The demographic transition and the asset supply channel

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Abstract

This paper examines the macroeconomic consequences of a demographic transition in an environment where a producer's capital structure is relevant, thereby introducing an asset supply channel. Producers are heterogeneous with respect to how productive they are in different states of the world, and may pursue different combinations of safe and/or risky securities issuance when financing projects. I simulate a demographic transition calibrated to replicate the US experience starting in 1880. This transition results in modest increases in output, larger increases in saving as a whole and, particularly, in a relative increase in saving in the form of safe assets. Lower capital costs lead to producer entry (and more issuance) and to a tilt towards safe issuance. I show that omitting this asset supply channel, as standard representative firm models do, results in a quantitatively important overestimation of the transmission effects of the demographic transition, with larger output gains despite smaller interest rate reductions.

Keywords: Demographic Transition; Overlapping Generations; Asset supply; Financing Policy

JEL codes: E21, E43, E44, G32, J11

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1 Introduction

The macroeconomic impact of demographic transitions – the process by which societies move from a context of high birth and death rates to one where both rates are significantly lower – has been extensively studied by both demographers and economists. Important transmission channels – like the evolution of the labor force, changes in savings and interest rates, implications for average labor productivity, and the impact on ideas, innovation, and business startup rates – have been identified, but the debate is as alive as ever, fueled by population aging concerns in developed economies and by the uncertain economic future of some of the world's youngest economies.[1]

In most of these models the credit channel plays an important role, but it often centers around households that face a life-cycle problem where saving is done in the form of capital claims issued by a representative firm (or government bonds, in some instances). I depart from this setting by introducing an environment in which producers are heterogeneous and their capital structure is consequential, that is, the Modigliani and Miller (1958) equivalence breaks down, enabling producers to optimally choose between issuing safe and/or risky liabilities.

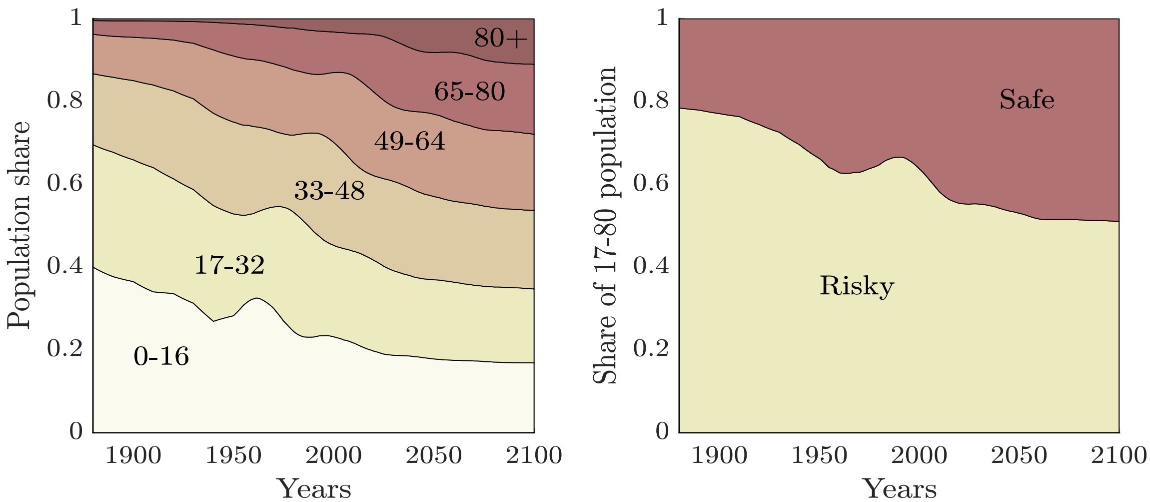
This asset supply endogeneity is important in a context where the demand for different asset types is moving in a systematic way because of demographic changes. In a recent study using Norwegian household tax records, Fagereng et al. (2017) find that, as they approach retirement, households substantially reduce their exposure to risky assets, increasing their exposure to safe ones.[2] To capture the effect of the demographic transition on asset demand, I assume that as households go through their life-cycle, their risk-aversion (exogenously) shifts, in a way to be made clear in the model, and it becomes optimal for younger cohorts to save in the form of risky assets and for older cohorts to save in the form of safe ones. Figure 1 shows the evolution of the share of different age cohorts in the U.S. from 1880 to 2100 (projected) as well as the implied constituency for the two asset types (risky and safe).[3] The producers' ability to respond to time-varying asset demand by adjusting the type of assets they issue creates a mechanism – the asset supply channel – through which the demographic transition affects macroeconomic outcomes. I find this channel, which is absent from representative-firm economies, is not only quantitatively meaningful for macroeconomic variables like output and interest rates, but also introduces a further motive – related to producer entry/exit - through which population aging leads to a productivity slowdown.

[1] For each of the channels mentioned, see Gagnon et al. (2021), Carvalho et al. (2016), Feyrer (2007), Acemoglu and Restrepo (2017), and Karahan et al. (2019). For surveys see Bloom et al. (2001), Birdsall et al. (2003), Lee (2016), and Iparraguirre (2020).

[2] Using consecutive cohorts, Fagereng et al. (2017) are able to sidestep two problems that have hitherto plagued attempts to measure how household portfolios vary with age: cohort effects (previous studies, like the evidence cited in Guiso et al. (2002), looked mostly at cross-sectional data) and endogenous stock-market participation.

[3] As discussed in detail in Section 2, I divide a household's lifetime into 6, 16-year-long, periods and assume their preferences are such that it is optimal for pre-retirement and just-retired households to buy safe assets, while young and prime-aged adults find it optimal to acquire risky assets. In reality things are much more complex, with households holding diversified portfolios, but this is simply a modeling device designed to capture the fact that older households' portfolios emphasize safer assets, while avoiding computational difficulties.

Figure 1: U.S. Population aging and implied asset-type constituency



Sources: Please see Appendix section A.

The demographic transition simulation is characterized by an increase in household saving and, particularly, by a relative increase in the demand for safe assets.[4] This, in turn, lowers interest rates (safe rates, in particular) and the cost of capital, something that is standard in most of the literature cited here. In contrast, by modeling heterogeneous producers that optimally issue both risky and safe securities, the model emphasizes two effects that are largely absent in the literature, but may have important implications for asset markets and macro variables: (i) the equilibrium reduction in interest rates leads to the entry of lower skilled managers and therefore lowers measured total factor productivity (TFP); (ii) producers change their financing asset mix towards safe issuance in a way that results in an equilibrium increase in the risk spread. These *asset supply* effects matter in a quantitatively important way, in the sense that ignoring them leads to a substantially different impact of the transition on interest rates and the growth of capital and output.[5]

Another way in which the model departs from most of the literature is in allowing for aggregate uncertainty within an overlapping-generations (OLG) framework: most OLG models with a full transition between steady-states feature either perfectly foresighted households or idiosyncratic uncertainty. The presence of aggregate uncertainty is necessary to deal with state-contingent and safe assets simultaneously. This means that interest rates and firm financing policies are time-varying for reasons that extend beyond the changing demographic variables, like recessions and booms. To be sure, there are studies in the literature that model a demographic transition and feature both risky

[4]To my knowledge, Bernanke et al. (2011) was the first to refer to a *savings glut*, while Caballero et al. (2017) emphasized that the supply of safe assets, in particular, have not kept up with increases in demand.

[5]For recent examples, besides the ones already cited, of research emphasizing the impact of the demographic transition on macroeconomic variables through changes in savings and interest rates, see Lisack et al. (2017), Auclert et al. (2019), Kopecky and Taylor (2020), and Liu and Poonpolkul (2020).

and safe assets, like Kopecky and Taylor (2020) and Liu and Poonpolkul (2020), but they do not emphasize the asset supply channel: the supply of safe assets is assumed to be a constant share of output in the former, while safe assets are in zero net supply in the latter.

What evidence is there for the channel I am introducing? It is difficult to tease out the effects of variables that change at such low frequencies as demographics since other, higher frequency, economic shocks or regulatory changes confound their impact. Nonetheless, given long enough data sets, researchers have been able to identify some important consequences of these long term movements. Two of the most important in the recent literature are the impact on safe rates and productivity. Using US data from 1890 to 2016, Lunsford and West (2019) find that changes in demographic variables are important long-term correlates of safe real interest rates, while Aksoy et al. (2019) find, using a panel VAR for 21 OECD economies from 1970–2014, that demographic effects are important drivers of productivity. While these studies provide empirical support for the effects of demographic transitions, they do not help in distinguishing the effects of the channels I am introducing – production heterogeneity and asset supply – from more standard models.

There is however, empirical ground on which these mechanisms can stand. One of the most distinct outcomes of the model is that demographic changes are important drivers of issuance. In the model, along the transition, producers increase the issuance of safe securities, and the vast majority switches to issuing both risky and safe securities. One can make the argument that the rise of modern corporate debt markets in the 20th century U.S. has helped achieve just this.[6] More recently, the rise in Collateralized Loan Obligations (CLO) issuance has served precisely the same role. CLOs are securities backed by commercial loans that have a tiered structure where debt tranches have priority over equity tranches. While these securities are largely originated by the shadow banking sector and not issued directly by producers, from the perspective of the model this distinction is immaterial and one can think of it as a way for producers to securitize risky cash flows (uncertain profits).[7]

Another important model outcome, and one that is necessarily absent from representative firm models, has to do with risk spreads. The simulation results imply that demographic factors have contributed to an increase in risky spreads. Using U.S. data between 1900 and 1990, Bakshi and Chen (1994) find that an increase in the average age predicts an increase in the risk premium, measured as stock returns (S&P 500) and the T-bill returns. Using data from 1926 to 1998, Goyal (2004) finds that increases in the share of middle-aged people (45 to 64) increase the risk premium, while increases in the fraction of retirees (65+) and younger workers (25 to 44) decrease it. I am able to replicate these egression results using model-simulated data.

Finally, the model predicts (measured) TFP should fall as a consequence of the demographic transition. One should be careful in interpreting this result, as there are many, unmodeled, mechanisms that could lead to TFP improvements and overturn the effects emphasized here over such

[6] According to data from the Securities Industry and Financial Markets Association (SIFMA), the US corporate debt outstanding w. as 16.4 percent of GDP in 1980, and by 2020 it had risen to 46.8 percent.

[7] According to SIFMA, the amount of CLOs outstanding in the U.S. rose from 1.1 billion USD in 1993 to 686 billion in 2019. See Amaral et al. (2020) for more details.

long periods of time. Nonetheless, there is a large literature on the productivity slowdown in the U.S. and other Western economies.[8] To wit, existing models of demographic transitions can deliver productivity slowdowns that come about because of demographic changes in the labor force, as in Krueger and Ludwig (2007). While such mechanism is present in my framework, there is another one that owes directly to one of the channels I am introducing: the heterogeneous nature of production. The declining cost of capital along the transition leads to the optimal endogenous entry of previously sub-marginal producers, reducing average TFP. This is in line with the findings of Decker et al. (2017), that worsening allocative efficiency can account for much of the decline in aggregate productivity growth.

Let us start by formalizing the model.

2 The environment

Time is discrete and infinite and there is aggregate uncertainty about the state of the economy. The aggregate shock $\eta \in \{L, H\}$ can be low (L) or high (H) and follows a first-order Markov process with known transition function $M : \{L, H\} \to \{L, H\}$. I assume that M is irreducible, hence globally ergodic.

There are two types of finitely-lived agents in the economy: households and producers. The latter are endowed with an idiosyncratic project ability that depends on the aggregate shock in a way to be made clear below, but lack funds to start a project. In order to do so, they sell securities to the households, whom I now cover in more detail.[9]

2.1 Households

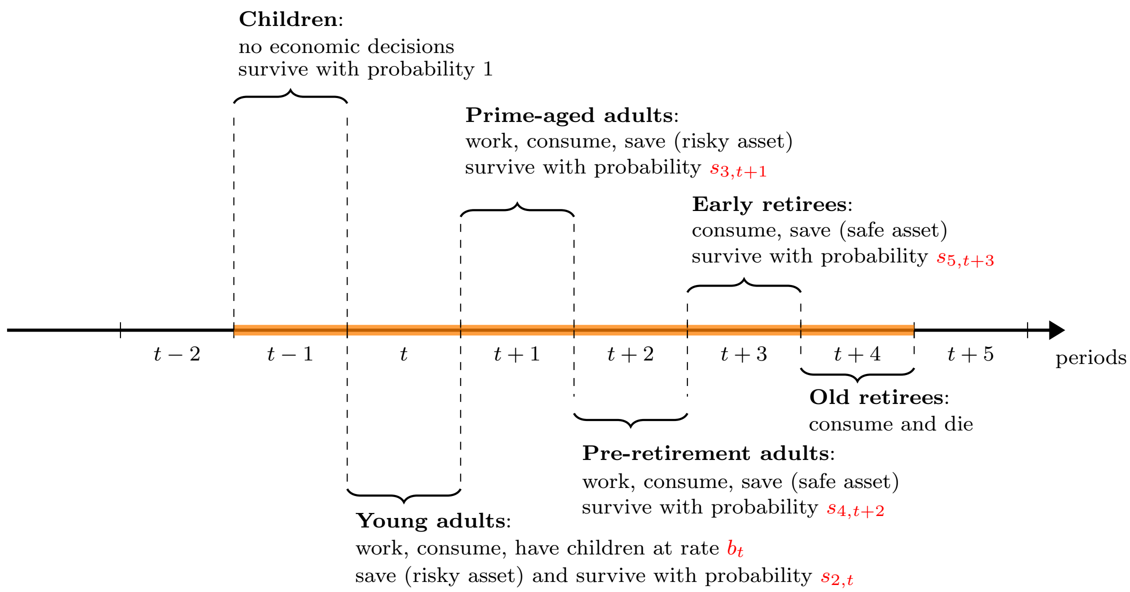
Each period, a new cohort of households, who may live for a maximum of six periods, is born. The size of each consecutive cohort is time-varying, as birth rates and survival rates may change over time. Given the measure of each cohort at time t, $N_{j,t}$, for $j = 1, \cdots, 6$, denote the total household population alive at time t by $N_t = \sum_{j=1}^{6} N_{j,t}$ and the growth rate of this population by $g_t = N_t/N_{t-1} - 1$. The share of households in each cohort j alive at time t is given by $sh_{j,t} = \frac{N_{j,t}}{N_t}$. Each cohort faces a time-varying survival probability given by $s_{j,t}$, with $s_{1,t} = 1$ and $s_{6,t} = 0$.

Households start out their lives with a childhood period when they make no meaningful economic decisions: they do not work and their consumption is determined by their parents. For the next three periods, households are endowed with one unit of labor per period which they deliver inelastically in return for wage earnings that they use for consuming and saving (borrowing is not allowed). In the first of these working periods, households also have children at rate b_t. Finally, there are two retirement periods; in the first of these periods, household face a simple consumption/saving decision

[8] See, for examples, Byrne et al. (2016), Gordon (2016), and Cette et al. (2016).

[9] A more streamlined, two-period, version of this setup is studied in Amaral and Quintin (2021).

Figure 2: Household timeline



whereas in the last period of their lives they simply consume the return on their savings. Figure 2 helps envision the households' life cycle.[10]

Households value their own lifetime consumption and that of their progeny (while they are children), but their instantaneous period preferences changes as they age. While young and prime-aged adults, households are risk-neutral with respect to consumption bundles and order them according to $u(c) = \log\left(E_t(c|\eta_t)\right)$, but in their pre- and early-retirement periods they become infinitely risk averse and their preferences are represented by $u(c) = \log\left\{\min\left(c(L), c(H)\right)\right\}$.

As discussed in the Introduction, this is a modeling device designed to capture the kind of life-cycle differences in portfolio composition found in Fagereng et al. (2017). Note that there is no internal inconsistency as far as intertemporal decision-making is concerned: households fully anticipate their preferences will change and plan accordingly.[11] It helps to think of households, in their capacity as savers, as being of two types: one type in their young and prime-aged adult periods, and another type in their pre- and early-retirement periods. Below, in Section 2.2, I assume that producers sell securities to each type separately. I then show that, given household preferences, the securities producers optimally sell to old, infinitely risk-averse, households pay a non-contingent dividend. As a result, I call these securities *safe* (S), while the securities younger agents buy are called *risky* (R), as their dividends may be contingent on the state of the world.

A young adult household in period t observes the period's aggregate shock, η_t, before making

[10] Note that while I am assuming that age 1 cohorts survive with probability 1, mortality at young ages is still captured in the calibration below through appropriately lower birth rates.

[11] I make this assumption because the alternative – having households optimally hold different types of securities simultaneously and having the shares of those holdings change with age – is much more complicated to model.

any decisions.[12] It takes as given wage rates $\{w_{t+j}\}_{j=0}^{2}$ and the menus of security returns offered by different producers $\left\{R^i_{t+j}(z)\right\}_{j=1}^{4}$ for the two different security types $i = R, S$ and chooses a consumption profile $c_{1,t}$, $\{c_{j,t+j-2}\}_{j=2}^{6}$, and security holdings from each producer z, $\{a_{j,t+j-2}(z)\}_{j=2}^{5}$, so as to solve:

$$\max E_t \Bigg\{ \gamma\left(1+b_t\right)\log c_{1,t} + \log c_{2,t} + s_{2,t}\beta \log\left\{E_t\left(c_{3,t+1}|\eta_t\right)\right\} + s_{3,t+1}\beta^2 \log\left\{E_{t+1}\left(c_{4,t+2}|\eta_{t+1}\right)\right\}$$

$$+ s_{4,t+2}\beta^3 \log\left\{\min\left(c_{5,t+3}(L), c_{5,t+3}(H)\right)\right\} + s_{5,t+3}\beta^4 \log\left\{\min\left(c_{6,t+4}(L), c_{6,t+4}(H)\right)\right\} \Bigg\}$$

$$\begin{aligned}
s.t. \quad c_{1,t} + c_{2,t} + \int_{Z_t} a_{2,t}(z)d\mu \quad & \leq (1-\tau)(1+\pi_2)w_t + b_{2,t}, \\
c_{3,t+1} + \int_{Z_{t+1}} a_{3,t+1}(z)d\mu \quad & \leq (1-\tau)(1+\pi_3)w_{t+1} + b_{3,t+1} + \int_{Z_t} a_{2,t}(z)R^R_{t+1}(z)d\mu, \\
c_{4,t+2} + \int_{Z_{t+2}} a_{4,t+2}(z)d\mu \quad & \leq (1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + \int_{Z_{t+1}} a_{3,t+1}(z)R^R_{t+2}(z)d\mu, \\
c_{5,t+3} + \int_{Z_{t+3}} a_{5,t+3}(z)d\mu \quad & \leq b_{5,t+3} + p_{t+3} + \int_{Z_{t+2}} a_{4,t+2}(z)R^S_{t+3}(z)d\mu, \\
c_{6,t+4} \quad & \leq p_{t+4} + \int_{Z_{t+3}} a_{5,t+3}(z)R^S_{t+4}(z)d\mu, \\
a_{2,t}, a_{3,t+1}, a_{4,t+2}, a_{5,t+3} \quad & \geq 0 \text{ and } c_{i,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} > 0.
\end{aligned}$$

In the first period of their adult lives, households choose a consumption bundle for themselves, $c_{2,t}$, and their children $c_{1,t}$, along with purchases of securities from the different z producers, $a_{2,t}(z)$.[13] These are financed by after-tax $(1-\tau)$, age-productivity weighted $(1+\pi_2)$, wage earnings and by incidental bequests $b_{2,t}$ to be defined below. As prime-aged adults, consumption, $c_{3,t+1}$, and assets purchases, $a_{3,t+1}(z)$, are financed not only by wage earnings and bequests, but also by interest income $a_{2,t}(z)R^R_{t+1}(z)$ received from the different z producers. Anticipating the equilibrium, I write the returns to risky securities bought while young and prime-aged adults as R^R and the returns to safe securities bought in pre- and early-retirement as R^S. In the pre-retirement period, the state of affairs is the same except for the fact that in equilibrium, the securities purchased, $a_{4,t+2}(z)$, have non-state-contingent returns. Finally, during the two retirement periods, households receive a pension in addition to interest income and bequests, and they do not save in the last period of their lives since death is certain. As pointed out above, wage income is subject to a proportional tax τ that is used to fund pension payments. Aggregate tax proceedings are invested for one period at the safe rate of

[12] To keep notation light I am omitting how variables depend on the aggregate state, but to be clear, consumers know the aggregate shock before making decisions (therefore we have $c_{2,t}(\eta_t)$), while producers, as the next section makes clear, make decisions before knowing the current shock (therefore we have $R^R_{t+1}(z,\eta_t)$).

[13] Child consumption is weighed by the relative size of the child cohort $(1+b_t)$ as well as by a filial care term, γ, to be calibrated below.

return and then disbursed equally among retirees.[14] Formally, pensions are given by:

$$p_{t+3} = \frac{\displaystyle\int_{Z_{t+2}} t_{t+2}(z,\eta_{t+2}) R^S_{t+3}(z) d\mu}{(1+g_{t+3})\sum_{j=5}^{6} sh_{j,t+3}},$$
$$p_{t+4} = \frac{\displaystyle\int_{Z_{t+3}} t_{t+3}(z,\eta_{t+3}) R^S_{t+4}(z) d\mu}{(1+g_{t+4})\sum_{j=5}^{6} sh_{j,t+4}},$$

where $t_t(z,\eta)$, the amounts invested by the social security fund across producers that issue safe securities must sum up to the total tax revenues:

$$\int_{Z_{t+2}} t_{t+2}(z,\eta_{t+2}) d\mu = \tau w_{t+2} \sum_{j=2}^{4} sh_{j,t+2}(1+\pi_j),$$
$$\int_{Z_{t+3}} t_{t+3}(z,\eta_{t+3}) d\mu = \tau w_{t+3} \sum_{j=2}^{4} sh_{j,t+3}(1+\pi_j).$$

Finally, the incidental bequests mentioned above arise because death is probabilistic. For simplicity, I assume they are equally distributed among the previous cohort:

$$b_{2,t} = \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1-s_{2,t-1}}{1+g_t} \int_{Z_{t-1}} a_{2,t-1}(z) R^R_t(z) d\mu,$$
$$b_{3,t+1} = \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1-s_{3,t}}{1+g_{t+1}} \int_{Z_t} a_{3,t}(z) R^R_{t+1}(z) d\mu,$$
$$b_{4,t+2} = \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1-s_{4,t+1}}{1+g_{t+2}} \int_{Z_{t+1}} a_{4,t+1}(z) R^S_{t+2}(z) d\mu,$$
$$b_{5,t+3} = \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1-s_{5,t+2}}{1+g_{t+3}} \int_{Z_{t+2}} a_{5,t+2}(z) R^S_{t+3}(z) d\mu,$$

In making decisions, households must form expectations regarding future variables, possibly several periods ahead. For example, when young adults decide how much to save, they need to conjecture what their income will be for the remainder of their lifetime. The rational expectations solution to OLG models with aggregate uncertainty and various cohorts is much more computationally challenging than the perfect foresight solution.[15] To make such challenge less burdensome, and because the main computational contribution of this piece is on solving the securities supply problem of heterogeneous producers described below, I posit an alternative expectation formation mechanism. Households form expectations by regressing variables of interest on their information set, which includes the path of all past variables up to the decision moment. In practice, and as shown in Appendix section F,

[14] The assumption that there is a one-period delay in the pay-as-you go scheme is important for Proposition 1 below.
[15] See, for example, Reiter (2015).

I start by assuming households have perfect foresight over demographic variables and regress future discounted wages and interest rates on lags of the state variables: the aggregate shock and aggregate asset holdings of each security type. In the appendix, I also argue that the resulting estimates share a crucial aspect with their rational expectations counterparts: the fact that agents are very close to being "right" on average.

2.2 Producers

There is a large mass of two-period-lived producers whose size changes at the same rate as the household population. Each is endowed with ability $z = (z_L, z_H) \in I\!R^2_+$, indexing their skills in operating a project in the two states of the world during the first period of their lives. Let $\mu(Z)$ denote the mass of producers in a given Borel set $\mathcal{Z} \subset I\!R^2_+$. In the upcoming calibration, I set this distribution μ in such a way that $z_H > z_L$ for most producers, which means the equilibrium aggregate producer profits and overall production are higher, on average, in the high state (H) compared to the low one (L) – hence the names. But the economy also contains producers whose profits are counter-cyclical, namely those that happen to be very productive when the aggregate state is low.

In the first period of their lives, producers can choose to operate a project before the aggregate shock is realized. In order to do so, they must pay an entry cost (proportional to population) of $N_t e \geq 0$ units of the consumption good, and commit whatever operational capital $k_t \geq 0$ they plan to use. A project that is activated and operated by a producer of type z with $k_t \geq 0$ units of capital and $n_t \geq 0$ units of labor yields gross output

$$y\left(k_t, n_t; z(\eta_t)\right) = z(\eta_t)\left(k_t^{\alpha} n_t^{1-\alpha}\right)^{\nu}$$

at the end of the period once state η_t is realized, where $\alpha, \nu \in (0, 1)$.

Producers value consumption in both periods of their lives. The consumption bundle of a time t-born producer is a non-negative vector: $(c_{y,t}, c_{o,t+1}(L), c_{o,t+1}(H))$ where $c_{y,t}$ is their consumption at the start of the first period of their life, before the period t shock is realized and production takes place, while $(c_{o,t+1}(L), c_{o,t+1}(H))$ is their second-period consumption, which depends on the realization of the aggregate shock at the end of time t. These consumption profiles are ranked according to linear preferences:

$$c_{y,t} + \epsilon E\left(c_{o,t+1}(\eta_t)|\eta_{t-1}\right),$$

where ϵ is small, but strictly positive. Following the realization of the aggregate shock, conditional on having activated a project with capital k_t, and taking the wage rate w_t as given, a producer of skill z chooses the labor input n_t by solving

$$\Pi(k_t, w_t; z(\eta_t)) \equiv \max_{n_t > 0} y\left(k_t, n_t; z(\eta_t)\right) - n_t w_t,$$

where Π denotes net operating income.

Since they lack funds to operate and finance their project, active producers obtain external funds by selling securities – claims to their output – to households. The fundamental friction in this process is that selling securities to one type of household (older or younger) is costless, whereas selling securities to both types simultaneously carries a cost (proportional to population) $N_t\zeta > 0$. One way to rationalize this is to think of the markets for the two types as being segmented and subject to different regulations, for example related to retirement, in such a way that operating in multiple security markets is more onerous than operating in a single one. But more broadly, one can think of ζ as proxying for costs associated with managing a more complex capital structure.

I follow Allen and Gale (1988) in assuming that producers take the households' willingness to pay for different securities as given when making security issuance decisions. A security is a mapping from the aggregate state to non-negative dividends.[16] Denote by $q_{i,t}(x_{i,t}(L), x_{i,t}(H))$ the price households are willing to pay for a marginal amount of a security of type $i = R, S$ with payoffs $(x_{i,t}(L), x_{i,t}(H)) \geq (0, 0)$ at date t in each respective state of the world. Note that so far, I am not imposing that securities sold to older households (type $i = S$) have non-contingent payoffs: that will be a consequence of the assumptions on preferences. All I am requiring at this point is that the securities sold to the two different types of households (older versus younger) may be different and that selling to both types simultaneously carries a cost.

Active producers of type (z_L, z_H) choose capital $k_t > 0$ and non-negative security payoffs $(x_{i,t}(L), x_{i,t}(H))$ for $i = R, S$, in order to solve:

$$\max \quad c_{y,t} + \epsilon E\left(c_{o,t+1}(\eta_t)|\eta_{t-1}\right)$$

subject to

$$\begin{aligned}
c_{y,t} &\leq q_{S,t}\left(x_{S,t}(L), x_{S,t}(H)\right) + q_{R,t}\left(x_{R,t}(L), x_{R,t}(H)\right) - k_t - N_t\left(e + \zeta 1_{\{x_{S,t}>0, x_{R,t}>0\}}\right), \\
c_{o,t+1}(L) &\leq \Pi(k_t, w_t(L); z_L) - x_{S,t}(L) - x_{R,t}(L), \\
c_{o,t+1}(H) &\leq \Pi(k_t, w_t(H); z_H) - x_{S,t}(H) - x_{R,t}(H), \\
c_{y,t} &\geq 0, c_{o,t+1}(L) \geq 0, c_{o,t+1}(H) \geq 0,
\end{aligned}$$

where the indicator function $1_{\{x_{S,t}>0, x_{R,t}>0\}}$ takes the value one if both types of securities are issued, in which case the producer must bear cost $N_t\zeta$, and zero otherwise. The first constraint in this problem states that the producer gets to consume whatever is left of their revenue from selling securities, $q_{S,t}\left(x_{S,t}(L), x_{S,t}(H)\right) + q_{R,t}\left(x_{R,t}(L), x_{R,t}(H)\right)$, after all costs, $k_t + N_t\left(e + \zeta 1_{\{x_{S,t}>0, x_{R,t}>0\}}\right)$, have been covered. Producers only become active if they can meet this constraint, as in that case they can enjoy non-negative consumption compared to their opportunity cost of not becoming active, which is zero. The second and third constraints (combined with the non-negativity constraints on old consumption) are no-default constraints and require that, in each state of the world, producers

[16] Allowing for negative dividends (short-selling) would pose existence problems even in one-period versions of the model. See Allen and Gale (1988).

must be able to cover their security payments with their operating income.

2.2.1 Security pricing and equilibrium

A key feature of the equilibrium I am about to define is that the households' willingness to pay for securities has to coincide with the pricing functionals implied by the security payments set by producers.

Recall that households take as given the set of securities available at the start of a particular period. From their point of view, they face a menu of security returns:

$$R_t^i(z,\eta) = \frac{x_{i,t}(z,\eta)}{q_{i,t}(x_{i,t}(z,L), x_{i,t}(z,H))},$$

for securities of type $i = \{R, S\}$ issued by producer type $z = (z_L, z_H)$, with the convention that $R_t^i(z,\eta) = 0$ if type z is inactive.

Since they have risk-neutral preferences, young and prime-aged adults purchase securities from those producers offering the highest expected return. Letting

$$\bar{R}_t^R = \max_z M\left(L|\eta_{t-1}\right) R_t^R(z,L) + M\left(H|\eta_{t-1}\right) R_t^R(z,H),$$

these agents are willing to pay:

$$q_{R,t}(x(L), x(H)) = \frac{M\left(L|\eta_{t-1}\right) x(L) + M\left(H|\eta_{t-1}\right) x(H)}{\bar{R}_t^R}$$

for a marginal investment in a security with payoff $(x(L), x(H))$ at date t.

Pre-retirement and recently retired agents, on the other hand, have Leontieff preferences over their remaining consumption plans. Consider such agents alive at time t and define

$$\bar{R}_{4,t}^S \equiv \frac{\min\left(c_{5,t}(L), c_{5,t}(H)\right)}{a_{4,t-1}}, \text{ and } \bar{R}_{5,t}^S \equiv \frac{\min\left(c_{6,t}(L), c_{6,t}(H)\right)}{a_{5,t-1}},$$

as the effective return these agents realize on their investment at the optimal solution to their problem. Anticipating the definition of equilibrium to simplify exposition, note that because producers sell the same securities to cohorts $j = 4, 5$, we will have $\bar{R}_t^S = \bar{R}_{4,t}^S = \bar{R}_{5,t}^S$.

Because they only value marginal payoffs in the lowest consumption state, if $c_{j,t}(H) > c_{j,t}(L)$ at the optimal solution, the willingness to pay for a marginal investment in a security with payoffs $(x(L), x(H))$ of an agent of cohort $j = 4, 5$ would be

$$q_{S,t}(x(L), x(H)) = \frac{x(L)}{\bar{R}_t^S}.$$

The symmetric condition must hold when $c_{j,t}(H) < c_{j,t}(L)$. Finally, when $c_{5,t}(L) = c_{5,t}(H)$ and

$c_{6,t}(L) = c_{6,t}(H)$, which, I will argue below, must hold in equilibrium at all dates,

$$q_{S,t}(x(L), x(H)) = \frac{\min(x(L), x(H))}{\bar{R}_t^S}.$$

Let agents of ages $j = 2, \cdots, 5$ enter date 0 with wealth $a_{j,-1} > 0$. Then, the state of the economy at date 0 is fully described by $\Theta_0 = \{a_{j,-1}, \eta_{-1}\}$, $j = 2, \cdots, 5$, where $\eta_{-1} \in \{L, H\}$ is the aggregate shock at date $t = -1$. Producers only produce when young hence do not accumulate resources. All active producers must therefore raise all the funds they use from households.

To define a stationary equilibrium it will be helpful to express variables in per capita terms, where $\hat{x}_t = \frac{x_t}{N_t}$. A stationary equilibrium is then defined, for all dates and for all possible histories of aggregate shocks, as a list of:

- consumption plans $\{\hat{c}_{j,t}(\eta)\}$, for $j = 1, \cdots, 6$ and security purchases $\{\hat{a}_{j,t}(\eta, z)\}$, for $j = 2, \cdots, 5$ for households;
- a set $Z_t \in \mathcal{Z}$ of active producers and their corresponding consumption plans $(\hat{c}_{y,t}(z), \hat{c}_{o,t+1}(\eta, z))$, capital $\left\{\hat{k}_t(z)\right\}$, labor $\{\hat{n}_t(z)\}$, and a menu of security payoffs $\{\hat{x}_{i,t}(z, \eta_t)\}$ for security types $i = R, S$;
- social security purchases of safe securities from each producer z: $\hat{t}_t(z, \eta_t)$;
- a list of prices: wage rates $\{\hat{w}_t(\eta)\}$, payoff pricing functionals $\{\hat{q}_{S,t}, \hat{q}_{Rt}\}$, and the associated returns $\{\bar{R}_t^R(z, \eta_t)\}$ and $\{\bar{R}_t^S(\eta_t)\}$,

such that:

1. Security purchases and consumption plans solve each household's problem;
2. Security menus, capital and labor choices, and consumption plans solve each producer's problem;
3. The goods market clears:

$$\begin{aligned} \int_{Z_t} \hat{y}(\hat{k}_t(z), \hat{n}_t(\eta); z) d\mu \quad &= \quad \sum_{j=1}^{6} sh_{j,t}\hat{c}_{j,t} + \int_{Z_t} \hat{c}_{y,t}(z) + \hat{c}_{o,t}(z) d\mu \\ &+ \quad \int_{Z_{t+1}} (1 + g_{t+1})\hat{k}_{t+1}(z) + e + \zeta 1_{\{x(z)_{R,t+1}>0, x(z)_{S,t+1}>0\}} d\mu; \end{aligned}$$

4. The market for labor clears:

$$\int_{Z_t} \hat{n}(\hat{w}_t(\eta); z) d\mu = \sum_{j=2}^{4} sh_{j,t}(1 + \pi_j);$$

5. The market for each security type clears, i.e., for μ-almost each producer type z:

$$\begin{aligned} sh_{2,t}\hat{a}_{2,t}(z) + sh_{3,t}\hat{a}_{3,t}(z) &= \hat{q}_{R,t}\left(\hat{x}_{R,t}(z,L), \hat{x}_{R,t}(z,H)\right), \text{ and} \\ \hat{t}_t(z) + sh_{4,t}\hat{a}_{4,t}(z) + sh_{5,t}\hat{a}_{5,t}(z) &= \hat{q}_{S,t}\left(\hat{x}_{S,t}(z,L), \hat{x}_{S,t}(z,H)\right); \end{aligned}$$

6. Social security purchases of safe assets equal tax revenues:

$$\int_{Z_t} \hat{t}_t(z) d\mu = \tau \hat{w}_t \sum_{j=2}^{4} sh_{j,t}(1+\pi_j);$$

7. Pricing functionals are consistent with the household's willingness to pay for marginal payoffs:
   (a) $\hat{q}_{R,t}(\hat{x}(L), \hat{x}(H)) = \frac{M(L|\eta_{t-1})\hat{x}(L) + M(H|\eta_{t-1})\hat{x}(H)}{\bar{R}_t^R}$,
   (b) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\min(\hat{x}(L),\hat{x}(H))}{\bar{R}_t^S}$ if $\hat{c}_{j,t}(L) = \hat{c}_{j,t}(H)$ for both $j = 5, 6$,
   (c) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\hat{x}(H)}{\bar{R}_t^S}$ if $\hat{c}_{j,t}(L) < \hat{c}_{j,t}(H)$ for both $j = 5, 6$,
   (d) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\hat{x}(L)}{\bar{R}_t^S}$ if $\hat{c}_{j,t}(L) > \hat{c}_{j,t}(H)$ for both $j = 5, 6$,

   for all possible securities $(\hat{x}(L), \hat{x}(H)) \geq (0, 0)$ where:

$$\bar{R}_t^R = \max_z M\left(L|\eta_{t-1}\right) R_t^R(z, L) + M\left(H|\eta_{t-1}\right) R_t^R(z, H),$$

   while

$$\bar{R}_t^S = \frac{\min\left(\hat{c}_{j,t}(L), \hat{c}_{j,t}(H)\right)}{\hat{a}_{j,t-1}} \quad \text{for both } j = 5, 6.$$

The properties of a two-period version of this type of equilibrium are studied in Amaral and Quintin (2021). Some of the results carry through to the present environment. In particular, the following proposition establishes that in equilibrium, retired household consumption is non-state contingent, and therefore, pre- and early-retired households only buy safe securities. It also establishes that the equilibrium return on risky securities has to be larger than that of safe securities, implying that earlier in life, households choose to purchase risky securities.

Proposition 1. *In any equilibrium, the consumption of retired agents is risk-free and they only purchase safe securities the period before. Furthermore, in any equilibrium, $\bar{R}_t^R \geq \bar{R}_t^S$, with a strict inequality whenever $\zeta > 0$ and a positive mass of producers issue two securities.*

The intuition for the first part of the result is as follows: if the retired agents' consumption was such that for $j = 5, 6$, $\hat{c}_j(H) > \hat{c}_j(L)$ then, a period before, these agents would pay nothing for the H state payoff on any security, as their marginal valuation of consumption in that state would be zero. Nonetheless, in order for $\hat{c}_j(H) > \hat{c}_j(L)$ to hold, a positive mass of securities with higher payoffs in state H than in state L must be sold to these agents a period before. But the producers selling those securities would be strictly better-off either selling the state H payoff to younger agents, or simply consuming it themselves. The case in which $\hat{c}_j(H) < \hat{c}_j(L)$ for $j = 5, 6$, can be similarly ruled out.[17]

The second part of the result also follows by contradiction: if it were the case that $\bar{R}_t^R < \bar{R}_t^S$, then it would not be profitable for any producer to issue securities to pre-retirement agents as they would always be better off selling risk-free assets to younger (risk-neutral) investors and consuming excess profits, but this contradicts market clearing for safe securities.

Another important equilibrium property (again, see Amaral and Quintin (2021) for a formal argument) is that producers who choose to issue safe securities, issue as much of it as possible. That is, their non-contingent dividend payment equals their minimum profit across states. What do they do with the remainder of their profits in the state when their profit is highest? If ζ is low enough they can pay the security creation cost and issue risky securities. If this is too expensive, they simply consume the remainder in the second period of their lives.[18]

In conclusion, at any date t, besides producers that are inactive, there can be three types of active producers as far as their financing goes. Some of them only sell safe securities to older, infinitely risk-averse, cohorts. This makes sense for producers whose productivities across states are sufficiently similar, that is businesses that are less sensitive to the cycle, like consumer staples, or utilities. Others only sell risky securities to younger cohorts in the risk-neutral periods of their lives. These are producers whose productivities are sufficiently different across states and, importantly, their productivity in one of the states is so low that it does not make financial sense to pay the security creation fee to sell both types of securities. Examples of these would be highly cyclical businesses, like consumer discretionary. Finally, the remaining active producers sell both types of securities and pay the security creation cost because even though their business is cyclical they are productive enough even in even bad times.

Having established the main theoretical properties of the model, we can now turn to the main exercise.

3 Simulating a demographic transition: U.S. (1880-2100)

The main experiment simulates the effects of demographic changes in the U.S. from 1880 to 2100. While this is not a full demographic transition – by 1880 the U.S. was already well into its transition – it is long enough for it to have important macroeconomic implications.[19]

[17] This argument relies on having contemporaneous consumption depend only on the contemporaneous shock through the rate of return. Indeed, if today's pension benefits were purely pay-as-you-go and depended on today's wages (instead of yesterday's, as it is the case), then the rate of return that would fully stabilize consumption across states would no longer necessarily be non-contingent.

[18] Note that ϵ is a very small but strictly positive number, meaning that producers who only sell securities to infinitely risk-averse cohorts are better off consuming any excess profits in the second period of their lives, rather than selling the right to those cash-flows in the form of contingent securities to infinitely risk-averse cohorts at zero price.

[19] Delventhal et al. (2019) report transition lengths between 50 and 200 years.

I start by assuming that each *model period* lasts for 16 years and map the age distributions from the data into 6 model age-cohorts indexed by j: 0:15, 16:31, 32:47, 48:63, 64:79, and 80+, for 14 consecutive 16-year periods indexed by t: 1880:1895, 1896:1911, ... , 2088:2103. I use data from various sources described in Appendix Section A to compute the four demographic model inputs over time: (i) the share of population in each cohort: $sh_{j,t}$; (ii) the population growth rate: g_t; (iii) the survival rates for each cohort: $s_{j,t}$ and; (iv) the birth rates: b_t.

I use population data to compute the cohort share evolution shown in panel A of Figure 3.[20] There are some salient characteristics to note. First, and at the risk of pointing out the obvious, the population aged significantly. In 1880, youths outnumbered retirees ten-to-one and the median age was 20; cut to 2100 when there will be 50 percent more retirees than youths and the median age will be 45. Second, despite the ageing, the dependency ratio in 2100 will be roughly the same as it was in 1880, when the dependency ratio was roughly 0.75. It has, since then, exhibited a W-like behavior, as the dependency switched from mostly youths to mostly retirees.[21] Finally, the baby-boomer generation was large enough to have caused an echo that can be seen rippling through the cohort shares.

I also use these population data to compute population growth rates, g_t, shown in panel B of Figure 3. As is characteristic of the latter stages of a demographic transition, the U.S. population growth rate has been on a long-run downward trend from a yearly rate of 2.5 percent in 1881, to a paltry 0.2 percent expected in 2100.

Ideally one would like to use the cohort shares over time to compute survival rates, $s_{j,t}$, and birth rates, b_t. The problem is that the underlying population data includes migration and, as is well known, the U.S. had very large net immigration flows throughout this period. These flows are not age-independent, which means they give rise to implied survival rates, $\hat{s}_{j,t}$, and birth rates, $\hat{b}_t$, that are inaccurate from a vital statistics or a fertility sense.[22] It is important to get accurate measures of survival and birth rates because these are inputs into the households' expected utility and therefore condition household-level decision making.

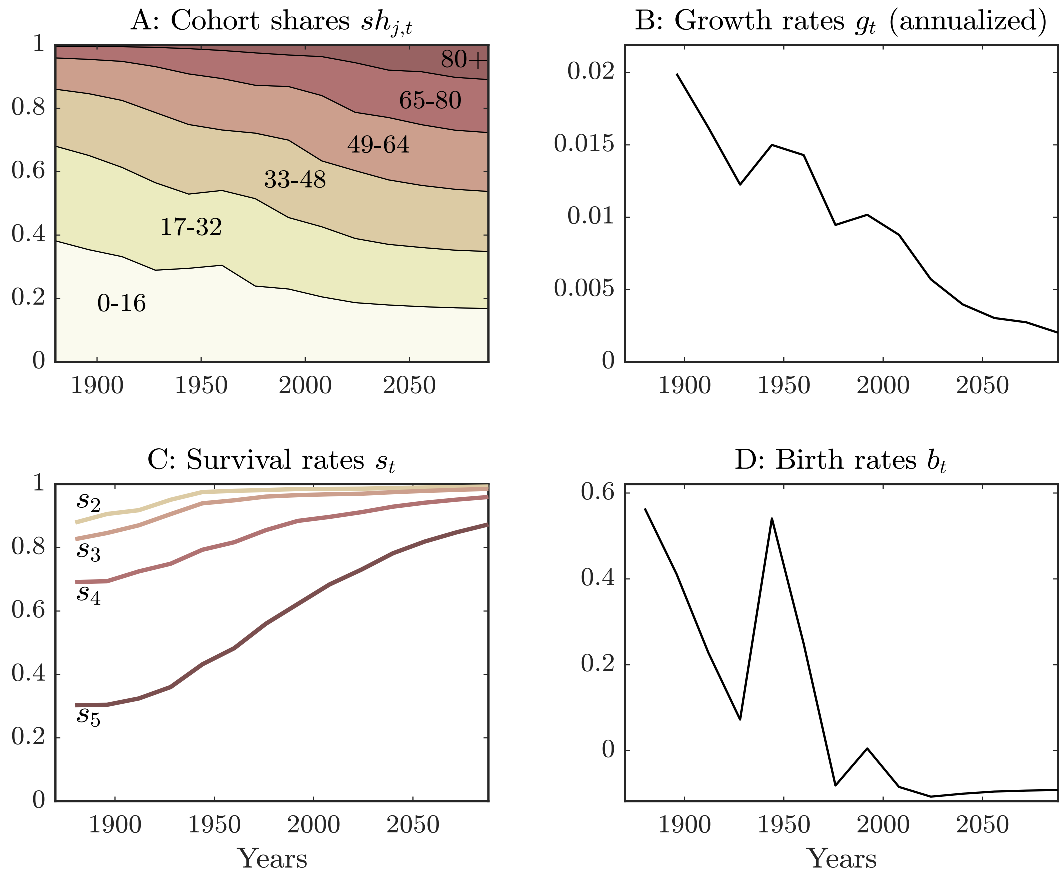
To circumvent this problem I use vital statistics data to compute survival rates and total fertility rate (TFR) data to compute birth rates. This raises the concern that these independently derived measures are not consistent with the evolution of cohort shares and population growth rates I obtained above. Indeed they are not; in general we get $s_{j,t} \neq \hat{s}_{j,t}$ and $b_t \neq \hat{b}_t$. The reason this does not raise a consistency problem is that in forming expectations, households are assumed to know the full path of all demographic variables: $\{g_t, sh_{j,t}, s_{j,t}, b_t\}_{t=1880:1895}^{2088:2103}$, and therefore they do not derive population

[20] This is the same as in the left panel of Figure A, but in model periods (16 years) instead of actual years.

[21] Here I am using a common definition of the dependency ratio as the number of youths (0-16) and retirees (65+) as a share of working-age households.

[22] I use hats to denote measures implied by total population data. Letting $P_{j,t}$ denote the number of people in cohort j in period t, we have $\hat{s}_{j,t} = \frac{P_{j+1,t+1}}{P_{j,t}}$ and $\hat{b}_t = \frac{P_{1,t}}{P_{2,t}}$. These may differ from *actual* measures in the presence of non-zero net migration. In some instances, implied survival rates are larger than one, for example.

Figure 3: Demographic model inputs



growth rates and cohort shares from survival and birth rates.[23] The former are important for market clearing purposes, while the latter are important for optimization purposes.

The cohort survival rates are computed using actuarial life-tables adjusted for the model's age cohorts and are shown in panel C of Figure 3. Note that I assume that $s_1 = 1$ (not shown in the chart), but child mortality up to 16 is being captured in the way birth rates are computed in the next paragraph. Survival rates improve across all cohorts, but the improvements in old-age survivorship are particularly large. It is precisely the fact that people are living well beyond age 80 more frequently that led me to include two 16-year-long retirement periods instead of just one.

In the context of the model, young adults weigh their children's utility with the term $\gamma(1 + b_t)$, where b_t represents how much larger their progeny is relative to the current household and γ is a filial care term to be calibrated below to match expenditures with children. I use TFR data and child mortality to compute model-adjusted birth rates, b_t. I normalize TFR by 2 (meaning that a household with 2 children has $b_t = 0$) and adjust for survival rates up to 16 years of age. These birth

[23] See Appendix Section F for details on the expectation formation mechanism.

rates are shown in panel D of Figure 3. [24] It is worthwhile noting the large spike in births from the baby-boomer generation. TFR was not as high then as at the turn to the 20the century, but child mortality rates were considerably lower, so the adjusted birth rates are almost as high.

3.1 Calibration

The non-demographic part of the calibration targets pertinent moments of the US economy between 2008 and 2024, the ninth period of the transition. A subset of parameters are set exogenously. Given a period length of 16 years, a low state is a rare, but necessarily protracted, event: a disaster in the sense of Barro and Ursua (2008), who define it as a drop in output, from peak to trough, of 10% or more. They find economies spend, on average, 12% of time in those depressed states, but their dataset does not include disasters longer than 16 years. To match this, I set the elements of the aggregate shock's transition matrix M so that the model economy almost never spends more than one period in the low state ($M_{LL} = 0.001$), and the probability of remaining in the high state is set to $M_{HH} = 0.82$, so that the model economy spends 12% of time in the low state.

I set the support of managerial talent to $\mathcal{Z} = [0,1] \times [0,1]$, and assume that μ is distributed according to a truncated bivariate log-normal with mean $\bar{z} = (\bar{z}_L, \bar{z}_H)$ and variance-covariance matrix

$$\Phi = \begin{bmatrix} (\varsigma \bar{z}_L)^2 & \sigma \\ \sigma & (\varsigma \bar{z}_H)^2 \end{bmatrix}$$

where $\varsigma > 0$ is calibrated below. That is, I take the two skill levels to be correlated at the population level (controlled by $\sigma > 0$, which is also calibrated below) and normalize the two variance terms so that the coefficient of variation of managerial talent is the same in the two aggregate states. I set the mean producer productivity in the good state to $\bar{z}_H = 0.05$, a simple normalization.

The income tax rate is set to $\tau = 0.124$, replicating the U.S. social security tax (6.2% on the employee and 6.2% on the employer), but assuming full incidence on the worker. The production function coefficients are ν, which regulates the income share of producer rents, and α, which determines the share of the remaining income accruing to capital. I set the latter to $\alpha = 0.4$ following a vast literature that puts the capital income share in the 35% to 45% range, and calibrate the former below. I set $\epsilon = 10^{-6}$, a number small enough so that producers strictly prefer consuming left-over output to selling it for nothing. Finally, to discipline the human capital accumulation, I normalize $\pi_2 = 0$ and set $\pi_3 = 0.28$ and $\pi_4 = 0.16$ so that the earnings gains in the model match those implied by the findings of Rupert and Zanella (2015) and Lagakos et al. (2018), a process detailed in Appendix B.

These parameters are summarized on the top panel of Table 1. The remaining parameters are set jointly so that selected moments of the model economy in transition period 9 (corresponding to 2008-2024) match their U.S. data counterparts, and appear on the bottom panel of the table.

[24] Please see the Appendix section A for more details.

Table 1: Calibration

Parameters	Targets
$M_{HH} = 0.82$	12% of time spent in low state
$M_{LL} = 0.001$	Two consecutive periods in low state extremely rare
$\bar{z}_H = 0.05$	Normalization
$\tau = 0.124$	Social security statutory tax rate
$\pi_2 = 0$	Normalization
$\pi_3 = 0.28$	See Appendix B
$\pi_4 = 0.16$	See Appendix B
Parameters	**Joint targets**
$\beta = 0.808$	Risk-free rate: 2.25%
$\bar{z}_L = 0.046$	Drop in output: 17%
$\sigma = 0.703$	Risk premium: 4.5%
$\varsigma = 16.8$	Employment share in 50% smallest projects: 5%
$\nu = 0.81$	Producer rents: 10% of GDP
$\gamma = 0.80$	Share of child spending on parental income: 32.3%
$e = 0.156$	Entry costs: 0.75% of GDP
$\zeta = 0.071$	Security creation costs: 0.18% of GDP

I set the discount factor, β, (together with the other parameters) so that the risk-free rate is 2.25% in yearly terms. This is the average (from 1997 to 2019) real prime corporate bond yield as measured by the ICE BofA AAA US Corporate Index Effective Yield minus inflation expectations from the Survey of Professional Forecasters.

The mean managerial talent in the low state, $\bar{z}_L$, is set so that the fall in output (peak to trough) when a bad state occurs is 17%, which is the value I obtain from detrending US output in the Barro and Ursua (2008) dataset using an exponential trend.

I set the off-diagonal coefficient of the variance-covariance matrix the for skill distribution, σ, such as to obtain an annual risk premium of 4.5%, the average yield spread between the aforementioned AAA yield and the ICE BofA Single-B US High Yield Index Effective Yield, my proxy for risky securities. The cross-sectional variance of managerial talent depends on ς, which I set so that the model economy's share of employment in the 50% smallest projects is 5%, as in the US establishment data collected by the Census Bureau in its 2017 County Business Patterns Survey.

The production function parameter regulating managerial profits, ν, is set so that the ratio of producer rents to output in the model is 10%, matching the approximation for this moment obtained by Corbae and Quintin (2016) using US private corporate sector data.

How much parents care for their progeny depends on parameter γ, which I set so that in the model economy parents spend the same fraction of their wage income net of taxes as the average

US household. In 2015, the USDA estimated that a middle-class married couple spent 16% of pre-tax income per child.[25] A model household with one child is roughly equivalent to a US household with two children. Allowing for a modest degree of economies of scale in child rearing, I assume that raising two children costs 30% of pre-tax income. Finally, OECD (2020) estimates that the tax wedge in the U.S. for an average married worker with two children was at 18.8% in 2019.[26] This implies that γ is set such that in the model, child consumption to second cohort income ratio is $c_1/w = \frac{(1-\tau)}{1-0.188}0.3 = 0.323$.

Regarding entry costs, the World Bank's *Doing Business project* estimates that the cost of business start-up procedures as a fraction of GNI per capita in the U.S. in 2018 was 1%. On the other hand, Djankov et al. (2002) estimate these costs to be roughly half of that, at 0.5% of GDP. Splitting the difference, I set e such that entry costs to GDP in the model are 0.75% of output.

Active projects also pay a cost, ζ, if they choose to issue both types of securities simultaneously. Underwriting fees for corporate debt in the U.S. average roughly 88 basis points (see Manconi et al. (2018)). At the same time, corporate debt issuance in the U.S. has averaged roughly $2 trillion from 2016 to 2019, according to Moody's, implying that underwriting fees represented roughly 0.09% of GDP.[27] I take a conservative view and assume other implicit security creation costs – whether in terms of governance, disclosure, or managing a more complex capital structure – double these costs to 0.18% of GDP.

3.2 Results

The experiment consists of a large number of simulations all starting out in a steady-state characterized by the demographic features of the U.S. economy in 1880 and evolving according to Figure 3.[28] Each simulation is characterized by a different sequence of aggregate shocks and remains for 32 model periods in this steady-state before the transition lasting 14 model periods (representing the years 1880-2103) ensues.[29]

As panel A in Figure 3 makes clear, the demographic transition results in a large increase in the share of retirees – from 4 to 28 percent – almost entirely at the expense of the share of children in the population that declines by 22 percentage points. Since early retirees continue to save, this means, first, that the share of the population saving (in any form) increases from 62 percent to 72 percent, and second, that most of this increase is in the fraction of older savers: the share of savers buying

[25] See "Expenditures on children by families" (USDA, 2015). A middle-class couple is defined as having earned between $59,200 and $107,400 in 2015.

[26] Note that this differs from the model tax rate $\tau = 0.124$ since in the model, tax revenues are used exclusively for social security funding.

[27] See Moody's Analytics *Weekly Market Report*, November 14, 2019.

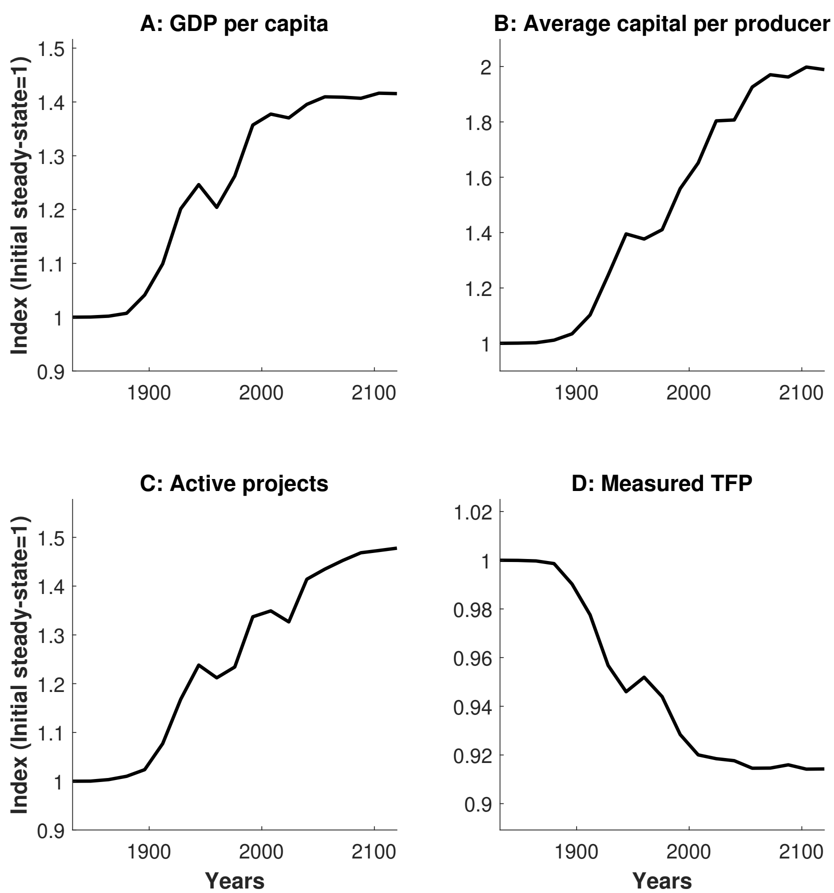
[28] Given a set of parameters and a large enough sequence of aggregate shocks, η_t, the model economy converges to a stochastic steady-state characterized by an invariant distribution. See, for a standard argument, Brock and Mirman (1972).

[29] The simulations require the economy to stay in the original steady-state for some time so that households have enough data to form their expectations.

riskless assets jumps from 22 to 48 percent.

The transition results in an increase of roughly 41 percent in GDP per capita, as shown in panel A of Figure 4. Given that it lasts 14 model periods and each model period corresponds to 16 years, this translates to a yearly increase of 0.15 percent in GDP per capita. Since 1880, U.S. GDP per capita has averaged 1.6 percent according to Bolt and van Zanden (2020), which puts the contribution of demographic factors at slightly less than 10 percent. This accords well with the evidence in Kim (2016). Having noted this, I want to de-emphasize the *absolute* magnitudes because this is, for all its features, a somewhat stylized model. Among the effects the model is not designed to capture are the demographic transition's potential effect on entrepreneurship (see Liang et al. (2018) and Aksoy et al. (2019)) and the adoption of labor-substituting automation technologies that allow economies to reverse a potential labor scarcity trend, as in Acemoglu and Restrepo (2017).

Figure 4: **Aggregate outcomes**



I do want to emphasize the magnitudes *relative* to a version of the model without the asset supply channel, which I simulate in the next section. For now, it will be instructive to understand in more detail where this increase is coming from: it is the net result of five proximate channels. There are two channels involving the labor factor that are present in most OLG models of demographic changes – *cf.* Krueger and Ludwig (2007). On one hand, the employment to population ratio decreases slightly, from 58 percent in 1880 to 56 percent in 2100. On the other hand, the average worker is marginally (3 percent) more productive, meaning these two effects roughly cancel each other out.[30]

The last three channels are direct consequences of the fact that overall saving in the economy increases. This results, first, in an increase in the average capital each project uses, which can be gauged in panel B of Figure 4. Second, because the increase in saving reduces the cost of capital, as argued below, it leads to an increase in the share of active projects, as shown in panel C of the same figure. These two channels are the main reason behind the increase in output per capita. Finally, the fifth channel operates against the net increase in income: the newly-activated projects were infra-marginal in the original steady-state, that is, they are operated by less skilled managers and, as such, lower overall measured TFP, as seen in Panel D of Figure 4. Note that the last two channels, producer entry and TFP reduction, are necessarily absent in representative firm frameworks.

The model has, therefore, something to add to the large literature on the U.S. productivity slowdown.[31] Extant models of demographic transitions have implications for (labor) productivity that come about because the share of workers in more or less productive ages changes. In addition to this, my framework with production heterogeneity opens up a channel from demographics to TFP. Because the cost of capital declines along the transition, worse managers find it optimal to enter, reducing average TFP. This is in line with the findings of Decker et al. (2017), that worsening allocative efficiency can account for much of the decline in aggregate productivity growth.

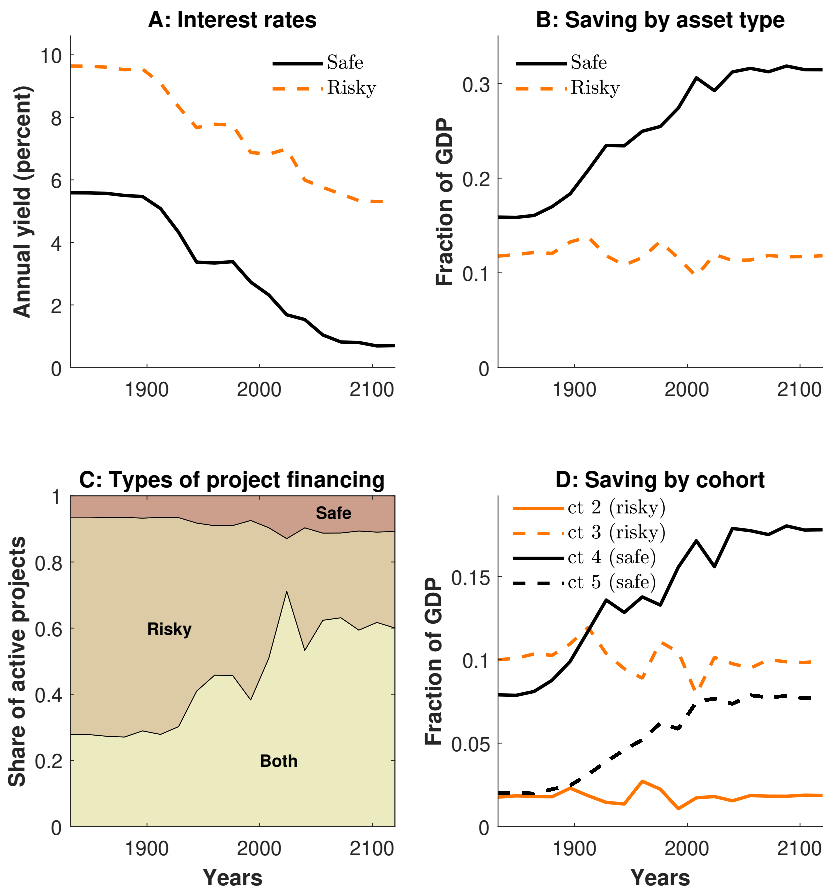
As mentioned above, the increase in aggregate saving and overall project financing is associated with a reduction in the cost of capital: both the safe and risky rates fall markedly, as shown in panel A of Figure 5. Although it is hard to see from the chart, the risk premium increases by about 50 basis points. While the risk premium in the 2008-2023 model period is targeted, its dynamics before and after are not. Is there any evidence that demographic transitions cause increases in the risk premium? The effects of low frequency movements like demographic changes can be hard to tease out in the presence of higher frequency shocks like policy and regulatory changes, requiring long data sets. Using U.S. data from 1926 to 1998, Goyal (2004) finds that the share of middle-aged people (45 to 64) is positively associated with the risk premium, while the shares of retirees (65+) and younger people (25 to 44) are negatively associated.

I replicate this regression exercise using simulated data. In particular, I take the risk premium that results from the demographic transition simulation – the difference between the two lines in panel A of Figure 5 – and regress it on (one-period) lagged cohort shares. As shown in the Appendix Table 2, I am able to replicate the results in Goyal (2004). I find that the share of households aged 48 to 63 (cohort 4) is positively, and significantly, associated with the risk premium while the shares

[30] In Appendix B.1 I confirm, in a more detailed way, the relatively small importance of the age-skill profile for the overall increase in output by simulating the demographic transition assuming no skill change with age ($\pi_2 = \pi_3 = \pi_4 = 0$).

[31] See, for examples, Gordon (2016) and Cette et al. (2016).

Figure 5: **Saving and asset markets**



of 32- to 47-year-olds (cohort 3) and 65- to 79-year-olds (cohort 5) are negatively, and significantly, associated with it.[32]

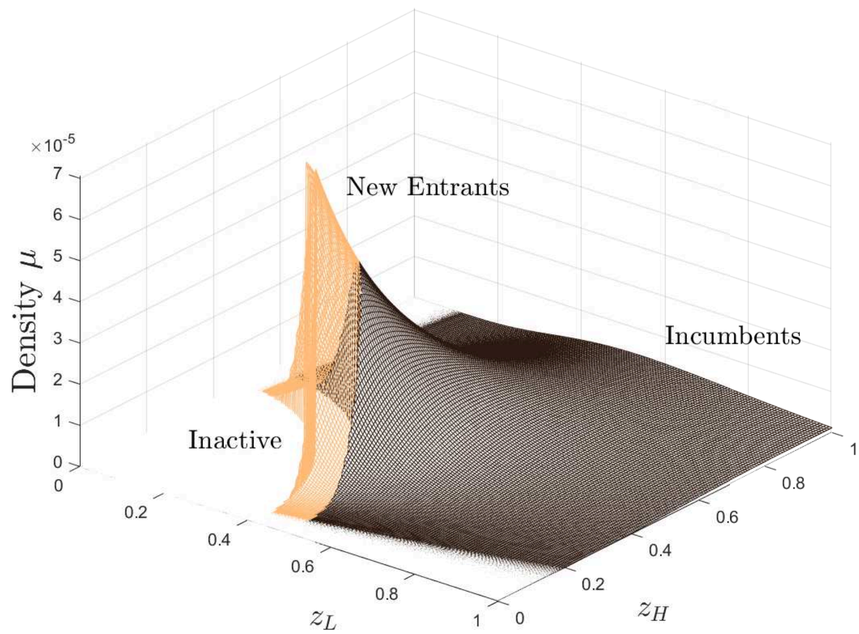
Even though both rates fall, saving in the form of risky securities remains roughly constant, as panel B shows, implying that it is the rise in riskless saving that is generating the whole of the increase in overall saving. Panel D, showing cohort saving as a share of GDP, confirms that this difference is homogeneous across cohorts – both cohorts that save in the form of risky assets keep their saving roughly constant, while the older cohorts increase it. Despite the fact that safe rates fall, older cohorts save more, not only because they are a larger share of the population, but also because their individual household-level saving also increases. This is because marginal propensities to save out of current income increase at the same time that incomes increase.[33]

But this is not the end of the story surrounding asset markets; and this is where modeling the asset

[32]Please see Appendix Section C for details.

[33]Marginal propensities to save increase because life expectancy increases following the demographic transition (see Carvalho et al. (2016) for a similar effect). Importantly, marginal propensities to save out of future income drop (for the same reason) as expected future incomes increase, but this effect is smaller because of discounting.

Figure 6: **Growth in active projects**



supply side becomes important. Since the safe rate falls relative to the risky rate, incumbent producers shift financing sources more towards safe issuance. In particular, a large mass of producers starts to issue safe securities *in addition to* risky ones (see panel C of Figure 5), securitizing their risky cash-flows. Note that risky rates go down even though household demand for risky securities falls. This is the result of the fact that risky asset issuance contracts even more, as more producers substitute away from risky into safe issuance. Absent securitization costs, safe issuance would be preferred by producers since households are willing to pay more for it. With strictly positive securitization costs, as the demand for safe assets increases, the risk premium widens. As a result, some of the producers that were issuing risky assets in exclusive in the initial steady-state, realize enough capital cost savings by starting to issue safe assets (in addition to risky ones) to cover the additional securitization cost.

Crucially, as interest rates fall, some projects that were idle before the start of the transition find that capital costs become low enough to justify entry. These new entrants are less productive than incumbents, as can be seen in Figure 6. This not only leads to the reduction in measured TFP that we had already noted in panel D of Figure 4, but also has important implications for the behavior of equilibrium interest rates. One of the main points this study makes is that modeling a demographic transition in the context of a representative firm economy will lead to an underestimation of the fall in interest rates. While I show that by comparing simulations in the next section, here we can already understand the intuition behind this result.

In a representative firm economy, the extra demand for securities stemming from the demographic transition is met by more issuance and lower interest rates along a stable (representative) firm capital demand schedule. In the present model, in addition to this mechanism, the extra demand is also met by the issuance of less productive entrants that in the original steady-state could not turn a profit because interest rates were too high. Being lees productive, their entry puts downward pressure on interest rates because it lowers the economy-wide capital demand schedule.

One key implication of the model is that we should see an increase in the amount of mixed-finance, that is, firms that issue both risky and safe(r) assets. Historically, the rise in corporate bond issuance sporting less risk than equity is evidence of this. More recently, the increased issuance of collateralized loan obligations (CLOs) also constitutes prima-facie evidence of this mechanism. CLOs are asset-backed securities (pools of commercial bank loans to firms) bundled together by an originator (usually in the shadow-banking sector) and sold in tranches that vary in risk and expected return: debt (or mezzanine) tranches receive coupon payments and get paid first – within debt tranches there are also different priorities with different returns – and equity tranches that get paid last. In Amaral and Quintin (2021) we use data from the Securities Industry and Financial Markets Association to document the large increase in U.S. CLO issuance. To be sure, these securities are issued by financial intermediaries and not by firms directly, as in the model, but this setup is ultimately equivalent to one where a financial intermediary channels household savings to producers and earns the profits we are attributing to producers (recall their outside opportunity is zero). Therefore one can interpret real-world CLO issuance as a way for producers to securitize risky cash flows (uncertain profits).

3.3 A representative firm economy

As established in the previous section, projects react to the demographic transition-induced saving glut by adjusting their financing policies and entering decisions. In particular, the fact that less productive projects are being activated puts additional downward pressure on interest rates and output (as TFP falls) relative to an economy where financing policies and entry decisions are held fixed. Since this is the central mechanism this article highlights, it is important to ascertain its magnitude. To do this, I simulate the same demographic transition, but under the assumption that there is a representative firm that requires financing and issues risky debt only. To be clear, although this is the more standard way of modeling the production side, I refer to the model developed in the previous section (with heterogeneous producers) as the *benchmark*.

I follow the same setup and equilibrium concept and, in the interest of brevity, I only show the household and producer problems, highlighting the differences. I relax the assumption that older cohorts are infinitely risk-averse and keep the same risk-neutral period log preferences throughout a household's lifetime. I assume the same expectation formation mechanism as in the benchmark economy. As a result, given expectations for future wages and interest rates, the household problem is very similar. In particular, marginal propensities to consume and save are the same as in the benchmark economy, and largely the result of changes in demographic characteristics: survival probabilities and population growth rates. Formally, households now solve:

$$\max E_t \Big\{ \gamma(1+b_t)\log c_{1,t} + \log c_{2,t} + s_{2,t}\beta \log\{E_t(c_{3,t+1}|\eta_t)\} + s_{3,t+1}\beta^2 \log\{E_t(c_{4,t+2}|\eta_{t+1})\}$$

$$+ s_{4,t+2}\beta^3 \log\{E_t(c_{5,t+3}|\eta_{t+2})\} + s_{5,t+3}\beta^4 \log\{E_t(c_{6,t+4}|\eta_{t+3})\} \Big\}$$

$$\begin{aligned}
s.t. \quad & c_{1,t} + c_{2,t} + a_{2,t} && \leq (1-\tau)(1+\pi_2)w_t + b_{2,t}, \\
& c_{3,t+1} + a_{3,t+1} && \leq (1-\tau)(1+\pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}, \\
& c_{4,t+2} + a_{4,t+2} && \leq (1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}, \\
& c_{5,t+3} + a_{5,t+3} && \leq p_{t+3} + b_{5,t+3} + a_{4,t+2}R_{t+3}, \\
& c_{6,t+4} && \leq p_{t+4} + a_{5,t+3}R_{t+4}, \\
& c_{i,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} && > 0 \text{ and } a_{2,t}, a_{3,t+1}, a_{4,t+2}, a_{5,t+3} \geq 0,
\end{aligned}$$

where the incidental bequests are given by

$$\begin{aligned}
b_{2,t} &= \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1-s_{2,t-1}}{1+g_t} a_{2,t-1}R_t, \\
b_{3,t+1} &= \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1-s_{3,t}}{1+g_{t+1}} a_{3,t}R_{t+1}, \\
b_{4,t+2} &= \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1-s_{4,t+1}}{1+g_{t+2}} a_{4,t+1}R_{t+2}, \\
b_{5,t+3} &= \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1-s_{5,t+2}}{1+g_{t+3}} a_{5,t+2}R_{t+3},
\end{aligned}$$

and the social security benefits are given by:

$$\begin{aligned}
p_{t+3} &= \frac{t_{t+2}(\eta_{t+2})R_{t+3}}{(1+g_{t+3})\sum_{j=5}^{6} sh_{j,t+3}}, \\
p_{t+4} &= \frac{t_{t+3}(\eta_{t+3})R_{t+4}}{(1+g_{t+4})\sum_{j=5}^{6} sh_{j,t+4}},
\end{aligned}$$

where $t_t(\eta)$, the amount invested by the social security fund must sum up to the total tax revenues:[34]

$$\begin{aligned}
t_{t+2}(\eta_{t+2}) &= \tau w_{t+2} \sum_{j=2}^{4} sh_{j,t+2}(1+\pi_j), \\
t_{t+3}(\eta_{t+3}) &= \tau w_{t+3} \sum_{j=2}^{4} sh_{j,t+3}(1+\pi_j).
\end{aligned}$$

The output produced when K_t units of capital and N_t units of labor are used is given by the

[34] Although I technically no longer need tax revenues to be invested for one period before being disbursed as social security benefits, I keep this for comparability purposes with the benchmark.

following schedule:

$$Y\left(K_t, N_t; z(\eta_t)\right) = z(\eta_t) K_t^{\alpha} N_t^{1-\alpha},$$

where $\eta \in \{L, H\}$, $0 < \alpha < 1$ and $0 < z_L < z_H < 1$. After the aggregate shock is realized and conditional on using capital K_t, the representative producer takes the wage rate w_t as given and chooses its labor input by solving:

$$\Pi(K_t, w_t; z(\eta_t)) \equiv \max_{N_t > 0} Y\left(K_t, N_t; z(\eta_t)\right) - w_t N_t,$$

where, anticipating labor market clearing, N_t needs to coincide with the mass of working-age individuals.

Like in the benchmark economy, the representative producer cannot self-finance and needs to borrow from households. To this end it takes the interest rate R_t as given and chooses capital K_t so that its first-order condition with respect to capital holds, recalling that relative to the benchmark economy, the representative firm issues at most one security type and therefore does not pay security creation costs.

Next, I simulate the same demographic transition as before, subject only to the changes highlighted above. The model's calibration is much simpler given the reduced number of parameters. The discipline is the same though, as the values of the remaining parameters are chosen to match the relevant targets in Table 1.[35] There is one exception: because the modified model only features one interest rate, I set β such that the capital-output ratio in the initial steady-state is the same as in the benchmark economy.[36]

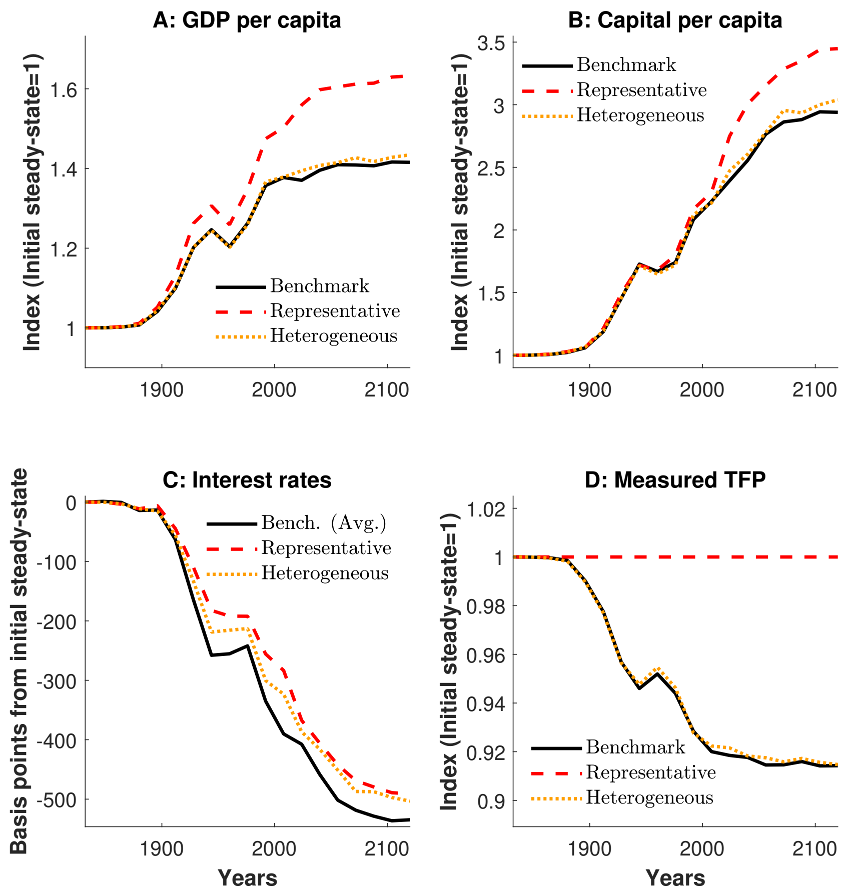
The comparison between the two economies is in Figure 7 (*benchmark* vs. *representative*). Even though the driving force behind the transition is the same, the different transmission mechanisms give rise to quantitatively important differences in interest rates (panel C) and output (panel A). The main reason for the difference should now be apparent. In the representative firm economy, the schedule for the supply of funds increases (a direct consequence of the demographic transition) and slides down along a static capital demand schedule lowering the interest rate. In the benchmark economy, this downward pressure on interest rates resulting from the increase in the supply of funds is further *amplified* by a change in financing policies at the project level and by the activation of less productive projects (detailed in the previous section) that shift the capital demand schedule down, ultimately resulting in an average fall in interest rates that is 45 basis points larger than in the representative firm economy.[37] Not only does the representative firm model underpredict the fall in

[35]The parameters σ, ν, ς, e, and ζ, along with the targets associated with them in Table 1 are not relevant for the representative firm economy.

[36]I also simulated a version where I set β such that the interest rate in the final steady-state is equal to the (asset weighted) average of the two interest rates in the final steady-state of the benchmark model and the results differ very little.

[37]In panel C, the interest rate in the benchmark economy is the (asset-weighted) average of the two interest rates and should be interpreted as the interest paid on the average unit of saved funds. The reason it falls even more than the safe rate in panel A of Figure 5 is because some weight is transferred from risky securities, whose interest rate falls

Figure 7: **Comparison to representative-firm economy**



interest rates, but it also overpredicts the increase in output (panel A). Importantly, it does so not only because it overpredicts the increase in capital (panel B) but also because, by construction, it is a model where TFP is constant, whereas in the benchmark economy average TFP decreases (panel D) with the entry of less productive projects. The output difference, like in the case of interest rates, is quantitatively significant: the representative firm model predicts an increase in output 22 percentage points above that of the benchmark model (panel A).

The large increase in output in the representative firm version of the model may seem exaggerated in face of some of the results in the literature. For example, Krueger and Ludwig (2007) predict a fall of 7 percent in output per adult (20+) between 2000 and 2080. The discrepancy is illusory and owes to the fact that I am looking at a different time horizon (1880-2100) and reporting output per capita. Indeed, if I compute output per adult between 2000 and 2080 in the representative firm economy, I get a fall of 2 percent. Other simulations in the literature report slightly increasing to flat output profiles in this period, as in Gagnon et al. (2021) or Cooley and Henriksen (2018).

relatively little, to safe securities, whose interest rate falls relatively more.

While Figure 7 makes clear the quantitative importance of the asset supply channel, it is instructive to separate the changes caused by the introduction of heterogeneous producers (which may affect asset issuance through entering and exiting) from those arising from the introduction of a safe security – which change the security issuance mix projects use to finance production. In order to do this decomposition, I simulate another version of the economy where producers are heterogeneous in the same way as in the benchmark, but there is just one (state-contingent) asset and all household cohorts have the same preferences, like in the representative firm economy. In this *heterogeneous* model economy, producers can entry and exit but cannot finance themselves by issuing different security types.[38]

The results in Figure 7 show that the bulk of the differences in output are coming about because of the skill heterogeneity assumption (panel A), but that the ability to switch financing sources plays a non-negligible role in interest rate determination (panel C), allowing more business entry and resulting in lower interest rates and (slightly) lower TFP than in the *heterogeneous* economy.

3.4 Discussion

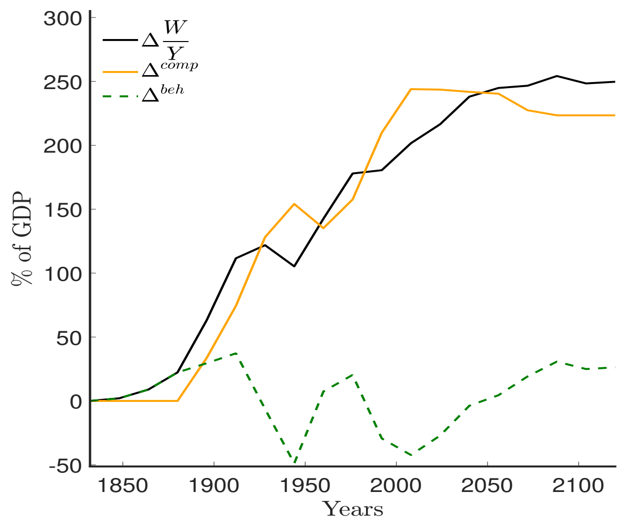
The model economy, may seem too stylized to be able to make accurate quantitative statements. In particular, one might worry that while the model is designed to capture changes in asset holdings that stem from demographic shifts, it may fail to capture important changes in holdings *conditional* on age given its stark preference structure assumption. To lend further credibility to the framework, consider the following piece of external validation. In recent work, Auclert et al. (2019) use a shift-share approach to decompose the evolution of wealth-to-GDP ratios into a *compositional effect* – that owes to the demographic changes while holding individual asset ownership constant – and a *behavioral effect* – coming from changes in individual asset holdings. Looking at cross-country data, they find large shifts in wealth-to-GDP ratios, with the compositional effect overwhelming the behavioral effect.

Figure 8 shows the equivalent decomposition for the benchmark economy. Despite the fact that the calibration does not target any moment related to these data, the change in wealth-to-GDP ratio is large and mostly accounted for by the compositional effect. This means that, at least along this dimension, the corners the model cuts – namely the fact that different cohorts hold only one asset type and there is only aggregate, and not idiosyncratic, risk – seem to be of limited relevance.

The model's implication that the share of safe assets rises along the demographic transition stands in contrast to US evidence that the share of safe assets has remained remarkably constant even as the population has aged, as noted by Gorton et al. (2012). Note though, that this pattern is not ubiquitous, Japan being the main exception, as Gourinchas and Jeanne (2012) note, while conjecturing precisely that "This may reflect demographic characteristics (as an aging society will prefer to shift its wealth towards safer assets)...". In fact, while in the simulation there are only demographic forces

[38]This economy's parameters are calibrated so as to match the targets in Table 1 except for the interest rate targets. Instead, the discount factor β is calibrated so that the capital-output ratio in the initial steady-state is the same as in the benchmark economy (which is also the same as in the representative firm economy, for that matter).

Figure 8: **Decomposing the wealth-to GDP ratio**



at work, real economies are buffeted by all kinds of other shocks, from innovations and regulation changes in their domestic financial markets, to changes in foreign demand for domestic assets, all of which ultimately impact the size of the safe asset share.

Speaking of open-economy considerations, by not modeling a source of foreign demand for domestic assets (or the possibility to save in the form of foreign assets, for that matter) I do not mean to imply that such effects are not important. In fact, Krueger and Ludwig (2007) show exactly how important they can be, but adding open-economy considerations to an already complex model is beyond the scope of the analysis I would like to carry out.

In the model economy, securities are issued directly by producers, but it is important to note that for the main model mechanism to work, it need not be the case that asset issuance is done by producers. In particular, the model would be isomorphic to one where, because of information asymmetries between households and producers, fully informed intermediaries with market power would take household savings and use them to buy a pool of projects, driving entrepreneur profit to zero (their outside cost). These intermediaries would then offer households a menu of state-contingent and non-contingent securities, just like in the current version of the model. The only difference would be that what we now call producer profits would constitute instead financial sector profits. In this case, non-contingent assets would simply be bank deposits, while the contingent securities would be equity in banks. Alternatively still, intermediaries would not even have to acquire projects, and they could simply serve as fully competitive pass-through entities (making zero profits) between households and projects. In this case, the savings of the older cohorts would be akin to bank loans, while the savings of the younger cohorts would be akin to equity.

A final aspect worth discussing is the fact that for a model that emphasizes safe asset supply, shortterm government securities – the securities that epitomize the safe asset class, at least in developed economies – are conspicuously absent. The implication being that the share of household savings that are tied up in government securities would not be available for business financing, and therefore the model could be overestimating the amount of safe business financing. Marx et al. (2021) find that while safe interest rates have been falling over the last 3 decades, the return on capital has been fairly stable. This is not something the model can replicate, as the return on capital falls along with interest rates, but I conjecture this is precisely because safe government securities supply is absent. If that were not the case, the increase in demand for safe securities brought about by the demographic transition could be channeled to government securities, thus lowering safe interest rates, while leaving corporate return on capital largely unchanged. In defense of the model, such a simplification – mapping savings in the form of government securities in the data, to savings used as productive capital in the model – is a common one in macro models for a good reason: it is cumbersome to model simultaneous holdings of different securities with a common expected return absent other security characteristics like liquidity, for example. Moreover, to the extent that this type of government funding eventually ends up financing downstream projects much in the same way, then little is lost in the abstraction.

4 Conclusion

Asset markets are a key transmission mechanism for the macroeconomic impact of demographic changes. As such, the quantitative evaluation of such impact depends on the modeling assumptions surrounding said markets. I argue that omitting (i) producer heterogeneity with respect to productivity in different states of the world and; (ii) producers' ability to finance themselves by issuing different types of securities, gives rise to quantitatively important differences when evaluating a demographic transition's impact. In particular, such models overestimate the impact on output and underestimate the accompanying interest rate reduction.

While post-industrial economies like the US are in the last phase of their demographic transitions, much of the developing world today looks a lot more, demographically speaking, like what the US looked like in the late 1800s than what it looks like now. For these countries, the points raised in this work are all the more important going forward. Moreover, the demographic evolution of post-industrial economies has not stopped. In recent work, Doepke et al. (2022) show that some trends regarding fertility's relationship with income and women's labor force participation have started to change, which no doubt will have important implications for future demographic characteristics and their economic impact as studied in the present paper.

A Demographic data

A.1 Cohort shares and population growth rates

From 1880 to 1940 I use data for the US population by single age for the decennial US census years from U.S. Department of Commerce (1930) and U.S. Department of Commerce (1940), which I interpolate to get yearly data between 1880 and 1949. From 1950 to 2040 I use yearly data for the US population by single age from United Nations (2019). I aggregate to get total population by year and compute yearly population growth rates, which I then average over the 16-year periods.

For the cohort shares I start by summing up over individual years of age within each cohort to get cohort populations. I go on to compute the share of population in each cohort for each year. Finally, I take the average share of each cohort for each consecutive 16-year period.[39] This gives me the cohort shares that appear in panel A of Figure 3.

Ideally, one would use these population data to compute survival rates and birth rates. The problem with this is that these data do not net out immigration and as a result we have instances where age cohorts grow over time instead of decreasing.[40] This gives rise to survival probabilities greater than one as well possibly inaccurate birth rates. To circumvent this problem, below I use life table data and total fertility rate data (TFR) to compute survival and birth rates.

It is important to note that the potential inconsistency between growth rates and cohort shares on one hand and birth and survival rates on the other, does not lead to any internal consistency problems because agents are aware of the full demographic path of the economy. That is, while they use accurate, data-derived, birth and survival rates that matter in the computation of their expected utility, they do not use this information to compute future cohort shares and growth rates (which are important for market clearing) because they know the full path of these objects themselves.

A.2 Survival probabilities

Survival probabilities are important model objects in that they discipline expected utility at different ages, playing a crucial role in individual intertemporal decision-making. I derive survival probabilities from vital statistics data, and not from the model-compatible population cohorts, as I discussed.

From 1890 to 1940 I use life tables from U.S. Department of Commerce (1921), U.S. Department of Commerce (1923), U.S. Department of Commerce (1936), and Federal Security Agency (1947). These contain data on survival rates by single age-year for decennial census years that I interpolate across years and backcast (using the 1890-1900 average growth) to 1880. For the 1950 to 2100 period, I use abridged life tables from United Nations (2019), containing survival probabilities for 5-year age groups for every five years from 1950 to 2100 (projected). I interpolate these data by year of age and calendar year. Finally, I convert them to 16-year (age) survival rates and take averages over the consecutive 16-year (date) periods. The results are shown in panel C of Figure 3.

A.3 Birth rates

The model-adjusted birth rates, b_t, represent how much more numerous the progeny is relative to the parents. Together with γ (which is discussed in the calibration section), they represent the weight put on the progeny's utility. I start by computing TFRs. I have data from 1880 to 1933 for

[39] For completeness, I assume years 2101-2103 are the same as year 2100.

[40] As an example, the UN data predict there will be 4.62 million 21 year-olds in 2080 and 4.66 million 22 year-olds in 2081.

intermittent years from Haines (2006) and from 1950 to 2100 (estimates) every five years from United Nations (2019). I take linear approximations for missing years and splice the two series together. This produces a yearly TFR series from 1880 to 2100. Finally, I normalize TFR by 2 (meaning that a household with 2 children has $b_t = 0$).

Because of child mortality, parents do not, on average, take care of children for the whole 16-year youth period. To account for this, I use the same decadal Census data as in section A.1 to compute the probability that a child survives to age 16 and take linear approximations for the remaining (date) years. Next, I compute the model-compatible birth rate series, b_t, as the product of the TFR and this probability of survival up to age 16. Finally, I take 16-year averages, which appear in panel D of Figure 3.

As an example, from 1880 to 1895, the TFR averaged roughly 4 (which normalizes to 2). Relatively high child mortality implied that, in expected value, parents were taking care of each child for only 12.5 years, implying an adjustment factor of $0.76 \simeq 12.5/16$, and a birth rate of $b_t = 0.76\frac{4}{2} - 1 \simeq 0.56$.

B Lifetime earnings profiles

I normalize the human capital of young adults to $\pi_2 = 0$ and assume that human capital accumulation throughout an agent's lifetime evolves exogenously and in such a way that prime-aged earnings are $(1 + \pi_3)w$ and middle-aged earnings are $(1 + \pi_4)w$. I then calibrate π_3 and π_4 so that the earnings gains in the model match the earnings gains implied by the findings of Rupert and Zanella (2015) and Lagakos et al. (2018). Here I provide some more details about this calibration.

Lagakos et al. (2018) estimate experience-wage profiles for a set of rich and poor countries. They are able to disentangle experience, time, and cohort effects by assuming that there should be no experience effects on wages near the end of the life-cycle when the incentives to invest in human capital, or look for better jobs, are much smaller than earlier in life. Under the assumption that there are no experience effects in the last 10 years of potential experience and no depreciation, they report (see their Table 4, panel A) that wage gains for the 5-9 experience group are 40.3% larger than for the 0-4 group, while gains are 79.3% for the 20-24 group and 80.8% for the 35-39 group. These results are for an average of 4 rich countries (United States, Canada, Germany and the United Kingdom). I will take them to reflect the U.S economy, as the data in Lagakos et al. (2018), suggest (see their Figure 5, for example) that the U.S. is close to the average.

I use a piecewise linear function of weakly increasing wages to match these 3 relative differences. This is shown in panel A of Figure 9. Since the model does not have an extensive margin (workers supply labor inelastically), I map model's labor compensation to earnings. In order to convert the above wage profile to earnings I use the results in Rupert and Zanella (2015). They find that hours worked do not significantly vary with age up until age 50. This means that life-cycle earnings track life-cycle wages up until age 50. From then on, workers significantly reduce their working hours in anticipation of retirement. Figure 6 in Rupert and Zanella (2015) suggests that yearly hours decline from about 2200 to 1700. I assume this decline occurs linearly for the last 15 years of a worker's career. This results in the earnings pattern shown in panel B of Figure 9.

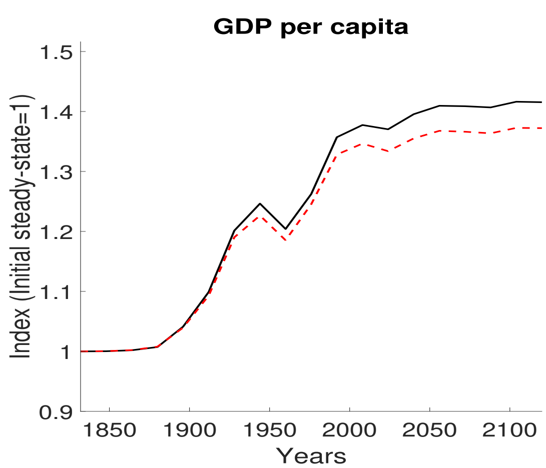
Using this earnings profile I compute the average earnings for each of the three 16-year periods of a worker's career and find that prime-aged workers (16-31 years of experience) earn 28% more than young workers (0-15 years of experience), while middle aged workers (32-47 years of experience) earn 16% more than young workers. I therefore set $\pi_3 = 0.28$ and $\pi_4 = 0.16$.

Figure 9: **Life-cycle wages and earnings**

B.1 Robustness: no lifetime worker productivity changes

To gauge how robust the results are to my estimates for age-specific skills, I rerun the demographic transition assuming that workers are equally productive throughout their lifetime. In terms of the model's parameters, this simply means setting $\pi_2 = \pi_3 = \pi_4 = 0$. As Figure 10 shows, even with such a large change in parameters, final output differs only by 4 percentage points (an annualized growth rate difference of 0.013 percent). I take this to mean results are robust to my estimates of age-specific skills π_3 and π_4.

Figure 10: **Robustness: no lifetime worker productivity changes**



C Demographics and the risk-premium

The risk premium, the return difference between risky and safe securities, may fluctuate for a variety of reasons that have to do with the business cycle or regulatory changes, for instance. In the presence of these relatively higher-frequency movements, it is hard to identify the influence of lower-frequency variables, like demographic changes. Long data sets are needed for such purpose. Goyal (2004) uses annual data on the S&P500 and T-bill returns from 1926 to 1998. He finds that changes in the share of 45- to 64-year-olds are positively, and significantly, associated with the risk premium, while changes in the shares of 25- to 44-year-olds and 65-plus are negatively, and significantly, associated with it.

Table 2: **Risk premium and cohort shares**

	(1)	(2)
$ct_2 + ct_3$	0.0169	
	(0.0256)	
$ct_4 + ct_5$	0.0404 *	
	(0.0145)	
ct_3		-0.0593 *
		(0.0264)
ct_4		0.171 ***
		(0.0295)
ct_5		-0.0599 **
		(0.0196)
Adj.R^2	0.56	0.84
F	11.4	27.4
N	16	16

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

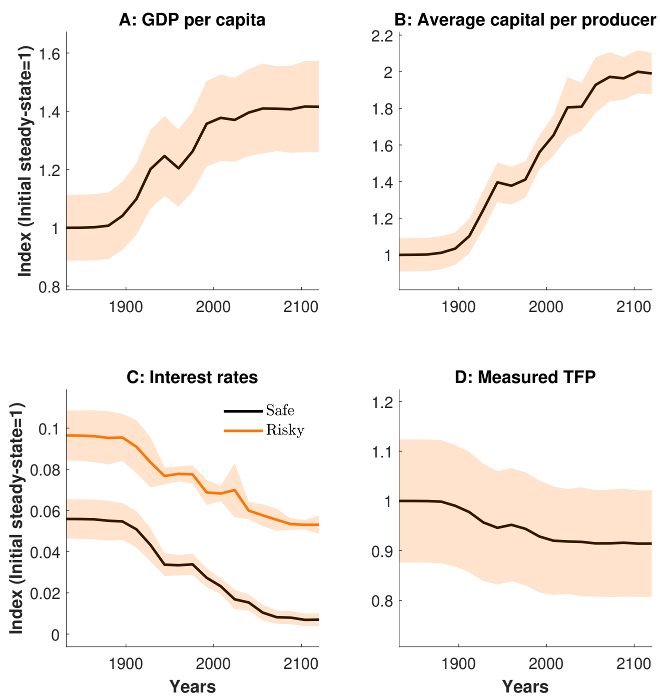
To verify whether the model can reproduce these results, I use data from the simulated transition: a total of 16 periods (14 in the transition and 2 in the initial steady-state) averaged over a large number of simulations. In the model, cohorts 2 and 3 save in the form of risky securities, while cohorts 4 and 5 save in the form of riskless ones. Given this, a natural starting point is to ask how the shares of people saving in each type of asset affect the risk premium. To do this I regress the risk premium on the one-period lagged population shares. Note that I do not need to control for the aggregate shock since the data are averaged over different simulations.

The results are in column (1) of Table 2. While the share of people saving in the form of riskless securities is negatively, and significantly, associated with the risk premium, the share of people saving in risky assets is not statistically associated with movements in the risk premium. The reason for this is that saving by cohort also change over time (see panel D in Figure 5).

To hone in more precisely on the age cohorts Goyal (2004) finds relevant, I regress the risk premium on the one-period lagged shares in cohort 3 (33 to 47), cohort 4 (48 to 63), and cohort 5 (64 to 79). The results are shown in column (2). They line up precisely with the findings in Goyal (2004) and are statistically significant.

D Uncertainty

In the main text I present model outcomes as averages taken over a large number of simulations. Nonetheless, the stochastic nature of the model means there is uncertainty surrounding these averages. Recall that the aggregate shock is calibrated such that the model spends 12 percent of the time in the low state and the severity of the shock is such that output declines by 17%. In Figure 11 I add confidence bands capturing 2 standard deviations for the model's main variables.



In some instances, confidence bands become narrower (for example, towards the latter years for interest rates) but this does not mean there is less uncertainty, it is simply because the units are becoming smaller and standard deviations are not unit-free. That is, in proportional terms uncertainty is roughly uniform over time.

E The household problem

In this section I solve the household problem and derive expressions for cohort-specific saving: equations (E.6)-(E.9).

These expressions are contingent on a set of expected variables whose calculation I make clear in the next section.

$$\max E_t \Bigg\{ \gamma\left(1+b_t\right)\log c_{1,t}(\eta_t) + \log c_{2,t}(\eta_t) + s_{2,t}\beta \log\left\{E_t\left(c_{3,t+1}(\eta_{t+1})|\eta_t\right)\right\} + s_{3,t+1}\beta^2 \log\left\{E_{t+1}\left(c_{4,t+2}(\eta_{t+2})|\eta_{t+1}\right)\right\}$$
$$+ s_{4,t+2}\beta^3 \log\left\{\min\left(c_{5,t+3}(L), c_{5,t+3}(H)\right)\right\} + s_{5,t+3}\beta^4 \log\left\{\min\left(c_{6,t+4}(L), c_{6,t+4}(H)\right)\right\} \Bigg\}$$

$$
\begin{aligned}
s.t. \quad c_{1,t}(\eta_t) + c_{2,t}(\eta_t) + \int_{Z_t} a_{2,t}(z,\eta_t) d\mu \quad & \leq (1-\tau) w_t + b_{2,t}, \\
c_{3,t+1}(\eta_{t+1}) + \int_{Z_{t+1}} a_{3,t+1}(z,\eta_{t+1}) d\mu \quad & \leq (1-\tau)(1+\pi_3) w_{t+1} + b_{3,t+1} + \int_{Z_t} a_{2,t}(z,\eta_t) R^R_{t+1}(z) d\mu, \\
c_{4,t+2}(\eta_{t+2}) + \int_{Z_{t+2}} a_{4,t+2}(z,\eta_{t+2}) d\mu \quad & \leq (1-\tau)(1+\pi_4) w_{t+2} + b_{4,t+2} + \int_{Z_{t+1}} a_{3,t+1}(z,\eta_{t+1}) R^R_{t+2}(z) d\mu, \\
c_{5,t+3}(\eta_{t+3}) + \int_{Z_{t+3}} a_{5,t+3}(z,\eta_{t+3}) d\mu \quad & \leq b_{5,t+3} + p_{t+3} + \int_{Z_{t+2}} a_{4,t+2}(z,\eta_{t+2}) R^S_{t+3}(z) d\mu, \\
c_{6,t+4}(\eta_{t+4}) \quad & \leq p_{t+4} + \int_{Z_{t+3}} a_{5,t+3}(z,\eta_{t+3}) R^S_{t+4}(z) d\mu, \\
c_{i,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} \quad & > 0 \text{ and } a_{2,t}, a_{3,t+1}, a_{4,t+2}, a_{5,t+3} \geq 0,
\end{aligned}
$$

where the incidental bequests are given by

$$
\begin{aligned}
b_{2,t} &= \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1-s_{2,t-1}}{1+g_t} \int_{Z_{t-1}} a_{2,t-1}(z) R^R_t(z) d\mu, \\
b_{3,t+1} &= \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1-s_{3,t}}{1+g_{t+1}} \int_{Z_t} a_{3,t}(z) R^R_{t+1}(z) d\mu, \\
b_{4,t+2} &= \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1-s_{4,t+1}}{1+g_{t+2}} \int_{Z_{t+1}} a_{4,t+1}(z,\eta_{t+1}) R^S_{t+2}(z) d\mu, \\
b_{5,t+3} &= \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1-s_{5,t+2}}{1+g_{t+3}} \int_{Z_{t+2}} a_{5,t+2}(z,\eta_{t+2}) R^S_{t+3}(z) d\mu,
\end{aligned}
$$

and the social security benefits are given by:

$$
\begin{aligned}
p_{t+3} &= \frac{\displaystyle\int_{Z_{t+2}} t_{t+2}(z,\eta_{t+2}) R^S_{t+3}(z) d\mu}{(1+g_{t+3}) \sum_{j=5}^{6} sh_{j,t+3}}, \\
p_{t+4} &= \frac{\displaystyle\int_{Z_{t+3}} t_{t+3}(z,\eta_{t+3}) R^S_{t+4}(z) d\mu}{(1+g_{t+4}) \sum_{j=5}^{6} sh_{j,t+4}}
\end{aligned}
$$

where $t_t(z,\eta)$, the amounts invested by the social security fund across producers that issue safe securities must sum up to the total tax revenues:

$$
\begin{aligned}
\int_{Z_{t+2}} t_{t+2}(z,\eta_{t+2}) d\mu &= \tau w_{t+2} \sum_{j=2}^{4} sh_{j,t+2}(1+\pi_j), \\
\int_{Z_{t+3}} t_{t+3}(z,\eta_{t+3}) d\mu &= \tau w_{t+3} \sum_{j=2}^{4} sh_{j,t+3}(1+\pi_j).
\end{aligned}
$$

In what follows, I will abuse notation and introduce some equilibrium results in order to make the notation lighter. In particular, let $R^S_t = R^S_t(z)$ and $R^R_t = R^R_t(z)$ for all active projects $z \in Z$ denote, respectively, the equilibrium safe and risky rates, and let $a_{j,t} = \int_{Z_t} a_{j,t}(z) d\mu$ denote total savings of cohort j. I will also write $\underline{c}_{5,t+3} \equiv \min\left(c_{5,t+3}(L), c_{5,t+3}(H)\right)$, $\underline{c}_{6,t+4} \equiv \min\left(c_{6,t+4}(L), c_{6,t+4}(H)\right)$ for the, riskless, consumption of infinitely risk-averse cohorts.

The first-order conditions are

$$\frac{c_{1,t}}{c_{2,t}} = \gamma\left(1 + b_t\right) \tag{E.1}$$

$$\frac{E_t c_{3,t+1}}{c_{2,t}} = \beta s_{2,t} E_t R^R_{t+1} \tag{E.2}$$

$$\frac{E_t c_{4,t+2}}{E_t c_{3,t+1}} = \beta \frac{s_{3,t+1}}{s_{2,t}} E_t R^R_{t+2} \tag{E.3}$$

$$\frac{E_t \underline{c}_{5,t+3}}{E_t c_{4,t+2}} = \beta \frac{s_{4,t+2}}{s_{3,t+1}} E_t R^S_{t+3} \tag{E.4}$$

$$\frac{E_t \underline{c}_{6,t+4}}{E_t \underline{c}_{5,t+3}} = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_t R^S_{t+4}, \tag{E.5}$$

for all t and for all aggregate states $\eta_t \in \{L, H\}$.

Note that the reason we can write the FOC (E.3) as $\frac{E_t c_{3,t+1}}{c_{2,t}} = \beta s_{2,t} E_t R^R_{t+1}$, instead of the usual $\frac{1}{c_{2,t}} = \beta s_{2,t} E_t\left(\frac{R^R_{t+1}}{c_{3,t+1}}\right)$, is that the utility argument is $E_t c_{3,t+1}$ and not $c_{3,t+1}$. This means the FOC is $\frac{1}{c_{2,t}} = \beta s_{2,t} E_t\left(\frac{R^R_{t+1}}{E_t c_{3,t+1}}\right)$, which means that conditional on time t information, $E_t c_{3,t+1}$ is a constant and can, therefore, be brought out of the expectation operator: $E_t\left(\frac{R^R_{t+1}}{E_t c_{3,t+1}}\right) = \frac{1}{E_t c_{3,t+1}} E_t R^R_{t+1}$.

From the perspective of a household in the last period of its life: $c_{6,t+4} = p_{t+4} + a_{5,t+3} R^S_{t+4}$, for all η_{t+4}. Abusing notation and using the equilibrium result that $c_{6,t+4} = \underline{c}_{6,t+4}$, and $c_{5,t+3} = \underline{c}_{5,t+3}$, we can use the analogous of FOC (E.5) in period $t+3$ to substitute out $c_{6,t+4}$:

$$\frac{E_{t+3} p_{t+4} + a_{5,t+3} E_{t+3} R^S_{t+4}}{c_{5,t+3}} = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_{t+3} R^S_{t+4}.$$

Further substituting out $c_{5,t+3} = p_{t+3} + a_{4,t+2} R^S_{t+3} + b_{t+3} - a_{5,t+3}$ from the early retirees budget constraint:

$$E_{t+3} p_{t+4} + a_{5,t+3} E_{t+3} R^S_{t+4} = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_{t+3} R^S_{t+4}\left(p_{t+3} + a_{4,t+2} R^S_{t+3} + b_{t+3} - a_{5,t+3}\right).$$

Solving for the saving of the early-retiree cohort:

$$a_{5,t+3} = \frac{\beta \frac{s_{5,t+3}}{s_{4,t+2}}\left(p_{t+3} + b_{5,t+3} + a_{4,t+2} R^S_{t+3}\right) - \frac{E_{t+3} p_{t+4}}{E_{t+3} R^S_{t+4}}}{1 + \beta \frac{s_{5,t+3}}{s_{4,t+2}}}. \tag{E.6}$$

Letting $d_{5,t+3} \equiv \beta \frac{s_{5,t+3}}{s_{4,t+2}}$, note that optimal saving is a fraction $\frac{d_{5,t+3}}{1+d_{5,t+3}}$ of current income minus discounted future income.

We can now replace this optimal solution back in the early retirees budget constraint and solve for consumption:

$$c_{5,t+3} = p_{t+3} + a_{4,t+2} R^S_{t+3} + b_{t+3} - \frac{d_{5,t+3}}{1 + d_{5,t+3}}\left(p_{t+3} + b_{5,t+3} + a_{4,t+2} R^S_{t+3}\right) - \frac{\frac{E_{t+3} p_{t+4}}{E_{t+3} R^S_{t+4}}}{1 + d_{5,t+3}}$$

$$c_{5,t+3} = \left(\frac{1}{1+d_{5,t+3}}\right)\left(p_{t+3} + \frac{E_{t+3}p_{t+4}}{E_{t+3}R^S_{t+4}} + a_{4,t+2}R^S_{t+3} + b_{5,t+3}\right),$$

Replacing the expression for the optimal $c_{5,t+3}$ we just found, as well as the expression for $c_{4,t+2}$ from the pre-retiree cohort's budget constraint, into the analogue of FOC (E.4) for the pre-retirees in period $t+2$ we get:

$$\begin{aligned} &E_{t+2}\left\{\left(\frac{1}{1+\beta\frac{s_{5,t+3}}{s_{4,t+2}}}\right)\left(\frac{p_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{E_{t+3}p_{t+4}}{E_{t+2}R^S_{t+3}E_{t+3}R^S_{t+4}} + \frac{a_{4,t+2}R^S_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{b_{5,t+3}}{E_{t+2}R^S_{t+3}}\right)\right\} \\ &\qquad = \beta\frac{s_{4,t+2}}{s_{3,t+1}}\left((1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2} - a_{4,t+2}\right) \end{aligned}$$

Using the law of iterated expectations and recalling that households know the full evolution of the survival probabilities, $s_{j,t}$, we get:

$$\begin{aligned} \left(\frac{1}{1+\beta\frac{s_{5,t+3}}{s_{4,t+2}}} + \beta\frac{s_{4,t+2}}{s_{3,t+1}}\right)a_{4,t+2} =& \beta\frac{s_{4,t+2}}{s_{3,t+1}}\left((1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2}\right) \\ &- \frac{1}{1+\beta\frac{s_{5,t+3}}{s_{4,t+2}}}\left(\frac{E_{t+2}p_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R^S_{t+3}E_{t+2}R^S_{t+4}} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R^S_{t+3}}\right) \end{aligned}$$

Letting $d_{4,t+2} \equiv \beta\frac{s_{4,t+2}}{s_{3,t+1}} + \beta^2\frac{s_{5,t+3}}{s_{3,t+1}}$ we get:

$$\begin{aligned} a_{4,t+2} =& \frac{d_{4,t+2}}{1+d_{4,t+2}}\left((1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2}\right) \\ &- \frac{1}{1+d_{4,t+2}}\left(\frac{E_{t+2}p_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R^S_{t+3}E_{t+2}R^S_{t+4}} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R^S_{t+3}}\right) \end{aligned} \tag{E.7}$$

We can now get consumption $c_{4,t+2}$ from the pre-retiree cohort's budget constraint:

$$c_{4,t+2} = \left(1 - \frac{d_{4,t+2}}{1+d_{4,t+2}}\right)\left((1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2}\right) + \frac{\frac{E_{t+2}p_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R^S_{t+3}E_{t+2}R^S_{t+4}} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R^S_{t+3}}}{1+d_{4,t+2}}$$

$$c_{4,t+2} = \frac{(1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2} + \frac{E_{t+2}p_{t+3}}{E_{t+2}R^S_{t+3}} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R^S_{t+3}E_{t+2}R^S_{t+4}} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R^S_{t+3}}}{1+d_{4,t+2}}$$

Continuing backwards, in period $t+1$, consider a household in their second period of work, with information set $\mathcal{I}_{t+1}$. Replacing the expression for the optimal $c_{4,t+2}$ we just found, as well as the expression for $c_{3,t+1}$ from the prime-age cohort's budget constraint, into FOC (E.3) we get:

$$\begin{aligned} &E_{t+1}\left\{\frac{(1-\tau)(1+\pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R^R_{t+2} + \frac{p_{t+3}}{R^S_{t+3}} + \frac{p_{t+4}}{R^S_{t+3}R^S_{t+4}} + \frac{b_{5,t+3}}{R^S_{t+3}}}{1+d_{4,t+2}}\right\} \\ =& \beta\frac{s_{3,t+1}}{s_{2,t}}E_{t+1}R^R_{t+2}\left[(1-\tau)(1+\pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R^R_{t+1} - a_{3,t+1}\right] \end{aligned}$$

$$\frac{(1-\tau)(1+\pi_4)\frac{E_{t+1}w_{t+2}}{E_{t+1}R^R_{t+2}}+\frac{E_{t+1}b_{4,t+2}}{E_{t+1}R^R_{t+2}}+a_{3,t+1}+\frac{E_{t+1}p_{t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+4}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}E_{t+1}R^S_{t+4}}+\frac{E_{t+1}b_{5,t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}}{1+d_{4,t+2}}$$
$$=\beta\frac{s_{3,t+1}}{s_{2,t}}\left[(1-\tau)(1+\pi_3)w_{t+1}+b_{3,t+1}+a_{2,t}R^R_{t+1}-a_{3,t+1}\right]$$

Solving for $a_{3,t+1}$

$$a_{3,t+1}\left(\frac{1}{1+d_{4,t+2}}+\beta\frac{s_{3,t+1}}{s_{2,t}}\right)=\beta\frac{s_{3,t+1}}{s_{2,t}}\left[(1-\tau)(1+\pi_3)w_{t+1}+b_{3,t+1}+a_{2,t}R^R_{t+1}\right]$$
$$-\frac{(1-\tau)(1+\pi_4)\frac{E_{t+1}w_{t+2}}{E_{t+1}R^R_{t+2}}+\frac{E_{t+1}b_{4,t+2}}{E_{t+1}R^R_{t+2}}+a_{3,t+1}+\frac{E_{t+1}p_{t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+4}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}E_{t+1}R^S_{t+4}}+\frac{E_{t+1}b_{5,t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}}{1+d_{4,t+2}}$$

Note that $\frac{1}{1+d_{4,t+2}}+\beta\frac{s_{3,t+1}}{s_{2,t}}=\frac{1}{1+\beta\frac{s_{4,t+2}}{s_{3,t+1}}+\beta^2\frac{s_{5,t+3}}{s_{3,t+1}}}+\beta\frac{s_{3,t+1}}{s_{2,t}}=\frac{\beta\frac{s_{3,t+1}}{s_{2,t}}+\beta^2\frac{s_{4,t+2}}{s_{2,t}}+\beta^3\frac{s_{5,t+3}}{s_{2,t}}}{1+\beta\frac{s_{4,t+2}}{s_{3,t+1}}+\beta^2\frac{s_{5,t+3}}{s_{3,t+1}}}$, and therefore, letting $d_{3,t+1}\equiv\beta\frac{s_{3,t+1}}{s_{2,t}}+\beta^2\frac{s_{4,t+2}}{s_{2,t}}+\beta^3\frac{s_{5,t+3}}{s_{2,t}}$

$$a_{3,t+1}=\frac{d_{3,t+1}}{1+d_{3,t+1}}\left[(1-\tau)(1+\pi_3)w_{t+1}+b_{3,t+1}+a_{2,t}R^R_{t+1}\right] \tag{E.8}$$
$$-\frac{(1-\tau)(1+\pi_4)\frac{E_{t+1}w_{t+2}}{E_{t+1}R^R_{t+2}}+\frac{E_{t+1}b_{4,t+2}}{E_{t+1}R^R_{t+2}}+\frac{E_{t+1}b_{5,t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+4}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}E_{t+1}R^S_{t+4}}}{1+d_{3,t+1}}$$

We can now get consumption $c_{3,t+1}$ from the prime-aged cohort's budget constraint:

$$c_{3,t+1}=\left(\frac{1}{1+d_{3,t+1}}\right)\Bigg((1-\tau)(1+\pi_3)w_{t+1}+b_{3,t+1}+a_{2,t}R^R_{t+1}+(1-\tau)(1+\pi_4)\frac{E_{t+1}w_{t+2}}{E_{t+1}R^R_{t+2}}$$
$$+\frac{E_{t+1}b_{4,t+2}}{E_{t+1}R^R_{t+2}}+\frac{E_{t+1}b_{5,t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+3}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}}+\frac{E_{t+1}p_{t+4}}{E_{t+1}R^R_{t+2}E_{t+1}R^S_{t+3}E_{t+1}R^S_{t+4}}\Bigg)$$

Continuing backwards, in period t, consider a household in their first period of work, with information set $\mathcal{I}_t$. Replacing the expression for the optimal $c_{3,t+1}$ we just found into FOC (E.2) we get:

$$\left(\frac{1}{\beta s_{2,t}\left(1+d_{3,t+1}\right)}\right)\Bigg((1-\tau)(1+\pi_3)\frac{E_tw_{t+1}}{E_tR^R_{t+1}}+\frac{E_tb_{3,t+1}}{E_tR^R_{t+1}}+a_{2,t}+(1-\tau)(1+\pi_4)\frac{E_tw_{t+2}}{E_tR^R_{t+1}E_tR^R_{t+2}}$$
$$+\frac{E_tp_{t+3}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}}+\frac{E_tp_{t+4}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}E_tR^S_{t+4}}+\frac{E_tb_{4,t+2}}{E_tR^R_{t+1}E_tR^R_{t+2}}+\frac{E_tb_{5,t+3}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}}\Bigg)=c_{2,t}$$

From the FOC E.1: $c_{1,t}=\gamma(1+b_t)c_{2,t}$. Replacing this in the young adult cohort budget constraint: $a_{2,t}=(1-\tau)w_t+b_{2,t}-(1+\gamma(1+b_t))c_{2,t}$. This gives us:

$$a_{2,t}=(1-\tau)w_t+b_{2,t}-\Bigg[\left(\frac{1+\gamma(1+b_t)}{\beta s_{2,t}\left(1+d_{3,t+1}\right)}\right)\Bigg((1-\tau)(1+\pi_3)\frac{E_tw_{t+1}}{E_tR^R_{t+1}}+a_{2,t}+(1-\tau)(1+\pi_4)\frac{E_tw_{t+2}}{E_tR^R_{t+1}E_tR^R_{t+2}}$$
$$+\frac{E_tp_{t+3}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}}+\frac{E_tp_{t+4}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}E_tR^S_{t+4}}+\frac{E_tb_{3,t+1}}{E_tR^R_{t+1}}+\frac{E_tb_{4,t+2}}{E_tR^R_{t+1}E_tR^R_{t+2}}+\frac{E_tb_{5,t+3}}{E_tR^R_{t+1}E_tR^R_{t+2}E_tR^S_{t+3}}\Bigg)\Bigg]$$

Letting $d_{2,t} \equiv \beta s_{2,t}(1 + d_{3,t+1}) = \beta s_{2,t} + \beta^2 s_{3,t+1} + \beta^3 s_{4,t+2} + \beta^4 s_{5,t+3}$ and solving for $a_{2,t}$, we get:

$$a_{2,t} = \frac{d_{2,t}[(1-\tau)w_t + b_{2,t}]}{1 + d_{2,t} + \gamma(1+b_t)} - \frac{1+\gamma(1+b_t)}{1+d_{2,t}+\gamma(1+b_t)} \Bigg((1-\tau)(1+\pi_3)\frac{E_t w_{t+1}}{E_t R^R_{t+1}} + (1-\tau)(1+\pi_4)\frac{E_t w_{t+2}}{E_t R^R_{t+1} E_t R^R_{t+2}} \tag{E.9}$$
$$\frac{E_t p_{t+3}}{E_t R^R_{t+1} E_t R^R_{t+2} E_t R^S_{t+3}} + \frac{E_t p_{t+4}}{E_t R^R_{t+1} E_t R^R_{t+2} E_t R^S_{t+3} E_t R^S_{t+4}} + \frac{E_t b_{3,t+1}}{E_t R^R_{t+1}} + \frac{E_t b_{4,t+2}}{E_t R^R_{t+1} E_t R^R_{t+2}} + \frac{E_t b_{5,t+3}}{E_t R^R_{t+1} E_t R^R_{t+2} E_t R^S_{t+3}} \Bigg)$$

Equipped with the expressions for cohort saving we can solve for expectation formation.

F Expectations formation

To solve for saving decisions $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$ using equations (E.6)-(E.9) above, one needs instances for the following expectations: $E_t w_{t+1}$, $E_t R^R_{t+1}$, $E_t R^S_{t+1}$, $E_t p_{t+1}$, $E_t w_{t+2}$, $E_t R^R_{t+2}$, $E_t R^S_{t+2}$, $E_t p_{t+2}$, $E_t b_{5,t+2}$, $E_t b_{4,t+2}$, $E_t R^S_{t+3}$, $E_t p_{t+3}$, $E_t b_{5,t+3}$, $E_t R^S_{t+4}$, and $E_t p_{t+4}$. Notice that expectations on bequests and pensions depend only on expectations of future assets, interest rates and wage rates.

The assumption is that households have perfect foresight over the evolution of demographic characteristics b_t, g_t, $s_{j,t}$, and $sh_{j,t}$, for $j = 1, \ldots, 6$ and $t = 0, \ldots$, and form expectations regarding next period's wages and interest rates by regressing these on a subset of their information set at time t containing the history of states $\mathcal{I}_t = \{\eta_0, \ldots, \eta_{t-1}, a^R_0, \ldots, a^R_{t-1}, a^S_0, \ldots, a^S_{t-1}\}$. Since the economy's states in period t are $\{\eta_{t-1}, a^R_{t-1}, a^S_{t-1}\}$, the regressions are:

$$\begin{aligned} w_t &= \beta^w_0 + \beta^w_\eta \eta_{t-1} + \beta^w_R a^R_{t-1} + \beta^w_S a^S_{t-1} + \varepsilon^w_t, \\ R^R_t &= \beta^R_0 + \beta^R_\eta \eta_{t-1} + \beta^R_R a^R_{t-1} + \beta^R_S a^S_{t-1} + \varepsilon^R_t, \\ R^S_t &= \beta^S_0 + \beta^S_\eta \eta_{t-1} + \beta^S_R a^R_{t-1} + \beta^S_S a^S_{t-1} + \varepsilon^S_t. \end{aligned}$$

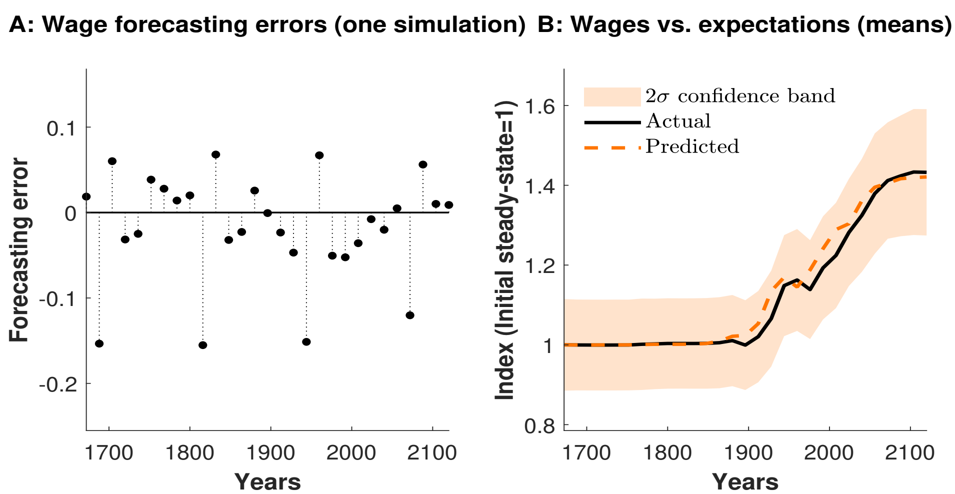
This gives me estimated coefficients $\hat\beta^w_0$, $\hat\beta^w_\eta$, $\hat\beta^w_R$, $\hat\beta^w_S$, for the wage rate and its counterparts for the risky interest rate: $\hat\beta^R_0$, $\hat\beta^R_\eta$, $\hat\beta^R_R$, $\hat\beta^R_S$, and the safe interest rate $\hat\beta^S_0$, $\hat\beta^S_\eta$, $\hat\beta^S_R$, $\hat\beta^S_S$. I then use these coefficients and the actual states to compute a one-period-ahead forecast, which I take to be the households' expectations: $E_t w_{t+1}$, $E_t R^R_{t+1}$, and $E_t R^S_{t+1}$:

$$\begin{aligned} E_t w_{t+1} &\equiv \hat\beta^w_0 + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)]\,\hat\beta^w_\eta + \hat\beta^w_R a^R_t + \hat\beta^w_S a^S_t, \\ E_t R^R_{t+1} &\equiv \hat\beta^R_0 + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)]\,\hat\beta^R_\eta + \hat\beta^R_R a^R_t + \hat\beta^R_S a^S_t, \\ E_t R^S_{t+1} &\equiv \hat\beta^S_0 + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)]\,\hat\beta^S_\eta + \hat\beta^S_R a^R_t + \hat\beta^S_S a^S_t. \end{aligned}$$

To be able to compute their forecast, period t households need to estimate η_t, which they do by using the Markov transition matrix M, and η_{t-1}. A further wrinkle to the problem has to do with simultaneity. To compute households forecasts I need a^R_t and a^S_t, but these depend on $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$, which are the variables I am trying to solve for, and for which I need the expectations in the first place. To solve this problem I set up a system of 4 equations (E.6)-(E.9) on 4 unknowns: $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$. I can write the right-hand-side of these four equations as functions of past variables and expectations, which I can in turn write as functions of the unknowns I am looking for. For this last step, I need a Martingale assumption on aggregate asset types: $E_{t+k} a_{j,t+k} = a_{j,t}$ for $k = 1, 2, 3$. Note that this still allows me to take into account the direct (if not the indirect) effect of the demographic transition on future variables because when I aggregate cohort savings to obtain, for example, $E_{t+k}a^R_{t+k}$, I take into account the future evolution of cohort sizes, which households are assumed to know. Therefore, $E_{t+k}a^R_{t+k} = sh_{2,t+k}a_{2,t} + sh_{3,t+k}a_{3,t}$, for example. I also use this to compute higher order expectations for wages and interest rates like, for example, $E_t w_{t+2}$.

In a stochastic steady-state, the solution that results from this expectation formation mechanism, like the rational expectations solution, has agents making very close to zero average forecast mistakes, even if this is an empirical, not theoretical result. I illustrate this in Figure 12 for wage expectations (the results are similar for interest rates expectations). Panel A shows the forecast errors for a particular simulation. Statistically I cannot reject, at the 5% level, the null that the mean forecast error is zero. Panel B shows wages and respective expectations averaged over multiple simulations – the invariant distribution means. Again, one can see that in the initial steady-state, expectations are very accurate, on average (basically the same point that panel A makes), but that some of this accuracy is lost in the transition period and is gradually recovered as time in the new steady-state increases. This is because agents take into account all the history they have available, so when the transition starts, most of their sample pertains to the previous steady-state.

Figure 12: **Expectation formation performance**



Note: Shaded area represents 2 standard deviations confidence interval.

G Algorithm

The algorithm proceeds in two large blocks. In the first block I solve for the equilibrium wage rate, w_t, and interest rates for the two assets: R^R_t and R^S_t. I do this by solving the producers' problem each period given current states: the aggregate shock, η_{t-1}, risky funds supplied, a^R_{t-1}, and safe funds supplied, a^S_{t-1}. Armed with prices, in a second block, I solve the households' problem and find saving decisions for next period $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$, which allows me to redo the whole process for the next period. In more detail, here are the steps:

(1) Supply initial asset holdings $a_{2,-1}$, $a_{3,-1}$, $a_{4,-1}$, and $a_{5,-1}$ (in practice this is an informed choice close to steady-state);

(2) Start simulation with $\eta_0 = 1$ and draw a sequence of shocks $\{\eta_t\}_{t=1}^{S+T}$ using the Markov transition matrix M;

(3) For every period $t = 0, \ldots, S+T$, given states η_{t-1}, a^R_{t-1}, and a^S_{t-1} solve for labor market and asset markets clearing:

(3.1) Interest rates loop: Guess interest rates $R^R_{0,t}$, $R^S_{0,t}$: if in first period provide informed guess, otherwise start with interest rates from previous periods:

(3.1.1) Wage rate loop: guess a wage rate, $w_{0,t}$ and solve producers' problem to determine demand for labor;

(3.1.2) Iterate on wage rate until labor market clears: if labor demand larger than labor supply (given by exogenous population evolution) adjust wage down; if larger adjust wage up; otherwise done.

(3.2) Iterate on interest rates until both asset markets clear. This is a two-dimensional problem (unlike the labor market problem) that is more complicated and time consuming:

(3.2.1) Given risky rate guess $R^R_{0,t}$ find a safe rate target $R^S_{T0,t}(R^R_{0,t})$ that clears safe securities market;

(3.2.2) Given safe rate guess $R^S_{0,t}$ find a risky rate target $R^R_{T0,t}(R^S_{0,t})$ that clears risky securities market;

(3.2.3) Check whether "guesses" and "targets" coincide. If so, done. If not, use an adjustment factor $\alpha \in (0,1)$ and update guesses: $R^R_{1,t} = \alpha R^R_{0,t} + (1-\alpha)R^R_{T0,t}$ and $R^S_{1,t} = \alpha R^S_{0,t} + (1-\alpha)R^S_{T0,t}$ until convergence.

(4) Given aggregate state η_{t-1}, current savings $a_{2,t-1}$, $a_{3,t-1}$, $a_{4,t-1}$, $a_{5,t-1}$, and equilibrium prices w_t, R^R_t, R^S_t, solve the households' problem to find saving for next period $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, $a_{5,t}$. See details in sections E and F;

(5) Go back to (3) while $t \leq T+S$; and

(6) Repeat steps (1) through (5) for N simulations, dropping the first S periods from each simulation. Take averages over the N simulations, resulting in an invariant distribution with T periods.

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