The Stolper-Samuelson Trade Effects Trigger the Rybczynski Trade Effects Negatively

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- Studying Trade Effect Based on Price-Trade Equilibrium

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Abstract – Most literature talks about trade effects of price changes on outputs in international economics as that a price increase of a good will lead to an expansion of the output of that good and a reduction in the output of the other good (see Bhagwati, Panagariya, and Srinivasan, 1998, p. 62). It only tells the story from the supply side. It is not in line with the fundamental economic principle, the law of demand, that says that there is an inverse (or negative) relationship between the price of a good (or service) and the quantity demanded. This study investigates it again based on the price-trade equilibrium from integrated world equilibrium (IWE). The paper shows that the overall result of supply and demand by the equilibrium is that a price increase of a good leads to a reduction in the output of that good and an increase in the output of another good. It is just a process of the Stolper-Samuelson trade effects negatively triggering the Rybczynski trade effects. The study proves the law of demand analytically.

Keywords: Factor price equalization; Heckscher-Ohlin Model; Equilibrium price; Trade Effects; General Trade Equilibrium

JEL Classification: F10, F20.

1. Introduction

Paul Samuelson and Lionel McKenzie are pioneers both in general equilibrium theory and in international trade theory. McKenzie (1987) described the importance of the general trade equilibrium as

> "Walras set of major objectives of general equilibrium theory as they have remained ever since. First, it was necessary to prove in any model of general equilibrium that the equilibrium exists. Then its optimality properties should be demonstrated. Next, it should be shown how the equilibrium would be attained,

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> that is, the stability of the equilibrium and its uniqueness should be studied. Finally, it should be shown how the equilibrium will change when conditions of demand, technology, or resources are varied".

The general trade equilibrium is at the center place of international trade theory. It is also a pre-condition to process full analyses of trade effects of one variable change on all other variables.

The Rybczynski theorem illustrates the trade effect of factor endowment changes on outputs by holding prices unchanged. The Stolper-Samuelson theorem tells the trade effects of good price changes on factor price by holding outputs unchanged. They are on partial equilibrium analyses. The equilibrium result is not available at the time. The trade effects of price change on output should be an extension of the Stolper-Samuelson trade effects on goods outputs. It is helpful to extend it to full equilibrium analyses, based on the general trade equilibrium.

The law of demand and supply says that there is an inverse (or negative) relationship between the prices of a good or service and the quantity of it that consumers are willing to purchase (See Gwartney, Stroup, Sobel, and Macpherson, 2006, p.58). The current common understanding of the trade effect of price changes on output, in textbooks, is that "an increase in the price of good j along will increase, or leave unchanged, the output of good j." It is consistent with the reciprocity relationship Samuelson proposed. However, it is odd to the basic economic principle, the law of demand and price.

Dixit and Norman's (1980) Integrated World Equilibrium (IWE) is remarkable to characterize equalized factor price from the mobility of factor endowments. They illustrated that if the allocation of factor endowment of two countries changes within the factor-price equalization (FPE) set in the IWE diagram, world prices will remain the same. It implies that the ratio of wage/ rental rate is constant.

Helpman and Krugman (1985) first confirmed Dixit and Norman's (1980) integrated world equilibrium. And they moved further to explore the equilibrium property by trade volume defined by domestic factor endowments. They abstracted an insight into the logic of the trade volume, that the differences in factor composition from the world consumption composition are the sole basis for trade. Guo (2015) visualized the sole basis of trade in the IWE diagram, as a border of the trade box specified by the goods price diversification cone[1], and used it to reach the price trade equilibrium within the integrated world economy. This study processes comparative statics and trade effects based on the equilibrium.

This study concludes that an increase in the price of good j along will decrease, or leave unchanged, the quantity of output of good j. It is consistent with the law of demand.

This paper is divided into four sections. Section 2 first reviews the equilibrium solution of Guo (2015). It then provides another independent approach to confirm the solution. Section 3 shows the trade effects of price changes on outputs based on the equilibrium. Section 4 discusses the negative feedback and its importance for economic system stability. The last one is the concluding remark.

2. Structure of Integrated World Equilibrium

With the normal assumptions, we denote a standard $2 \times 2 \times 2$ Heckscher-Ohlin model as the following. The production constraint of full employment of resources is

$$AX^h = V^h \qquad (h = H, F) \tag{2-1}$$

where $A(w/r)$ is the 2×2 technology matrix, X^h is the 2×1 vector of goods of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A are $a_{ki}, k = K, F, i = 1,2,$

The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \tag{2-2}$$

where $W^h$is the 2×1 vector of factor prices, its elements are r^h, rental rate and w^h, wage, And P^h is the 2×1 vector of good prices.

The balance condition[2] for factor content of trade is

$$\frac{w^*}{r^*} = -\frac{F_K^h}{F_L^h} = -\frac{s^h K^W - K^h}{s^h L^W - L^h} \qquad (h = H, F) \tag{2-3}$$

where F_L^h and F_K^h are exports of capital and labor services separately. $K^W$and L^W are world factor endowments; s^h is the share of the GNP of country h to the world GNP.

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[1] For the goods price diversification cone, see Fisher (2011).

[2] The trade balance of goods outputs is

$$\frac{p_2^*}{p_1^*} = -\frac{T_2^h}{T_1^h} = -\frac{s^h x_2^W - x_2^h}{s^h x_1^W - x_1^h} \qquad (h = H, F)$$

It is not a independant condition with trade balance condition of factor content of trade (2-3). They are same condition mathematically. We will only use (2-3) in equlibrium analysis.

Denote two parameters, which are the shares of country h's factor endowments to the world factor endowments respectively,

$$\lambda_L^h = \frac{L^h}{L^W} \qquad (h = H, F) \qquad (2\text{-}4)$$

$$\lambda_K^h = \frac{K^h}{K^W} \qquad (h = H, F) \qquad (2\text{-}5)$$

Eq. (2-3) can be rewritten as

$$\frac{w^*}{r^*} = \frac{(\lambda_K^h - s^h)}{(s^h - \lambda_L^h)} \frac{K^W}{L^W} \qquad (h = H, F) \qquad (2\text{-}6)$$

2.1 Pre-Equilibrium by the IWE

Dixit and Norman (1980, chapter 4) illustrated the whole FPE set in the IWE diagram shares the same world prices. It implies that world prices are constant within the FPE set. Therefore, the wage/rental ratio, $\frac{w^*}{r^*}$, is a constant. Guo (2015) introduced a Dixit-Norman constant,

$$\varphi = \frac{(\lambda_K^h - s^h)}{(s^h - \lambda_L^h)} \qquad (2\text{-}7)$$

The trade balance of factor content (2-6) can be expressed as

$$\frac{w^*}{r^*} = \varphi \frac{K^W}{L^W} \qquad (2\text{-}8)$$

This constant characterizes the world integration equilibrium that the world prices remain the same no matter how the world factors are distributed within the FPE set. We now use Walras's law to drop one market-clearing condition by assuming[3]

$$r^* = L^W \qquad (2\text{-}9)$$

Substituting it into (2-8) yields

$$w^* = \varphi K^W \qquad (2\text{-}10)$$

Substituting (2-9) and (2-10) into the price function (2-2) yields

$$p_1^* = a_{k1} L^W \ + a_{L1} \varphi K^W \qquad (2\text{-}11)$$

$$p_2^* = a_{k2} L^W + a_{L2} \varphi K^W \qquad (2\text{-}12)$$

We get a pre-equilibrium with one unknown variable. Eqs. (2-9) through (2-12) reduce the mystery of the structures of equalized factor prices. Eq. (2-8) bridges the production system (2-1) and cost system (2-2). It provides a condition to process the analyses of trade effects of one variable change on other variables within the Heckscher-Ohlin model, fully.

[3] Bhagwati, etc. (1998, p.139) used this way in their equilibrium analysis for small open economy. They mentioned "By Walras's law, we can drop one of these market-clearing conditions. By the same token, we can choose one of the goods as the numericaire and set its price equal to unity.

Guo (2015) provided the three approaches to illustrate that $\varphi = 1$. One is by the optimization of the trade volume. Another is using the trade volume defined with domestic factor endowments by Helpman and Krugman (1985, p.23). The last one is to show that if $\varphi \neq 1$, it will cause a conflicted relationship within the model. To be cautious, we use a different approach to derive the equilibrium solution, although it also uses Helpman and Krugman's trade volume. It is more direct and simpler. See Appendix for details.

2. Trade Effects of The Price Changes on Outputs

The price changes will affect world production output. The world production functions can be expressed as

$$a_{K1}x_1^W + a_{K2}x_2^W = K^W \tag{3-1}$$

$$a_{L1}x_1^W + a_{L2}x_2^W = L^W \tag{3-2}$$

World unit cost functions after factor price equalization are

$$a_{K1}r^* + a_{L1}w^* = p_1^* \tag{3-3}$$

$$a_{K2}r^* + a_{L2}w^* = p_2^* \tag{3-4}$$

where w^* and r^* are the factor price equalized, p_1^* and p_2^* are the world prices of goods.

At the equilibrium $\varphi = 1$, we can express factor price equalized as

$$r^* = L^W \tag{3-5}$$

$$w^* = K^W \tag{3-6}$$

Substituting them into (3-3) and (3-4) yields[4]

$$a_{K1}L^w + a_{L1}K^w = p_1^* \tag{3-7}$$

$$a_{K2}L^w + a_{L2}K^w = p_2^* \tag{3-8}$$

Eqs. (3-1), (3-2), (3-7), and (3-8) compose the equilibrium relationship among output prices, factor endowments, and good outputs[5]. We will use them to process comparative statics of world productions and costs.

International trade is with four physical market equilibriums: two factors and two outputs by two sets of prices: factor prices and good prices. Eqs (3-5) through (3-6) bridge

[4] The relative good price is the same as $\frac{p_1^*}{p_2^*} = \frac{a_{K1}L^W + a_{L1}K^W}{a_{K2}L^W + a_{L2}K^W}$ for assuming $r^* = 1$ or for assuming $r^* = L^W$.

[5] If we use Dixit and Norman's pre-equilbrium (2-9) through (2-12), all analyses in the following hold also.

world cost equilibriums (3-1) and (3-2) with world production physical equilibriums (3-3) and (3-4).

We will differentiate on Eqs. (3-1), (3-2), (3-7), and (3-8), with respect to p_1^*, to obtain the trade effects $\frac{\partial x_1^W}{\partial p_1^*}$ and $\frac{\partial x_2^W}{\partial p_1^*}$.

Firstly, differentiating (3-3), and (3-4) yields

$$\begin{bmatrix} a_{K1} & a_{L1} \\ a_{K2} & a_{L2} \end{bmatrix} \begin{bmatrix} \frac{\partial L^W}{\partial p_1^*} \\ \frac{\partial K^W}{\partial p_1^*} \end{bmatrix} + \begin{bmatrix} L^W \frac{\partial a_{K1}}{\partial p_1^*} + K^W \frac{\partial a_{L1}}{\partial p_1^*} \\ L^W \frac{\partial a_{K2}}{\partial p_1^*} + K^W \frac{\partial a_{L2}}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{3-10}$$

The unit-factor requirements $a_{ij}(w^*, r^*)$ are on the optimal levels derived from the cost minimization exercise, which are the functions of wage, w^*, and rental, r^*. The output price changes will cause the wage and rental rate changes, which will affect changes in the unit-factor requirements $a_{ij}(w^*, r^*)$. The second term on the right of (3-10) reflects the changes. Substituting (3-5) and (3-6) into it yields

$$\begin{bmatrix} L^W \frac{\partial a_{K1}}{\partial p_1^*} + K^W \frac{\partial a_{L1}}{\partial p_1^*} \\ L^W \frac{\partial a_{K2}}{\partial p_1^*} + K^W \frac{\partial a_{L2}}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} r^* \frac{\partial a_{K1}}{\partial p_1^*} + w^* \frac{\partial a_{L1}}{\partial p_1^*} \\ r^* \frac{\partial a_{K2}}{\partial p_1^*} + w^* \frac{\partial a_{L2}}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3-11}$$

It will be zero by the envelope theorem[6].

Simplify (3-11) as

$$\begin{bmatrix} a_{K1} & a_{L1} \\ a_{K2} & a_{L2} \end{bmatrix} \begin{bmatrix} \frac{\partial L^W}{\partial p_1^*} \\ \frac{\partial K^W}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{3-12}$$

Rewrite it to

$$\begin{bmatrix} \frac{\partial L^W}{\partial p_1^*} \\ \frac{\partial K^W}{\partial p_1^*} \end{bmatrix} = \frac{1}{b} \begin{bmatrix} a_{L2} & -a_{L1} \\ -a_{K2} & a_{K1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{b} \begin{bmatrix} a_{L2} \\ -a_{K2} \end{bmatrix} \tag{3-13}$$

where $b = (a_{K1} a_{L2} - a_{L1} a_{K2})$.

Secondly, differentiate (3-1), and (3-2) with respective to p_1^*

[6] The derivation of the Stolper-Samuelson theorem used this condition before (see Suranovic, 2010 chapter 115).

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1^W}{\partial p_1^*} \\ \frac{\partial x_2^W}{\partial p_1^*} \end{bmatrix} + \frac{\partial \begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix}}{\partial p_1^*} \begin{bmatrix} x_1^W \\ x_2^W \end{bmatrix} = \begin{bmatrix} \frac{\partial K^W}{\partial p_1^*} \\ \frac{\partial L^W}{\partial p_1^*} \end{bmatrix} \quad (3\text{-}14)$$

Generally, the factor endowments changes will not affect the changes of input request, a_{ij}. Eq. (3-14) involves the price change. we need to check the second item on the left side of the equation to see the effect of possible substitution.

$\frac{\partial a_{ij}}{\partial p_1^*} x_i^W$ in above summarizes the substitution of factor i due to a change in the price of good 1 across the economy. For the simple, we assume

$$\frac{\partial \begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix}}{\partial p_1^*} \begin{bmatrix} x_1^W \\ x_2^W \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3\text{-}15)$$

This assumption is majorly based on the consideration that the substitution and elasticities are comparatively small, they will not overturn the change direction of factors in Eq. (3-14).

Simplify (3-14) as

$$\begin{bmatrix} a_{K1} & a_{K2} \\ a_{L1} & a_{L2} \end{bmatrix} \begin{bmatrix} \frac{\partial x_1^W}{\partial p_1^*} \\ \frac{\partial x_2^W}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} \frac{\partial K^W}{\partial p_1^*} \\ \frac{\partial L^W}{\partial p_1^*} \end{bmatrix} \quad (3\text{-}16)$$

Rewrite it to,

$$\begin{bmatrix} \frac{\partial x_1^W}{\partial p_1^*} \\ \frac{\partial x_2^W}{\partial p_1^*} \end{bmatrix} = \frac{1}{b^2} \begin{bmatrix} a_{L2} & -a_{K2} \\ -a_{L1} & a_{K1} \end{bmatrix} \begin{bmatrix} \frac{\partial K^W}{\partial p_1^*} \\ \frac{\partial L^W}{\partial p_1^*} \end{bmatrix} \quad (3\text{-}17)$$

It implies that

$$\frac{\partial K^W}{\partial p_1^*} = \frac{-a_{K2}}{b} \quad (3\text{-}18)$$

$$\frac{\partial L^W}{\partial p_1^*} = \frac{a_{L2}}{b} \quad (3\text{-}19)$$

Thirdly, substituting (3-18) and (3-19) into (3-17) yields[7]

[7] Note here that the elements in (3-13) switch their order when substituting it into (3-16).

$$\begin{bmatrix} \frac{\partial x_1^W}{\partial p_1^*} \\ \frac{\partial x_2^W}{\partial p_1^*} \end{bmatrix} = \frac{1}{b}\begin{bmatrix} a_{L2} & -a_{K2} \\ -a_{L1} & a_{K1} \end{bmatrix}\begin{bmatrix} -\frac{a_{K2}}{b} \\ \frac{a_{L2}}{b} \end{bmatrix} = \frac{1}{\mathrm{b}^2}\begin{bmatrix} -a_{L2}a_{K2} - a_{K2}a_{K2} \\ a_{L1}a_{K2} + a_{K1}a_{K2} \end{bmatrix} \tag{3-20}$$

It shows

$$\frac{\partial x_1^W}{\partial p_1^*} < 0 \tag{3-23}$$

$$\frac{\partial x_2^W}{\partial p_1^*} > 0 \tag{3-24}$$

Eqs. (3-21) and (3-22) imply that an output price increase will cause a decrease in the quantity of that output and an increase in the quantity of another output. This is just opposite to the existing trade effect between outputs and their prices.

In Eq. (3-17), there are two items on the right side, we name them as

$$ZY = \frac{1}{b}\begin{bmatrix} a_{L2} & -a_{K2} \\ -a_{L1} & a_{K1} \end{bmatrix} \tag{3-25}$$

$$SS^* = \frac{1}{b}\begin{bmatrix} -a_{K2} \\ a_{L2} \end{bmatrix} \tag{3-26}$$

The elements in ZY are the Rybczynski trade effects, their signs can be written as[8]

$$ZY = \begin{bmatrix} \frac{\partial x_1^W}{\partial K^W} & \frac{\partial x_1^W}{\partial L^W} \\ \frac{\partial x_2^W}{\partial K^W} & \frac{\partial x_2^W}{\partial L^W} \end{bmatrix} = \begin{bmatrix} + & - \\ - & + \end{bmatrix} \tag{3-27}$$

The elements in SS^* and their signs can be written as

$$SS^* = \begin{bmatrix} \frac{\partial w^*}{\partial p_1^*} \\ \frac{\partial r^*}{\partial p_1^*} \end{bmatrix} = \begin{bmatrix} - \\ + \end{bmatrix} \tag{3-28}$$

It is not the typical Stolper-Samuelson trade effect. It switches the order of two elements. The original Stolper-Samuelson trade effect by the notation of this study is

$$SS \quad = \begin{bmatrix} \frac{\partial r^*}{\partial p_1^*} \\ \frac{\partial w^*}{\partial p_1^*} \end{bmatrix} = \frac{1}{b}\begin{bmatrix} a_{L2} \\ -a_{K2} \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix} \tag{3-29}$$

We see that SS^* reverses the sign of SS. Identifying this reverse leads to an interesting result. Substituting (3-27) and (3-28) into (3-17) yields

[8] See Thompson (2007) for trade effect items in Eq. 8.

$$\begin{bmatrix}\frac{\partial x_1^W}{\partial p_1^*}\\ \frac{\partial x_2^W}{\partial p_1^*}\end{bmatrix}=\begin{bmatrix}\frac{\partial x_1^W}{\partial K^W} & \frac{\partial x_1^W}{\partial L^W}\\ \frac{\partial x_2^W}{\partial K^W} & \frac{\partial x_2^W}{\partial L^W}\end{bmatrix}\begin{bmatrix}\frac{\partial w^*}{\partial p_1^*}\\ \frac{\partial r^*}{\partial p_1^*}\end{bmatrix}=\frac{1}{b^2}\begin{bmatrix}-a_{K2}a_{L2}-a_{L2}a_{L2}\\ a_{L1}a_{L1}+a_{K1}a_{L2}\end{bmatrix} \tag{3-30}$$

Its sign expression by (3-27) and (3-28) is

$$\begin{bmatrix}\frac{\partial x_1^W}{\partial p_1^*}\\ \frac{\partial x_2^W}{\partial p_1^*}\end{bmatrix}=\begin{bmatrix}+ & -\\ - & +\end{bmatrix}\begin{bmatrix}-\\ +\end{bmatrix}=\begin{bmatrix}-\\ +\end{bmatrix} \tag{3-31}$$

Eqs. (3-30) show that the trade effect of price change on outputs is a complex combination of the Rybczynski trade effects and the Stolper-Samuelson trade effect. Briefly, it is a process that the Stolper-Samuelson trade effects trigger the Rybczynski trade effects negatively.

The output price changes will cause changes in all other variables on the model. We present them by Jones's (1965) magnification effects as

$$\Delta p_1^* \uparrow: \quad \hat{w}^* \downarrow > 0 \; > \; \hat{p}_1^* \uparrow > \hat{r}^* \uparrow \qquad \rightarrow \qquad \hat{x}_1^H \downarrow < 0 < \hat{L}^H \uparrow < \hat{x}_2^H \uparrow$$

4. The importance of negative feedback on the price-trade equilibrium

The trade effect between price and output in the current literature is consistent with Samuelson's reciprocity relationship, which is

$$\frac{\partial w_i}{\partial p_j}=\frac{\partial x_j}{\partial V_i} \tag{4-1}$$

It is true in quantity. *And it is only true in quantity*. Samuelson obtained this relationship by single country analysis. Jones and Scheinkman (1977) restated it by dual analysis, which is a single country analysis too. It is a partial equilibrium result.

By the price-trade equilibrium, prices can be expressed as a function of world factor endowments. And outputs can be expressed as a function of prices. The trade effect between prices and outputs in the last section shows that both the Stolper-Samuelson trade effects and the Rybczynski trade effects are involved but they engage negatively.

With the trade equilibrium, we can use numerical simulation to illustrate the effect of price changes on output. The result does not favor the relationship (4-1) and the existing logic of the trade effect price changes on output in textbooks.

The trade effects between prices and outputs by (3-30) show negative feedback between output quantities and output prices. It is an important property for equilibrium. Negative feedback generally is a fundamental feature of system stability. It tends to promote a settling to equilibrium and reduces the effects of perturbations. The existing trade effect between price and output is positive which tends to lead to instability via exponential growth or chaotic behavior, which does not reflect trade equilibrium and comparative statics.

Negative feedback is widely accepted and used in many fields, like mechanical and electronic engineering or living organisms to reset and keep their equilibriums. Market mechanics for international trade should be in a process of negative feedback also.

Conclusion

The trade effect of price changes on outputs is a classical topic in international economics. The novelty of this study is to use the IWE price-trade equilibrium to revisit this topic. The conclusion of this paper replenishes the trade effect in the current literature with the view of demand and prices. It shows a negative trade effect between good prices and good outputs: the Stolper-Samuelson trade effects trigger the Rybczynski trade effects negatively.

The result of this paper is also useful in providing the first analytical case for the demand law.

Appendix - *Specifying the equilibrium solution by the trade volume defined by Helpman and Krugman*

Solve out s^H by (2-7)

$$s^H = \frac{(\lambda_K^h + \varphi\lambda_L^h)}{(\varphi+1)} = \frac{K^h}{(\varphi+1)K^W} + \frac{\varphi L^h}{(\varphi+1)L^W} = \frac{K^h L^W + \varphi K^W L^h}{(\varphi+1)L^W K^W} \tag{A-1}$$

Rewrite the factor content of trade, by (A-1), as

$$F_K^h = s^h K^{hW} - K^h = \frac{K^h L^W + \varphi K^W L^h}{(\varphi+1)L^W} - \frac{(\varphi+1)L^W}{(\varphi+1)L^W} K^h = -\frac{(K^h L^W - K^W L^h)}{(\varphi+1)L^W} \quad h = (H, F) \tag{A-2}$$

$$F_L^h = s^h L^{hW} - L^h = \frac{K^h L^W + \varphi K^W L^h}{(\varphi+1)K^W} - \frac{(\varphi+1)K^W}{(\varphi+1)K^W} L^h = \frac{\varphi(K^h L^W - K^W L^h)}{(\varphi+1)K^W} \quad h = (H, F) \tag{A-3}$$

We express exports by a negative sign in (A-2) and (A-3).

Helpman and Krugman (1985, p.23) showed that a line parallel to the diagonal line in the IWE diagram is an equal trade volume line. They came up with an insightful idea of the equal trading volume line defined by domestic factor endowments. They illustrated that there are some variables (γ_L , γ_K) for all equal trade volumes lines, which satisfy the following relationships:

$$VT = \gamma_L L^H + \gamma_K K^H \tag{A-4}$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \tag{A-5}$$

They argued (A-4) that the trade volume is a linear function of K^H and L^Heventually and that the differences in factor composition are the sole basis of trade (see Helpman and Krugman 1985, pp24-24, pp175). It implies that the equal volume of trade curves in the IWE diagram are straight parallel lines to the diagonal. The two equations also ensure that a higher difference in factor composition leads to a higher trade volume; trade volume is zero if factor endowments distribute at the diagonal line in the diagram of Integrated World Equilibrium. They showed that one of γ_L, γ_K is negative. If country H is capital abundant, its two variables satisfy $\gamma_K > 0$ and $\gamma_L < 0$.

Assume country H is capital abundant, and assume

$$\gamma_K = L^W \tag{A-6}$$

We obtain, by (A-5)

$$\gamma_L = -K^W \tag{A-7}$$

Substituting them into (A-4) yields

$$VT = K^H L^W - K^W L^H \tag{A-8}$$

The trade volume can be written as

$$VT = 2|F_K^H|\, r^* = -2F_K^h r^* \tag{A-9}$$

Substituting (2-9) and (A-2) into it yields

$$VT = 2\frac{(K^h L^W - K^W L^h)}{(\varphi+1)L^W} \cdot L^W \tag{A-10}$$

Substituting (A-8) into (A-10) yields

$$2\frac{K^h L^W - K^W L^h}{(\varphi+1)} = K^h L^W - K^W L^h \tag{A-11}$$

It can be simplified as

$$2 = \varphi + 1 \tag{A-12}$$

We obtain

$$\varphi = 1 \tag{A-13}$$

The vector of factor prices is orthogonal to the vector of factor content of trade,

$$\vec{W} \cdot \vec{F}^h = 0 \tag{2-26}$$

And the vector of world factor endowments is orthogonal to the vector of Helpman and Krugman's variables in their trade volume

$$\vec{V}^W \cdot \vec{\gamma}^h = 0 \qquad (2\text{-}27)$$

where $\vec{V}^W = \begin{bmatrix} K^W \\ L^W \end{bmatrix}$ and $\vec{\gamma}^h = \begin{bmatrix} \gamma_K \\ \gamma_L \end{bmatrix}$. We can show them together as

$$\frac{w^*}{r^*} = -\frac{F_K^h}{F_L^h} = -\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \qquad (2\text{-}28)$$

The equilibrium solution is a merge of Dixit and Norman's idea of factor price equalization set (FPE set) and Helpman and Krugman's idea of the equal trade volume line. The lines of equal wage-rental ratio, $\frac{w^*}{r^*}$, are parallel to the anti-diagonal line in the IWE diagram.

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