau_t^K\,(R_t/\pi)\,s_{t-1}\pi N_{t-1} + B_{t+1} = R_tB_t + G_t$, where $\tau_t w_t l_t N_t$ is aggregate labor income tax revenue, $\tau_t^K R_t s_{t-1} N_{t-1}$ is aggregate capital income tax revenue, B_{t+1} is newly issued public debt, R_tB_t is debt repayment, and G_t is aggregate public expenditure. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the constraint by N_t, we can obtain a per- capita expression of the government budget constraint:

$$\tau_t w_t l_t + \frac{\tau_t^K R_t s_{t-1}}{1+n} + (1+n)b_{t+1} = R_t b_t + \frac{(1+n)+\pi}{1+n} g_t, \tag{7}$$

where $b_t \equiv B_t/N_t$ is per-capita debt and $g_t \equiv G_t/(N_t + \pi N_{t-1})$ is the per-capita public expenditure.

Capital market-clearing condition

Public debt is traded in the domestic capital market. The market-clearing condition for capital is $K_{t+1} + B_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged agents in period t, $N_t s_t$, to the sum of the stocks of aggregate physical capital and aggregate public debt at the beginning of period $t+1$. We can rewrite this condition as

$$(1+n)(k_{t+1} + b_{t+1}) = s_t. \tag{8}$$

Economic Equilibrium

Hereafter, we drop the usage of time subscripts and use the notation z' (e.g., k', b', g', τ', and $\tau^{K\prime}$) to denote the next period of z. To define an economic equilibrium in the present framework, we reduce the conditions (1) - (8) to a system of two difference equations, one representing the government budget constraint and the other representing the capital market-clearing condition, for two state variables, physical capital k and public debt b. To show this, consider the labor supply in (3), the savings in (4), and factor prices in (5) and (6). We write these as functions of physical capital, k, and the labor income tax rate, τ as follows:[8]

$$l = l(\tau, k) \equiv \left[(1-\tau)(1-\alpha)A\,(k)^{\alpha}\right]^{v/(1+\alpha v)}, \tag{9}$$

$$s = s\left(\tau, k, l(\tau, k)\right) \equiv \frac{\beta\pi\mu}{1+\beta\pi\mu}\cdot\frac{1/v}{1+1/v}\left[(1-\tau)w\left(k, l(\tau, k)\right)\right]^{1+v}, \tag{10}$$

$$w = w\left(k, l(\tau, k)\right) \equiv (1-\alpha)A\,(k)^{\alpha}\left[l(\tau, k)\right]^{-\alpha}, \tag{11}$$

$$R = R\left(k, l(\tau, k)\right) \equiv \alpha A\,(k)^{\alpha-1}\left[l(\tau, k)\right]^{1-\alpha}. \tag{12}$$

Using the labor supply function in (9) and the factor prices in (11) and (12), we can reformulate the government budget constraint in (7) in terms of the state variables, k and b, and the government policy variables, τ, τ^K, and g as follows:

$$TR(\tau, l(\tau, k), k) + TR^K\left(\tau^K, l(\tau, k), k, b\right) + (1+n)b' = R\left(k, l(\tau, k)\right)b + \frac{(1+n)+\pi}{1+n}g,$$

or,

$$\begin{aligned} b' &= b'\left(\tau, \tau^K, l(\tau, k), g, k, b\right) \\ &\equiv \frac{1}{1+n}\left[\frac{(1+n)+\pi}{1+n}g + R\left(k, l(\tau, k)\right)b - TR(\tau, l(\tau, k), k) - TR^K\left(\tau^K, l(\tau, k), k, b\right)\right], \end{aligned} \tag{13}$$

where we define $TR(\tau, l(\tau, k), k)$ and $TR^K\left(\tau^K, l(\tau, k), k, b\right)$, representing the tax revenues from labor and capital income, respectively, as follows:

$$TR(\tau, l(\tau, k), k) \equiv \tau w\left(k, l(\tau, k)\right) l(\tau, k),$$
$$TR^K\left(\tau^K, l(\tau, k), k, b\right) \equiv \tau^K R\left(k, l(\tau, k)\right)(k+b).$$

We can also reformulate the capital market-clearing condition in (8) as $(1+n)(k'+b') = s\left(\tau, k, l(\tau, k)\right)$, or:

$$k' = k'\left(\tau, \tau^K, g, l(\tau, k), k, b\right) \equiv \frac{1}{1+n}\cdot s\left(\tau, k, l(\tau, k)\right) - b'\left(\tau, \tau^K, l(\tau, k), g, k, b\right). \tag{14}$$

We should note that the next period physical capital and public debt, k' in (14) and b' in (13), respectively, depend on the current policies only, not on future ones. Thus, the economic equilibrium in the present framework is defined as follows:

[8] The derivation of (9) - (12) is as follows. First, we substitute (6) into (3) to write the optimal labor supply as a function of τ_t and k_t, as in (9). Second, we reformulate the saving function in (4) using (6) and (9), as in (10). Third, we use firms' profit maximization with respect to L_t in (6) and the labor supply function in (9) to obtain the labor market-clearing wage rate, as in (11). Finally, firms' profit maximization with respect to K_t in (5) and the labor supply function in (9) lead to (12).

Definition 1 Given current values of the policy variable, (τ, τ^K, g), an *economic equilibrium* is a mapping Ψ^{ECON} from the current state (k, b) to the next state (k', b'). That is,

$$(k', b') = \Psi^{ECON}\left(\tau, \tau^K, g, k, b\right). \tag{15}$$

The economic equilibrium is simply given by (13) and (14).[9] In the equilibrium, we can express the indirect utility of the current middle-aged, V^M, and that of the current older adults, V^O, as functions of policy variables, physical capital, and public debt. V^M becomes:

$$\begin{aligned} V^M\left(\tau, g, \tau', \tau^{K\prime}, g', k'(\cdot), k\right) = \ln\left[c\left(\tau, l(\tau,k), k\right) - \frac{(l(\tau,k))^{1+1/v}}{1+1/v}\right] + \theta \ln g \\ + \beta\pi\left[\mu \ln d'\left(\tau, \tau^{K\prime}, l(\tau,k), l(\tau', k'(\cdot)), k'(\cdot)\right) + \theta \ln g'\right], \end{aligned} \tag{16}$$

where $k'(\cdot)$ is defined in (14). We define $c\left(\tau, l(\tau,k), k\right)$ and $d'\left(\tau, \tau^{K\prime}, l(\tau,k), l(\tau', k'(\cdot)), k'(\cdot)\right)$, representing consumption in middle age and older adulthood, respectively, as follows:

$$\begin{aligned} c\left(\tau, l(\tau,k), k\right) &\equiv (1-\tau) w\left(k, l(\tau,k)\right) l(\tau,k) - s\left(\tau, k, l(\tau,k)\right), \\ d'\left(\tau, \tau^{K\prime}, l(\tau,k), l(\tau', k'(\cdot)), k'(\cdot)\right) &\equiv \left(1-\tau^{K\prime}\right) \frac{R\left(k'(\cdot), l(\tau', k'(\cdot))\right)}{\pi} s\left(\tau, k, l(\tau,k)\right). \end{aligned}$$

The indirect utility function of the older adult in period t, V^O, is

$$V^O\left(\tau, \tau^K, g, k, b\right) = \mu \ln d\left(\tau^K, l(\tau,k), k, b\right) + \theta \ln g, \tag{17}$$

where $d\left(\tau^K, l(\tau,k), k, b\right)$ is defined as[10]

$$d\left(\tau^K, l(\tau,k), k, b\right) \equiv \left(1-\tau^K\right) \frac{R\left(k, l(\tau,k)\right)}{\pi} (1+n)(k+b).$$

3 Political Equilibrium

In this section, we consider voting on fiscal policy. We employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As Persson and Tabellini (2002) demonstrated, the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

[9]One could write $\Psi^{ECON}(\cdot)$ as

$$(k', b') = \Psi^{ECON}\left(\tau, \tau^K, g, k, b \mid \Psi^{POL}\right),$$

where $(\tau, \tau^K, g) = \Psi^{POL}(k, b)$ denotes the mapping that governs policy for a given state variables, (k, b). In the present framework, future policies do not affect current economic outcomes. Thus, Ψ^{POL} does not affect Ψ^{ECON}, and one can eliminate Ψ^{POL} from the expression of Ψ^{ECON} and write Ψ^{ECON} as in (15). In general, in a Markov-perfect equilibrium, one would have to distinguish between on-equilibrium rules and off-equilibrium deviations, but this distinction is unnecessary in the present model because of the abovementioned special property.

[10]The arguments of $d'(\cdot)$ differs from those of $d(\cdot)$. The reason for the difference is that in the expression of d, s_{-1} that denotes the saving in the previous period is replaced by $(1+n)(k+b)$ by using the capital market clearing condition.

In the proposed framework, both the middle-aged and older adults have an incentive to vote. Thus, the political objective is the weighted sum of the utility of the middle-aged and older adults, given by $\pi\omega V^O + (1+n)(1-\omega)V^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older adult and the middle-aged, respectively. A larger value of ω implies greater political power of older adults. We use the gross population growth rate, $(1+n)$ to adjust the weight of the middle-aged and life expectancy (i.e., the probability of living in older adulthood), π to adjust the weight of older adults, to reflect their share of the population. To obtain the intuition behind this result, we divide the objective function by $(1+n)(1-\omega)$ and redefine it, denoted by Ω, as follows:

$$\Omega = \frac{\pi\omega}{(1+n)(1-\omega)}V^O + V^M,$$

where the coefficient $\pi\omega/(1+n)(1-\omega)$ of V^O represents the relative political weight of older adults.

We substitute V^M in (16) and V^O in (17) into Ω and obtain

$$\Omega\left(\tau, \tau^K, g, k, b \mid \Psi^{POL}\right) = \frac{\pi\omega}{(1+n)(1-\omega)}V^O\left(\tau, \tau^K, g, k, b\right) + V^M\left(\tau, g, \tau', \tau^{K\prime}, g', k'(\cdot), k\right), \tag{18}$$

where $\left(\tau', \tau^{K\prime}, g'\right) = \Psi^{POL}(k', b')$ denotes the mapping that governs policy for a given pair of state variables, (k, b), and

$$\left(k', b'\right) = \Psi^{ECON}\left(\tau, \tau^K, g, k, b\right). \tag{19}$$

Notice that $\Psi^{ECON}(\cdot)$ is defined by (13) and (14).

We can now define a political equilibrium in the present framework as follows.

Definition 2 A *political equilibrium* is a mapping Ψ^{POL} that solves the following fixed point problem

$$\Psi^{POL}(k, b) = \arg\max_{\tau, \tau^K, g} \Omega\left(\tau, \tau^K, g, k, b \mid \Psi^{POL}\right) \textit{ for all } k, b.$$

For each period, the government selects τ, τ^K, and g, given state variables k and b. The public debt issue, b', is determined as a residual from the government budget constraint.

3.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following policy functions in the next period:

$$1 - \tau^{K\prime} = \frac{\bar{T}^K}{\alpha} \cdot \frac{1}{1 + b'/k'}, \tag{20}$$

$$\tau' = 1 - \bar{T}, \tag{21}$$

$$g' = \bar{G} \cdot \left[A(k')^\alpha\right]^{\frac{1+v}{1+\alpha v}}. \tag{22}$$

The conjectures in (20) – (22) suggest that at the aggregate level, the after-tax capital income, after-tax labor income, and government expenditure are all linearly related to GDP:

$$
\begin{aligned}
\left(1-\tau^{K\prime}\right) R^{\prime} s N &= \bar{T}^{K} Y^{\prime}, \\
(1-\tau^{\prime}) w^{\prime} l^{\prime} N^{\prime} &= \bar{T}\left(1-\alpha\right) Y^{\prime}, \\
G^{\prime} &= \left[\left(1+n+\pi\right)/\left(1+n\right)\right] \bar{G} Y^{\prime} / \left[\bar{T}\left(1-\alpha\right)\right]^{v(1-\alpha)/(1+\alpha v)} .
\end{aligned}
$$

The conjectures that satisfy this property are based on the result of Uchida and Ono (2021) who show the linear relation of the policy functions to GDP under the inelastic labor supply. Our conjectures here indicate that the same property holds under elastic labor supply.

Given the conjectures in (20)–(22), we solve the optimization problem described in Definition 2 and obtain the following first-order conditions:

$$
\begin{aligned}
\tau :& \frac{\pi\omega\mu}{(1+n)(1-\omega)} \cdot \frac{d_{\tau}}{d} + \frac{c_{\tau} - (l)^{1/v} l_{\tau}}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi\mu \frac{d_{\tau}^{\prime}}{d^{\prime}} + \beta\pi \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{k^{\prime}}^{K\prime} + d_{k^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{k^{\prime}}^{\prime}}{g^{\prime}} \right) k_{\tau}^{\prime} \\
& + \beta\pi \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) b_{\tau}^{\prime} = 0, \qquad (23) \\
\tau^{K} :& \frac{\pi\omega\mu}{(1+n)(1-\omega)} \cdot \frac{d_{\tau^{K}}}{d} + \beta\pi \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) b_{\tau^{K}}^{\prime} = 0, \qquad (24) \\
g :& \left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} + \beta\pi \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) b_{g}^{\prime} = 0, \qquad (25)
\end{aligned}
$$

where a variable with a subscript x represents a derivative with respect to x (e.g., $d_{\tau} = \partial d / \partial \tau$).

Given the definition of $b^{\prime}$ in (13), we have

$$
\begin{aligned}
b_{\tau}^{\prime} &= \frac{1}{1+n} \left(-TR_{\tau} - TR_{\tau}^{K} + R_{\tau} b \right), \\
b_{\tau^{K}}^{\prime} &= -\frac{1}{1+n} TR_{\tau^{K}}^{K}, \\
b_{g}^{\prime} &= \frac{1}{1+n} \cdot \frac{(1+n)+\pi}{1+n}.
\end{aligned}
$$

Thus, we can summarize the first-order conditions in (23)–(25) by focusing on the policy trade-offs as follows:

$$
\frac{\beta\pi}{1+n} \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) = \frac{\frac{\pi\omega\mu}{(1+n)(1-\omega)} \cdot \frac{d_{\tau}}{d} + \frac{c_{\tau} - (l)^{1/v} l_{\tau}}{c - \frac{(l)^{1+1/v}}{1+1/v}} + \beta\pi\mu \frac{d_{\tau}^{\prime}}{d^{\prime}} + \beta\pi \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{k^{\prime}}^{K\prime} + d_{k^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{k^{\prime}}^{\prime}}{g^{\prime}} \right) k_{\tau}^{\prime}}{TR_{\tau} + TR_{\tau}^{K} - R_{\tau} b}, \qquad (26)
$$

$$
\frac{\beta\pi}{1+n} \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) = \frac{\frac{\pi\omega\mu}{(1+n)(1-\omega)} \cdot \frac{d_{\tau^{K}}}{d}}{TR_{\tau^{K}}^{K}}, \qquad (27)
$$

$$
\frac{(-1)\beta\pi}{1+n} \left(\mu \frac{d_{\tau^{K\prime}}^{\prime} \tau_{b^{\prime}}^{K\prime} + d_{b^{\prime}}^{\prime}}{d^{\prime}} + \theta \frac{g_{b^{\prime}}^{\prime}}{g^{\prime}} \right) = \frac{\left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g}}{\frac{(1+n)+\pi}{1+n}}. \qquad (28)
$$

The expressions in (26)-(28) suggest that the following three effects shape policy: the effect through the next period's state variables k' and b'; the effect through the capital income tax rate, $\tau^{K\prime}$; and the effect through public goods provision, g'. To understand how these effects work, first consider (26). The numerator on the right-hand side, representing the net marginal costs of the labor income tax, includes the following four effects. First, the term $[\pi\omega\mu/(1+n)(1-\omega)]\cdot(d_\tau/d)$ shows the marginal cost of the labor income tax for older adults; raising the tax rate induces the middle-aged to reduce labor supply. This in turn lowers the return from savings and decreases the consumption of the older adult.

Second, the term $\left(c_\tau - (l)^{1/v} l_\tau\right) / \left(c - (l)^{1+1/v} / (1+1/v)\right)$ includes the marginal costs and benefits for the middle-aged. An increase in the labor income tax rate causes disposable income to decrease, and thus reduces the consumption of the middle-aged, as represented by the term c_τ. Simultaneously, the increase in the labor income tax rate discourages the labor supply of the middle-aged and thus lowers their disutility of labor, as the term $(l)^{1/v} l_\tau$ represents. Third, the term $\beta\pi\mu d'_\tau/d'$ shows the marginal cost of the labor income tax for the middle-aged. The rise in the tax rate reduces the disposable income of the middle-aged and thus their savings, which in turn decreases consumption in their older adulthood.

Finally, the term $\beta\pi\left(\mu\left(d'_{\tau^{K\prime}}\tau^{K\prime}_{k'} + d'_{k'}\right)/d' + \theta g'_{k'}/g'\right) k'_\tau$ includes the marginal costs and benefits of the labor income tax for the middle-aged through the next period physical capital, k'. As mentioned above, the increase in the tax rate decreases savings, which degrades physical capital accumulation. This, in turn, produces the following three effects on the middle-aged: first, a rise in the capital income tax rate in the next period, thus reducing their consumption in their older adulthood, as represented by the term $d'_{\tau^{K\prime}}\tau^{K\prime}_{k'}/d'$. Second, an increase in the return from savings, that is, the consumption in their older adulthood, as represented by the term $d'_{k'}/d'$. Finally, a decrease in public goods provision in the next period, as represented by the term $\theta g'_{k'}/g'$. The right-hand side of (26) evaluates the above four effects based on the change in the tax revenue through labor income taxation, as represented by the term $TR_\tau + TR^K_\tau - R_\tau b$ in the denominator.

The left-hand side of (26) shows the marginal costs and benefits of public debt issuance associated with the decision on the labor income tax. As we can see in (26), the issue of public debt increases the capital income tax rate in the next period, $\tau^{k\prime}$. Simultaneously, the issue of public debt crowds out physical capital accumulation, and thus has the same effect to that observed on the fourth term in the right-hand side of (26). The expression in (26) suggests that the government selects the labor income tax rate to balance the abovementioned costs and benefits.

Subsequently, consider the expression in (27). The right-hand side shows the marginal costs of the capital income tax. A rise in the capital income tax rate lowers the consumption of the older adult and thus makes them worse off. We evaluate this effect based on the change in the tax revenue from capital income taxation represented by the term $TR^K_{\tau^K}$ in the denominator.

The left-hand side represents the marginal costs and benefits of public debt issuance associated with the decision on the capital income tax. The expression in (27) suggests that the government selects the capital income tax rate to balance these marginal costs and benefits.

Finally, we consider the expression in (28). The right-hand side shows the marginal benefit of public goods provision, normalized by the dependency (i.e., beneficiary-contributor) ratio. The left-hand side is equal to that of (26) multiplied by minus one, and so includes the marginal costs and benefits of public debt reduction associated with the decision on public goods provision. The expression in (28) suggests that the government selects public goods provision to balance the marginal costs and benefits arising from the choice of public goods provision.

We can obtain the policy functions that are the solutions to the government's optimization problem by solving (26)-(28) and the government's budget constraint in (13) for τ, τ^K, g, and b'. To simplify the presentation of the policy functions, we introduce the following notations:

$$\bar{T}^K \equiv \frac{1-\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)\frac{D_3}{D_1}}{\left[1+\frac{1}{\mu}\theta+\frac{(1+n)(1-\omega)}{\pi\omega\mu}\left(\theta+\frac{\beta\pi(\mu+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)\frac{D_2}{D_1}},$$

$$\bar{T} \equiv \frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1},$$

$$\bar{G} \equiv \frac{1+n}{(1+n)+\pi}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\frac{1}{\mu}\theta\left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^K,$$

$$\bar{B} \equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^K+(1-\alpha)\bar{T}-1\right]+\frac{(1+n)+\pi}{1+n}\bar{G},$$

where D_1, D_2, and D_3 are defined by

$$D_1 \equiv \left[\frac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)-\beta\pi\left(\mu+\theta\right)\alpha(1+v)\left[(-1)\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+1+\frac{v(1-\alpha)}{1+\alpha v}\right],$$

$$D_2 \equiv \left[\frac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\frac{1}{\mu}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\frac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},$$

$$D_3 \equiv \left[\frac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\frac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.$$

The following proposition describes the optimal policy functions in the present framework.

Proposition 1 *There is a political equilibrium characterized by the following policy functions:*

$$\tau^K = 1-\frac{\bar{T}^K}{\alpha}\cdot\frac{1}{1+b/k},$$

$$\tau = 1-\bar{T},$$

$$g = \bar{G}\cdot\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)},$$

$$(1+n)b' = \bar{B}\cdot\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)},$$

Proof. See Appendix A.1.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt, but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. However, a higher level of physical capital lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the levels of public goods provision and public debt issues are linear functions of the output. This implies that the government finds it optimal to provide public goods and issue public debt in proportion to the output. Third, the government borrows in the capital market as long as $\bar{B} > 0$; if this is the case, then the government finds it optimal to shift a part of the burden onto the future generations.

Having established the policy functions, we are ready to demonstrate the accumulation of physical capital. We substitute the policy functions of b' and τ in Proposition 1 into the capital market-clearing condition in (14) and obtain

$$k' = \frac{1}{1+n}\left\{\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}\left[(1-\alpha)\bar{T}\right]^{(1+v)/(1+\alpha v)} - \bar{B}\right\}\left[A\,(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{29}$$

where k' denotes the next-period capital stock. Given the initial condition k_0, (29) determines the equilibrium sequence $\{k_t\}$. A steady state is defined as an equilibrium sequence with $k = k'$. In other words, per-capita capital is constant in a steady state. Equation (29) indicates a unique, stable steady-state equilibrium of k.

3.2 Inelastic Labor Supply

The result established in Proposition 1 indicates that increased life expectancy (i.e., an increase in π) and declining population growth rate (i.e., a decrease in n) affect the policy functions. We consider those effects in the inelastic labor supply case, $v = 0$.

Proposition 2 *Suppose that labor supply is inelastic: $v = 0$.*

(i) If $\mu(1-\alpha) - \alpha\theta > 0$ such that the government borrows in the capital market (i.e., $b' > 0$), then the ratio of public debt to GDP increases in life expectancy and the population growth rate: $\partial\left(B'/Y\right)/\partial\pi > 0$ and $\partial\left(B'/Y\right)/\partial n > 0$.

(ii) The capital income tax rate decreases in life expectancy, and increases in the population growth rate: $\partial\tau^K/\partial\pi < 0$ and $\partial\tau^K/\partial n > 0$.

(iii) The labor income tax rate decreases in the population growth rate: $\partial\tau/\partial n < 0$; it increases (decreases) in life expectancy if $\beta\mu\,(1+\theta) < (>)\,(\mu+\theta)\left[\omega/(1+n)(1-\omega) + \beta\alpha\right]$:

$$\partial\tau/\partial\pi \gtreqless 0 \text{ if } \beta\mu\,(1+\theta) \lesseqgtr (\mu+\theta)\left[\omega/(1+n)(1-\omega) + \beta\alpha\right].$$

(iv) The ratio of government expenditure to GDP decreases (increases) in the population growth rate if $\mu < (>) 1 + \beta\pi\,(\mu+\theta)\,\alpha$ and increases (decreases) in life expectancy if $1 > (<) \left[(1+n)\,(1-\omega)/\omega\right]\beta\,(\mu+\theta)\,\alpha + \mu$:

$$\partial\left(G/Y\right)/\partial n < (>) 0 \text{ if } \mu < (>) 1 + \beta\pi\,(\mu+\theta)\,\alpha,$$
$$\partial\left(G/Y\right)/\partial\pi > (<) 0 \text{ if } 1 > (<) \left[(1+n)\,(1-\omega)/\omega\right]\beta\,(\mu+\theta)\,\alpha + \mu.$$

Proof. See Appendix A.2.

Proposition 2 shows that the labor income tax rate increases as the population growth rate declines. Moreover, when the political weight of older adults is large, the tax rate increases with an increase in life expectancy. These results are generally consistent with the evidence from Figure 2. Proposition 2 also shows that when the life expectancy is high such that $\mu < 1 + \beta\pi\left(\mu + \theta\right)\alpha$ holds, the ratio of government expenditure to GDP increases as the population growth rate declines; and that when the population growth rate is low such that $1 > \left[\left(1+n\right)\left(1-\omega\right)/\omega\right]\beta\left(\mu+\theta\right)\alpha + \mu$ holds, the ratio increases as life expectancy increases. These results are also consistent with the evidence from Figure 2. However, we should note that under other conditions, the result is contrary to the evidence from Figure 2. This point is discussed again in Section 4.

For the ratio of public debt to GDP, there is some discrepancy between theory and evidence. An aging population, represented by an increase in life expectancy and a decline in the population growth rate, works to increase the political weight of older adults. As public debt reallocates resources from today to the future and would lower the tax burdens of older adults, one would expect an increase in debt issuance in response to the aging population. This expectation is confirmed by the evidence observed from Figure 2. The result in Proposition 2 implies that the model prediction is consistent with the evidence for the increase in life expectancy. However, it is not consistent for the decline in population growth rate: the result shows that the ratio of public debt to GDP decreases as the population growth rate declines.

To understand the reason for this counterintuitive result, we consider the marginal net cost of public debt issuance, which appears on the left-hand side of (26). When labor supply is inelastic such that $v = 0$ holds, it is reduced to:

$$\begin{aligned}
\mu\frac{d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'} &= \frac{\left(-1\right)\left(\mu+\theta\right)\alpha}{k'} \\
&= \frac{\left(-1\right)\left(\mu+\theta\right)\alpha}{\underbrace{\frac{1}{1+n}\cdot\frac{\beta\pi\mu}{1+\beta\pi\mu}}_{1.a}\left[w(k) - Rb - \underbrace{\frac{1+n+\pi}{1+n}}_{1.b}g + TR^{K}\left(\tau^{K},\tau,k,b\right) + \underbrace{(1+n)b'}_{1.c}\right] - b'},
\end{aligned} \tag{30}$$

where the second equality comes from the capital-market-clearing condition in (14) and the government budget constraint in (13). The expression in (30) indicates that the population growth rate and life expectancy have effects on the marginal net cost of public debt issuance through the three terms, denoted by (1.a), (1.b), and (1.c). We see below in detail how the two causes of population aging, that is, a decline in population growth rate and an improvement in life expectancy, affect the marginal net cost of public debt issuance through these three terms.

The term (1.a) indicates that the two causes of population aging work to increase capital equipment per capita and thus lower the marginal net cost of public debt issuance. The term (1.b) implies that the two causes of population aging work to increase public spending per capita.

This in turn lowers savings through an increase in the tax burden on households, thus reducing capital per capita and increasing the marginal net cost of public debt issuance. Thus, from the terms (1.a) and (1.b), we observe that the two causes of population aging have quantitatively the same effects on public debt issuance.

However, the decline in population growth rate has an additional impact on debt issuance, as observed in the term (1.c). Recall that public debt issuance implies a shift of financial resources from taxes to public debt. This reduces the labor income tax burden of the middle-aged, which in turn stimulates their savings and thus capital accumulation. This positive effect on capital becomes weaker as the population growth rate declines. That is, the decline in the population growth rate has an additional negative impact on capital, which in turn increases the marginal net cost of public debt issuance. The cost-raising effect observed in the term (1.c) moderates the effects observed in the term (1.a), resulting in an increase in the marginal net cost of public debt issuance. Thus, the decline in the population growth rate works to lower the ratio of public debt to GDP.

Proposition 2 also suggests a discrepancy between theory and evidence for the capital income tax rate. The result in Proposition 2 shows that the tax rate decreases as life expectancy increases and the population growth rate decreases. This result, implying a negative association between the tax rate and aging, seems to be intuitive at first glance, because such changes in demographic factors lead to an increase in the political weight of older adults, which in turn provides incentives for the government to select policies favoring older adults who bear the capital income tax burden. However, the cross-country evidence from Figure 2 shows the positive association. In the following analysis, we show that assuming elastic labor supply could solve the discrepancy between theory and empirical findings.

4 Elastic Labor Supply

For the analysis, we take a numerical approach owing to the limitations of the analytical approach in the presence of elastic labor supply. Our strategy is to calibrate the model economy in such a manner that the steady-state equilibrium matches some key statistics of the average OECD countries over the time period 1995-2016. We then use the calibrated economy to run some quantitative experiments. Appendix A.3 describes the sources of the data used in calibration.

We introduce young age into the model. During youth, individuals make no economic decision and depend on their parents for their livelihood. This extension does not affect the result of the analysis up to the previous section. Each period lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (Gonzalez-Eiras and Niepelt, 2008; Song et al., 2012; Lancia and Russo, 2016). The probability of living in older adulthood, π, is taken from the average life expectancy at birth. The average life expectancy in OECD countries is 78.052 years; therefore, individuals will, on average, live 18.052(=78.052–60) years into older adulthood. In other words, individuals are expected to live 18.052/30 of their

30 years of older adulthood, thus, $\pi = 0.602$. The net population growth rate, n, is taken from the average annual (gross) population growth rate, 1.0055. The net population growth rate for one period is $(1.0055)^{30} - 1 \simeq 0.179$.

We fix the share of capital at $\alpha = 0.4$ following Mateos-Planas (2008). We calibrate the parameters θ, β, ω, μ, and v to match the average statistics of the ratios of government deficit $((B' - B)/Y)$, government expenditure (G/Y), and gross saving to GDP (sN/Y), the labor income tax rate (τ), and the annual gross interest rate $(R^{1/30})$ for the OECD economies during the period 1995-2016. The ratios of government deficit, government expenditure, and gross saving to GDP are set to the values 2.02%, 22.4%, and 23.2%, respectively. The labor income tax rate is set at 27.8%, and the gross interest rate per year is set at 1.054. Jointly matching these values yields $(\theta, \beta, \omega, \mu, v) = (0.988, 0.960, 0.761, 3.394, 0.237)$. Table 1 summarizes the estimated parameter values. Appendix A.4 provides details on calibration.

β : Discount factor	0.960
ω : Political weight of the elderly	0.761
v : Frisch elasticity of labor supply	0.237
θ : Preferences for public goods	0.988
μ : Preferences for old-age consumption	3.394
π : Probability of living in old age	0.602
n : Population growth rate	0.179
α : Capital share of output	0.4

Table 1: Calibration
Note: β is the 30-year period discount factor. Its annual number is $(0.960)^{1/30} \approx 0.9986$.

4.1 Comparative Statics

We numerically investigate the effects of population aging on the fiscal policy variables such as labor and capital income tax rates and the ratios of public debt and government expenditure to GDP. In particular, we first present the effects of the two aging factors, π and n, on the fiscal policy variables, and then demonstrate the association between the share of older adults in the population and the fiscal policy variables.[11]

4.1.1 Labor Income Tax Rate

The numerical results in Figures 3 and 4 show that the labor income tax rate shows a monotone increase against the declining population growth rate, and an inverted U-shaped pattern against

[11] We define the ranges of π and n in the comparative statics analysis as follows: For the range of π, we estimate π for each country based on the observed data of the life expectancy and find that the maximum value is 0.7432 of Japan and the minimum value is 0.3869 of Latvia. Considering these maximum and minimum values and the expected increase in π in the future, comparative statics are performed within a range of (0.38,1). To determine the range of n, we estimate n for each country and find that the maximum value is 0.8555 of Israel and the minimum value is -0.2873 of Latvia. Considering these maximum and minimum values and the expected decrease in n in the future, the range of n is set to (-0.5,0.9).

an increase in life expectancy.[12] The former result is qualitatively similar to that observed under the inelastic labor supply case, but the latter differs from that observed under inelastic labor supply.

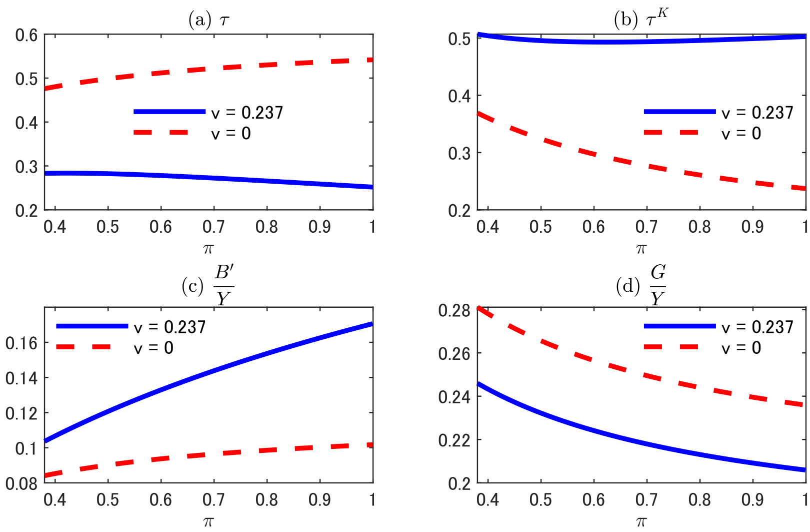


Figure 3: Changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), the ratio of public debt to GDP (Panel (c)), and the ratio of government expenditure to GDP (Panel (d)) against changes in π. The dotted and solid curves plot the results when $\upsilon = 0$ (inelastic labor supply case) and $\upsilon = 0.237$ (elastic labor supply case), respectively.

To understand the mechanism behind the different outcomes, we consider the consumption of older adults, given by

$$d = \left(1 - \tau^K\right) \frac{R\left(k, l(\tau, k)\right)}{\pi}(1+n)(k+b). \tag{31}$$

From the expression in (31), we find that an increase in the labor income tax rate lowers the labor supply of the middle-aged. This in turn lowers the interest rate, R, and thus reduces consumption of older adults. The negative effect on the consumption of older adults via the labor supply becomes larger as life expectancy increases. The negative effect, which is not observed under inelastic labor supply, incentivizes the government to select a lower labor income tax rate. In the proposed framework, this negative effect on the tax outweighs the positive effect common to both inelastic and elastic labor supply cases when life expectancy is above a certain threshold. This is the mechanism by which the labor income tax rate exhibits an inverse U-shaped pattern for increasing life expectancy.

4.1.2 Ratio of Public Debt to GDP

The comparative statics results in Figures 3 and 4 show that when labor supply is elastic ($\upsilon > 0$) the ratio of public debt to GDP shows a strong tendency to increase in response to increased life

[12]The solid curve depicted in Panel (a) of Figure 3 achieves a maximum between 0.4 and 0.5 of π, although the curve appears to decrease monotonically with increasing π at first glance.

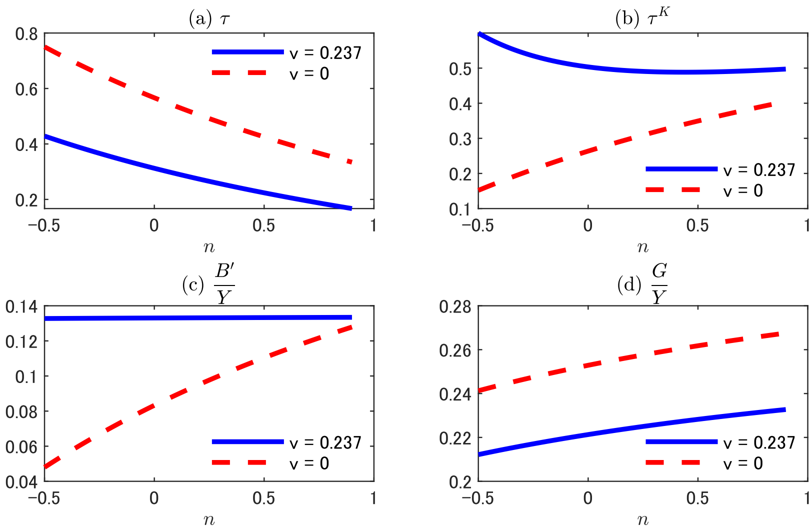


Figure 4: Changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), the ratio of public debt to GDP (Panel (c)), and the ratio of government expenditure to GDP (Panel (d)) against changes in n. The dotted and solid curves plot the results when $v = 0$ (inelastic labor supply case) and $v = 0.237$ (elastic labor supply case), respectively.

expectancy, while it remains almost unchanged as the population growth rate declines. Both results differ significantly from those obtained in the inelastic labor supply case ($v = 0$), where the ratio increases weakly as life expectancy improves and it decreases as the population declines.

To obtain the intuition of the difference between the two cases, consider the marginal net cost of debt issuance that appears on the left-hand side of (26). It is rewritten as follows:

$$\mu \frac{d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}}{d'} + \theta \frac{g'_{b'}}{g'} = (-1)\left(\mu + \theta\right) \frac{(1+v)\alpha}{1+\alpha v} \cdot \frac{1}{k'},$$

where the term k', appearing on the right-hand side, is rewritten by using the capital-market-clearing condition in (14) and the government budget constraint in (13) as follows:

$$k' = \underbrace{\frac{1}{1+n} \cdot \frac{\beta\pi\mu}{1+\beta\pi\mu}}_{2.a} \cdot \left[w(k) - Rb - \underbrace{\frac{(1+n)+\pi}{1+n}}_{2.b} g + TR^K\left(\tau^K, k, b\right) + \underbrace{(1+n)b'}_{2.c} \right]^{\overbrace{1+v}^{2.d}} - b'. \tag{32}$$

The terms (2.a), (2.b), and (2.c) in (32), including the life expectancy, π, and population growth rate, n, correspond to the terms (1.a), (1.b), and (1.c) under the inelastic labor supply in (30). Under elastic labor supply, the negative effects on capital through the terms (2.b) and (2.c) are diminished by the elasticity of labor supply represented by the term (2.d) in (32). This in turn works to lower the marginal net cost of public debt issuance. Therefore, the effect through the term (2.a) is outweighed by the effects through the terms (2.b) and (2.c) under the elastic labor supply case. This is why the population decline has an insignificant effect but increased life expectancy has a strong positive effect on the ratio of public debt to GDP under elastic labor supply.

4.1.3 Capital Income Tax Rate

As for the capital income tax rate, the numerical results in Figures 3 and 4 show that the capital income tax rate exhibits a U-shaped pattern against the declining population growth rate and increased life expectancy. The mechanism behind this U-shaped pattern, which diverges from the pattern under inelastic labor supply, is as follows. Recall that the capital income tax rate is given by $\tau^K = 1 - \left(\bar{T}^K/\alpha\right)/(1 + b/k)$ (see Proposition 1). This expression indicates that changes in life expectancy (π) and population growth rate (n) impact the capital income tax rate through $\bar{T}^K$ and $\alpha(1 + b/k)$ as illustrated in Figure 5.

The term $\bar{T}^K$ increases as π increases and n decreases, regardless of the labor supply status, v. The negative effect on the capital income tax rate through the term $\bar{T}^K$ reflects the preferences of older adults who want to reduce their fiscal burden of capital income taxation. The term $\alpha(1+b/k)$ indicates that the capital income tax rate is influenced by the burden of debt repayment, b/k. A higher level of public debt results in a larger burden of debt repayment, and a lower level of capital leads to a higher interest rate, also resulting in a larger burden of debt repayment. Therefore, a higher level of public debt and a lower level of capital together incentivize the government to raise the capital income tax rate. It should be noted that the term b/k is independent of π and n when labor supply is inelastic ($v = 0$), while it increases as π increases and n decreases when labor supply is elastic ($v > 0$). This suggests that population aging prompts the government to increase the capital income tax rate in response to increased burden of debt repayment under elastic labor supply.

Overall, the effects through the two terms, $\bar{T}^K$ and $\alpha(1 + b/k)$ suggest that when labor supply is inelastic, $v = 0$, the capital income tax rate decreases as π increases and n decreases. However, when labor supply is elastic, $v > 0$, the negative effect of the term $\bar{T}^K$ surpasses the positive effect of the term $\alpha(1 + b/k)$ for low values of π and high values of n. Conversely, for high values of π and low values of n, the opposite outcome is observed. Consequently, an increase in life expectancy and a decrease in population growth rate lead to a decrease, followed by an increase in the capital income tax rate when $v > 0$.

To explore the effects of life expectancy (π) and population growth rate (n) on the term $\alpha(1 + b/k)$ in more detail, recall the policy function of public debt presented in Proposition 1 and the capital market clearing condition in (29). As $b'/k' = b/k$ holds in a steady state, the ratio of b/k in the steady state becomes:

$$\frac{b}{k} = \frac{\bar{B}}{\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}\left[(1-\alpha)\bar{T}\right]^{(1+v)/(1+\alpha v)} - \bar{B}}. \tag{33}$$

This expression shows that π and n affect the ratio of b/k and thus the capital income tax rate through the three terms, $\beta\pi\mu/(1 + \beta\pi\mu)$, $\bar{T}$ and $\bar{B}$.

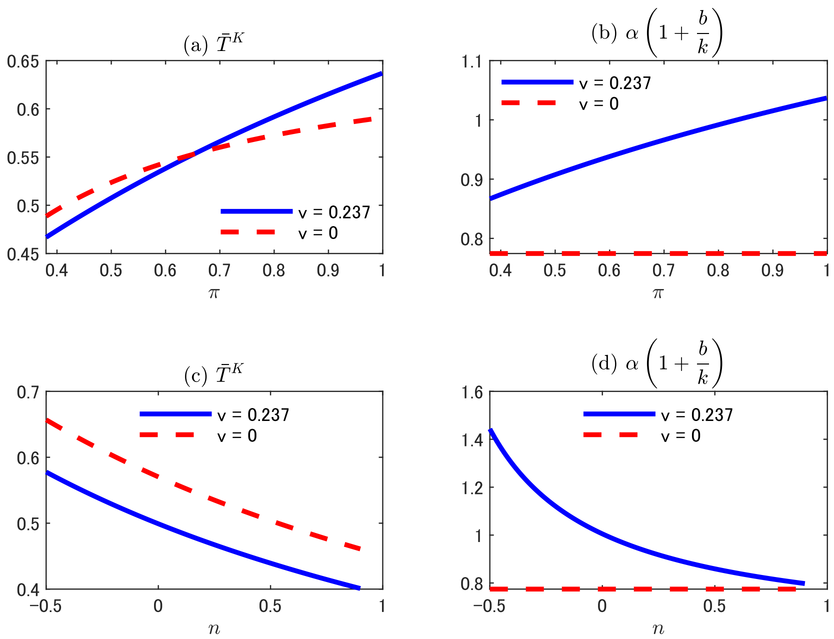


Figure 5: Changes in $\overline{T}^K$ (Panel (a)) and $\alpha(1+b/k)$ (Panel (b)) against changes in π, and changes in $\overline{T}^K$ (Panel (c)) and $\alpha(1+b/k)$ (Panel (d)) against changes in n. The dotted and solid curves plot the results when $v=0$ (inelastic labor supply case) and $v=0.237$ (elastic labor supply case), respectively.

The roles of the three terms in the determination of the capital income tax rate is as follows. First, the term $\beta\pi\mu/(1+\beta\pi\mu)$, representing the saving rate, indicates that an increased life expectancy raises the saving rate, stimulates physical capital accumulation, and thus lowers the ratio of b/k. This is a negative effect on the capital income tax rate, τ^K, which is common to the inelastic ($v=0$) and elastic ($v>0$) labor supply cases.

Second, consider the term $\bar{T}=1-\tau$, representing the after-tax labor income. A decline in population growth raises the labor income tax rate, impedes physical capital accumulation, and thus increases the ratio of b/k. This creates a positive effect on the capital income tax rate, τ^K, which is shown in both inelastic ($v=0$) and elastic ($v>0$) labor supply cases. As for the life expectancy, its effect on the capital income tax rate depends on the status of labor supply. When the labor supply is inelastic ($v=0$), increased life expectancy raises the labor income tax rate and, therefore, positively affects the capital income tax rate. When the labor supply is elastic ($v>0$), increased life expectancy produces an inverse U-shaped pattern of the labor income tax rate; thus, an increase in π initially raises and later lowers the labor income tax rate and the ratio of b/k, resulting in a U-shaped effect on the capital income tax rate.

Finally, we consider the term $\bar{B}$, representing the debt issuance. A decrease in n lowers $\bar{B}$ and the ratio of b/k, thereby negatively affecting the capital income tax rate when the labor supply is inelastic ($v=0$). However, such an effect almost disappears; hence, a decrease in n has little effect on the capital income tax rate when labor supply is elastic ($v>0$). As for the life expectancy, π, its increase raises $\bar{B}$ and the ratio of b/k, and thus positively affects the capital income tax rate when labor supply is inelastic ($v = 0$), and this positive effect is strengthened under elastic labor supply ($v > 0$).

Overall, the quantitative results suggest that the effects through the three terms, $\beta\pi\mu/(1+\beta\pi\mu)$, $\bar{T}$, and $\bar{B}$, offset each other when labor supply is inelastic ($v = 0$). However, when labor supply is elastic ($v > 0$), the results change drastically. As for an increase in π, the positive effect through the term $\bar{B}$ dominates the other negative effects. As for a decrease in n, the positive effect through the term $\bar{T}$ dominates the weak negative effect through the term $\bar{B}$. This is why an increase in π and a decrease in n produce an initial decrease followed by an increase in the capital income tax rate.

4.1.4 Ratio of Government Expenditure to GDP

The numerical results in Figures 3 and 4 show that the ratio of government expenditure to GDP exhibits a downward trend against the aging of the population, suggesting the divergence between theory and reality. There are three potential explanations for this divergence. First, labor and capital income tax rates rise with the aging of society, resulting in higher tax revenues. This incentivizes the government to increase expenditure. However, debt issuance increases simultaneously, and the resulting increase in debt redemption costs exceeds the increase in tax revenues as depicted in Figure 6. Consequently, the ratio of government expenditure to GDP declines as the population ages from the perspective of balancing the government budget.

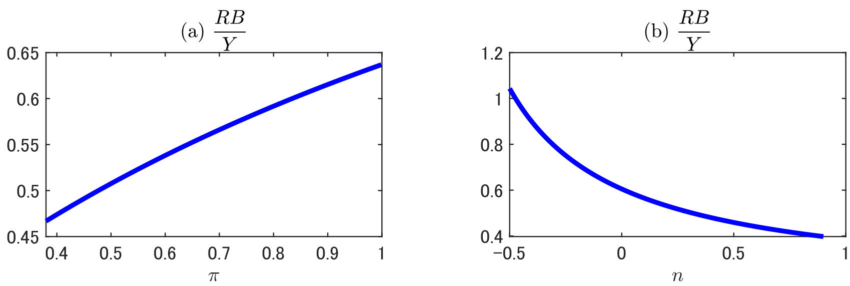
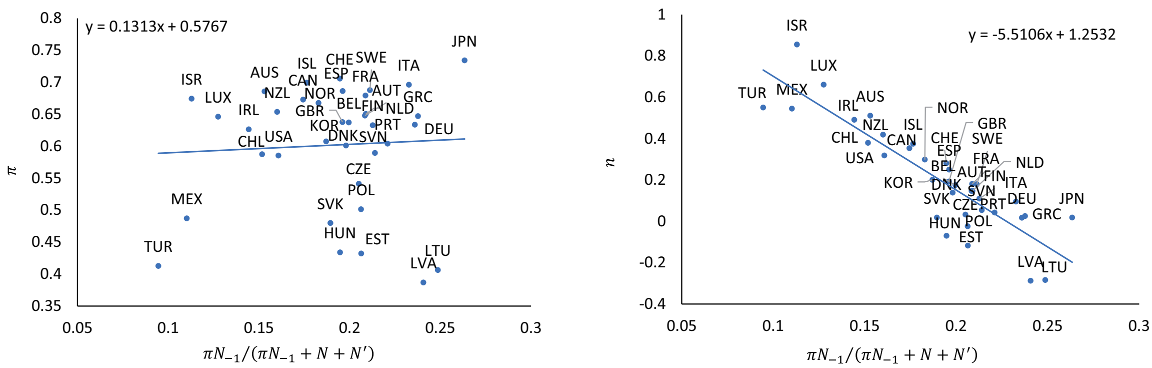


Figure 6: Changes in the ratio of debt redemption costs to GDP, RB/Y, against changes in π (Panel (a)) and n (Panel (b)).

Second, when we look at the values of the calibrated parameters, the weight on consumption utility in old age, $\mu = 3.394$, is significantly larger than the weight on public goods utility in old age, $\theta = 0.988$. This means that the government has an incentive to increase its allocation to consumption rather than public goods as the weights of consumption and public goods in old age become stronger with aging. Third, the current model assumes that the government spending is limited to public goods (e.g., public infrastructure and services) that are enjoyed by both middle-aged and older adults and does not include spending targeted to older adults (e.g., social security including public pension and long-term care). Owing to the above factors, our analysis finds a discrepancy between theoretical predictions and observed data regarding the government spending-GDP ratio. However, this discrepancy does not significantly impact the outcomes of other fiscal policy measures, as demonstrated in the sensitivity analysis presented in Section 4.1.6.

4.1.5 Association Between the Share of Older Adults in the Population and Fiscal Policy Variables

To further evaluate the model's fit to the data, we compute the share of older adults in the population, $\pi N_{-1}/(\pi N_{-1} + N + N')$, in the model for each country from population growth and life expectancy data, and illustrate in Figure 7 the scatter plots of the share and life expectancy (Panel (a)) and population growth rates (Panel (b)). The illustration of Figure 7 shows the association between the share of older adults in the total population and life expectancy or population growth rate in the present framework. This illustration corresponds to Figure 1, which plots the correlation between the share of the population aged 65 and over in the total population and life expectancy or population growth rate obtained from the data.



(a) (b)

Figure 7: Each panel plots model based demographic variables for OECD countries during 1995–2016. The horizontal axis represents the average share of the older adult in the population. The vertical axis represents the average life expectancy (Panel (a)) and the average population growth rate (Panel (b)). Source. OECD.Stat (`https://stats.oecd.org/`) (accessed in April 6, 2021).

To demonstrate the association between the fiscal policy variables and the share of older adults in the population in the numerical analysis, we estimate the fiscal policy variables ($\tau, \tau^K, B'/Y$, and G/Y) for each country in the present framework. In estimation, we use the data for the population growth rate and life expectancy for each country, whereas the values in Table 1 are commonly used for the other parameters. We then illustrate the scatter plots of the association between the share of older adults in the population and each fiscal policy variable in Figure 8.

The result in Figure 8 shows that a larger share of older adults in the population is associated with higher labor and capital income tax rates. This association, which is consistent with the evidence observed in Figure 2, emerges through the following mechanisms. As shown in Figures 3 and 4, the labor income tax rate increases as the population growth rate (n) decreases (Panel (a)

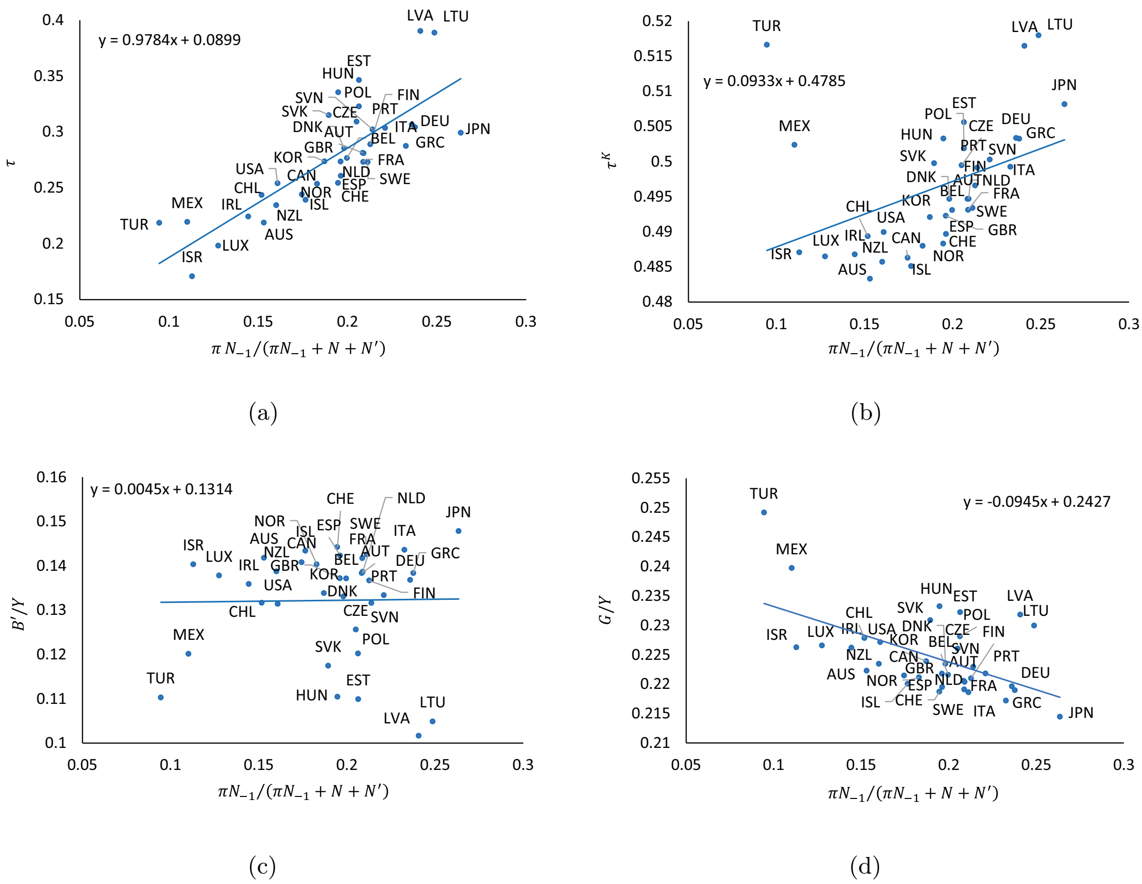


Figure 8: Correlation between the share of the older adult in the population, $\pi N_{-1}/(\pi N_{-1} + N + N')$, and the fiscal policy variables, τ (Panel (a)), τ^K (Panel (b)), B'/Y (Panel (c)), and G/Y (Panel (d)).

of Figure 4) and displays an inverse U-shape pattern in relation to an increase in life expectancy (Panel (a) of Figure 3). Conversely, the capital income tax rate increases as the population growth rate (n) decreases (Panel (b) of Figure 4) and exhibits a U-shape pattern in relation to an increase in life expectancy (Panel (b) of Figure 3). The results in Figure 8 suggest that for both capital and labor income tax rates, the positive impact of a decrease in the population growth rate outweighs the negative aspect of the increased life expectancy or resonates with its positive aspect, resulting in a positive association between population aging and the tax rates for labor and capital income.

The result in Figure 8 indicates a weak but positive association between the aging population and the ratio of public debt to GDP, which is also generally consistent with the findings shown in Figure 2. The increase in the ratio can be attributed to an increase in life expectancy (as seen in Panel (c) of Figure 3), while it remains relatively stable when the population growth rate decreases (as seen in (Panel (c) of Figure 4). The positioning of Greece, Italy, and Japan in Panel (c) of Figure 8 aligns with the findings in Panel (c) of Figure 2, indicating that the model used in Figure 8 accurately reflects high ratios of public debt to GDP in these countries.

However, two observations emerge from the numerical results in Figure 8. First, the results suggest that there is a positive association between the share of older adults in the population and the ratio of public debt to GDP, though it is weaker than the association seen in the data from Figure 2. One reason for this weaker association in the model is the inclusion of certain Eastern European countries, such as Latvia, Estonia, Hungary, Lithuania, Poland, and Slovakia, which have relatively low life expectancies. These countries have low ratios of public debt to GDP because an increase in life expectancy significantly impacts the increase in the ratio, while a decrease in population growth has insignificantly affects it, as shown in Figures 3 and 4. Therefore, the inclusion of these Eastern European countries weakens the positive association between the debt-GDP ratio and the share of older adults in the population.

Second, the results in Figure 8 show a negative association between the share of older adult in the population and the ratio of government expenditure to GDP. This contradicts the evidence depicted in Figure 2. This discrepancy arises from the fact that in the model economy utilized in this study, an increase in life expectancy and a decline in the population growth rate leads to a decrease in the ratio of government expenditure to GDP, as demonstrated in Panel (d)s of Figures 3 and 4. The underlying causes for this outcome are outlined in Subsection 4.1.4. In the next subsection, we examine the impact of this counterintuitive result on the other variables related to fiscal policy. In Section, 5, we suggest potential avenues for reconciling these conflicting theoretical and empirical results.

4.1.6 Sensitivity Analysis

We have thus far assumed that the government expenditure is endogenous, and it is determined via voting. However, under this assumption, an aging population leads to a decline in the ratio of government expenditure to GDP, which is counterfactual. To check whether the counterfactual behavior of the government expenditure affects the result of other fiscal policy variables, we undertake a sensitivity analysis by assuming an exogenous sequence of the ratio of government expenditure to GDP and selecting it to fit the data. The calibration is modified to (i) focus on the four parameters, μ, β, ω, and v; (ii) match the four parameters with the average statistics of the ratio of government deficit to GDP, $(B' - B)/Y$, gross saving to GDP, sN/Y, the labor income tax rate, τ, and the annual gross interest rate, $R^{(1/30)} = 1.053$; and (iii) set the value of the remaining parameter θ at 0.988 through calibration where government spending (G) is considered an endogenous variable. Appendix A.4 provides details on calibration.

Through this modification, we examine the influence of decreasing population growth, rising life expectancy, and a rising proportion of older adults on labor and capital income tax rates and the ratio of public debt to GDP. The findings, presented in Figures 9, 10, and 11, reveal that this modification does not significantly alter the results obtained in the endogenous government expenditure scenario. The labor and capital income tax rates continue to increase as population growth diminishes, life expectancy improves, and the share of older adults in the population increases. Regarding the ratio of public debt to GDP, improving life expectancy leads to an increase in the ratio, while decreasing population growth has a minimal impact on the ratio, similar to the case of endogenous government spending. Furthermore, while an increase in the share of older adults in the population decreases the ratio of debt to GDP, the effect is minimal and the resulting regression line is nearly horizontal, similar to the case of endogenous government spending.[13] Consequently, it can be inferred that the assumption of endogenous government expenditure leads to unrealistic government expenditure results, while this assumption insignificantly impacts the results for the other fiscal policy variables.

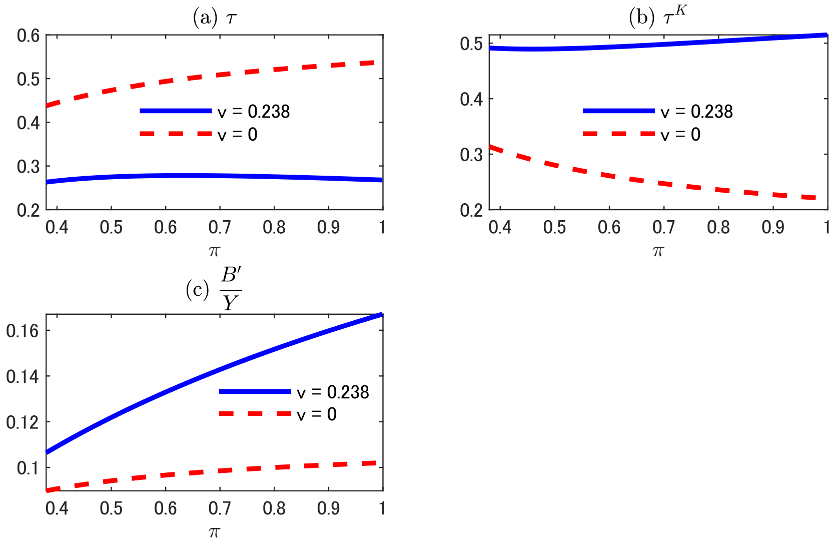


Figure 9: Changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), and the ratio of public debt to GDP (Panel (c)) against change in π when the government expenditure (G) is exogenous.

4.2 Demographic Projections and Their Policy Implications

To further facilitate the analysis, we undertake a model-based time series analysis predicting how fiscal policy changes over time as the population ages. We focus on two contrasting countries: one is Japan that is experiencing a declining population growth rate and the highest life expectancy at age 65 among OECD countries, and the other is the United States that has a higher fertility than other developed countries and is expected to show high population growth over the next few decades. By performing a comparative analysis of these two contrasting countries, we can assess the impact of population aging on fiscal policy over time and provide quantitative estimates of the predicted impacts.

The analysis proceeds as follows. We compute three sequences of model predictions, with a period length of 30 years each. In the first sequence, the periods correspond to the years 2000, 2030, and 2060; in the second sequence, to the years 2010, 2040, and 2070; and in the third

[13]Eastern European countries are the main factor disrupting the positive relationship between the share of older adults in the population and the ratio of public debt to GDP.

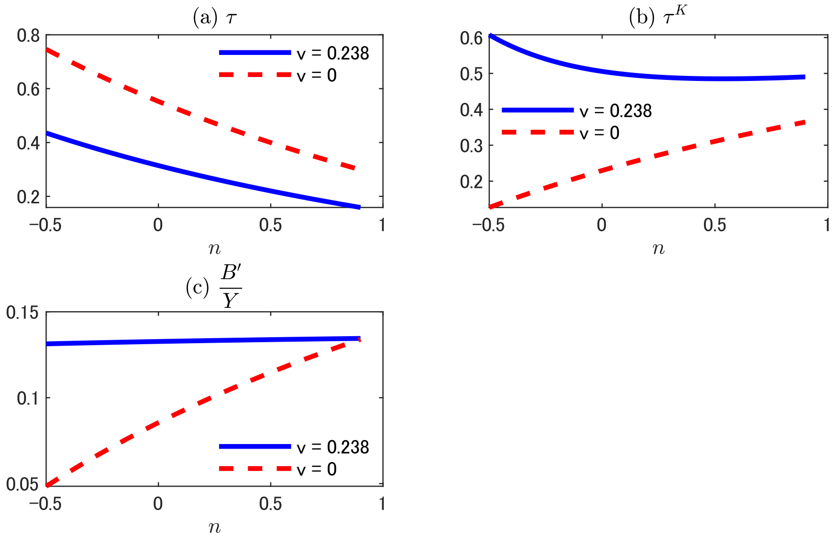


Figure 10: Changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), and the ratio of public debt to GDP (Panel (c)) against change in n when the government expenditure (G) is exogenous.

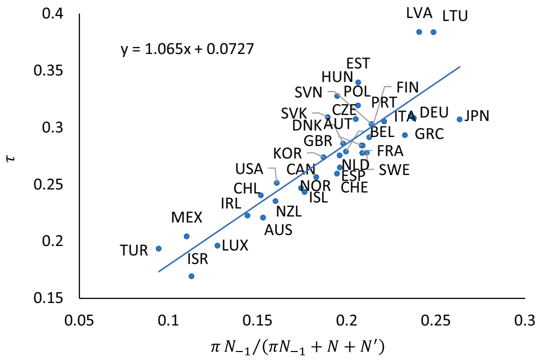
sequence, to the years 2020 and 2050. In the third sequence, we exclude the third year, 2080, because the model-based prediction of fiscal policy in 2080 requires life expectancy estimates for the year 2110, but the available estimates are limited up to the year 2100. When reporting time series predictions, we list the three sequences in a single time series. This procedure follows that of Gonzalez-Eiras and Niepelt (2012).

The population and life expectancy estimates for Japan and the United States are taken from the United Nations World Population Prospects.[14] Figure 12 takes years on the horizontal axis and plots the predicted series of the population growth rates ($n_t = N_t/N_{t-1} - 1$) and the probability of living in old age (π_t) for Japan and the United States.[15] We utilize the values of the other parameters obtained from the calibration conducted at the beginning of Section 4. We should note that π_t in Japan is stuck at the upper limit of 1 since 2070. This is because the upper limit of life expectancy is 90 years in the model, while the estimate of life expectancy is above 90 years after 2070.

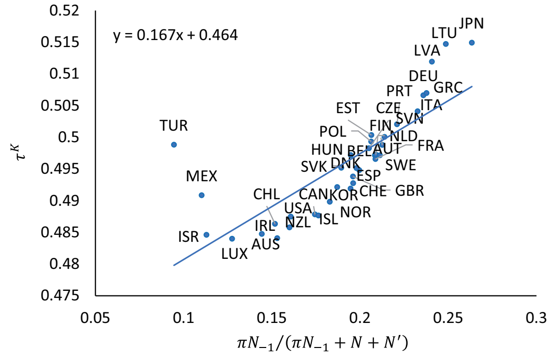
The initial values for the analysis are established as follows: The ratios of capital to public debt for the years 2000, 2010, and 2020 are represented by b_{2000}/k_{2000}, b_{2010}/k_{2010}, and b_{2020}/k_{2020}, respectively. These ratios serve as the initial values for the sequences that begin in the years 2000, 2010, and 2020. To calculate these initial values, we use the policy function of public debt outlined in Proposition 1 and the capital market clearing condition in (29), and compute the ratio of public debt to capital as outlined in (33). We then use the calibrated values of

[14]Source: https://population.un.org/wpp/ (accessed March 18, 2022).

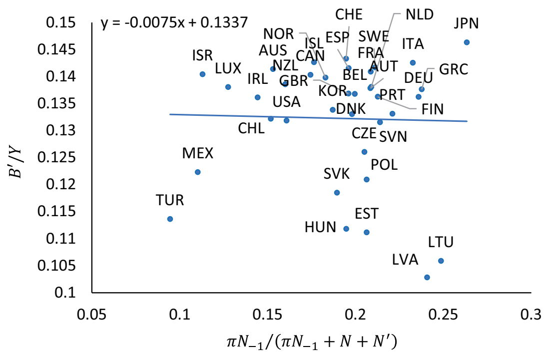
[15]It should be noted that n_t represents the net population growth rate from the period-$t-1$ middle-aged to the period-t middle-aged, and π_t represents the probability of living in older adulthood for the period-$t-1$ middle-aged.



(a)



(b)



(c)

Figure 11: Correlation between the share of the older adult in the population, $\pi N_{-1}/(\pi N_{-1} + N + N')$ and the fiscal policy variables, τ (Panel (a)), τ^K (Panel (b)), and B'/Y (Panel (c)) when the government spending (G) is exogenous.

the structural parameters, along with life expectancy and population growth rate data, for each sequence. For example, to determine b_{2000}/k_{2000}, we need data on the life expectancy in 2020, represented by π_{2020}, the life expectancy in 1990, represented by π_{1990}, and the average annual population growth rate between 1960-1990, represented by n_{1990}. We use the life expectancy as of 2020 (1990) instead of the average life expectancy between 1990 and 2020 (1960 and 1990) as it is more logical to consider the life expectancy at that specific point in time, rather than the average of the past 30 years, when examining household decision making in 2020 (1990) (see Gonzalez-Eiras and Niepelt, 2012). The initial ratios of public debt to capital for the sequences beginning in 2000 and 2010 are calculated in a similar manner.[16]

Figure 13 takes years on the horizontal axis and plots the policy responses to predicted demographic changes in Japan and the United States. All simulations are based on the calibration described in the beginning of Section 4 and differ only with respect to the demographic series fed into the model. Thus the simulation results clearly show the impact of differences in population growth rates and life expectancy on the formation of fiscal policy. The following

[16]Owing to missing data, we use the average annual population growth rate between 1950 and 1970 instead of between 1940 and 1970 to obtain n_{1970}.

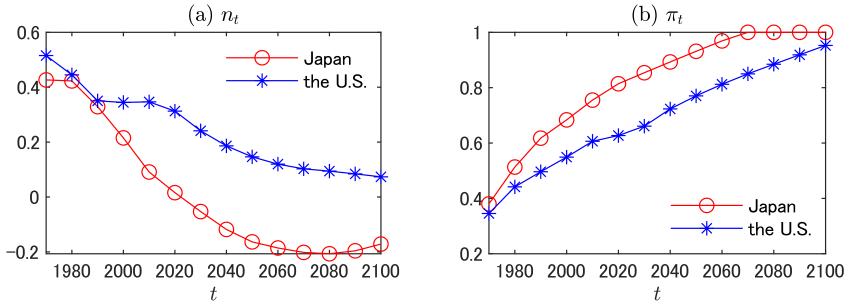


Figure 12: Expected changes in the population growth rate (Panel (a)) and the survival rate of living in old age (Panel (b)) in Japan and the United States from 2000-2100.

two characteristics are observed from the result in Figure 13. First, labor and capital income tax rates as well as the ratio of public debt to GDP are expected to increase in both countries. Second, with its lower population growth rate and higher life expectancy, Japan is expected to experience a higher tax rate for labor income, higher ratio of public debt to GDP, and lower ratio of government expenditure to GDP.

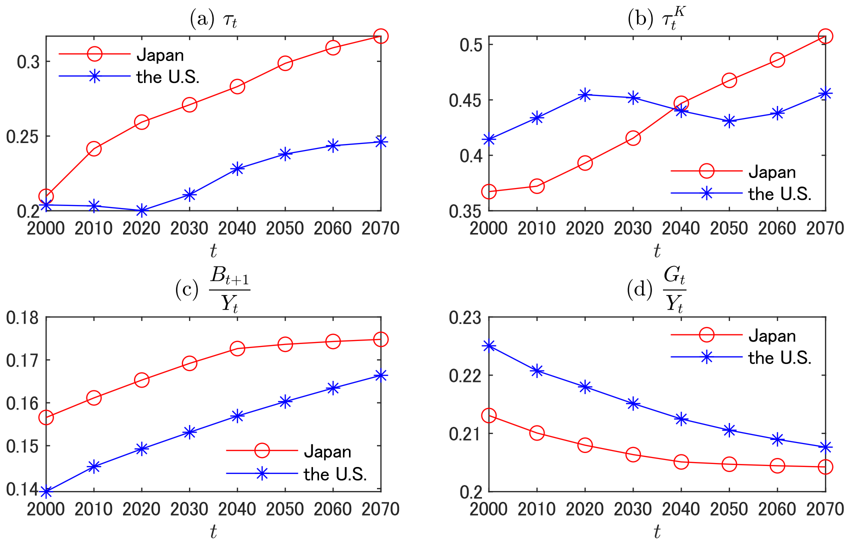


Figure 13: Expected changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), the ratio of public debt to GDP (Panel (c)), and the ratio of government expenditures to GDP (Panel (d)) in response to demographic changes in Japan and the United States from 2000 to 2070.

Two factors are notable with respect to the labor and capital income tax rates in the United States and Japan. First, the labor income tax rate in the United States is predicted to decrease temporarily because of the steady population growth rate and increasing life expectancy from 1990 to 2010. Second, the capital income tax rate in the United States is anticipated to be higher than that of Japan between 2000 and 2030. This is because the variable $\bar{T}^K$, which reflects the preferences of the older adults for a lower capital income tax rate, in Japan is higher than that in the United States (see Panel (a) of Figure 14), although the public debt-capital ratio, represented by b/k, is relatively similar between the two countries (see Panel (b) of Figure 14). Despite these observations, the public debt-capital ratio in Japan is expected to rise more rapidly after 2030, indicating that Japan, with a significantly older population, will likely face a greater burden of debt repayment and provide less public spending for future generations in comparison to the United States.

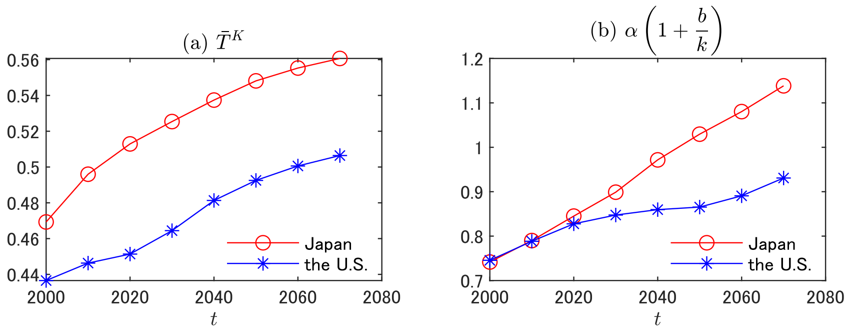


Figure 14: Expected changes in $\bar{T}^K$ (Panel (a)) and $\alpha(1 + b/k)$ (Panel (b)) in response to projected demographic changes in Japan and the United States from 2000 to 2070.

The results shown in Figure 13 reflect the combined effects of both an increase in life expectancy and a decrease in population growth rate. To further understand the impact of these variables on fiscal policy, we consider two separate scenarios: one in which the population growth rate is fixed at the initial level and only life expectancy changes (see Figure 15), and the other in which life expectancy is fixed at the initial level and only the population growth rate changes (see Figure 16). Two points can be derived from the two figures. First, in the United States, the increase in life expectancy has a minimal impact on the labor income tax rate (as shown in Figure 15), whereas the decrease in the population growth rate has a significant effect (as shown in Figure 16). Therefore, it might be concluded that the main reason for the increase in labor income tax rate depicted in Figure 13, which is due to the increase in the share of older adults, is the decline in the population growth rate. Second, for the public debt-GDP ratio in both Japan and the United States, the increase in life expectancy leads to an increase in the ratio (as shown in Figure 15), whereas the decrease in population growth rate has insignificantly impacts the ratio (as shown in Figure 16). Therefore, the main cause of the increase in the public debt-GDP ratio observed in Figure 13 can be attributed to the increase in life expectancy in both countries.

5 Conclusion

This study analyzed the distribution of the fiscal burden across generations in a political economy model of fiscal policy. The model includes the two elements: (i) the two tax instruments, capital and labor income taxes, accompanied by debt finance; and (ii) household decisions on labor supply. The first element enables us to investigate the impact of population aging on the

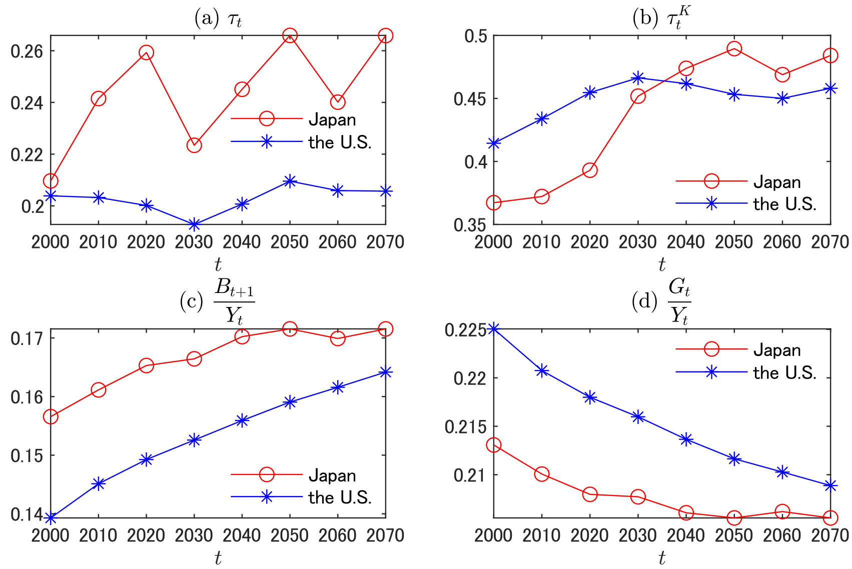


Figure 15: Expected changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), the ratio of public debt to GDP (Panel (c)), and the ratio of government expenditures to GDP (Panel (d)) in response to projected changes in life expectancy in Japan and the United States from 2000 to 2070.

distribution of the fiscal burden across generations; the second element allows us to present the effects of population aging on fiscal policy variables via households' labor decisions.

Given these features, we showed the two aging factors, a decline in the population growth rate and an improvement in the life expectancy, have the following effects. First, the two aging factors work together to raise the labor income tax rate as well as the ratio of public debt to GDP. Second, the capital income tax rate exhibits a U-shaped pattern against the declining population growth rate and the increased life expectancy. These results imply that population aging first produces a shift of the fiscal burden from older to younger generations, but as the population ages further, the fiscal burden on both younger and older generations increases.

Mateos-Planas (2010) also predicts a similar pattern, but limits his analysis to the balanced government budget case. This study instead allowed for government deficits, showing that the ratio of public debt to GDP increases as population ages. The result suggests that when debt finance is allowed, a shift of the fiscal burden from older to younger generations could prove to be stronger in the early stage of population aging. However, further aging leads to an increased fiscal burden on both younger and older generations. The increased fiscal burden is an inevitable consequence of population aging in the long run.

A caveat to our result is that the model's predicted effect of aging on the ratio of government expenditure to GDP does not fit well with the evidence observed from OECD countries. One method for resolving this discrepancy between theory and evidence is to include government spending targeted toward the elderly, such as public pension and long-term care. It is anticipated that adding this type of the spending to the model would improve the model predictions, but it

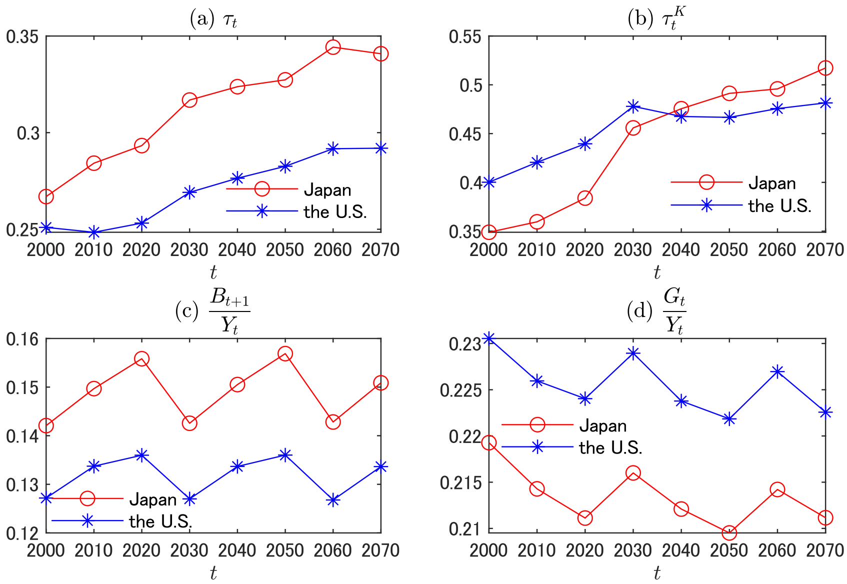


Figure 16: Expected changes in the labor income tax rate (Panel (a)), the capital income tax rate (Panel (b)), the ratio of public debt to GDP (Panel (c)), and the ratio of government expenditures to GDP (Panel (d)) in response to projected changes in population growth in Japan and the United States from 2000 to 2070.

is beyond the scope of this study. The task is left for future research.

A Mathematical Appendix

A.1 Proof of Proposition 1

Based on the specification of the utility and production functions, we can reformulate the first-order conditions in (23)-(25) as follows:

$$\frac{(-1)\pi\omega\mu}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}+\frac{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}{1+\alpha v}$$
$$\times\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{(1+n)k'}=0, \tag{A.1}$$

$$(-1)\left[\frac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\frac{1}{1+\alpha v}\frac{1}{1-\tau}$$
$$+\frac{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}{1+\alpha v}\cdot\frac{(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}}{(1+n)k'}$$
$$\times\left\{(-1)\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+\frac{1}{1-\tau}\frac{v}{1+\alpha v}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}=0, \tag{A.2}$$

$$\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\frac{\theta}{g}-\beta\pi\left(\mu+\theta\right)\frac{\alpha(1+v)}{1+\alpha v}\frac{\frac{(1+n)+\pi}{1+n}}{(1+n)k'}=0. \tag{A.3}$$

We present the derivation of (A.1)–(A.3) in Appendix B.

The procedure to find the optimal policy functions is as follows. First, substitute the first-order condition with respect to τ^K in (A.1) into the first-order condition with respect to g in (A.3) to write g as a function of τ^K and $\tau : g = g(\tau^K, \tau)$. Second, substitute $g = g(\tau^K, \tau)$ into the capital market-clearing condition in (14) to write k' as a function of τ^K and $\tau : k' = k'\left(\tau^K, \tau\right)$. Third, substitute $k' = k'\left(\tau^K, \tau\right)$ into the first-order condition with respect to τ^K in (A.1) and τ in (A.2) to obtain the two optimal relations between τ^K and τ, and solve them for τ^K and τ. Fourth, substitute the solutions for τ^K and τ into $g = g(\tau^K, \tau)$ to obtain the optimal policy function of g. Finally, substitute the optimal policy functions of τ^K, τ, and g into the government budget constraint in (13) to obtain the optimal policy function of b'.

Recall the first-order condition with respect to g in (A.3), which we rewrite as

$$\frac{(1+n)+\pi}{1+n}g=\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{1+\alpha v}{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}(1+n)k'.$$

We substitute the first-order condition with respect to τ^K in (A.1) into the above expression to obtain $g = g(\tau^K, \tau)$, or

$$\frac{(1+n)+\pi}{1+n}g=\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}}. \tag{A.4}$$

Next, we substitute (A.4) into the capital market-clearing condition in (14) to obtain

$$
\begin{aligned}
(1+n)k' &= \frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
&\quad -\alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{b}{k} \\
&\quad -\left(\frac{\pi\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{1}{1-\tau^K}} \\
&\quad +\frac{\tau}{1-\tau}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{1/(1+\alpha v)}\cdot\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{v/(1+\alpha v)} \\
&\quad +\tau^K\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right).
\end{aligned}
$$

Rearranging the terms, we have

$$
\begin{aligned}
(1+n)k' &= \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
&\quad\times\left\{\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right. \\
&\quad\left.-\left[1+\frac{1}{\mu}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}. \qquad \text{(A.5)}
\end{aligned}
$$

Eq. (A.5) shows that we can express $(1+n)k'$ as a function of τ^K and τ.

Third, we substitute (A.5) into the first-order condition with respect to τ^K in (A.1) and obtain

$$
\begin{aligned}
&\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{1}{1-\tau^K} \\
&= \frac{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}{1+\alpha v}\times\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{X} \\
&= \frac{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}{1+\alpha v} \\
&\times\frac{\alpha\left(1+\frac{b}{k}\right)}{\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\frac{1}{\mu}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1},
\end{aligned}
$$

where X is defined by

$$
\begin{aligned}
X &\equiv \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
&\times\left\{\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\frac{1}{\mu}\left(1+\frac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}.
\end{aligned}
$$

Rearranging the terms, we have

$$
\begin{aligned}
&\left(\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)+1 \\
&= \left[1+\frac{1}{\mu}\theta+\frac{(1+n)(1-\omega)}{\pi\omega\mu}\left(\theta+\frac{\beta\pi\left(\mu+\theta\right)\alpha(1+v)}{1+\alpha v}\right)\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right). \qquad \text{(A.6)}
\end{aligned}
$$

This equation describes the optimal relationship between τ^K and τ.

Third, we substitute (A.5) in the first-order condition with respect to τ into (A.2) to obtain

$$
\begin{aligned}
&\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left\{\left(\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right.\\
&\left.-\left[1+\frac{1}{\mu}\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}\\
&=\beta\pi\left(\mu+\theta\right)\alpha(1+v)\left(1-\tau\right)\left(1-\alpha\right)\\
&\times\left\{(-1)\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+\tfrac{v}{1+\alpha v}\tfrac{1}{1-\tau}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}\\
&=\beta\pi\left(\mu+\theta\right)\alpha(1+v)\left(1-\alpha\right)\\
&\times\left\{\left[(-1)\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right](1-\tau)+\tfrac{v}{1+\alpha v}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\tfrac{v}{1+\alpha v}\right\}.
\end{aligned}
$$

Rearranging the terms, we have

$$
\begin{aligned}
&\underbrace{\left\{\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}-1\right)-\beta\pi\left(\mu+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right]\right\}}_{D_1}\\
&\times(1-\tau)(1-\alpha)\\
&=\underbrace{\left\{\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\frac{1}{\mu}\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\tfrac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_2}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)\\
&-\underbrace{\left\{\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_3},
\end{aligned}
\tag{A.7}
$$

where D_1, D_2, and D_3 are defined by

$$
\begin{aligned}
D_1&\equiv\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}-1\right)-\beta\pi\left(\mu+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right],\\
D_2&\equiv\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\frac{1}{\mu}\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\theta\right]+\tfrac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},\\
D_3&\equiv\left[\tfrac{\pi\omega\mu(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta\pi(\mu+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.
\end{aligned}
$$

Eqs. (A.6) and (A.7) characterize the optimal τ and τ^K. Substituting (A.7) into (A.6) yields

$$
1-\tau^K=\underbrace{\frac{1-\left(\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_3}{D_1}}{\left[1+\tfrac{1}{\mu}\theta+\tfrac{(1+n)(1-\omega)}{\pi\omega\mu}\left(\theta+\tfrac{\beta\pi(\mu+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\tfrac{\beta\pi\mu}{1+\beta\pi\mu}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_2}{D_1}}}_{=\bar{T}^K}\cdot\frac{1}{\alpha}\cdot\frac{1}{1+b/k},
\tag{A.8}
$$

thereby verifying the conjecture of τ^K in (20). In addition, we substitute (A.8) into (A.7) to obtain

$$
1-\tau=\frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1}\equiv\bar{T},
\tag{A.9}
$$

thereby verifying the conjecture of τ in (21).

Fourth, we substitute (A.8) and (A.9) into (A.4) to derive the policy function of g :

$$
g=\underbrace{\frac{1+n}{(1+n)+\pi}\left(1+\tfrac{(1+n)(1-\omega)}{\pi\omega}\right)\frac{1}{\mu}\theta\left[(1-\alpha)\,\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^K}_{=\bar{G}}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)},
\tag{A.10}
$$

thereby verifying the conjecture in (22).

Finally, substituting τ^K, τ, and g into the government budget constraint in (13) leads to the following policy function of b':

$$(1+n)b' = \bar{B}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{A.11}$$

where $\bar{B}$ is defined by

$$\bar{B} \equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^K + (1-\alpha)\bar{T} - 1\right] + \frac{(1+n)+\pi}{1+n}\bar{G}.$$

■

A.2 Proof of Proposition 2

Suppose that $v = 0$ holds. The policy functions of b', τ^K, and τ presented in Proposition 1 then reduce to

$$b' = \frac{1}{1+n}\cdot\frac{\beta\pi\left[\mu\left(1-\alpha\right)-\alpha\theta\right]}{\frac{\pi\omega}{(1+n)(1-\omega)}\left(\mu+\theta\right)+\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]}A\left(k\right)^{\alpha}, \tag{A.12}$$

$$\tau^K = 1 - \frac{\frac{\pi\omega\mu}{(1+n)(1-\omega)}}{\frac{\pi\omega}{(1+n)(1-\omega)}\left(\mu+\theta\right)+\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]}\cdot\frac{1}{\alpha\left(1+b/k\right)}, \tag{A.13}$$

$$\tau = 1 - \frac{1}{1-\alpha}\cdot\frac{1+\beta\pi\mu}{\frac{\pi\omega}{(1+n)(1-\omega)}\left(\mu+\theta\right)+\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]}. \tag{A.14}$$

The ratio of B' to Y in (A.12) becomes

$$\frac{B'}{Y} = \frac{(1+n)b'N}{A\left(k\right)^{\alpha}N} = \frac{\beta\pi\left(\mu-\alpha\left(\mu+\theta\right)\right)}{\frac{\pi\omega}{(1+n)(1-\omega)}\left(\mu+\theta\right)+\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]}.$$

The expression above indicates that the ratio of B'/Y is positive if $\alpha\left(\mu+\theta\right) < \mu$, and that the ratio is increasing in π and n if $\alpha\left(\mu+\theta\right) < \mu$.

The capital income tax rate in (A.13) becomes

$$\tau^K = 1 - \frac{\mu}{\left(\mu+\theta\right)+\frac{(1+n)(1-\omega)}{\pi\omega}\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]}\cdot\frac{1}{\alpha\left(1+b/k\right)}.$$

In period 0, given the initial conditions of k_0 and b_0, the equation indicates that τ_0^K is decreasing in π and increasing in n. In period $t \geq 1$, we have $1+b/k = \mu/\alpha\left(\mu+\theta\right)$. Thus, τ^K is decreasing in π and increasing in n.

The labor income tax rate in (A.14) indicates that τ is decreasing in n. To see the effect of a higher π on τ, we take the first derivative of τ with respect to π and obtain

$$\frac{\partial\tau}{\partial\pi} = \frac{(-1)\left[\beta\mu\left(1+\theta\right)-\left(\mu+\theta\right)\left(\frac{\omega}{(1+n)(1-\omega)}+\beta\alpha\right)\right]}{\left(1-\alpha\right)\left\{\frac{\pi\omega}{(1+n)(1-\omega)}\left(\mu+\theta\right)+\left[1+\theta+\beta\pi\left(\mu+\theta\right)\alpha\right]\right\}^2}.$$

Thus, $\partial\tau/\partial\pi \gtrless 0$ if $\beta\mu\left(1+\theta\right) \lessgtr \left(\mu+\theta\right)\left(\omega/(1+n)(1-\omega)+\beta\alpha\right)$ holds.

When $v = 0$, the ratio of G to Y becomes

$$\frac{G}{Y} = \frac{g}{A(k)^{\alpha}}\left(1 + \frac{\pi}{1+n}\right) = \frac{\left(\frac{\pi\omega}{(1+n)(1-\omega)} + 1\right)\theta}{\frac{\pi\omega}{(1+n)(1-\omega)}(\mu+\theta) + [1 + \theta + \beta\pi(\mu+\theta)\alpha]}.$$

A direct calculation leads to

$$\frac{\partial(G/Y)}{\partial n} = \frac{(1-\omega)\theta\pi\omega[\mu - 1 - \beta\pi(\mu+\theta)\alpha]}{\{\pi\omega(\mu+\theta) + (1+n)(1-\omega)[(1+\theta) + \beta\pi(\mu+\theta)\alpha]\}^2},$$

$$\frac{\partial(G/Y)}{\partial \pi} = \frac{\theta\left[\frac{\pi\omega}{(1+n)(1-\omega)}(1-\mu) - \beta(\mu+\theta)\alpha\right]}{\left\{\frac{\pi\omega}{(1+n)(1-\omega)}(\mu+\theta) + [(1+\theta) + \beta\pi(\mu+\theta)\alpha]\right\}^2}.$$

Thus, we obtain

$$\frac{\partial(G/Y)}{\partial n} \gtrless 0 \Leftrightarrow \mu \gtrless 1 + \beta\pi(\mu+\theta)\alpha,$$

$$\frac{\partial(G/Y)}{\partial \pi} \gtrless 0 \Leftrightarrow 1 \gtrless \frac{(1+n)(1-\omega)}{\omega}\beta(\mu+\theta)\alpha + \mu.$$

■

A.3 Sources of Data

Data on average life expectancy, average population growth rate, and average government deficit are sourced from OECD.stat (https://stats.oecd.org/) (accessed on April 6, 2021). Data on labor and capital income tax rates are from Professor McDaniel's data archive (https://www.caramcdaniel.com/) (accessed on February 17, 2022).

We define government expenditure as the sum of general government consumption expenditure and general government gross fixed capital formation. Gross savings are calculated as gross national income less total consumption, plus net transfers. Data on government expenditure are from OECD.stat (https://stats.oecd.org/) (accessed on April 6, 2021), and on gross saving-GDP ratio are from Word Development Indicators (https://datatopics.worldbank.org/world-development-indicators/) (accessed on March 13, 2022).

A.4 Technical Details on Calibration

A.4.1 Choice of α

We fix the share of capital at $\alpha = 0.4$ following Mateos-Planas (2008). The assumption of $\alpha = 0.4$ is slightly higher than the assumption of $\alpha = 1/3$ made by Song et al. (2012) and Lancia and Russo (2016). We take the assumption of $\alpha = 0.4$ rather than $\alpha = 1/3$ to generate simulated results that match the evidence of the debt-GDP ratio and the capital income tax rate reported in Figure 2. From the empirical viewpoint, our assumption of $\alpha = 0.4$ reflects the recent global trend of a declining labor share, $1 - \alpha$, reported by Karabarbounis and Neiman (2014). Such a trend is also reported by International Labor Organization and the Organization for Economic Co-operation and Development (ILO and OECD, 2015).

A.4.2 Choice of R

We select a target of 1.054 for the gross interest rate per year using the following procedure. In some previous studies, the gross interest rate per year is targeted in the range of 1.04 (Song et al., 2012) to 1.07 (Mateos-Planas, 2008). We explore the target for R within this range. Specifically, we tentatively assign various values of μ and calibrate the remaining four parameters, θ, ω, β, and v under each value of μ. We then undertake comparative statics analysis of the impacts of population growth rate and life expectancy on the policy variables and find that when $\mu = 3.394$, the results of the comparative statics analysis best match the data in Figure 1. Since the R at which $\mu = 3.394$ is realized is $(1.054)^{30}$, we choose $R = (1.054)^{30}$ as the target for the gross interest rate per year.

A.4.3 Choice of Targets

We calibrate the parameters θ, β, ω, μ, and v to match the average statistics of the ratios of government deficit, government expenditure, and gross saving to GDP, the labor income tax rate, and the annual gross interest rate. It is natural to target the public debt-GDP ratio and the capital income tax rate instead of the budget deficit-GDP ratio and the savings-GDP ratio because we focus on the debt-GDP ratio and the capital income tax rate observed from Figure 2. However, we take the budget deficit-GDP ratio and the savings-GDP ratio as alternative targets because the values of the parameters of v and/or ω obtained by calibration do not fall within a plausible range when targeting the public debt-GDP ratio and the capital income tax rate.

A side effect of this choice is that the simulated results diverge from the observations in Figure 2 in the following two aspects. First, the value of the public debt-GDP ratio resulting from the simulation is lower than the actual value. We calibrate the parameters using the average deficit-GDP ratio to obtain empirically plausible values of the parameters, while we focus on the debt-GDP ratio in finding the evidence and in doing simulation. This discrepancy between the targeted and predicted variables works to derive a lower simulated value of the public debt-to-GDP ratio than the actual one.

Second, in the simulation, the ratio of government spending to GDP decreases rather than increases in response to an increase in the elderly as a percentage of the population. This suggests the divergence between prediction and reality; this divergence can be explained as described in the main text (see Section 4). Despite these two theoretical and real-world discrepancies, the simulation results are generally consistent with the observations in Figure 2 regarding the impact of aging on the public debt-GDP ratio, capital income tax rate, and labor income tax rate.

A.4.4 Role of μ

The parameter μ is introduced to improve the estimation of the parameters in calibrations. At first glance, the μ appears to be a parameter that constitutes the discount factor in the utility function. The total discount factor for the consumption utility of the older adult is then $\beta\pi\mu \simeq 1.5$, which means that individuals value their future consumption utility higher than their current consumption utility. This property may be disconcerting to some readers because it is different from the standard presumption that individuals discount future consumption utility. One interpretation of this somewhat strange property is to regard μ as a relative weight on the utility of private consumption goods in older adulthood. Since the expected utility in older adulthood can be rewritten as $\beta\pi \ln(d)^{\mu}(g)^{\theta}$, μ and θ can be regarded as the relative weights of the utilities obtained from private and public goods consumption in older adulthood, respectively.

To support the abovementioned interpretation, the two parameters, μ and θ, must be set within a range $(0, 1)$. However, in the present framework, the parameters that can be adjusted in the calibration are limited, so we could not obtain estimation results where both are less than 1. In particular, μ and β are negatively related, and μ needs to be set above 3.39 to keep β below 1. Alternatively, one might introduce a weight λ for the utility of the old-age public good. In the current framework, the elasticity of labor supply, v, takes a negative value under the values of λ that achieve $\beta < 1$. Due to this limitation, we introduced μ instead of λ to perform the calibration although the estimated value of μ is quite high relative to the standard assumption.

A.4.5 Sensitivity Analysis

The data used for calibration is the same as that employed in the examination of endogenous government expenditure. The values of the parameters α, n, and π are in line with those found in the endogenous government expenditure analysis. The exogenous government expenditure-GDP ratio is set to match the 1995-2016 average of the OECD, specifically with a value of 0.224 for G/Y. As for the remaining parameters θ, β, ω, μ, and v, we maintain the value previously used in the endogenous government expenditure analysis for θ specifically 0.988. The other four parameters are determined by targeting the labor income tax rate, the budget deficit-GDP ratio, the savings-GDP ratio (using the 1995-2016 averages of the OECD), and the annual gross interest rate. The annual gross interest rate, targeted at 1.053, is established following the same procedure as that applied in the endogenous government expenditure analysis. Assuming a 30-year period results in $R = (1.053)^{30}$. The values of the parameters obtained through calibration are: (β, ω, μ, v)=(0.989, 0.767, 3.305, 0.238).

■

B Online Appendix

B.1 Reformulation of V^M in (16) and V^O in (17)

The utility function for the middle-aged in period t, V^M, is

$$V^M = \ln\left(c - \frac{(l)^{1+1/v}}{1+1/v}\right) + \theta \ln g + \beta\pi\left(\mu \ln d' + \theta \ln g'\right).$$

We rewrite the term $c - (l)^{1+1/v}/(1+1/v)$ as follows:

$$\begin{aligned} c - \frac{(l)^{1+1/v}}{1+1/v} &= (1-\tau)wl - s - \frac{(l)^{1+1/v}}{1+1/v} \\ &= (1-\tau)w\left(k, l(\tau,k)\right) l(\tau,k) - s\left(\tau, k, l(\tau,k)\right) - \frac{(l(\tau,k))^{1+1/v}}{1+1/v}, \end{aligned} \tag{B.1}$$

where the first line comes from the budget constraint in middle age in (1), and the second line comes from the labor market-clearing wage rate in (11), the labor supply function in (9), and the saving function in (10). Rearranging the terms, we can reduce the expression in (B.1) to

$$c - \frac{(l)^{1+1/v}}{1+1/v} = \frac{1}{1+\beta\pi\mu} \cdot \frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}. \tag{B.2}$$

We rewrite the term d' as follows:

$$\begin{aligned} d' &= \left(1-\tau^{K\prime}\right) R's \\ &= \left(1-\tau^{K\prime}\right) \frac{R\left(k', l(\tau',k')\right)}{\pi} s\left(\tau, k, l(\tau,k)\right), \end{aligned} \tag{B.3}$$

where the equality in the second line comes from (10) and (12).

With (9), (10), (11), and (12), we can reformulate the equation in (B.3) further as follows:

$$\begin{aligned} d' &= \left(1-\tau^{K\prime}\right) \cdot \frac{\alpha}{\pi}\left[(1-\tau')(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A\left(k'\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \frac{1}{k'} \\ &\times \frac{\beta\pi\mu}{1+\beta\pi\mu} \cdot \frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A(k)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}}. \end{aligned} \tag{B.4}$$

Thus, with (B.2) and (B.4), we can reformulate the expression in (16) as

$$\begin{aligned} V^M &= V^M\left(\tau, g, \tau', \tau^{K\prime}, g', k'; k\right) \\ &\simeq \underbrace{(1+\beta\pi\mu)\frac{1+v}{1+\alpha v}\ln(1-\tau)}_{(\#1)} + \theta\ln g + \beta\pi\mu\ln\left(1-\tau^{K\prime}\right) \\ &+ \underbrace{\beta\pi\mu\frac{(1-\alpha)v}{1+\alpha v}\ln(1-\tau')}_{(\#2)} + \underbrace{(-1)\beta\pi\mu\frac{1-\alpha}{1+\alpha v}\ln k'}_{(\#3)} + \beta\pi\theta\ln g', \end{aligned} \tag{B.5}$$

where we omit the irrelevant terms from the expression in (B.5). Term (#1) includes the effects of the period-t labor income tax rate on $c - (l)^{1+1/v}/(1+1/v)$ and s_t; term (#2) includes the effect of the period-$t+1$ labor income tax rate on the interest rate R' through the labor supply l_{t+1}; and term (#3) includes the effect of physical capital on the interest rate R'.

Using (9) and (12), we reformulate the expression in (17) as follows:

$$V^{O} = V^{O}\left(\tau, \tau^{K}, g, k, b\right) \simeq \mu \ln\left(1-\tau^{K}\right) + \mu \frac{(1-\alpha)\, v}{1+\alpha v} \ln(1-\tau) + \theta \ln g, \tag{B.6}$$

where we omit the irrelevant terms from the expression.

■

B.2 Derivation of (A.1)

We reformulate the terms d_{τ^K}/d, $TR^K_{\tau^K}$, $\left(d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}\right)/d'$, and $g'_{b'}/g'$ in (27) as follows. First, consider the terms d_{τ^K}/d and $TR^K_{\tau^K}$. Given $d = \left(1-\tau^K\right) R\left(k, l(\tau,k)\right)(1+n)(k+b)$ and $TR^K = \tau^K R\left(k, l(\tau,k)\right)(k+b)$, we have

$$d_{\tau^K} = -R\left(k, l(\tau,k)\right)(1+n)(k+b) \Rightarrow \frac{d_{\tau^K}}{d} = \frac{-1}{1-\tau^K}, \tag{B.7}$$

$$TR^K_{\tau^K} = R\left(k, l(\tau,k)\right)(k+b). \tag{B.8}$$

Next, consider the term $\left(d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}\right)/d'$. Note that we can rewrite d' as

$$\begin{aligned} d' &= \left(1-\tau^{K\prime}\right)\frac{R\left(k', l(\tau',k')\right)}{\pi} s\left(\tau, k, l(\tau,k)\right) \\ &= \frac{\bar{T}^K}{\alpha}\frac{(1+n)k'}{s\left(\tau,k,l(\tau,k)\right)}\frac{1}{\pi}\alpha A\left(k'\right)^{\alpha-1}\left[\left(1-\tau'\right)(1-\alpha)A\left(k'\right)^{\alpha}\right]^{(1-\alpha)v/(1+\alpha v)} s\left(\tau,k,l(\tau,k)\right) \\ &= \frac{\bar{T}^K}{\alpha}(1+n)\frac{\alpha A}{\pi}\left[\left(1-\tau'\right)(1-\alpha)A\right]^{(1-\alpha)v/(1+\alpha v)}\left(k'\right)^{\alpha(1+v)/(1+\alpha v)}, \end{aligned} \tag{B.9}$$

where the equality in the second line comes from (10), (12), and (20). The capital market clearing condition, $(1+n)(k'+b') = s$, implies $\partial k'/\partial b' = -1$. Thus, we have

$$\frac{d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}}{d'} = \frac{\partial d'}{\partial k'}\frac{\partial k'}{\partial b'}\frac{1}{d'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}\frac{1}{k'}. \tag{B.10}$$

Finally, consider the term $g'_{b'}/g'$. Based on the conjecture of the policy function in (22), we have

$$\frac{g'_{b'}}{g'} = \frac{\partial g'}{\partial k'}\frac{\partial k'}{\partial b'}\frac{1}{g'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}\frac{1}{k'}. \tag{B.11}$$

With (B.10) and (B.11), we obtain

$$\mu\frac{d'_{\tau^{K\prime}}\tau^{K\prime}_{b'} + d'_{b'}}{d'} + \theta\frac{g'_{b'}}{g'} = (-1)\frac{(1+v)\alpha}{1+\alpha v}(\mu+\theta)\frac{1}{k'}. \tag{B.12}$$

By using (B.7), (B.8), (B.10), and (B.12), we can reformulate (27) as

$$\frac{\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{-1}{1-\tau^K}}{R\left(k, l(\tau,k)\right)(1+n)(k+b)} + \frac{\beta\pi}{1+n}\frac{(1+v)\alpha}{1+\alpha v}(\mu+\theta)\frac{1}{k'} = 0,$$

or as in (A.1).

■

B.3 Derivation of (A.2) and (A.3)

Equation (A.3) is immediate from substituting (B.12) in (28). The derivation of (A.2), which is equivalent to (26), is as follows.

We reformulate the terms in (26) as follows. First, consider the term $\frac{\pi\omega\mu}{(1+n)(1-\omega)} \cdot \frac{d_\tau}{d}$, which expresses the first derivative of $\frac{\pi\omega\mu}{(1+n)(1-\omega)} \ln\left(1-\tau^K\right) R(k, l(\tau,k))s$ with respect to τ. From (9) and (12), R is given by

$$R = \alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{1}{k}.$$

We substitute this into the term $\frac{\pi\omega\mu}{(1+n)(1-\omega)} \ln\left(1-\tau^K\right) R(k, l(\tau,k))s$ and obtain

$$\begin{aligned}&\frac{\pi\omega\mu}{(1+n)(1-\omega)}\ln\left(1-\tau^K\right)R(k,l(\tau,k))s\\ &=\frac{\pi\omega\mu}{(1+n)(1-\omega)}\ln(1-\tau^K)\alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{1}{k}(1+n)(k+b).\end{aligned}$$

Differentiation with respect to τ leads to

$$\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{d_\tau}{d} = (-1)\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}. \tag{B.13}$$

Next, consider the term $\left[c_\tau - (l)^{1/v} l_\tau\right] / \left[c - (l)^{1+1/v}/(1+1/v)\right]$, which expresses the first derivative of $\ln\left\{c(\tau,k,l(\tau,k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ. Using (B.2), we have

$$\ln\left\{c(\tau,k,l(\tau,k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\} = \ln\frac{1}{1+\beta\pi\mu}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}.$$

Differentiating $\ln\left\{c(\tau,k,l(\tau,k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ leads to

$$\frac{c_\tau - (l)^{1/v} l_\tau}{c - (l)^{1+1/v}/(1+1/v)} = \frac{1+v}{1+\alpha v}\frac{-1}{1-\tau}. \tag{B.14}$$

Third, consider the term $\beta\pi\mu d'_\tau/d' + \beta\pi\left(\mu\left(d'_{\tau^{K'}}\tau^{K'}_{k'} + d'_{k'}\right)/d' + \theta g'_{k'}/g'\right)k'_\tau$, which expresses the first derivative of $\beta\pi\mu\ln d' + \beta\pi\theta\ln g'$ with respect to τ. Using (9) and (12) and the conjecture of the policy function in (22), we can write

$$\begin{aligned}\beta\pi\mu\ln d' + \beta\pi\theta\ln g' &= \beta\pi\mu\ln\frac{\bar{T}^K}{\alpha}\frac{(1+n)k'}{s}\frac{R'}{\pi}s + \beta\pi\theta\ln\left(k'\right)^{\alpha(1+v)/(1+\alpha v)}\\ &\simeq \beta\pi\frac{\alpha(1+v)}{1+\alpha v}(\mu+\theta)\ln k'.\end{aligned}$$

Thus, we have

$$\beta\pi\mu\frac{d'_\tau}{d'} + \beta\pi\left(\mu\frac{d'_{\tau^{K'}}\tau^{K'}_{k'} + d'_{k'}}{d'} + \theta\frac{g'_{k'}}{g'}\right)k'_\tau = \beta\pi\left(\mu+\theta\right)\frac{\alpha(1+v)}{1+\alpha v}\frac{k'_\tau}{k'}. \tag{B.15}$$

Fourth, consider the term $TR_\tau + TR^K_\tau - R_\tau b$, which expresses the first derivative of $TR + TR^K - Rb$ with respect to τ. Using (9), (11), and (12), we can reformulate the term $TR +$

$TR^K - Rb$ as follows:

$$
\begin{aligned}
TR + TR^K - Rb &= \tau w\left(k, l(\tau, k)\right) l(\tau, k) + \tau^K R\left(k, l(\tau, k)\right)(k + b) - R\left(k, l(\tau, k)\right) b \\
&= \left[(1-\alpha) A (k)^{\alpha}\right]^{(1+v)/(1+\alpha v)} (1-\tau)^{(1-\alpha)v/(1+\alpha v)} \left[\tau + \tau^K \frac{\alpha}{1-\alpha}\left(1 + \frac{b}{k}\right) - \frac{\alpha}{1-\alpha}\frac{b}{k}\right].
\end{aligned}
\tag{B.16}
$$

Differentiating $TR + TR^K - Rb$ in (B.16) with respect to τ leads to

$$
\begin{aligned}
TR_\tau + TR^K_\tau - R_\tau b = &\left\{(-1)\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}\left[\tau + \tau^K \frac{\alpha}{1-\alpha}\left(1+\frac{b}{k}\right) - \frac{\alpha}{1-\alpha}\frac{b}{k}\right] + 1\right\} \\
&\times (1-\tau)^{(1-\alpha)v/(1+\alpha v)} \left[(1-\alpha) A (k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}.
\end{aligned}
\tag{B.17}
$$

Using (B.12) and (B.13)–(B.17) derived so far, we can rewrite (26) as

$$
\begin{aligned}
&(-1)\frac{\pi\omega\mu}{(1+n)(1-\omega)}\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau} + \frac{1+v}{1+\alpha v}(-1)\frac{1}{1-\tau} + \beta\pi\left(\mu+\theta\right)\frac{\alpha\left(1+v\right)}{1+\alpha v}\frac{k'_\tau}{k'} \\
&= \left\{(-1)\frac{(1-\alpha)v}{1+\alpha v}\frac{1}{1-\tau}\left[\tau + \tau^K \frac{\alpha}{1-\alpha}\left(1+\frac{b}{k}\right) - \frac{\alpha}{1-\alpha}\frac{b}{k}\right] + 1\right\} \\
&\times (1-\tau)^{(1-\alpha)v/(1+\alpha v)} \left[(1-\alpha) A (k)^{\alpha}\right]^{(1+v)/(1+\alpha v)} (-1)\frac{\beta\pi}{1+n}\frac{\alpha\left(1+v\right)}{1+\alpha v}\left(\mu+\theta\right)\frac{1}{k'}.
\end{aligned}
\tag{B.18}
$$

The remaining task is to compute k'_τ. Recall the capital market clearing condition in (14). Differentiating k' with respect to τ yields

$$
k'_\tau = (-1)\frac{1+v}{1+\alpha v}\frac{1}{1+n}\frac{\beta\pi\mu}{1+\beta\pi\mu}\frac{1/v}{1+1/v}(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha) A (k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}. \tag{B.19}
$$

Substituting (B.19) into (B.18) and rearranging the terms, we obtain (A.2).

■

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