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Kantian Epistemology in Examination of the Axiomatic Principles of Economics: the Synthetic a Priori in the Economic Structure of Society

Mirza Adil Ahmad Mughal

Abstract—Transcendental Analytic, in Critique of Pure Reason, combines the space and time as conditions of the possibility of phenomenon from Transcendental Aesthetic with the pure magnitude-intuition notion. The property of continuity as a qualitative result of the additive magnitude brings the possibility of connecting with experience, even though only as a potential because of the a priori necessity from assumption, as syntheticity of the a priori task of a scientific method of philosophy given by Kant, which precludes the application of categories to something not empirically reducible to the content of such a category's corresponding and possible object. This continuity as the qualitative result of a priori constructed notion of magnitude lies as a fundamental assumption and property of, what in Microeconomic theory is called as, 'choice rules' which combine the potentially-empirical and practical budget-price pairs with preference relations. This latter result is the purest qualitative side of the choice rules', otherwise autonomously, quantitative nature. The theoretical, barring the empirical, nature of this qualitative result is a synthetic a priori truth, which, if at all, it should be, if the axiomatic structure of economic theory is held to be correct. It has a potentially verifiable content as its possible object in the form of quantitative price-budget pairs, yet, the object that serves the respective Kantian category is qualitative itself which is utility. This article explores the validity of Kantian qualifications for this application of 'categories' to the economic structure of society.

Keywords— Categories of Understanding, Continuity, Convexity, Psyche, Revealed Preferences, Synthetic a priori,

I. INTRODUCTION

Kantian Subtitle :

"It is remarkable that of magnitudes in general we can know a

priori only one *quality*, namely, *continuity*, while with regard to all quality (the real of appearances) nothing more can be known to us a priori than the intensive *quantity* of appearances, that is, the fact that they have a degree. Everything else is left to *experience*."

- Critique of Pure Reason [1]

Continuity taken qualitatively in itself can be examined synthetic a priori, as can be seen in the common properties between continuous functions and compact sets [1], yet, *utility* gained from an experience of consumption as a behaviour and as psyche, it seems, is only experiential that cannot be universalised and made to be seen as a stable *quantity* in itself; therefore the *transcendental* possibility of gaining an intuitive *magnitude* of utility, by explicitly *not* taking utility as a thing in itself as in the *Kantian transcendental* and in economic theory, from the underlying *continuity in psychic preference structure*, brings the *quantitative from the qualitative*. Which is here proposed to be an instance of a 'transcendental induction' on the quantitative from the qualitative through continuity.

[Lemma: That is, if the quantitative gathered through a transcendental magnitude deduces the quality of continuity, the case of economic theory turns the qualitative of psyche into an induction on the quantitative, which is proposed here as a possible case of 'transcendental induction', to be considered within the Kantian epistemology. (This statement, apart from the rest of this paper, is an original *problematique* as a consequence to what this paper is about.)]

Almost unwittingly, the economic theory employs explicitly the transcendentially taken continuities and magnitudes of something like *psyche* and puts them in the *mathematical* apparatus of the *synthetic a priori*. The budget-price pairing, that maps preferences as psychic potential of choice rules into *revealed preferences*, makes utility, the experiential, into the knowable in the synthetic a priori through *continuity* (under the category of reality) if not as the contemplation of utility itself.

The additive "*intensive*" magnitude of consumption makes itself less valuable in the form of the psychological fact and the *concept* of diminishing marginal utility whereas this diminishing valuability of *added* consumption of a *consumed* and its qualitative result of the *nonempty*¹ and therefore

¹ The counter-argument to *nonemptiness* is as healthy and therefore as unhealthy as the prevalent objections to Axiom of Choice regarding the continuity of continuous functions and compact sets in mathematics. These

convex nature of utility as a psychic *experience* is the method of *convex, and therefore continuous*, preference structure with respect to choice-relation sets. Our task here is to see this:

1 The criterion of the truth of this *experience* be *a priori* thus *universal and necessary*.

2 It be synthetic (*even though there's much that must be thrashed through its analytic*).

3 It be parse-able under the *categories*, say, of necessity, causality and reality; with a special care towards not applying a category to a *content* that is not its possible *object*. (So that the axiomatic structure of economic theory is not taken as what Kant calls "*dialectical illusion*" which better belongs in "reason" than in the *Analytic*.)

II. REVEALED PREFERENCES AND TRANSCENDENTAL MAGNITUDES

The quantitative of budget-price pairs can be continuous because of the magnitude having the inherence of continuity as a quality. What we are getting here at is that through continuity of preferences, the psychic is being transformed into the quantitative as the quantitative side of the *itself-qualitative* continuity which has been transcendently inferred from the psychic-qualitative notion of *preferences*. What about *preferences in-themselves*? Are they things-in-themselves? *If so*, what can be turned into a transcendental *space-and-time* magnitude measure from them to turn them into *phenomena*? The answer lies in *rendering* preferences and this time *not* in budget-price pairs of choice rules (which would've been the revealed preferences) *directly* but in the form of *im-mediacy* of preferences which *lies* in their *consumption*.²

Now, consumption is additive - quantitative magnitude - and is a possibility of that in the *number* of units consumed, and then the *succession* in consumed units. The latter synthesizes time as a proper synthetic here because from *prior* experience it can

objections are linearly a questioning of the phenomenon itself (*like in Alain Badiou's, Being & Event* [16]). Now, a priori and pure intuition of space as a Kantian 'condition', of all possibility of pure and empirical conceptions of experience and possibility of experience itself, just makes phenomenon a derived but a necessary attribute of this '*condition*'. And because this condition is a priori therefore it's universal and necessary. Thus in such a reverse framing, the objections, to Axiom of Choice and the necessity and universality of a possible phenomenon, are wrong.

² Utility function, a *real-valued continuous function*, as a monotonic increase in a *preordered preferences*' set [11].

be even a priori built that every additional unit consumed gives a diminishing marginal utility which is a continuous of the utility itself as well as the additive-quantitative of the consumption. Diminishing marginal utility is a category of *negation*; as utility's continuity through revealed preferences comes under the category of *reality*. Because continuity can be conceived either in space or in time, the choice rules case of *revealed preferences* is about synthetic a priori case of *space* as magnitude but that of *preferences* directly rendered through utility of consumption is a case of continuity, synthetic and a priori, *in time*. In the former case continuity comes as an additive price of consumption utility from the latter case of preferences of utility of consumption. Whereas it is the latter case of preferences in themselves from utility of consumption that, in the prime, builds the choice rules of budget-price pairs. *And this is where both kinds of continuities merge. The time continuity of non-linear or diminishing marginal utility actually builds the possibility of the continuity in choice rules; which is really important to understand. In the time-continuity of preferences of utility from direct consumption, the higher the consumption of the good *a*, the lower the utility and thereby a continuously increasing preference for the good *b*, whereby *a* can be foregone as, and as an *implied, price* for buying *b*. The diminishing utility of *a* here is like an implied³ budget set for buying *b*, that is, to pay *a*, to buy *b*. Now let's regather what we just saw: in consumption utility, the *number* of units consumed is the implication as *conception* of time here from its *intuition, that is : the time itself is being taken only transcendently as an intuition-condition - internally as succession - of the conception of magnitude as a number with diminishing utility of each succeeding unit, which is not taking time as a thing in itself.**

And because every individual is at an individual good's utility preference from consumption, there can be many individuals at *almost every point or slope* of the marginal utility of consumption curve so that there are many *price-budget* pairs possible for all goods *a, b, c, ... n*, such that the choice rules have a *linear space* continuity. The *continuity* in revealed preferences is the synthesis of these two continuities that are synthetic as well as a priori where the questions of universality and necessity are successfully implied too.

Now we turn to the problem of examining utility through continuity under Kantian categories of understanding.

Of the four classes of categories namely, of Quantity, of Quality, of Relation and of Modality, Kant puts the first two

³ This is here a suggested result of an implication of envelope theorem and that of indifference curves too.

classes as of *intuition*, "either pure or empirical" that relate with the "*mathematical*"-ity of magnitude as intuition in extensive or intensive space (and time) from which we derived continuity. The latter two classes are put as "*dynamical*"; it is notable that the class of categories under Quality is not taken as partaking in the dynamic classes of Relation and Modality. In so far as the continuity as *qualitative* is enumerated as a result of the quantitative as an *analytic* property of the synthetics of magnitude in *Transcendental Analytic*, we should take it so at face value because the movement into the Relational and the Modal is rife with the dangers of *dialectical illusion*, at least for the case at hand. Being careful also for not turning "criteria of *thought*" into "properties of *objects*" which is to remind the fundamental method of transcendental logic of not taking appearances as representations of objects for the objects as things in themselves. For the transcendental *cognition* of an object, not the object *itself*, Kant gives for the class of Quantity, the categories of *Unity*, *Plurality* and *Totality* with their subtitles of *qualitative unity*, *qualitative plurality* and *qualitative completeness* (think of *convexity* in economic theory) respectively for an attribution to criteria of a *cognition of an object*, not the *object of cognition in itself*. The categories, insofar as they are *categories of understanding*, under the class of *Quantity* have *qualitative* syntheses of *unity*, *plurality* and *completeness*, of sensibility from intuitions to conceptions in which objects are cognized. The *unity of conception* in *qualitative unity* makes possible the qualitative result from the quantitative category of *Unity*; for instance, the *measure* conception of magnitudes taken transcendently of a content that is just *psychic*, for the economic case at hand. The *truth* of it determines its 'objective reality' in the form of *qualitative plurality* of the instances of rendering that truth, turning the *quantity* category of *Plurality* into a *quality* of conception; for instance the synthetic *truth* of diminishing marginal utility in its *a priori and synthetic universality*. Whereas it is the conception through *qualitative completeness* that renders the *extent* or *perfection* of judgement through understanding enabled through the quality taken from the quantity category of *Totality*; for instance, the *linear space* conception of the market wherein the global moment of budget-price pairs is possible as a *continuous measure* 'spanning' the revealed preferences linearly.

In the categorical analysis of continuity we have synthesized, in the cases of that in space with budget-price pairs and of that in time with preference from utility of consumption, into *revealed preferences*, the point of departure is here: the *content* of *utility* or of revealed preferences is not *cognized of*, in itself as a thing in itself; that's why the rendition of continuity in economic theory, as per the synthetic a priori method of mathematics, is *mathematical*; because

transcendental deduction preserves only a *possibility of experience*, not experience in the empirical. As Kant goes [3]:

"§10" (*Transcendental Logic*):

"The whole aim of the transcendental deduction of all *à priori* conceptions is to show that these conceptions are *à priori* conditions of the possibility of all experience. Conceptions which afford us the objective foundation of the possibility of experience, are for that very reason necessary. But the analysis of the experiences in which they are met with is not deduction, but only an illustration of them, because from experience they could never derive the attribute of necessity."

The *conception* of continuity that "affords" us the "possibility" of *this* experience, even without experience itself, is *if, necessary* for the validity of a given experience then at rough it may seem to be a *reduction* of an experience itself to its conception which as we said makes that experience possible in the first place; which then can be framed with a *charge* of a "*material idealism*", an inculcation not any infrequent in these times too especially regarding the elaborate theoretical apparatus of *Economics*. In *Refutation of Idealism* section [4] in *Critique of Pure Reason* Kant refutes this idealism; specifically, the one kind of it he calls the "*problematic*" idealism of Descartes - that declares anything other than "*I am*" as "doubtful" - and the other kind as the "*dogmatical*" one of Berkeley - wherein *space* itself and anything possible in it is held as just an "imagination". The case here at hand is about reducing an experience in its possibility *to* its conception. It seems useful to explore both of the kinds of material idealism given by Kant for the implications for the case at hand.

Given that the claim under contest is that experience is nothing but a *conception* thereof, it is quite comparable with the dogmatic idealism that space and *the things in it* are just an *imagination*, for instance, *magnitudes* taken as an extended notion of *space*. In dogmatic idealism there is a *denial of space itself in taking it to be* a thing in itself - *whereby positing it to be unknowable* by implication⁴. This denial in the first does not differentiate between the *things* in space and *the space*; because Kant requires space in a priori intuition to be a *condition* of the possibility of things in space - keeping in mind that the *intuition-condition* of space is the condition of the *possibility* of the *conception of magnitudes*. Thus, as per Kant, we can say that the assertion, of calling 'experience itself

⁴ If space is a thing in itself then it must be closed to us insofar as in an experience of space we must only resort to its empirical intuition of it without a pure intuition but we know that even calling the *experience* of an empirical recourse with space as a mere "imagination" must confirm the presence of the a priori intuition of it which makes even the empirical intuition itself possible if it is still held that *we* ourselves are *not* space.

to be just a conception', and like calling 'space to be just an imagination', is akin to calling 'experience to be a thing in itself in the empirical and therefore it being unknowable a priori' which again is like calling space to be only experiential and empirical, not knowable a priori. The question must now re-examine the Kantian charge that such a material idealism actually means a *characterization* of space *not* as a *condition* of external spatial reality but as a *thing in itself* with its concomitant implications. Kant takes space to be an *external*, as time to be *internal*, *intuition*, not as a *conception* but as the *foundation* of a conception of things in space. In *Metaphysical Exposition of this Conception [Of Space]*, Kant [5]:

"Space is not a conception which has been derived from outward experiences...in order that I may represent them [things in space] not merely as without of and near to each other, but also in separate places, the representation of space must already exist as a foundation. Consequently, the representation of space cannot be borrowed from the relations of external phenomena through experience;".

Which means *if* space is held to be a determination of *relation* of things in space then it *is* a *property* of those things *in themselves*. Because, as Kant says about Geometry, the synthetic a priori *conception* of space must have an *intuition* of space a priori because a conception just has an "internal necessity" that does not give much about the *object* of conception, and *still* that the cognition of the conception of objects is not a cognition of those objects *themselves*. That's why the a priori intuition of space must be *pure*, not empirical. Now, *if*, the conception at hand about the *reality* or *imaginariness* of experience, and, the denial of space, treat space as *part* of the conception of things, as a *property* of things in themselves resorted to empirically, and *not* as a pure a priori intuition of mind about the external spatiality, *then* it itself is a *denial* of the *separation* of 'an *intuition* pure and a priori' from 'a *conception*'. Such a denial of this separation means that there is no *internal* (about time) or *external* (about space) *sense* as a pure intuition existing in a subject, which, as we know, as per Kant, *does* exist. Thus, the said denial is wrong.

Let's now move towards examining this *first separation* of *intuition a priori* from the *conception* to the *second* case of the *separation* of a *conception* from its *experience* (or the *object of the conception*). If a conception of an experience is the experience itself then the experience is the conception that must access the pure intuition *directly*. But the *pure* intuition being *simple* and a *priori sense of space* (as a '*possibility space*' for *magnitudes*) coming in direct contact with experience must become *empirical*, not a priori, which should then lead to an *empirical conception* if the *first separation* of a

conception from the pure intuition is held to be *preserved*, *but*, if this *preservation* is still held to be true about the *first separation* then the *second separation* is *also* true; because how can a pure intuition (from the first separation) lead to an empirical conception *directly* without an *external* experience and a *conception* which is not empirical? *Either the intuition is not 'pure a priori', or, the conception is not empirical! That is, the empirical conception cannot meaningfully and phenomenally access the 'pure intuition a priori' directly. And therefore the intuition of space a priori, as a sense of magnitude, gives the qualitative result of continuity from the quantitative nature of the sense of extensive magnitude in general with pure intuition a priori which imparts intensive magnitude of quality of appearances in the form of the quality of continuity. Finally on this, the a priori pure intuition as the condition regarding conceptions and experience of space is universal and necessary because of its locational primitiveness precisely arising in the a priori.*

Having done the Kantian exposition of the falsity of Berkeley's dogmatical *material* idealism in Kantian terms we now move towards such an exposition on Cartesian problematic material idealism. But before that, there is a parallel synthesis that should be achieved alongside establishing the very need and *scope* of this contesting of a material idealism charge regarding conceptions creating the very possibility of experience, which we just established to be necessary and universal in being a priori in their conditions for experience of space. Because the revealed preferences synthesis of utility through *continuity* is essentially a question of whether such a synthesis running through *continuity* is a *material idealism* by other means, or *not*. If it is so, the qualitative result in the form of *continuity* coming from the quantitative governed by a pure intuition and then a conception of *magnitude* means, in a reverse progressing implication that, the magnitude that gives continuity *is only possible empirically* which in turn means that '*either*': there is no possibility of the conception of a magnitude even in the empirical '*or*': the experience gives the conception of magnitude without the a priori and pure intuition of space. If the *either*-part is correct then it denies the *empirical inner sense* of *himself* for a subject headlong whereby the *either*-part stands refuted. If the *or*-part is correct then the conceptions *come* from experience whereby the above established *first separation* of 'a priori intuition as a condition of a conception' from 'the conception itself' is violated which makes the *or*-part be refuted too.

With this, it is thus conveniently declared that the refutation of *either*-part is a direct refutation of the Cartesian *material idealism*, *also*.

Now we come to the above mentioned *scope* of the *commoned-in mathematical* conception of *continuity* between *Economics* and *Kantian epistemology*. The intent here is to vet the reverse engineering implications of economic theory, through the mediacy of mathematical truths being synthetic a priori, onto *Kantian epistemology*.

Value in economic theory is - in terms of the revealed preferences' framework which is the precise way of *not taking* something as abstract and *psychic* as utility *to be* a thing in itself - *inter-subjective*. Is it rigorous to take value as inter-subjective and then find the result that the inter-subjectiveness is an *induction* on the a priori pure intuition of magnitude thereby mutually acceding the *necessity and universality* of the a priori intuition *to* this induction? *And* also, is it, that such, is a possible case of a *transcendental induction*?

III. TRANSCENDENTAL MAGNITUDES; ADDITIVITY AND INDEPENDENCE FOR INTERSUBJECTIVE VALUE

Let's examine now the *additivity* implied in continuity and vice versa. The *intersubjective* methodicity in value seen through the Kantian lens is not to be taken as a *thing in itself*, that is, it must be taken as a matter of *magnitudes* taken *transcendentally* after which the synthetic a priori treatment should guarantee the result only mathematically and *as valid*; and that result should then be treated as per the economic theory to be, if at all, in agreement with that result. That the magnitudes once taken transcendently should imply continuity qualitatively and be as *additive* as the countably finite additivity of a *measurable* function. It is the synthesis of intersubjective value through individual *subjectivities* themselves which are *interdependent* and therefore dependent, *intersecting* and *nonadditive*. But those subjectivities, of themselves, treat of the object of utility or that of desire as of a thing in itself *as of* some intrinsic value; not a value in the sense of a transcendental magnitude. This latter conception, though, is the way, of an ordered magnitude of value through prices, that the *good* is actually *given* in the market. Because, as above remarked, the *direct* experience of utility, as if it were of a thing in itself, barring an instance of intersubjective magnitude of value, is itself *mediate* through the *additive* experience of consumption *which*, even itself being *interdependent* in the price formation, is *only* so in being given in the market, and then is there in the market as being a *linear* product of revealed preferences in the form of *moneyed* magnitude that combines choice rules with preference relations. *And* this is why [6]'s objection to the *Euclidean*

space formulation of value is not warranted for a magnitude which, though made intersubjectively, is made of transcendental magnitude as a *conception* of it in the market and not of things in themselves. This *linearity* of transcendental magnitudes is mathematically plausible given the underlying nature of the assumption of *large number of buyers and sellers* - implying a *law of large numbers*⁵ -, and the *Lebesgue* measure conception [7] [9] wherein the *measure of a single point is zero*; and this puts [8]'s objection in perspective to fads and fashions *seeming* to make aggregate market demand curve of a product, which is more *elastic*⁶ than the individual demand curve, *non-independent* and therefore non-additive.⁷ This objection is answered this way: tastes, fads and fashions are *not* extraneous to demand and prices per se even if it is *ceteris paribus* assumed that they be on hold while the price acts on a quantity demanded in a demand curve. *Because* the price *itself*, as a measure of demand for a product, is made of *desires* for a good which *may or may not* stem from a *need*, a *usefulness*, a *snobbery*, a *neighbourly competition*, or just as a *function of income*. "Preferences are almost *always, to some extent*, induced [2]."⁸ Because the latter enumeration is a delving into the *intrinsic* notions of goods and their value *which* the economic theory *explicitly debars* and *that's why* the essential method of value conception in economic theory, through revealed preferences, is *transcendental and Kantian*.⁹ The law of large numbers and

⁵ "whereby convergence in distribution (denoted $D \rightarrow$) for a functional defined on a sequence of finite probabilistic objects (in this case, rescaled marked point processes) is established by showing that these probabilistic objects themselves converge in distribution to an infinite probabilistic object (in this case, a homogeneous marked Poisson process) and that the functional of interest is continuous." (M D Penrose 2007 [17])

⁶ That the idiosyncrasies of tastes and fads do not make the demand space into a case of *tastes' friction*.

⁷ "A sufficient condition for market demand to satisfy the Law of Demand is that the mean of all households' income effect matrices be positive definite. We show how this mean income effect matrix can be estimated from cross section data under metonymy, an assumption about the distribution of households' characteristics. The estimation procedure uses the nonparametric method of average derivatives. Income effect matrices estimated this way from U.K. family expenditure data are in fact positive definite." (Härdele, Wolfgang, Werner Hildenbrand, and Michael Jerison. "Empirical Evidence on the Law of Demand." *Econometrica* 59, no. 6 (1991): 1525–49. <https://doi.org/10.2307/2938277>.)

⁸ "Consider, for example, preferences for lotteries over amounts of money available tomorrow. Unless the individual's preferences over consumption today and tomorrow are additively separable, his decision of how much to consume today—a decision that must be made before the resolution of the uncertainty concerning tomorrow's wealth—affects his preferences over these lotteries in a manner that conflicts with the fulfillment of the independence axiom."

⁹ Austrian school economists starkly object to the Marshallian explanation of the long run being a case of costs determining prices instead of the current prices *always* determining costs such and so that the long run *never exists*. [18]

asymptotics of large samples imply, likewise, *divisibility, additivity, convexity and therefore continuity*. [10]

IV. MATHEMATICAL ADDITIVITY IS ITSELF KANTIAN

Possibility of Inaction and Additivity [11] :

Let T be a convex cone with vertex 0, and T being a set of p prices in commodity space R^N of profit maximizers. Given that Y is the total production set;

Then,

" $0 \in Y$, (possibility of inaction). Given p in T , 0 may be a maximizer (inaction may be optimal), it may even be the unique maximizer. In any case the maximum profit is clearly non-negative."

In the above, the mathematical synthetic a priori is being treated as an analytic for taking magnitudes mathematically; and through the common continuity synthetic from the mathematically-taken analytic of continuity the economic synthetic is built a priori.

" $(Y_j + Y_j) \subset Y_j$ (additivity). Given p in T_j , the maximum profit is non-positive (author's addition: but not negative which implies zero when read with possibility of inaction). (If a possible y_j gave a positive profit, $2y_j$ would also be possible and give a twice larger profit.) 'Additivity and possibility of inaction' therefore implies that the maximum profit is null if it exists. This covers the case of a free entry industry."

Here above, the *no maximum profit* analytical observation is constructed from a synthetical a priori through the mathematical relation of magnitudes such that the axiomatically correct transcendental intuition of magnitude is then treated as a mathematico-economic synthetic a priori as such.

Given that Y is also the total technological knowledge 'because' it is the total production set insofar as the total production implies the possibility and expansion of its frontiers through technological knowledge too. And, "it is, in general, no longer contained in a relatively small coordinate subspace of R^N ". This implies that even though Y is convex and subadditive alongside being additive it still can be greater

The prices adjust so much intersubjectively that the realized transactions only reveal the *impersonal, non-intrinsic and in-the-market magnitudes*.

than R^N which then implies the evolution of technological knowledge in the form of non-convex and even *superadditive* increasing returns to scale.¹⁰ Below here is a case of abstracting *time, location, and commodities*, all into a *transcendental magnitude of intersubjective value, through and for, the implied pricing of uncertainty - call it opportunity cost - which is a non-value itself taken additively*.

"The definition of a certain commodity may require several dates and several locations" Which is to say that the same good at different times is a wholly different commodity than the same good at different locations. In terms of dates instead of locations the commodities become contingent on events which imply the time uncertainty and opportunity costs arising: "the concept of uncertain commodity is derived from the concept of certain commodity by substituting the tree structure of events for the line structure of dates and replacing everywhere "date" by "event."" [11]

Additivity and Independence [12]:

"Theorem 26 Let (G, \leq) be an independent and connected mixture with respect to an algebra A with more than two disjoint non-null sets. Then there exist functions $F: G \text{ in } R$ $f: G \times A\text{-algebra in } R$ such that F is a mean groupoid homomorphism. $f(-, A)$ is strictly monotonic on $G|A$ and $f(g, \cdot)$, is additive on $A\text{-algebra}$.

And, Definition 113: (subjective probability) Let \leq be a total pre-order on A . An additive $a: A \rightarrow [0,1]$ such that, $a(A) > a(B) \Leftrightarrow A > B$ is called subjective probability (representing the order relation on A).

¹⁰ "Constant returns to scale (g) together with additivity (e) implies that Y , is a convex cone with vertex 0. In the case of constant returns to scale, convexity is therefore easily justified. Note that, conversely, "convexity (f), additivity (e), and possibility of inaction (h)" implies "constant returns to scale (g)." Also, but this is of less interest, "convexity (f) and constant returns to scale (g)" implies "additivity (e)." [11]

And then, Definition 114 (independence): \leq a total preorder on A -algebra is independent if for all $A, A_1, A_2, C \in A$ -algebra, $C \subset A^c, A_1 \cup A_2 \subset A$

$$A_1 \leq A_2 \Leftrightarrow A_1 \cup C \leq A_2 \cup C$$

Corollary 9: Let $(X, A$ -algebra, \leq) be an uncertainty space, where \leq is a total preorder. Let \leq be independent and let A -algebra A (in A -algebra) be connected. Then there exists a subjective probability.
Proof. Special case of theorem 26. "

The general Kantian corollary we gather from [11] and [12] is that the synthetic a priori from transcendental magnitudes dealing with the quantitative through the quality of continuity from the qualitative of *psyche*, as the closest possibility to the thing in itself for the case at hand here, is, either an *only possible* magnitude in quantity with the *only possible* quality of continuity in that same magnitude, or we cannot know anything about things in phenomena *at all*. The only validity in Kantian, and *also* in economic theory's, terms possible is of the soundness of the synthetical gathered a priori. But the synthetic proposition itself cannot move beyond magnitude intuitions and conceptions which then, put in mathematical formulations, are as sound as the mathematical synthetic a priori truths.

V. WHITHER THE ANALYTIC OR THE SYNTHETIC?

General Equilibrium [7]:

"An essential point in the proof and in the economic application of the First and Second Fundamental Theorems is the absence of external effects (external economies and diseconomies). This notion shows up mathematically in specifying the possible consumption sets of the households, of the household sector, the possible production sets of individual firms and of the production sector. All of the relations are additive. That is, each household's tastes and opportunities are independent of the others' and of the firms'. Each firm's technology is independent of other firms. When external effects, issues like water and air pollution (diseconomies) or beneficial effects of a neighbor's garden (external economies), are significant, the theorem does not correctly apply." [7]

This rather seemingly strange passage is what exactly might seem like the objections [13] raises, namely, those relating to whether the assertions in economic theory come as *analytical* or *synthetic*. Firstly, her argument per se confuses the synthetic with the empirical, whereas, given the Kantian framework, the

synthetic knowledge that comes to us in being available, is not *through* experience, but from *prior* experience taken transcendently which implies that it abstracts the experience itself and retains the synthetical as *a priori* as synthetical conceptions made possible by the intuitions for those conceptions; so, much more than only being "logical", the Kantian method approaches the problem in terms of the *transcendental logic*, which makes the whole deduction one of the *psychological* kind - which is yet another clue into the *psychologico-transcendental* nature of magnitudes and their qualitative continuities in economic theory. As regards her reference to *reality* and *certainty* of economic theorems this again implies the synthetic a priori being confused with the empirical *yet* as far the empirical *concerns* matter per se they themselves are *possible* as knowledge only because of the synthetic a priori of the economic theory. *And* any deviations that *possibility* itself produces in the empirics just takes us back to the passage of [7], which we must eventually vet. Secondly, what is implied by her of *action* being an *offshoot* of *reason* by Von Mises [13], at least in terms of the Kantian frame, relates to reason as of the dialectics which is not the proper area of Transcendental Analytic and Logic in the *Critique* and which can tend more towards the *categories of understanding* with 'no objects being their *possible* content' of empirical reduction (*say action as reason*), thus such an application of categories is not a valid deployment of them, as per Kant.

Finally, before addressing the analytic-synthetic dichotomy and their imputed confounding in the economic theory, the *psychological-logical* divide of the synthetic a priori itself needs to be sorted out perspectively which, as will be made plain, is of the essence for parsing what the *transcendental* is. For which Pritchard's [13]-[14] '*Kant's Theory of Knowledge*' is counter examined.

"Time is a necessary representation, lying at the foundation of all our intuitions." [15]

- 'Of Time', *Critique of Pure Reason*

Pritchard did a rather deficient job of dealing with the *Transcendental Aesthetic* when he asserted quite wrongly that *as if*, the Kantian intuitions of space and time as conditions of phenomenon and the requisite conceptions for it, *were* a rendering of space and time *as conceptions which would imply taking space and time as things in themselves*. This seems to be a fundamental error in critiquing the *Aesthetic*. *Nay*, it should be counter-posed to Pritchard's claim that how could the said "*directness*" [14], of the relation between reality and the knower be so obvious, while examining the *Critique* itself,

when the very *impossibility* of such a directness is the *whole point* of the Kantian *œuvre*? Because Kant posits an impossibility of experience itself without the necessary conceptions for *experiencing* it. Kant's *psychologico-transcendental* is not *logical* as implied by [13] precisely because in order not to take things as things in themselves they are taken in transcendental abstraction where the content of the thing in itself is abstracted and intuitions of magnitudes thus created are then subjected to the logical which is to say is the method of *transcendental logic*. First, this method does not take things to be only *mental* if those are being *held to be unknowable as of in themselves* explicitly. That is, the *unknowability* of a thing as a thing in itself *does not* mean that the thing is only imaginary. Second, the implied *psychological* is not as such *per se in the transcendental*¹¹ because as far as the transcendental is concerned the things are taken in abstraction as intuitions and conceptions of magnitudes while the remaining object-content is abstracted away into the things in themselves which are declared as unknowable. Thirdly, the magnitudes are deployed only as far as the categories permit while these themselves must not be wrongly applied to things of an object-content with no possible empirical reduction in sight. In essence, Kantian epistemology, like economic theory, is neither dialectical nor a positive science; it, like economic theory which its detractors put as if it *were* dialectical, is a *negative science*. Now finally, we come to the alleged *analytic-synthetic* confounding that economic theory is incriminated in. Given the problems like that of *the identity of the indiscernibles* and the analytical difficulty of *rendering $a = a$* without the synthetic intervention of *b* synthetically *identified* to be *a* to render *a*, for example, in *$a - b = 0$ implying $a = b$* , it is the synthetic *per se* that *delivers* identities of *analytical* kind. Economic theory is analytical in its *assertions* and *axioms* but the development of new *analytical identities* is done through the *synthetic a priori* which only, like Kant says about philosophy and mathematics, makes *even* the experience in the empirical *possible*. Kantian philosophy replies in the *negative* when we say we can *know* directly through experience; likewise, economic theory also gives the *negative* when we think we can directly *act* in the economic sense other than when only being subject to and within the market mechanisms.

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¹¹ This is an alternative *Mathematical Psychics* approach from that of Edgeworth who took it from Hamilton's *principle of stationary action* : " *all the unknowns in a system can be reduced to one unknown and that single unknown is connected with the known* ". This characterization in *Kantian* terms at least takes us to *reason and dialectics* which are not thus held to be healthy foundations for vetting the axiomatics of economic theory.

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