Karl Marx's Theory of Capitalism
Exposition, Critique, and Appraisal

Escudé, Guillermo J.

2021
This book gives a clear synthesis of Marx’s theory of Capitalism and its relation with economic theory as it evolved over the course of the last 300 years. The analytical aspects of Marx’s theory are rigorously expressed by means of the technique of Input-Output Analysis, which is explained starting from the most elementary level. Parts I and II of the book address the philosophical-methodological foundations of Marx’s scientific endeavor (his Historical Materialism and his Dialectical Method) and his scientific theory of capitalist society, respectively. Part III contains our critique of Marx’s theory of Capitalism and contrasts it to Walras’ influential theory. Finally, Part IV contains a synthesis of Marx’s political thought and praxis, which had enormous effects over the course of the 20th century, and our critique thereof.

Guillermo J. Escudé holds a Ph.D. in Economics from the University of Buenos Aires (UBA-1981). He has been a Professor of Economics at UBA and Visiting Professor at U.C. Berkeley (1987 and 1991). He has been National Director at the Ministry of Finance (1993-98) and Research Manager at the Central Bank of Argentina (1998-2016).
Guillermo J. Escudé

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KARL MARX’S THEORY
OF CAPITALISM

EXPOSITION, CRITIQUE, AND APPRAISAL

Guillermo J. Escudé
I dedicate this book to the memory of
my parents,
Juan Carlos Escudé and Graziela Celina Carvajal
and my teacher,
Professor Julio H. G. Olivera
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PREFACE

This book is the result of two periods of study and writing separated by more than thirty years. I began to write a formal exposition of Marx’s theory of Capitalism in the late 1970s, as I worked for my doctoral thesis at the University of Buenos Aires, but it did not finally become a part of my thesis. After a career as an economist and two years before retiring I happened to find that manuscript of more than a hundred pages which I had completely forgotten. When I read it I decided it was finally time to finish that aborted enterprise. This embarked me on a project that was to take around five years of intensive work.

Due to my father’s electronic engineering career most of my elementary school education was in the U.S. After four years of high school in Tucumán, Argentina, Icoursed the last two years of high school in Lexington, Massachusetts, and graduated in mid-1968. The Vietnam War was at its climax (it was the year of the Tet offensive) and I had the obligation of signing up for military service, which implied some risk of being drafted. I quickly decided (with my parent’s backing) to return to Argentina and start college there. Both my anti-imperialist feelings on account of the horrors of the Vietnam War and the political climate prevailing in the University of Buenos Aires in the 1970s led me to read the three volumes of *Capital* with great interest, as I coured for an Economics major. I was also greatly interested in Philosophy and History, and found Marx’s Historical Materialism quite convincing. My basic mathematical training at the Faculty of Exact Sciences (before deciding for Economics) made me realize how feasible it was to give a formal mathematical expression to *Capital*, which I later confirmed upon reading Bródy (1970) and Morishima (1973) in postgraduate seminars.

When I was working on my doctoral thesis on linear economic models I consulted with my thesis advisor, the late Professor Julio H. G. Olivera, on the convenience of extending the content of my doctoral thesis to include a mathematical formalization of Marx’s theory. With his usual good humor he smiled and said it was not a good idea. I was not surprised, since academic self-censorship was a fact of life in Argentina during this tumultuous period in which a merciless (first civil, and then military) government was in the process of annihilating several guerrilla groups bent on taking over state power with little more backing than sheer will, audacity, and hazy revolutionary ideologies. I completed my Ph.D. in 1981, two years before democratic government was resumed in Argentina in the aftermath of the military defeat in the Falklands/Malvinas War. Although my thesis advisor’s opinion eased my decision to cast aside my manuscript, I was fully aware that I lacked the necessary intellectual maturity for making a serious balance of the merits and deficiencies of Marx’s *Capital*.

Since then, I have worked as lecturer and researcher on economics for several universities with the backing of the CONICET (National Council for Technical and Scientific Research), as economist in several private firms, and especially in the public sector (Ministry of Economics and Central Bank). When I found my long forgotten manuscript I was working on a critical essay on certain aspects of mainstream economics (published later as Escudé (2017)), such as its a-historicity and its tendency to omit the representation of social stratification while painting rosy pictures of government actions (‘altruistic’, ‘benevolent’, etc.). Some of the same characteristics of nineteenth century Political Economy had been criticized by Marx. I decided it was time to finish the task I had suspended for more than 30 years. This book, written in Spanish and translated by me to English, is the end product of that project.
INTRODUCTION

Marx’s works have three distinct strands that he himself tried to weld together: 1) his philosophically based contribution to the interpretation of history (Historical Materialism), 2) his scientific analysis of the rise, evolution and functioning of Capitalism, and 3) his political-millenarian project for the achievement of a communist society devoid of social classes, markets, money, capital, and a State understood as a  machinery that ensures the subordination of the non-dominant classes to the interests of the dominant class. Jointly with Engels, he forged an ideology of ‘scientific socialism’ that aimed to procure a foundation for 3) based on 1) and 2). But there was never a necessary link between 1) and 2), on the one hand, and 3) on the other. Indeed, the foundation of the Communist project on Marx’s interpretation of history and his theory of Capitalism was always flimsy; and the course of historical events during the last 150 years proved that the project was misleading, since it could at best lead to the organization of a society that was in the antipodes of the one Marx desired, i.e., one in which there would be an abundance of produced goods and services and spare time to enjoy and create, and a minimum of authority and inequality. This book is mainly concerned with 2), in its lengthy Part II, and my critique, in Part III. But it also addresses 1) in Part I, and 3) in Part IV (along with its critique).

The most developed part of Marx’s thought, his theory of Capitalism as expressed in Capital, was far more sold than read and far more read than understood by the followers of the many political parties that eventually developed under the inspiration of ‘scientific socialism’. That book was sui generis. Marx meticulously studied the evolution of economic thought (Aristotelian, Mercantilist, Physiocratic, and Classical), and though his theory was significantly influenced by several of the Classical Political Economists, he generated a very original theory that diverged from the mainstream of existing theory in the direction of socialist and communist thinkers that criticized the exploitative aspects of Capitalism but lacked a coherent theory. The theoretical part of Capital (which also contains lengthy sections of social history and the history of economic thought) has two noteworthy characteristics. First, Marx made his theory more scientific (though only slightly more mathematical) than that produced by previous political economists. It was constructed to account for a huge mass of empirical material he had gathered from many sources; and it coherently explained many features of the economic and social functioning of several centuries of Capitalism by means of a carefully constructed hypothetico-deductive structure endowed with precise definitions and explicit simplifying hypotheses that were gradually lifted as the theoretical edifice took form. Second, he gave his work a special twist by expressly writing from the point of view of the least privileged classes, a feature existing in previous much less sophisticated socialist and communist literature. His repeated manifestations of authentic indignation with the suffering of the dispossessed attracted readers inclined to progressive social change but at the same time generated repulsion on the part of more conservative intellectuals with a propensity to render their services to established power. At both ends there was scant comprehension of the theory as a whole. This was probably inevitable in a period (the last decades of the 19th century and first few of the 20th) in which many had doubts on the future possibilities of Capitalism while many others were convinced that it was the very essence of modernity. But the difficulties in understanding Marx’s theory of Capitalism also resulted from the fact that during his lifetime he was only able to finish and publish Book I of Capital to his complete satisfaction. He left drafts with widely differing degrees of completion for Books II, III and IV. The first two of these were polished and published posthumously by Engels in
1885 and 1894, respectively, but he died before being able to do the same with Book IV. Karl Kautsky took on the job and published it in three volumes between 1905 and 1910 under the title *Theories of Surplus Value*. Since the first German edition of Book I was published in 1867, Books I-III (which contain the theoretical part of *Capital*) were published over a period of 27 years and Books I-IV over a period of 43 years. The corresponding periods for other languages were even longer.

One of the attractive features of Marx’s *magnum opus* is that it was gestated before the process of specialization consolidated the division of the various individual ‘social sciences’ as well as their respective corporate-academic interests. His work reflects an overall vision of the functioning of human society that combines the social, economic, political, and ideological aspects as well as its historical evolution. In *Capital* one finds only a glimpse of the relation between ‘civil society’ and the State under Capitalism, which was an integral part of his research plans, largely because Marx had to leave it aside as time passed and his health deteriorated. But it is also true that in his view the comprehension of the specific functions of the State in Capitalism required a previous understanding of the economic functioning of ‘civil society’. And he had an early interpretation that the “executive of the modern State” is “a committee for managing the common affairs of the whole bourgeoisie” (Manifesto, MEWC 6, 486). This was consistent with the philosophical underpinnings of the ‘materialist interpretation of history’ which he elaborated in his youth, according to which ever since human society generated a State it mainly represented the interests of the dominant class.

The organizing principle of Marx’s work was to try to understand present reality as the consequence of a historical development, the tendencies of which he believed could be discovered, with the goal of transforming this reality in the direction of a society devoid of a dominant class in which authentic freedom could flourish. The prophetic-millenarian aspect of Marx’s thought was persistent throughout his intellectual development and is ubiquitous in his works, even though it only extends over few pages. In his worldview society’s advance towards the elimination of the oppression of some classes over others was inexorable. And he believed it was necessary to educate workers in order to ease the labor pains of the upcoming new society. Marx believed this process would take place in the countries with most advanced capitalist development, already equipped with large capitalist firms practicing internal economic planning and a State with increasing regulatory functions. But the complexity of human society makes its future impossible to forecast, even for the most powerful minds. Revolutionary structural change along the lines drafted by Marx did become a reality in the 20th century, but in countries with scant capitalist development, such as Russia in 1917 and (even less developed) China in 1949. In both cases its leaders eventually came to the conclusion that completely replacing private firms with state production and markets with centralized state planning was so inefficient that it prevented a successful competition with the advanced capitalist societies of the ‘First World’. Furthermore, many socialists in the capitalist world gradually began to realize that (real) Communism generated a concentration of power even greater than that of the capitalist societies they criticized and that increasing the living standards of the working class could not do without the networks of hierarchical authority in firm organizations, in both the private and public sectors. Marx’s conception of an ‘association of workers’ devoid of hierarchical authority and conflicting interests, and working under arrangements of production and planning devoid of markets, was not compatible with the increasingly complex productive, commercial, financial, and administrative structures of modern society. Even if centralized planning was implemented, the functioning of markets continued to be indispensable, and there was no valid reason for trying to repress
the emergence of private entrepreneurs, regardless of whether or not they hired wage laborers.

Marx’s works have occupied a very special place in the socioeconomic-political life of the world during the last century and a half. His anti-capitalist stance in favor of marketless economic planning had a profound influence in the political and economic life of many of the most developed countries in the last decades of the 20th century; and an even more substantial practical influence in many other countries starting with the 1917 Russian Revolution that gave birth to the Soviet Union. There they had a very deep influence on the views of Lenin and the party he led, and with the triumph of the revolution they inspired such structural changes as state ownership of the means of production and natural resources, production by state firms, and central economic planning. After World War II the Soviet-style revolutionary changes were extended to several countries of East Europe as well as China and other ‘Third World’ countries. But Marx’s works also inspired, or at least anticipated, various reformist structural changes in many capitalist countries, especially in the aftermath of the depression of the 1930s. Marx’s works exerted influence (by rejection) in some countries, as was the case of the U.S.A., where anti-Marx rhetoric became the mainstay of domestic and foreign policies before and after World War II. Most of the principal influences were derivative of the least developed strand of Marx’s thought, the one that can be dubbed as political-prophetic or millenarian. Lenin had sufficient pragmatism and ability to apply Marx’s political project under extreme circumstances (2 to 3 million Russian soldiers had died in World War I before the October 1917 Revolution and the long 1918-1923 Civil War that followed). The difficulties encountered during the phase of War-Communism made Lenin avoid an even greater economic catastrophe by reinstating market incentives with his New Economic Policy (in March 1921). However, his aggravating health problems (possibly related to the failed attempt against his life in 1918) and premature death (in January 1924) gave rise to a power struggle at the very top of the Party, making it impossible to know whether a continuation of his leadership would have led to more permanent market friendly policies. We do know that the political power struggle led to Stalinism and what the anarchist Bakunin had anticipated when he criticized Marx’s political project: the consolidation of a new dominant class.

The very successful pro-capitalist economic revolution from the top in China (dubbed ‘reforms’) that began in 1979 and the surprising implosion of the Soviet Union in 1989-91 were irrefutable evidence of the conceptual mistakes in Marx’s political-prophetic posture. The liberation of Eastern European countries from Soviet domination and the reformulation of a capitalist Russian Federation generated a brief period of illusion that the Cold War was coming to an end. But events soon proved that the Cold War had never actually been a confrontation between Capitalism and Communism but only a new form for the old struggle for hegemony between imperial powers adapted to the Age of Hiroshima, in which total war (necessarily nuclear) is (normally) avoided because it would lead to mutually assured destruction. Hence, inter-imperial struggles at the military level are carried out mainly through proxies and in territories not completely controlled by the real players. But the risk of an unexpected sequence of events unleashing a nuclear devastation is always present and ominously increases as more and more countries acquire nuclear arsenals and each delegates the nuclear button to an expanding circle of individuals on account of the risk of decapitation in their chain of commands.1

1Hence the probability that a derailed general appears (such as Jack D. Ripper in Stanley Kubrick’s 1964 film Dr. Strangelove) increases over time. See Ellsberg (2017).
At this point the reader may wonder why well into the 21st century there may still be interest in a book on Marx’s theory of Capitalism. We believe that though Marx made grievous errors of judgment, especially but not exclusively in the political-prophetic sphere, he also had many insightful ideas that have not been adequately assessed notwithstanding the fact that he is probably the thinker on whose ideas more has been written (for and against) in history. This book tries to formulate a precise exposition of his theory of Capitalism, focusing attention especially on its economic aspects but not omitting its philosophical, historical, social, and ideological aspects. We clear up several aspects of his theory that still deserve a clearer and more precise treatment. For the expression of the more analytical aspects of Marx’s theory we make extensive use of ‘Input-Output analysis’, which has a long tradition in the economic literature. In order to make the book self-contained we include a chapter that explains this instrument starting from the most elementary level and also summarizes the theory of Perron and Frobenius on non-negative square matrices. In a sense, this allows us to make the strongest possible case in favor of the analytical coherence of Marx’s theory, which implies correcting some of his imprecision and approximations that the formal language of (elementary) mathematics allows and mandates. But it also allows us to pinpoint what is wrong in his theory.

After the exposition of Marx’s theory in the central body of the book (Part II) we show in Part III why the theory of surplus value is invalid, precisely the part of his theory most precious to Marx because it was the foundation for his theory of the exploitation of wage workers in capitalist society. We show that the Gordian knot of this invalidity hinges on the non-representation of entrepreneurial activity in his formal theory (even though it was present in much of his conceptual analysis). The fact that this has only been mentioned in passing by a few scholars is probably due to the fact that mainstream economic theory does something very similar, but for very different reasons. In Marx’s case it enabled the definition of surplus value in terms of the ‘non paid labor’ of wage workers. In the case of mainstream economic theory, my opinion is that the apologetic desire to avoid the representation of even the most elementary class structure of capitalist society prevailed. The entrepreneur is replaced by the ‘firm’, a black box that maximizes profit and the interior of which is the subject matter of the discipline of ‘business administration’ but is mostly absent in (macro and micro) economic theory.

Nevertheless, Marx’s theory of Capitalism is sufficiently rich and redundant that even after completely eliminating his theory of labor-value and surplus value there subsists a theoretical structure of great interest that is capable of reflecting, with slight modifications, many of the most important aspects of the economic functioning of capitalist society, including multi-sector (balanced) economic growth. The areas that Marx left in the most unsatisfactory state from the formal point of view consist in: 1) assuming fixed structures for consumption baskets, 2) not formally integrating ground rent with his production prices, and 3) not formally representing the monopoly power of large corporations in modern industrial Capitalism. Shortly after Marx stopped developing his theory (but before his death) Neoclassical economic theory made it possible to lift the assumption in 1) through the development of a (subjective) theory of individual demands dependent on market prices. With respect to topics 2) and 3), Marx was on the right track but was unable to develop a minimally satisfactory theory. In the case of 2), Marx criticized Ricardo for limiting his theory to the ‘differential’ ground rent and explained why with private property of land there had to exist an ‘absolute’ rent even in the least fertile or most unfavorably situated lands. In the case of 3), Marx pointed out that the great corporations of modern industry had the power
to set prices that eschewed the process of equalization of profit rates. But Marx lacked
the necessary instruments to enable working analytically in these three areas. We show
in a specific chapter how topics 1) and 2) were developed by Léon Walras in his theory
of general equilibrium under ‘perfectly free competition’. However, in the process he
eliminated the entrepreneur as a human agent per se and hence hindered the possibility
of having an adequate representation of the class structure of Capitalism. Economists
were to require several decades of efforts in order to make a significant advance in topic
3), notwithstanding the notable early contribution of Cournot (1897 [1838]). But many
of the tendencies of the capitalist production and circulation process described by Marx
are still very interesting and relevant; and some of his analyses inspired non-Marxian
economists that became famous, in some cases without due recognition of their source
of inspiration.

The path we follow

Let us now sketch the path this book follows. Because the approach is historical-
genetic, as Marx’s, I did not want to leave out a biographical sketch (which the informed
reader can readily omit), as well a synthesis of the philosophical and methodological
platform from which Marx would elaborate his theory of Capitalism. During his early
twenties Marx developed his ‘materialist conception of history’, which was to be the
foundation of all his subsequent investigations (as well as his political activity). To limit
the extension of this book I have had to leave out many of the details of the genesis of
Marx’s worldview, summarizing those aspects I deemed most relevant. This synthesis
may also be omitted by readers not attracted to philosophical matters. Therefore,
the reader mainly interested in Marx’s theory of Capitalism can jump to Part II and
eventually go back to Part I if he wishes to read Part IV, which is its complement.

A distinctive characteristic of Marx was that, having had an excellent philosophical
training at the University of Berlin, when he was around 32 years old he decided
to focus his studies especially on Political Economy, convinced that it was in this
field that he should seek the fundamental elements for the analysis of Capitalism.
Hence, his readings in Economics went through a very personal filter that differed from
those of all the ‘economists’ of his time, even those most attracted to socialist ideas.
Another distinctive characteristic of Marx’s intellectual development is that there is a
marked contrast between his careful empirical enquiry of social, economic, and political
reality, both historical and contemporary, and his detailed construction of models and
extensive conceptual and numerical analyses of the functioning and various tendencies
in capitalist society, on the one hand, and his ideological conviction that the Capitalism
of the most advanced nations was sufficiently mature to be replaced by a more advanced
and very different ‘mode of production’. ‘Class struggle’ would lead to ‘Communism’ by
means of the collective property of the means of production and natural resources, and
this required the abolition of not only of capital, but also markets, money, the division
of labor, and stratification based on social classes. The working class (‘proletariat’) would be the social force behind this accomplishment. He had this conviction since he
was young, it varied little throughout his life, it was scantily analyzed and argued, but
it was the guide of all his political activity and the main motivation for his scientific
inquiry. As mentioned, I have left this aspect of Marx’s thought and praxis for Part IV
of this book in order to focus primarily on the exposition and criticism of his theory of
Capitalism. Although it can be argued that Marx’s ‘materialist conception of history’ includes his political-millenarian view, I believe it is convenient and even necessary to
clearly distinguish between his interpretation of the past (and, to a certain extent, the
present) and his view of a future that he considered both desirable and possible and
which he used as a guide for his political praxis.
In a nutshell, Part I of this book contains as preliminaries a biographical sketch of Marx, the gestation of his Historical Materialism, and his scientific methodology. In Part II there is a rigorous and detailed exposition of Marx’s theory of Capitalism. Part III has my critique of Marx’s theory of surplus value, a synthesis of the theoretical treatment of the entrepreneur and profit before and after Marx, and a comparison of the most enduring part of (the static part) of Marx’s theory with the theory of León Walras, the leading representative of the Neoclassical economic theory that developed during the last phase of Marx’s life. Part IV follows the evolution of Marx’s political thought and, in particular, his evaluation of (and participation in) the European revolutions of 1848. The last chapter (Part V) has my concluding thoughts. For the text processing, numerical exercises, and figures, I have used Scientific WorkPlace 5.5 (MacKichan Software Inc.).
GLOSSARY

Abbreviations

SCP     Simple Commodity Production
CCP     Capitalist Commodity Production
SR      Simple Reproduction
ER      Extended Reproduction
MECW    Marx & Engels Collected Works

Works by Marx

B1, B2, B3 Books I, II, and III of Capital (MECW 35, 36, 37)
B4.x    Theories of Surplus Value (MECW 30, 31, 32, 33)
Theories Theories of Surplus-Value, Progress Publishers, Moscow
Contribution Contribution to the Critique of Political Economy (MECW 29)
Introduction (Discarded) Introduction to Contribution (MECW 28)
Poverty The Poverty of Philosophy (MECW 6)
Manuscripts Economic and Philosophical Manuscripts (MECW 3)
Class Struggles The Class Struggles in France 1848-1850 (MECW 10)
Brumaire Eighteenth Brumaire of Louis Bonaparte (MECW 11)
Civil War Civil War in France (MECW 22)
Gotha    Critique to the Gotha Programme (MECW 24)

Works by Marx and Engels

MECW x Marx & Engels Collected Works, Vol. x
Yearbook French-German Yearbook (MECW 3)
Family The Holy Family (MECW 4)
Ideology The German Ideology (MECW 5)
Manifesto Manifesto of the Communist Party (MECW 6)
Letters Karl Marx/Friedrich Engels Letters (MECW 38-50)
Part I

Philosophical foundations of Marx’s *Capital*
Chapter 1  MARX AND HIS ENCOUNTER WITH PHILOSOPHY

Karl Marx: a biographical sketch

Karl Heinrich Marx was born in 1818 in Trier, a city of the Rhineland, the westernmost province of the Kingdom of Prussia (currently a few kilometers from the Germany-Luxemburg border). Fifteen years before Marx’s birth Trier had been occupied by the French and soon after Napoleon made it part of the Confederation of the Rhine, which welded several German client states into an alliance with the First French Empire. After Napoleon’s defeat, the Congress of Vienna (1815) allocated the Rhineland to Prussia. The French domination over parts of Germany had introduced progressive (or liberal) measures such as the abolition of serfdom, trials with due process, the softening of the Catholic Church’s power, as well as a certain concentration of the great political dispersion of German states. But after Napoleon’s defeat, the (still semi-feudal) kings and princes of the German states suppressed some of the institutions and ideas introduced by the French. Foremost among these authorities was Frederick William III of Prussia. Faced with censorship, many German writers decided to exile.

German Jews had benefitted greatly with the French liberalization measures and, in particular, with the introduction of the Napoleonic Code, since it had given them the possibility of practicing certain liberal professions from which they had been excluded. And they were frustrated when many of the old restrictions were re-introduced. Both of Karl’s parents were of Jewish ascendancy. His mother, Henrietta Pressman, was Dutch and died when he was 45 years old. His father, born Herschel Marx, had Germanized his name to Heinrich and died when Karl was 20. Although he was a Rabbi’s son, Heinrich was not very religious (in contrast with his brother Samuel who followed the family rabbincal tradition). And to be able to practice as a lawyer he had to ‘assimilate’ by converting to Christianity before Karl was born (McLellan 1980, 30).1 The couple had four sons and five daughters, born between 1815 and 1826. Karl was the third to be born, and the second of the boys, but the first boy died in 1819, leaving Karl as the eldest of the boys, with Sophie as his elder sister (Gemkow 1975, 11). All the children were baptized on the same day of August 1824 (except another yet unborn) when Karl was 6, and so was their mother Henrietta the following year, after her father died (McLellan 1980, 31). Two more of Karl’s brothers and two of his sisters died between 1837 and 1847. Hence, since he was 29 years of age he was left with only three sisters, all of which survived him.

When he was 17 Karl started his studies at the University of Bonn but shortly after he changed for the University of Berlin. There he studied Law, Philosophy, and History, and got involved with a group of young Hegelians of which Ludwig Feuerbach and Bruno Bauer were prominent disciples of Hegel. Marx broke with all religion at an early age, despite this father’s theistic beliefs. He had troublesome family conflicts related to the differences of origin with his girlfriend and bride Jenny von Westphalen, of aristocratic descent. In a letter (of March 13, 1843) to Arnold Ruge, Marx informs him he is about to marry and adds: “I have been engaged for more than seven years, and for

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1 According to McLellan (1980), his baptism must have taken place before August 1817, when the Protestant parish was established in Trier. Therefore, he was probably baptized in a Catholic Church. Catholicism was predominant in the Rhineland and especially in Trier. In 1844 half a million Catholic faithful participated in a pilgrimage to Trier to see the tunic that, according to tradition, Jesus Christ had worn on the road to crucifixion and somehow ended up in this city.
my sake my fiancée has fought the most violent battles, which almost undermined her health, partly against her pietistic aristocratic relatives, for whom ‘the Lord in heaven’ and the ‘lord in Berlin’ are equally objects of religious cult, and partly against my own family, in which some priests and other enemies of mine have ensconced themselves. For years, therefore, my fiancée and I have been engaged in more unnecessary and exhausting conflicts than many who are three times our age” (MECW 1, 399). There is clear evidence that such conflicts did not originate in the attitude of Jenny’s father, Baron Ludwig von Westphalen. It was probably at least partially due to the Baron’s son from a previous marriage –15 years Jenny’s senior– who was as much of a reactionary as the Baron was a liberal. The Marxes and the von Westphalens were neighbors in Trier and Sophie Marx was an intimate friend of Jenny since childhood. According to McLellan (1980), a long time before the engagement “The Baron devoted much of his time to the young Marx” and “As well as being a man of culture, the Baron was keen on progressive political ideas and interested Marx in the personality and work of Saint-Simon”, the French socialist (McLellan 1980, 39). Around ten years before Karl was born the Baron had been imprisoned due to his opposition to some of the policies of the French occupation in the town where he lived.

The letters of Heinrich Marx to Karl express his hopes for his favorite son, whom he saw as gifted with great talent, as well as his worries that he might squander them and his reprobation of his careless use of the funds sent to him. In his letter of September 16, 1837, Heinrich consoles his son for the long time elapsed without a letter from Jenny, assuring him that “her attitude to you is one of the most self-sacrificing love, and she was not far from proving it by her death” (MECW 1, 681), probably referring to an illness she contracted in the midst of the conflict over their engagement. And Karl’s mother’s Postscript to that letter reveals that the engagement had finally been approved by Jenny’s family. After graduating from the University of Berlin, in April 1841 he presented his doctoral thesis to the University of Jena (which was approved) on a specific topic of Greek Philosophy: “Difference Between the Democritean and Epicurean Philosophy of Nature in General.” By then it was already three years since the Baron had given his consent to his daughter’s engagement. Karl’s respect and admiration for the Baron are reflected in the carefully written dedication of his thesis:

You will forgive me, my dear fatherly friend, if I set your name, so dear to me, at the head of an insignificant brochure. I am too impatient to await another opportunity of giving you a small proof of my love. May everyone who doubts of the Idea be so fortunate as I, to be able to admire an old man who has the strength of youth, who greets every forward step of the times with the enthusiasm and the prudence of truth and who, with that profoundly convincing sun-bright idealism which alone knows the true word at whose call all the spirits of the world appear, never recoiled before the deep shadows of retrograde ghosts, before the often dark clouds of the times, but rather with godly energy and manly confident gaze saw through all veils the empyreum which burns at the heart of the world. You, my fatherly friend, were always a living argumentum ad oculos to me, that idealism is no figment of the imagination, but a truth (MECW 1, 28).

This dedication reveals the filial attitude Karl had towards his bride’s father. Possibly foreseeing his coming death (which occurred a year later), he did not want to waste the opportunity of transmitting words he could no longer direct to his own father, who had died more than three years before after sending Karl a letter that communicated his illness and was full of reprimands for not having yet settled down to a career that
could bring happiness to “this angelic girl.” The dedication also reveals how much Karl appreciated the moral fabric of his fiancée’s father and his own romantic feelings when invoking “the empyreum which burns at the heart of the world.”

Shortly after the death of the Baron, in May of 1842 Karl started to publish articles in the *Rhenish Newspaper (Rheinische Zeitung)*, a publication of the Rhineland liberal bourgeoisie. They made such a good impression that only five months later he was offered (and he accepted) the editorship, notwithstanding his youth (he was 25). For this he moved to the city of Cologne. But only five months later the periodical was closed by the government. Due to the incompatibility between the university policies of the monarchical Prussian government and Karl’s cravings for intellectual independence, he gradually came to the conclusion that he had little chances of following an academic career in Germany.

Karl and Jenny were married in June 1843 and in October they moved to Paris, where Karl would assist Arnold Ruge –16 years his senior– in the production and publication of a *Frank-German Yearbook (Deutsch-Französische Jahrbücher)*, thus escaping from German censorship. In Paris Karl’s first daughter Jenny was born and he befriended another participant in the project: his compatriot Friedrich Engels. Marx contributed two articles to the sole issue finally published (in February 1844): *Contribution to the Critique of Hegel’s Philosophy of Law. Introduction, and On the Jewish Question*. The latter was a critique of two articles by Bruno Bauer on the subject of whether full political rights should be granted to the Jews, in which Marx took the opportunity to give his own view of ‘human emancipation’. Engels also contributed two articles, one of which was *Outlines of a Critique of Political Economy*, which made a very good impression on Marx. The Yearbook also published a series of letters interchanged between Ruge and Marx, Bakunin, and Feuerbach, respectively, as well as *Letters from Paris* by Moses Hess, another participant who had previously published in the *Rhenish Newspaper* and was a good friend of Marx and Engels. Hess was 6 year older than Marx, which makes his profound admiration for the latter’s intellectual talent even more amazing, as revealed in a letter he sent to his novelist friend Berthold Auerbach (in September, 1841). He describes his friend as “the greatest, perhaps the only, genuine philosopher now alive, who will soon... attract the eyes of all Germany... Dr. Marx...will give medieval religion and politics their coup de grâce. He combines the deepest philosophical seriousness with the most biting wit. Imagine Rousseau, Voltaire, Holbach, Lessing, Heine, and Hegel fused into one person –I say fused not juxtaposed– and you have Dr. Marx” (McLellan 1980, 71).² But for various reasons the *Frank-German Yearbook* project had to be abandoned after the publication of its initial issue.

Towards mid-1844 Marx wrote some manuscripts in which he addressed matters of Political Economy for the first time. They were published posthumously as *Economic-Philosophic Manuscripts*. That same year, he wrote with Engels (jointly for the first time) a book with a critique of the *Literary Gazette* that Bruno Bauer and his brothers

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²Moses Hess’ intellectual evolution is noteworthy. When he wrote this letter he had already published two books. He had communist ideas before Marx and Engels and for several years he collaborated with them. But they increasingly diverged in their respective political positions. First Hess became a German ‘true socialist’ (criticized in *The German Ideology*) and later he followed the Lasalle version of German socialism. Finally, he turned passionately towards what was later called proto-Zionism. Enraged by the persistence of anti-Semitism, he rejected the assimilation of Jews in the countries in which they lived. In 1862 he published *Rome and Jerusalem*, proposing the emigration to Palestine. This book so impressed the Hungarian Theodor Herzl –the great organizer of the Zionist movement– that he once asserted that if he had read it he would not have written *The Jewish State*, his most significant book.
Edgar and Egbert had been publishing. The book was published early in 1845 in Frankfurt (and in German) as The Holy Family or Critique of Critical Critique, Against Bruno Bauer and Company.\(^3\) Notwithstanding the polemical character of the book, of which we know which parts were written by each of the authors, Marx sketches some of the elements of Philosophy and Political Philosophy that would soon constitute an integral part of both his ‘materialist conception of history’ and his communist political project for the future.

After the Yearbook project came to an end, Marx had begun to contribute to the socialist biweekly paper Forward! (Vorwärts!) that was published in Paris in German. But in 1845 the French government, under pressure from that of Prussia, not only closed the paper but also expelled Marx from France. He therefore moved with his family to Brussels, where he stayed during the next three years. Engels also moved to Brussels and they jointly began to write a lengthy book: The German Ideology, Critique of Modern German Philosophy According to Its Representatives Feuerbach, B. Bauer and Stirner, and of German Socialism According to Its Various Prophets. A large part of this book is a critique of the three ‘Neo Hegelians’\(^4\) named in the subtitle and of the most popular version of German socialism. But more important for us is the fact that this book contains an exposition of the ‘materialist conception of history’ (or Historical Materialism) as well as some ideas on the Communist political project that the authors adopted on such a basis. Due to various circumstances they were unable to publish the book, which remained unpublished until 1932 (except for an article published in a journal of Westphalia in 1847 with a critique of German Socialists).

But in 1847 Marx did succeed in publishing in Brussels and in Paris (and in French) a new book, The Poverty of Philosophy, Answer to the Philosophy of Poverty by M. Proudhon, which was a critique of the Socialist theory of Pierre-Joseph Proudhon as he expressed it in his book System of Economical Contradictions or, the Philosophy of Poverty. Towards the end of the year Marx gave some lectures that were published in 1849 as Wage Labor and Capital and, jointly with Engels, participated in a London meeting of the “League of Communists”, to which they affiliated. They agreed to write a manifesto that would express the League’s political agenda (which they had begun to lead). The outcome was the famous pamphlet Manifesto of the Communist Party, in which they proclaim the objective of Communists of leading the working class to wresting political power from the bourgeoisie and converting their private property into collective property.

During 1848 there were popular uprisings in various European cities. In France this led to the end of the July Monarchy (inaugurated after the revolution of July, 1830) and to the short-lived Second Republic, thanks to which Marx was able to return to Paris when the Belgium government ordered him to leave the country. However, in Germany (as in all continental Europe) democratic anti-monarchical protests and uprisings were also taking place. Hence Marx soon moved to Cologne, were he was active analyzing the European events as editor of the New Rheinish Newspaper (Neue Rheinische Zeitung) and campaigning for a unified democratic German state. Several of the articles Marx wrote for the Newspaper were compiled and published posthumously by Engels (in 1895) as The Class Struggles in France 1848-1850. During the first half of 1849 the German democratic government was defeated and the Prussian government closed de

\(^3\)The first part of the title (The Holy Family) was suggested by the editor, in the belief it would attract buyers.

\(^4\)We use the same terminology as in MECW, distinguishing ‘Old Hegelians’ from ‘Neo Hegelians’. Löwith (1968), in contrast, distinguishes “right wing old Hegelians” from “left wing young Hegelians”, preserving “Neo Hegelians” for the future renovators of Hegelianism (that were his contemporaries).
Newspaper and ordered Marx to leave the country. Hence, in June 1849 he returned to Paris.

But the reactionary forces also prevailed in France and Marx was again expelled. Hence, in August 1849 he moved to London, followed shortly by his wife and children (Jenny, Laura, and Edgar), where he was to live for the rest of his life. The first years in England were painful for the Marx family, who as resourceless political refugees had to survive in conditions of extreme poverty. Jenny had traveled pregnant to London, where another son was born but died a year later. A few months later another daughter was born (Franciska) who also died after a year. In 1855, a few months after Eleanor was born Edgar, who was 8 years old, died. In 1857 a seventh child was born but died before being named. Hence, only Jenny, Laura, and Eleanor survived childhood. Many years later Eleanor quotes some biographical notes left by her mother Jenny Marx on the circumstances Franciska’s death (in April 1852): “The death of the dear child came in the time of our bitterest poverty. Our German friends could not help us; Engels, after vainly trying to get literary work in London, had been obliged to go, under very disadvantageous conditions, into his father’s firm, as a clerk, in Manchester... In the anguish of my heart I went to a French refugee who lived near, and who had sometimes visited us. I told him our sore need. At once with the friendliest kindness he gave me £2. With that we paid for the little coffin in which the poor child now sleeps peacefully” (Marx 1897, vii; Note by the Editor, Eleanor Marx Aveling). Marx was rescued from extreme poverty when in mid-1853 he began to contribute articles for the New York Daily Tribune.

The turmoil in France ended with the December 1851 military coup by Louis Bonaparte (Napoleon’s nephew) that put an end to the brief Second Republic (1848-51), of which he was President. Shortly after, he became Emperor –following the footsteps of his uncle– and thus inaugurated the Second French Empire (1852-70). During the first months of 1852 Marx wrote The Eighteenth Brumaire of Louis Bonaparte, where he again analyzed, but with a little more historical perspective, the events in France between 1848 and 1851. It was published (in German) in New York. From 1853 to 1862 Marx (and Engels) contributed articles to the New York Daily Tribune on various topics, among which were the Crimean War (1853-54) and the British colonial rule over India (1853-58), including the great Indian uprising of 1857 and its repression. He also wrote articles on the American Civil War (1861-65), especially in the Viennese periodical Die Presse. During these years he principally focused on his studies of Political Economy, writing in 1857-58 a long manuscript that was published in the 20th century as Grundrisse: Foundations of the critique of Political Economy. It includes a section with the title Forms which precede capitalist production that awakened great academic interest and has also been published separately. Marx published in 1859 his first book on Political Economy after his intensive studies: A Contribution to the Critique of Political Economy. During 1862-63 Marx prepared a much longer manuscript under the same title which was a continuation of his published book. This manuscript was “the first systematically worked out draft –though still only rough and incomplete– of all four volumes of Capital”, as specified in the Preface to the first of the three volumes of Theories of Surplus Value written by the Moscow-based Institute of Marxism-Leninism. In successive letters Marx referred to this part of his work as “Book IV. On the History of the Theory” (Letter to Kugelmann of October 13 1866, MECW 42, 328) and as “Volume III [of Capital] the history of political economy from the middle of the 17th century” (Letter to Siegfried Meyer, April 30 1867, MECW 42, 367).^5

^5At that time Marx still believed his Volume II would include what later became Volumes II and
Marx again had severe economic difficulties when he lost his main source of income (the New York Daily Tribune) due to the American Civil War. On December 28, 1862, he wrote to Kugelmann to explain why he had been unable to send for publication another portion of his book after his initial Contribution of 1859: “In 1861, I lost my chief source of income, the New-York Tribune, as a result of the American Civil War. My contributions to that paper have remained in abeyance up till the present. Thus, I have been, and still am, forced to undertake a large amount of hackwork to prevent myself and my family from actually being relegated to the streets” (MECW 41, 435-6). He then explains he tried to get a job in a railway office but was rejected due to his bad handwriting.

After the publication of Book I of Capital in 1867, Marx’s creative work was impeded by his political activities and his health problems, so he was unable to make his drafts publishable in spite of how much they meant to him. He expresses this in his letter of April 30, 1867 (quoted above) to Siegfried Meyer: “Why then did I not answer you? Because I was the whole time at death’s door. I thus had to make use of every moment when I was capable of work to complete my book, to which I have sacrificed my health, happiness, and family. I hope this explanation suffices. I laugh at the so-called ‘practical’ men and their wisdom. If one wanted to be an ox, one could, of course, turn one’s back on the sufferings of humanity and look after one’s own hide. But I should really have thought myself unpractical if I had pegged out without finally completing my book, at least in manuscript” (MECW 42, 366).

Before dying, in 1883, Marx asked Engels to polish for publication the immense material he was leaving unfinished. Thanks to Engels’ intensive work, Books II and II were published in 1885 and 1894, respectively. Engels intended to compile in Volume IV Marx’s extensive critical analysis of the theoretical developments of Political Economy most related to his theory of Capitalism. But his declining health did not allow him to undertake this task, dying in 1895. The material was edited by Karl Kautsky between 1905 and 1910, and published in three volumes as Theories of Surplus Value. Decades later, the Institute of Marxism-Leninism of Moscow published a new edition of this work that more closely reflected the original manuscripts than Kautsky’s edition. It was published in three volumes between 1956 and 1962. Also, a section of the manuscript left by Marx that was not included in Books I-IV of Capital (and written sometime in 1863-66) was published in 1933 (in Moscow and in Russian) as Results of the Direct Production Process (Chapter 6 of Capital).

Since the 1864 founding in London of the “International Workingmen’s Association” (IWA), also known as the “First International”, Marx actively participated as member of its General Council. He was especially involved in political struggles against the anarchist factions of Proudhon and Bakunin. An event that shocked Europe and the IWA was France’s defeat by the Prussians in the Franco-Prussian War of 1870-71 which resulted in the occupation of Paris by the Prussians, the end Louis Bonaparte’s Second Empire, and the popular insurrection in Paris that led to the brief but intense ‘Paris Commune’. Marx wrote several manifestos of the IWA addressing both the functioning of the Commune and the bloody repression that followed. They were compiled 20 years later into the book The Civil War in France. One of Marx’s last political documents was his critique of the program written for the fledgling Social Democratic Party of Germany (that resulted from the fusion of two pre-existing parties) at its initial 1875 party congress held in the city of Gotha. It was published posthumously as Critique of the Gotha Program, and contains some of Marx’s last written ideas on a hypothetical Communist society.

III of Capital.
The last years of Marx’s life were marked by his illness and that of his wife Jenny, who died in December 1881. In January 1883 their eldest daughter Jenny died of cancer at the age of 38, and two months later Marx died at 64. He was survived only by his daughters Eleanor and Laura. Eleanor committed suicide in 1898 at 43 after being abandoned by her longtime lover, the socialist Edward Aveling; and Laura committed suicide in 1911 (at 66) jointly—and by mutual agreement—with her husband, the socialist Paul Lafargue.

Marx’s encounter with Philosophy

Marx approached Political Economy armed with a high-level university education in Law, Philosophy, and History. As he writes in the Preface to the Contribution: “Although jurisprudence was my special study, I pursued it as a subject subordinated to philosophy and history” (MECW 29, 261). He would gradually deepen his studies of Economic, Social, and Political History throughout the rest of his life. However, his studies of Philosophy and Law, which had started in an environment impregnated by German Idealism, and mainly Hegelianism, had to go through a process of creative destruction that would lead Marx to elaborate his own Philosophical, Political, and Scientific stance, and thereafter completely abandon any type of Philosophical enquiry in order to concentrate exclusively on scientific research and political activity.6

Since Philosophy (with History) was at the core of his university studies, he chose for his doctoral thesis a strictly philosophical subject matter, as we have seen. He also mingled in the company of ‘Neo Hegelians’ that were quite older than he, including Arnold Ruge, Bruno Bauer, and Ludwig Feuerbach. Germany was more backward in its economic and institutional development than Britain and France. These had traversed profound political, social, and economic revolutions (Britain in 1642-1651 and France in 1789-1799) which had resulted (after various ups and downs) in forms of government that were more adequate for the functioning and development of industrial Capitalism. In Germany, on the other hand, many independent political entities coexisted (Kingdoms, Principalities, Duchies, Electorates, etc.), the largest and most powerful of which was the Kingdom of Prussian. The Austrian Empire stood apart, dominating over many peoples of various languages, including German. The power of the landowning aristocracy was still very strong in Germany and the industrial bourgeoisie was underdeveloped and lacked political power. This implied that there was a relatively weaker demand for scientific knowledge to be used in industry in comparison to England and, correspondingly, intellectuals were markedly more attracted to Philosophy, a field that had a long tradition in Germany.

To understand at present many of the vicissitudes of philosophical thought in the period in which Marx’s ideas were developing it is necessary to bear in mind the difficulties faced by thinkers since the Renaissance who tried to break away from the grip of the religiously inspired philosophical thought that predominated since the Middle Ages. The scientific revolutions of the 16th and 17th centuries gradually subverted many of the assumptions that had previously bounded philosophical thought, making it less theological and more daring in its defiance towards traditional religious authority, as reflected in the works of Hobbes, Locke, Descartes, Bayle, Leibniz, and Spinoza (Bristow 2017). However, this was a gradual process, with advances and retreats. One must recall that during the last decades of the 16th and most of the 17th century (but especially the first half) there were throughout Europe bouts of burnings at the stake of ‘witches’ after ‘confessions’ were obtained by means of torture. And this

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6 This was not the case of Engels, who did write on various Philosophical matters during a lengthy period of time.
happened both where the Catholic Church dominated (where the process occurred in parallel with the last stage of the persecution of ‘heretics’ by the Inquisition) as well as where Protestant Reforms had taken place. The European Enlightenment, which approximately spans the second half of the 17th and the 18th century, took place on the shoulders of the advances made by the scientific revolutions. Its thinkers tended to be optimistic about the possibilities of sustained human progress, and to highlight the possibility of increasing the welfare of individuals if they could learn to be free to use their reason. The Enlightenment started after the English Civil War (1642-51) and during its existence other great transforming political processes were taking place: The English ‘Glorious Revolution’ (1688), the American Revolutionary War (1775-83) almost a century later and, lastly, the French Revolution (1789-99) and ensuing Napoleonic wars (1799-1815). The Enlightenment culminates and ends with the latter social and political maelstrom in France, which had profoundly transforming effects throughout all of Europe. Romanticism is the intellectual movement that replaced the Enlightenment in Europe towards the end of the 18th century.

1. The philosophers of the Enlightenment and some of their predecessors

Empiricism, rationalism, and skepticism, were some of the main features that the philosophers of the Enlightenment inherited from their intellectual predecessors (Bristow 2017). Empiricism appeared very early in England with Francis Bacon (1561-1626), generally considered the father of experimentation based on empirical data and the use of induction to obtain laws from the data instead of deducing propositions from first principles. Descartes (1596–1650) was a leading French skeptic and rationalist, though he spent a large part of his life in Holland due to the Thirty Years War (1618-48). doubting everything was the method that inspired him. In particular, he doubted that the senses could be the foundation of knowledge. For Descartes the knowledge about God and the soul was derived from innate ideas and not the senses. There was a certain dualism between the body and the mind: whereas the senses could be used to study the body, knowledge of the mind could only be obtained by means of introspection. The Dutch philosopher Spinoza (1632-1677) developed a rationalist Philosophy that was monist, not dualist like that of Descartes. For him there existed a unique substance in the world that was indistinctly Nature or God, and which however had two attributes: body and mind. The mind, perception, and thought were the products of corporeal organization. The identification of God with Nature implied pantheism at the very least, and many accused him of atheism. Not only was he expelled from his Jewish community in Amsterdam but the Catholic Church prohibited his books. Although he never explicitly negated the existence of God, his writings suggest that for him there existed no Supreme Being above the rest.

Hobbes (1588-1679) was an early empiricist, rationalist, and materialist whose Leviathan (1651) upholds the convenience of absolute monarchy: the imperative need for a very strong central authority able to avoid discord and civil war (of which there was plenty in Britain at the time). He postulated that the ‘natural condition’ of the human species is such that each person seeks to appropriate what belongs to others in a “war of all against all.” In such conditions economic activity cannot exist since all would live in fear and constant danger. It was to avoid such a situation that people accepted the existence of a State (Commonwealth, or Leviathan) that “hath the use of so much power and strength conferred on him that, by terror thereof, he is enabled to form the wills of them all, to peace at home, and mutual aid against their enemies abroad” (Hobbes 1839 [1651], 158).
Bacon’s empiricism had great influence on the development of the natural sciences and, in particular, on that of its great representative Isaac Newton (1643-1727), who began his investigations by observing data and using induction found laws or principles that could explain the data. Bacon’s influence on John Locke (1632-1704) was important. Quite differently from Descartes, Locke held that people are born without innate ideas (being their minds at birth a tabula rasa) and that knowledge is obtained through the perception of the senses, a central idea of empiricism, and the incorporation of the culture of the society they are born into. In his Second Treaty on Civil Government (1689) he explains that political power is “a right of making laws” and imposing penalties “for the regulating and preserving of property, and of employing the force of the community, in the execution of such laws, and in the defence of the commonwealth from foreign injury” (Locke 2003, §3). The establishment of a Civil Government remedied the inconveniences of the ‘state of nature’ but did not imply absolute power as in Hobbes’ theory.

Locke’s ideas on the role of the senses in the pursuit of knowledge were later adopted by two Frenchmen of the Enlightenment: Condillac (1714-1780) and Helvétius (1715-1771). The latter held that all human beings are the result of the physical attributes of the specie, on the one hand, and the special circumstances of their environment and education, on the other. Hence, education was the means to reform. Helvétius also held that human beings act on the basis of their desire to obtain pleasure and to avoid pain. In this he had great influence on the utilitarianism of Bentham (1748-1832).

The ideas of Hobbes and Locke that societies began to form governments by means of a covenant, a pact, or a contract, were further developed by the Genevan Jean-Jacques Rousseau (1712-1778) in Discourse on the Origin and Foundations of Inequality Among Mankind (1754), Discourse on Political Economy (1755), and especially The Social Contract (1762). His ideas had a strong influence on the leaders of the French Revolution (especially the Jacobin wing: Robespierre and Saint-Just). He held that in the ‘natural state’ there prevailed the strongest and that they subjugated the weak, and that the formation of an association by means of social contract could, under favorable circumstances, preserve individual liberty and prevent tyranny. According to Rousseau, the essence of the social contract was that “Each of us puts in common his person and all his power under the supreme direction of the general will; and in return each member becomes an indivisible part of the whole” (Rousseau 2002, 164). This ‘whole’ was the Republic; and the associate members were the citizens that participated in sovereign power. Although the initial formulation of the social contract was supposed to take place by unanimous decision, from then on the minority was constrained to accept the majority decisions through an electoral process. Rousseau drew inspiration from the Genevan institutions. He believed it was difficult for a Republic to function well in large states like France. For in such states it was less probable that the participation of the citizenship could force those who governed to act on behalf of the interests of all.

2. The evolution of philosophical thought in Germany

Leibniz (1646-1716) was the foundational thinker of the German Enlightenment. He invented infinitesimal calculus simultaneously with, and independently from, Isaac Newton, but the latter’s claim (filed through the courts) for the paternity of the invention darkened the last years of his life. He is famous for establishing the ‘principle of sufficient reason’, according to which everything that exists has a reason or cause. This confidence in the strength of reason for understanding the world that surrounds us was
characteristic of the Enlightenment. He was critical of Descartes’ (1596-1650) notion that space was infinitely divisible and held, on the contrary, that there existed indivisible units—monads—that combined to form more complex forms of matter. He was very critical of Hobbes (1588-1679) and Spinoza (1632-1677), since he believed that their theories were dangerously impregnated with materialism, atheism and determinism, to which he was opposed since he was convinced of God’s role in the creation of the world and man’s free will. Wolff (1679-1754), a disciple of Leibniz, introduced a classification of philosophical stances that distinguished between the ‘skeptics’, who did not believe that there could be knowledge of ultimate reality and the ‘dogmatics’, who believed that there could. He also classified the ‘dogmatics’ into two types: those who were ‘monists’ because they held that there is a single ultimate reality and those who were ‘dualists’, who believed that there were two. Dualists believed that ‘body’ and ‘mind’ (or ‘matter’ and ‘spirit’) coexisted, while monists where classified as ‘materialists’ or ‘idealists’, according to whether for them ultimate reality was corporeal (or material) or spiritual (or mental), respectively. For example, among Greek philosophers Wolff classified Plato as an idealist while (the post-Aristotelian) Epicurus was a materialist.\(^7\)

Kant (1724-1804), held by many as the central figure of modern Philosophy, gradually separated from the ideas of Leibniz and Wolff with his critiques of ‘Pure Reason’ (1787), ‘Practical Reason’ (1785) and ‘Judgment’ (1790). In the Preface to the second edition of his Critique of Pure Reason Kant highlights the success accumulated by mathematics during centuries, as well as the accomplishments of the natural sciences since Bacon based them firmly on empirical principles. In contrast, metaphysics had “not been so favored by fate as to have been able to enter upon the secure course of a science, even though it is older than all other sciences” (Kant 1998, 109). Kant praises Wolff as “the greatest among all dogmatic philosophers” (Ibid., 119-20) but he criticizes him for not having prepared the field for a critique of pure reason, which is what Kant set out to do with his book. He proposes to take a leap as Copernicus did when, after “he did not make good progress in the explanation of the celestial motions if he assumed that the entire celestial host revolves around the observer, tried to see if he might not have greater success if he made the observer revolve and left the stars at rest” (Ibid., 110). In the case of metaphysics, Kant writes that “Up to now it has been assumed that all our cognition must conform to the objects but all attempts to find out something about them a priori through concepts that would extend our cognition have, on this presupposition, come to nothing. Hence let us once try whether we do not get farther with the problems of metaphysics by assuming that the objects must conform to our cognition” (Ibid., 110).

Kant distinguished essences (things ‘in themselves’) from phenomena (or appearances), and held that the characteristics of our cognitive structure constrain us to the world of phenomena, not allowing us to discern if what we experiment as an object is not simply a product of our mind (and thus pure, theoretical, or a priori). Kant named his doctrine that appearances should be considered mere representations and not things ‘in themselves’ ‘transcendental idealism’. And he distinguished it from the ‘idealist’ stance of those who, not necessarily denying the existence of external objects of sense, did “not admit that it is cognized through immediate perception and infers from this that we can never be fully certain of their reality from any possible experience” (Kant 1998, 426).\(^8\)

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\(^7\)As we have seen, in his doctoral thesis Marx analyzed key aspects of Epicurus’ thought; and in the sketch of a Preface for its eventual publication he praised the contempt of Epicurus towards the superstitious attitude of those who believed that the gods intervened in human affairs.

\(^8\)His characterization of ‘idealism’ is what many today call ‘epistemological idealism’, reserving the
Much of what the philosophers of Idealism wrote was related to their (psychological) need to avoid ‘materialism’, arising from the fact that it was interpreted to imply determinism and was hence held to be incompatible with the notion of ‘free will’, cornerstone of the moral and religious conceptions of the times. Kant is diaphanous in this. He affirms that if the distinction were not made between things as objects of experience and things ‘in themselves’, “then the principle of causality, and hence the mechanism of nature in determining causality, would be valid of all things in general as efficient causes” (Ibid. 115) and “freedom and with it morality (...) would have to give way to the mechanism of nature” (Ibid. 116). He affirms that “I cannot even assume God, freedom and immortality for the sake of the necessary practical use of my reason unless I simultaneously deprive speculative reason of its pretension to extravagant insights” (Ibid. 117). Hence, he candidly admits that he “had to deny knowledge in order to make room for faith” (Ibid.).

Kant’s works had a profound impact on the main representatives of German Idealism (or ‘Absolute Idealism’): Fichte (1762-1814), Hegel (1770-1831), and Schelling (1775-1854). With variants among them, and each individually over his lifetime, it can be said that these philosophers tended to hold that there was no real opposition between a world independent of any subject and a world conformed by cognitive elements that arise from some type of subjective activity (Guyer and Horstmann 2018). A new conception of idealism was constructed that was monist (instead of dualist) and was based on the postulate that thought and being are inseparable. They introduced a dynamic notion according to which reality was the result of the development of self-consciousness; and to learn about the nature of that reality it was necessary to enquire about the activity that generated this development (Guyer 2000).

Fichte was strongly impacted by Kant’s Critiques. He constructed his own version of transcendental idealism and published the book The Science of Knowledge. But he was too forthcoming in his ideas for the times, and after being accused of atheism in an anonymous pamphlet that had wide circulation he had to quit his position in the University of Jena. As Fichte, Hegel also drew heavily from Kant while making substantial changes. According to Hegel, Kant’s (epistemological) Idealism was “a subjective idealism because even our most secure knowledge reflects the nature of the human subject rather than the essence of the objects of knowledge themselves” (Guyer 2000, 37). In contrast, Hegel’s Absolute Idealism held “that human thought reflects the nature of reality itself, not its own subjectivity, although since the deepest fact about the nature of reality is that it is a product of God’s thought this absolutism is still, in Hegel’s view, a form of idealism rather than any kind of absolute realism or materialism” (Ibid.). In the Introduction to Encyclopedia of the Philosophical Sciences in Basic Outline Hegel writes: “It is true that philosophy initially shares its objects with religion. Both have the truth for their object, and more precisely the truth in the highest sense, in the sense that God and God alone is the truth. Moreover, both treat the sphere of finite things, the sphere of nature and the human spirit, their relation to each other and to God as their truth” (Hegel 2010 [1830], §1).

Hegel called his Philosophy ‘speculative’. And in it the ‘dialectics’ occupies a central position (Maybee 2016). He answered Kant’s skepticism with his theory that we are not limited by our cognitive faculties because the world has the same rationality as our minds. Hegel’s ‘logic’, hence, refers not only to logic in its traditional sense but to the development of reality itself. The contradictions generated by means of the

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term ‘ontological idealism’ for a more extreme conception of idealism as that of Berkeley (1685-1753), for whom the foundation of reality is mental (Guyer and Horstmann 2018). Berkeley and Kant are usually considered to be the main representatives of Philosophical Idealism in the 18th century.
dialectics made reason generate more and more universal concepts that contained the
previous ones, leading ultimately to the ‘Absolute Idea’ (in the *Science of Logic*) and
to the ‘Absolute Spirit’ (in the *Phenomenology of Mind*).9 In his dialectics there
were three aspects, or ‘moments’, which drew inspiration from Fichte’s dialectics.10
First there was the moment of understanding, second the dialectical (or ‘negatively
rational’) which is opposite to the first. But this moment at the same time negates
and conserves the initial moment. The third moment (the ‘negation of the negation’)
was the ‘speculative’ (or ‘positively rational’) moment, which included both the unity
and the opposition of the first two. But since the details of this dialectical process were
specific to each field of application, Hegel could hold that it constituted a ‘scientific’
method. In the field of the natural sciences this dialectic was similar to the process of
the formulation of a hypothesis (as first moment), the questioning of that hypothesis
(second), and the modification of the original hypothesis to obtain a superior one (third
moment). And just as it could be applied to the movement of the planets, it could be
applied to the field of ‘spirit’ in religion, art, and social mores.

Hegel died in November 1831, when Marx was 13 years old. By then, some had
already characterized his system as pantheistic, an accusation of which his disciple
Ludwig Feuerbach (1804–1872) was also made of after his 1830 publication of his
*Thoughts on Death and Immortality*. Feuerbach had started as a strong supporter
of Hegelian Philosophy. But even though he was able to publish four more books on
Philosophy (or the history of Philosophy) during the decade he was unable to obtain an
academic position. He only started to distance himself from Hegel with his publication
in 1839 of the essay *Toward a Critique of the Hegelian Philosophy*, where he states that
in Hegel’s dialectics there is a ‘self-monologue’ disconnected from the real world and
that it is necessary to return to Nature. The Protestant theologian David Friedrich
Strauss (1808–1874) had published in 1835-36, *The Life of Jesus Critically Examined*
in which he held that Jesus Christ was mortal and that the Gospels that elevated him
to divinity reflected the myths and poetic activity of the early Christians. His stance
(and intellectual honesty) led to his losing his position at the University of Tubingen
and precluded him from getting an appointment at the University of Zurich in 1839.
Hegel’s disciple Bruno Bauer (1809–1882) published in 1838 his *Critical Exhibition of
the Religion of the Old Testament* in which he interpreted miracles in Naturalistic
terms. He lectured at the University of Bonn from 1839 to 1842, year in which he too
lost his position on account of his heterodox writings on the Gospels.

In 1840 young Frederick William IV succeeded to the throne of Prussia. He seemed
to have more liberal ideas than his deceased father (Frederick William III) and initially
even attenuated censorship somewhat. However, his conservative ministers soon stif-
fed the repression against the liberal press. Nevertheless, censorship and repression
were unable to prevent the radicalization of German philosophical thought. Feuerbach
published *The Essence of Christianity* in 1841 (the year in which Marx presents his
doctoral thesis) and *Principles of Philosophy of the Future* in 1843 (year in which Marx
marries and moves to Paris).

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9 *The Science of Logic* and *The Phenomenology of Mind* are Hegel’s main theoretical works. The
first may be said to be mainly ontological (the study of ultimate reality) and the second mainly
epistemological (the study of knowledge). But only in his *Encyclopedia of the Philosophical Sciences
in Basic Outline* did Hegel make a grand synthesis of his philosophical thought. It had three editions,
the last of which he published in 1830, shortly before his death.

10 Hegel avoided, however, using Fichte’s terminology of ‘thesis’, ‘antithesis’ and ‘synthesis’.
3. Young Marx’s evolving thought

**Marx and Hegel** From his earliest manuscripts and publications Marx expresses his perception of both the positive and negative aspects of Hegel’s Philosophy. In his *Manuscripts* of 1844 he writes that “The outstanding achievement of Hegel’s Phänomenologie and of its final outcome, the dialectic of negativity as the moving and generating principle, is thus first that Hegel conceives the self-creation of man as a process, conceives objectification as loss of the object, as alienation and as transcendence of this alienation; that he thus grasps the essence of labour and comprehends objective man—true, because real man—as the outcome of man’s own labour” (*Manuscripts*, MECW 3, 332-3). Hegel’s standpoint, writes Marx, “is that of modern political economy. He grasps labour as the essence of man.” Although “The only labour which Hegel knows and recognises is abstractly mental labour” (Ibid., 333), one of the “positive aspects of the Hegelian dialectic within the realm of estrangement is that “this movement of self-genesis... is the absolute, and hence final, expression of human life” (Ibid., 342). Furthermore, “This movement, in its abstract form as dialectic, is therefore regarded... as a divine process, but as the divine process of man” (Ibid.). However, “This result—the subject knowing itself as absolute self-consciousness— is therefore God, absolute Spirit, the self-knowing and self-manifesting idea.” Hence, for Hegel “Real man and real nature become mere predicates... a mystical subject-object or a subjectivity reaching beyond the object” (Ibid.). And “The absolute idea, the abstract idea... this whole idea which behaves in such a strange and bizarre way, and which has given the Hegelians such terrible headaches, is from beginning to end nothing else but abstraction (i.e., the abstract thinker)” (Ibid., 343-4). And hence Nature itself is an emanation of thought. “This entire transition from logic to natural philosophy is nothing else but the transition... from abstracting to intuiting” (Ibid., 344). According to Marx, “Hegel separates thought from the subject” and cannot grasp thought “as an expression of man as a human and natural subject endowed with eyes, ears, etc., and living in society, in the world, and in nature” (Ibid.).

In *The Holy Family* (written jointly with Engels and published in 1845) Marx refers to Hegel in the process of responding to his ex neo-Hegelian fellow traveler Bruno Bauer, but we are here interested in what he writes about Hegel. He states that Hegel’s Philosophy contains three elements: “Spinoza’s Substance, Fichte’s Self-Consciousness and Hegel’s necessarily antagonistic unity of the two, the Absolute Spirit.” Whereas Spinoza’s Substance is nature separated from man, Fichte’s Self-Consciousness is spirit separated from nature, and Hegel’s Absolute Spirit is the unity of “real man and the real human species” (MECW 4, 139). Also, concerning *The Phenomenology of Mind* he writes: “Because Hegel here substitutes self-consciousness for man, the most varied manifestations of human reality appear only as definite forms, as determinateness of self-consciousness... In Hegel’s *Phänomenologie* the material, sensuously perceptible, objective foundations of the various estranged forms of human self-consciousness are allowed to remain. The whole destructive work results in the most conservative philosophy because it thinks it has overcome the objective world, the sensuously perceptible real world, by transforming it into a ‘Thing of Thought’... The *Phänomenologie* is therefore quite consistent in that it ends by replacing human reality by ‘absolute knowledge’... The whole of the *Phänomenologie* is intended to prove that self-consciousness is the only reality and all reality” (Family, MECW 4, 192).

Marx again refers to Hegel’s Philosophy in his 1847 *The Poverty of Philosophy*, his critique of the French socialist Proudhon, who had tried to use the Hegelian dialectics to elaborate on his economic ideas but, according to Marx, had not understood Hegel.
He writes: “Economists express the relations of bourgeois production, the division of labour, credit, money, etc., as fixed, immutable, eternal categories” and Proudhon “wants to explain to us the act of formation, the genesis of these categories, principles, laws, ideas, thoughts...” And both were wrong. On the one hand, “Economists explain how production takes place in the above-mentioned relations, but what they do not explain is how these relations themselves are produced, that is, the historical movement which gave them birth” (Poverty, MECW 6, 162). And on the other, “M. Proudhon’s material is the dogmas of the economists. But the moment we cease to pursue the historical movement of production relations, of which the categories are but the theoretical expression, the moment we want to see in these categories no more than ideas, spontaneous thoughts, independent of real relations, we are forced to attribute the origin of these thoughts to the movement of pure reason” (Ibid.). Hence, Proudhon had to fail when he tried to use dialectics to work with the categories of the economists. Not only did he fail to understand in depth the Hegelian dialectics but that dialectics was useless. Marx writes:

Impersonal reason, having outside itself neither a base on which it can pose itself, nor an object to which it can oppose itself, nor a subject with which it can compose itself, is forced to turn head over heels, in posing itself, opposing itself and composing itself. Or, to speak Greek –we have thesis, antithesis and synthesis. For those who do not know the Hegelian language, we shall give the ritual formula: affirmation, negation and negation of the negation... it is the language of this pure reason, separate from the individual. Instead of the ordinary individual with his ordinary manner of speaking and thinking we have nothing but this ordinary manner purely and simply –without the individual (Poverty, MECW 6, 162-3)

Thus, for Hegel, all that has happened and is still happening is only just what is happening in his own mind. Thus the philosophy of history is nothing but the history of philosophy, of his own philosophy. There is no longer a “history according to the order in time”, there is only “the sequence of ideas in the understanding.” He thinks he is constructing the world by the movement of thought, whereas he is merely reconstructing systematically and classifying by the absolute method the thoughts which are in the minds of all (Ibid., 165).

**Marx and Feuerbach**

As we have seen, after his brief but intense experience as editor of the Rhenish Newspaper, Marx closely collaborated from Paris with Arnold Ruge in the organization of the German-French Yearbooks. However, after the publication of the first issue the project had to be suspended due to lack of financing and also to quarrels over the first issue. In one of the two articles that Marx contributes to the 1844 issue (under the title Contribution to the Critique of Hegel’s Philosophy of Law. Introduction11) he states

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11 The title does not adequately reflect the content of this article, which consists of an evaluation of what Marx considered a critique should accomplish in Germany, given its backwardness. He had planned it as an introduction to a much more ambitious work that would critically analyze (at least) the part of Hegel’s Elements of the Philosophy of Right that referred to the State. Long after Marx’s death a draft of that work was found, written in 1843 and to which Marx referred in the German-French Yearbook. It was published by D. Rjazanov in 1927 as Contribution to the Critique of Hegel’s Philosophy of Law. But neither is this title precise since in it Marx only addresses §§261-313 of Elements of the Philosophy of Right, all included within the section The State (§§257-360), which is the third section of Part 3 on Ethical Life, following the sections on The Family and Civil Society,
that “The criticism of religion is therefore in embryo the criticism of the vale of tears, the halo of which is religion” (MECW 3, 176). But for Marx the critique of religion (the opium of the people) had already been accomplished in Germany. He underlines how backward Germany was in comparison with England and France, not having had, as them, a revolution that established conditions favorable for industrial development. And he upholds the need to radically modify the social conditions in Germany, “the categorical imperative to overthrow all relations in which man is a debased, enslaved, forsaken, despicable being” (Ibid., 182).

Marx never met Hegel, but he did have a direct relationship with Feuerbach (who was 14 years his senior), and his books had a strong impact on him when he was giving form to his own ideas. A series of letters that Marx exchanged with Ruge during 1843, in which they discuss the situation in Germany and the content they should give to the projected Yearbook, are indicative of the stance he was developing. In his letter of March 13 Marx writes: “Feuerbach’s aphorisms’ seem to me incorrect only in one respect, that he refers too much to nature and too little to politics. That, however, is the only alliance by which present-day philosophy can become truth” (MECW 1, 400). In another letter (of September 1843) he returns to the same theme with greater extension. After writing that “we must try to help the dogmatists to clarify their propositions for themselves” – referring to the socialist/communist teachings of Cabet, Dezamy, and Weitling— he states “This communism is itself only a special expression of the humanistic principle, an expression which is still infected by its antithesis— the private system”, and it is confronted by “other socialist doctrines – such as those of Fourier, Proudhon, etc.” The socialist principle of the abolition of private property was only one of the aspects of reality that had to be addressed in the Yearbook; there was also the critique of religion, science, and politics. “In the first place religion, and next to it, politics are the subjects which form the main interest of Germany today. We must take these, in whatever form they exist, as our point of departure, and not confront them with some ready-made system.” “Hence, nothing prevents us from making criticism of politics, participation in politics, and therefore real struggles, the starting point of our criticism, and from identifying our criticism with them. In that case we do not confront the world in a doctrinaire way with a new principle: Here is the truth, kneel down before it! We develop new principles for the world out of the world’s own principles. We do not say to the world: Cease your struggles, they are foolish; we will give you the true slogan of struggle. We merely show the world what it is really fighting for” (MECW 3, 144).

In October 1843 Marx wrote a letter to Feuerbach, introducing himself as Ruge’s partner in the Yearbook that was to be produced from Paris. Ruge had asked for Feuerbach’s collaboration in this project a few months earlier. Marx specifically suggested that he contribute a critical article on Schelling, for whom Marx expressed a deep contempt. Schelling had denounced the progressive views he initially had, turning to religious mysticism. Having been invited in 1841 by the Prussian authorities to the University of Berlin, he had turned against the influence of the Young Hegelians. “Thus an attack on Schelling is indirectly an attack on our entire policy, and especially on Prussian policy... You would therefore be doing a great service to our enterprise, but even more to truth, if you were to contribute a characterisation of Schelling to the very first issue” (MECW 3, 350). However, the first and only issue of the Yearbook was published in February 1844 without an article by Feuerbach. Nevertheless, his name was made present with the inclusion of a brief letter Feuerbach had written to Ruge (and is the only letter published there without a corresponding answer).
In the Yearbook Marx published his *Introduction* to a critique of Hegel’s Philosophy of Law to which we have referred above, and *On the Jewish Question*. Furthermore, since April 1844 he was working on his *Manuscripts*. In August he sent a second letter to Feuerbach, along with a copy of his recently published *Introduction*. With unusual modesty and admiration, and in contrast with the condescending tone of his previous letter, Marx writes: “I don’t attribute any exceptional value to this essay but I am glad to have an opportunity of assuring you of the great respect and—if I may use the word—love, which I feel for you. Your *Principles of the Philosophy of the Future*, and your *Essence of Faith in Luther’s sense*¹³, despite their small size, are certainly of greater weight than the whole of contemporary German literature put together. In these writings you have provided—I don’t know whether intentionally—a philosophical basis for socialism and the Communists have immediately understood them in this way. The unity of man with man, which is based on the real differences between men, the concept of the human species brought down from the heaven of abstraction to the real earth, what is this but the concept of society!” (MECW 3, 354).

Feuerbach highlighted the real as object of the senses and, in particular, he emphasized the feeling of love to others (peculiarly neglecting altogether the feeling of hate). For example, in his *Principles of Philosophy of the Future* he writes: “The Christian God himself is only an abstraction from human love and an image of it (Feuerbach 1972 [1843], §32), and “The new philosophy bases itself on the truth of love, on the truth of feeling” (§34). He writes “Empiricism is therefore perfectly justified in regarding ideas as originating from the senses; but what it forgets is that the most essential sensuous object for man is man himself; that only in man’s glimpse of man does the spark of consciousness and intellect spring. And this goes to show that idealism is right in so far as it sees the origin of ideas in man; but it is wrong in so far as it derives these ideas from man understood as an isolated being, as mere soul existing for himself; in one word, it is wrong when it derives the ideas from an ego that is not given in the context of its togetherness with a perceptibly given You. Ideas spring only from conversation and communication. Not alone but only within a dual relationship does one have concepts and reason in general. It takes two human beings to give birth to a man, to physical as well as spiritual man; the togetherness of man with man is the first principle and the criterion of truth and universality” (§41). The problematic aspect of these ideas for Marx was that the relations between human beings that Feuerbach elevated to “the first principle and the criterion of truth and universality” were lacking a structure based on the insertion of such human beings in politics and the economy. When Marx sent this letter to Feuerbach he had already published months earlier (in the article of the *Yearbook* he enclosed with the letter):

The weapon of criticism cannot, of course, replace criticism by weapons, material force must be overthrown by material force; but theory also becomes a material force as soon as it has gripped the masses. Theory is capable of gripping the masses as soon as it demonstrates ad hominem,

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¹²The *Yearbook* also contains two articles by Engels: *Outlines of a Critique of Political Economy* and *The Condition of England. Past and Present* by Thomas Carlyle. Marx would characterize the first of these in his Preface to the *Contribution* as a “brilliant essay on the critique of economic categories.” There Engels gave a stunning sample of the optimism underlyiug the Communist project in gestation: “But the economist does not know himself what cause he serves. He does not know that with all his egotistical reasoning he nevertheless forms but a link in the chain of mankind’s universal progress. He does not know that by his dissolution of all sectional interests he merely paves the way for the great transformation to which the century is moving—the reconciliation of mankind with nature and with itself” (MECW 3, 424).

¹³These titles are in German in the original and abbreviated.
and it demonstrates ad hominem as soon as it becomes radical. To be radical is to grasp the root of the matter. But for man the root is man himself. The evident proof of the radicalism of German theory, and hence of its practical energy, is that it proceeds from a resolute positive abolition of religion. The criticism of religion ends with the teaching that man is the highest being for man, hence with the categorical imperative to overthrow all relations in which man is a debased, enslaved, forsaken, despicable being (MECW 3, 182).

And in the letter he nearly begged Feuerbach for his opinion on his published article or even any reply at all: “It would be of the greatest value to me if you would let me know in advance your opinion, and in general some speedy sign of life from you would make me happy” (MECW 3, 356-7). The quoted passage clearly connected Feuerbach’s ‘new philosophy’ based on the notion that “man is the highest being for man” with Marx’s conviction of the need to “overthrow all relations in which man is a debased, enslaved, forsaken, despicable being”, and to do this by transforming theory into “a material force” by bringing clarity to “the masses.” He seemed to be luring Feuerbach towards taking his philosophical “criticism of religion” one step forward towards the “categorical imperative” of radical revolution.

In The Holy Family Marx inserts some references to Feuerbach. In the first sentence of the Foreword he and Engels name their own philosophical stance ‘real humanism’, where the adjective ‘real’ is employed as synonymous to ‘material’, as in Feuerbach, that is, as opposite to ‘ideal’ (or ‘spiritual’). Marx attributes to Feuerbach the representation of materialism (qua humanism) in the domain of theory similarly to how “French and English socialism and communism represent materialism coinciding with humanism in the practical domain” (Family, MECW 4, 125). Feuerbach, who had “criticised Hegel from Hegel’s point of view by resolving the metaphysical Absolute Spirit into ‘real man on the basis of nature’, was the first to complete the criticism of religion by sketching in a grand and masterly manner the basic features of the criticism of Hegel’s speculation and hence of all metaphysics” (Ibid., 139). But only a few months later, in The German Ideology (written in 1845-46 but only published posthumously) Marx and Engels propose the abandonment of philosophical speculation and undertaking the task of devoting “oneself like an ordinary man to the study of actuality, for which there exists also an enormous amount of literary material, unknown, of course, to the philosophers”, since “Philosophy and the study of the actual world have the same relation to one another as onanism and sexual love” (Ideology, MECW 5, 236). Marx had reached these conclusions (with which Engels agreed since a certain point in time, at least in what concerns human reality) through a series of stages concentrated in a period of at most three years, leading him to polemicize and break with several of his ‘neo-Hegelian’ fellow travelers as he would later continue doing with the socialists he termed Utopian.

The writings of Feuerbach had a strong impact on Marx during the period immediately preceding his elaboration of the ‘materialist conception of history’. Consequently, Engels would later take the publication of Feuerbach’s The Essence of Christianity as a landmark for the “end of classical German philosophy” (MECW 26, 364). But although Feuerbach’s philosophical thought had a decisive influence on the development of Marx’s Historical Materialism until 1844, he quickly acknowledged its weaknesses, and the (very condensed) critique found in his Theses on Feuerbach marks a

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14 Although this book was written jointly by Marx and Engels, the Contents specified which sections had been written by each. Here we consider only some of those written by Marx.
critical point in his definitive transition to Historical Materialism and to his activist practical-revolutionary stance. Nevertheless, his criticisms of Feuerbach never attained the markedly aggressive style of some of his other ruptures with thinkers to which he had been previously attracted to.

**Marx’s ‘new materialism’** A few months after the publication of *The Holy Family* Marx writes down his very condensed critiques to Feuerbach, to “all previous materialism”, and to Philosophy in general, in eleven bullet points that Engels published posthumously (in 1888) as an Appendix to his *Ludwig Feuerbach and the End of Classical German Philosophy* under the title “Theses on Feuerbach.”¹⁵ In the famous last and very brief ‘thesis’ 11 Marx writes: “The philosophers have only interpreted the world in various ways; the point is to change it” (MECW 5, 5). Whereas ‘contemplative materialism’ or ‘old materialism’ could only contemplate single individuals and civil society, the *new materialism* comprehended ‘sensuousness as practical activity’ and adopted the standpoint of ‘human society, or social humanity’ (‘theses’ 9 and 10). The first of these ‘theses’ included Feuerbach in its critique of ‘all previous materialism’ on account of its contemplative character:

The chief defect of all previous materialism (that of Feuerbach included) is that things [Gegenstand], reality, sensuousness are conceived only in the form of the object, or of contemplation, but not as sensuous human activity, practice, not subjectively. Hence, in contradistinction to materialism, the active side was set forth abstractly by idealism, which, of course, does not know real, sensuous activity as such. Feuerbach wants sensuous objects, really distinct from conceptual objects, but he does not conceive human activity itself as objective activity. In *The Essence of Christianity*, he therefore regards the theoretical attitude as the only genuinely human attitude, while practice is conceived and defined only in its dirty-Jewish form of appearance. Hence he does not grasp the significance of ‘revolutionary’, of ‘practical-critical’, activity (MECW 5, 3).

By means of the expression ‘dirty-Jewish form of appearance’ (of ‘practice’), Marx was referring in an elliptical way (and in his personal jargon) to the commercial, banking, and industrial activities.¹⁶ According to Marx, although Feuerbach conceived the economic practice of men in society he did not understand the significance of exerting revolutionary activity on the foundations on which those practices are based, nor did he conceive of the practical-critical attitude as a “genuinely human” one. Marx had recently written in his *Manuscripts*: “It is easy to see that the entire revolutionary movement necessarily finds both its empirical and its theoretical basis in the movement

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¹⁵In his Preface, Engels writes: “We have expressed ourselves in various places regarding our relation to Hegel, but nowhere in a comprehensive, coherent account. To Feuerbach, who after all in some respects forms an intermediate link between Hegelian philosophy and our conception, we never returned” (MECW 26, 519), thus giving the reason for his new book.

¹⁶This can be seen in Marx’s critique of two articles by Bruno Bauer on the question of the full political emancipation of the German Jews, who after the defeat of Napoleon had lost some of their newly acquired rights: *The Jewish Question* and *The Capacity of Present-Day Jews and Christians to Become Free*. Marx’s critiques were expressed in the *Yearbook* and later also in some subsections of *The Holy Family*. In the *Yearbook*, for example, Marx writes “What is the worldly religion of the Jew? Huckstering. What is his worldly God? Money. Very well then! Emancipation from huckstering and money, consequently from practical, real Judaism, would be the self-emancipation of our time” (MECW 3, 170).

Some critics have (absurdly) accused Marx of anti-Semitism. For an excellent criticism of this opinion see Leopold (2007; 163-182).
of private property – more precisely, in that of the economy. The positive transcendence of private property, as the appropriation of human life, is therefore the positive transcendence of all estrangement – that is to say, the return of man from religion, family, state, etc., to his human, i.e., social, existence” (MECW 3, 297).

A few months after his Theses on Feuerbach, Marx starts to write, with Engels, another polemical book: The German Ideology, on which they work during a year or so in Brussels. The subtitle of the book is Critique of modern German philosophy according to its representatives Feuerbach, B. Bauer and Stirner, and of German socialism according to its various prophets. The book was structured in two volumes. The first had three parts: I Feuerbach, II Saint Bruno (Bauer), and III Saint Max (Stirner); and the second contained a critique of German Socialism, or ‘True Socialism’. All three parts of Volume I contain commentary on Feuerbach (either by either the authors or by Bauer or Stirner). Part I centers mainly on the presentation of the new conception of history proposed by the authors. Part III has more than 350 pages, i.e., more than half of the complete book, and is mainly a critique of Stirner’s book The Ego and his Own, which may be described as a foundation for an extremely individualistic version of anarchism.

In Part I (of Volume 1) the authors distinguish Feuerbach’s materialism both from ‘pure materialism’, and from ‘communist materialism’. Feuerbach ‘realises that man too is an ‘object of the senses’. But... he knows no other ‘human relations’ ‘of man to man’ than love and friendship, and even then idealised’, which makes him “relapse into idealism at the very point where the communist materialist sees the necessity, and at the same time the condition, of a transformation both of industry and of the social structure... With him materialism and history diverge completely” (MECW 5, 41). Also: “Feuerbach’s whole deduction with regard to the relation of men to one another is only aimed at proving that men need and always have needed each other... Feuerbach, in endeavouring to produce consciousness of just this fact, is going as far as a theorist possibly can, without ceasing to be a theorist and philosopher.” But as Marx had expressed in his ‘theses’, it was not simply a matter of producing consciousness of this reality: “for the real Communist it is a question of overthrowing the existing state of things” (MECW 5, 57-58).

Furthermore, when in part III they criticize Stirner, they highlight the need to empirically observe the “actual material premises” of the world: “Owing to the fact that Feuerbach showed the religious world as an illusion of the earthly world... German theory too was confronted with the question which he left unanswered: how did it come about that people ‘got’ these illusions ‘into their heads’?” thus paving the way to “the materialistic view of the world, a view which is not without premises, but which empirically observes the actual material premises as such and for that reason is, for the first time, actually a critical view of the world” (MECW 5, 236). They add that such a path was “already indicated” in the two articles Marx had published in the Yearbook.

4. From philosophical empiricism/materialism to the first versions of Socialism and Communism

An interesting contribution of Marx to The Holy Family is the synthetic chronology he makes of how a philosophic materialist thought was developed in England and France that departed from the existing (theological) metaphysics, while metaphysics was restored in Germany with the triumph of philosophic Idealism. The chronology reveals the extent to which Marx attributed to English and French empiricism (which he preferred to call materialism) a decisive influence on the very origins of socialist
and communist currents. He writes: “the French Enlightenment of the eighteenth century, and in particular French materialism, was not only a struggle against the existing political institutions and the existing religion and theology; it was just as much an open, clearly expressed struggle against the metaphysics of the seventeenth century”, referring among the latter to Descartes, Malebranche, Spinoza, and Leibniz. However, seventeenth century metaphysics “experienced a victorious and substantial restoration in German philosophy, particularly in the speculative German philosophy of the nineteenth century” (MECW 4, 124-5).

According to Marx, French materialism/empiricism had a double origin: the physics of Descartes (1596-1650) and the influence of English empiricism. Descartes’ materialism was limited to his studies in the natural sciences, which was completely separate from his metaphysical philosophical stances in other fields. He had many successful disciples in the natural sciences. And the main opponent of 17th century metaphysics was the French Protestant (Huguenot) Pierre Bayle (1647-1706), who through his religious doubts and his critiques of Spinoza and Leibnitz had shown “that a society consisting only of atheists is possible, that an atheist can be a man worthy of respect, and that it is not by atheism but by superstition and idolatry that man debases himself” (Ibid., 127).

The second source of materialism in France according to Marx, English materialism/empiricism, was of great importance and culminated with Locke’s treatise An Essay Concerning Human Understanding, which contributed “a positive, anti-metaphysical system.” But Locke was the culmination of the development of English empiricist thought which had begun almost a century earlier with Bacon (1561-1626), the “real progenitor of English materialism and all modern experimental science”, for whom “natural philosophy is the only true philosophy, and physics based upon the experience of the senses is the chiefest part of natural philosophy... Induction, analysis, comparison, observation, experiment, are the principal forms of such a rational method” (Ibid., 128). Marx points out that it was Hobbes (1588-1679) who systematized Bacon’s empiricism; but he also “shattered the theistic prejudices of Baconian materialism” since for him “It is impossible to separate thought from matter that thinks. This matter is the substratum of all changes... Only material things being perceptible, knowable to us, we cannot know anything about the existence of God” (Ibid., 129). Finally, Locke’s Essay had furnished a proof for Bacon’s principle that the origin of all human knowledge and ideas is in sensations and had “founded the philosophy of bon sens, of common sense; i.e., he said indirectly that there cannot be any philosophy at variance with the healthy human senses and reason based on them.”

The Abbot Condillac (1714-1780) had not only translated Locke to French but in his Essay on the origin of human knowledge (1746) developed his thought further by proving that “not only the soul, but the senses too, not only the art of creating ideas, but also the art of sensuous perception, are matters of experience and habit” and that “The whole development of man therefore depends on education and external circumstances” (Ibid., 129). Another French philosopher (and also an Abbot) that took inspiration from Locke was Helvétius (1715-1771), for whom “The sensory qualities and self-love, enjoyment and correctly understood personal interest are the basis of all morality. The natural equality of human intelligences, the unity of progress of reason and progress of industry, the natural goodness of man, and the omnipotence of education, are the main features in his system” (Ibid., 130).

According to Marx, French materialism led, on the one hand, to the development of the natural sciences through Descartes and his disciples and, on the other, to Socialism and to Communism (a term invented by the Frenchman Cabet (1788-1856)) through
the influence of Helvétius (1715-1771) and Fourier (1772-1837). In England, Socialism and Communism were introduced, respectively, by Bentham (1748-1832) and Owen (1771-1858): “Bentham based his system of correctly understood interest on Helvétius’ morality, and Owen proceeded from Bentham’s system to found English communism.” And Cabet, while he was exiled in England, “came under the influence of communist ideas there and on his return to France became the most popular, if the most superficial, representative of communism.” On the other hand, the more scientific French communists, like Dezamy (1808-1850), followed Owen by developing “the teaching of materialism as the teaching of real humanism and the logical basis of communism” (Ibid., 131).

Marx stresses as an essential aspect of Socialism and Communism the French materialist idea that a society should be forged in which education would allow the young to become “truly human”: “man’s private interest must be made to coincide with the interest of humanity.” That way, circumstances could be generated so that men could be formed according to “correctly understood interest”, which is “the principle of all morality”:

There is no need for any great penetration to see from the teaching of materialism on the original goodness and equal intellectual endowment of men, the omnipotence of experience, habit and education, and the influence of environment on man, the great significance of industry, the justification of enjoyment, etc., how necessarily materialism is connected with communism and socialism. If man draws all his knowledge, sensation, etc., from the world of the senses and the experience gained in it, then what has to be done is to arrange the empirical world in such a way that man experiences and becomes accustomed to what is truly human in it and that he becomes aware of himself as man. If correctly understood interest is the principle of all morality, man’s private interest must be made to coincide with the interest of humanity. If man is unfree in the materialistic sense, i.e., is free not through the negative power to avoid this or that, but through the positive power to assert his true individuality, crime must not be punished in the individual, but the anti-social sources of crime must be destroyed, and each man must be given social scope for the vital manifestation of his being. If man is shaped by environment, his environment must be made human... These and similar propositions are to be found almost literally even in the oldest French materialists (MECW 4, 130-1).

Months after finishing The Holy Family, Marx’s third ‘thesis’ on Feuerbach revisits these materialist topics in his specification that it is ‘revolutionary practice’ that can modify circumstances, including among these education: “The materialist doctrine concerning the changing of circumstances and upbringing forgets that circumstances are changed by men and that the educator must himself be educated. This doctrine must, therefore, divide society into two parts, one of which is superior to society. The coincidence of the changing of circumstances and of human activity or self-change can be conceived and rationally understood only as revolutionary practice” (MECW 5, 4).

And though the materialist Feuerbach “resolves the essence of religion into the essence of man”, the latter “is the ensemble of the social relations” which Feuerbach does not address critically, which leads him to “abstract from the historical process” that generates them and only consider social relations in their natural sense, i.e. those that deal with man as a member of a species (‘thesis’ 6), and not to those that have to do with the complexities of the economic life of man.
Chapter 2  HISTORICAL MATERIALISM AND MARX’S METHODOLOGY

Historical Materialism
First exposition of Historical Materialism: *The German Ideology*

*The German Ideology* was conceived by its authors as a critique of some of the most prominent ‘Young Hegelians’ (Ludwig Feuerbach, Bruno Bauer, and Max Stirner) and of the version of Socialism then prevalent in Germany (‘True Socialism’). Here we focus on the sections that contain the positive exposition of Historical Materialism, often inserted within critical commentary on the writings of the Young Hegelians. The authors distinguish the ‘Old Hegelians’, who “understood everything as soon as it was reduced to a Hegelian logical category,” from the ‘Young Hegelians’, who “criticised everything by ascribing religious conceptions to it or by declaring that it is a theological matter” (*Ideology*, MECW 5, 29-30). And they address the views of Feuerbach somewhat differently from those of Bauer (who specialized in the critique of ideas, especially religious ones) and Stirner (who advocated for freedom of the individual from any mental or physical constraint marked by others, whether it be family, religion, or the State). For Marx and Engels the basic problem with the ‘Young Hegelians’ was that they considered “conceptions, thoughts, ideas, in fact all the products of consciousness, to which they attribute an independent existence, as the real chains of men (just as the Old Hegelians declare them the true bonds of human society).” Hence, they would “put to men the moral postulate of exchanging their present consciousness for human, critical or egoistic consciousness” (respectively, for Feuerbach, Bauer, and Stirner) “and thus of removing their limitations.” But “in spite of their allegedly ‘world-shattering’ phrases” they were the staunchest conservatives because they were “in no way combating the real existing world” (Ibid., 30).

In contrast, Marx and Engels proposed a radical change in the interpretation of history, the ‘materialist conception of history’, which was to be the foundation for their political project.¹ For them philosophical speculation per se should cease and be replaced by an adequate exposition of man’s development process, which was the essence of social science: “Where speculation ends, where real life starts, there consequently begins real, positive science, the expounding of the practical activity, of the practical process of development of men. Empty phrases about consciousness end, and real knowledge has to take their place. When the reality is described, a self-sufficient philosophy loses its medium of existence” (Ibid., 37).

**Ingredients of the new conception of history** *The German Ideology* presents a sketch of both a theory of the evolution of human society, and a theory of how one should analyze and present that history. Its authors would later develop this sketch in their joint and individual works, in different directions and with (sometimes significant) variations, but its central nucleus would be present over the creative careers of each. The following paragraph synthesizes their rupture with ‘German philosophy’ and some of the basic ingredients of their new conception of history:

¹The political project and the political praxis of Marx and Engels are addressed in Part IV of this book.
In direct contrast to German philosophy which descends from heaven to earth, here it is a matter of ascending from earth to heaven. That is to say, not of setting out from what men say, imagine, conceive... but setting out from real, active men, and on the basis of their real life-process demonstrating the development of the ideological reflexes and echoes of this life-process. The phantoms formed in the brains of men are also, necessarily, sublimates of their material life-process, which is empirically verifiable and bound to material premises. Morality, religion, metaphysics, and all the rest of ideology as well as the forms of consciousness corresponding to these, thus no longer retain the semblance of independence. They have no history, no development; but men, developing their material production and their material intercourse, alter, along with this their actual world, also their thinking and the products of their thinking. It is not consciousness that determines life, but life that determines consciousness (Ideology, MECW 5, 37).

By decomposing the Hegelian ‘self-consciousness’ into differentiated and hierarchical strata, such as the socioeconomic (the ‘material life-process’) and the ideological (‘Morality, religion, metaphysics’), they held that the latter does not possess a history of its own because it is conditioned by the former. This means that it is at the socioeconomic (or ‘material’) level where one should seek the ‘determinants’ (in the lax sense of ‘conditioning factors’) of moral, religious, political, artistic, or philosophical thought. And they find in the ‘division of labor’—understood in a hierarchical manner (slave-owner, serf-lord, worker-capitalist) power and authority is concentrated in the upper level—the foundation for “the unequal distribution, both quantitative and qualitative, of labour and its products, hence property” (Ibid., 46). They find “the nucleus” of power and property in the primitive family, “where wife and children are the slaves of the husband” and hence the latter can decide on the use of the resulting products.

But along with the hierarchical ‘division of labor’ there is a horizontal one in which a man is “a hunter, a fisherman, a shepherd, or a critical critic, and must remain so if he does not want to lose his means of livelihood” (Ibid., 47). There hence arises a “contradiction between the interest of the separate individual or the individual family and the common interest of all individuals who have intercourse with one another”, a common interest given by “the mutual interdependence of the individuals among whom the labour is divided.” Such common interest “assumes an independent form as the state... always based, however, on the real ties existing in every family conglomeration and tribal conglomeration—such as flesh and blood, language, division of labour on a larger scale, and other interests—and especially, as we shall show later, on the classes, already implied by the division of labour, which in every such mass of men separate out, and one of which dominates all the others” (Ibid., 46). The authors conclude that the struggles for the establishment of particular forms of government (such as democracy, aristocracy, or monarchy) and even subforms (like the extension of suffrage in democracy) take place on the foundation of the actual (i.e. social, economic, and political) struggles between classes. Furthermore, “every class which is aiming at domination... must first conquer political power in order to represent its interest in turn as the general interest” (Ibid., 47). The authors distinguish the interest of the individual (or particular interest) form both the (actual) common interest and the ‘illusive common interest’, also referred to as the ‘general interest’. The state intervenes practically, as a biased referee, in the struggles among interests. Hence, “the practical struggle of these particular interests, which actually constantly run counter to the common and illusive common interests, necessitates practical intervention and restraint.
by the illusory ‘general’ interest in the form of the state” (Ibid.).

Quite differently from the “Young Hegelians”, Marx and Engels believed that one should not begin with the analysis of religion or Philosophy but instead with “the material conditions”, both natural and social, that condition men’s lives. “The premises from which we begin... are the real individuals, their activity and the material conditions of their life, both those which they find already existing and those produced by their activity. These premises can thus be verified in a purely empirical way” (Ibid., 41). Since men “begin to distinguish themselves from animals as soon as they begin to produce their means of subsistence... The way in which men produce their means of subsistence depends first of all on the nature of the means of subsistence they actually find in existence and have to reproduce... Hence what individuals are depends on the material conditions of their production” (Ibid., 32). And depending on these material conditions of production, as producers individuals contract among themselves certain economic, social, and political relations. The insistence on the empirical is repetitive in The German Ideology: “Empirical observation must in each separate instance bring out empirically, and without any mystification and speculation, the connection of the social and political structure with production.” And empirical observation led them to the conclusion that “The social structure and the State are continually evolving out of the life-process of definite individuals, but of individuals... as they really are; i.e. as they operate, produce materially, and hence as they work under definite material limits, presuppositions and conditions independent of their will” (Ibid., 35-6; italics added).

Their was a new ‘conception of history’ which differed from ‘the conception of history held hitherto, which neglects the real relations and confines itself to spectacular historical events.” They, in contrast, believed it necessary to first study “The form of intercourse determined by the existing productive forces”, which in each historical stage was determined by ‘civil society’, which “embraces the whole material intercourse of individuals within a definite stage of the development of productive forces. It embraces the whole commercial and industrial life of a given stage and, insofar, transcends the state and the nation” (Ibid., 89). This “material intercourse” transcended the political limits of the tribe or nation (according to the stage of development) to the extent that these tribes or nations were interconnected. For them ‘civil society’3 “in all ages forms the basis of the state and of the rest of the idealistic superstructure” (MECW 5, 89). Succinctly, the new “conception of history thus relies on expounding the real process of production –starting from the material production of life itself– comprehending the form of intercourse connected with and created by this mode of production, i.e., civil society in its various stages, as the basis of all history; describing it in its action as the state, and also explaining how all the different theoretical products and forms of consciousness, religion, philosophy, morality, etc., etc., arise from it, and tracing the process of their formation from that basis; thus the whole thing can, of course, be depicted in its totality (and therefore, too, the reciprocal action of these various sides on one another)” (MECW 5, 53). In contrast with the “idealist view of history”, the

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2The word ‘intercourse’ (Verkehr in German) used in The German Ideology refers in general to social relations, and ‘material intercourse’ to specifically economic relations (in production or in the circulation of what is produced). They often use the expression ‘relations of production and intercourse’. In Capital, Marx uses the expression ‘mode of production and circulation’.

3The German term ‘bürgerliche Gesellschaft’ may be translated both as ‘civil society’ and as ‘bourgeois society’. And it is used in both ways by the authors, in different contexts. Hence, the translators have used ‘bourgeois society’ whenever the context indicates that they are dealing with capitalist society. Otherwise they use ‘civil society’. In Capital Marx no longer uses this expression in the first sense, using instead ‘mode of production’. 
materialist view "does not explain practice from the idea but explains the formation of ideas from material practice." Since the idealists did not part from the economic foundations of the historical process, they had "only been able to see in history the spectacular political events and religious and other theoretical struggles, and in particular with regard to each historical epoch they were compelled to share the illusion of that epoch. For instance, if an epoch imagines itself to be actuated by purely 'political' or 'religious' motives, although 'religion' and 'politics' are only forms of its true motives, the historian accepts this opinion" (Ibid., 55). With such methodology, "The 'fancy', the 'conception' of the people in question about their real practice is transformed into the sole determining and effective force, which dominates and determines their practice" (Ibid.). The authors also express this critique quite graphically, writing: "Whilst in ordinary life every shopkeeper is very well able to distinguish between what somebody professes to be and what he really is, our historiography has not yet won this trivial insight. It takes every epoch at its word and believes that everything it says and imagines about itself is true" (MECW 5, 62).

For Marx and Engels, although "Men are the producers of their conceptions, ideas... they are conditioned by a definite development of their productive forces and of the intercourse corresponding to these" (Ibid., 36, italics added). For them, "a certain mode of production, or industrial stage, is always combined with a certain mode of co-operation, or social stage, and this mode of co-operation is itself a 'productive force'." And "the aggregate of productive forces accessible to men determines the condition of society, hence, the 'history of humanity' must always be studied and treated in relation to the history of industry and exchange" since there exists "a materialist connection of men with one another, which is determined by their needs and their mode of production, and which is as old as men themselves. This connection is ever taking on new forms, and thus presents a 'history' irrespective of the existence of any political or religious nonsense which would especially hold men together" (Ibid., 43). The authors recognize that "The French and the English, even if they have conceived the relation of this fact with so-called history only in an extremely one-sided fashion... have nevertheless made the first attempts to give the writing of history a materialistic basis by being the first to write histories of civil society, of commerce and industry" (Ibid., 43).4

It is only after taking into consideration that the degree of development of the productive forces in any given society conditions the relations that the individuals that form it establish between themselves, "do we find that man also possesses 'consciousness'" and produces and transforms ideas. But even this consciousness is intimately conditioned by material determinants such as the language that is used, since, "The 'mind' is from the outset afflicted with the curse of being 'burdened' with matter, which here makes its appearance in the form of agitated layers of air, sounds, in short, of language. Language is as old as consciousness, language is practical, real consciousness that exists for other men as well, and only therefore does it also exist for me; language, like consciousness, only arises from the need, the necessity, of intercourse with other men" (Ibid., 44). We thus have, as of 1845, a succinct summary of the basic ingredients of the 'materialist conception of history', or Historical Materialism.

**Historical forms of social organization** Marx always gave a very significant role to the 'development of the productive forces', i.e., all the different innovations and

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4In a letter to Borgius (of 1894), Engels writes: “If it was Marx who discovered the materialist view of history, the work of Thierry, Mignet, Guizot and every English historiographer prior to 1850 goes to show that efforts were being made in that direction.”
inventions that had the effect of increasing the productivity of labor, as well as to its two-way relation with the division of labor, the organization of production, and the relation between the worker and the material with which he works, the instruments he uses, and the resulting product. The latter relation was intimately related with the forms of property. In *The German Ideology* he and Engels show how over history “The various stages of development in the division of labour are just so many different forms of property, i.e., the existing stage in the division of labour determines also the relations of individuals to one another with reference to the material, instrument and product of labour” (MECW 5, 32). They distinguish in European history a succession of forms of social organization that correspond to as many forms of property. The first is tribal property, in which “The division of labour is at this stage still very elementary and is confined to a further extension of the natural division of labour existing in the family. The social structure is, therefore, limited to an extension of the family: patriarchal chieftains, below them the members of the tribe, finally slaves” (Ibid., 33). As a second form of social organization they introduce “the ancient communal and state property, which proceeds especially from the union of several tribes into a city by agreement or by conquest, and which is still accompanied by slavery” (Ibid.). In this form of social organization there is, beside communal property, the gradual development of private property, first as movable property and later also as immovable, or real estate, property. The laboring slaves constitute “the communal private property of the active citizens who, in relation to their slaves, are compelled to remain in this spontaneously derived form of association.” But as real estate property develops, this form of social organization declines.

The development of private property generally brings about the concentration of property. But different social forms appear according to the degree of advance of the productive forces. The authors take as examples Roman society and the later development of European feudal institutions. In the expansion of Rome through the conquest and enslavement of neighboring Germanic peoples, the concentration of private real estate property led to “the transformation of the plebeian small peasantry into a proletariat, which, however, owing to its intermediate position between proprietied citizens and slaves, never achieved an independent development” (Ibid.). The decomposition of Roman society originated a third form of property: “feudal or estate property”, which had its beginnings in the rural areas owing to the depopulation of the cities after the conquests by various Germanic tribes (Ostrogoths, Visigoths, Vandals, Lombards, Franks, etc.) during the 5th and 6th centuries. “The last centuries of the declining Roman Empire and its conquest by the barbarians destroyed a considerable part of the productive forces; agriculture had declined, industry had decayed for want of a market, trade had died out or been violently interrupted, the rural and urban population had decreased. These conditions and the mode of organisation of the conquest determined by them, together with the influence of the Germanic military constitution, led to the development of feudal property.”

As in the cases of tribal property and the communal property of Antiquity, the feudal form of property of the Middle Ages was “an association against a subjected producing class; but the form of association and the relation to the direct producers were different because of the different conditions of production”, and the subjected producing class was no longer “as in the case of the ancient community, the slaves, but the enserfed small peasantry” (Ibid., 34). Corresponding to the hierarchical feudal structure of land ownership in the countryside there developed “in the towns in the shape of corporative property, the feudal organisation of trades”, where various factors contributed to the formation of guilds, such as the “necessity for associating against
the association of the robber-nobility, the need for communal covered markets in an age when the industrialist was at the same time a merchant, the growing competition of the escaped serfs swarming into the rising towns.” In the towns the form of property was directly related to labor. From “The gradually accumulated small capital of individual craftsmen and their stable numbers, as against the growing population, evolved the relation of journeyman and apprentice, which brought into being in the towns a hierarchy similar to that in the country” (Ibid.). This hierarchy of estates\(^5\) was formed in the towns by masters, journeymen, apprentices, as in the countryside there were princes, nobility, clergy and enserfed peasants. There was little division of labor in the feudal social organization because of the “restricted conditions of production—the small-scale and primitive cultivation of the land, and the craft type of industry.” “In agriculture it was rendered difficult by the strip-system, beside which the cottage industry of the peasants themselves emerged”, whereas “In industry there was no division of labor at all in the individual trades themselves, and very little between them.” The feudal structure led to the “grouping of larger territories into feudal kingdoms”, which was “a necessity for the landed nobility as for the towns. The organisation of the ruling class, the nobility, had, therefore, everywhere a monarch at its head” (Ibid., 35).

They consider it is generally recognized that “The relations of different nations among themselves depend upon the extent to which each has developed its productive forces, the division of labour and internal intercourse”, but hold also that “the whole internal structure of the nation itself depends on the stage of development reached by its production and its internal and external intercourse” (Ibid., 32). And they stress that there is a close connection between the degree of development of the productive forces and the division of labor: “How far the productive forces of a nation are developed is shown most manifestly by the degree to which the division of labour has been carried. Each new productive force, insofar as it is not merely a quantitative extension of productive forces already known (for instance, the bringing into cultivation of fresh land), causes a further development of the division of labour” (Ibid.). Also, the division of labor within a country “leads at first to the separation of industrial and commercial from agricultural labour, and hence to the separation of town and country and to the conflict of their interests” and continues with “the separation of commercial from industrial labour”, along with the division of labor within these branches and “among the individuals co-operating in definite kinds of labour” (Ibid.).

The division of labor further evolved, according to the authors, with the separation of commerce from production by means of the generation of a class of merchants that linked the various cities and allowed for the specialization of these in certain industrial products. The development of manufactures, especially weaving, led to the accumulation of capital by industrial capitalists and merchants. The discoveries of America and the sea route to the East Indies facilitated the creation of a world market and the colonization of many peoples and to increased competition and wars between the colonizing countries. Finally, large-scale industry was first developed in England with the introduction of machinery that further extended the division of labor and the invention of which was made possible by the advances in the natural sciences. We will not enter here into the details of the process described by Marx and Engels since we will return to the subject of the genesis and evolution of Capitalism when we deal with

\(^5\)In Ideology the authors make a distinction between classes and estates. The latter term was especially used for the feudal organization of society where, for example, the burghers of a certain city were initially an estate and later developed into a class when the towns of a certain territory united within a single state. In Capital Marx no longer makes this distinction and uses the concept of social class for all forms of society.
Marx’s *Capital*.

**Political power, the state, law, and the historical dynamics** The causal relations between political power, the state, law, and the economic conditions formed an integral part of the new conception. The authors state that in history there was an opposition between “those theoreticians who regarded might as the basis of right” and “those who looked on will as the basis of right.” “If power is taken as the basis of right, as Hobbes, etc., do, then right, law, etc., are merely the symptom, the expression of other relations upon which state power rests” (*Ideology*, MECW 5, 329). Marx and Engels are clearly in favor of the Hobbesian perspective that *might* is the foundation of jurisprudence and not *will*, as in the theories of Rousseau (*The Social Contract*) and previous contractualist theorists, according to which there was a primeval ‘contract’ of social coexistence. But for Marx and Engels there were “other relations upon which state power rests” and these were those relations that originate in the “material life of individuals”, in frank opposition to the spheres of ideas and of will. Both the power of the state in any time period and the predominant ideas have as basis the prevailing “mode of production and the form of intercourse”, which is independent of the will of individuals, since these are born within a previously determined social structure, which only changes according to certain patterns that Marx, in the case of Capitalism, tried to decipher during the next three decades. According to Marx and Engels in all stratified societies there is a class that is dominant and which, conditioned by the existing web of relations of production and circulation, expresses and imposes the interests of its members by means of the state and the law:

The material life of individuals, which by no means depends merely on their ‘will’, their mode of production and form of intercourse, which mutually determine each other—this is the real basis of the state and remains so at all the stages at which division of labour and private property are still necessary, quite independently of the will of individuals. These actual relations are in no way created by the state power; on the contrary they are the power creating it. The individuals who rule in these conditions—leaving aside the fact that their power must assume the form of the state—have to give their will, which is determined by these definite conditions, a universal expression as the will of the state, as law, an expression whose content is always determined by the relations of this class, as the civil and criminal law demonstrates in the clearest possible way (Ibid, 329).

There is also in *Ideology* a sketch of a theory of the dynamics of history that is based on the degree of correspondence (or lack of correspondence, or ‘contradiction’) between the level of development reached by the forces of production and the economic-social organization of society, or existing ‘mode of production and form of intercourse’. Each generation inherits a given ‘mode of production and form of intercourse’ as well as certain productive forces. But while the former tend to crystallize, the productive force of human labor tends to advance by means of the activity of society in the economic sphere, with varying speed according to the characteristics of the social organization and the modalities of ‘intercourse’, which include trade, plunder, and conquest. As society develops the productive forces, in the long run there eventually appears a lack of correspondence, or ‘contradiction’ between them and the relatively static set

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6In *Capital* Marx replaces this expression with ‘mode of production and circulation’ or simply ‘mode of production’.
of relations of production and intercourse. This tension may eventually lead to a revolution. The latter may have the effect, in a period of time of varying length, of adapting the structure of society so that it is no longer in contradiction with the existing productive forces. In their words:

What appears accidental to a later age as opposed to an earlier... is a form of intercourse\(^7\) which corresponded to a definite stage of development of the productive forces... The conditions under which individuals have intercourse with each other, so long as this contradiction is absent, are conditions appertaining to their individuality, in no way external to them... The definite condition under which they produce thus corresponds, as long as the contradiction has not yet appeared,

to the reality of their conditioned nature... Then this condition appears as an accidental fetter, and the consciousness that it is a fetter is imputed to the earlier age as well... These various conditions, which appear first as conditions of self-activity, later as fetters upon it, form in the whole development of history a coherent series of forms of intercourse, the coherence of which consists in this: an earlier form of intercourse, which has become a fetter, is replaced by a new one corresponding to the more developed productive forces... Since these conditions correspond at every stage to the simultaneous development of the productive forces, their history is at the same time the history of the evolving productive forces taken over by each new generation (Ibid, 82).

This conception of the fundamental factors in the dynamics of history was not meant to be mechanical nor deterministic.\(^8\) It tried to reflect the complexity of its most fundamental aspects, not the acts of ‘great men’ but the conditioning forces that create the circumstances that allow certain men to stand out in any given historical juncture. On the one hand is the interrelation between nations through their competition in the world market, which generate ‘contradictions’ in countries which are not those in which the fundamental contradictions of the epoch occur. Although “all collisions in history have their origin, according to our view, in the contradiction between the productive forces and the form of intercourse”, “to lead to collisions in a country, this contradiction need not necessarily have reached its extreme limit in that particular country”, since “The competition with industrially more advanced countries, brought about by the expansion of international intercourse, is sufficient to produce a similar contradiction in countries with a less advanced industry” (Ibid, 74-5). And even within a country there could be long periods of time in which the power of the state represented mainly the interests of a class that is no longer dominant from the point of view

\(^7\)The authors often use ‘form of intercourse’ to refer to the relations that individuals adopt in the process of production. In Capital Marx will call them ‘relations of production’.

\(^8\)Notwithstanding, many ‘Marxists’ have so presented it with propagandistic, political, or misconstrued pedagogical aims. And this was so even during Engels’ life who, in the already quoted letter to Bloch (of September 21, 1890) writes: “If some younger writers attribute more importance to the economic aspect than is its due, Marx and I are to some extent to blame. We had to stress this leading principle in the face of opponents who denied it, and we did not always have the time, space or opportunity to do justice to the other factors that interacted upon each other. But it was a different matter when it came to depicting a section of history, i. e. to applying the theory in practice, and here there was no possibility of error. Unfortunately people all too frequently believe they have mastered a new theory and can do just what they like with it as soon as they have grasped—not always correctly—its main propositions. Nor can I exempt from this reproach many of the more recent ‘Marxists’ who have, indeed, been responsible for some pretty peculiar stuff” (MECW 49, 36).
of its insertion in economic activity: “Furthermore, this development proceeds only very slowly; the various stages and interests are never completely overcome, but only subordinated to the prevailing interest and trail along beside the latter for centuries afterwards. It follows from this that even within a nation... an earlier interest, the peculiar form of intercourse of which has already been ousted by that belonging to a later interest, remains for a long time afterwards in possession of a traditional power in the illusory community (state, law), which has won an existence independent of the individuals; a power which in the last resort can only be broken by a revolution” (Ibid., 83). These comments allude particularly to the situations of England and France before their respective ‘bourgeois’ revolutions (of the 17th and 18th centuries, respectively), in which the state and law still mainly reflected the interests of the landowning nobility to a larger degree than that which was compatible with the ascendant economic power of the capitalist class. But they also allude to the contemporary situation of other countries (such as Germany) where there had not yet taken place a ‘bourgeois’ revolution. The authors tried to obtain, by induction, regularities that explained the historical dynamics in various places and times, and that could also serve as a guide for political action that could be compatible with the historical tendencies that derived from such regularities.

**Marx’s 1846 letter to Annenkov**  According to the new conception of history, the “mode of production of life, and the form of intercourse coupled with it” (MECW 5, 45) of any society was intimately related to the state of advance of the ‘productive forces’, that is, the degree to which society is capable of transforming nature for its benefit (usually called ‘productivity’ in modern economics), state of advance which each generation leaves as inheritance to the next in cumulative fashion and the study of which was key for the comprehension of the ultimate conditioning forces of the dynamics of history. In a letter written shortly after his work on *Ideology* to the Russian writer P. W. Annenkov, Marx expresses in a very synthetic way his vision of the conditioning role that the productive forces have with respect to key aspects of social organization and, finally, to the state: “If you assume a given state of development of man’s productive faculties, you will have a corresponding form of commerce and consumption. If you assume given stages of development in production, commerce or consumption, you will have a corresponding form of social constitution, a corresponding organisation, whether of the family, of the estates or of the classes – in a word, a corresponding civil society. If you assume this or that civil society, you will have this or that political system, which is but the official expression of civil society” (MECW 38, 96).

The letter synthetically expresses how the activity of individuals is conditioned by the structure of society, which is the fruit of the preceding generations, and how the degree of development of the productive forces constitutes a unifying thread in history: “Needless to say, man is not free to choose his productive forces – upon which his whole history is based— for every productive force is an acquired force, the product of previous activity. Thus the productive forces are the result of man’s practical energy, but that energy is in turn circumscribed by the conditions in which man is placed by the productive forces already acquired, by the form of society which exists before him, which he does not create, which is the product of the preceding generation. The simple fact that every succeeding generation finds productive forces acquired by the preceding generation... engenders a relatedness in the history of man, engenders a history of mankind” (Ibid.).

The productive forces of society tend to increase progresssively through time (but
not always, since at times they can diminish). Sometimes a further increase is possible and needed but is impeded by the rigidity of the institutions derived from the social relations in the sphere of production. Then, “If he is not to be deprived of the results obtained or to forfeit the fruits of civilisation, man is compelled to change all his traditional social forms as soon as the mode of commerce ceases to correspond to the productive forces acquired.” Marx clarifies that he is using the word commerce “in its widest sense—as we would say Verkehr in German.” And he gives the following illustrative example taken from the historical dynamics of Britain: “privilege, the institution of guilds and corporations, the regulatory system of the Middle Ages, were the only social relations that corresponded to the acquired productive forces and to the pre-existing social conditions from which those institutions had emerged. Protected by the corporative and regulatory system, capital had accumulated; maritime trade had expanded, colonies had been founded—and man would have lost the very fruits of all this had he wished to preserve the forms under whose protection those fruits had ripened. And, indeed, two thunderclaps occurred, the revolutions of 1640 and of 1688.”

As a consequence of these events “all the earlier economic forms, the social relations corresponding to them, and the political system which was the official expression of the old civil society”, i.e., all the corporations and regulations of the ‘old civil society’ that constituted fetters for the further development of Capitalism, including the relative independence of the King with respect to Parliament (like the ability to convolve or close it at will), “were destroyed.” In a succinct and encompassing description of long historical periods, Marx writes: “With the acquisition of new productive faculties man changes his mode of production and with the mode of production he changes all the economic relations which were but the necessary relations of that particular mode of production (Ibid.). The existing productive forces in England were capable of sustaining relations of production and circulation more adequate for the further development of Capitalism, but this progress was hindered by inherited institutions that no longer corresponded to society’s needs. The brusque changes that took place in a short period of time put in place the institutions that allowed the remarkable further development of Capitalism in England.

The later evolution of Marx’s thought

Marx’s ideas evolved after The German Ideology, getting more concrete as he further pursued his studies of political economy, social history, and contemporary affairs. His urgent need for an income made him stay keenly aware of all sorts of empirical data. In the Preface to the Contribution to a Critique of Political Economy (of 1859) Marx explains that when he moved to London “the imperative necessity of earning my living” had led him to contribute during eight years to the New York Tribune. “Since a considerable part of my contributions consisted of articles dealing with important economic events in Britain and on the Continent, I was compelled to become conversant with practical details which, strictly speaking, lie outside the sphere of political

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9The revolution of 1640 he mentions includes the whole period the British call the ‘Civil War’ (1642-1651). In 1640 the Scottish troops twice defeated the English. King Charles I (of the Stuart dynasty) was a Scotchman and wanted to increase taxing and make certain changes in the religious sphere that aimed to achieve a greater religious unity between Scotland and England. But he was decapitated in 1649 and then came the dictatorship (‘Protectorate’) of Cromwell. This period ended with the Restoration of the Stuarts with Charles II. The revolution of 1688, called ‘The Glorious Revolution’ by the British, was produced by means of an invasion by the Protestant William of Orange of Holland in alliance with an important group of Englishmen who wanted to get rid of the Catholic King James (a Stuart), whose daughter Mary was married to William. William and Mary thus became the last British monarchs of the Stuart dynasty.
economy” (Contribution, MECW 29, 265).

Aside from The Poverty of Philosophy (1847) which, though it had some economic content was in essence a polemical book, the first book Marx published on Economics was the 1959 Contribution. It was the product of several years of study focused mainly on political economy. He there concentrates on the study of commodities producing society and the genesis and functioning of money in the circulation of commodities. He planned to continue the publication of his research in the form of periodic installments. In the Preface he synthesizes the results to which he had arrived –after “a critical re-examination of the Hegelian philosophy of law” (part of which was published in the Yearbook) – on the conditioning role of the “material conditions of life.” His results also reaffirmed much of the content of Ideology he had written jointly with Engels some 14 years previously. But since the latter remained unpublished there was no reason for mentioning it in the Preface. Marx explains his conclusion that “neither legal relations nor political forms could be comprehended whether by themselves or on the basis of a so-called general development of the human mind, but that on the contrary they originate in the material conditions of life, the totality of which Hegel, following the example of English and French thinkers of the eighteenth century, embraces within the term ‘civil society’; that the anatomy of this civil society, however, has to be sought in political economy” (Ibid., 262). The following paragraph is the best synthesis available of his view of Historical Materialism at this stage of his investigations (a stage in which he had already written a first rough draft of a portion of Capital, i.e., Grundrisse):

In the social production of their existence, men inevitably enter into definite relations, which are independent of their will, namely relations of production appropriate to a given stage in the development of their material forces of production. The totality of these relations of production constitutes the economic structure of society, the real foundation, on which arises a legal and political superstructure and to which correspond definite forms of social consciousness. The mode of production of material life conditions the general process of social, political and intellectual life. It is not the consciousness of men that determines their existence, but their social existence that determines their consciousness. At a certain stage of development, the material productive forces of society come into conflict with the existing relations of production or –this merely expresses the same thing in legal terms– with the property relations within the framework of which they have operated hitherto. From forms of development of the productive forces these relations turn into their fetters. Then begins an era of social revolution. The changes in the economic foundation lead sooner or later to the transformation of the whole immense superstructure. In studying such transformations it is always necessary to distinguish between the material transformation of the economic conditions of production, which can be determined with the precision of natural science, and the legal, political, religious, artistic or philosophic –in short, ideological forms in which men become conscious of this conflict and fight it out. Just as one does not judge an individual by what he thinks about himself, so one cannot judge such a period of transformation by its consciousness, but, on the contrary, this consciousness must be explained from the contradictions of material

\[10\] The word ‘commodity’ in Marx refers to goods (or services) that are produced to be sold in a market. It does not have the modern connotation of highly homogeneous goods that can be standardized.
life, from the conflict existing between the social forces of production and the relations of production (Contribution, MECW 29, 263).

Leaving aside small changes in terminology (such as “mode of production of material life conditions” instead of “mode of production and form of intercourse”) and a increased clarity in the exposition, it seems clear that Marx had not substantially modified his ‘materialist conception of history’ from the time of The German Ideology.

During the years following the publication of Contribution Marx dedicated himself intensely to the writing of Capital. An important part of that process consisted in furthering his studies of political economy, seeking in particular the specific differences between the theory he was elaborating and the theories of the economists that preceded him. But his readings were much broader. For example, a book that strongly impacted both him and Engels was Darwin’s The Origin of Species by means of Natural Selection or, The Preservation of Favoured Races in the Struggle for Life (published in 1859). In a letter to Engels (of December 19, 1860) he writes: “this is the book which, in the field of natural history, provides the basis for our views.” Shortly later he writes to Lassalle (January 16, 1861) that “Darwin’s work is most important and suits my purpose in that it provides a basis in natural science for the historical class struggle... Despite all shortcomings, it is here that, for the first time, ‘teleology’ in natural science is not only dealt a mortal blow but its rational meaning is empirically explained.” But Marx did not mechanically use any part of Darwin’s theory. It merely served to verify that there were certain analogies between the field of biological evolution and the field of human history.11

In Chapter 15 of Book I of Capital (on “Machinery and Modern Industry”), where Marx seeks to investigate “how the instruments of labour are converted from tools into machines,” he manifests the need for a “critical history of technology,” since “Technology discloses man’s mode of dealing with Nature, the process of production by which he sustains his life, and thereby also lays bare the mode of formation of his social relations, and of the mental conceptions that flow from them.” And he makes an analogy between tools and machines and the organs of plants and animals: “Darwin has interested us in the history of Nature’s Technology, i. e., in the formation of the organs of plants and animals, which organs serve as instruments of production for sustaining life. Does not the history of the productive organs of man, of organs that are the material basis of all social organisation, deserve equal attention?” Marx here finds in Darwin’s theory an opportunity to find analogies to his ‘materialist conception of history’. As in the case of plants and animals, the study of human society also has a fundamental technological dimension. And the latter goes beyond the biological fact that men have legs and arms which allow him to walk and grasp objects, since there is a cultural dimension that explains the transmission from generation to generation of the pre-existing and the newly acquired technological knowledge, in a cumulative process that involves education. Marx highlights the acquired level of technological knowledge as a conditioning factor for the production process and for the relations that the individuals involved in that process adopt. And, through that mediation, technological knowledge is also a conditioning factor (and a source of inspiration) for the ideas and ideological representations that men generate in any given time period. In the present case Marx centers on religion, stating that “the only materialistic, and therefore the only scientific” method of writing histories of religions is “to develop from the actual relations of life the corresponding celestialised forms of those relations” rather than, conversely,

11 This contrasts with the quite mechanical application that some modern ‘evolutionists’ in the field of Economics make when they use mathematical models developed for Biology.
to “discover by analysis the earthly core of the misty creations of religion.”

It was very difficult for Darwin to challenge the religious beliefs of his contemporaries, even at the personal level since his wife—as David Ricardo decades earlier—was affiliated to the Unitarian version of Protestantism. The great majority of British scientists took the biblical tales of Creation and Noah’s Ark literally and experienced confusion as more and more caverns were discovered with bones of defunct species, some with distinctly humanoid features. When Darwin published *The Origin of Species* he was aggressively attacked by the Church. And the attacks became more virulent when in 1870 he published *The Descent of Man, and Selection in Relation to Sex*, in which he dared to state that, far from having been created by God, the human species had evolved from the predecessors of existing monkeys. In the scientific community there was a very gradual expansion of the range of those who accommodated their beliefs to a theoretical structure that explained the biological evidence better than those that preceded it (as that of the French naturalist Lamarck (1744–1829)) and, of course, than the tale of Creation of the Old Testament. But the new ideas gradually gained consensus, something that certainly never occurred with the challenge posed by Marx. Marx did not face a comparable difficulty at the personal level when he posed his theoretical challenge, but his conception of how social scientists should theorize found multiple obstacles, partly because, as in the case of Darwin, it broke with the prevailing ideas, but especially because, in contrast with Darwin, Marx not only used his new conception of history to develop his theory of Capitalism but also to give foundations to a political project that aimed to radically change the established order.

Darwin’s ideas were abominable for theologians and for the great mass of believers in the predominant religions, but they began to gain more and more support among scientists who began to make way for this theory in their worldview. It took much longer to gain acceptance among the Churches and their practitioners. But Marx and Engels’ project of applying their ‘materialistic’ scientific notions to better the capacity of the working class to better their situation by negatively affecting the interests of the capitalist entrepreneurial class and, to top it all, with the aim of eventually taking over political power and expropriating them altogether, was quite another thing. Even if Marx and Engels’ political project gained many followers among the working class and leftist militants and intellectuals, those who understood that it was meant to harm their economic interests certainly did not accept its ideological foundations, and such people had a predominant influence on the state and on the established academic community. Even among those who wanted to study the economy from a more or less scientific stance, the majority was in one way or another linked to the dominant class and was hence highly motivated to demonize the new conception *in toto*. Marx was conscious of the nature of the impediments to a generalized acceptance of his theory. In the Preface to the first edition of *Capital* (1867) he wrote: “In the domain of political economy, free scientific inquiry meets not merely the same enemies as in all other domains. The peculiar nature of the material it deals with, summons as foes into

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12 When Engels gave a speech at Marx’s funeral (in March 1883), he made a parallel between the discoveries of Marx and Darwin: “Just as Darwin discovered the law of development of organic nature, so Marx discovered the law of development of human history: the simple fact, hitherto concealed by an overgrowth of ideology, that mankind must first of all eat, drink, have shelter and clothing, before it can pursue politics, science, art, religion, etc.; that therefore the production of the immediate material means of subsistence and consequently the degree of economic development attained by a given people or during a given epoch form the foundation upon which the state institutions, the legal conceptions, art, and even the ideas on religion, of the people concerned have been evolved, and in the light of which they must, therefore, be explained, instead of vice versa, as had hitherto been the case” (MECW 24, 467-8).
the field of battle the most violent, mean and malignant passions of the human breast, the Furies of private interest” (B1, 9). The strength of the self-interest of those who analyze economic, sociological, or political matters, which is inevitably related to the interests of those whom they depend on to earn a living, has not significantly changed since Marx’s death almost 140 years ago.

What Marx did not elucidate, perhaps for political reasons, is that there was no necessary relation between his interpretation of history and his concrete political project, that is, his political proposal of introducing very radical changes in the basic structure and functioning of society with the aim of obtaining a better functioning and more just society, starting with the seizure of political power by an organization representing the working class. This lack of a necessary relation between the two, and indeed the unsurmountable difficulties and risks Marx’s project involved, was increasingly perceived by reformist socialists of the most advanced countries a few decades later. And what was valid of Marx’s scientific works was either mostly ignored by those who were hostile to his project or increasingly distorted by those who continued to (non critically) vindicate aspects of his works (and especially of his political project) that were increasingly called into question by a reality (inevitably) more complex than the analytic and forecasting abilities of Marx. This complexity, of course, also exceeded the capacity of any other researcher of the time period during which Marx wrote his works. However, there were thinkers of lesser intellectual caliber than Marx that gradually became aware that the political project of Marx and Engels was extremely risky for the working class. According to the anarchist Bakunin, if that project succeeded it would lead to a new kind of fierce dictatorship and not to the democracy of associated workers Marx had in mind. And according to the (bourgeois) reformist John Stuart Mill, who sympathized with various aspects of the socialist cause, prudence should prevail against the risk that a project like that of Marx and Engels, far from correcting many of the deficiencies of Capitalism, could aggravate them. But we must leave these matters at this point. As we have already anticipated, we have chosen to leave the analysis of Marx and Engel’s political project to Part IV of this book in order to concentrate on the nomothetic aspects of Marx’s application of Historical Materialism to Capitalism, i.e., on his search for regularities and tendencies in the development of the capitalist phase of human history.

We tend to agree with anthropologist Marvin Harris (1968) that: “Marx formulated a scientific principle at least as powerful as Darwin’s natural selection – a general principle that showed how a science of human history might be constructed” (Harris 1968, 219). We also agree with his statement, closer to the strictly anthropological field, that Marx and Engels’ “formulation of the principles of cultural evolution was intended as a contribution to the explanation of cultural differences and similarities with respect to all cultural types. It is in this regard that their contribution was strictly analogous to Darwin’s principle of ‘natural selection’ – as an explanatory principle, applicable not just to one phylum or genus, but to the evolution of all life forms” (Ibid., 229). As Harris states, Historical Materialism constitutes a ‘general principle’, not the application of this principle to any specific historical case. The concrete case that Marx did develop, with many successes and some important failures, was his theory of the genesis and functioning of Capitalism, and its tendencies. This will be the focus of our attention for the greater part of this book. But one can agree or not on the correctness of the application of this principle (or research strategy) to any concrete case by any particular researcher without this invalidating the principle or research strategy as such.

Marx conceived the study of human society as a ‘science of history’. The diachronic
reconstruction of human affairs required an adequate understanding of the synchronic functioning of human society. The latter required the study of the reciprocally conditioning relations between the technological sphere (the ‘productive force’ of human labor), the sphere of the social structure understood as the socioeconomic relations between the individuals that compose the social classes (starting from a certain level of development of social life in which ‘stratification’ begins) and, lastly, the spheres of political organization and construction of various ideological expressions: art, religion, philosophy, etc. Marx held that the way these reciprocally conditioning relations had been understood in the past was erroneous, that it was vitally important to understand that the technological sphere conditioned both the social structure and the ideological superstructure, while at the same time the social structure conditioned the production of ideas. Such conditioning relations were asymmetric but not unidirectional since, for example, the principle did not exclude the possibility that a certain social structure favor technological changes, or that a certain ideology favor changes in the social structure. Only by means of the study of concrete cases (in space and in time) could such conditioning relations, asymmetries, and feedbacks, be made precise. That is why Marx placed such emphasis on the importance of empirical data and why they are so abundant in Capital.

As Harris correctly understood, Historical Materialism should be considered a strategy for addressing the empirical data and not a precise method, and this strategy is based on the assumption that it is more probable to find causal relations form the technological to the social structure than the reverse, or from the social structure to the production of ideas than the reverse. Marx had to rebel against the different view (particularly the Hegelian view predominant in Germany) that sought to explain ideas by means of previous ideas without leaving the sphere of ideas. And he also had to confront the many idealistic and romantic historians of his time that explained the events of an epoch on the basis of the ‘spirit’ of that epoch and the acts of exceptional individuals.13

The asymmetric and hierarchical class relations

When Marx observes the historical development of human society he is alert to the different ways in which production is organized. Although in every concrete society there are always different ‘modes of production’, there tends to be one that predominates. Each mode of production is defined by a pattern of social relations that fundamentally rest in the –usually asymmetric and hierarchical– relations that individuals establish between them in the production process. And Marx finds that there is a basic thread that connects the succession of predominant modes of production: the progressive development of the productive force of human labor, i.e., the increasing capacity of man in society to transform the products of nature to his advantage. The concept of ‘productive force’ encompasses the technological knowledge that makes the transformation of the products of nature possible, the social capacity to organize the labor of various individuals, and the ability to produce increasingly elaborate instruments of production that enhance human labor. The productive force of human labor gradually

13 An example is Victor Hugo’s book Napoléon le Petit on the same events studied by Marx in Class Struggles and in Brumaire. In the Preface to the second edition of the latter Marx wrote (in 1869): “Victor Hugo confines himself to bitter and witty invective against the responsible producer of the coup d’état. The event itself appears in his work like a bolt from the blue. He sees in it only the violent act of a single individual. He does not notice that he makes this individual great instead of little by ascribing to him a personal power of initiative such as would be without parallel in world history” (MECW 21, 56-7).
increased through history but the process was not unidirectional since it had advances and sometimes retreats. It was the result of key technological or organizational innovations produced within certain communities and the interaction between communities, both pacific and bellicose, that generate the diffusion of the new methods and sometimes the loss of the productive force of labor (e.g. when a more advanced society is conquered by another that is more primitive). The possibility of retreat was exemplified by Marx and Engels by the case of the inventions of the Phoenicians: “That even with a relatively very extensive commerce, highly developed productive forces are not safe from complete destruction, is proved by the Phoenicians, whose inventions were for the most part lost for a long time to come through the ousting of this nation from commerce, its conquest by Alexander and its consequent decline” (Ideology, MECW 5, 67).

To survive, every society needs to reproduce in time its individual and group components. For this it must be able to consume ‘means of subsistence’ which must first be produced (which includes hunting, fishing, and gathering in the more primitive communities). And production requires the combination of workers and means of production, which may occur in different ways that for Marx define the socioeconomic structure of society. This structure must be coherent with the existing level of development of the productive force of labor: “Whatever the social form of production, labourers and means of production always remain factors of it. For production to go on at all they must unite. The specific manner in which this union is accomplished distinguishes the different economic epochs of the structure of society from one another” (B2, 41).

According to Marx, ever since societies stratified in social classes exist, there is a “relationship of rulers and ruled” in the process of production, where the latter constitute a class of “direct producers” that must produce for their own consumption but also a surplus that is appropriated by the members of the dominant or ruling class (and possibly ultimately also by other classes and subclasses that provide various services to the dominant class) because it exercises dominion over certain key conditions of production (such as the land, other natural resources, and other means of production). The specific form in which the surplus (which is always ‘unpaid labor’) is obtained constitutes for Marx “the hidden basis of the entire social structure”:

The specific economic form, in which unpaid surplus labour is pumped out of direct producers, determines the relationship of rulers and ruled, as it grows directly out of production itself and, in turn, reacts upon it as a determining element. Upon this, however, is founded the entire formation of the economic community which grows up out of the production relations themselves, thereby simultaneously its specific political form. It is always the direct relationship of the owners of the conditions of production to the direct producers – a relation always naturally corresponding to a definite stage in the development of the methods of labour and thereby its social productivity – which reveals the innermost secret, the hidden basis of the entire social structure, and with it the political form of the relation of sovereignty and dependence, in short, the corresponding specific form of the state (B3, 777-8).

Far from having a unilinear or deterministic conception of history, Marx ends the preceding paragraph emphasizing the multilinear character of cultural development when he states that the same ‘economic basis’ can show ‘infinite variations’ in its concrete manifestations according to circumstances: “This does not prevent the same economic
basis—the same from the standpoint of its main conditions—due to innumerable different empirical circumstances, natural environment, racial relations, external historical influences, etc., from showing infinite variations and gradations in appearance, which can be ascertained only by analysis of the empirically given circumstances” (B3, 778). And for Marx the accelerated process of technological advance and destruction of old modes of production that distinguished the last centuries of human history from previous epochs were a manifestation of the specifically capitalist way of combining “labourers and means of production”, i.e., of organizing production by means of the ‘exploitation of salaried workers’ by entrepreneurs that own or have access to ‘capital’.14 For “only the capitalist production of commodities has become an epoch-making mode of exploitation, which, in the course of its historical development, revolutionises, through the organisation of the labour process and the enormous improvement of technique, the entire economic structure of society in a manner eclipsing all former epochs” (B2, 43).

In each stage of historical social development, characterized by a certain level of development of the ‘productive force’ (or productivity) of labor, certain social relations crystallize in the process of production. These ‘production relations’ depend on the division of labor and on the assumption of certain key social functions by certain groups of individuals, functions that are related to their control (or lack of control) over certain material conditions of the labor process (such as land, water, buildings in which production is carried out, the tools and materials used, etc.). For example, in European (or Japanese) feudal society there were the relations between the serfs who toiled the land (and were legally tied to it) and the noble landowner who had control over the land and, hence, over the serfs. In the capitalist society that grew out of the bowels of feudal society there were the production relations between free wage workers that lacked the economic means to acquire means of production and work independently, and capitalist entrepreneurs that did have that capacity and also the capacity to purchase the labor power to combine with the means of production.

The production relations in each concrete society are cemented by means of customs, ideology, and the law, thus gaining a certain resistance to change, and conforming the division of society into social classes, each one of which includes the individuals that are similarly inserted in the productive process and in the process that deals with the circulation of commodities, and even outside of these (such as the servants that work in the homes of the well to do).15 And as long as the social structure formed by these production relations allow the subsistence of society and do not fetter its progress—which normally includes some population growth— it is reproduced over time. However, as societies develop their productive forces (either directly by means of their own innovations or indirectly by importing technology and organizational forms), history shows that eventually a situation is reached in which the existing relations of production cease to be functional to the maintenance (and growth) of large portions of society and, on the contrary, become impediments to social development. At such historical junctures large social and economic transformations take place, often accompanied by political revolutions and internal and external wars which (in the best case) end by adapting

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14The precise meaning that Marx gives to ‘capital’ will be studied in Part II of this book.
15In modern anthropological terminology the term ‘stratification’ is used instead of ‘division in social classes’ but the idea is the same. Classical Political Economy openly analyzed questions pertaining to the distribution of income in terms of a hierarchy of social classes. But modern Economics has tended to ignore (and conceal) as much as possible the hierarchical stratification of society by using ‘firms’ as a basic agent instead of the persons that compose them and the State as an impersonal entity that raises taxes and spends on public goods thus leaving the matter entirely to Sociology and Political Science.
the structure of production relations to the needs of ulterior social development.

In Marx’s historical horizon there existed before the formation of social classes in the remote past a ‘primitive’ Communism. And he perceived in the future a superior form of Communist society. Contrasting with (the very long) period of ‘primitive’ Communism, all the following known forms of society were divided in social classes structured hierarchically by power and authority. Marx distinguished within the multiplicity of social classes of any concrete society, the ‘big classes’ that defined the basic structure of the predominant mode of production. In the feudal society of the early Middle Age, for example, there were the two big classes of peasant serfs and landowning nobility. All the serfs had in common their being bound to the soil and having the obligation to work the lands of the feudal noble, and the members of the nobility had in common that they controlled a certain extension of land and had the right to require that the enserfed peasants produce a surplus.

In Marx’s perspective, class societies have one class that is dominant in the sense that it exerts control over the material conditions necessary for the realization of the production process. This class normally has political hegemony through its preferential access to the state apparatus, but its ultimate power base lies in its economic role. Members of the dominant class often entertain functions in government (i.e., the king and his court in the case of feudal Europe), including military functions. But for class society to be possible the productivity of labor has to be sufficiently high that the workers generate a surplus product beyond their own consumption, which also grows along with economic-social development and the increase in the productive force of labor. The surplus created by the working class sustains the dominant class (and its subsidiary classes) and its activities in the political, military, administrative, and cultural spheres (including religion, science, and arts).

According to Marx this appropriation of surplus was quite evident in precapitalist societies such as those based on slavery or serfdom. For it was based on the direct submission of the workers and the consequent limitation in their rights in comparison with those of other classes that did not function within the production process, and especially in comparison with the dominant class. For Marx it was a characteristic of capitalist society that the phenomenon of exploitation was mystified by the fact of the personal freedom of the wage worker and his voluntary participation in the productive process through the recurrent sale of his labor power (i.e., his capacity to generate labor during a certain period of time). But lifting this veil through analysis it became clear that, as in previous modes of production, wage workers produced more than sufficient for their maintenance and that the surplus was appropriated by the owning classes. As in previous modes of production, in the capitalist mode of production there prevailed an asymmetric and hierarchical relation between workers and capitalists.

However, in contrast with previous modes of production in which the “strictly regulating authority” emanated from the “political or theocratic rulers”, in Capitalism this authority reached its bearers “only as the personification of the conditions of labour”, i.e., their property of the capital invested in industrial, commercial, and banking enterprises. And in contrast with the ‘complete hierarchy’ within the factory, the commercial establishment, etc., there was a ‘complete anarchy’ outside, in the competition between capitalists where “the social interrelations of production assert themselves only as an overwhelming natural law in relation to individual free will.” The following paragraph of the second to last chapter of Capital expresses these ideas:

The authority assumed by the capitalist as the personification of capital in the direct process of production, the social function performed by him in
his capacity as manager and ruler of production, is essentially different from the authority exercised on the basis of production by means of slaves, serfs, etc. Whereas, on the basis of capitalist production, the mass of direct producers is confronted by the social character of their production in the form of strictly regulating authority and a social mechanism of the labour process organised as a complete hierarchy—this authority reaching its bearers, however, only as the personification of the conditions of labour in contrast to labour, and not as political or theocratic rulers as under earlier modes of production—among the bearers of this authority, the capitalists themselves, who confront one another only as commodity owners, there reigns complete anarchy within which the social interrelations of production assert themselves only as an overwhelming natural law in relation to individual free will (B3, 867-8).

After Marx moved to London one of his goals was to understand the functioning of the “overwhelming natural law” that was intimately related to Adam Smith’s ‘invisible hand’. To understand how markets functioned, what factors explained the exchange values of commodities, how money was generated as a peculiar commodity and capital as a peculiar way of using money. For this he undertook a detailed study of the available works of Political Economy. He reached the conclusion that the ‘production relations’ of Capitalism are different both from the symmetric and non-hierarchical production relations in the Simple Commodity Production (SCP) that independent producer/workers engage in (as the artisans of towns who were free from the fetters of the guilds and the peasants who were free of feudal bonds—yeomen) to then sell their products in the market, as well as from the asymmetric and hierarchical relations of production in the Asiatic, Slave, or Feudal modes of production. In a monetary society with a mode of production based on slavery, for example, money can be used to purchase slaves because the institution of slavery so allows. In Marx’s words: “The purchase and sale of slaves is formally also a purchase and sale of commodities. But money cannot perform this function without the existence of slavery. If slavery exists, money can be invested in the purchase of slaves. On the other hand the mere possession of money by a buyer cannot make slavery possible” (B2, 38). Similarly, a society of independent producers that sell their goods in markets and in which the institution of wage labor does not exist would be a mercantile (commodity-producing) society but not a capitalist society.

Models of Simple and Capitalist Commodity Production

Marx develops an elaborate model of a society of independent commodity producers/workers in where there is no wage labor nor cooperation in the work process, i.e., a model of Simple Commodity Production (SCP), as a stepping stone for the construction of a model of pure Capitalist Commodity Production (CCP).\textsuperscript{16} In SCP many commodities are bought and sold by means of one of the commodities that has the role of being money. But labor power is not one of them since this model represents a society in which the institution of wage work does not exist. Marx tells us that certain historical conditions had to be met for this institution to appear and propagate. In his theory, the capitalist mode of production is the most evolved form of commodity

\textsuperscript{16}The reason we use the adjective ‘pure’ here is that there are many instances in Marx’s analyses in which he combines more than one mode of production. This can in principle be reflected in a corresponding model, as we do for example in Chapter 16 where we have a model that combines CCP with SCP, whereby CCP is not pure.
production, i.e., a mode of production in which produced goods (and services) have to be sold in a market before being able to satisfy their social goal of consumption (human or productive). What is specific of CCP is that the labor power of workers is also a commodity that must be sold before it can be consumed in production by means of a labor process that is controlled by a capitalist entrepreneur and in which many workers cooperate.

The *pure* SCP and CCP models are based on corresponding historical realities, although the historical SCP only existed in a very limited way. According to Marx’s theory, after a long period (involving many centuries) in which the profit motive predominated in the spheres of commerce and banking (or usury), i.e., in which ‘capital’ (as money that is disbursed with the goal of retrieving it engrossed by profit) dominated these spheres while production was still done in a non-capitalist way (either as commodity or non-commodity production), CCP started to develop, that is, production in which wage laborers were contracted by a capitalist entrepreneur in an asymmetric and hierarchical way that was inexistent in SCP. Both SCP and CCP are commodity-producing societies. But in the latter case it is the capitalist entrepreneur that has the control over the production process, purchases the needed inputs (among these, labor power), and owns the resulting product which he sells.

The process of *circulation* of commodities and commercial and financial capital had existed for thousands of years before the emergence of the capitalist mode of production. Marx gave great importance to the relations established by people in the social process, especially if they performed different functions in the economic processes, which includes the production and the allocation (or distribution) of commodities. This allocation, when we refer to commodities, is done through the process of circulation, where money and the concept of value play a fundamental role. A considerable proportion of the products produced had been converted to commodities thousands of years ago, i.e., were sold in markets. Commercial and financial (or loan) capital had hence generated large fortunes many centuries before the rise of the capitalist mode of production, which implied the extension of the profit motive to the sphere of production as well as the generalization of the institution of wage labor. Marx writes:

> not commerce alone, but also merchant’s capital, is older than the capitalist mode of production, is, in fact, historically the oldest free mode of existence of capital... Since merchant’s capital is penned in the sphere of circulation, and since its function consists exclusively in promoting the exchange of commodities, it requires no other conditions for its existence –...– outside those necessary for the simple circulation of commodities and money. Or rather, the latter is the condition of its existence... The extent to which products enter trade and go through the merchants’ hands depends on the mode of production, and reaches its maximum under the full development of capitalist production, where the product is produced solely as a commodity, and not as a direct means of subsistence (B3, 323).

For Marx only when capitalist industrial production expanded strongly –gradually substituting (commodity and non-commodity producing) pre-capitalist modes of production, did production for the markets become generalized. The very dynamics of capitalist production, where the profit motive is predominant, gradually destroys (with varying speeds among countries, whether kingdoms, principalities or empires) the pre-commodity modes of production (such as the self-subsisting feudal mode of production) as well as the pre-capitalist commodity production based on slave or serf labor or on
the own labor of free peasants and town artisans. In particular, the increase in productivity achieved by capitalist entrepreneurs rent obsolete these more primitive social forms of production that sooner or later were forced to either convert to capitalist production or become extinct. In Marx’s words:

... the same conditions which give rise to the basic condition of capitalist production, the existence of a class of wage workers, facilitate the transition of all commodity production to capitalist commodity production. As capitalist production develops, it has a disintegrating, resolvent effect on all older forms of production, which, designed mostly to meet the direct needs of the producer, transform only the excess produced into commodities. Capitalist production makes the sale of products the main interest, at first apparently without affecting the mode of production itself. Such was for instance the first effect of capitalist world commerce on such nations as the Chinese, Indians, Arabs, etc. But, secondly, wherever it takes root capitalist production destroys all forms of commodity production which are based either on the self-employment of the producers, or merely on the sale of the excess product as commodities. It first makes the production of commodities general and then, by degrees, transforms all commodity production into capitalist production (B2, 42).

These processes occurred with very uneven speeds in different countries, empires, or colonies, and had much to do with the social conflicts, civil wars, and international wars within and among these societies.

Two of the principle defects Marx detected in “apologetic economics” were “the identification of the circulation of commodities with the direct barter of products, by simple abstraction from their points of difference” and “the attempt to explain away the contradictions of capitalist production, by reducing the relations between the persons engaged in that mode of production to the simple relations arising out of the circulation of commodities” (B1, 124). He points out that “The production and circulation of commodities are, however, phenomena that occur to a greater or less extent in modes of production the most diverse.” In particular, in ancient Rome the circulation of commodities was extremely developed, as well as the accumulation of huge fortunes. But the fact that the institution of slavery predominated was an impediment for the development of the capitalist mode of production. In contrast, when the feudal mode of production began to be subverted by the emergence of burgs where free workers concentrated, some immersed in guilds and others not –especially runaway serfs from the rural areas–, the conditions for the rise of capitalist (or bourgeois) production began to predominate.

One of the important achievements of Marx’s work is to very effectively concentrate on the historic-genetic process of the emergence and gradual imposition and generalization of the capitalist institutions, which encompasses both its genesis starting with pre-capitalist modes of production and circulation as well as its progressively predominant dynamics through the gradual destruction of subsisting pre-capitalist modes of production. Although in Marx’s time this regime was already predominant in the most advanced countries, especially England, it still coexisted and linked with pre-capitalist modes of production, both within those countries as well as through trade with other countries and the colonial subjection of the most backward societies. In Russia, in particular, the czar only nominally liberated more than 20 million serfs in 1861, after its humiliating defeat in the Crimean War (1853-1856) made the ruling class understand the practical consequences of having backward institutions. The effective liberating
process, however, lasted at least two more decades (Hellmann et al., 1975), especially because the serfs had to purchase their own freedom, which was in most cases virtually impossible. Another example of the enormous interlinking of modes of production in Marx’s time is the case of the U.S.A. before the Civil War (1861-1865), where a predominantly capitalist North combined with a predominantly slavery-based South that exported cotton to be processed by England’s textile industry. The emancipation of around 4 million slaves was announced in January 1863 as a war tactic by the North and was made effective after the South’s defeat. Their integration to the revolutionized economic life, however, took decades of famines and miseries by a population that was freed from slavery but not from the need to eat for survival. Such huge social events were taking place during the period in which Marx was writing Capital.

Although the Neoclassical economic theory that developed after the ‘Classics’ and after Marx produced very important advances in the formalization of many of the characteristics of the functioning of a ‘purely’ capitalist economic system, it was impoverished in its breadth by completely leaving aside most of the richness of the historical—economic and social—material that was so present in the theories of the ‘Classics’ and Marx. This impoverishment was parallel to the simultaneous process of unconnected specialization of the different disciplines that emerged out of the more holistic social science that existed in the era of Classical Political Economy. Thus emerged the present disciplines of Economics, Sociology, Political Science, Anthropology, etc., which became almost watertight compartments unable to provide a holistic vision of the functioning of human society. This book aims to discern to what extent Marx was able to shed light on the functioning of the capitalist society of his time, to what extent he was mistaken and why, and to give some hints on how certain aspects of his theory could be modified in order to have a reasonable representation of some of the main features of Capitalism even taking into account the century and a half of additional history that has since elapsed.

Marx’s methodology

The method of research and the exposition of results

For Marx it was important to distinguish between the method of research and the exposition of research results. In the Afterword (of 1873) to the Second German edition of Book I of Capital he writes: “the method of presentation must differ in form from that of inquiry. The latter has to appropriate the material in detail, to analyse its different forms of development, to trace out their inner connection. Only after this work is done, can the actual movement be adequately described. If this is done successfully, if the life of the subject-matter is ideally reflected as in a mirror, then it may appear as if we had before us a mere a priori construction” (B1, 20). Since his research object was fundamentally the Capitalism of his time, the research had to begin with a thorough knowledge of basic historical information on the origins, the functioning, and the development in time of capitalist society. The writings of the political economists were a fundamental part of the raw material of research. He set out to subject the economic categories they used to a critical analysis that would lead him to forge the theoretical terms with which he would be able to study capitalist society from the perspective of his Historical Materialism. To expose the laws of movement of that society he had to ideally –i.e. theoretically– reconstruct its structure and functioning by means of adequate economic categories. In his view the exposition of results should reconstruct the concrete and complex starting from abstract and simple categories. Hence, matters like the state, international trade, or the world market could only be dealt with dealing
with labor, the division of labor, exchange values, and the control over the means of production. These last categories were ‘abstract determinations’ and ‘simple concepts’. The state, international trade and the world market, in contrast, were ‘concrete’, because they were “a synthesis of many determinations” which appeared in thought “as a process of summing-up, as a result, not as the starting point.” In the *Introduction* he sketched for the *Contribution* and later decided to omit because it would be “confusing to anticipate results which still have to be substantiated” (MECW 29, 265) – especially since that publication did not even address his concept of ‘capital’ –, he states that “The 17th-century economists, for example, always started with the living whole, the population, the nation, the State, several States, etc., but analysis always led them in the end to the discovery of a few determining abstract, general relations, such as division of labour, money, value, etc. As soon as these individual moments were more or less clearly deduced and abstracted, economic systems were evolved which from the simple [concepts], such as labour, division of labour, need, exchange value, advanced to the State, international exchange and world market. The latter is obviously the correct scientific method” (MECW 28, 37-8; text within square brackets introduced by the editors).

Marx distinguished two ways of intellectually appropriating reality: in one empirical data were recollected: “It would seem right to start with the real and concrete, with the actual presupposition, e.g. in political economy to start with the population, which forms the basis and the subject of the whole social act of production” (Ibid.). But this would be a unilateral way of representing the way in which knowledge of social reality is produced. For “Population is an abstraction if, for instance, one disregards the classes of which it is composed. These classes in turn remain an empty phrase if one does not know the elements on which they are based, e.g. wage labour, capital, etc.” (Ibid.). Following this course, “one would arrive analytically at increasingly simple concepts; from the imagined concrete, one would move to more and more tenuous abstractions until one arrived at the simplest determinations.” This was the course “taken by political economy historically at its inception”, i.e., the economists of the 17th century. But “the correct scientific method” is two-way. Since if the simple categories forged by political economists after observing the recollected data in detail are complemented with new observed data and new and appropriate categories, one can reconstruct the concrete starting from simplest, most abstract, determinations. Since the concrete “is a synthesis of many determinations... it therefore appears as a process of summing-up, as a result, not as the starting point” (Ibid., 38). The course the scientist should follow was hence always two-way. There being no neutral, unbiased way of observing and analyzing ‘the data’, he should always start from categories that society has previously forged. But in the *exposition* of the results it is best to follow the second course, elaborating the more concrete categories from the more abstract ones.

On the other hand, the second course was problematic if one did not part from the materialist view of history. In his absolute idealistic view, Hegel had “arrived at the illusion that the real was the result of thinking synthesising itself within itself, delving ever deeper into itself and moving by its inner motivation”, whereas “actually, the method of advancing from the abstract to the concrete is simply the way in which thinking assimilates the concrete and reproduces it as a mental concrete. This is, however, by no means the process by which the concrete itself originates” (Ibid.). In what concerns the scientific method of addressing the study of Political Economy, Marx returns to his criticism that in Hegel’s view “the movement of categories appears as the real act of production –which unfortunately receives an impulse from outside– whose
result is the world.” However, “the concrete totality... is by no means a product of the self-evolving concept whose thinking proceeds outside and above perception and conception, but of the assimilation and transformation of perceptions and images into concepts... [it] is a product of the thinking mind, which assimilates the world” (Ibid.). In contrast with “the artistic-, religious- and practical-intellectual assimilation of this world”, in the scientific-theoretical way of assimilating the world the “real subject remains outside the mind and independent of it... Hence the subject, society, must always be envisaged as the premise of conception” (Ibid., 38-9). Therefore, “Individuals producing in a society –hence the socially determined production by individuals– is of course the point of departure” and the ‘Robinsonades’ such as “The individual and isolated hunter and fisherman, who serves Adam Smith and Ricardo as a starting point” are “unimaginative fantasies”, as is “Rousseau’s contrat social, which by means of a contract establishes a relationship and connection between subjects that are by nature independent” (Ibid., 17).

It is in the light of these ideas that Marx’s statement in the Afterword to the Second Edition of Capital should be understood: “My dialectical method is not only different from the Hegelian, but is its direct opposite”, as well as his critique of political economists and Rousseau for ignoring that “society, must always be envisaged as the premise.” For “Man... is not only a social animal, but an animal that can isolate itself only within society” (Ibid., 18). Hence, for Marx “when we speak of production, we always have in mind production at a definite stage of social development, production by social individuals (Ibid., 23). For his research project it was necessary to “declare at the very beginning that we are dealing with one particular historical epoch... modern bourgeois production, which is indeed our real subject-matter” (Ibid.).

The ‘dialectical method’

Marx conceived of world society in any given historical period as the juxtaposition of various concrete communities interlinked (starting from some very remote times) by trade (and, later, also be finance) as well as conflict (wars, conquest, colonies, etc.). And in each historical juncture each individual community had a dominant mode of production: “In every form of society there is a particular [branch of] production which determines the position and importance of all the others, and the relations obtaining in this branch accordingly determine those in all other branches” (MECW 28, 43). In particular, “In all forms in which landed property rules supreme, the nature relationship still predominates” whereas “in the forms in which capital rules supreme, the social, historically evolved element predominates” (Ibid., 44)). The relations between individuals involved in production (which corresponded to a certain level of development of the forces of production) defined the ‘mode of production’, and there was one that predominated in each period. But each society also had other, subordinated, modes of production, some of which were residual from earlier periods and others that were vestiges of their possible future evolution. Accordingly, he held that the theoretical exposition of the predominant economic relations in a society had to differ from the historical exposition. For in the theoretical construction the categories had to appear, not in the order of their historical appearance, but in the order of importance they had in the dominant mode of production: “It would therefore be inexpedient and wrong to present the economic categories successively in the order in which they played the determining role in history” (Ibid.). For societies in which Capitalism predominates “Rent cannot be understood without capital, but capital can be understood without rent. Capital is the economic power that dominates everything in bourgeois society. It
must... be analysed before landed property... Their order of succession is determined rather by their mutual relation in modern bourgeois society, and this is quite the reverse of what appears to be their natural relation or corresponds to the sequence of historical development” (Ibid.).

This view determined the way Marx organized the structure of Capital. It led him, in particular, to relegate to a second level any consideration of the classes and modes of production subordinate to the capitalist mode of production and concentrate on the “three big classes of modern society based upon the capitalist mode of production”: wage workers, capitalists and landowners, who respectively owned labor power, capital, and land, and “whose respective sources of income are wages, profit and ground rent” (B3, 870). And since landowners had the limited role of rentiers in advanced Capitalism, he could introduce the private property of land and landowners after having given a thorough treatment to wage work and capital and their mutual relations. Furthermore, capital was the “dominant category”, representing the “determining production relation” (B3, 814), but had to be treated after “the simplest categories of the capitalist mode of production, and even of commodity production... commodities and money” (Ibid., 812; bold emphasis added).

Notwithstanding the primacy of the theoretical construction of the ‘production and circulation of capital’ in Capital, the wealth of empirical and historical material it contains is enormous. Approximately one half of the extension of Book I is empirical and historical. There are abundant references to pre-capitalist modes of production, both contemporary (such as the slavery-based mode of production in Southern U.S.A.) and before the emergence of Capitalism (such as the Ancient, Asiatic, and Feudal modes of production). Much of what Marx wrote on pre-capitalist modes of production remained unpublished until decades after his death, even ignoring Books II and III.17

Marx held that there were no general ‘laws’ (in the nomothetic sense) valid for all human societies, that the ‘dominant mode of production’ of each concrete society had its own ‘laws’. Possibly one of the clearest statements on his methodological view on this is the commentary of a Russian that Marx himself quotes in his Afterword to the Second German Edition of Book I of Capital. It was written in 1873, when there had already appeared some adverse comments on the Hegelian phrases used in some parts of the book. To show that there were people who interpreted his book quite correctly, Marx extensively quotes some paragraphs of the (apparently anonymous) Russian article published the year before on the methodology of Capital. According to Marx, the article satisfactorily describes his ‘dialectical method’. We here reproduce a selection of Marx’s long quotation because he certainly would not have included it in his Afterword if he had not been impressed by its accuracy:

But it will be said, [...] the general laws of economic life are one and the same, no matter whether they are applied to the present or the past. This Marx directly denies. According to him, such abstract laws do not exist. [...] On the contrary, in his opinion every historical period has laws of its own.... As soon as society has outlived a given period of development, and is passing over from one given stage to another, it begins to be subject also to other laws. In a word, economic life offers us a phenomenon analogous to the history of evolution in other branches of biology. [...] The old economists misunderstood the nature of economic laws when they likened

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them to the laws of physics and chemistry. [...] A more thorough analysis of phenomena shows that social organisms differ among themselves fundamentally as plants or animals. Nay, one and the same phenomenon falls under quite different laws in consequence of the different structure of those organisms as a whole, of the variations of their individual organs, of the different conditions in which those organs function, &c. Marx, e. g., denies that the law of population is the same at all times and in all places. He asserts, on the contrary, that every stage of development has its own law of population.... With the varying degree of development of productive power, social conditions and the laws governing them vary too. Whilst Marx sets himself the task of following and explaining from this point of view the economic system established by the sway of capital, he is only formulating, in a strictly scientific manner, the aim that every accurate investigation into economic life must have. [...] The scientific value of such an inquiry lies in the disclosing of the special laws that regulate the origin, existence, development, death of a given social organism and its replacement by another and higher one. And it is this value that, in point of fact, Marx’s book has (B1, 18-9).

Marx continues stating that this Russian author adequately described his ‘dialectical method’, which was the ‘direct opposite’ of Hegel’s. Whilst the writer pictures what he takes to be actually my method, in this striking and [as far as concerns my own application of it] generous way, what else is he picturing but the dialectic method? Of course the method of presentation must differ in form from that of inquiry. The latter has to appropriate the material in detail, to analyse its different forms of development, to trace out their inner connection. Only after this work is done, can the actual movement be adequately described. If this is done successfully, if the life of the subject-matter is ideally reflected as in a mirror, then it may appear as if we had before us a mere a priori construction. My dialectic method is not only different from the Hegelian, but is its direct opposite. To Hegel, the life process of the human brain, i.e., the process of thinking, which, under the name of ‘the Idea’ he even transforms into an independent subject, is the demiuergs of the real world, and the real world is only the external, phenomenal form of ‘the Idea’. With me, on the contrary, the ideal is nothing else than the material world reflected by the human mind, and translated into forms of thought” (Ibid., 19). Notice that this clarification by Marx is completely congruent with what Marx had written in The German Ideology 27 years earlier, making it plain that his philosophical rupture with Hegel and Philosophy in general in order to focus on his peculiar combination of social science and political activism did not change over the long gestation period of his main theoretical work. Marx explains in this Afterword that the unjust criticisms of Hegel made in Germany at the time he was writing Book I of Capital made him decide to declare himself his disciple and “even here and there, in the chapter on the theory of value, coquetted with the modes of expression peculiar to him” (Ibid.)

18 We notice here a defect Marx sometimes had in his writing when he wanted to stress a property, giving a (false) impression of reductionism. It is manifested here in the use of ‘is nothing else than’ instead of simply ‘is’. We have checked that it is also so in the German original.

19 Although Hegel disagreed with the philosophy of Spinoza, in his Encyclopaedia he defended him against the ‘dead dog’ treatment he had received by some. In an analogous fashion, though Marx disagreed with Hegel’s philosophy he defended him against “the peevish, arrogant, mediocre epigones who now talk large in cultured Germany,” and who began “to treat Hegel in same way as the brave Moses Mendelssohn in Lessing’s time treated Spinoza, i.e., as a ‘dead dog’” (Ibid.).
political posture, Marx never stopped admiring him for his erudition, for the elaborate construction of his philosophical system, and for his knowledge of universal history and the prominent role he gave to it in his works. 20

In a letter to Kugelmann (of June 27, 1870) Marx clearly expressed the intimate connection between the empirical and his ‘dialectical method’. Referring to the 1865 book by Friedrich Albert Lange The Labor Question and its Significance for the Present and the Future, which is very complimentary to Marx and makes references to his ample use of empirical data, Marx criticizes Lange for (among other things) not understanding the deep link there is between his methodology and the use of empirical information: “And what this Lange has to say about the Hegelian method and my application of the same is simply childish. First, he understands rien about Hegel’s method and, therefore, second, still less about my critical manner of applying it... Lange is naïve enough to say that I ‘move with rare freedom’ in empirical matter. He has not the slightest idea that this ‘free movement in matter’ is nothing but a paraphrase for the method of dealing with matter –that is, the dialectical method” (MECW 43, 528). Marx denominated ‘dialectical method’ a way of investigating human society that could have had quite different names but can be synthesized as a) a search for, assimilation, and analysis of a great mass of empirical data of all kind (historical, statistic, official reports, etc.), including the theories previously formulated that use them, b) the construction of various models for trying to understand the relations between certain variables deemed important, and c) the use of the data and the models for trying to discern the tendencies (which, as was usual in his time, he called ‘laws’) of society in multiple aspects.

Notwithstanding the abundant references in various literatures to Marx’s ‘dialectical materialism’, he never used this expression. Apparently it was Joseph Dietzgen, an autodidact German master tanner who was greatly influenced by the works of Marx and Engels, who first used this expression. Later Georgi Plekhanov introduced it into Russia and in 1908 Lenin used it as synonymous to “the philosophy of Marxism” in his Materialism and Empirio-criticism. Engels used very similar expressions like “a conception of nature which is dialectical and at the same time materialist” or “modern materialism is essentially dialectic” (Anti-Dühring) or “materialist dialectic” (Ludwig Feuerbach), and wrote extensively on his conception of ‘dialectics’ applied to the natural sciences. Marx, in contrast, always avoided philosophical speculation applied to the natural sciences and preferred to concentrate on the fields on which he had in-depth knowledge: human social affairs. After all, he had repudiated Philosophy as such as early as 1845. In his Misery of Philosophy (of 1847) he had criticized both Hegel’s dialectics and the incorrect use Proudhon made of it in his System of Economical Contradictions: or, the Philosophy of Misery. In a letter (of January 24, 1865) to J. B. Schweitzer Marx writes that in his book he had demonstrated how little Proudhon “had penetrated into the secret of scientific dialectics and how, on the contrary, he shares the illusions of speculative philosophy, for instead of regarding economic categories as the theoretical expression of historical relations of production, corresponding to a particular stage of development in material production, he garbles them into pre-existing eternal ideas” (MECW 20, 29). In this letter Marx calls ‘scientific dialectics’ what he would later refer to as his ‘dialectical method’ (and by ‘speculative philosophy’ he was referring to Hegel’s philosophy).

Marx had a very personal way of linking the construction of theory with the exposition of factual and historical material. And in his theoretical construction he rigorously

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20 As expressed by Karl Löwith: “Not only does Hegel’s work include a philosophy of history and a history of philosophy, but his entire system is historically oriented to an extent which is true of no previous philosophy” (Löwith 1968, 31).
used what today is called the ‘hypothetico-deductive method’. He always makes his assumptions explicit when he constructs his theory. But he also frequently interpolates examples taken from contemporary and historical socioeconomic reality. And there are whole chapters dedicated to specific historical socioeconomic matters that are supported by abundant bibliographic references. The permanent *dialogue* between the theoretical construction and the references to social reality certainly has a character that we could call ‘dialectic’ if it were not for the fact that Marx’s ‘dialectical method’ is so much more sophisticated than this simple metaphor. But Marx’s methodology was very personal and hardly possible to emulate. And because it is personal, the interlinking of the empirical-historical with the theoretical construction that is found in *Capital* does not have an objectively necessary sequencing. Different researchers could exhibit the same contents using a different interlinking. And in Part II of this book we make an exposition of the more formalizable aspects of Marx’s theory that differs in some ways from that found in *Capital* using mathematical tools that, though elementary today, were not available in Marx’s time. On the other hand, Marx’s considerable academic training was humanistic and had not put him in contact with the incipient (and completely peripheral) mathematical currents that were only beginning to develop in economic theory when he wrote the main manuscript for *Capital* in the 1860’s.\(^{21}\)

**Appendix to Chapter 2**

**Bibliographic Notes**

**Marvin Harris and his ‘cultural materialism’** Marvin Harris was a great American anthropologist who became well known with his 1968 book *The Rise of Anthropological Theory, A History of Theories of Culture*. One of its merits is that it gives ‘anthropology’ an encompassing meaning. The Introduction begins with the statement: “Anthropology began as the science of history”, and the first chapter is on the social thinkers of the Enlightenment: Locke, Helvétius, Turgot, Rousseau, Montesquieu, Condorcet, Voltaire, and many others. For Harris, “Inspired by the triumphs of the scientific method in the physical and organic domains, nineteenth-century anthropologists believed that sociocultural phenomena were governed by discoverable lawful principles.” However, in the 20th century “there arose in England, France, Germany, and the United States, schools of anthropology that in one way or another rejected the scientific mandate. It came to be widely believed that anthropology could never discover the origins of institutions or explain their causes. In the United States, the dominant school flatly asserted that there were no historical laws and that there could not be a science of history” (Harris 1968, 1-2). These schools “denied historical determinism in general, and above all, they denied the determinism of the material conditions of life. By emphasizing inscrutable values, vain prestige, irrational motives, they discredited the economic interpretation of history. Anthropology came increasingly to concern itself with idiographic phenomena, that is, with the study of the unique and the nonrepetitive aspects of history” (Ibid., 2).

Such deficiencies led Harris to “reassert the methodological priority of the search for the laws of history in the science of man.” For he believed that “the analogue of the Darwinian strategy in the realm of sociocultural phenomena is the principle of technoenvironmental and techno-economic determinism. This principle holds that similar technologies applied to similar environments tend to produce similar arrangements of

\(^{21}\) However, Marx was very conscious of the importance of mathematics for science. The last years of his life he studied calculus with great interest, as evidenced by Marx (1983).
labor in production and distribution, and that these in turn call forth similar kinds of social groupings, which justify and coordinate their activities by means of similar systems of values and beliefs.” Hence Harris adopted as research strategy the “principle of techno-environmental and techno-economic determinism”, assigning “priority to the study of the material conditions of sociocultural life, much as the principle of natural selection assigns priority to the study of differential reproductive success” (Ibid., 3). He called this approach to anthropology the “strategy of cultural materialism”, and though he believed it would be pusillanimous to omit the word ‘materialism’ on account of the “reflexive opprobrium which it elicits”, he felt it was important to carefully separate his ‘cultural materialism’ both from ‘philosophical materialism’ and from ‘dialectical materialism’. The former, which dealt with the question of the ontological primacy of matter over mind, Harris left to philosophers. And the latter was, according to Harris, “a sub-case of cultural materialism, which has been incorporated into the political creed of Marxist communism” (Ibid., 4).

Harris was right to carefully distinguish his stance, the foundation of which –he recognizes– is Marx’s Historical Materialism, from ‘Marxist’ schools that flourished when he was writing and which were based, in general, on schematic and often deeply distorted versions of Marx’s thought. Such schools had not made a conscientious critique of Marx’s political project nor, often but not always, of the societies in which it had materialized in the Soviet Union and other countries. In his book *Cultural Materialism*, published 12 years later, Harris polemicized with various ‘alternative’ currents, among them the two he called, respectively, ‘dialectical materialism’ and ‘structural Marxism’. His definition of the former is somewhat confusing. Whereas in his 1968 book he called ‘dialectical materialism’22 what we –following Marx– have been calling Historical Materialism, in the later book he used this term to refer to what should more correctly be called ‘post-Marx Marxism’. He tended –incorrectly we believe– to emphasize the Hegelian roots of Marx (which could perhaps be more correct in the case of Engels and other epigones). Harris gave an inordinate importance to certain images Marx clearly used (but sparsely) and seemed to ignore hundreds of pages of analysis in *Capital* that have nothing Hegelian about them, even if they sometimes culminate in ‘Hegelian’ phrases like ‘the negation of the negation’. As we have seen in this chapter, what Marx called his ‘dialectical method’ could perhaps be called ‘genetic, empirical, and hypothetico-deductive’ method. And though Marx sometimes did adopt, especially in the first section of *Capital*, a style strongly reminiscent of Hegel, as he himself admitted, it was due to his indignation with the ‘dead-dog’ treatment given to the great Hegel by intellectually inferior thinkers. Nevertheless, it is understandable that Harris made an effort to differentiate his practice and methodological proposal applied to anthropology from the various sterile currents of ‘Marxism’, or ‘Marxism-Leninism’, etc., and one should admire his intellectual honesty in striving strongly to defend his Marx-inspired methodological proposal in such an anti-Marx environment as that of the U.S.A. in the Cold War, in which any defense of the more enduring aspects of Marx’s work was destined to be either ignored or repudiated.23

**Bertrand Russell’s stance on Historical Materialism**  Bertrand Russell (1872–1970) was a great British thinker (philosopher, logician, essayist, and social critic) who

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22 Such is the name of the book’s chapter on Marx.
23 We admire his courageous criticism of the authorities of the University of Columbia (in which he worked) and his defense of the students that were attacked and evicted form the campus by the police in April 1968 on account of their demonstrations against the University’s collaboration with the government in the development of weapons for the Vietnam War (Harris 1968b).
had significant fluctuations in some of his opinions in matters of social and political organization along his long and fruitful life. One of the books that gave him fame was *A History of Western Philosophy and Its Connection with Political and Social Circumstances from the Earliest Times to the Present Day*. There, despite some mis-givings apparently related to an erroneously deterministic interpretation of Historical Materialism, he states that Marx’s thesis is ‘very important’ and admits it influenced his ideas on the historical development of Philosophy: “The politics, religion, philosophy, and art of any epoch in human history are, according to Marx, an outcome of its methods of production, and, to a lesser extent, of distribution. I think he would not maintain that this applies to all the niceties of culture, but only to its broad outlines. The doctrine is called the ‘materialist conception of history.’ This is a very important thesis; in particular, it concerns the historian of philosophy. I do not myself accept the thesis as it stands, but I think that it contains very important elements of truth, and I am aware that it has influenced my own views of philosophical development as set forth in the present work” (Russell 1945, 785).

The summary description Russell makes here of Historical Materialism is faulty, for Marx nowhere states that “The politics, religion, philosophy, and art of any epoch in human history are... an outcome of its methods of production” (italics added). Instead, he stressed the conditioning role of the productive forces and the relations between the social classes involved in production, whereby any history of ideas and/or politics should be written with references to such conditioning factors (which change over time), which is very different. And *this* is precisely what Russell did –to some extent– in his lively history of Western Philosophy.
Part II

Marx’s theory of Capitalism
Chapter 3  COMMODITY PRODUCTION AND CAPITALISM

The historical gestation of Capitalism
Requisites for the rise of industrial capital

During a very long historical period the production and distribution of most of the social product was for self-subsistence and only a small community surplus was traded with neighboring or far-away communities. The existing trade was mostly of luxuries (silk, species, or jewels) or stones or metals that were important as inputs for production (obsidian, copper, tin, iron), and each community produced to satisfy most of its needs. Natural resources were freely used and the distribution of output was in accordance with tradition and had a strong patriarchal component. With the gradual development of the division of labor within communities and between different communities, and the subsequent development of productive forces and mercantile activity, the strong personal links that characterized self-subsistence communities began to erode and, correspondingly, market linkages began to grow within and between communities. Gradually, on the basis of pre-capitalist social relations in production –such as slavery and serfdom– market relations and the profit-motive in trade and lending (or usury) developed, as well as the use of certain commodities (like gold or silver) as money. After many centuries during which the land was communal and movable private property –such as cattle– was firmly established, the private property of land developed. Through the hoarding of money, treasures were formed that would eventually serve to finance the purchase or lease of the means of production characteristic of capitalist production: workshop grounds, buildings, tools, raw materials, etc., as well as the labor power that could use them in the labor process. But that required the development of the institution of wage labor.

In ancient Rome, commodity production and markets were greatly developed, to a large extent through the conversion of independent peasants to slavery in plantations. But much of the great increase in production was based on military-based territorial expansion and the consequent increased provision of slaves. Although the scale of production and hence productivity increased in agriculture by the transformation of smallholdings into large estates, the extant social relation of slavery impeded the development of new and more efficient methods of production, while the continued wars that were an integral component of the regime devoured much of the society’s resources. Roman decadence gave way to the conquest of its western territories during the 5th and 6th centuries by various warring agrarian communities (Franks, Visigoths, Ostrogoths, Lombards, etc.) that were being chased westward by the torrent of Huns led by Attila who had themselves been expelled from East Asia by the Chinese. And there followed centuries of urban depopulation and rural feudal production based on serfdom with institutional continuity restricted to the eastern part of the Roman Empire, later called Byzantine Empire, that would last for an incredible thousand years to be finally taken over by Turkish peoples. Only very gradually were European cities developed during the Middle Ages and with them the increase in commodity production and interurban trade. Meanwhile, the development of monetary relations and commodity production in rural areas gradually undermined the feudal social relations, which often led to the expulsion of the peasants from the lands they had habitually tilled as well as the retainers that had made their living at the expense of the feudal
lord.

In Western Europe commodity production developed on the basis of the increasing personal freedom of workers both in the rural areas (independent peasantry: yeomen in England) as in the burgs (the artisans). Given the concentration of wealth by commercial and banking entrepreneurs, a class of entrepreneurs emerged eager to obtain profit and increase their capital by engaging in industry. While displaced rural peasants and retainers as well as artisans free from the guilds needed their labor power to be demanded, those capital-bearers needed to find labor power that could engage in production in workshops or factories. The production process based on the purchase of labor power generated an enterprising and austere (in comparison with the decadent nobility) capitalist class with increasing political influence that would revolutionize production and inaugurated the modern era. In the historical chapters of Capital Marx highlights the fact that only with the rise of the capitalist mode of production, beginning in the 16th century, did commodity production generalize at an increasing pace until reaching, much later, a predominant—but far from exclusive—character.

The development of commodity production implied the development of money, that is, the spontaneous selection of one or two commodities with convenient properties (such as durability, divisibility, and homogeneity as in the case of precious metals) that facilitated the exchange of commodities. For Marx, “The capitalist mode of production—...—can assume greater dimensions and achieve greater perfection only where there is available in the country a quantity of money sufficient for circulation and the formation of a hoard (reserve fund, etc.) promoted by it” (B2, 342). Hence “the increased supply of precious metals since the sixteenth century is an essential element in the history of the development of capitalist production” (Ibid.).

Some of the few non-military ways of getting rich in European medieval society was through commerce and the lending of money for interest. Great fortunes were generated this way over the centuries, giving rise to the increasing political power of the inhabitants of ‘burgs’, i.e., the ‘bourgeoisie’, that had been granted charters of political and economic independence by the sovereign. That independence was increasingly granted because it gave monarchs the possibility of obtaining the financial support of the richest bourgeois to counter the centrifugal forces of the landowning nobility, always seeking to undermine its political power. Hence, the political consolidation of the monarchy was eased if the towns were able to accumulate wealth. Although some Italian cities developed the first centers of capitalist manufacturing production, the development on a greater scale of capitalist commodity production (CCP) necessitated extended territories that were politically unified by powerful (‘absolute’) monarchies with the towns gradually losing their political independence. Hence, “Although we come across the first beginnings of capitalist production as early as the 14th or 15th century, sporadically, in certain towns of the Mediterranean, the capitalistic era dates from the 16th century. Wherever it appears, the abolition of serfdom has been long effected, and the highest development of the Middle Ages, the existence of sovereign towns, has been long on the wane” (B1, 706-7). Furthermore, “The modern history of capital dates from the creation in the 16th century of a world-embracing commerce and a world-embracing market” (B1, 157).

As capitalism developed, the profit motive became increasingly generalized among those who organized economic activities. Marx holds that it was in commerce and finance where the method of disbursing money-capital with a profit motive first took hold, calling these ‘antediluvian forms of capital’, leading to the increased importance of the role of money as a reserve of wealth. The profit to be obtained by disbursing capital was desired not merely to ensure present consumption but also to accumulate
as large a fortune as possible. Commerce and banking (or usury) were often exercised under non-competitive conditions by means of monopolies granted by the crown (which economists such as Malthus or Ricardo called ‘artificial monopolies’), thus generating immense fortunes. Since some of the trading activities were often subject to great risks they would not have been undertaken if very substantial profits were not to be expected. Merchant ships were usually strongly armed or escorted by armed ships for defense against piracy; and often commerce and piracy were combined (either illegally or legally as in the case of privateer patents). The slave trade was for many centuries a very lucrative business. The activity of small scale lending usually had racketeering characteristics and lending to the crown could have a high risk of expropriation, especially when it financed wars.

Marx considered that for the emergence of industrial capital to be possible it was necessary, first, that a significant accumulation of wealth be achieved by people willing to invest in industry and, second, that there exist sufficiently many people willing (or obliged by circumstances) to work for a wage. The latter required that workers not be subject to pre-capitalist production relations such as slavery or serfdom and also that they not be in a position to work independently as commodity producers, whether in the towns as artisans free from the guilds or in rural areas as free peasants or shepherds.

The primitive accumulation of capital

Marx dedicates the last Part (VIII) of Book I of Capital to the subject of ‘primitive accumulation of capital’, i.e., the historical process of “divorcing the producer from the means of production”. The long process of liberation of workers from their personal bonds without them being able to make an independent living forms for Marx “the pre-historic stage of capital and of the mode of production corresponding with it” (B1, 706). Fundamental aspects of this process were the discoveries of great deposits of gold and silver in America (especially in present day Mexico and Bolivia) and their extraction through the slave labor of the Indian population, the beginning of the conquest and looting of the East Indies, and the subjection to slavery of great numbers of Black Africans. For Marx, during the process of primitive accumulation “great masses of men are suddenly and forcibly torn from their means of subsistence, and hurled as free and ‘unattached’ proletarians on the labour market. The expropriation of the agricultural producer, of the peasant, from the soil, is the basis of the whole process” (B1, 707).

Marx finds abundant historical evidence on how wage labor came into existence in Europe and points to two different paths for this process: “In so far as it is not immediate transformation of slaves and serfs into wage labourers, and therefore a mere change of form, it only means the expropriation of the immediate producers, i.e., the dissolution of private property based on the labour of its owner” (B1, 748). As an example of the first path he points to the case of Italian cities: “In Italy, where capitalistic production developed earliest, the dissolution of serfdom also took place earlier than elsewhere. The serf was emancipated in that country before he had acquired any prescriptive right to the soil. His emancipation at once transformed him into a free proletarian, who, moreover, found his master ready waiting for him in the towns, for the most part handed down as legacies from the Roman time” (B1, 707, footnote 1). He further notes that “When the revolution of the world market, about the end of the 15th century, annihilated Northern Italy’s commercial supremacy, a movement in the reverse direction set in. The labourers of the towns were driven en masse into the country, and gave an impulse, never before seen, to the petite culture, carried on
in the form of gardening.” This would explain the late development of modern industrial Capitalism in Italy (in comparison to England) as a consequence of the greater development of agriculture based on smallholdings.

**The expulsion of peasants from their ancestral lands** But for Marx the fundamental path to the origins of the capitalist mode of production and the diffusion of wage labor was the second path, based on “the expropriation of the immediate producers.” In the feudal manor the peasant worker was not free to leave or to work for anyone else than the landowner; and in the burgs, to sell his labor power the craftsman “must further have escaped from the regime of the guilds, their rules for apprentices and journeymen, and the impediments of their labour regulations” (B1, 706). Marx reproaches ‘bourgeois historians’ for only describing this aspect of ‘primitive accumulation’. For there was another fundamental aspect of the process that was ignored: “these new freedmen became sellers of themselves only after they had been robbed of all their own means of production, and of all the guarantees of existence afforded by the old feudal arrangements. And the history of this, their expropriation, is written in the annals of mankind in letters of blood and fire.” The newly freed workers had to be sufficiently poor that wage labor became their salvation.

Since the most advanced capitalist country was England (which also had the best statistics) Marx centered on that country’s historical process. There the Norman conquest of the 11th century had produced a tremendous concentration of landed property. But servitude had practically disappeared already by the end of the 14th century. The rural population was then composed mostly of free peasants that owned the land they worked on, though this landholding could have different denominations (some feudal) and no written deed. Although there already existed a class of agricultural wage workers, it was relatively small and the class of peasants that worked on their own land and in their spare time worked for big landowners for a wage was much bigger. This ‘own’ land was agricultural land (of at least 4 acres) that had been assigned along with a cottage in the feudal lands. Some of these assigned lands also enjoyed the usufruct of communal lands, “an old Teutonic institution which lived on under cover of feudalism”, where they could graze cattle and find firewood. Marx uses various historical sources (such as William Harrison, Francis Bacon, Thomas More, Frederic Morton Eden, Mirabeau, etc.) to describe the milestones of the process that resulted in the expulsion of the peasantry from the lands in which they had traditionally worked either independently or dependent on a landowner but endowed with the right to work for their own benefit at least part time. According to Marx, both the monarchy and the great feudal lords created a ‘proletariat’ “by the forcible driving of the peasantry from the land, to which the latter had the same feudal right as the lord himself, and by the usurpation of the common lands.” A driving factor was the “rapid rise of the Flemish wool manufactures, and the corresponding rise in the price of wool in England” that led to the transformation of arable land into pastures for sheep, which required a much smaller work force. This process was described by William Harrison in mid-16th century. According to Francis Bacon, and quoted by Marx, “Inclosures at that time (1489) began to be more frequent, whereby arable land (which could not be manured

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1But he also made frequent references to the historical process in other European and even Asian countries. For example, in a footnote he writes: “Japan, with its purely feudal organisation of landed property and its developed petite culture, gives a much truer picture of the European Middle Ages than all our history books, dictated as these are, for the most part, by bourgeois prejudices” (B1, MECW 35, 708, footnote 2).

2However, “In Scotland, the abolition of servitude took place some centuries later than in England” (B1, MECW 35, footnote 2).
without people and families) was turned into pasture, which was easily rid by a few herdsman; and tenancies for years, lives, and at will (whereupon much of the yeomanry lived) were turned into demesnes. This bred a decay of people, and (by consequence) a decay of towns, churches, tithes, and the like.” Owing to generalized protests, Henry VII in 1489 “forbade the destruction of all ‘houses of husbandry’ to which at least 20 acres of land belonged”, stating that “many farms and large flocks of cattle, especially of sheep, are concentrated in the hands of a few men, whereby the rent of land has much risen and tillage has fallen off, churches and houses have been pulled down, and marvellous numbers of people have been deprived of the means wherewith to maintain themselves and their families” (Ibid., 710). On this topic Marx refers to Frederic Morton Eden’s, *The State of the Poor: or a history of the labouring classes in England, from the Conquest to the present period* (1797): “The due proportion between arable land and pasture had to be established. During the whole of the 14th and the greater part of the 15th century, there was one acre of pasture to 2, 3, and even 4 of arable land. About the middle of the 16th century the proportion was changed of 2 acres of pasture to 2, later on, of 2 acres of pasture to one of arable, until at last the just proportion of 3 acres of pasture to one of arable land was attained” (B1, 718).

Another source of the “process of forcible expropriation of the people” was the religious Reformation of Henry VIII in the 16th century. The Catholic Church was the “feudal proprietor of a great part of the English land” and when they were expropriated many people that lived in convenits had to abandon them. Consequently, the customary assignment of a part of the Church’s tithes to the poor was abandoned. The 17th century legislation still required that a minimum of 4 acres of land be assigned to any new house within 10 miles of London, and was still valid during the reign of Charles I (beheaded in 1649, at the beginning of the English Civil War) and then during the rule of the Puritan ‘Protector’ (i.e., dictator) Oliver Cromwell. But gradually these laws were abandoned and the size of the land allocated to workers was reduced until it became a mere orchard. The Restoration of the Stuarts (with Charles II in 1660) brought about the abolition of the feudal land regime, and the peasants began to have to pay taxes to the Crown. The class of independent yeomanry that was predominant during Cromwell’s time and supported him in his struggles practically disappeared. Furthermore, ‘Laws of settlement’ were passed in England which in essence allowed a parish to kick out anyone who came from another parish that might eventually need poverty relief. Rural workers needed settlement certificates from some parish to be accepted in seasonal work (like harvesting) as a guarantee that they would have somewhere to return to and not become a public burden. The reign of William and Mary (1688)

inaugurated the new era by practising on a colossal scale thefts of state lands, thefts that had been hitherto managed more modestly. These estates were given away, sold at a ridiculous figure, or even annexed to private estates by direct seizure... without the slightest observation of legal etiquette. The Crown lands thus fraudulently appropriated, together with the robbery of the Church estates... form the basis of the today princely domains of the English oligarchy. The bourgeois capitalists favoured the operation with

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3 According to the Merriam-Webster online dictionary (in this context) a demesne is “manorial land actually possessed by the lord and not held by tenants.”

4 According to Marx, F. Morton Eden was “the only disciple of Adam Smith during the eighteenth century that produced any work of importance.” But he reproached him for his lack of indignation over “The whole series of thefts, outrages, and popular misery, that accompanied the forcible expropriation of the people, from the last third of the 15th to the end of the 18th century” (B1, 717-8).
the view, among others, to promoting free trade in land, to extending the
domain of modern agriculture on the large farm system, and to increasing
their supply of the free agricultural proletarians ready to hand. Besides,
the new landed aristocracy was the natural ally of the new bankocracy,
of the newly-hatched haute finance, and of the large manufacturers, then
depending on protective duties (B1, 714).

Until then the usurpation of state lands “was carried on by means of individual
acts of violence against which legislation, for a hundred and fifty years, fought in
vain.” During the 18th century, however, it was done by law by means of the Bills
for Enclosures of Commons, “in other words, decrees by which the landlords grant
themselves the people’s land as private property, decrees of expropriation of the people”
(B1, 715). Hence, while the yeomen were substituted by small tenant farmers that
paid rent to the landowner, “the systematic robbery of the Communal lands helped
especially, next to the theft of the State domains, to swell those large farms, that
were called in the 18th century capital farms or merchant farms, and to ‘set free’ the
agricultural population as proletarians for manufacturing industry” (Ibid.). The last
stage of the expropriation of the agricultural population was called the Clearing of
Estates, by which “the agricultural labourers do not find on the soil cultivated by
them even the spot necessary for their own housing.” Marx illustrates in detail how
in the Highlands of Scotland this process was particularly severe. There the clan
chiefs, who were only ‘owners’ of the communal lands as representatives of their clans,
“On their own authority they transformed their nominal right into a right of private
property, and as this brought them into collision with their clansmen, resolved to drive
them out by open force” (B1, 718-9). In the 18th century, “the hunted-out Gaels
were forbidden to emigrate from the country, with a view to driving them by force
to Glasgow and other manufacturing towns.” The “usurpation of the common lands
and the revolution in agriculture accompanying this” had such an adverse effect on the
agricultural labourers that, “according to Eden, between 1765 and 1780, their wages
began to fall below the minimum, and to be supplemented by official poor-law relief”
(B1, 717).

Laws persecuting vagabonds and reducing wages Marx details the repressive
measures taken by the 16th century monarchies to deal with the enormous social prob-
lems they had contributed to generate. A ‘free’ proletariat had been formed by the
“breaking up of the bands of feudal retainers” and by the forcible expropriation of the
people from the soil” that “could not possibly be absorbed by the nascent manu-
factures as fast as it was thrown upon the world.” Hence they “were turned en masse
into beggars, robbers, vagabonds, partly from inclination, in most cases from stress of
circumstances” (B1, 723). To address the problem (not only in England, but in all
Western Europe) legislation was passed against vagabondism. In England, Henry VIII
decreed that beggars who were old or incapacitated for work be given a beggar’s license
while the rest be subject to whipping and imprisonment, and sent back to their places
of origin. Twice offenders were to have an ear sliced off and those arrested for a third
time were to be hanged. A few years later Edward VI decreed that anyone unwilling
to work be given as slave to whoever denounced him. If a slave was absent for more
than a fortnight he was to be branded with an S on his forehead or back. Running

5 In the feudal system a retainer was a person granted the use of land, in return for rendering
homage, fealty, and usually military service (or its equivalent) to a lord.
away for a third time was to be punished by execution. Similar penalties were decreed
by Elizabeth I and James I.

Marx describes how the nascent industrial capitalist class employed the power of
the state to regulate wages and lengthen the working day. In England, as early as 1349
Edward III started regulating wages by means of the *Statute of Labourers*: “A tariff
of wages was fixed by law for town and country, for piece work and day work. The
agricultural labourers were to hire themselves out by the year, the town ones ‘in open
market’ ” (B1, 727). Both the payment and the taking of higher wages than those
in the *Statute* were punished by imprisonment “but the taking of higher wages was
more severely punished than the giving them.” This type of legislation continued for
centuries. Only in 1796 a *minimum* wage for agricultural laborers was proposed but
rejected, and only in 1813 were the laws regulating wages derogated. Furthermore, any
coalition of workers was “treated as a heinous crime from the 14th century to 1825,
the year of the repeal of the laws against Trades’ Unions” (and when Marx was 7 years
old).

**The capitalist farmer and the capitalist manufacturer**

The rise of the capitalist farmer that employed rural wage workers and paid a rent to
the landowner was a long and complex process. According to Marx, in England up to
the first half of the 14th century the *bailiff* ⁶ of a manor, who was himself a serf, was
the first form of farmer since he was in charge of overseeing the property and running
its farms. “During the second half of the 14th century he is replaced by a farmer,
whom the landlord provides with seed, cattle and implements” (B1, 731). Later,
this type of farmer evolved towards the sharecropper (or *métayer*), who advanced a
part (often a half) of the agricultural stock (seeds, instruments, wages of laborers)
while the landlord advanced the rest and received as rent a proportional share of the
crop. And finally, this form of farmer gave place to the “farmer proper, who makes
his own capital breed by employing wage labourers, and pays a part of the surplus
product, in money or in kind, to the landlord as rent” (B1, 732). The revolution
in the conditions of landed property (which included the conversion of large tracts
of arable land into pasture and the eviction of peasants) was accompanied by better
methods of cultivation, increased hours of labor and more intense labor. Hence, there
was a veritable agricultural revolution that began during the last third of the 15th
century and continued during all of the 16th. This revolution was spurred by the
increased inflation produced by the depreciation of gold and silver, which had the
effect of reducing real wages as well as real rents. Hence, there was a marked increase
in productivity and in the real profits of the new class of capitalist farmers.

But this process was complex, since “hand in hand with the expropriation of the self-
supporting peasants, with their separation from their means of production”, went “the
destruction of rural domestic industry, the process of separation between manufacture
and agriculture.” Also, while the manufacturing industry grew, “a new class of small
villagers” emerged who, “while following the cultivation of the soil as an accessory
calling, find their chief occupation in industrial labour, the products of which they sell
to the manufacturers directly, or through the medium of merchants.” The genesis of the
industrialist capitalist was, according to Marx, faster than that of the farmer capitalist.
What prevented the conversion of commercial or lending capital into industrial capital
was the feudal regime in the rural areas and the guild organization in the towns. Thus,

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⁶According to Collins dictionary, “A bailiff is a law officer who makes sure that the decisions of a
court are obeyed.”
“The new manufactures were established at sea-ports, or at inland points beyond the control of the old municipalities and their guilds” (B1, 738-9). The expansion of the colonial system and the consequent expansion of commerce played an important role in the development of manufacturing. “The colonies secured a market for the budding manufactures, and, through the monopoly of the market, an increased accumulation.” Such monopolies were granted by “the State, the concentrated and organised force of society.” “The treasures captured outside Europe by undisguised looting, enslavement, and murder, floated back to the mother-country and were there turned into capital.” Hence, it was commercial supremacy that generated industrial development, which is the opposite of what later happened with the development of modern industry. “The discovery of gold and silver in America, the extirpation, enslavement and entombment in mines of the aboriginal population, the beginning of the conquest and looting of the East Indies, the turning of Africa into a warren for the commercial hunting of blackskins, signalised the rosy dawn of the era of capitalist production. These idyllic proceedings are the chief momenta of primitive accumulation” (B1, 739). Marx gives as an example the case of the English East India Company, which “obtained, besides the political rule in India, the exclusive monopoly of the tea trade, as well as of the Chinese trade in general, and of the transport of goods to and from Europe” (B1, 740).7

Manufacturing industry

The manufacturing period extended from mid-16th century to the end of the 18th century. For Marx the manufacturing industry exhibits the classical form of cooperation between multiple workers based on the division of labor, and it came about in two different ways. On the one hand, manufacturing “arises from the union of various independent handicrafts, which become stripped of their independence and specialised to such an extent as to be reduced to mere supplementary partial processes in the production of one particular commodity.” On the other, “it arises from the co-operation of artificers of one handicraft; it splits up that particular handicraft into its various detail operations, isolating, and making these operations independent of one another up to the point where each becomes the exclusive function of a particular labourer” (B1, 343; italics added). In the first there is an “assemblage, in one workshop under the control of a single capitalist, of labourers belonging to various independent handicrafts, but through whose hands a given article must pass on its way to completion” (B1, 341). It is a sequential process. The second arises when a capitalist employs “simultaneously in one workshop a number of artificers who all do the same or the same kind of work” (B1, 342). There are here parallel processes. But no matter how it comes about, in manufacturing “each operation has to be done by hand, retains the character of a handicraft, and is therefore dependent on the strength, skill, quickness, and sureness, of the individual workman in handling his tools” (B1, 343).

7The English East India Company was created in 1600, when Queen Elizabeth granted a Royal Charter to “George, Earl of Cumberland, and 215 Knights, Aldermen, and Burgess”, which gave them a monopoly on English trade with all countries east of the Cape of Good Hope and west of the Straits of Magellan. During more than a century the Company centered on trade. Starting from its first foothold in Bengal, it gradually began to control vaster territories. By mid 18th century it had taken control of all of the Indian subcontinent, and at the beginning of the 19th century it had a (private) army of 260 thousand soldiers, much more than the British Army. After the bloody repression of the Great Indian Rebellion of 1857 the Crown took direct control of the whole territory, forming the British Raj or British Indian Empire, which at its peak spanned the actual territories of India, Pakistan, Bangladesh, Sri Lanka and Myanmar.
Marx describes the production of a (horse drawn) carriage as an example of the first way in which manufacturing arose: “At first, carriage manufacture is a combination of various independent handicrafts. By degrees, it becomes the splitting up of carriage-making into its various detail processes, each of which crystallises into the exclusive function of a particular workman.” And the workers of various trades “now exclusively occupied in carriage-making, each gradually loses, through want of practice, the ability to carry on, to its full extent, his old handicraft” because they start to specialize in the application of their trade to the specific production of carriages.

As an example of the second way in which manufacture developed Marx addresses the case in which the same capitalist gathers in the same workshop many artisans (possibly assisted by a few officials) that execute the same (or analogous) work: some produce paper, others types, and other needles. These artisans work as they previously did in their respective workshops. But “external circumstances cause a different use to be made of the concentration of the workmen on one spot, and of the simultaneousness of their work.” There is a redistribution of the labor of artisans and officials. “Instead of each man being allowed to perform all the various operations in succession, these operations are changed into disconnected, isolated ones, carried on side by side; each is assigned to a different artificer, and the whole of them together are performed simultaneously by the co-operating workmen.” Hence, the commodity that used to be the product of an independent worker becomes the social product of a group of workers; each specialized in a partial operation.

Modern industry

Whereas the development of manufacturing was characterized by the concentration of workers and means of production in a single workshop, that of modern industry was characterized by the revolutionizing of the instruments of labor by means of machinery. Modern industry adapted the workshop for the use of machinery, transforming it into a factory. According to Marx, the transformation of the instruments of labor from simple tools to machine-tools marked the beginning of modern industry, and this process started in the last third of the 18th century (B1, 374, 283), continuing into the 19th century with the transformation of handicraft or manufacture “into an industry carried on by machinery” (B1, 376). In modern industry, the tools of labor “are the implements of a mechanism, or mechanical implements” instead of being simply the implements of the worker. With the invention of machine-tools, the worker “instead of working with an implement on the subject of his labour, becomes merely the motive power of an implement-machine.” But “it is a mere accident that motive power takes the disguise of human muscle; and it may equally well take the form of wind, water or steam.” This substitution emancipated production from the limitations of human strength.

“One motive mechanism was now able to drive many machines at once. The motive mechanism grows with the number of the machines that are turned simultaneously, and the transmitting mechanism becomes a widespread apparatus” (B1, 381).

Marx distinguishes between the cooperation of a number of machines of one kind from “a complex system of machinery.” In the case of several machines in cooperation he gives the example of a weaving factory formed by many power looms working side by side, “all the machines receiving their impulse simultaneously, and in an equal degree, from the pulsations of the common prime mover, by the intermediary of the transmitting mechanism; and this mechanism, to a certain extent, is also common to them all, since only particular ramifications of it branch off to each machine.” In contrast, the material processed in a machinery system, instead of going through a
series of independent machines, “goes through a connected series of detail processes that are carried out by a chain of machines of various kinds, the one supplementing the other.” “As soon as a machine executes, without man’s help, all the movements requisite to elaborate the raw material, needing only attendance from him, we have an *automatic system of machinery*” (B1, 384; italics added). Marx takes the British paper mill as an example of the automatic system.

Since there was an increasing demand for cost reducing specialized machinery, it became common in the first decades of the 19th century to use machines in the production of machines. And the speed of technological innovation was distinctive of the modern industrial phase of Capitalism. In previous handicraft and manufactures, after slowly finding the empirically best suited way to produce a given item it could crystallize and stay the same for hundreds of years. Marx points out that “even down into the eighteenth century, the different trades were called ‘mysteries’ (mystères); into their secrets none but those duly initiated could penetrate.” But modern industry had “rent the veil that concealed from men their own social process of production, and... created the new modern science of technology.” It “never looks upon and treats the existing form of a process as final. The technical basis of that industry is therefore revolutionary, while all earlier modes of production were essentially conservative. By means of machinery, chemical processes and other methods, it is continually causing changes not only in the technical basis of production, but also in the functions of the labourer, and in the social combinations of the labour process” (B1, 489). Modern industry also completed the divorce between agriculture and rural domestic industry. And “In the sphere of agriculture, modern industry has a more revolutionary effect than elsewhere, for this reason, that it annihilates the peasant, that bulwark of the old society, and replaces him by the wage labourer.” The irrational, old-fashioned methods of agriculture are replaced by scientific ones (B1, 506).

**Cooperation, division of labor, and private property**

**Cooperation, the division of labor, and the concentration process**

In Marx’s theory important roles are played by the cooperation of workers and their division of labor within the work process, the division of labor outside the workshop or factory through the market mechanism, and the fact that the control of the conditions of labor (tools, buildings, land, etc.) and of the resulting output is in the hands of capitalist entrepreneurs. Common property of the conditions of production, such as land, forests, etc., and the firm implantation of the individual in his community were, according to Marx, the foundation on which the cooperation in the labor process flourished from the remotest times: “Co-operation, such as we find it at the dawn of human development among races who live by the chase or, say, in the agriculture of Indian communities, is based, on the one hand, on ownership in common of the means of production, and on the other hand, on the fact, that in those cases each individual has no more torn himself off from the navel-string of his tribe or community, than each bee has freed itself from connexion with the hive” (B1, 339). This contrasts with the reality of petty industry (reflected in the SCP model), in which artisans working independently, own their means of production, and can develop their individuality with little cooperation in the labor process and a scantily developed social division of labor. This type of simple industrial production existed to some degree “under slavery, serfdom, and other states of dependence. But it flourishes... only where the labourer is the private owner of his own means of labour set in action by himself: the peasant of the land which he cultivates, the artisan of the tool which he handles as a virtuoso” (B1, 749). “The
private property of the labourer in his means of production is the foundation of petty
industry, whether agricultural, manufacturing, or both; petty industry, again, is an es-
sential condition for the development of social production and of the free individuality
of the labourer himself” (Ibid.). But this mode of production precludes the concentra-
tion of the means of production and of workers since it “presupposes parcelling of the
soil, and scattering of the other means of production”, and it “excludes co-operation,
division of labour within each separate process of production, the control over, and
the productive application of the forces of Nature by society, and the free development
of the social productive powers” (Ibid.). Hence, such a mode of production was only
feasible under scant development of the productive forces and was not an adequate
arrangement for their further development. Eventually, “new forces and new passions
spring up in the bosom of society; but the old social organisation fetters them and
keeps them down. It must be annihilated; it is annihilated.” And this annihilation was
achieved by means of “the transformation of the individualised and scattered means
of production into socially concentrated ones, of the pigmy means of production into
socially concentrated ones, of the pigmy property of the many into the huge property
of the few”, i.e., the property of industrial capitalists. Historically, this was accom-
plished through the ‘primitive accumulation of capital’ described above in the case of
Great Britain. And it could only have been accomplished through the transformation
of petty producers of commodities, slaves and/or serfs into wage workers concentrated
in workshops or factories run and owned by industrial capitalists.

Accumulation, concentration, and centralization of capital

For Marx the process of concentration is both the concentration of many workers in
a workshop or factory and the concentration of means of production as the private
property of an industrial capitalist. This concentration is suited for the development of
the division of labor in the workshop as well as the cooperation among the concentrated
workers. Once Capitalist Commodity Production (CCP) is in place, in each sphere of
production social capital “is divided among many capitalists who face one another as
independent commodity producers competing with each other” (B1, 620). For Marx
each individual industrial capital implies a concentration of means of production and
a unified command over a group of workers who cooperate in the labor process. And
the accumulation of capital through the reinvestment of a part of the profits obtained
increases the concentration of the capital of the individual capitalists. Hence, the
“simple concentration of the means of production and of the command over labour” is
“identical with accumulation” (B1, 621). Global capital grows along with the growth
of the individual capitals and the concentration of the means of production. On the
other hand, the individual capitals can also grow by the fusion of several individual
capitals to form a greater one. But there are also mechanisms that work in the opposite
direction, since “the increase of each functioning capital is thwarted by the formation
of new and the sub-division of old capitals” (B1, 620-1). The former takes place when
capital accumulated in one sector is used to make a capital investment in another
sector; and the latter when there is a division in the capital of a deceased capitalist
due to its splitting up among his heirs.

On the other hand, the development of modern industry brings about a new type
of ‘expropriation’: that of some capitalists by other capitalists, a process that responds
to “immanent laws of capitalistic production itself” (B1, 750). The competition be-
tween capitalists produces the disappearance of the firms least capable of generating
profitability as their productions are replaced by the existing more profitable ones, a
process that Marx calls the *centralization* of capital. Marx invokes what later economic theory has named ‘scale economies’ (based on increasing returns to scale): “The battle of competition is fought by cheapening of commodities. The cheapness of commodities depends, *ceteris paribus*, on the productiveness of labour, and this again on the scale of production. Therefore, the larger capitals beat the smaller” (B1, 621).

The centralization of capital brings about various related phenomena which include the increasing cooperation of workers within the factory, the increasing application of scientific knowledge to the production process, the increasingly rational use of natural resources, the use of increasingly complex machinery that must be used collectively, as well as the increasing internationalization of the institutions of Capitalism as the world market develops. Additionally, “with capitalist production an altogether new force comes into play—the credit system. Not only is this itself a new and mighty weapon in the battle of competition. By unseen threads it, moreover, draws the disposable money, scattered in larger or smaller masses over the surface of society, into the hands of individual or associated capitalists. It is the specific machine for the centralisation of capitals” (Ibid., 621-2). For Marx the credit system is “the principal basis for the gradual transformation of capitalist private enterprises into capitalist stock companies” and it “offers the means for the gradual extension of cooperative enterprises on a more or less national scale” (B3, 438). Both forms of organization of production, joint-stock companies and cooperatives were for Marx evidence of an increasing socialization of production that would eventually lead to a new kind of society with centrally planned production and distribution and without the private property of the means of production. This aspect of Marx’s thought is addressed in Part IV of this book.

Both the accumulation and the centralization of capital enable the growth in the size of industrial firms and the rational and scientific organization of production. But the centralization process can be much quicker than the usual accumulation process since it “only redistributes the social capital already to hand, and melts into one a number of old capitals” (B1, 622). A consequence of the formation of huge enterprises (like the railroads) thanks to the credit system and the institution of joint-stock companies is that it tends to create a separation between the “manager, administrator of other people’s capital”, and the owners. And whereas the dividends received by stockholders “include the interest and the profit of enterprise”, both of which are retributions to property, “the salary of the manager is, or should be, simply the wage of a specific type of skilled labour, whose price is regulated in the labour market”. Thus, “In stock companies the [managerial] function is divorced from capital ownership, hence also labour is entirely divorced from ownership of means of production and surplus labour” (B3, 434; term within brackets added for clarity). Furthermore, “a new swindle develops in stock enterprises with respect to wages of management, in that boards of numerous managers or directors are placed next and above the actual director, for whom supervision and management serve only as a pretext to plunder the stockholders and amass wealth” (B3, 388). Finally, another consequence of the formation of stock companies is the introduction of monopoly power (though Marx does not use this expression). As the number of firms in any given branch of production diminishes due to the large size of the existing ones, the latter can elude the pressure of competition.

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8In Marx’s own words: “Hand in hand with this centralisation, or this expropriation of many capitalists by few, develop, on an ever-extending scale, the co-operative form of the labour process, the conscious technical application of science, the methodical cultivation of the soil, the transformation of the instruments of labour into instruments of labour only usable in common, the economising of all means of production by their use as the means of production of combined, socialised labour, the entanglement of all peoples in the net of the world market, and with this, the international character of the capitalistic régime” (B1, 750).
Marx highlights the “economically important fact” that such firms tend to stem “the fall of the general rate of profit” since “such undertakings... do not necessarily enter into the equalisation of the general rate of profit” (B3, 435).

We deal more specifically with the principal tendencies that Marx perceived in the Capitalism of his time in Chapter 17.
Chapter 4  THE STRUCTURE OF CAPITAL AND MARX’S THEORY OF VALUE

As we have seen, Marx maintained the historical exposition separate from the theoretical construction. The latter required a hypothetico-deductive structure that could order the (stylized) material according to the degree of importance it had in Capitalism. And for this it was important to begin with “the simplest categories of the capitalist mode of production... commodities and money” (B1, 813). And that is why Part I of Book I is titled “Commodities and Money” and contains a theoretical study of a commodities producing society which is not capitalist and in which money appears as a commodity that has been selected by the participants in the process of exchange of commodities in a previous historical process. It is assumed that commodity producers produce individually (without help from wage workers, slaves, or serfs), each commodity being produced by many different producers, and that they sell their products in a market. It is implicit that these producers make free use of natural resources like land, rivers, etc. In the remaining theoretical Parts of Book I (Parts II-VII, since Part VIII is purely historical) the capitalists are introduced. These invest their capitals in the purchase of means of production and the labor power of wage workers, maintaining implicitly the assumption that there is free use of natural resources. Hence, Book I includes two broad income categories: the wages of workers and the profits of industrial capitalists. The private property of land is only introduced in Book III, giving rise to the study of ground rent as the income category of a class of landowners. In this third stage of the theoretical construction the commercial capitalists and money-dealing capitalists are sub-classes of the capitalist class (along with the industrial capitalist sub-class). Finally, at the most concrete stage of Marx’s theoretical construction other social classes are also introduced, such as the managers, administrators, or directors of capitalist enterprises, who may differ from the owners of the capital tied up to the firm’s activities, especially when joint-stock companies and the stock market are introduced.

For Marx “As a matter of history, capital, as opposed to landed property, invariably takes the form at first of money” and “appears as moneyed wealth, as the capital of the merchant and of the usurer” (B1, 157; italics added). And landed property gradually evolved from its original feudal form to its capitalist, rent-bearing, form. But as a matter of theory, Marx deemed it better to abstain from treating landed property in Book I and, when he introduces it from a theoretical viewpoint in Book III he treats it as completely integrated to the capitalist mode of production, whereby landowners who do not participate in agriculture or livestock activities simply receive a money rent for allowing tenant farmers to use their land in the capitalist firms they exploit. In a footnote Marx refers to the contrast between the power based on the personal relations of dominion and servitude in the feudal mode of production and the impersonal relations of money-capital: “The contrast between the power, based on the personal relations of dominion and servitude that is conferred by landed property, and the impersonal power that is given by money, is well expressed by the two French proverbs, ‘No land without its lord’, and ‘Money has no master’ ” (Ibid.).
Labor, commodity production, and Capitalism

The labor process in general

Marx carefully defines the terms he uses in his analyses and precisely distinguishes which of these terms are generally valid and which are applicable only to specific modes of production and circulation. The clarity of his definitions reveals his philosophical training. Since man in society is at the center of his analyses, human labor is a particularly important category for any mode of production: “The fact that the production of use values, or goods, is carried on under the control of a capitalist and on his behalf, does not alter the general character of that production. We shall, therefore, in the first place, have to consider the labour process independently of the particular form it assumes under given social conditions” (B1, 187). Labor is a process in which man and nature interact and in which man “of his own accord starts, regulates, and controls the material reactions between himself and Nature.” And as man acts on Nature, at the same time he transforms his own nature, since he must submit his instincts to a discipline that allows him to attain an end. The simple factors (or aspects) of the labor process are “1, the purposeful activity or work itself, 2, its object, and 3, its means”1. The means of labor are the things “which the labourer interposes between himself and the object of his labour, and which serves as the conductor of his activity” (B1, 189).

The earth is man’s “original larder” of consumption goods as well as the “original tool house” that supplies him “stones for throwing, grinding, pressing, cutting, etc.” But as soon as labor develops somewhat it requires “specially prepared instruments” like the stone implements and weapons often found in caves dating from the earliest times of human history. And in the labor process man uses instruments of labor to obtain an alteration in the material upon which he works, culminating in a product which has a use-value since it can satisfy human needs. Hence, looked at from the point of view of the end product of the labor process “both the instruments and the object of labour, are means of production”, and “labour itself is productive labour” (B1, 191). The objects of labor are either those directly provided by Nature (such as the timber from a forest or the fish caught in the water) or those objects already transformed by labor denominated raw material (such as “ore already extracted and ready for washing”).

The following table summarizes Marx’s definition of the simple aspects of the labor process as well as some of the examples he mentions:

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1The corresponding sentence in MECW 35 is: “The elementary factors of the labour process are 1, the personal activity of man, i. e., work itself, 2, the subject of that work, and 3, its instruments.” However, under the suspicion that instead of ‘subject’ it should read ‘object’ (as in the Spanish translation of Roces: ‘objeto’) we checked the German original. Both the Google and the Microsoft translators, translate 2 and 3 as ‘its object and its means’. Our quoted text corresponds to the Google translation of the German sentence. Below we make other similar changes from ‘subject’ to ‘object’.
Table 1

<table>
<thead>
<tr>
<th>Subject of labor (who works)</th>
<th>Means of labor (using what means)</th>
<th>Objects of labor (applying them to what objects)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worker</td>
<td>Means of production</td>
<td></td>
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<tr>
<td></td>
<td>found on or underneath the earth, or under water</td>
<td></td>
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<tr>
<td></td>
<td>–the stone he throws</td>
<td>–tree trunks, minerals, fish produced</td>
</tr>
<tr>
<td></td>
<td>Instruments</td>
<td>Raw Materials</td>
</tr>
<tr>
<td></td>
<td>–the sharpened stone</td>
<td>–wood for carpentry</td>
</tr>
<tr>
<td></td>
<td>–domesticated animals</td>
<td>–extracted ore ready for washing</td>
</tr>
</tbody>
</table>

The labor process thus characterized “is the necessary condition for effecting exchange of matter between man and Nature... and therefore is independent of every social phase of that existence, or rather, is common to every such phase.” Hence, there was no need above “to represent our labourer in connection with other labourers.” But thence this simple process does not “tell you of itself what are the social conditions under which it is taking place, whether under the slave-owner’s brutal lash, or the anxious eye of the capitalist” (B1, 194). In the beginnings of CCP, “The general character of the labour process is evidently not changed by the fact that the labourer works for the capitalist instead of for himself.” The capitalist “must begin by taking the labour power as he finds it in the market, and consequently be satisfied with labour of such a kind as would be found in the period immediately preceding the rise of capitalists” and hence “the particular methods and operations employed in bootmaking or spinning are not immediately changed by the intervention of the capitalist.” Only in a more mature period of Capitalism do changes in the methods of production take place by the effects of its own dynamics. Also, even though in this abstract discussion of the labor process “labor is productive labor”, we will see in Chapter 15 that for Marx not all wage labor is ‘productive labor’ in Capitalism; because in his theory only the wage labor that produces surplus value is deemed ‘productive’ and surplus value is only produced in the industrial sphere.

Labor in commodity production and its productive force

*Capital* begins with a paragraph in which Marx quotes himself (in simple quotes here): “The wealth of those societies in which the capitalist mode of production prevails, presents itself as ‘an immense accumulation of commodities’, its unit being a single commodity. Our investigation must therefore begin with the analysis of a commodity.” By quoting himself in the first sentence, Marx was pointing out that in the first chapters of his new book he was returning to the topics of his *Contribution to the Critique of Political Economy* published eight years earlier. That book was entirely dedicated to the analysis of commodities and money. But given the time that had elapsed and taking into account it had not been widely read, Marx re-wrote the substantial contents of that book in Part I (of Book I), extending, modifying, and leaving out some parts of its contents (in particular, the sections on the history of economic theories). There was also a certain change in the writing style. Chapter I (‘Commodities’) is somewhat more complicated reading than the *Contribution* in part due to some
intellectual juggling with economic ‘categories’ as if they had a life of their own\(^2\). Since this chapter condenses his theory of commodities and their values, which is the base for the theoretical edifice of *Capital*, this was rather unfortunate.

Like Smith and Ricardo, in *Contribution* Marx parted from the distinction between *use value* and *exchange value*. But unlike them from the beginning he makes it explicit that he is studying a certain society, capitalist society, which is the commodity producing society *par excellence*. And he puts that society in a historical perspective making frequent references to past societies. In pre-commodity modes of production goods were produced for their direct utility to the community of producers or their utility to those who exploited the producers. The producers were subject to strong social links that determined both the division of labor and the distribution of the resulting product.\(^3\) In contrast, production for the market implies that for the producer the goods are above all exchange values, or goods that are destined to be exchanged for other goods.

In the *SCP* that Marx takes as theoretical reference point in the explanation of the nature of Capitalism, the producers’ labor is not predetermined by the existing social order. Instead, each producer is an independent private entity who needs to have his output, his commodity, actually sold in the market in order for it to really participate in the *social* labor process. Only the confrontation of commodities in the exchange process and their sale can ensure the social character of the individual producers’ labor. The labor that results in a good that cannot be sold does not have such a social character and is socially unnecessary. And since the producers act independently of one another and guide their production actions by the demand they expect to find in the market for their commodities, their expectations can be frustrated. “On the other hand, under the rural patriarchal system of production, when spinner and weaver lived under the same roof—the women of the family spinning and the men weaving, say for the requirements of the family—yarn and linen were social products, and spinning and weaving social labour within the framework of the family... the product of labour bore the specific social imprint of the family relationship with its naturally evolved division of labour” (*Contribution*, MECW 29, 274-5). And going even further back in time, in the case of the “communal labour in its naturally evolved form as we find it among all civilised nations at the dawn of their history... The communal system on which [this mode of] production is based prevents the labour of an individual from becoming private labour and his product the private product of a separate individual; it causes individual labour to appear rather as the direct function of a member of the social organisation” (Ibid., 275). Only in commodity production do the products of human labor have to be associated with the notion of *exchange values*. For in societies in which the work is not done by independent units, in the last instance regulated by a market, labor is directly destined to satisfy a social need without the producers having to confront one another by competing in the exchange process, i.e., without a seller having to find a buyer willing to purchase his product. It is this exchange process that requires the generation of the notion that the product of labor has a value.

**Capitalist commodity production and the productive force of labor**

In Marx’s theory the capitalist mode of production is one in which 1) independent firms run by entrepreneurs produce in order to sell in a market, i.e., their output are

\(^2\)This is probably what Marx referred to when he admitted to having “coquetted with the modes of expression peculiar to” Hegel (B1, 19).

\(^3\)In his *Grundrisse* Marx calls the production of goods directly for consumption without market mediation ‘production of immediate use values’ (*Grundrisse* 1, MECW 28, 131).
commodities, 2) work is performed by workers who are not owners of the firms they work in nor of the necessary conditions of the labor process (land, means of labor, raw material, etc.) that would enable them to work on their own, so they need to sell their labor power in exchange for a wage, 3) the necessary conditions for the labor process and the (final or intermediate) output are always under the control of the capitalist that invests money-capital in their purchase (or lease) and in the purchase of labor power (the subjective condition of the labor process), 4) the aim of the capitalist when he invests his capital is not only to consume and preserve his capital but, fundamentally, to increase it, to become wealthier.

**CCP spreads commodity production and wage labor** In historical reality commodity production and the circulation of commodities and money by means of commerce existed centuries before the rise of Capitalism. However, before the rise of CCP commodity production never had a generalized character. Commerce developed initially through the sale of the surplus production above the own consumption of communities where different modes of production and distribution prevailed, such as slavery in Antiquity or in the plantations, serfdom in the Middle Ages, and petty commodity production by rural peasants or urban artisans. Only with the rise, propagation, and development of wage work in industrial establishments managed by capitalist entrepreneurs did industrial production reach the great impulse that led to the progressive dissolution of pre-capitalist modes of production and, in the mature phase of modern capitalist production, to the continued revolution in productive methods with the consequent sustained increase in output due to increased productivity.

Marx observes the diversity of modes of production worldwide. The existence of a world market implies that all commodity production, whether capitalist or not, is linked by the circulation process: “Within its process of circulation... the cycle of industrial capital, whether as money capital or as commodity capital, crosses the commodity circulation of the most diverse modes of social production, so far as they produce commodities” (B2, 115). This ‘crossing’ tends to transform all commodity production into capitalist commodity production, i.e., to convert workers who were previously either subject to constraints on their personal freedom or worked independently for selling in the market, to wage workers. The labor transformed to wage labor was in both cases of low productivity. Since industrial Capitalism substituted non-commodity production with (capitalist) commodity production and also greatly increased productivity (hence producing more commodities with the same amount of labor), with the rise of industrial Capitalism there was a widespread diffusion of commodity production.

It is important to bear in mind that in Marx’s time serfdom and slavery were very common. At the end of the 18th century slavery was the norm in the plantations of North America and the Caribbean. Citing a report by Henry Brougham (An inquiry into the Colonial Policy of the European Powers, Edinburgh, 1803), Marx points out that “In 1790, there were in the English West Indies\(^4\) ten slaves for one free man, in the French fourteen for one, in the Dutch twenty-three for one” (B1, 747, footnote 1). Although Great Britain abolished the slave trade in 1807, it did not free the slaves. Only after the 1831 slave rebellion in Jamaica was enough support mustered to pass the Slavery Abolition Act of 1833 (when Marx was 15 years old). Also, serfdom was still very common in East Europe during the greater part of the 19th century. In Russia, the nominal liberation of the serfs was decreed in 1861 but, as we have seen,

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\(^4\)The British West Indies included a series of British territories in the Caribbean, among them the Cayman Islands, the Virgin Islands, The Bahamas, Barbados, Grenada, Jamaica, and Trinidad and Tobago.
they had to buy their own freedom, making the process highly ineffective. Before the Civil War (1861-65) slavery was predominant in the South of the U.S.A., where 3 and a half million slaves constituted 38% of the population. Northern capitalists understood that the South’s economic integration with the British textile industry through its slave based cotton industry was an obstacle for the development of their own industry. And according to Marx slave production was not suited for industrial capitalism, since the slave “takes care to let both beast and implement feel that he is none of them, but is a man... by treating the one unmercifully and damaging the other *con amore.*** Hence slave labor was confined to the employment of only “the rudest and heaviest implements and such as are difficult to damage” (B1, 207).

The circulation in the world market of commodities produced by the main industrial centers (such as England) created a tendency to further increase in the productive force of labor by the mere fact that the commodities produced by pre-capitalist forms of commodity production could not compete with those of wage labor Capitalism and hence were gradually phased out. This occurred both within countries which already had a significant capitalist industrial production and within less developed countries in which capitalist industrial production was minimal but had wealthy individuals who could invest in production employing wage labor to increase their wealth. There was hence a widespread diffusion of the institution of wage labor and consequently of the capitalist mode of production of commodities.

**The structure of Capital**

As we have seen, *Capital* is structured in four Books, the last of which was published as *Theories of Surplus Value*, in three volumes. The latter may appropriately be grouped along with the sections on the history of economic thought in the *Contribution* and the various comments of a similar nature contained in the first three Books of *Capital*. They are an invaluable source for evaluating the individual influences of various authors on the formation of Marx’s theory of Capitalism. But they do not contain any part of the actual exposition of this theory. Therefore, in the rest of the present chapter and in the next chapters we focus on the theoretical parts of Books I-III.

**The theoretical edifice of Capital**

Marx’s exposition of the theory of the capitalist mode of production and circulation begins with commodity production in general, or Simple Commodity Production (SCP), which is built on the assumption that the natural conditions of production, such as land and rivers are freely available to the individual producers of the community. Also, in SCP there is no money lending nor credit markets and hence no interest nor debts. As we will see, in such a society it is only labor that in the last instance explains the exchange values of commodities. On the basis of these assumptions Marx develops his *theory of commodities*, aspects of which are his theory of *value* and his *theory of money* as a commodity that adopts certain monetary functions such as the measure of values, medium of circulation, means of payment, and means of hoarding, that may be substituted by a simple symbol (e.g. paper) at an advanced stage. Finally, Marx

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5These sections are: A) *Historical Notes on the Analysis of Commodities*, B) *Theories of the Standard of Money*, and C) *Theories of the Medium of Circulation and of Money.*

6From this point on we will write the word ‘value’ in bold characters whenever we refer to Marx’s concept of ‘labor-value’. Similarly, we will use ‘surplus value’ to refer to a certain surplus in *value* that we will define below. Without bold characters the word ‘value’ will have the usual meaning relating to the valuation of commodities using prices. All quotes from Marx, however, will be transcribed without such bold characters.
develops his theory of capital, by introducing the features that distinguish Capitalist Commodity Production (CCP) from SCP, i.e., the cleavage of the producer/worker of SCP into the two polar figures of the capitalist and the wage worker. After the introduction of CCP, the rest of the theoretical construction of Book I (implicitly) maintains the assumption that land and other natural resources are freely accessible and that there are neither loans nor debts. It is only in Book III that the private property of land, commerce, and money lending and banking are introduced (with the corresponding incomes of rent, commercial profit, interest, and bank profit). A substantial part of the reason for this postponement is that Book I was dedicated to the process of production of capital, which is tantamount to saying the production of surplus value and its reinvestment. And in Marx’s theory this only occurs in the sphere of production (or industry). The details of how this surplus value is distributed through the circulation process among the industrial capitalists and to money lenders, commercial and banking capitalists, and landowners, could thus be postponed to Book III, which dealt with the more global aspects.

One of the basic characteristics of CCP is that industrial capitalists have the incentive (based on their profit motive) and the need (due to competition and its effect on the risk of failure) for continually expanding production on the basis of their reinvestment of a large part of their profit. Hence, they use their profit not only to finance their consumption but also, and fundamentally, to finance the expansion of their capital. The continued production of additional capital by means of the reinvestment of a significant part of the profit, i.e., capital accumulation, is for Marx a fundamental characteristic of the capitalist mode of production. But Marx’s theory of capital is much more than the theory of the production of capital that is contained in Book I. For it also includes the theory of the circulation of capital, which is only insinuated in Book I and developed in Book II, which is titled ‘The process of circulation of capital’. There the intersectoral aspects of transactions are extensively studied, i.e., various schemes of Simple and Extended ‘Reproduction and Circulation of Aggregate Social Capital’, which implied a very significant advance over the Tableau Économique of the Physiocratic School and many years later inspired the development of ‘Input-Output analysis’. Finally, both the production and circulation of capital are considerably extended and integrated in Book III, titled ‘The process of capitalist production as a whole’. There many of the complexities which had been set aside in Book I through various simplifying assumptions are addressed.

Summarizing, SCP is a theoretical abstraction that Marx uses as foundation for the theoretical edifice he constructs in order to explain the functioning of capitalism. It is a first model of commodities and their equilibrium monetary prices (which he shows are derived from their values). This model presupposes the study of the process of exchange of commodities, which leads to the selection of a commodity that has special characteristics that turn it into money. Commodities and money are themselves the base on which a second level of the edifice is built: capital. This second level is itself constructed in stages by making simplifying assumptions that are later relaxed. Books I and II assume that the commodity used as money is gold. But both Contribution and Capital develop a historical-genetic theory of money that reaches the level of paper money and credit money.

Marx illustrates the circulation process of commodities and the currency of money using visual schemes like the following:

\[ C - M - C - M - C - M - C - \ldots \]

\(^7\)In contrast, the historical sections of Book I do contain considerations on rent and interest.
where the $C$s are amounts of (valued) commodities and $M$ the amount of money (gold) that has the same value as the $C$s. One can see here that each group of commodities has a short cycle since the sale made by the producers of the first $C$ to buyers who give them $M$ in return allow them to use this money to purchase other commodities reflected in the second appearance of $C$ (which are produced by other producers). The producers of the commodities in the first appearance of $C$ are not interested in their use values but only their exchange value, i.e., the price at which they can sell it and, hence, the quantity of money $M$ they can obtain and use to purchase the commodities in the second appearance of $C$. It is the use value of the latter that they are interested in. The role of the producers of the commodities in the first $C$ ends with this short cycle $(C - M - C)$. The same can be said of the producers of the second appearance of $C$, who are interested in the use value of commodities in the third appearance of $C$. But the money given by the producer of the commodities in the first $C$ to the producers of the commodities in the second $C$ follows its course, going “further and further away from its starting-point” (B1, 124), since the latter use it to purchase the commodities in the third appearance of $C$, the commodities whose use value they are interested in.\footnote{In Chapter 7 we present a more thorough exposition of Marx’s monetary theory.}

In this scheme the prices of the various commodities involved play a fundamental role in determining the quantities of commodities and money involved in each step. And those prices are founded on the exchange values of the commodities, where money is one of the commodities (gold) and the price of a commodity is the exchange value of that commodity in terms of gold. As we show in Chapter 6, in SCP that exchange value is founded on the values of the commodities, which are given by the amount of labor that is directly or indirectly necessary to produce the commodities under average conditions of organization and technology. That in SCP the exchange values are founded on the values of commodities can be expressed in modern economic language by saying that the ‘equilibrium’ monetary price of a commodity is given by its value divided by the value of money. However, this is not so in CCP—even continuing with the assumption that there is free access to land—in which the foundation of the exchange values is to be found in the ‘prices of production’ of commodities, which in general differ from their values divided by the value of money. This is due to the fact that capital tends to flow between industrial sectors until the rates of profit are the same in all sectors. And when the private property of land is present, the equilibrium exchange value of commodities is to be found in ‘modified production prices’ that take into account the existence of ground rent. In Chapter 16 we deal with Marx’s theory of ground rent and its effect on prices of production. Finally, the process of centralization of capital and the generation of huge firms endowed with monopoly power imply that the exchange values of commodities in the capitalism of modern industry are even more complex due to the fact that these firms can avoid the process of equalization of profit rates and hence can have higher profit rates than other firms.

The goal of human actions plays an important role in Marx’s theory. In the specific case of simple circulation of commodities we represented above, the aim of the simple commodity producers involved in the process is simply to satisfy their needs, for which they sell their produced commodities in order to purchase the means of production they have used up and the means of consumption they need to live. This is above symbolized by Marx by the ‘formula’ $C - M - C$. In the case of the circulation of capital, that is, of commodities resulting from CCP, Marx characterizes the process from the capitalist point of view as “buying to sell dearer”, which he symbolizes by means of the formula $M - C - M'$, where $M' > M$. Hence, the capitalist invests
money capital $M$ in the purchase of commodities $C$ that after a certain period of time he sells (possibly transformed) in exchange for a greater sum of money than he initially had. Whereas in the simple circulation of commodities the aim of the producer is the consumption of commodities (qualitatively) different from those that he produces on account of the division of labor and specialization, in the circulation process of capital the aim the capitalist has when he disburses his ‘money as capital’ (or money capital) is to (quantitatively) increase his initial capital.

Capital has different forms. The “antediluvian forms” precede CCP and include merchant capital and financial capital. The former is characterized by the above ‘formula’ $M - C - M'$ where $C$ is the value of the commodities purchased by paying $M$, to be sold with a profit. And the latter is characterized by Marx by the abbreviated formula $M - M'$ where a certain amount of money is lent and (after a certain period of time) returned with interest. According to Marx “Interest-bearing capital, or, as we may call it in its antiquated form, usurer’s capital, belongs together with its twin brother, merchant’s capital, to the antediluvian forms of capital, which long precede the capitalist mode of production and are to be found in the most diverse economic formations of society” (B3, 588).

These forms of capital participate in the commodities’ circulation process. The commodities involved may have been produced by simple commodity producers or by proprietors that use slave or serf labor and sell at least a part of their output with the aim of getting a surplus value, i.e., the money value of a surplus amount of commodities generated by workers that are not wage workers and which are sold in markets. For centuries before the rise and diffusion of CCP, merchant capital and financial capital regularly obtained surplus value (or profits) by dealing with commodities (including the money commodity) that were produced in pre-capitalist modes of production. There existed two “characteristic forms, however, in which usurer’s capital exists in periods antedating capitalist production” which later also existed on the basis of capitalist production “as mere subordinate forms”: money loaned to members of the upper classes, particularly landowners, and to artisans and peasants “who possess their own conditions of labour.” But once industrial capital comes to existence and becomes widespread the surplus value is increasingly produced by wage labor in workshops or factories as the surplus of value generated in excess of the value of the means of consumption on which the workers spend their wages. In his theoretical construction Marx builds a model of pure CCP (after building a model of pure SCP). Hence, there the surplus value is exclusively generated by wage workers since it is assumed that there are neither slaves nor serfs.

In the case of industrial capital, the cycle of money capital includes the ‘sphere of production’ and the two phases of the ‘sphere of circulation’. Marx represents the cycle of industrial capital by means of the formula $M - C...P...C' - M'$, where the dots that surround the production process $P$ indicate that it is in production that a change in value takes place through the labor process. In contrast, in the two stages of the circulation process, $M - C$ and $C' - M'$ no value is produced, for these are simply ‘changes in form’ (literally, ‘metamorphoses’) of capital produced in the ‘sphere of circulation’ (from the money capital form to the commodity capital form and vice versa). The industrial capitalist disburses money capital for the purchase of two types of commodities included in $C$: the labor power of wage workers $LP$ and the means of production that workers need to be able to engage in the labor process $MP$. And the

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9 Notice that the term ‘industrial’ is very general for Marx, and includes transportation and communication, agriculture and livestock, entertainment, etc., that is, any non-commercial and non-financial activity in which there is capitalist production.
commodities $C'$ that result from the process of production are the material elements of the commodity capital that must be sold in the market for the capitalist to recover his money capital plus a surplus value (or profit). In Marx’s theory no systematic changes in the value of commodities (or capital) occur in the circulation process and he makes (in Book I) the assumption that all exchanges are of equivalent amounts of value, which implies the assumption that all transactions are done using equilibrium prices.

Once CCP spreads, merchant capital and financial capital, which pre-existed industrial capital by many centuries, become forms that are subordinate to industrial capital. The profits obtained in the commercial and financial spheres are detachments of the gross profits generated in the industrial sphere. The latter is initially appropriated by industrial capitalists when they sell their commodities but end up being redistributed among the all the owners of capital (industrial, commercial, and financial) and landowners (when all of these are introduced in Book III) by means of their payments to their providers of inputs.

The cyclical character of capital

Marx highlights that in any human society the production process can be viewed as a cyclical process that must “continue to go periodically through the same phases” (B1, 565). Furthermore, the production process is one in which both the agents that produce and the productive inputs they use must be reproduced. Hence: “Whatever the form of the process of production in a society, it must be a continuous process, must continue to go periodically through the same phases. A society can no more cease to produce than it can cease to consume. When viewed, therefore, as a connected whole, and as flowing on with incessant renewal, every social process of production is, at the same time, a process of reproduction” (B1, 565).

The means of production that are consumed in the cyclical process necessarily have to be reproduced, which implies that a part of the yearly production cannot be destined to the consumption of individuals. The only way a society can reproduce its material “wealth, and maintain it at one level, is by replacing the means of production –i.e., the instruments of labour, the raw material, and the auxiliary substances consumed in the course of the year– by an equal quantity of the same kind of articles; these must be separated from the mass of the yearly products, and thrown afresh into the process of production” (B1, 566).

And that cyclical process of production and reproduction which exists in any human society adopts particular forms in commodity producing societies. In order to understand these specificities Marx decomposes the exposition of the results of his investigations in a sequence of progressive concreteness: commodities, money, and finally capital. The circulation process is the whole set of exchange relations that are established between possessors of commodities. Money arises out of the circulation of commodities, as a commodity that has been specialized for specific functions, and capital is defined on the basis of the circulation of commodities and money and the profit making aim of those who disburse it. Commodities, money, and capital, all circulate and have a repetitive cycle.

As we have shown for the case of industrial capital, Marx uses a scheme that emphasizes the ‘valorization of capital’ that takes place in the production process: $M - C...P...C' - M'$, where the final money capital $M' = M + m$ includes a surplus value $m$. Both when he presents his theory of the ‘production of capital’ in Book I as when he presents his theory of the ‘circulation of capital’ in Book II, where the
transactions are thought of as "changes of forms" (or metamorphoses), Marx adopts the simplifying assumptions that commodities are sold according to their values and under conditions that are unvarying. For: "In order to conceive these forms in their pure state, one must first of all discard all factors which have nothing to do with the changing or building of forms as such" (B2, 31). The cycle\(^{10}\) of capital with a little more detail is represented by Marx as follows:\(^{11}\)

\[
M - C < \frac{LP}{MP} \left\{ \ldots \right. C + c \left. - (M + m). \right. 
\]

In a first stage, "The capitalist appears as a buyer on the commodity and the labour market" (B2, 31), disbursing his money capital \(M\) in the purchase of labor power \(LP\) and means of production \(MP\) (the value of which add to \(C\)). In the second stage these commodities are consumed productively in the production process: the capitalist "acts as a capitalist producer of commodities", and the "result is a commodity of more value than that of the elements entering into its production." And in the third stage the capitalist returns to the market as a seller of the produced commodities that have the value \(C + c\). These commodities are pregnant with a 'superior value', the monetary expression of which is \(M + m\). With the sale of these commodities the initially disbursed money capital \(M\) is reimbursed with an additional \(m\) which is the monetary expression of the surplus value. In established industrial Capitalism the majority of the sales of industry are purchased by commercial capital, which disburses its capital when it pays the industrial capitalist for the purchase of its finished goods. And it obtains its corresponding part of \(m\) when it produces the final sale of the commodities (to either individuals or firms). Furthermore, another portion of \(m\) may correspond to financial capital, that may have lent to the industrial capitalist a portion of the disbursed capital, and possibly another portion to the landowner that may have leased him the buildings in which the factory functions. In the last stage (possibly after going through other capitalist hands that engage in transportation or retail commercial capital), the commodities sold for the sum \(M + m\) end up in the 'sphere of consumption', i.e., either the consumption of individuals or the productive consumption of other capitalist firms that purchase them as means of production.\(^{12}\)

Marx’s analysis of the cycle of capital is much richer (and much more extensive) than what we have seen so far or describe below. We must necessarily limit ourselves to a short summary. Assuming the Simple Reproduction (SR) of capital (or absence of growth), the cyclical process of capital can be seen as an infinite chain like the following:

\[
\ldots M - C < \frac{LP}{MP} \left\{ \ldots \right. \frac{C}{c} \left. M' < \frac{LP}{MP} \right\} \ldots \frac{C'}{m - c} \ldots 
\]

In the first metamorphosis of capital, money capital \(M\) is transformed into productive capital which is composed of labor force \(LF\) and means of production \(MP\). Then takes place the production process (\(\ldots \)), during which value is produced, resulting

\(^{10}\)MECW 36 uses the word ‘circuit’ instead of ‘cycle’. However, we believe the latter is closer to Marx’s intention. Furthermore, both the Google and Microsoft translators yield “The Cycle of Money Capital” as the translation of Marx’s Der Kreislauf des Geldkapitals.

\(^{11}\)MECW uses \(L\) instead of \(LP\) for labor power.

\(^{12}\)“Even when the commodity is sold over and over again... it falls, when definitely sold for the last time, out of the sphere of circulation into that of consumption, where it serves either as means of subsistence or means of production” (B1, 125, footnote 1).
is commodity capital \((C' = C + c)\) the value of which is greater than that of the productive commodity capital consumed \((C)\). Hence, when the commodity capital is sold the capitalist obtains the originally disbursed money capital \(M\) plus \(m\), the money equivalent of the surplus value, that is, the value of the surplus product \(c\). In SR \(m\) is entirely spent by the propertied classes (capitalists and landowners) in consumption, i.e., their physical reproduction as individuals until the next period. And the reimbursed money capital \(M\) restarts the cycle with a new metamorphosis that is the same as the initial one. In contrast, in the Extended Reproduction (ER) of capital a part of \(m\), instead of being spent in consumption, is invested in the expansion of the productive capital. Hence, the new productive capital \(C'' = (LF' + MP')\) is larger than the initial one and one would need to place \(\ldots P'\ldots C''\) at the end of the scheme above instead of \(\ldots P\ldots C'\).

The wage worker owns his labor force, i.e., the commodity he sells to the capitalist in the transaction \(LF = M\) for the money wage he receives in return. And this is the first stage of the complete cycle for the wage worker, which is \(LF = M - C\), since he thence uses this money to purchase means of subsistence \((C)\) with which he reproduces his labor force. This is a particular case of the “general form of the simple circulation of commodities, \(C - M - C\). Money is in this case merely a passing means of circulation, a mere medium in the exchange of one commodity for another” (B2, 35), as it is in SCP, where there is no distinction between worker and producer. But in the case of the worker in CCP “The productive application of his labour power is not possible until the moment when it is sold and brought into connection with means of production” (B2, 36-7). And these are controlled by the capitalist, as the process as a whole is.

Marx analyzes this cyclical process from three alternative points of view, according to the initial and final points. In the case of SR he describes the following partial cycles, which he respectively calls:

- cycle of money capital: \(M - C\ldots P\ldots C' - M' = M + m\)
- cycle of productive capital: \(P\ldots C' - M - C\ldots P\)
- cycle of commodity capital: \(C' - M' - C\ldots P\ldots C''\).

“The actual cycle of industrial capital in its continuity is therefore not only the unity of the processes of circulation and production but also the unity of all its three cycles” (B2, 108). The cycle presents many complexities that Marx addresses in detail. On the one hand, although he describes a cycle that seems to repeat itself regularly, he highlights that in occasions there are interruptions that temporarily freeze capital, either in its phase of money capital, assuming “the rigid form of a hoard”, or in its phase of productive capital where “the means of production lie without functioning on the one side, while labour power remains unemployed on the other”, or in its phase of commodity capital, as “piles of unsold commodities accumulate” (B2, 58). In his basic analysis Marx assumes that the cycle goes on regularly, and for this to happen “\(C'\) must be sold at its value and in its entirety” and also the formula \(C - M - C\) must include “not merely replacement of one commodity by another, but replacement with value relations remaining the same” (B2, 79). But these assumptions are made merely to facilitate the analysis. “As a matter of fact, however, the values of the means of production vary. It is precisely capitalist production to which continuous change of value relations is peculiar, if only because of the ever changing productivity of labour that characterises this mode of production” (Ibid.). Finally, there is the additional complication that a part of capital, “the labour instruments proper (e. g., machinery), continually serve anew, with more or less numerous repetitions of the same processes of
production, hence transfer their values piecemeal to the products”, a theme he develops in Part II of Book II (and we address in Chapter 12).

The economists that most influenced Marx

The Physiocrats and capital

On several occasions Marx points to his debt (and generally Political Economy’s debt) to the Physiocrats. For example: “The analysis of capital, within the bourgeois horizon, is essentially the work of the Physiocrats. It is this service that makes them the true fathers of modern political economy” (B4.30, 352). They had set the foundations for the analysis of capitalist production. However, the Physiocrats thought in terms of the material components of capital and labor, and had not developed the notion of value nor had they entertained the necessary social conditions for the appearance of capital in production. They had been the first to “explain surplus value by the appropriation of the labour of others, and in fact to explain this appropriation on the basis of the exchange of commodities.” But for them surplus value was the rent paid to landowners and only agricultural labor was productive, that is, labor that produced surplus value, that produced rent. They “did not see that value in general is a form of social labour and that surplus value is surplus labour” since they “conceived value merely as use value, merely as material substance, and surplus value as a mere gift of nature” (Ibid., 360).

Marx also attributes to the Physiocrats the very idea of the cyclical character of capital. Although they confused the cycle of money as an instance of the turnover of capital with the currency of money, “which expresses its steady departure from the starting-point by changing hands again and again” (B2, 340), they were “the first to emphasise the reflux of money to its starting-point as the essential form of circulation of capital, as that form of circulation which promotes reproduction” (Ibid., footnote 33). As evidence he quotes two of the foremost Physiocrats. François Quesnay, who in his Dialogues sur le Commerce et les travaux des artisans (‘Dialogues on Commerce and the work of artisans’) writes: “Cast a glance at the Tableau Economique and you will see that the productive class provides the money with which the other classes buy products from it, and that they return this money to it when they come back next year to make the same purchases... You see, then, no other circle here but that of expenditure followed by reproduction, and of reproduction followed by expenditure, a circle described by the circulation of money, which measures expenditure and reproduction” (Ibid.). For Quesnay the ‘productive class’ was that of agricultural workers. Marx also quotes Turgot, Quesnay’s famous pupil, who in his Réflexions sur la formation et la distribution des richesses (‘Reflections on the formation and distribution of wealth’) writes: “It is this continual advance and return of capitals which should be called the circulation of money, this useful and fertile circulation which gives life to all the labours of society, which maintains the activity and life of the body politic, and which is quite rightly compared to the circulation of blood in the animal body” (Ibid.).

Adam Smith and surplus value

Adam Smith inherited many ideas from the Physiocrats; among them “the forms which capital assumes in circulation (fixed capital, circulating capital...), and in general the connection between the process of circulation and the reproduction process of capital” (B4.30, 353). But in contrast with the Physiocrats he took as starting point the production and interchange of commodities where —before Capitalism— it was labor time that regulated the exchange of commodities. According to Smith, that ceased
to be so “from the moment when the conditions of labour confront the wage labourer in the form of landed property and capital” (B4.30, 379-80). But Marx points out that he should have inferred that where there is capitalist production “the expressions ‘quantity of labour’ and ‘value of labour’ are now no longer identical”. Nevertheless, Marx finds in Smith the virtue of having conceived “surplus value—that is, surplus labour, the excess of labour performed and realised in the commodity over and above the paid labour, the labour which has received its equivalent in the wages— as the general category, of which profit proper and rent of land are merely branches” (B4.30, 388). This merit is however ameliorated by Smith’s not having explicitly defined “surplus value as such as a category on its own, distinct from the specific forms it assumes in profit and rent” (B4.30, 388-9). This had been an even greater source of error for Ricardo. Smith at least “senses that somehow” the law of value “is suspended: more labour is exchanged for less labour... the law of value changes into its opposite”, that is, he senses but does not specify the nature of the exploitation of wage labor that takes place in the production process. On the other hand, the great advance of Smith over the Physiocrats is that for the latter only agricultural labor created surplus value whereas for the former it is “the mere quantity of necessary labour, which creates value” and “rent, profit and interest are only different forms of surplus value” (B4.1, 391-2).

David Ricardo and his theory of value

During the period in which Marx establishes the foundations of his principal theoretical work—basically the 1850s—David Ricardo’s Principles of Political Economy and Taxation was the book of greatest influence in Great Britain in economic matters. It was first published in 1817, and in 1821 a third edition was published with some extensions. It had the merit of having achieved a considerable analytical progress with respect to preceding works on the subject and, in conjunction with a series de pamphlets by the same author, it had also reached a notorious political influence. Hence, it is not surprising that in specifically economic matters (in the present term of the word) Ricardo had great influence on Marx. Ricardo’s Principles was Marx’s principal source for the development of the economic aspects of his theory of Capitalism. An evidence of this is the fact that he dedicated more than 300 pages (out of a total of around 1200) of his Theories of Surplus Value to the analysis of only a part of this book (since he did not address taxation).

Classical Political Economy gave two meanings to the word value. First, it used the word for the property of goods or services of satisfying human needs. In this sense it was used as use value. Second, implicitly assuming the existence of either bartering or buying or selling in exchange for money, the word referred to the power of commodities of being exchanged for other commodities. In this sense it was also used as exchange value. In his Wealth of Nations, Adam Smith distinguished between these two senses: “The word VALUE, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys” (Smith 2005, 30). One of the objectives of classical economists was to explain the exchange value of commodities on the assumption that they had use value. Based on his predecessors, Ricardo centered his attention of the exchange value of reproducible commodities. He held that the exchange value of goods that could not be produced in additional quantities (such as certain paintings, or the grapes of a particular soil) could only be due to their scarcity. However, the majority of commodities could be freely produced “by the exertion of
human industry.”

Since gold (or silver) was the commodity used as money, the *price* of a commodity was its exchange value with respect to gold. Ricardo distinguished two types of prices. The *natural price* of a commodity was the price around which the *market price* fluctuated, according to its momentary scarcity or abundance. The deviations of the market price with respect to the natural price were considered ‘accidental and temporary’. Consequently, Ricardo restricted his analysis to “the laws which regulate natural prices, natural wages and natural profits” (Ricardo 2004, Vol. 1, 92). In order to construct a coherent theory of the determination of natural prices Ricardo had to elucidate the general principles that regulated the exchange value of reproducible commodities, and, in particular, the principles that regulated the exchange value of commodities with gold.

Taking as starting point Adam Smith’s book, though polemicizing with some of his ambiguities, Ricardo states that the fundamental explanation of the exchange value of reproducible commodities lies in the quantity of labor that is necessary to produce them: “If the quantity of labour realized in commodities, regulate their exchangeable value, every increase of the quantity of labour must augment the value of that commodity on which it is exercised, as every diminution must lower it” (Ibid., 13). When referring to the “quantity of labor realized in commodities”, Ricardo included not only the labor used directly in the production process but also the quantity “bestowed on the implements, tools, and buildings, with which such labour is assisted.” This may be expressed as saying that the exchange value of commodities depends not only on the quantity of labor directly necessary to produced them but, also, the labor bestowed on the means of production consumed in their production. If we call the labor bestowed on the consumed means of production ‘labor indirectly necessary’ for the production of a commodity, we can say that for Ricardo the exchange value of commodities depends on the amount of labor directly or indirectly necessary for their production. For brevity, in the context of Ricardian theory we will call the latter concept the *labor value* of commodities.

On the other hand, Ricardo held that the fact that there were different types of labor did not modify the general principle, since he believed that there was a stable relation between the wages of workers of different skills. This relation was established by the market and depended on the amount of learning required in the various professions and on the different intensities (and hence discomfort) of the respective labors. Ricardo believed that this scale for the natural wages of different skills was practically invariant in the short run and hence labors of different skills could be aggregated by weighing them according to their natural wages. This way all labors could be reduced to one and the general principle that the quantity of labor necessary for the production of a commodity determined its exchange value was still valid.

However, since at least the third edition of *Principles* Ricardo no longer believed that the total quantity of labor used up in the production of commodities was an

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13 "We may fairly conclude, that whatever inequality there might originally have been in them, whatever the ingenuity, skill, or time necessary for the acquirement of one species of manual dexterity more than another, it continues nearly the same from one generation to another; or at least, that the variation is very inconsiderable from year to year, and therefore, can have little effect, for short periods, on the relative value of commodities" (Ricardo 2004, 22).

14 "The estimation in which different qualities of labour are held, comes soon to be adjusted in the market with sufficient precision for all practical purposes, and depends much on the comparative skill of the labourer, and intensity of the labour performed. The scale, when once formed, is liable to little variation. If a day’s labour of a working jeweler be more valuable than a day’s labour of a common labourer, it has long ago been adjusted, and placed in its proper position in the scale of value" (Ricardo 2004, 20-21).
exact rule for the determination of their exchange values. In Section I of Chapter 1 he writes: “In the early stages of society, the exchangeable value of these commodities, or the rule which determines how much of one shall be given in exchange for another, depends almost exclusively on the comparative quantity of labour expended on each” (Ricardo 2004, Vol. 1, 12; italics added). In Section III of that chapter, where he introduces the means of production in the “early stages of society”, Ricardo makes the additional assumption that means of production of equal value and durability are used in the different occupations (hunting and fishing in this case). Since we will refer to this assumption several times, for brevity we say he assumes there are equal cost compositions in the different occupations. Ricardo states that when such conditions hold “the value of the deer, the produce of the hunter’s day’s labour, would be exactly equal to the value of the fish, the produce of the fisherman’s day’s labour. The comparative value of the fish and the game, would be entirely regulated by the quantity of labour realized in each; whatever might be the quantity of production, or however high or low general wages or profits might be” (Ibid., 26). If, say, in one day a fisherman normally fishes 20 salmon and the hunter hunts 10 deer, the ‘natural price’ of a deer is 2 salmon, and any changes in wages will bring about an opposite and equal change in profits but will not affect the relative value of the two commodities.

However, Ricardo recognizes that the assumption of equal cost compositions is not realistic. He knows that different occupations, given a certain capital invested, may require durable means of production of different values and different durabilities. And this introduces a ‘considerable modification’ to the principle that exchange values are regulated by labor-values. In the general case, the exchange values will vary not only when the labor-values change but also when wages change. The reason for this new cause of variation in exchange values was that there was a tendency for profit rates to be approximately equal in different industries, for there a propensity for capital to leave industries with a lower profit rate and flow towards those with high profit rates. The natural prices were those that prevailed when the profit rate are equalized across industries.15

Starting from a situation in which the ‘natural prices’ prevail, a wage increase will affect the costs of those industries that have a greater proportion of capital invested in labor relatively more than the cost of others with a smaller proportion. This will imply, at first, a reduction in the profit rate prevailing in the former industries. But this situation cannot last; since capital would flow out of the industries with lower profit rates to those where it is higher, reducing the production of goods in the former industries and increasing their market price (given an unchanged demand) and the opposite would occur in the latter industries. This process continues until the prices in industries with greater wage costs have increased sufficiently in relation to the rest that equalities in profit rates are restored (or their differences again compensate their real or imaginary advantages or disadvantages). In the final situation the market prices are once again equal to the natural prices and the general profit rate is lower than it was initially. But these final natural prices are necessarily different from the original ones. Hence, the changes in the distribution of income between wage workers and capitalists were for Ricardo a second explanatory factor of the relative exchange values

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15Ricardo was quite flexible on this issue. He believed there could be stable differences between the profit rates of different industries since they could reflect asymmetries in “the security, cleanliness, ease, or any other real or fancied advantage which one employment may possess over another”. He writes: “This restless desire on the part of all the employers of stock, to quit a less profitable for a more advantageous business, has a strong tendency to equalize the rate of profits of all, or to fix them in such proportions, as may in the estimation of the parties, compensate for any advantage which one may have, or may appear to have over the other” (Ricardo 2004, 88-90).
of commodities that had to be appended to their relative labor-values.

If changes in the distribution of income (between workers and capitalists) without changes in the labor-values necessarily led to changes in the exchange values of commodities, labor-values could not be the exclusive explanation of exchange value. Since there were different proportions of total capital invested in wages in different industries and there tended to be the same profit rate (except stable differences in profit rates that could compensate for the perception of advantages or disadvantages in the different industries), natural prices were necessarily different from labor-values. Hence, even in the "early stages of society" the law of labor-values was only an approximation to the determination of exchange values. Nevertheless, Ricardo believed that the distribution of income between wage workers and capitalists was very stable in the short run and only varied in the lapse of several years. In contrast, he believed that changes in labor-values occurred more frequently. Therefore, he held that in short-run analysis one could dispense with the second factor in the determination of exchange value and assume that the exchange values of commodities are determined by their labor-values. And Ricardo explicitly makes this assumption: "in the subsequent part of this work, though I shall occasionally refer to this cause of variation, I shall consider all the great variations which take place in the relative value of commodities to be produced by the greater or less quantity of labour which may be required from time to time to produce them" (Ibid., 36-37).

Hence, for Ricardo the ratios between the labor-values of commodities are a reasonable first approximation to their exchange values which would have to be discarded if a better approximation (or the exact rule') was found. On October 9, 1820, when he was probably working on the modifications for the third edition of his Principles, he wrote to Malthus: "You say that my proposition ‘that with few exceptions the quantity of labour employed on commodities determines the rate at which they will exchange for each other, is not well founded’. I acknowledge that it is not rigidly true, but I say that it is the nearest approximation to truth, as a rule for measuring relative value, of any I have ever heard" (Ricardo 2004, Vol. 8, 241-42).

The Ricardian Socialists

As we have seen, Marx attributed the origin of his theory of surplus value to the Physiocrats, to Adam Smith, and to Ricardo's very compact and somewhat confusing exposition in the first two chapters of Principles. But he also recognized the contributions of several post-Ricardian economists whose critiques of Political Economy were based on Ricardo. Marx dedicates Chapter 21 of his Theories to the works of various thinkers usually grouped as ‘Ricardian Socialists’ by historians of economic thought. He addresses two pamphlets: one of 1821 that is anonymous (The Source and Remedy of the National Difficulties) and one of 1824 by Piercy Ravenstone (Thoughts on the funding system and its effects); three works by Thomas Hodgskin (of 1825, 1827 and 1832); and the 1839 book by John Francis Bray, Labour’s Wrongs and Labour’s Remedy. For Marx such works "seize on the mysteries of capitalist production which have been brought to light in order to combat the latter from the standpoint of the industrial proletariat” (B4.32, 374). On the anonymous pamphlet, Marx finds that it "contains an important advance on Ricardo. It bluntly describes surplus-value—or ‘profit’, as Ricardo calls it (often also ‘surplus produce’), or ‘interest’, as the author of the pamphlet terms it— as ‘surplus labour’, the labour which the worker performs gratis, the labour he performs over and above the quantity of labour by which the value of his labour-power is replaced, i.e., by which he produces an equivalent for his
wages." (Ibid.) 16 Although these authors made contributions that had a strong influence on Marx, he streamlined them and integrated them into a completely different and much more organic theory. The category of value that Marx constructed allowed him to define precisely his concept of surplus value after introducing the institutional assumptions that were necessary for defining CCP on the foundations of SCP.

Marx’s theory of value
Marx drew strongly from Ricardo in his analysis of exchange value, but he submitted Ricardo’s theory to an exhaustive analysis and a careful definition of the economic categories involved. Marx’s category of value is a re-elaboration of what we called above labor-value in the context of Ricardo’s theory. However, for Marx the concept of value is not merely a theoretical construction that helps to explain (at least approximately) the exchange values of commodities, but a theoretical concept of its own that he uses as foundation to his theory of the exploitation of labor in the capitalist mode of production, i.e., his theory of surplus value. In his theory of Capitalism, the income categories of the various proprietor classes are portions of the global surplus value, which is exclusively generated in the sphere of industry (in a wide sense that includes agriculture and livestock, transportation, communications, etc.). Marx visualized capitalist production as a process of production of surplus value by workers and initially appropriated by the capitalists in whose factories they worked. The latter then redistributed this surplus value by making payments to other capitalist firms (whether industrial, commercial, or banking), to proprietors of real or financial assets (such as land or loanable funds, respectively), or to the State through tax payments. In contrast with Ricardo, Marx maintains value as a fundamental theoretical category even after explaining (in Book III) that under CCP relative values are no longer the (average) relative equilibrium prices of the commodities produced. Hence, except in Book I where, mainly for communicational reasons, he completely omits the subject matter, Marx keeps a ‘double accounting’ system in large portions of Books II and III. In Marx’s conception, the accounts using values and surplus values was deeper and was not visible to the naked eye; and the visible accounts using prices, wages, profits, rents, etc. were those that concentrated the attention of economists.

As in Ricardo’s theory, in Marx’s theory the propensity to invest in sectors where higher profits rate prevail generates a tendency for the equalization of the profit rates in all sectors. Instead of what we above called ‘cost composition’ in the case of Ricardo, Marx carefully defines his concept of the ‘value composition’ of capital as the ratio between the value of the means of production and the value of labor power, the two components of the capital invested. Since in general these compositions differ among industrial sectors and, in Marx’s view the rate of surplus value (defined as the ratio between the surplus value and the value of the labor power) tends to equality among sectors, those that have a greater proportion of their capital invested in labor power (i.e., have a lower value composition of capital) generate more surplus value than those that have a lower proportion (i.e., have capital of a higher value composition). But the profit rate is defined by Marx as the ratio between the surplus value generated and the capital invested, which Marx also measures in terms of values (even in Book

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16 This chapter of *Theories* is evidently unfinished. The last section, on Bray, only transcribes extensive quotations of his book and hardly has any analysis or commentary. Marx had extensively quoted from Bray’s book in his early *Poverty*, where he refers to *Labour’s Wrongs and Labour’s Remedy* as a “remarkable work”. However, he inserts surprisingly poor comments on its contents, taking as evident that it inspired Proudhon’s theory (the object of his critique). He also quotes from Bray in *Contribution* and, almost in passing, in *Capital*. Bray’s book probably had an influence on some aspects of the sketches Marx made of the two phases of a future communist society.
III, as we shall see). Hence, since the aggregate (or global) profit is equal to aggregate surplus value (leaving aside for the moment the rent on land, commercial and banking capital, and taxes), if the rates of profit in the various industrial sectors are equalized by the flows of capital among sectors, in that process the surplus value generated in the various sectors must necessarily be redistributed from sectors where the value composition is lower (where more surplus value is generated for a given capital) to those where it is higher (where less surplus value is generated for the same amount of capital).

But the process is more complex, since value and surplus value are only generated in the industrial sectors and hence part of the surplus value must turn into commercial and banking profits, interest, rents, and taxes. Hence, the surplus value generated globally is redistributed among capitalists in the industrial, commercial and banking sectors, non-bank lenders, landowners, and the State. In summary, Marx conceives of various stages in the formation of incomes in capitalist society. In the first stage, the industrial production process generates a first broad division of the aggregate net value produced (after subtracting the value of the means of production consumed productively) into the value of the labor power that become the industrial workers’ wages and the surplus value appropriated by industrial capitalists immediately after selling the produced output. In a second stage there is the subsequent redistribution of a part of that surplus value towards the rest of the owning classes and the State by means of the payments they make to the various services consumed in the productive process (commercial commissions, banking or loan capital interests, rents, etc.) and the taxes paid to the State. And in a third stage these various incomes are spent not only in the purchase of goods and services for consumption but also in the wages of dependent workers (servants, chauffeurs, etc.). When there is Extended Reproduction, this third stage also includes the expenditure of a part of the profits in the expansion of the means of production and in the purchase of the corresponding additional labor power.

We have already mentioned that Book I of Capital deals with “the process of production of capital.” Since ‘capital’ in Marx’s sense of the word is produced in an ever increasing scale by means of Extended Reproduction, i.e., the reinvestment of surplus value in the purchase of additional labor power and means of production, that Book deals primarily with the process of production of surplus value, a part of which is channeled to the consumption of the proprietor classes (and the tax collection of the State) and the rest is normally reinvested for the expansion of capital. There was no need to show in Book I that the average prices around which market prices fluctuated could not, in general, be proportional to values (as Ricardo did with his labor-values already in Chapter I of Principles). Furthermore, since Marx was interested in showing that the foundation of Capitalism was the process of production of surplus value that took place in the process of production and wanted to refute those who believed that the exploitation of wage workers was due to workers not being paid the ‘true value’ of their work, he believed his best strategy was to explain the production and accumulation of surplus value on the assumption that all commodities, including labor power, were bought and sold according to their values. If exploitation was compatible with the circulation of commodities according to their values, then its explanation could not reside in the exchange of unequal amounts of value in the circulation process but in a deeper asymmetrical mechanism that took place in the production process.

And that is why in Book I Marx makes the simplifying assumption that commodities are exchanged in proportion to their values. As we will see in Chapters 6 and 8, the latter are proportional to the ‘equilibrium prices’ in SCP but not in CCP, where the
‘equilibrium prices’ are the ‘production prices’ (assuming away other complications like the private ownership of land or monopoly power). In order to avoid in Book I the enormous complications derived from keeping a system of double accounting, in values to obtain surplus values, and in ‘production prices’ to work with ‘equilibrium prices’ under pure CCP with free access to land, he chose to assume throughout Book I that commodities are transacted according to their values. When he took this path he already had very advanced drafts of Books II and III. And he did not imagine in 1867 (when he 49 years old) that his deteriorating health would prevent him from finishing and publishing them. As we will see in Chapter 8, Marx could have simply made in Book I the simplifying assumption that the value compositions of capital are the same in all industrial sectors, which implies that production prices are proportional to values.

When Marx explains that money becomes (industrial) capital when it is invested in the purchase of labor power, he writes: “The conversion of money into capital has to be explained on the basis of the laws that regulate the exchange of commodities, in such a way that the starting point is the exchange of equivalents” (B1, 176). He adds in a footnote: “the formation of capital must be possible even though the price and value of a commodity be the same; for its formation cannot be attributed to any deviation of the one from the other. If prices actually differ from values, we must, first of all, reduce the former to the latter, in other words, treat the difference as accidental in order that the phenomena may be observed in their purity, and our observations not interfered with by disturbing circumstances” (Ibid., footnote 1).

It should be noted that in Marx’s theory the divergence between prices and values are due to several reasons. First, the ‘market price’ of a commodity fluctuates around the average price, or ‘regulating price’ (which we would now call ‘equilibrium price’). Whereas the ‘regulating prices’ in SCP are proportional to the values, in CCP they are proportional to their ‘production prices’ \textsuperscript{17}, which in general are not proportional to their values because different industrial sectors tend to have different value compositions of capital and there is a tendency for the equalization of profit rates in the various sectors (as we explained above). But there is an additional dimension in Marx’s theory, both in his theory of the equilibrium prices in SCP and his theory of the equilibrium prices in CCP. This dimension tends to be ignored (in both cases), and springs from the empirical fact that different producers of a commodity in SCP and different firms that produce the same commodity in CCP normally use different production or organizational techniques. Hence, in SCP individual values differ from market values (or average values) and in CCP individual production prices are different from market prices of production (or average prices of production). \textsuperscript{18}

In Book I Marx leaves aside all these divergences in order to concentrate on the phenomena related to the creation of capital under the simplest of circumstances. The “creation of capital” was accomplished, first, by the creation of surplus value and, second, by the reinvestment of a part of the resulting profits that are neither spent on consumption nor hoarded. \textsuperscript{19} To study the basic features of the creation of capital it

\textsuperscript{17} Notice that this is so only before introducing the private property of land, and hence, rent. When it is introduced the ‘production prices’ must be modified to obtain the ‘equilibrium prices’. This issue is studied in Chapter 16. It is also previous to the process of centralization of capital in modern industry, by which there tends to be monopoly power which also makes the ‘equilibrium prices’ deviate from the ‘production prices’. This issue is addressed in Chapter 17.

\textsuperscript{18} The last part of Chapter 6 addresses this issue for the case of SCP and serves as an introduction to this topic. Chapter 9 addresses this issue for the case of CCP.

\textsuperscript{19} We will see in Chapters 12, 13 and 14 that Marx’s theory of the industrial cycle was based, to a certain extent, on the fluctuations in hoarding.
was unnecessary to go into the complicated issues of the exact relation between values and production prices or the relation between the rate of surplus value and the rate of profit. That is why the expression “production price” does not appear at all in Book I.

As mentioned above, the fact that Marx had already written advanced drafts for Books II and II when he published Book I is proof that he was well aware that the average prices in CCP could not in general be proportional to the average values. But even in Book I the above quote ends with the following: “How can we account for the origin of capital on the supposition that prices are regulated by the average price, i.e., ultimately by the value of the commodities? I say ‘ultimately’, because average prices do not directly coincide with the values of commodities” (B1, 176-7). Hence, there can be no doubt that when Marx published Book I he was very well aware that relative prices did not fluctuate around relative values in CCP except under highly restrictive assumptions. Nevertheless, like Ricardo he believed that “in the last instance” values regulated prices because their changes (brought about exclusively by technical or organizational changes that impinged on the necessary amounts of labor) had an important influence of the production prices, to which he referred indirectly by using “average prices” in Book I because he had not yet defined the concept.

When Marx studies the “process of circulation of capital” in Book II he again makes the assumption that commodities exchange according to their values. Shortly before the publication of the first edition of Book I Engels warned Marx (in his letter of June 26, 1867) that he would be criticized (by “the manufacturer, and with him the vulgar economist”) for not elucidating the relation between the wage, the value of labor power, and surplus value (and hence between surplus value and profits and between value and production price). The next day Marx answered that to elucidate how “the value of the commodity transformed into its price of production” presupposed having elucidated not only “the transformation of, for example, the value of a day’s labour-power into wages or the price of a day’s labour” but also “the transformation of surplus-value into profit, and of profit into average profit, etc.” (MECW 42, 390). And the latter presupposed “that the process of the circulation of capital has been previously explained, since the turnover of capital, etc., plays a part here. This matter cannot therefore be treated prior to the 3rd book.” With respect to the criticism he might reap for not having dealt with all these matters in Book I Marx defended himself writing “Now if I wished to refute all such objections in advance, I should spoil the whole dialectical method of exposition” (Ibid.). Marx’s ‘dialectical method of exposition’ required introducing the complications step by step, that is, eliminating the simplifying assumptions as the theoretical edifice was constructed, starting from the embryonic simple commodity production to the most concrete and complex mode of production and circulation in the era of modern industry.

As we will see in Chapter 14, when Marx successfully addresses the quantitative issues of Extended Reproduction he makes the simplifying assumption that the capitals in the two great spheres of production in which he divides the economy have equal

---

20 In order to conceive these forms in their pure state, one must first of all discard all factors which have nothing to do with the changing or building of forms as such. It is therefore taken for granted here not only that the commodities are sold at their values but also that this takes place under the same conditions throughout” (B2, 31-2).

“Let us, then, consider first the simple reproduction of productive capital, assuming that, as in the first chapter, conditions remain constant and that commodities are bought and sold at their values” (B2, 71).

“It is furthermore assumed that products are exchanged at their values and also that there is no revolution in the values of the component parts of productive capital” (B2, 392).
value compositions. And when he tries to develop Extended Reproduction without this assumption he encounters algebraic difficulties that he manages to cope with by making arbitrary (but intelligent) assumptions on the fraction of surplus value reinvested in each sphere for the expansion of capital. We will also prove that if the various (two or more) sectors of production have equal value compositions of capital (and there is neither private property of land nor monopoly power) the values of commodities are necessarily proportional to their prices of production. Hence, instead of simply assuming that commodities are sold according to their values, Marx could just as well have used the simplifying assumptions that 1) all the firms in each industrial sector use the same productive and organizational techniques, and 2) all the industrial sectors have equal value composition of capital (which makes the production prices proportional to the values). On account of 1), "individual prices" would be the same as "average prices"; and on account of 2) "average prices" would be proportional to values.

Marx criticized the faulty architecture of Ricardo’s Principles. After presenting a detail of the contents of Principles he adds: “The Ricardian theory is therefore contained exclusively in the first 6 chapters of the work. It is in respect of this part of the work that I use the term faulty architectonics” (B4.31, 393). The economic most concrete and complex categories were not defined constructively and with precision starting from the most abstract and simple, as in his own 'dialectical method of exposition', since already in the first chapter “not only are commodities assumed to exist—and when considering value as such, nothing further is required— but also wages, capital, profit and even, as we shall see, the general rate of profit, the various forms of capital as they arise from the process of circulation, and also the difference between ‘natural and market price’” (Ibid.).

Marx exposit the process of production of capital (i.e., of production and reinvestment of surplus value) before the process of circulation of capital because in his ‘materialist conception of history’ the fundamental structure of all societies was given by the whole web of relations that individuals establish in the production process. It was important to explain in Capital the genesis of the specifically capitalist version of the surplus product that all class societies generate and is appropriated by their dominant class. In Capitalism, the value of this surplus product was the surplus value. This put the capitalist mode of production in perspective with respect to pre-capitalist modes of production. The transformation of aggregate surplus value into the incomes of the various proprietor classes could be dealt with after explaining the more fundamental topic of the exploitation of wage workers in the capitalist mode of production and the accumulation of capital based on it.

In a way similar but conceptually more precise than Ricardo’s procedure in Section IV of the first chapter of Principles, in Book III Marx shows that when the value compositions in the various industrial sectors differ commodities cannot be exchanged according to their relative values since this was not compatible with having the same profit rate in all the sectors. He defined the price of production (or production price) by adding profit to the “cost price”, where the elements of cost price were measured in values, profit was the global profit rate times the cost price, and the global profit rate was the ratio between aggregate surplus value and aggregate capital (the elements of which were also measured in values). Thus, the different capitals would redistribute aggregate surplus value through the process of circulation according to the magnitude of their respective capitals. In contrast to Ricardo, however, for Marx this did not weaken the theoretical function of the concept of value since, as we said, for him its fundamental role was that of deriving a precise definition of the concept of surplus
value. And this is why for Marx, in contrast to Ricardo, value in capitalist society is not simply an approximation to the equilibrium price but a fundamental category in itself underlying the more superficial ones of Political Economy.

Convinced that the category of value was important to figure out “the physiology, so to speak, of the bourgeois system” (B4.31, 390), Marx criticizes Ricardo for not maintaining the concepts of value and production price neatly separated. In a letter to Engels (of August 9, 1862), for example, he refers to the “theoretically false dogma deriving from A. Smith—the supposed identity of cost-prices and values of commodities” (MECW 41, 403).21 Marx also criticizes Ricardo for not having precisely distinguished surplus value from profit: “In the critique of Ricardo, we have to separate what he himself failed to separate. His theory of surplus value, which of course exists in his work, although he does not define surplus value as distinct from its particular forms, profit, rent, interest. Secondly, his theory of profit” (B4.31, 397). But beyond such criticisms, Marx recognized Ricardo’s merit in having laid the scientific foundations of his own concept of value. Smith had written “with great naïveté in a perpetual contradiction” between the ‘esoteric’ and the ‘exoteric’ parts of his book. In the esoteric part he traced “the intrinsic connection existing between economic categories or the obscure structure of the bourgeois economic system”, while in the exoteric part he would merely set forth “the connection as it appears in the phenomena of competition and thus as it presents itself to the unscientific observer” (B4.31, 390). In contrast, Ricardo “steps in and calls to science: Halt! The basis, the starting-point for the physiology of the bourgeois system—for the understanding of its internal organic coherence and life process—is the determination of value by labour time. Ricardo starts with this and forces science to get out of the rut... and in general, to examine how matters stand with the contradiction between the apparent and the actual movement of the system. This then is Ricardo’s great historical significance” (B4.31, 391). The first two chapters of Ricardo’s Principles presented “the whole bourgeois system of economy as subject to one fundamental law” (B4.31, 394).

As mentioned in the preceding section, in Contribution Marx begins with the distinction between the concepts of use value and exchange value. In that work he did not yet emphasize the distinction between use value and value, as he would in Part I of Book I of Capital.22 In contrast with exchange value, which is an exchange ratio, value is an absolute measure which is given by a certain amount of (abstract) labor. More specifically, the value of a commodity is an attribute of a commodity as a representative of a class of commodities that all satisfy the same needs (or have the same use value) but can be produced in different ways, i.e., using different technologies and organizational forms. Hence, the magnitude of the value of a commodity does not depend on what commodity is used as a standard of reference, as is true of exchange value. The values of commodities are the exclusive determinants of their mutual exchange values in SCP, since there the equilibrium exchange values are the ratios between the values of the respective commodities, as we will see in Chapter 6.

Also, in Capital Marx bases the distinction between value and value of commodities in the distinction between useful labor and abstract labor. He believed that this distinction was “the pivot on which a clear comprehension of political economy turns” (B1, 51). Useful labor was the specific labor of a certain type (or skill, or specialty) and quality that stems from the division labor and the characteristics of the

---

21 Smith’s cost-price was conceptually equivalent to Marx’s production price and not to his concept of cost-price since the latter did not include the normal profit on capital.

22 In Contribution, for example, Marx refers to the “substance of exchange value” (MECW 29, 271) which in Capital would become the “substance of value”.
worker. Hence, useful labor existed in all social regimes as “an eternal nature-imposed necessity, without which there can be no material exchanges between man and Nature” (B1, 53). For Marx useful labor produces use value, that is, “a thing that by its properties satisfies human wants of some sort or another” (B1, 45). Abstract labor, in contrast, is a theoretical concept that aims to determine human labor as an elementary use of human energy, since “all labour is, speaking physiologically, an expenditure of human labour power” (B1, 56). As its name indicates, abstract labor makes abstraction of the peculiarities of different concrete labors that produce commodities of the same kind, in order to reduce them to a common, elementary, labor that can be aggregated; it is a concept of labor that simply represents human effort. For Marx the value of a commodity is determined by a certain quantity of abstract labor: the quantity that is socially necessary to produce it. To see what this means, let us consider all the different production processes by means of which a certain commodity is produced in a given society (in a given period of time), processes that may differ by the technical and organizational methods employed, by the skills involved in production, by the aptitude and intensity of the labor exerted and by the extension of the labor day.

For Marx, labor creates value\(^\text{23}\), adding it to the value previously materialized in the means of production that are consumed in the production process to conform the value of the end product of the labor process. To quantitatively determine the value that is created Marx theoretically transforms (and holds that this is what the market does) the various concrete labors that participate in the various production processes into a simple and homogeneous labor, that is, abstract labor. This transformation implies, on the one hand, reducing skilled (or complex) labor into non-skilled (or simple) labor and, on the other, obtaining the quantity of that simple labor that is used in average in all of society to produce the quantity demanded and effectively sold of the commodity. This quantity of abstract labor is the quantity that is socially necessary. The substance of value of a commodity is the quantity of abstract labor that is socially necessary to produce it. This substance’ is bestowed on the commodities in the production process. On the other hand, the magnitude of the value of a commodity is the quantity of abstract labor that is socially necessary for the production of one unit of the commodity.\(^\text{24}\) This magnitude is fundamentally determined by the productive power of labor, which itself “is determined by various circumstances, amongst others, by the average amount of skill of the workmen, the state of science, and the degree of its practical application, the social organisation of production, the extent and capabilities of the means of production, and by physical conditions” (B1, 50).

Value is for Marx a theoretical term by means of which the process of production and circulation of commodities can be understood, a process in which commodities are exchanged and the exchange ratios of which must be explained. However, the value of commodities only becomes visible, only manifests itself, and hence can only be perceived cognitively to the extent that it rules the proportions in which commodities exchange, their exchange values. And this only happens in SCP. As already noted, Marx felt that in the capitalistic mode of production the value of commodities ruled their exchange proportions only ‘in the last instance’ because the tendency for the equalization of profit rates in all branches of industry led to the formation of ‘production prices’ which are not in general proportional to values. These production prices

\(^{23}\) All “labour is, speaking physiologically, an expenditure of human labour power, and in its character of identical abstract human labour, it creates and forms the value of commodities” (B1, 56; italics added).

\(^{24}\) Although these concepts may appear blurry and even metaphysical, we will see in Chapters 6 to 9 that they have very precise mathematical representations.
were the true centers upon which market prices fluctuated (assuming away the private property of land and the monopoly power of huge firms in modern Capitalism). We will show in the following chapters that this determination ‘in the last instance’ is not strictly correct and that what Marx should have expressed instead is that the productivity (or productive power) of labor is the ultimate determinant of both values and production prices.

Appendix to Chapter 4
Some contrasts between the lives of David Ricardo and Karl Marx

Ricardo was born in Gloucestershire, England, in 1772, i.e. 46 years before Marx. In contrast with Marx he did not have a formal academic education. He became a theorist of Political Economy starting from the profession of stock exchange broker he inherited from his father. His parents were religious Jews. His paternal grandfather, descendant from a Portuguese family of Sephardic Jews that migrated to Holland in 1680, adopted the profession of dealer in the Stock Exchange of Amsterdam. One of his sons, David’s father, migrated to London in 1760, and a few years later was allocated one of the 12 dealerships reserved for Jews in London. He had 17 children, the third of which was the economist.

An interesting contrast (but also coincidence) between Marx and Ricardo concerns their attitude towards religion. As we have seen in Chapter 1, Marx’s father converted to Christianity when Karl was a child, and Karl rejected all religion since he was young. He had family conflicts that derived from the differing backgrounds between Karl and his future wife. Ricardo, on the other hand, had to overcome a difficult rupture with his family owing to his attitude of religious and social independence. He had the audacity, when 21 years of age, to disobey his paternal mandates by marrying a Quaker girl and furthermore himself becoming a Unitarian Christian25. His mother never forgave him (dying when David was 29 years old) and his father separated him from the family business (King 2013), which made him seek a job elsewhere.

Practical and political affairs, such as the depreciation of the pound, customs tariffs, taxation, etc., led Ricardo to theorize on economics when he had already been very successful in finance. In contrast, Marx became involved in political economy when he was already endowed with a vast historical and philosophical perspective obtained in prestigious German universities and after having been a political exile in several countries. While Ricardo’s intellectual interests involved current economic matters that were of interest in the highest spheres of the United Kingdom, Marx was led by his desire to establish the link between the fundamental conclusions of political economy (adequately re-elaborated) on the functioning of capitalist society and the great social transformations of human history. While Ricardo especially wanted to explain the distribution of income between the great social classes of his time, particularly in the short run, Marx wanted to demonstrate that Capitalism was founded on the exploitation of wage labor by capitalists and that it would eventually be replaced by another mode of production, as had been the case with all the preceding dominant modes of production in human history.

Ricardo wanted to understand the capitalist institutions of his time with not only intellectual and moral aims (like making them better perform for the benefit of all) but also to take advantage of his knowledge for making an immense fortune by shrewdly

25 The Unitarians were Christians that did not believe in the Trinity (Father, Son, and Holy Spirit). Until the enactment in 1813 of the Doctrine of the Trinity Act, denying the Trinity was considered a punishable offense in the United Kingdom.
speculating in the Stock Exchange, which he achieved when he gambled with foresight on Napoleon’s defeat in Waterloo. This allowed him to dedicate time to his “favorite science” and also, after purchasing a seat in the House of Commons, to political activity during the last years of his short life (dying when he was 51 years old from an ear infection). Marx, in contrast, wanted to explain the historical and theoretical foundations of capitalist society and help organize a proletarian party capable of leading the masses towards its suppression and its replacement by a more just society. And he remained poor throughout his life, going through some periods of severe economic hardship.
Chapter 5  INTRODUCTION TO INPUT-OUTPUT ANALYSIS

Despite his exceptional analytical abilities, Marx was limited by his scant knowledge of mathematics. His training had not included the natural sciences. Nevertheless, the mathematics of his time was little developed in comparison to its present state. We can now use mathematical instruments generated after Marx’s death which greatly facilitate the formulation of the more analytical aspects of his theoretical corpus. This chapter presents an introduction to ‘Input-Output analysis’, a matrix representation of the interrelations in an economy which is divided in various sectors that transact among them that is particularly useful for our purposes. We endeavor to make the exposition of this technique as close as possible to the way in which it will be used in the following chapters.

One produced commodity and one type of labor
Consider a community of homogeneous producers/workers, each of which produces a single consumption commodity with a certain amount of hours of his work. We first construct a very simple table that represents a single production process and a single reproductive process, i.e., the consumption process that makes the producers subsist until the next period. It is just a stepping-stone for our representation of SCP, showing the essence of the method in as simple a manner as possible so that we can later introduce the complications that emerge. There is only one kind of producer/worker, and, similarly, there is a single consumption basket for all of them. And since in this most simple version only a single kind of commodity is produced, the consumption ‘basket’ only has that one commodity.

The social matrix and the quantities of commodities and work
Chart 2 exemplifies in numerical form the quantities involved in the global social process, where the symbol ‘⊕’ indicates that the quantities of wheat and work are combined and the symbol ‘→’ indicates that this combination yields what appear on the right hand side after a certain period of time that is given. It is assumed that only wheat is produced and that the production of 25 kilograms (kgs) of wheat requires the productive consumption of 10 kgs of wheat and the use of 20 hours (hs) of work. On the other hand, the producers/workers reproduce their capacity to generate 20 hs of work by consuming 15 kgs of wheat.

Table 2

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10 kgs wheat ⊕ 20 hs work</td>
<td>→</td>
<td>25 kgs wheat</td>
</tr>
<tr>
<td>15 kgs wheat</td>
<td></td>
<td>→</td>
</tr>
<tr>
<td>25 kgs wheat</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice that if we know that the total quantities of wheat and work produced are the elements of the vector \( q = [25 \text{ kgs wheat}, 20 \text{ hs work}] \), the ‘normalized’ processes of Table 2’ give the same information as the processes of Table 2, where the numbers now indicate the inputs that are necessary to produce a unit in each process (the productive and reproductive process, respectively):
The coefficients of the table may be expressed as the elements of a matrix $M$:

$$M = \begin{bmatrix} 0.4 \text{ kgs wheat/kg wheat} & 0.8 \text{ hs work/kg wheat} \\ 0.75 \text{ kgs wheat/h work} & 0 \text{ hs work/h work} \end{bmatrix}.$$  

And if we also keep tacit the units in which the elements of $q$ and $M$ are measured (as we do from here on), all the information of Table 2 can be expressed by the following equality:

$$\begin{bmatrix} 25 & 20 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.75 \end{bmatrix} = 25 \begin{bmatrix} 0.4 \\ 0.75 \end{bmatrix} + 20 \begin{bmatrix} 0.8 \\ 0 \end{bmatrix} = 25,$$

that is,

$$25 \times 0.4 + 20 \times 0.75 = 25,$$

$$25 \times 0.8 + 20 \times 0 = 20.$$

In algebraic notation, this can also be expressed as

$$\begin{bmatrix} q^Q & q^L \end{bmatrix} \begin{bmatrix} A \\ c_L \end{bmatrix} = \begin{bmatrix} q^Q \\ q^L \end{bmatrix},$$

where $q^Q$ and $q^L$ are the quantities of wheat and work produced, $A$ and $\ell$ are the quantities of wheat and work, respectively, that are necessary to produce a kg of wheat, and $c_L$ is the quantity of wheat that is necessary to produce an hour of work. In an even more compact form we have $qM = q$, where the ‘quantities vector’ $q$ and the ‘social matrix’ $M$ are:

$$q \equiv \begin{bmatrix} q^Q \\ q^L \end{bmatrix}, \quad M \equiv \begin{bmatrix} A & \ell \\ c_L & 0 \end{bmatrix}.$$  

Separating the two equations in $qM = q$ we have:

$$q^Q A + q^L c_L = q^Q,$$

$$q^Q \ell = q^L.$$

Also, if the second is replaced in the first and the result is divided by $q^Q$ we get $A + \ell c_L = 1$. In the numerical example we can check that $0.4 + 0.8 \times 0.75 = 1$.

**Effects of some changes in the data**

Let us see how certain changes in the data affect $q$ and $M$.

1. **Change in the unit used for work**  If instead of measuring work in hours, we used days (ds) of 10 hours of duration, Table 2 and (5.1) would be the following:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10 kgs wheat</td>
<td>2 ds work</td>
</tr>
<tr>
<td>15 kgs wheat</td>
<td>2 ds work</td>
</tr>
<tr>
<td>25 kgs wheat</td>
<td>2 ds work</td>
</tr>
</tbody>
</table>
\[
\begin{bmatrix}
25 & 2 \\
7.5 & 0
\end{bmatrix}
\begin{bmatrix}
0.4 & 0.08 \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
25 & 2 \\
0 & 0
\end{bmatrix}.
\]

Changing the unit in which work is measured changes the last row and the last column of the social matrix. Whereas the column corresponding to the technical coefficient of work is divided by 10, the row corresponding to the consumption necessary to produce an hour of work is multiplied by 10. Also, the quantity of work produced is 2 days.

2. **More working hours per day** Going back to Table 2, if we assume that the day of work is extended from 10 to 12 hours, i.e., a 20% increase, and it is further assumed that this has the effect of proportionally increasing production (i.e., there are ‘constant returns to scale’), we have instead of Table 2 and (5.1) the following:

<table>
<thead>
<tr>
<th>Table 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 (=10<em>1.2) kgs + 24 (=20</em>1.2) hs → 30 (=25*1.2) kgs</td>
</tr>
<tr>
<td>18 (=15<em>1.2) kgs + 24 (=20</em>1.2) hs → 42 (=26*1.2) kgs</td>
</tr>
<tr>
<td>30 kgs + 24 hs</td>
</tr>
</tbody>
</table>

\[
\begin{bmatrix}
30 & 24
\end{bmatrix}
\begin{bmatrix}
0.4 & 0.8 \\
0.75 & 0
\end{bmatrix}
= \begin{bmatrix}
30 & 24 \\
0 & 0
\end{bmatrix}. \tag{5.6}
\]

Since more is produced in the simple commodity producing community and the producers consume all the additional (net) production (and there are constant returns to scale), there is no change in the social matrix. The only changes are that the production (and consumption) of wheat and the exertion of work increase. Also, due to the assumption of increasing returns to scale, each increased in the same proportion as the expansion in the day of labor (i.e., 20%). This assumption may seem quite restrictive, especially when the working day is already long. But the simplicity that the assumption yields to the analysis may more than compensate for the lack of realism. On the other hand, Marx in many cases presents examples where the coefficients in the top row of the matrix change over time. For example, when considering a year with bad harvests there is an increase in the coefficient of direct labor requirements because more work is needed per unit of output simply because less is produced.

Let us now assume that \( q^L \) represents the population of producers/workers, instead of the number of hours of work exerted. In this case \( q^L = 20 \) both before and after the expansion of the labor day. But since 20% more is produced, each producer consumes \( 0.75 \times 1.2 \) kgs of wheat, and hence the consumption basket is \( c_L = 0.75 \times 1.2 \). And the number of workers per kg of wheat produced diminishes from 0.8 to 0.8/1.2. Hence, after the expansion of the working day instead of (5.6) we have:

\[
\begin{bmatrix}
30 & 20
\end{bmatrix}
\begin{bmatrix}
0.4 & 0.8/1.2 \\
0.75 \times 1.2 & 0
\end{bmatrix}
= \begin{bmatrix}
30 & 20 \\
0 & 0
\end{bmatrix}.
\]

3. **Technological progress** The equality in (5.2) indicates that all that is produced is consumed in the very same processes (of production and reproduction). Hence, we can say that the community of simple commodity producers is in a state of Simple Reproduction (SR), a term coined by Marx. But the quantities system could be altered
by an exogenous change. For example, if the discovery of a new process for producing wheat makes it possible to productively consume less wheat with the same amount of work in order to produce the same quantity of wheat, instead of Table 2 and (5.1) we would have, say:

Table 5

| 8 kgs | ⊕ | 20 hs | → | 25 kgs |
| 15 kgs | → | 20 hs |
| 23 kgs | 20 hs |

\[
\begin{bmatrix} 25 & 20 \end{bmatrix} \begin{bmatrix} 0.32 & 0.8 \\ 0.75 & 0 \end{bmatrix} = \begin{bmatrix} 23 & 20 \end{bmatrix} \leq \begin{bmatrix} 25 & 20 \end{bmatrix}
\] (5.7)

that is,

\[
qM^* \leq q
\] (5.8)

where \(M^*\) is the changed social matrix. There is now a surplus of 2 kgs of wheat that can be used either to increase the level of consumption of the producers and/or to increase the scale of production and reproduction. We can alternatively write (5.7) as:

\[
\begin{bmatrix} 25 & 20 \end{bmatrix} \begin{bmatrix} 0.32 & 0.8 \\ 0.75 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} 25 & 20 \end{bmatrix}
\] (5.9)

or, with \(\Delta q = [2 \ 0]\):

\[
qM^* + \Delta q = q.
\] (5.10)

If the surplus produced is used to increase the level of consumption of the producers, there is still SR. In that case, the reproduction process of producers changes since they begin to consume 17 kgs of wheat instead of 15, a 13.33% increase. Table 2 now becomes:

Table 6

| (1 - 0.2) * 10 = 8 kgs | ⊕ | 20 hs | → | 25 kgs |
| (1 + 0.1333) * 15 = 17 kgs | → | 20 hs |
| 25 kgs | 20 hs |

The new social matrix is:

\[
M^{**} = \begin{bmatrix} (1 - 0.2) * 0.4 & 0.8 \\ (1 + 0.1333) * 0.75 & 0 \end{bmatrix} = \begin{bmatrix} 0.32 & 0.8 \\ 0.85 & 0 \end{bmatrix}
\]

and we have \(qM^{**} = q\), that is, SR is preserved:

\[
\begin{bmatrix} 25 & 20 \end{bmatrix} \begin{bmatrix} 0.32 & 0.8 \\ 0.85 & 0 \end{bmatrix} = \begin{bmatrix} 25 & 20 \end{bmatrix}.
\] (5.11)

Comparing with (5.1) we see that the quantity vector (\(q\)) is the same. But \(A\) has diminished by 20% while \(c_L\) has increased 13.33%. As long as all the increase in the

\(^1\) If \(a\) and \(b\) are vectors, \(a \leq b\) means that all the elements of \(a\) are less than or equal to the corresponding elements of \(b\) but they are not all equal, i.e., \(a \leq b\) and \(a \neq b\).
net product produced by the technological advance is used to increase consumption, the consumption coefficient of the social matrix can be increased until equality in (5.8) is reestablished. Algebraically, we have

\[
\begin{pmatrix}
q^Q & q^L \\
\end{pmatrix}
\begin{pmatrix}
(1 - \delta) A & \ell \\
(1 + \mu) c_L & 0 \\
\end{pmatrix} =
\begin{pmatrix}
q^Q & q^L \\
\end{pmatrix},
\]

where \(\delta\) is the proportion in which \(A\) diminishes and \(\mu\) is the proportion in which \(c_L\) increases while preserving SR.\(^2\)

But if the surplus produced were used, at least partially, to increase the scale of production (instead of increasing as much as possible the consumption of the existing producers), instead of SR we could have Extended Reproduction (ER), a term also coined by Marx. Thus more workers could be incorporated to the production process, with the consequent expansion of the production of commodities and the reproduction of workers. Because it is more complex, we will leave ER for Chapter 14 and in the rest of this chapter we concentrate on Input-Output analysis under SR.

**Multiple produced commodities and one type of labor**

So far we have considered the production of a single commodity and the reproduction of a single type of producer. In this subsection we generalize the analysis to the production of multiple commodities, each produced by means of the action of a producer on already produced commodities (inputs). Assume there is a certain number of independent private producers who sell their output (whether they are goods or services) in the market. Assume also that each one produces a *single* output and that all the producers that produce the same output use the same technique of production. In modern technical language, we can say that all the processes, whether productive or reproductive, produce a *simple* output (that is, there is no joint production of commodities, or equivalently, no process produces multiple outputs), with constant proportions between inputs, and constant returns to scale. The last two assumptions make it possible to specify each process by means of constant technological coefficients. But we have already seen that one can still study the effects of an exogenous change in one or more coefficients. In the sphere of production, hence, there are as many *productive* processes as there are commodities, and each process can be operated in parallel by many individual producers that mutually compete. Each producer uses inputs produced by other processes (and possibly also by their own productive process) and his own work to produce a commodity. Also, each producer consumes commodities that, jointly, form his consumption basket. This constitutes the *reproductive* process of his working capacity.\(^3\) We further assume here that the reproduction (or consumption) process does not require work for its realization.

Table 7 is analogous to Table 2 except that it has two produced commodities instead of one. When all processes are normalized as we did above we obtain a square matrix \(A\) of technical coefficients where element \(A_{ij}\) represents the quantity of \(j\) that is necessary

---

\(^2\)In the numerical example \(q\) remained unchanged. This is due to the simplicity of the example. Further below \(A\) will be a matrix and \(q^Q\) a row vector. And the changes in the elements of \(A\) can be arbitrarily complex. Hence, the structure of \(q^Q\) (and hence that of \(q\)) will normally change even if RS is preserved.

\(^3\)Obviously, we are dealing with the reproduction of the producer and his dependents from one period to the next, and not the reproduction of the producer as a member of the human species. When we consider ER, however, we will additionally take into account the (biological) growth of the population and, hence, of working capacity.
to produce a unit of $i$.\footnote{The reader is warned that it has been more usual in the literature to define $A_{ij}$ as the quantity of $i$ necessary to produce a unit of $j$. We have preferred to have quantities as row vectors and prices as column vectors. Mathematically, the two procedures are equivalent.} We also have a column vector of labor coefficients, $\ell$, where $\ell_i$ is the quantity of work that is necessary to produce a unit of commodity $i$, and a row vector of consumption coefficients $c_L$, where $c_{Li}$ is the quantity of $i$ necessary to reproduce a unit of the producer.

Table 7

<table>
<thead>
<tr>
<th></th>
<th>2 scythes</th>
<th>8 kgs wheat</th>
<th>5 hs work</th>
<th>→</th>
<th>5 scythes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 scythes</td>
<td>2 kgs wheat</td>
<td>15 hs work</td>
<td>→</td>
<td>25 kgs wheat</td>
<td></td>
</tr>
<tr>
<td>1 scythes</td>
<td>15 kgs wheat</td>
<td>→</td>
<td>20 hs work</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| 5 scythes | 25 kgs wheat | 20 hs work |

The numerical values that correspond to the elements of (5.2) are the following:

\[
A = \begin{bmatrix}
  2/5 & 8/5 \\
  2/25 & 2/25
\end{bmatrix}, \quad \ell = \begin{bmatrix}
  5/5 \\
  15/25
\end{bmatrix}, \quad c_L = \begin{bmatrix}
  1/20 & 15/20
\end{bmatrix}
\]

\[
q^Q = \begin{bmatrix}
  5 \\
  25 \\
  20
\end{bmatrix}, \quad q^L = 20,
\]

and we can check that the equalities hold:

\[
\begin{bmatrix}
  5 \\
  25 \\
  20
\end{bmatrix}
\begin{bmatrix}
  2/5 & 8/5 & 5/5 \\
  2/25 & 2/25 & 15/25 \\
  1/20 & 15/20 & 0
\end{bmatrix} = \begin{bmatrix}
  5 \\
  25 \\
  20
\end{bmatrix}.
\]

Table 8 represents the case in which $n$ commodities are produced. Each entry (except those of the last row and the last column) is decomposed as the product of the level of gross production of the process and an input-output coefficient. It is evident that each one of the rows may divided by the level of gross production of the process to obtain the row of technical coefficients that can produce one unit of the commodity, as we had above. Also, since in each one of the first $n$ columns there are quantities of the same commodity (that are inputs to the production of the various commodities and to the reproductive process), their elements can be added. Finally, column $n + 1$ contains the quantities of work used in the various productive processes and the sum of its elements gives $q^L$, the total working capacity exercised.

Table 8

| $q_1^Qa_{11}$ | $q_1^Qa_{12}$ | ... | $q_1^Qa_{1n}$ | $q_1^Q\ell_1$ | → | $q_1^Q$ |
| $q_2^Qa_{21}$ | $q_2^Qa_{22}$ | ... | $q_2^Qa_{2n}$ | $q_2^Q\ell_2$ | → | $q_2^Q$ |
| ... | ... | ... | $q_n^Qa_{n1}$ | $q_n^Qa_{n2}$ | ... | $q_n^Qa_{nn}$ | $q_n^Q\ell_n$ | → | $q_n^Q$ |
| $q^Lc_1$ | $q^Lc_2$ | ... | $q^Lc_n$ | 0 | → | $q^L$ |

\[
q_1^Q \quad q_2^Q \quad ... \quad q_n^Q \quad q^L
\]
Due to the fact that the columns can be added (because they represent the same physical units), the accounting aspect of Table 8 can be written in matrix format as:

$$
\begin{bmatrix}
q_1^Q & q_2^Q & \ldots & q_n^Q & q^L
\end{bmatrix}
\begin{bmatrix}
a_{11} & a_{12} & \ldots & a_{1n} & \ell_1 \\
a_{21} & a_{22} & \ldots & a_{2n} & \ell_2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
a_{n1} & a_{n2} & \ldots & a_{nn} & \ell_n \\
c_1^L & c_2^L & \ldots & c_n^L & 0
\end{bmatrix}
= \begin{bmatrix}
q_1^Q & q_2^Q & \ldots & q_n^Q & q^L
\end{bmatrix}.
$$

(5.14)

More compactly we have (5.2) and (5.3) where now \( q^Q = \left( q_1^Q \ldots q_n^Q \right) \) is a vector of dimension \( 1 \times n \). Also, in the social matrix \( M, A \) is now a square matrix of dimension \( n \times n \), \( \ell \) is a matrix (or column vector) of dimension \( n \times 1 \) and \( c_L \) is matrix (or row vector) of dimension \( 1 \times n \). The first \( n \) rows of \( M \) that is, the rows of \( [A \ \ell] \), represent the productive processes, and the last row \( [c_L \ 0] \) represents the consumption process, or reproductive process of the producers.

With this extended notation the components (5.4)-(5.5) of the quantity system (5.2) are still valid. (5.4) shows that total output of the different commodities, \( q^Q \), is the sum of the means of production consumed productively in the various sectors, \( q^Q A \), plus the net product that is consumed by the producers, \( q^L c_L \). And (5.5) shows how, by means of the vector \( \ell \), the different producers/workers are distributed among the various productive processes. As we saw in Table 3, the numerical value of \( q^L \) (as well as the elements of \( \ell \)) depend on the unit of measurement used for the work of the producers. Inserting (5.5) in (5.4) we get:

$$
q^Q(A + \ell c_L) = q^Q,
$$

(5.15)

where \( q^Q \) cannot now be eliminated from the equation because it is a vector, and \( \ell c_L \) is a square matrix of the same dimensions as \( A \):

$$
\ell c_L =
\begin{bmatrix}
\ell_1 \\
\ell_2 \\
\vdots \\
\ell_n
\end{bmatrix}
\begin{bmatrix}
c_1^L & c_2^L & \ldots & c_n^L
\end{bmatrix}
= \begin{bmatrix}
\ell_1 c_1^L & \ell_1 c_2^L & \ldots & \ell_1 c_n^L \\
\ell_2 c_1^L & \ell_2 c_2^L & \ldots & \ell_2 c_n^L \\
\vdots & \vdots & \ddots & \vdots \\
\ell_n c_1^L & \ell_n c_2^L & \ldots & \ell_n c_n^L
\end{bmatrix}.
$$

Hence,

$$
A + \ell c_L =
\begin{bmatrix}
a_{11} + \ell_1 c_1^L & a_{12} + \ell_1 c_2^L & \ldots & a_{1n} + \ell_1 c_n^L \\
a_{21} + \ell_2 c_1^L & a_{22} + \ell_2 c_2^L & \ldots & a_{2n} + \ell_2 c_n^L \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1} + \ell_n c_1^L & a_{n2} + \ell_n c_2^L & \ldots & a_{nn} + \ell_n c_n^L
\end{bmatrix}.
$$

(5.16)

(5.15) is a compact way of representing the productive interrelation between inputs and outputs in SCP obtained by eliminating the explicit formulation for the population of producers \( q^L \) by means of the extension of the input-output matrix \( A \) to matrix \( A + \ell c_L \). For some purposes it is convenient to start from equation (5.15) and then go to (5.2).

We saw that in Table 8 it is possible to add the elements of each column because they are all quantities of the same output. But the elements of the rows cannot be added. Each row shows how in each productive process the producers transform various inputs to obtain a certain quantity of a commodity and how in the reproductive process the producers transform quantities of commodities (that are consumed) for the reproduction of their working capacity. But if each input \( i \) is multiplied by a ‘price’ \( p_i \) and the amount of work exerted is multiplied by a ‘price’ \( w \) that represents the
producer’s income per unit of work, it becomes possible to also add the elements of the rows, as we see in Table 9:5

<table>
<thead>
<tr>
<th>$q_1^O a_{11} p_1$</th>
<th>$q_1^O a_{12} p_2$</th>
<th>$\ldots$</th>
<th>$q_1^O a_{1n} p_n$</th>
<th>$q_1^O \ell_1 w$</th>
<th>$q_1^O p_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2^O a_{21} p_1$</td>
<td>$q_2^O a_{22} p_2$</td>
<td>$\ldots$</td>
<td>$q_2^O a_{2n} p_n$</td>
<td>$q_2^O \ell_2 w$</td>
<td>$q_2^O p_2$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$q_n^O a_{n1} p_1$</td>
<td>$q_n^O a_{n2} p_2$</td>
<td>$\ldots$</td>
<td>$q_n^O a_{nn} p_n$</td>
<td>$q_n^O \ell_n w$</td>
<td>$q_n^O p_n$</td>
</tr>
<tr>
<td>$q^L c_{11}^T p_1$</td>
<td>$q^L c_{12}^T p_2$</td>
<td>$\ldots$</td>
<td>$q^L c_{1n}^T p_n$</td>
<td>$0$</td>
<td>$q^L w$</td>
</tr>
</tbody>
</table>

If we now divide each row by the corresponding quantity ($q_i^O$ or $q^L$) we obtain $n + 1$ equalities (corresponding to the first $n + 1$ rows of the Table) that can be put in the following matrix format:

$$
\begin{bmatrix}
  a_{11} & a_{12} & \ldots & a_{1n} & \ell_1 \\
  a_{21} & a_{22} & \ldots & a_{2n} & \ell_2 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  a_{n1} & a_{n2} & \ldots & a_{nn} & \ell_n \\
  c_{11}^T & c_{12}^T & \ldots & c_{1n}^T & 0
\end{bmatrix}
\begin{bmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_n \\
  w
\end{bmatrix}
= 
\begin{bmatrix}
  p_1 \\
  p_2 \\
  \vdots \\
  p_n \\
  w
\end{bmatrix}.
$$

(5.17)

Given a matrix $C$, we say that the equation $Cp = p$ is the dual of the equation $qC = q$. Hence (5.17) is the dual of (5.14). Finally, if we eliminate the sums and equalities of Table 9 we obtain a simplified version of what is presently called ‘intersectoral transactions’ table, or ‘input-output’ table:

<table>
<thead>
<tr>
<th>$q_1^O a_{11} p_1$</th>
<th>$q_1^O a_{12} p_2$</th>
<th>$\ldots$</th>
<th>$q_1^O a_{1n} p_n$</th>
<th>$q_1^O \ell_1 w$</th>
<th>$q_1^O p_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2^O a_{21} p_1$</td>
<td>$q_2^O a_{22} p_2$</td>
<td>$\ldots$</td>
<td>$q_2^O a_{2n} p_n$</td>
<td>$q_2^O \ell_2 w$</td>
<td>$q_2^O p_2$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td>$q_n^O a_{n1} p_1$</td>
<td>$q_n^O a_{n2} p_2$</td>
<td>$\ldots$</td>
<td>$q_n^O a_{nn} p_n$</td>
<td>$q_n^O \ell_n w$</td>
<td>$q_n^O p_n$</td>
</tr>
<tr>
<td>$q^L c_{11}^T p_1$</td>
<td>$q^L c_{12}^T p_2$</td>
<td>$\ldots$</td>
<td>$q^L c_{1n}^T p_n$</td>
<td>$0$</td>
<td>$q^L w$</td>
</tr>
<tr>
<td>$q_1^O p_1$</td>
<td>$q_2^O p_2$</td>
<td>$\ldots$</td>
<td>$q_n^O p_n$</td>
<td>$q^L w$</td>
<td></td>
</tr>
</tbody>
</table>

**Perron-Frobenius theory**

At this point it is convenient to introduce some mathematical tools that have proved to be of great utility in the analysis of the type of linear equation systems that often appear in economic theory and, in particular, in Marx’s theory. Readers with no mathematical training can simply omit this section or just glance over it. We begin with some basic algebraic concepts. If $x$ is a vector composed of $n$ real non-negative numbers that are not all zero (that is, $x \geq 0$, $x \neq 0$) we say that $x$ is semi-positive, which is represented by $x \geq 0$. Similarly, we say that a non-negative and non-null matrix $A$ is semi-positive, which is represented as $A \geq 0$. The matrices dealt with in this book are always real (i.e., have no complex entries) and semi-positive.

---

5We do not here go into the nature of these ‘prices’. They simply make it possible to express the ‘exchange value’ of each commodity in terms of one that so far is still unspecified.

6See Miller and Blair (2009).
Let $A$ be a square semi-positive matrix of dimension $n \times n$. If there exists a semi-positive vector $x \geq 0$ such that $Ax = \lambda x$, where $\lambda$ is a scalar (which in general may be a real or complex number), then it is said that $\lambda$ is an eigenvalue of $A$ and $x$ is an eigenvector of $A$ associated to $\lambda$. Evidently, any multiple $ax$ (where $a$ is a real non-zero number) of an eigenvector $x$ is also an eigenvector (associated to the same $\lambda$). Hence, when we refer to the uniqueness of an eigenvector we are usually neglecting scalar factors such as $a$, which can be expressed as ‘unique up to a scalar factor’. For $x$ to be unique without qualifications we must ‘normalize’ it by choosing a row vector $\beta$ with the same number of elements as $x$ and impose the equality $\beta x = 1$. The set of eigenvalues of a matrix $A$ (called the spectrum of $A$) is the same as that of the transposed matrix $A^T$, where $(A^T)_{ij} = A_{ji}$.

The theory of Perron-Frobenius (which we henceforth refer to simply as ‘Perron-Frobenius’) consists of a series of theorems that include advances made by Frobenius (1908, 1909, 1912) to the initial findings of Perron (1907). The basic theorem states that a) a real square non-negative matrix $A$ has at least one non-negative eigenvalue $\lambda \geq 0^*$. Let us call the greatest of these non-negative eigenvalues the dominant eigenvalue of $A$, and refer to it as $\lambda(A)$. Also, b) $A$ has at least one semi-positive eigenvector $x \geq 0$ associated with $\lambda(A)$. Hence, there exist an $x \geq 0$ and a $\lambda(A) \geq 0$ such that $Ax = \lambda(A)x$.

A stronger version of this theorem obtains when $A$ is an indecomposable matrix, that is, when it is not decomposable. A square matrix $A$ is said to be decomposable if it is possible to permute its rows and columns (with the same permutation for rows as for columns) in such a way that it adopts the following structure:

$$A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix},$$

(5.18)

where $A_{11}$ and $A_{22}$ are square and 0 is a submatrix of zeros. We know that the transpose $A^T$ is decomposable (indecomposable) if and only if $A$ is decomposable (indecomposable).

For the input-output matrices used in this book the permutation of rows and columns simply means renumbering the commodities, which makes the coefficients change their locations in a systematic way without any conceptual change from the economic point of view. Since $A_{ij}$ represents the quantity of $j$ necessary for the production of one unit of $i$, $A_{ij} > 0$ means that $j$ is a necessary direct input for the production of $i$. If there is a sequence $k_1, \ldots, k_4 \in \{1, 2, \ldots, n\}$ such that $A_{ik_1}A_{k_1k_2} \ldots A_{k_4j} > 0$ (which implies that each of these coefficients is positive), then $j$ is directly or indirectly necessary for the production of $i$. It can be proved that $A$ is indecomposable if and only if any commodity is directly or indirectly necessary for the production of any other commodity.

If $A \geq 0$ is indecomposable, Perron-Frobenius yields stronger (and for us more convenient) statements: a) $A$ has at least one positive eigenvalue $\lambda > 0$, the greatest of which is the dominant eigenvalue $\lambda(A)$, b) $A$ has a positive eigenvector $x > 0$ associated with $\lambda(A)$, which is the unique (up to a scalar factor) positive eigenvector associated with $A$. We will call $x$ the dominant eigenvector. An important property of $\lambda(A)$ when $A$ is indecomposable is that if one or more of the elements of $A$ increases (diminishes) $\lambda(A)$ also increases (diminishes). In contrast, if the matrix is decomposable, it can only be asserted that if one or more of the elements of $A$ increases (diminishes) $\lambda(A)$

---

7Notice that this implies that $\lambda$ is a real number (and hence, is not a complex number).

8There is more detail on this in the Mathematical Appendix to this chapter.
does not diminish (does not increase).\(^9\)

We have seen that if \(A\) is indecomposable there exists a unique dominant eigenvalue \(\lambda(A) > 0\) and a unique (up to a scalar factor) dominant right eigenvector \(x > 0\), that is, such that \(Ax = \lambda(A)x\). Since \(A^T\) is indecomposable if and only if \(A\) is, and they both have the same eigenvalues (the same spectrum) and, in particular, the same dominant eigenvalue, Perron-Frobenius ensures that there exists a unique (up to a scalar factor) \(z > 0\) such that \(A^Tz = \lambda(A)z\). Hence, defining \(y \equiv z^T\) and transposing, we have \(yA = \lambda(A)y\). That is, \(A\) also has a unique (up to a scalar factor) left dominant eigenvector \(y\).

Another concept that is useful in Input-Output analysis is that of productive matrix. An input-output matrix \(A\) is productive when \(\lambda(A) < 1\), that is, when it has a dominant eigenvalue that is less than one.\(^10\)

Production functions

Neoclassical economics developed the concept of ‘production function’, which is a mathematical way of representing the functional relation between inputs and the quantity of the output that they can produce. For this it would take labor as just another input, along with screws or motors. Classical economists generally avoided this procedure but their formal techniques were very primitive, merely numerical exercises. Marx had too good a philosophical training to fall into such simplistic reductionism but when he formulated his SR and ER schemes he too had to consider labor power as a resource managed by the capitalist. From a purely formal point of view it is possible to make abstraction of the specificity of human labor and assume it is just another ‘input’ that those in charge of the production process handle. This is obviously only an approximation to a much more complex reality: screws or motors do not go on strikes or shirk on the intensity of their use. And even for the representation of Marx’s SCP, where it is the worker who is in charge of the production process, it is possible to assume that the producer/worker, for certain purposes, considers his work time as just another input.

Although the fixed coefficients technology we use in this book is particularly simple, for certain purposes it is convenient to express it in the format of production functions, that is, in the format \(y = f(x_1, x_2)\), where the function \(f(.)\) indicates that the combination of quantities \(x_1\) of input 1 and \(x_2\) of input 2 can produce \(y\) units of output. The production function can be quite complicated. In this book we restrict the analysis to fixed coefficients, which can be represented by means of the function that chooses the minimum of various quantities. Let us start with the case of SCP in which only two goods are produced \((n = 2)\). Let \(q^Q_j\) be the quantity of commodity \(j\) used as input in the production of commodity \(i\), and \(q^T_i\) the quantity of work that uses it in the given time period. Let \(C_i\) be the quantity of commodity \(i\) that is consumed in the reproduction process of the producer/worker. Then the production functions of each of the two commodities produced and the reproduction function of the producers

\(^9\)For a more formal approach to the mathematical concepts here summarized see Gantmacher (1959), Vol. 2, and Nikaido (1978). Some of the theorems in these books (and some others) are reproduced in the Mathematical Appendix to this chapter.

\(^10\)Another (equivalent) way of characterizing a productive matrix is that it satisfies the so-called ‘Hawkins-Simon conditions’. Cfr. Nikaido (1978).
can be written as:

\[
q_1^Q = \min \left( \frac{q_{11}^Q}{a_{11}}, \frac{q_{12}^Q}{a_{12}}, \frac{q_1^L}{\ell_1} \right) \\
q_2^Q = \min \left( \frac{q_{21}^Q}{a_{21}}, \frac{q_{22}^Q}{a_{22}}, \frac{q_2^L}{\ell_2} \right) \\
q^L = \min \left( \frac{C_1}{c_{L1}}, \frac{C_2}{c_{L2}} \right).
\]

Here the eight coefficients \(a_{11}, a_{12}, a_{21}, a_{22}, \ell_1, \ell_2, c_{L1}, c_{L2}\), are considered fixed. The ‘min’ function chooses the minimum of the arguments (in this case quotients) between the parentheses. The function expresses that in principle there could be one or more inputs that limit the level of production and there can be inputs the available quantities of which exceed those necessary to complement the inputs that limit production. The expressions in (5.19) imply, respectively, the following inequalities:

\[
q_1^Q a_{11} \leq q_{11}^Q, \quad q_1^Q a_{12} \leq q_{12}^Q, \quad q_1^Q \ell_1 \leq q_1^L \\
q_2^Q a_{21} \leq q_{21}^Q, \quad q_2^Q a_{22} \leq q_{22}^Q, \quad q_2^Q \ell_2 \leq q_2^L \\
q_1^L c_{L1} \leq C_1, \quad q_2^L c_{L2} \leq C_2,
\]

where in each row at least one of the inequalities holds with equality.

On the other hand, the quantities produced of commodities 1 and 2 \((q_1^Q\) and \(q_2^Q\)) limit the uses they can be put to as inputs in the production or reproductive processes and the quantity or workers reproduced limits their application to the productive processes of commodities. Hence, the following inequalities also hold:

\[
q_{11}^Q + q_{21}^Q + C_1 \leq q_1^Q, \quad q_{12}^Q + q_{22}^Q + C_2 \leq q_2^Q, \quad q_1^L + q_2^L \leq q^L.
\]

Using the inequalities of (5.20) in the corresponding rows of (5.21) the following inequalities result, which only involve the fixed coefficients and the total quantities of produced commodities and reproduced producers:

\[
q_1^Q a_{11} + q_2^Q a_{21} + q^L c_{L1} \leq q_1^Q, \quad q_1^Q a_{12} + q_2^Q a_{22} + q^L c_{L2} \leq q_2^Q, \quad q_1^Q \ell_1 + q_2^Q \ell_2 \leq q^L.
\]

And these inequalities can be expressed in matrix form:

\[
\begin{bmatrix}
q_1^Q & q_2^Q & q^L \\
\end{bmatrix}
\begin{bmatrix}
a_{11} & a_{12} & \ell_1 \\
a_{21} & a_{22} & \ell_2 \\
c_{L1} & c_{L2} & 0 \\
\end{bmatrix}
\leq
\begin{bmatrix}
q_1^Q & q_2^Q & q^L \\
\end{bmatrix},
\]

or, in compact notation: \(qM \leq q\). If matrix \(M\) has one as dominant eigenvalue \((\lambda(M) = 1)\) and the quantities produced and reproduced are the elements of the dominant eigenvector, we know from Perron-Frobenius that we can replace the inequality by an equality. But if in (5.22) there is one or more strict inequality we will necessarily have \(\lambda(M) < 1\). In that case, assuming \(M\) is initially indecomposable, one or more
of the components of $c_L$ (the consumption basket of each producer) can be increased until $\lambda(M) = 1$.\footnote{Notice that if a matrix is indecomposable it will not cease to be so if any of its coefficients are increased.} In that case we reach an expression like that in (5.23) but with equalities instead of inequalities. And if we assume the quantity of available work $q^L$ is given, we also know that the eigenvector $q$ that satisfies $qM = q$ is unique.\footnote{Notice that $q$ is then normalized by $q(u_2/q^L) = 1$, where $u_2 = (0, 0, 1)^T$, i.e., the third element of $q$ is $q^L$.}

In order to represent the functional relation between outputs and available work differently, we now concentrate on one of the sectors of production, the first row of (5.20), assuming that the equalities hold. Hence, we have

$$q^Q_i a_{1i} = q^Q_i a_{12}, \quad q^Q_i a_{12} = q^Q_i \ell_1 = q^L_i.$$ 

Dividing term by term each of the first two equalities by the third, respectively, it is noted that each of the produced inputs in the production process for commodity 1 has a linear relation with the quantity of work exerted by the producers:

$$q^Q_{1i} = (a_{11}/\ell_1) q^L_i, \quad q^Q_{12} = (a_{12}/\ell_1) q^L_i.$$

### The quantities system

We first make some basic assumptions on the matrices/vectors $A$, $\ell$, $c_L$ that will be used in much of what follows in this and the following chapters.

**Basic Assumptions on $A$, $\ell$, $c_L$:**

1) $A \geq 0$ is an $n \times n$ indecomposable and productive matrix (which implies $0 < \lambda(A) < 1$),

2) $\ell > 0$ is an $n \times 1$ column vector (i.e., labor is needed in each of the productive processes),

3) $c_L \geq 0$ is a $1 \times n$ row vector (i.e., each worker consumes at least one produced commodity).

These assumptions imply that the composite matrix $A + \ell c_L$ (which has the same dimension as $A$, as seen in (5.16)) is also indecomposable, since adding non-negative elements to matrix $A$ can only make it have more positive elements but not less. Using Perron-Frobenius these assumptions also imply that the dominant eigenvalue of $A + \ell c_L$ is positive (since $\lambda(A + \ell c_L) > \lambda(A) > 0$) and that its associated dominant eigenvector is positive and unique up to a scalar factor. We can say the same about the transpose matrix $(A + \ell c_L)^T$ (since $\lambda((A + \ell c_L)^T) = \lambda(A + \ell c_L)$). Hence, $(A + \ell c_L)$ has left and right dominant eigenvectors that are unique (up to a scalar factor) and positive.

Although the assumption that $A$ is productive implies $\lambda(A) < 1$, $\lambda(A + \ell c_L)$ can be greater than, less than, or equal to one. We are here interested in obtaining a solution $q^Q$ to equation $q^Q (A + \ell c_L) = q^Q$ (i.e., (5.15)). We know by Perron-Frobenius that if $\lambda(A + \ell c_L) = 1$ such a solution exists and is positive and unique (up to a scalar factor). Notice that we can always adapt $c_L$ so that this equality is verified. That is, given the technological assumptions on the elements of $[A \ \ell]$, it is always possible to increase some of the components of $c_L$ if $\lambda(A + \ell c_L) < 1$ (which makes $\lambda(A + \ell c_L)$ increase)\footnote{We here use Theorem 2.3 and Theorem 9 of the Mathematical Appendix to this chapter.} and diminish some of them if $\lambda(A + \ell c_L) > 1$ in order to obtain $\lambda(A + \ell c_L) = 1$. From
the economic point of view, given the technology represented by \([A \ell]\), the consumption basket \(c_L\) must be such that \(\lambda (A + \ell c_L) = 1\) if there is to be Simple Reproduction (SR) in this society of Simple Commodity Production (SCP). In summary, given our Basic Assumptions on \(A, \ell, c_L\) (5.24), and the additional assumption that \(c_L\) is such that \(\lambda (A + \ell c_L) = 1\), there exist \(q^Q > 0\) such that \(q^Q (A + \ell c_L) = q^Q\) and \(v > 0\) such that \((A + \ell c_L) v = v\). We will see in the next chapter that \(v\) can be interpreted as the vector of Marx’s values in the case of SCP.

In order to normalize the quantities vector \(q^Q\) we can assume that production uses all available work: \(q^Q \ell = q^L\), where \(q^L\) is the population of producers. Hence, we have \(q^Q (A + \ell c_L) = q^Q A + q^L c_L = q^Q\), and \(q^Q \ell = q^L\) (that is, (5.4) and (5.5)), equations that can be expressed jointly in the quantities system for SCP (5.2) (or (5.14)) and is repeated here for the reader’s convenience:

\[
\begin{bmatrix}
q^Q & q^L
\end{bmatrix}
\begin{bmatrix}
A \\
\ell
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L
\end{bmatrix}
.
\]

Sometimes it is convenient to write this system in per capita terms, simply dividing vector \(q\) by \(q^L\):

\[
\begin{bmatrix}
q^Q \\
1
\end{bmatrix}
\begin{bmatrix}
A \\
\ell
\end{bmatrix}
= \begin{bmatrix}
q^Q & 1
\end{bmatrix}
,
\]

where \(q^Q \equiv q^Q / q^L\) represents the vector of gross per capita outputs.\(^{14}\)

**Appendix to Chapter 5**

**Bibliographical Note: Landmarks in the development of Input-Output analysis**

Since ‘Input-Output analysis’ will be used throughout much of this book, we summarize here some of the landmarks in its historical development. The name of Wassily Leontief (1906-1999) has become associated to this kind of analysis but both the ideas and the basic methods have a long history (as always happens in science). Marx highlighted the importance of the contribution of the French physician and ‘Physiocrat’ François Quesnay (1694-1774), who was the first to construct something that looks like an input-output table (in his *Tableau Économique*) showing the flows of incomes between economic sectors. Quesnay was very influenced by some thinkers of the ‘Mercantilist’ era, in particular, the Irishman Richard Cantillon (*circum* 1680-1734), who, though not having formulated a table, in his analysis came close to one (Breuer 2005). And Cantillon himself was influenced by the English physician William Petty (1623–1687), who Marx called “the father of Political Economy” (B1, 278).

It was apparently the Frenchman Achilles Nicolas Isnard (1748-1803) who first formulated a system of two (‘dual’) equations: one for quantities and another for prices.\(^{15}\) Isnard was the first of a list of French mathematical economists (that includes Cournot (1801-1877) and Walras (1834-1910)) who were long ignored by Anglo-Saxon economic literature. The latter preferred the reasoned discourse seasoned with numerical examples, whereas Walras explicitly used linear equations with fixed coefficients

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\(^{14}\)This way of expressing the quantities equation will be useful when we consider our first model of Extended Reproduction in Chapter, 14 based on population growth.

\(^{15}\)Kurz and Salvadori (2000) make an interesting historical reconstruction that goes from William Petty and Richard Cantillon to Wassily Leontief and Robert Remak (which includes Quesnay, Isnard, Torrens, Marx, Dmitriev, Bortkiewicz and Charasoff). See also Appendix C (“Historical Notes on the Development of Leontief’s Input-Output Analysis”) of Miller and Blair (2009), where there is also an interesting historical synthesis.
in the theory of production of his *Elements*. These equations can easily be expressed in matrix format, as we do in this book. Walras also showed how the fixed coefficients could be replaced by production functions like \( y = f(x_1, x_2) \) that allowed for a gradual substitution between inputs.

Recently it has been revealed that the French Jesuit priest, engineer, and mathematician Maurice Potron (1872-1942) conceived, starting as of 1911, an economic system that used fixed technological coefficients as well as dual equations of quantities and prices. Moreover, his system was more general than Input-Output theory in some respects, and he was also one of the first to explicitly use the theorems of Perron-Frobenius for the analysis of an economic system. Abrahim-Frois and Lendjel (2004) have compiled 12 of the articles published by Potron between 1911 and 1941, also writing a biographical preface on this until recently unknown thinker.

Leontief (1905-1999) started his long career with clear influence of the ideas of Marx and his followers. He was born in 1906 in Russia and started his university studies in 1921-25 in the University of Leningrad of the recently formed USSR. It is almost certain that he had to study Marx’s theory, and specifically his theory of Simple and Extended Reproduction. He was able to travel to Berlin to attend to a health problem and there decided not to return to the Soviet Union, continuing his studies at the University of Berlin. There, he had as teachers Professors Sombart and Bortkiewicz (Leontief 1973), two scholars very familiar with Marx’s theory. The very title of his doctoral thesis (of 1928): “The economy as a circular flow” (*Die Wirtschaft als Kreislauf*) reflects an idea that is ubiquitous in *Capital*. In his thesis he considered inappropriate to begin with the ‘homo economicus’, stating that economics should start from “the ground of what is objectively given” and that economic analysis should focus on the concept of a circular flow, which “expresses one of the fundamental ‘objective’ features of economic life” (Kurz and Salvadori 2003, 23-4).

After migrating to the U.S. and working for the NBER during 1931 Leontief was invited by Schumpeter (another exiled European who had been much influenced by Marx) to work at Harvard University in 1932, where he started to work on an input-output table for the American economy. During the Second World War he also worked for the Office of Strategic Services (OSS), the predecessor of the CIA, in the construction of an input-output table for the German economy (Miller and Blair 2008, Appendix C.5, 731). In the American postwar environment of McCarthyism and the Cold War, and especially for an immigrant from the USSR, it was ‘politically correct’ in academic life to hide his early influences as much as possible. It is suggestive that in his first book, *The Structure of American Economy, 1919-1929*, published during World War II (1941), Leontief only cites Quesnay, Ricardo, and Walras. Kurz and Salvadori (2003, 26) write:

While Leontief conceived of his early contribution as firmly rooted in the classical tradition, he called his Input-Output method developed in the 1930s and 1940s “an adaptation of the neo-classical theory of general equilibium to the empirical study of the quantitative interdependence between interrelated economic activities” (Leontief, 1966, p. 134). Scrutiny shows, however, that in his input-output analysis he preserved the concept of circular flow and did not, as is maintained by some interpreters, adopt the Walras-Cassel view of production. In the second edition of *The Structure of American Economy*, published in 1951, he even explicitly rejected the view of production as a one-way avenue that leads from the services of the ‘original’ factors of production: land, labour and capital—the ‘venerable trinity’—to final goods (Leontief, 1951, p. 112). Unlike the theories of
Walras and Cassel, in Leontief there are no given initial endowments of these factors. We shall refrain from speculating about the reasons for the change in Leontief’s characterization of his own approach which seems to have occurred after his move from Europe to the United States (Kurz and Salvadori 2003, 26).

Two decades later, however, at the beginning of the decade of détente with the Soviet Union, Leontief wrote a laudatory preface to the book *Proportions, Planning and Prices* written by the Hungarian economist András Bródy (1970), the subtitle of which is ‘A mathematical restatement of the labor theory of value’. This book was meant to mathematically express Marx’s theory and is explicitly based on the idea of a circular flow of commodities and duality, and impressed Leontief very favorably.\(^{16}\)

The mathematical tools that make it possible to now treat with ease the matrix systems of this book were only beginning to be developed by professional mathematicians in Marx’s time. Although it appears that the Chinese mathematicians of the Han dynasty (206 BC-220 AC) that wrote *The Nine Chapters on the Mathematical Art* were the first to use matrix methods, the theorems of Perron and Frobenius were formulated and published more than two decades after Marx’s death in 1883. Perron published in 1907 his theory of positive matrices (i.e., those with all their elements positive) and Frobenius made important advances with his publication in 1912 when he extended Perron’s theory to non-negative matrices.

Among the economists that made substantial contributions to various quantitative aspects of the linear analysis of the productive process are Dmitriev (1898), Bortkiewicz (1907), Charasoff (1910), von Neumann (1945 [1938]), Dorfman, Samuelson, and Solow (1958), and Sraffa (1960), several of which used the matrix format of Input-Output theory.

And among the economists that contributed to the understanding of various mathematical aspects of Marx’s theory are Winternitz (1948), Seton (1957), Morishima and Seton (1961), Samuelson (1957, 1967, 1970, 1971), Bródy (1970), Morishima (1973), Bowles and Gintis (1978), and Abraham-Frois and Berrebi (1979). Unfortunately, in many cases the clarification of various mathematical aspects of Marx’s theory was accompanied by varying degrees of confusion on what Marx wrote or intended to do at the theoretical level.

**Mathematical Appendix to Chapter 5**

In this Appendix we reproduce, without proofs, several theorems of Gantmacher (1959), Karlin (1959), Nikaido (1978) and Lax (2007).

First we specify some notation. All the matrices dealt with in this book are real (their elements are real numbers). Let \(A\) be a matrix and \(q\) a vector. Then \(A \geq 0\) (or \(q \geq 0\)) means that all the elements of \(A\) (or \(q\)) are non-negative and it is said that \(A\) (or \(q\)) is non-negative; \(A \geq 0\) (or \(q \geq 0\)) means that \(A \geq 0\) and \(A \neq 0\) (or \(q \geq 0\) and \(q \neq 0\)) and it is said that \(A\) (or \(q\)) is semipositive; \(A > 0\) (or \(q > 0\)) means that all the elements of \(A\) (or \(q\)) are positive and it is said that \(A\) (or \(q\)) is positive. \(A_{ij}\) is the element of \(A\) that is in row \(i\) and column \(j\). \(A^T\) is the transpose matrix of \(A\), that is, the matrix whose element \(A^T_{ij}\) is element \(A_{ji}\) of \(A\). \(|C|\) is the determinant of matrix \(C\).

An \(m \times m\) square matrix \(B\) is a principal submatrix of an \(n \times n\) square matrix \(A\) if \(B\) can be obtained from \(A\) by eliminating any \(n - m\) rows and the same \(n - m\) columns \((0 < m < n)\).

\(^{16}\)See our Bibliographical Note on Bródy’s book in the Appendix to Chapter 8.
Let $A$ be a real $n \times n$ matrix. If there exists a real $n \times 1$ vector $x \neq 0$ such that $Ax = \lambda x$ (where $\lambda$ is a real or complex scalar), then $\lambda$ is called an eigenvalue of $A$ and $x$ is called an eigenvector of $A$ associated to $\lambda$. Notice that $Ax = \lambda x$ can be written as $(\lambda I - A)x = 0$. If $\lambda$ is an eigenvalue of $A$, this equation can have a solution $x \neq 0$ only if the determinant of matrix $\lambda I - A$ is zero, that is, if $|\lambda I - A| = 0$. This equation is called the characteristic equation of $A$, and it is a polynomial equation in $\lambda$ of order $n$.

The Fundamental Theorem of Algebra says that it has, in general, $n$ complex roots. The roots of this equation are the eigenvalues of $A$. We will see below that when $A \geq 0$ it has at least one non-negative (and hence real) eigenvalue and to the greatest of such eigenvalues corresponds at least one semi-positive (and hence real) eigenvector $x \geq 0$.

An eigenvalue $\lambda$ of $A$ is simple if it is a simple root of the characteristic equation of $A$.

An $n \times n$ square matrix $A$ is decomposable (or reducible) if $N = \{1, 2, ..., n\}$ can be divided into two non-empty sets $I \neq \emptyset$ and $J \neq \emptyset$ that are disjoint ($N = I \cup J$ and $I \cap J = \emptyset$) such that $A_{ij} = 0$ for all $i \in I$, $j \in J$. If $A$ is not decomposable it said to be indecomposable (or irreducible).\(^\text{17}\) $A$ is indecomposable if and only if there is no permutation matrix\(^\text{18}\) $P$ such that

$$PAP^{-1} = \begin{bmatrix} A_1 & 0 \\ A_3 & A_2 \end{bmatrix}$$

where $A_1$ and $A_2$ are square submatrices, that is, such that it is not possible to make a permutation of the rows and columns of $A$ (with the same permutation for rows as for columns) such as to obtain the right hand side of (5.25).

If $A_1$ or $A_2$ (or both) are decomposable, similar permutations or row and columns can be made, and so successively until the normal form of a decomposable matrix is obtained (Gantmacher 1959)\(^\text{19}\):

$$A = \begin{bmatrix} A_1 & 0 & 0 & \ldots & 0 \\ 0 & A_2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & A_g & 0 \\ A_{g+1,1} & A_{g+1,2} & \ldots & A_{g+1,g} & A_{g+1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \ddots & \vdots \\ A_{s,1} & A_{s,2} & \ldots & A_{s,g} & A_{s,g+1} & \ldots & A_s \end{bmatrix}$$

where all the submatrices in the principal diagonal are square and indecomposable and in each one of the rows of submatrices starting from row $g + 1$ there is at least one submatrix (not counting the principal diagonal) which is different from zero. This normal form is unique with certain exceptions. For example, the normal form is not lost with any permutation among the first $g$ matrices of the principal diagonal, nor also with some other permutations that involve matrices $A_{g+1} \ldots A_s$.

In the following Theorems $A$ always represents an $n \times n$ non-negative matrix ($A \geq 0$) unless otherwise specified.

**Theorem 1** (Perron-Frobenius):

1) $A$ has at least one non-negative eigenvalue $\lambda \geq 0$. The greatest of these eigenvalues is called the dominant eigenvalue and is represented as $\lambda(A)$. The absolute values

\(^{17}\) Notice that this definition implies that any $1 \times 1$ matrix is indecomposable.

\(^{18}\) A permutation matrix is a square matrix of zeros and ones such that each row and each column has exactly one element equal to 1.

\(^{19}\) Notice that there have also been permutations of rows and columns such that all the isolated blocks (those that have only zero blocks to their left) are in the upper part of the matrix.
of all other eigenvalues of $A$ do not exceed $\lambda(A)$. Also, there exists a semi-positive
eigenvector $x \geq 0$ associated to $\lambda(A)$.

2) $\lambda(A)$ is an increasing function of all the elements of $A$, that is, if $B \geq A$ then
$\lambda(B) \geq \lambda(A)$.

3) If $\theta$ is a real number and $I$ the $n \times n$ identity matrix, then $\theta I - A$ has a non-
negative inverse $(\theta I - A)^{-1} \geq 0$ if and only if $\theta > \lambda(A)$.

4) The dominant eigenvalue of $A$ is the same as that of $A^T$, that is, $\lambda(A) = \lambda(A^T)$.

**Theorem 2** (Perron-Frobenius): Let $A$ be indecomposable with $n > 1$. Then:

1) The dominant eigenvalue $\lambda(A)$ is positive and has an associated positive eigenvector $x > 0$ that is unique up to a scalar factor and we will call the dominant eigenvector
of $A$. The absolute values of all the other eigenvalues of $A$ are less than $\lambda(A)$.

2) $Ay = \mu y$ ($y > 0$) has a unique solution $\mu = \lambda(A)$.

3) $\lambda(A)$ is a strictly increasing function of all the elements of $A$, that is, if $B \geq A$ then
$\lambda(B) > \lambda(A)$.

4) If $B$ is a principal submatrix of $A$ then $\lambda(B) < \lambda(A)$.

5) $\lambda(A)$ is a simple root of the characteristic equation $|\lambda I - A| = 0$.

6) $\lambda(A) \geq \min_i \sum_j A_{ij}$ and $\lambda(A) \leq \max_j \sum_i A_{ij}$.

**Theorem 3**: The dominant eigenvalue $\lambda(A)$ of $A$ has an associated positive eigenvector
if and only if when $A$ is put in the normal form for decomposable matrices: 1) each of $A_1 \ldots A_g$ has $\lambda(A)$ as an eigenvalue, and 2) if $g < s$ none of $A_{g+1} \ldots A_s$ have
$\lambda(A)$ as an eigenvalue.

**Theorem 4**: Both $A$ and $A^T$ have a positive eigenvector associated to $\lambda(A)$ if
and only if $A$ is indecomposable or can be put (by means of permutation of rows and
columns) in ‘block diagonal’ form, that is, the normal form for decomposable matrices
where $s = g$.

**Corollary**: $A$ is indecomposable if and only if its dominant eigenvalue $\lambda(A)$ is a
simple root of the characteristic equation and both $A$ and $A^T$ have a positive eigenvector
associated to $\lambda(A)$.

**Theorem 5**: Let $A$ be indecomposable. Then if $(\theta I - A)$ has a non-negative inverse
it is positive, that is $(\theta I - A)^{-1} > 0$.

**Theorem 6**: If $A$ has a non-negative but non-positive eigenvector associated to a
positive eigenvalue, it is decomposable.

**Theorem 7**:

1) If $\lambda(A) < 1/(1 + \rho)$ then the series $(1 + \rho) \left( I + (1 + \rho) A + (1 + \rho)^2 A^2 + \ldots \right)$ is
convergent and its sum is

$$
\left( \frac{1}{1 + \rho} I - A \right)^{-1} \equiv B(\rho),
$$

where we have introduced $B(\rho)$ simply to have a convenient notation that will be often
used in this book.

2) If the series in 1) is convergent for some $1/(1 + \rho) > 0$, then $\lambda(A) < 1/(1 + \rho)$
and the sum is $B(\rho)$.

**Corollary**: $\lambda(A) < 1$ if and only if the series $I + A + A^2 + \ldots$ is convergent and
equal to $(I - A)^{-1}$.

**Theorem 8**: A square matrix is decomposable if and only if its transpose is decomposable.

**Theorem 9**: Let $A(\rho)$ be a square matrix (at least some of) whose elements are dif-
fferentiable functions of a real parameter $\rho$. If $\lambda_0$ is an eigenvalue of $A(0)$ of multiplicity
one (in the sense that $\lambda_0$ is a simple root of the characteristic polynomial of $A(0)$).
then for $\rho$ sufficiently small $A(\rho)$ has an eigenvalue $\lambda(\rho)$ that depends differentially on $\rho$ and such that $\lambda(0) = \lambda_0$ (Lax 2007, 130).

**Theorem 10:** Let $A(\rho)$ be a square matrix (at least some of) whose elements are differentiable functions of a real parameter $\rho$ and $\lambda(\rho)$ an eigenvalue of $A(\rho)$ of multiplicity one. Then we can choose an eigenvector $y(\rho)$ of $A(\rho)$ corresponding to $\lambda(\rho)$ that depends differentially on $\rho$. The reason for saying here that “we can choose” is simply that an eigenvector is only specified in its structure, that is, up to multiplication by a scalar factor (Ibid.).
Chapter 6  COMMODITIES AND SIMPLE COMMODITY PRODUCTION

Our exposition of Marx’s theory in this book differs in its ordering of the material from that of Capital. For various reasons Marx chose to leave the development of certain topics to Books II and III. Among these was the process of circulation of capital (including his schemes of simple and extended reproduction), developed in Book II, and the topic of ‘production prices’, developed in Book III. The fact that the equilibrium relative prices could not in general be the relative values under Capitalism whereas Book I assumed all transactions were done according to the values of the commodities became problematic for many of Marx’s critics, who had to wait more than 30 years for the publication of Book III how Marx dealt with this topic. As we have explained, Marx did not want the exposition of the complexities of the price system to deviate attention from other topics he considered more basic. However, the use of Input-Output analysis and matrix theory lets us easily address the intersectoral aspects that for Marx were too cumbersome to include in the initial presentation of his complex theory. Hence, these topics will be addressed in this book as soon as we deal with capitalism. We do, however, respect Marx’s more basic ordering that implies dealing successively with commodities, money, and capital, and with industrial capital before dealing with commercial and financial capital and ground rent. It is necessary to respect this ordering because money is a particular commodity, and capital (in Marx’s sense) adopts the forms of commodities and money (among other forms), for which it is essential to have previously analyzed these categories. Furthermore, ground rent is the topic he left in the most unsatisfactory state. Hence, in the present chapter we make a formal representation of commodities and SCP, while we focus on money in Chapter 7, and capital and CCP in the following chapters.

In the global social-economic process the production sphere is distinguished from the reproduction (or consumption) sphere of the producers. The producers’ consumption process is taken as the reproduction process of their vital existence, and in SCP the aim of production is to satisfy the producers’ consumption needs. Such is the essence of the ‘cyclical’ process of production and reproduction in SCP. Neither the Classical economists nor Marx tried to model the decisions of individuals on whether to purchase, say, more apples or more pears according to the prices they found in the market. They considered the consumption basket as an average of the purchases resulting from the habits and preferences and possibilities of the different individuals. In this Marx followed the procedure used by his predecessors, taking as given the (average) basket of consumption goods of the various social classes.¹ And in SCP there is but one class: that of producers/workers.

The proportionality between the various non-labor inputs and labor in short run analysis and in topics like simple reproduction in more long term analysis is a recurrent theme in Marx’s theory. Although the following quotes refer to CCP, they are included here because they exemplify Marx’s use of proportionality in production and they are also valid for SCP and, more generally, for Input-Output analysis:²

¹We shall see in Chapter 12, however, that when Marx deals with the industrial cycle he allows these baskets to change (at least in their levels) according to the changes in incomes.
²The italics are ours.
... within the workshop, the iron law of proportionality subjects definite numbers of workmen to definite functions,... (B1, 361).

... the proportions which the expansion of the productive process may assume are not arbitrary but prescribed by technology... (B2, 84).

... under all circumstances the part of the money spent for means of production... must at the outset be calculated accordingly, must be procured in corresponding proportion. To put it another way, the quantity of means of production must suffice to absorb the amount of labour, to be transformed by it into products. If the means of production at hand were insufficient, the excess labour at the disposal of the purchaser could not be utilised; his right to dispose of it would be futile. If there were more means of production than available labour, they would not be saturated with labour, would not be transformed into products (B2, 33).

Assertions such as these, as well as Marx’s innumerable numerical exercises, amply justify the use of what are today called ‘fixed coefficients production functions’ in order to represent Marx’s conception of the transformation of inputs and labor into products. However, in Marx’s theory such technical coefficients can vary from one period to the next, sometimes cyclically and sometimes with a trend. They are only necessarily fixed during each time period. For example, when Marx considers the adverse effects of a year with exceptionally bad weather on agricultural production he states that it increases the amount of work that is necessary per unit of production. On the other hand, if there are technological innovations in a branch of production over time, some of the coefficients of the production process will be (perhaps constantly) decreasing over time. Hence, we will often make these exogenous coefficients vary, as did Marx. Finally, the last of the above quotes expresses clearly that Marx was referring to the underlying ideas of the fixed coefficients production function we considered in the previous chapter (and represented by means of the function min(,)) in the sense that if there were an excess of labor in relation to the available means of production such labor “could not be utilised.”

**Value, exchange value, and price in SCP**

Marx sketches the historical process that over the centuries led to the production of commodities. For there to exist an exchange process of alienable objects “it is only necessary for men, by a tacit understanding, to treat each other as private owners of those alienable objects, and by implication as independent individuals” (B1, 98). But this only happened as a result of a long historical process. For “such a state of reciprocal independence has no existence in a primitive society based on property in common, whether such a society takes the form of a patriarchal family, an ancient Indian community, or a Peruvian Inca State. The exchange of commodities, therefore, first begins on the boundaries of such communities, at their points of contact with other similar communities, or with members of the latter” (Ibid.). The objects produced are gradually transformed into commodities, first “in the external relations of a community” and gradually also in the “internal intercourse” of the community.

The constant repetition of exchange makes it a normal social act. In the course of time, therefore, some portion at least of the products of labour must be produced with a special view to exchange. From that moment the distinction becomes firmly established between the utility of an object for the purposes of consumption, and its utility for the purposes of exchange. Its use value becomes distinguished from its exchange value (Ibid.).
SCP represents in stylized form an isolated and self-sufficient community of producers in which the distinction between use value and exchange value of the produced commodities is firmly in place. Each producer/worker offers his product (which has no use value to him) in exchange for commodities produced by other producers/workers that he needs. The exchange value of his product lets him try to exchange it for other commodities. In SCP what motivates the producer’s exertion is the reproduction of his existence, the satisfaction of his needs. But he will not necessarily be successful in exchanging his good. It is the ‘exchange process’ that determines whether he will succeed in obtaining the commodities he needs. The labor spent on the production of his commodity “counts effectively, only in so far as it is spent in a form that is useful for others. Whether that labour is useful for others, and its product consequently capable of satisfying the wants of others, can be proved only by the act of exchange” (B1, 96). On the other hand, although the exchange ratios between two given commodities may fluctuate over time, “in the midst of all the accidental and ever fluctuating exchange relations between the products, the labour time socially necessary for their production forcibly asserts itself like an overriding law of Nature” (B1, 86). As we will see, this ‘law of Nature’ is what we would now call the ‘tendency towards equilibrium’ exchange ratios.

Value and the exchange value of commodities

Let us assume that the input-output matrix $A$ of SCP is indecomposable and productive. We saw in the preceding chapter (when considering the quantities system (5.14) and (5.15)) that our assumptions on the social and technological data of SCP implied that SR required $\lambda(A + \ell c_L) = 1$ and that (by Perron-Frobenius) this implied the existence of a positive vector $q^Q$ (unique up to a scalar factor) that satisfies $q^Q(A + \ell c_L) = q^Q$ and also the existence of a positive vector $v > 0$ (unique up to a scalar factor) that satisfies the dual equation $(A + \ell c_L)v = v$. Furthermore, we normalized $q^Q$ with the equation $q^Q\ell = q^L$ taking into account that the population of producers $q^L$ (or the total quantity of work available, depending on the units chosen) is exogenous. We show here that if $v$ is normalized such that $c_L v = 1$, then it represents the vector of Marx’s values in the case of SCP, since $v_i$ represents the “socially necessary labor time” for the production of $i$. The consumption of the producers/workers is the total surplus this economy produces above the reproduction of the means of production. That the value of the consumption basket of each producer be equal to one ($c_L v = 1$) can be considered the natural normalization of vector $v$ in the case of SCP. Consider the following dual system of the quantities system of SCP (5.2) that we shall call the values system of SCP:

\[
\begin{bmatrix}
A & \ell \\
c_L & 0
\end{bmatrix}
\begin{bmatrix}
v \\1
\end{bmatrix}
=
\begin{bmatrix}
v \\1
\end{bmatrix},
\]

(6.1)

Separating the two equalities we have

\[
Av + \ell = v
\]

(6.2)

\[
c_L v = 1.
\]

(6.3)
From (6.2) we obtain the following expression for the vector of values:

$$v = (I - A)^{-1} \ell > 0.$$  \hfill (6.4)

Expressing the inverse matrix as an infinite series we have:

$$(I - A)^{-1} = I + A + A^2 + ... \hfill (6.5)$$

Hence we have the following decomposition of the values vector:

$$v = \ell + A\ell + A^2\ell + .... \hfill (6.6)$$

Take the $i$-th element of this vector. The first term $\ell_i$ is the quantity of work that is directly necessary to produce a unit of $i$. The second term $A_i\ell$ is the quantity of work that is directly necessary to produce the commodities that are required to produce a unit of $i$ (where $A_i$ represents the $i$-th row of $A$). Similarly, $(A^2)_i\ell$ is the quantity of work that is directly necessary to produce the commodities that are required to produce the commodities that are required to produce a unit of $i$, etc. Then we may say the value $v_i$ of the $i$-th commodity is the quantity of work that is directly necessary to produce a unit of $i$. Hence, $c_Lv_i = 1$ expresses the fact that the quantity of labor directly necessary to produce the consumption basket that allows the worker to reproduce his existence during one period is 1, that is, the quantity of labor he exerts in that unit of time.

We can also get $c_Lv = 1$ from the quantities system (5.2). Its two equalities are:

$$q^Q A + q^Lc_L = q^Q, \quad q^Lc_L = q^L. \hfill (6.7) \hfill (6.8)$$

From the first we get $q^Lc_L(I - A)^{-1} = q^Q$, and multiplying by $\ell$ and using (6.4) and (6.8) we get $q^Lc_Lv = q^L$. Finally, dividing by $q^L$ we get (6.3). This confirms that the value of the consumption basket of each producer is necessarily equal to the work he generates during the period of time he can live off of that consumption.

Notice that the vector of values $v$ that was obtained as an integral part of the values system (6.1) naturally arises here as a vector that values the elements of the consumption basket from the structure of production itself (given by (6.7) and (6.8)). In Chapter 8 we will show that Marx’s theory of the exploitation of wage workers under CCP takes as reference this nuclear model of SCP but with the crucial difference that in Capitalism $c_Lv < 1$ is obtained, where $c_L$ will be the consumption basket of each wage worker (instead of the consumption basket of the producer/worker).

The exchange values of commodities are the ratios in which they exchange in the market. The ‘equilibrium’ exchange value of a commodity with another is the ratio between their values. For example, the equilibrium exchange value of commodity $i$ in terms of commodity $k$ is $v_i/v_k$. For Marx the relative values do not necessarily represent the exchange values with which all transactions are realized, although in Book I he assumes that this is the case in order to facilitate the presentation. For the true exchange values of transactions are related to the situation of supply and demand, making it possible to have an excess demand or excess supply that makes the actual

---

3 Since we have assumed $A$ is productive we have $\lambda(A) < 1$. Then (by the Corollary to Theorem 7 of the Mathematical Appendix to Chapter 5), $I - A$ is invertible and its inverse is non-negative. By Theorem 5 its inverse is positive $(I - A)^{-1} > 0$. This implies $v > 0$. Notice that $v$ would be positive even if we assumed that $\ell$ had zeros $(\ell \geq 0)$, that is, even if there were productive processes that do not require labor directly.
exchange value between two commodities be above or below their relative values. Only when demand is equal to supply are, in SCP, the exchange ratios between commodities, their 'exchange values', equal to the ratios between their values. On the other hand, we are here making the assumption that all the producers of the same commodity use the same technology. But Marx's model of SCP is more complex and producers of the same commodity can use different technologies, as we show below in this chapter.

The duality between quantities of use values and exchange values was conceptually well understood by Marx, who explicitly refers to it in various occasions in his works, though not explicitly from an algebraic point of view (with distinct quantities and values equations). An outstanding example is the following:

We saw earlier that the commodity must acquire a double mode of existence in order to be made fit for circulation. Not only must it confront the buyer as an article with particular useful qualities, as a particular use value which satisfies particular needs, whether of individual or of productive consumption. Its exchange value must have acquired a form different and distinct from its use value, independent of it, although only notionally. It must appear as the unity of use value and exchange value, but at the same time it must appear as this duality

4 Its exchange value acquires this independent form, a form entirely independent of its use value, as the pure existence of materialised social labour time, in its price, that expression in which exchange value is expressed as exchange value, i.e. as money (Results, MECW 34, 364).

[Box: Mathematical Note: the indecomposability of the social matrix M]

The Basic Assumptions on \( A, \ell, c_L \) (5.24) imply, as we saw in the preceding chapter, the indecomposability of \( A + \ell c_L \). We here show that adding a special assumption this implies that the social matrix \( M \) in (5.2) and (6.1) is indecomposable. We showed that the assumptions imply the existence of vectors \( q^Q > 0 \) and \( v > 0 \) such that the dual equations: \( q^Q (A + \ell c_L) = q^Q \) and \( (A + \ell c_L) v = v \) are valid, where \( q^Q \ell = q^L \) and \( c_L v = 1 \). Hence, (5.2) and (6.1) are obtained, which shows that both \( M \) and \( M^T \) have positive eigenvectors corresponding to their dominant eigenvalue \( \lambda (M) = 1 \). Using Theorem 4 of the Mathematical Appendix to Chapter 5 it follows that either \( M \) is indecomposable or it can be put in the 'block diagonal' form, where each of the blocks has a dominant eigenvalue equal to one. Since the last alternative implies that the economy can be totally separated into two (or more) parts that are completely unrelated and this lacks any empirical interest, we can add the explicit assumption that the social matrix \( M \) cannot be expressed in 'block diagonal' form by means of a permutation of rows and columns. In that case the social matrix \( M \) of SCP is necessarily indecomposable. We leave the case of the decomposability of matrix \( A \) for CCP. In particular, in Chapter 10 we will see that when there are 'luxury' consumption goods \( A \) is decomposable and so is \( M \).

4 These italics were added, but not the rest.
5 According to the Corollary to Theorem 4 of the Mathematical Appendix to Chapter 5 we could alternatively assume that the eigenvalue \( \lambda (M) = 1 \) is a simple root of the characteristic polynomial of \( M \). But we prefer to highlight the economic rather than the mathematical assumption.
Exchange value and price in SCP

According to Marx the historical process that developed the production and circulation of commodities gradually segregated among circulating commodities one that could function as general equivalent, that is, money:

The historical progress and extension of exchanges develops the contrast, latent in commodities, between use value and value. The necessity for giving an external expression to this contrast for the purposes of commercial intercourse, urges on the establishment of an independent form of value, and finds no rest until it is once and for all satisfied by the differentiation of commodities into commodities and money. At the same rate, then, as the conversion of products into commodities is being accomplished, so also is the conversion of one special commodity into money” (B1, 97).

The price of a commodity \( i \) is its exchange value as it is expressed in terms of the commodity money, say, commodity 1 (which, as Marx, we assume to be gold). As Marx writes: “The elementary expression of the relative value of a single commodity, such as linen, in terms of the commodity, such as gold, that plays the part of money, is the price form of that commodity. The price form of the linen is therefore 20 yards of linen = 2 ounces of gold, or, if 2 ounces of gold when coined are £2, 20 yards of linen = £2” (B1, 81). The last expression can be read: the price of 20 yards of linen is £2.

But for Marx the price as an exchange ratio can fluctuate according to accidental supply and demand factors. As Marx writes:

Magnitude of value expresses a relation of social production, it expresses the connection that necessarily exists between a certain article and the portion of the total labour time of society required to produce it. As soon as magnitude of value is converted into price, the above necessary relation takes the shape of a more or less accidental exchange ratio between a single commodity and another, the money commodity. But this exchange ratio may express either the real magnitude of that commodity’s value, or the quantity of gold deviating from that value, for which, according to circumstances, it may be parted with. The possibility, therefore, of quantitative incongruity between price and magnitude of value, or the deviation of the former from the latter, is inherent in the price form itself (B1, 111-2).

Only if the exchange ratio refers to the ratio between a commodity’s value and the value of gold does it express what we would now call “equilibrium monetary price.” The equilibrium (monetary) price of commodity \( i \) is the ratio between the values of \( i \) and 1 (assuming the latter is gold): \( p_i = v_i / v_1 \). Hence the (transposed) equilibrium price vector is \( p^T = (p_2...p_n) = (1 v_2/v_1...v_n/v_1) \), where the first component of \( p \) simply means that the value of gold divided by itself is one; but it also means that gold does not have a price, since gold is the commodity used to measure the prices of the rest of the commodities.\(^6\) Since \( p \) and \( v \) are proportional \( (p = (1/v_1) v) \), the equilibrium price vector is also a dominant eigenvalue of the social matrix (that is, \( (A + \ell c_L) v = v \) implies \( (A + \ell c_L) p = p \)). Furthermore, the value of the consumption

\(^6\) “The expression of the value of a commodity in gold -x commodity A = y money commodity- is its money form or price... But money itself has no price. In order to put it on an equal footing with all other commodities in this respect, we should be obliged to equate it to itself as its own equivalent” (B1, 104-5).
basket of the commodity producers expressed in equilibrium prices is \( c_{LP} = c_L v (1/v_1) \), where the r.h.s. can be abbreviated as \( w \). Notice that here \( w \) does not represent a wage, a category that does not exist in the world of SCP. It is simply the real equilibrium income (and consumption) of each producer expressed in gold. Hence, the \textit{equilibrium price system}, or simply \textit{price system} in SCP is:

\[
\begin{bmatrix}
A & \ell \\
c_L & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w
\end{bmatrix} =
\begin{bmatrix}
p \\
w
\end{bmatrix},
\]

(6.9)

where the first element of \( p \) is 1. Summing up, the exchange \textbf{values} of commodities in terms of the commodity that performs the role of money are their \textit{equilibrium prices}. And the \textit{market prices} are the actual prices used in transactions, which normally differ from the equilibrium prices according to the discrepancies between supply and demand. Marx disagreed with economists that simply conformed with the idea that equilibrium prices were those for which supply was equal to demand, since he believed that there is an objective mechanism that “forcibly asserts itself like an overriding law of Nature” (B1, 86). In the case of SCP it was the \textbf{values} of commodities that explained that objective mechanism. We will see in Chapter 8 how, according to Marx, the objective mechanism in CCP differs from the one prevailing in SCP.

The ‘\textbf{law of value}’ in SCP

Marx highlights that the division of labor between producers of different commodities in SCP and the transactions between them produce interlinkages between them, market bonds (in contrast to the personal bonds of earlier modes of production): “But what is it that forms the bond between the independent labours of the cattle-breeder, the tanner, and the shoemaker? It is the fact that their respective products are commodities... Division of labour in society is brought about by the purchase and sale of the products of different branches of industry” (B1, 360). He also stresses that in commodity production, in contrast with what may happen in pre-mercantile communities, there is no planning of the allocation of labor and means of production to different branches of production but only the imperfect mechanism of competition: “the division of labour within the society” becomes “an \textit{a posteriori}, nature-imposed necessity, controlling the lawless caprice of the producers, and perceptible in the barometrical fluctuations of the market prices... The division of labour within the society brings into contact independent commodity producers, who acknowledge no other authority but that of competition, of the coercion exerted by the pressure of their mutual interests” (B1, 361). Hence, “chance and caprice have full play in distributing the producers and their means of production among the various branches of industry” (Ibid.).

As we have seen, the equations of (6.9) only represent \textit{equilibrium} states. For Marx the states that the solutions to these equations represent are components of a \textit{model} that helps to represent in thought a reality that is much more complex, one in which there may simply be a \textit{tendency} towards such an equilibrium but in which a constant \textit{disequilibrium} prevails: “the \textit{law of the value} of commodities ultimately determines how much of its disposable working time society can expend on each particular class of commodities. But this constant \textit{tendency to equilibrium}, of the various spheres of production is exercised only in the shape of a reaction against the constant \textit{upsetting of this equilibrium}” (Ibid.).

Hence, the theory of \textbf{value} that Marx generates as a component of his theory of commodities in SCP develops the idea that the relative \textbf{values} of commodities represent theoretical \textit{equilibrium exchange values} of commodities produced by independent
producers in constant competition, while the underlying values are based on the quantities of labor that are necessary for their production. This is an example of the above mentioned interrelations between 1) the construction of models that help to understand the interrelation among certain variables, and 2) the construction of a genetic-historical theory of a specific socioeconomic mode of production, which in this case is pure pre-capitalist commodity production (in which there is neither private property of land nor wage work). This interrelation is reflected in the use Marx makes of the concept of ‘equilibrium’ and ‘constant upsetting of this equilibrium’. The models are auxiliary instruments that help to discern certain correlations and tendencies of the variables that are endogenous (in the model), given the values of the variables that (in the model) are exogenous. But in the historical process all the magnitudes of interest move simultaneously. We can conceive of this process as a constant perturbation of equilibrium if we look at it from the point of view of the model. The tendency towards equilibrium that the model reflects is constantly upset by the changes in the magnitudes of what are there considered exogenous variables. And the changes in these magnitudes can also affect the model’s equilibrium state, due to which the paths of the variables considered endogenous in the model are constantly (continually or discretely) perturbed.

In commodity production (in general, be it simple or capitalist), the allocation of labor and means of production is (deficiently) accomplished by means of the ‘barometrical fluctuations of the market prices’ that producers perceive. If the market price is above the equilibrium price the producers of that commodity can increase their production if they are willing to work more. But this has a limit. More generally, if producers of other commodities observe that the consumption capacity of the producers of a certain commodity is persistently greater than their own, some may decide to change to the production of that commodity, hence increasing its supply and making the price return to its equilibrium level. As we will see when we consider the ‘law of value’ in CCP, where many workers are concentrated in the same workshop or factory under the direction of a capitalist, Marx there contrasts the direct coordination that he (or his agents) exercise within the workshop or factory to the much more imperfect coordination, similar to the case of SCP, which is given by the ‘barometric’ price signals and, more specifically, the effects they have on profits.

Effects of an increase in the productive force of labor

Marx rightly gave a central role to the technological and organizational innovations that tended to be produced in the mature phase of CCP, where ‘modern industry’ prevails. But even in the context of SCP it is useful to observe the effects that the introduction of innovations that increase the productive force of labor have on the systems of quantities and values, respectively.

On the quantities system  If there is a technological or organizational innovation that results in the ability to produce (reproduce) the same quantities of commodities (labor) using lower quantities of inputs (commodities or labor), some or all of the coefficients of \( A \ell \) fall to, say, \( A' \ell' \), yielding a new composite matrix \( A' + \ell' c_L \). We here assume that the matrix maintains its indecomposability (either because none of the positive coefficients becomes zero or because even if some do the matrix does not become decomposable). Since we start from a situation of SR, we initially have \( \lambda (A + \ell c_L) = 1 \). Hence, if there is no change in \( c_L \) (by Perron-Frobenius) the innovation diminishes the dominant eigenvalue of the composite matrix: \( \lambda (A' + \ell' c_L) < 1 \).
Since we do not wish to lift the assumption of SR\(^7\), it is necessary that the consumption of the producers increase in at least one of its elements shifting to, say, \(c_L' \geq c_L\) in which \(\lambda (A' + \ell' c_L') = 1\). Using the same reasoning as above, there exists a unique quantities vector \(q^{Q'} > 0\) such that \(q^{Q'} (A' + \ell' c_L') = q^{Q'}\), and can be normalized such that all the population is active in production: \(q^{Q'} \ell' = q^L\). We thus have a new quantities system in which the same population of producers generates a different vector of gross production to satisfy their increased consumption thanks to the increase in the productive force of their labor:

\[
\begin{bmatrix}
q^{Q'} & q^L
\end{bmatrix}
\begin{bmatrix}
A' & \ell' \\
c_L' & 0
\end{bmatrix}
= \begin{bmatrix}
q^{Q'} & q^L
\end{bmatrix}.
\]

In the simple case of the greatest possible proportional increase in the consumption basket \(c_L\) that maintains SR, there must exist a scalar \(\mu > 0\) such that \(c_L' = (1 + \mu) c_L\). Hence, in this case:

\[
\begin{bmatrix}
q^{Q'} & q^L
\end{bmatrix}
\begin{bmatrix}
A' & \ell' \\
(1 + \mu) c_L & 0
\end{bmatrix}
= \begin{bmatrix}
q^{Q'} & q^L
\end{bmatrix}.
\]

**On the values system** After the innovation and the increase it produces in consumption, there is a new values vector \(v'\) (which is the right dominant eigenvector of \(A' + \ell' c_L\) that corresponds to \(\lambda (A' + \ell' c_L') = 1\)) that must satisfy \((A' + \ell' c_L') v' = v'\). Since the value of the consumption basket of each producer in the reference time period is equal to the unit of value generated in this period, we have \(c_L' v' = 1\). Hence, we have \((A' + \ell' c_L') v' = A' v + \ell' = v'\) and the following holds:

\[
\begin{bmatrix}
A' \\
\ell'
\end{bmatrix}
\begin{bmatrix}
v' \\
0
\end{bmatrix}
= \begin{bmatrix}
v'
\
1
\end{bmatrix}.
\]

In the case of the maximum possible proportional expansion of the consumption basket we have:

\[
\begin{bmatrix}
A' & \ell' \\
(1 + \mu) c_L & 0
\end{bmatrix}
\begin{bmatrix}
v' \\
0
\end{bmatrix}
= \begin{bmatrix}
v'
\
1
\end{bmatrix}.
\]

The second of these equations is

\[
(1 + \mu) c_L v' = 1.
\]

The value of the expanded consumption of producers is still equal to the reproduction of labor capacity that this consumption allows in the time period of reference. Also, since the changes produced in \(A\) and/or \(\ell\) are reductions in (some or all) their coefficients, we can check that the new values vector is strictly less than the original one:

\[
v' = (I - A')^{-1} \ell' = \left( I + A' + (A')^2 + \ldots \right) \ell' < (I + A + A^2 + \ldots) \ell = v,
\]

which implies that at least some of the elements of \(c_L\) must have increases (to \(c_L'\)); or all of them in the proportional increase case (\(\mu\) must become positive in (6.12)).

\(^7\)Following *Capital*, we address ER only when we consider CCP. From a purely logical point of view, however, there can be ER in SCP.
Marx's theory of commodities and SCP is actually quite more complex than what we have seen so far, since in includes the existence of producers with labors of different complexities and, within each production sector, the simultaneous utilization of different techniques of production. These complications will be addressed in the next section of this chapter. But first we show a graphical representation of the simple case of one type of labor and one technique for each commodity we have seen so far.

Graphical representation of SCP

Figure 1 represents the whole economy under the assumption that only two commodities are produced. On the axes are represented both the gross productions \( q^Q = (q_1^Q, q_2^Q) \) as well as the aggregate consumptions \( C = q^L c_L = (q^L c_{L1}, q^L c_{L2}) \equiv (C_1, C_2) \) of the two commodities. The line furthest to the northeast can be called the 'employment line' and represents \( q^Q \ell = q^L \), that is,

\[
q_1^Q \ell_1 + q_2^Q \ell_2 = q^L. \tag{6.13}
\]

This line takes as given the total available labor \( q^L \) and the technical coefficients \( \ell = (\ell_1, \ell_2)' \). It cuts the vertical axis at \( q_2 = q^L/\ell_2 \) and the horizontal axis at \( q_1 = q^L/\ell_1 \). The 'consumption line' (which is below the employment line) represents (6.7). It can be written as \( C = q^Q (I - A) \), that is,

\[
[ C_1 \quad C_2 ] = [ q_1^Q \quad q_2^Q ] \begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix}. \tag{6.14}
\]

If (hypothetically) only one of the two goods were produced, we would have one of the two following alternative cases:

\[
[ C_1 \quad C_2 ] = [ q_1^Q \quad 0 ] (I - A) = q_1^Q \begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix}. \tag{6.15}
\]

\[
[ C_1 \quad C_2 ] = [ 0 \quad q_2^Q ] (I - A) = q_2^Q \begin{bmatrix} 1-a_{11} & -a_{12} \\ (1-a_{21}) & 1-a_{22} \end{bmatrix}. \tag{6.16}
\]

Although everything that is productively consumed must be produced, which implies that neither of the two production processes can be operated without the other one also being operated, it is useful to represent these two extreme cases in order to afterwards combine them. Equalities (6.15) and (6.16) determine the rays \( P_1 \) and \( P_2 \), respectively, of Figure 1. For example, if all the labor were dedicated to operating the first process, we would have \( q_1^Q = q^L/\ell_1 \), and hence (6.15) would yield

\[
C_1 = (1-a_{11}) q^L/\ell_1, \quad C_2 = -a_{12} q^L/\ell_1,
\]

which defines point \( P_1 \) on ray \( P_1 \). Similarly, if all the labor were dedicated to operating the second process, we would have \( q_2^Q = q^L/\ell_2 \), and (6.16) would yield point \( P_2 \) on ray \( P_2 \), defined by

\[
C_1 = -a_{21} q^L/\ell_2, \quad C_2 = (1-a_{22}) q^L/\ell_2.
\]

Since the technology is linear, a single point on each of these two quadrants can determine rays \( P_1 \) and \( P_2 \), respectively, which represent the points generated by (6.15) and (6.16) when \( q_1^Q \) and \( q_2^Q \), respectively, vary between zero and infinity.

All the points on the line segment between \( P_2 \) and \( P_1 \) can (hypothetically) be produced with different allocations of labor between the two processes. But only the

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8 Figure 1 is based on Fig. 9-2 of Dorfman, Samuelson and Solow (1958), which after more than a decade was extended by Samuelson (1971) for his Figure 1.
positive consumptions (represented by a thicker line) are economically meaningful. Each point on the line segment of the consumption line between $\overline{C}_2$ and $\overline{C}_1$ can be satisfied by means of a gross production vector within the line segment between $q^Q_A$ and $q^Q_B$ on the employment line. To get the gross productions that can yield a consumption vector $C$ it is sufficient to invert matrix $I - A$ in (6.14), that is,

$$
\begin{bmatrix}
q^Q_A & q^Q_B
\end{bmatrix}
= \begin{bmatrix}
C_1 & C_2
\end{bmatrix}
\begin{bmatrix}
1 - a_{11} & -a_{12} \\
-a_{21} & 1 - a_{22}
\end{bmatrix}^{-1}
= \begin{bmatrix}
C_1 & C_2
\end{bmatrix}
\frac{1}{\Delta}
\begin{bmatrix}
1 - a_{22} & a_{12} \\
a_{21} & 1 - a_{11}
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
(1 - a_{22})C_1 + a_{21}C_2 & a_{12}C_1 + (1 - a_{11})C_2
\end{bmatrix}
$$

where $\Delta = (1 - a_{11})(1 - a_{22}) - a_{12}a_{21}$ is the determinant of $I - A$. These are the gross productions necessary to obtain any aggregate consumptions $C_1$ and $C_2$. To obtain consumptions $\overline{C}_1$ and $\overline{C}_2$, the gross production quantities $q^Q_A$ and $q^Q_B$ of Figure 1 are:

\begin{align*}
q^Q_A &= \frac{1}{\Delta}
\begin{bmatrix}
(1 - a_{22}) & a_{12}
\end{bmatrix}
\overline{C}_1 \\
q^Q_B &= \frac{1}{\Delta}
\begin{bmatrix}
a_{21} & (1 - a_{11})
\end{bmatrix}
\overline{C}_2.
\end{align*}

As the labor of the producers shifts from process 2 to process 1 the gross productions shift from $q^Q_B$ towards $q^Q_A$. On the employment line of Figure 1 the corresponding thicker segment is shown. And the consumptions shift from $(0, \overline{C}_2)$ towards $(\overline{C}_1, 0)$ along the only relevant segment of the consumption line.

Figure 1

As we have seen, (6.14) (or (6.7)) yields $q^Q = C(I - A)^{-1}$. Multiplying by the vector of direct labor requirements $\ell$ we get

$$
q^L = q^Q\ell = C(I - A)^{-1}\ell = Cv.
$$

(6.17)

The consumption line of Figure 1 is graphical representation of

$$
C_1v_1 + C_2v_2 = q^L
$$

(6.18)
if we take $q^L$, $v_1$, and $v_2$ as given. In summary, given the total quantity of labor $q^L$ and the direct labor requirements $\ell_1$ and $\ell_2$, we have the slope and location of the employment line (6.13). And given $q^L$ and the values $v_1$ and $v_2$, we have the slope and location of the consumption line (6.18).

Figure 2 complements Figure 1. Whereas Figure 1 shows a segment of the possible consumptions and the segment within which the gross productions must be, Figure 2 graphs a given consumption vector and shows the gross productions vector $q^Q$ that generates it. Given already determined employment and consumption lines, the parallelogram shows that the gross production vector $q^Q$ results from the vector sum of final consumption $C$ and the productive consumption of means of production $q^Q \lambda$.

**Figure 2**

### SCP with multiple labor skills and production techniques

In our basic exposition of the SCP model above we assumed there is homogeneous labor and only one production technique for each commodity. In this section we successively lift these simplifying assumptions in order to bring the model closer to Marx’s.

### SCP with multiple labor skills

Marx argues that measuring value by an amount of labor does not mean that the value of a commodity is greater if the producer is less skillful or works less intensely:

Some people might think that if the value of a commodity is determined by the quantity of labour spent on it, the more idle and unskillful the labourer, the more valuable would his commodity be, because more time would be required in its production. The labour, however, that forms the substance of value, is homogeneous human labour, expenditure of one uniform labour power (B1, 49).

In any given time period what matters for the formation of value is the average skill and the average intensity of labor that society employs in producing commodities. This is clear if there is only one kind of labor as so far as its complexity is concerned. But
Marx also considers the existence of differentiated labor skills based on their specialization in a specific branch of production (carpenters, metalworkers, etc.). And these may have different degrees of complexity because e.g. they require different lengths or costs of training periods that command a compensation through a greater purchasing power of their income. The producers that exert more complex labor skills can command higher consumption expenditures due to the higher exchange value of the commodities they produce. Marx addresses this topic in a very compressed and exclusively textual form. In the first chapter of Book I he writes:

...human labour. It is the expenditure of simple labor, which, on average, every ordinary person, without any particular development, possesses in his physical organism. Simple average labour, it is true, varies in character in different countries and at different times, but in a particular society it is given. Skilled labour counts only as simple labour intensified, or rather, as multiplied simple labour, a given quantity of skilled being considered equal to a greater quantity of simple labour. Experience shows that this reduction is constantly being made. A commodity may be the product of the most skilled labour, but its value, by equating it to the product of simple unskilled labour, represents a definite quantity of the latter labour alone. The different proportions in which different sorts of labour are reduced to unskilled labour as their standard, are established by a social process that goes on behind the backs of the producers, and, consequently, appear to be fixed by custom. For simplicity’s sake we shall henceforth account every kind of labour to be unskilled, simple labour; by this we do no more than save ourselves the trouble of making the reduction (B1, 54-5; italics added)⁹.

In this section we verify that in the context of SCP the matter of complex or skilled work does not present significant complications if we stick to Marx’s procedure of taking as given the consumption basket. But now there is a different basket for each kind of labor.¹⁰ Let us see how simple work can be ‘multiplied’ to obtain skilled work or, equivalently, how the ‘reduction’ of skilled work to simple work can be accomplished. Assume that each productive processes requires the use of different labor skills in given proportions that may differ from those of other branches of production. After normalizing the process starting from a table as we did for the case of simple, homogeneous work, we now get a technological matrix (A L) where L replaces our previous ℓ and represents a rectangular matrix that has as many columns ℓj (say m) as there are labor skills. It is also assumed that each kind of labor j has a specific consumption basket cLj. Hence the row vector cL must be replaced by a matrix CL where each of its rows is the labor basket of a distinct kind of labor. One of the reasons this must be so is that if the skills that require a longer or more costly period of training (or are deemed more disagreeable by the average worker) did not obtain a higher remuneration, that is, did not allow him to consume more goods or a wider range of goods, there would probably not be enough workers of those particular skills as needed for the operation of the productive processes that require such skills. This would not be a sustainable

⁹The first complete sentence here has been slightly modified using the Google translation of the German original: “Sie ist Voraussetzung einfacher Arbeitskraft, die im Durchschnitt jeder gewöhnliche Mensch, ohne besondere Entwicklung, in seinem leiblichen Organismus besitzt”.

¹⁰When we return to the question of complex labor in the context of CCP, however, we will see that there are alternative ways of formalizing the values system parting from the scant and insufficiently precise indications Marx left.
situation and the producer/workers would eventually reallocate themselves toward the sectors where remuneration is more in accordance with lower training requirements or more agreeable tasks. Hence, in a stable situation the purchasing power of the different skills must be in accordance with the training requirements (and the disagreeableness assessments) of each skill.

Continuing with the assumption that there is SR in SCP, the quantities, values, and price systems are the following:

\[
\begin{bmatrix}
  q^Q \\
  q^L
\end{bmatrix}
\begin{bmatrix}
  A \\
  C_L
\end{bmatrix}
\begin{bmatrix}
  L \\
  0
\end{bmatrix}
= \begin{bmatrix}
  q^Q \\
  q^L
\end{bmatrix}
\tag{6.19}
\]

\[
\begin{bmatrix}
  A \\
  C_L
\end{bmatrix}
\begin{bmatrix}
  L \\
  0
\end{bmatrix}
\begin{bmatrix}
  v \\
  z
\end{bmatrix}
= \begin{bmatrix}
  v \\
  z
\end{bmatrix}
\tag{6.20}
\]

\[
\begin{bmatrix}
  A \\
  C_L
\end{bmatrix}
\begin{bmatrix}
  L \\
  0
\end{bmatrix}
\begin{bmatrix}
  p \\
  w
\end{bmatrix}
= \begin{bmatrix}
  p \\
  w
\end{bmatrix}
\tag{6.21}
\]

where in (6.19) \(q^L = (q^L_1 \ldots q^L_m) > 0\) is the vector of populations of workers with the \(m\) different skills, in (6.20) \(z' = (z_1 \ldots z_m)\) is a vector of reduction coefficients for the labors of different skills, and in (6.21) \(w' = (w_1 \ldots w_m)\) is a vector of monetary incomes (per unit of labor) for the workers of different skills. The vector \(z\) makes the ‘reduction’ of skilled work to simple work once the vector is adequately normalized.

It is assumed as before that \(A\) is indecomposable and productive. We now assume that \(L \geq 0\) and \(C_L \geq 0\) and that, furthermore, \(L\) has no columns of zeros and \(C_L\) has no rows of zeros. Hence, \(A + LC_L\) is indecomposable and we can use the same argument we used for simple labor. \(C_L\) can always be accommodated so that \(\lambda (A + LC_L) = 1\). Therefore, in the three systems the social matrix \(M\) has its dominant eigenvalue equal to one: \(\lambda (M) = 1\). Hence, it has left and right dominant eigenvectors that are unique up to a scalar factor. Notice that (6.21) and (6.20) represent the same system of equations and that \((p w)'\) and \((v z)'\) can only differ by a scalar factor. That is, the prices and incomes vector \((p w)\)' is necessarily proportional to the vector of values and reduction coefficients for skilled labor \((v z)\)' as above, the vector of equilibrium prices of commodities \(p\) is proportional to the vector of their values \(v\). And now we have the additional fact that the vector of ‘reduction’ coefficients (of skilled to simple labor) \(z\) is also proportional to the vector of monetary incomes per unit of work \(w\). Since \((p w)^T\) can only differ from \((v z)^T\) by a scalar factor, we have \((p w)^T = \alpha (v z)^T\) for some \(\alpha > 0\). Let the first commodity be gold, as before. Then \(p^T = (p_1 \ldots p_n) = \alpha (v_1 \ldots v_n)\). This implies \(\alpha = 1/v_1\) and hence the monetary prices are the ratios of the values of commodities and the value of money \(p_i = v_i/v_1\) for \(i \neq 1\) and the monetary incomes are the ratios of the reduction coefficients and the value of money \(w_i = z_i/v_1\) for all \(i\). Given the interpretation of \(z\), it is natural to define as simple labor the kind of labor \(k\) for which \(z_k\) is lowest, i.e., for which \(z_k < z_i\) for all \(i \neq k\) (assuming that \(k\) is unique), which is the same as saying that simple labor is the labor with lowest income: \(w_k < w_i\ \forall i \neq k\). The remaining elements of \(z\) give the factors that multiply simple labor to obtain the equivalent in simple labor for each kind of skilled labor; and from the opposite point of view they reduce each kind of skilled labor to simple labor. This conversion of skilled to simple labor makes the aggregation of the different kinds of labor possible. Hence, the introduction of labors of differing skills does not present any difficulty under SCP.
The two component equations of (6.20) yield explicit formulas for $v$ and $z$:

\begin{align*}
v &= (I - A)^{-1} Lz \quad (6.22) \\
z &= C_L v. \quad (6.23)
\end{align*}

Since $Lz$ is the vector of labor coefficients in the production of the various commodities in terms of simple labor, it plays the same role as $\ell$ did when labor was homogeneous. The value of commodity $i$ ($v_i$) is the quantity of simple work that is (directly and indirectly) necessary to produce a unit of $i$. Also, (6.23) expresses the fact that the reduction of skilled to simple labor is accomplished in accordance to the value of the consumption baskets of the various labor skills, implying that the producers that exert more skilled work consume a basket of greater value.

The value that is generated in a period of time is the total amount of (simple) labor exerted in the production processes $q^L z$, where the total amounts of skilled labor $q^L_i$ ($i \neq k$) have been reduced to their equivalents in simple labor.\footnote{11} According to (6.19) and (6.20) the following equalities are valid:

$$q^Q L = q^L, \quad Lz = (I - A) v, \quad z = C_L v.$$ 

Hence, the following are equivalent expressions for the total value generated in the time period:

$$q^L z = q^Q Lz = q^Q (I - A) v = q^Q LC_L v.$$ 

**SCP with multiple productive techniques in each sector**

**Introduction** As we have seen, in SCP commodities tend to have ‘exchange values’ that are proportional to their values, that is, to the quantities of labor that are directly or indirectly necessary to produce them after having reduced the labors of the various skills to simple labor. But it is important to point out that for Marx value is defined on the basis of the labor that is ‘socially necessary’, which means not only that the average intensity and skillfulness of workers must be taken into account, but also that the labor is performed under average technological and organizational conditions. For Marx this implies that the SCP model must take into account the range of different techniques used for the production of each commodity. And this is a point in which Marx’s theory markedly differs from the later development of Neoclassical theory. It is also an issue that has very often been misinterpreted or completely ignored in the exegeses of Marx’s theory. In this subsection we try to clarify this issue. For this we leave aside any heterogeneity in intensity or skill.

In Marx’s model of SCP (and in his model of CCP, as we see in Chapter 9) there can be multiple productive techniques used in any given period for the production of the same commodity. That is why Marx distinguishes the market value from the individual values of commodities. The former measures the labor exerted under average technological and organizational conditions by the group of workers producing the same commodity. It is obtained as the average of the individual values of the commodities produced by workers that use different techniques for the production of the same commodity. In monetary terms, Marx similarly distinguishes the market price from the individual price of a commodity. The relative market values are the centers around the relative market prices fluctuate over time according to the discrepancies between supply and demand. And the difference between the market price of a particular producer.
and its individual price defines the producer’s extra (or extraordinary), normal (or ordinary), or less than ordinary (or infraordinary) income (according to whether the former is higher than, equal to, or lower than the latter).

In the preceding subsection we saw how the ‘homogeneous human labor’ that forms the ‘substance of value’ can be obtained when there are various labor skills. What is the ‘homogeneous human labor’ that forms the ‘substance of value’ when multiple productive techniques are employed? As Marx specifies in the first chapter of Book I, the amount of labor that defines the value of a commodity must be measured in terms of a socially necessary labor time that reflects the normal conditions of production:

The total labour power of society, which is embodied in the sum total of the values of all commodities produced by that society, counts here as one homogeneous mass of human labour power, composed though it be of innumerable individual units. Each of these units is the same as any other, so far as it has the character of the average labour power of society, and takes effect as such; that is, so far as it requires for producing a commodity, no more time than is needed on an average, no more than is socially necessary. The labour time socially necessary is that required to produce an article under the normal conditions of production, and with the average degree of skill and intensity prevalent at the time (Ibid.; italics added).

If a producer uses a less productive method than the average, his labor time is not equivalent to those of other producers with which he competes. Marx assumes in Book I, in order to simplify, that all producers produce under the average conditions, hence disregarding this matter in order to focus on others. In Chapter 3 of Book I he writes (referring to the labor of a weaver): “We suppose him to have spent on his product only that amount of labour time that is on an average socially necessary. The price then, is merely the money name of the quantity of social labour realised in his commodity” (B1, 117). And even in Chapter 3 of Part I of Book III he writes: “in this part of the work we also proceed from the premise that commodities are produced under normal social conditions and are sold at their values” (B3, 54). That simplifying assumption made the actual distribution of the technical and organizational conditions concentrate on a single point, allowing him to further postpone the distinction between the individual value and the market value of commodities to Part II of Book III. It is there that Marx explicitly introduces the heterogeneity of the individual values of commodities due to the use of different techniques of production by the producers of the same commodity. This leads him to define the market value as the regulator of production and as the base for obtaining the ‘center of gravity’ around which prices oscillate:

The assumption that the commodities of the various spheres of production are sold at their value merely implies, of course, that their value is the centre of gravity around which their prices fluctuate, and their continual rises and drops tend to equalise. There is also the market value —of which later— to be distinguished from the individual value of particular commodities produced by different producers. The individual value of some of these commodities will be below their market value (that is, less labour time is required for their production than expressed in the market value) while that of others will exceed the market value. On the one hand, market value is to be viewed as the average value of commodities produced in a single sphere, and, on the other, as the individual value of the commodities produced
under average conditions of their respective sphere and forming the bulk of the products of that sphere (B3, 176-7; italics added except for ‘market value’).

The ‘market price’ is the price at which all the commodities of the same kind are bought and sold.\textsuperscript{12} It may be above or below the corresponding market value (expressed as its ratio with the value of money) whenever demand is above or below supply, respectively. And it coincides with the market value when demand equals supply. Assuming this equality between supply and demand, the market price defines a frontier between producers that have extraordinary incomes because they produce with lower cost and those that have extraordinary income because they have greater costs. Also, the strength of competition between producers determines how much dispersion there is between the individual values around the market value (which is their average). An increase in competition reduces the dispersion. And the least competitive techniques are eventually abandoned (and new ones are also eventually introduced). But the market price can be above or below the monetary expression of the market value whenever effective demand (‘socially solvent demand’) is above or below aggregate production:

For a commodity to be sold at its market value, i.e., proportionally to the necessary social labour contained in it, the total quantity of social labour used in producing the total mass of this commodity must correspond to the quantity of the social want for it, i.e., the effective social want. Competition, the fluctuations of market prices which correspond to the fluctuations in the ratio of demand to supply, tend continually to reduce to this scale the total quantity of labour devoted to each kind of commodity (B3, 191; italics added).

For the market price of identical commodities, each, however, produced under different individual circumstances, to correspond to the market value and not to deviate from it either by rising above or falling below it, it is necessary that the pressure exerted by different sellers upon one another be sufficient to bring enough commodities to market to fill the social requirements, i.e., a quantity for which society is capable of paying the market value (B3, 179; italics added).

Our analysis has revealed how the market value (…) embraces a surplus profit for those who produce in any particular sphere of production under the most favourable conditions (B3, 197; Italics added).

In modern (‘competitive’) economic theory it is usual to either assume (often implicitly) that the technical coefficients are some kind of average or, if there are alternative methods of production, that all producers choose the one that generates lowest costs, discarding the rest. Consequently, ‘equilibrium’ prices are modeled by selecting for each commodity the method that generates the lowest cost, leaving the rest aside. This assumes implicitly (sometimes explicitly) that the producers have knowledge of all the alternative methods in existence and that they have no restrictions on quickly implementing the most profitable one. This way of simplifying economic reality is far from the way many of the Classical economists perceived reality. In the case of Marx

\textsuperscript{12}Marx writes, for example, “all commodities of the same sphere of production available on the market have the same price (assuming of course that they are of the same quality)” (\textit{Theories}, MECW 31, 429).
it is clear that in general he takes into account the simultaneous use of heterogeneous techniques, which in the case of SCP leads to extraordinary, average, and infraordinary incomes of the different producers. A richer, more concrete reality is represented by recognizing this heterogeneity. Contrariwise, we can say that the modern concept of immediately switching to the least cost technique by producers of the same commodity because they have knowledge of and access to the full spectrum of available technologies is based on scarcely realistic assumptions. Marx’s thinking in terms of a statistical distribution of ‘individual values’ that reflect the multiple methods the producers use, even if he concentrated on the first moment (the average) of the distribution, has a very modern flavor.  

The challenge here is to formally represent this important and usually ignored aspect of Marx’s theory using the type of matrix techniques we have been using. In this section we extend the model of the preceding sections using an aggregation technique that reasonably reflects Marx’s theorizing in the case of SCP for the determination of the market value of commodities on the basis of the labor that is ‘socially necessary’ to produce them. We will see in Chapter 9 that with a few changes the aggregation technique here developed can also be applied to CCP.

**Analytical formulation** To maintain continuity with the first section of this chapter we here also assume there are multiple labor skills. So far we assumed that there was only one way of producing each commodity and that there was one consumption basket for each kind of labor. In the rest of this chapter we assume there are multiple ways of producing each commodity, either because different inputs and labor skills are used or because the same inputs and skills are used in different proportions. Additionally, although this is not done explicitly by Marx, we allow producers of the same skill complexity to consume different consumption baskets (because they may thus have different real incomes according to the technique they use). With these generalizations the social matrix is still square. It is simply of larger dimensions.

We also make the assumption that (both for rows and columns of the social matrix) commodities-techniques of the same kind are adjacent, and that the producers-consumption baskets of the same skill are also adjacent. This can always be achieved by means of a simultaneous permutation of rows and columns. Hence, we can always partition the social matrix in such a way that the submatrix $A_{ij}$ reflects the commodities of kind $j$ that are inputs to the production processes of commodities of kind $i$, where the kind includes both the commodity and the technique used to produce it. And the same can be said for the submatrices $L_{ij}$ and $C_{L_{ij}}$.

For example, if one of two producers of tables uses nails that were produced by two producers of nails that use different techniques, it is specified in the table used for the construction of $A$ how many nails were purchased from the producer that uses technique 1 and how many from the one that uses technique 2. The same is done in the sphere of reproduction (or consumption). For example, we keep track of how many loaves of bread each blacksmith purchases from the baker that uses technique 1 and how many from the one that uses technique 2. Hence, $A$ will have as many rows and columns as there are commodities-techniques; $C_L$ will have the same number of columns as $A$ and as many rows as there are ‘kinds of labor’, where the latter are defined by worker’s skill and his consumption; and $L$ will have the same number of rows as $A$ and as many columns as there ‘kinds of labor’.

With such a social matrix we can specify the systems of quantities, values, and

---

13See, for example, Foley (1994).
prices the way we did in the first part of this chapter. And then the elements of component $v$ of the vector $(v z)^T$ that solves (6.20) are no longer Marx’s values, but his individual values, since commodities of the same kind produced with different techniques will in general have different individual values, that is, they will have different quantities of labor directly or indirectly necessary to produce them. If in the preceding subsection it was shown how skilled (or complex) labor can be reduced to simple labor, what we must show in the present section is how we can additionally reduce the various concrete labors to the ‘abstract socially necessary labor’ that forms Marx’s market value (and which, using Marx’s terminology, constitutes the substance of value). As we will see, this simply requires averaging the individual values as well as averaging the individual coefficients that reduce skilled labor to simple labor, using in both cases adequate weights.\footnote{With this approach matrix $A \geq 0$ will have many more zeros that before, so it could in principle be decomposable. In that case we can apply Theorem 1 of the Mathematical Appendix to Chapter 5 (instead of Theorem 2), by which it will still have a dominant eigenvalue $\lambda(A) > 0$, at least one associated right eigenvector $v \geq 0$, and at least one associated left eigenvector $q^Q \geq 0.$}

For simplicity we here assume that there are only 2 techniques available for the production of each of 2 different commodities, and that there are only 2 consumption baskets available for the consumption of each of 2 labor skills, leaving the general case for the Mathematical Appendix to this chapter. Hence, the total number of productive processes is 4 and the same can be said for the number of reproductive (consumption) processes. Since the same commodities produced with different techniques are adjacent, we can write the vector of the quantities produced and the vector of individual values, respectively, as:

$$q^Q = \begin{bmatrix} q_{11}^Q & q_{12}^Q & q_{21}^Q & q_{22}^Q \end{bmatrix},$$

$$v = \begin{bmatrix} v_{11} & v_{12} & v_{21} & v_{22} \end{bmatrix}'.$$

where, for example, $v_{12}$ is the individual value of commodity 1 produced using technique 2, and $q_{21}^Q$ is the quantity of commodity 2 produced using technique 1. We also define the vector of aggregate productions of the 2 commodities $q^*Q$ as

$$q^*Q = \begin{bmatrix} q_{11}^Q + q_{12}^Q & q_{21}^Q + q_{22}^Q \end{bmatrix}.$$

Let $Q^i_{1j}$ be the share of $j$ in the total production of commodity $i$:

$$Q^i_{1j} = \frac{q_{ij}^Q}{q_i^Q} = \frac{q_{ij}^Q}{q_{i1}^Q + q_{i2}^Q} \quad (i = 1, 2; j = 1, 2)$$

and define the following (quasi-diagonal) matrix of shares:

$$Q^1 = \begin{bmatrix} Q^1_{11} & 0 & Q^1_{12} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(6.24)

The effect of postmultiplying the vector of aggregate productions $q^*Q$ by matrix $Q^1$ is to disaggregate, yielding the vector of individual quantities:

$$q^*QQ^1 = q^Q.$$

(6.25)
Define the vector of market values \( v^* \) of the two commodities as the vector of weighted averages of the individual values of the commodities:

\[
v^* = \begin{bmatrix} v_1^* \\ v_2^* \end{bmatrix} = \begin{bmatrix} Q_{11}^1 v_{11} + Q_{12}^1 v_{12} \\ Q_{21}^1 v_{21} + Q_{22}^1 v_{22} \end{bmatrix} = \begin{bmatrix} \frac{q_{11}^1}{q_{11}^1 + q_{12}^1} v_{11} + \frac{q_{12}^1}{q_{11}^1 + q_{12}^1} v_{12} \\ \frac{q_{21}^1}{q_{21}^1 + q_{22}^1} v_{21} + \frac{q_{22}^1}{q_{21}^1 + q_{22}^1} v_{22} \end{bmatrix}
\] (6.26)

The effect of premultiplying the vector of individual values \( v \) by matrix \( Q^1 \) is to form their (weighted) average, yielding the vector of market values:

\[ Q^1 v = v^*. \]

Similarly, we assume that there are 2 possible consumption baskets for each of 2 labor skills. Hence, the total number of consumption baskets is 4. Define the vector of populations \( q^L \) and the vector of reduction coefficients of the corresponding skills to units of simple labor \( z \), respectively, as:

\[
q^L = \begin{bmatrix} q_{11}^L & q_{12}^L & q_{21}^L & q_{22}^L \end{bmatrix} \\
\begin{bmatrix} z_{11} & z_{12} & z_{21} & z_{22} \end{bmatrix}.
\]

Hence, we can define the vector of aggregate populations endowed with the 2 labor skills as

\[ q^{*L} = [q_{11}^L + q_{12}^L, q_{21}^L + q_{22}^L]. \]

Let \( Q_{ij}^2 \) be the share of workers \( j \) in the aggregate population of workers with skill \( i \):

\[ Q_{ij}^2 = \frac{q_{ij}^L}{q_i^{*L}} = \frac{q_{ij}^L}{q_{11}^L + q_{12}^L} \quad (i = 1, 2; j = 1, 2). \]

The (quasi-diagonal) matrix of labor shares is:

\[
Q^2 = \begin{bmatrix} Q_{11}^2 & 0 \\ 0 & Q_{22}^2 \end{bmatrix} = \begin{bmatrix} \frac{q_{11}^2}{q_{11}^2 + q_{12}^2} & 0 \\ 0 & \frac{q_{22}^2}{q_{21}^2 + q_{22}^2} \end{bmatrix}. (6.27)
\]

The effect of postmultiplying the vector of populations by skills \( q^{*L} \) by matrix \( Q^2 \) is to disaggregate, yielding the vector of individual populations:

\[ q^{*L} Q^2 = q^L. \] (6.28)

Now define the vector of reduction coefficients of the labor power of each skill as weighted averages of the individual reduction coefficients:

\[
z^* = \begin{bmatrix} z_1^* \\ z_2^* \end{bmatrix} = \begin{bmatrix} Q_{11}^2 z_{11} + Q_{12}^2 z_{12} \\ Q_{21}^2 z_{21} + Q_{22}^2 z_{22} \end{bmatrix} = \begin{bmatrix} \frac{q_{11}^2}{q_{11}^2 + q_{12}^2} z_{11} + \frac{q_{12}^2}{q_{11}^2 + q_{12}^2} z_{12} \\ \frac{q_{21}^2}{q_{21}^2 + q_{22}^2} z_{21} + \frac{q_{22}^2}{q_{21}^2 + q_{22}^2} z_{22} \end{bmatrix} \] (6.29)

The effect of premultiplying the vector of individual coefficients of reduction \( z \) by matrix \( Q^2 \) is to form their (weighted) average, yielding the average reduction coefficients:

\[ Q^2 z = z^*. \]
Hence, we can jointly express the averaging effects of matrices $Q^1$ and $Q^2$ by means of the following expression:

$$
\begin{bmatrix}
Q^1 & 0 \\
0 & Q^2
\end{bmatrix}
\begin{bmatrix}
v \\
z
\end{bmatrix}
= 
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix}.
$$

(6.30)

It can be written compactly as $Q^0 r = r^*$, where $r \equiv (v \ z)^T$ and $r^* \equiv (v^* \ z^*)^T$.

The ratios $V_{ij}$ between the individual and market values of the same commodities, and the ratios $Z_{ij}$ between the individual and average labor reduction coefficients are defined as

$$
V_{ij} = \frac{v_{ij}}{v_i^*}, \quad Z_{ij} = \frac{z_{ij}}{z_i^*},
$$

and can be put in respective (quasi-diagonal) matrices

$$
V = \begin{bmatrix}
V_{11} & 0 \\
V_{12} & 0 \\
0 & V_{21} \\
0 & V_{22}
\end{bmatrix}, \quad Z = \begin{bmatrix}
Z_{11} & 0 \\
Z_{12} & 0 \\
0 & Z_{21} \\
0 & Z_{22}
\end{bmatrix},
$$

(6.31) (6.32)

These are disaveraging matrices since it can easily be checked that $V v^* = v$ and $Z z^* = z$, which can be expressed in a single matrix equation:

$$
\begin{bmatrix}
V & 0 \\
0 & Z
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix}
= 
\begin{bmatrix}
v \\
z
\end{bmatrix}.
$$

(6.33)

And it can be written compactly as $X^0 r^* = r$.

We now use these instruments to obtain an adequate aggregation of the individual quantities and values systems. The disaggregated systems (6.19), (6.20) and (6.21) can be written compactly as:

$$
q^0 M = q^0, \quad M r = r, \quad M y^0 = y^0,
$$

(6.34)

where $q^0 \equiv (q^Q \ q^L)$, $r \equiv (v \ z)^T$, and $y^0 \equiv (p \ w)^T$.

Starting with the second system of (6.34), we can use (6.33) to obtain $MX^0 r^* = r$. And premultiplying by $Q^0$ and using (6.30) we have $Q^0 M X^0 r^* = Q^0 r = r^*$, that is, $M^* r^* = r^*$, where we have defined the aggregate social matrix $M^*$:

$$
M^* = \begin{bmatrix}
A^* & L^* \\
C_L^* & 0
\end{bmatrix} = Q^0 M X^0 = \begin{bmatrix}
Q^1 AV & Q^1 LZ \\
Q^2 C_L V & 0
\end{bmatrix}.
$$

The market values and labor reduction coefficients system ($M^* r^* = r^*$) is hence:

$$
\begin{bmatrix}
A^* & L^* \\
C_L^* & 0
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix}
= 
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix}.
$$

(6.35)

Notice that $M^*$ (as $M$) has a dominant eigenvalue equal to one and that the vector $(v^* \ z^*)^T$ is the dominant right eigenvector of $M^*$ associated with it. Assuming $z_1^* < z_2^*$,
the second kind of labor is skilled labor and the first is simple labor. Therefore, the eigenvector \((v^* z^* )^T\) can be normalized by means of \(z^*_1 = 1\). And \(z^*_2 > 1\) is the factor that expresses skilled labor in (expanded) units of socially necessary abstract labor.

From the two equalities in (6.35) we have:

\[
v^* = (I - A^*)^{-1} L^* z^*
\]

\[
C_L^* v^* = z^*.
\]

Observe that the vector of market values \(v^*\) is given by the amount of abstract labor socially necessary for the production of the respective commodities. The direct individual labor requirements \(L\) are transformed into average labors \(L^* = Q^1LZ\) for each kind of labor and reduced to units of (abstract) simple labor by means of the labor reduction coefficients vector \(z^*\). We can also observe that vector \(z^*\) is defined by the vector of market values \((C_L^* v^*)\) of the average consumption baskets \(C_L^* = Q^2C_L V\) of the two kinds of labor. Finally, the vector of market values \(v^*\) is given by the (direct and indirect) quantity of abstract labor socially necessary for the unit production of each type of commodity \(((I - A^*)^{-1} L^* z^*)\). As Marx writes:

Along with the useful qualities of the products themselves, we put out of sight both the useful character of the various kinds of labour embodied in them, and the concrete forms of that labour; there is nothing left but what is common to them all; all are reduced to one and the same sort of labour, human labour in the abstract... A use value, or useful article, therefore, has value only because human labour in the abstract has been embodied or materialised in it. How, then, is the magnitude of this value to be measured? Plainly, by the quantity of the value-creating substance, the labour contained in the article (Pt. 48; italics added).

We can also obtain the system of aggregate quantities. For this it is convenient to first express jointly the disaveraging effects of \(Q^1\) and \(Q^2\) ((6.25) and (6.28)):

\[
\begin{bmatrix} q^*Q & q^*L \\ Q^1 & Q^2 \end{bmatrix} \begin{bmatrix} Q_1^1 & 0 \\ 0 & Q_2^2 \end{bmatrix} = \begin{bmatrix} q^Q & q^L \\ Q^1 & Q^2 \end{bmatrix},
\]

which can be written compactly as \(q^{0*}Q^0 = q^0\), and second, verify that

\[
\begin{align*}
Q^1 V &= \begin{bmatrix} Q_{11} V_{11} + Q_{12} V_{12} & 0 \\ 0 & Q_{21} V_{21} + Q_{22} V_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
Q^2 Z &= \begin{bmatrix} Q_{11} Z_{11} + Q_{12} Z_{12} & 0 \\ 0 & Q_{21} Z_{21} + Q_{22} Z_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\end{align*}
\]

whereby

\[
\begin{bmatrix} Q^1 & 0 \\ 0 & Q^2 \end{bmatrix} \begin{bmatrix} V & 0 \\ 0 & Z \end{bmatrix} = \begin{bmatrix} I_{2 \times 2} & 0 \\ 0 & I_{2 \times 2} \end{bmatrix},
\]

which can be written compactly as \(Q^{0*}X^0 = I\).

Beginning with the system of individual quantities \(q^{0*}M = q^0\) we can use \(q^{0*}Q^0 = q^0\) on both sides of the equality to obtain \(q^{0*}Q^0 M = q^{0*}Q^0\). Multiplying by \(X^0\) we get \(q^{0*}Q^0 M X^0 = q^{0*}Q^0 X^0 = q^{0*}\), where (6.34) was used for the first equality and (6.37) for the second. Hence, the system of aggregate quantities is \(q^{0*}M^* = q^{0*}\), that is:

\[
\begin{bmatrix} q^*Q & q^*L \\ A^* & L^* \end{bmatrix} \begin{bmatrix} C_L^* \\ 0 \end{bmatrix} = \begin{bmatrix} q^*Q & q^*L \end{bmatrix}.
\]
The relative market values (formed from the elements of $v^*$) are for Marx the ‘centers of gravity’ around which the relative market prices fluctuate over time. The market prices are unique for each commodity at each moment of time. They fluctuate around the respective market values (relative to the market value of gold) and coincide exactly with them when the aggregate demand and aggregate supply of each commodity coincide. In modern mathematical language we can say that the market values (as ratios to the market value of gold) are ‘attractors’ for the corresponding market prices. The following paragraph expresses this:

On the one hand, market value is to be viewed as the average value of commodities produced in a single sphere, and, on the other, as the individual value of the commodities produced under average conditions of their respective sphere and forming the bulk of the products of that sphere. It is only in extraordinary combinations that commodities produced under the worst, or the most favourable, conditions regulate the market value, which, in turn, forms the centre of fluctuation for market prices. The latter, however, are the same for commodities of the same kind. If the ordinary demand is satisfied by the supply of commodities of average value, hence of a value midway between the two extremes, then the commodities whose individual value is below the market value realise an extra surplus value, or surplus profit, while those whose individual value exceeds the market value are unable to realise a portion of the surplus value contained in them (B3, 177; italics added).

As we will see in the following chapters, according to Marx’s theory of Capitalism the search for extra profits on the part of innovating industrial capitalists is a central aspect of the revolutionary dynamics of this mode of production. And these extra profits will be constructed in a way that is analogous to what we have seen in this section, with the important difference that instead of relative market values, it will be the relative market production prices that will be the ‘attractors’ for the relative market prices (if private landowners and rents are assumed away).

**Graphical representation** To graph the market value in SCP it is convenient to maintain the assumption that there are two techniques for the production of each of two commodities but to assume that there is only simple labor which consumes a single consumption basket. Hence, we have $q^*L = q^L$, $Q^2 = q^L/q^*L = 1$, $z = z^* = 1$, $Z = 1$, and (6.38) and (6.35) reduce to

$$
\begin{bmatrix}
q^O & q^L
\end{bmatrix}
\begin{bmatrix}
A^* & \ell^*
\end{bmatrix}
\begin{bmatrix}
v^*
\end{bmatrix}
= \begin{bmatrix}
q^O & q^L
\end{bmatrix}
\begin{bmatrix}
v^*
\end{bmatrix},
$$

where the matrix is

$$M^* = \begin{bmatrix}
A^* & \ell^*
cia L & 0
\end{bmatrix}
= \begin{bmatrix}
Q^1 & 0
\end{bmatrix}
\begin{bmatrix}
A & \ell
\end{bmatrix}
\begin{bmatrix}
v
\end{bmatrix}
= \begin{bmatrix}
Q^1AV & Q^1\ell
\end{bmatrix}.
$$

---

15 For the market price signifies that the same price is paid for commodities of the same kind, although they may have been produced under very different individual conditions and hence may have considerably different cost prices” (B3, 197).
Figure 3 (roughly) shows how Figure 1 changes with the introduction of multiple techniques for the production of each commodity. The two rays on each of the northwest and southeast quadrants show that there are two alternative techniques for the production of each commodity. The line most to the northeast is the ‘employment line’ \( q^*Q \ell^* = q^L \), where the slope is now given by the average direct labor requirements \( \ell^* \). The line that extends from one ray to the other shows where aggregate consumption would be located if only the most efficient techniques were used. This ‘efficiency frontier’ is typically emphasized by neoclassical theory on the assumptions that all producers know about and can implement all the available techniques, and minimize costs in a context of perfect competition, and lacking any additional constraints (such as the inability to raise enough money in order to use a technique that requires high expenditures). Under such assumptions it would be irrational to use any but the most efficient technique. But Marx’s perspective was completely different, accepting as an empirical fact that in any time period there is a range of techniques that are in use and that individual producers can face various restrictions that prevent their use of the most efficient one. Consequently, in Figure 3 we have highlighted a point that is below the ‘efficiency frontier’, because it is the sum of net productions that use techniques that are not both the most efficient. That point is located on the ‘consumption line’ represented in within the northeast quadrant and is given by \( C^*v^* = q^L \), where \( q^L \) is given and defines its location and where the slope is defined by the market values, that is, the elements of \( v^* \), which are simply the weighted averages of the individual values of each of the two produced commodities.

Figure 3

Appendix to Chapter 6
Bibliographic Notes

Dorffman, Samuelson and Solow (1958) and Samuelson (1957) The book *Linear Programming & Economic Analysis*, by Dorffman, Samuelson and Solow (1958), was an important contribution for the understanding of the relation between the linear models widely used in economic theory at the time (Input-Output analysis, Linear Programming, Game Theory) and standard Neoclassical theory. However, it is also
a book that almost completely eludes certain very important topics such as the distribution of income and wealth. Our Figure 1 is based on one of the figures of the two chapters of that book on the "Statical Leontief System." This is a model like our simplest version of SCP, with one kind of labor and one technique per sector. However, the authors call the income of the workers the 'wage rate' whereas the model cannot represent wage work because for that there would have to exist a social class whose members employ wage labor and pay these wages. They obtain the relative prices of the model as ratios between coefficients that represent "the total direct and indirect labor embodied in a unit of final consumption" of the respective commodities. But curiously they do not relate this in any way to the theories of Ricardo or Marx and much less do they note that such coefficients represent a very simplified version of Marx's values. Although Ricardo is mentioned in various places, Marx is only mentioned once (in passing) within a list of economists that built 'closed' systems. It is surprising that in a book of more than 500 pages income distribution is not even mentioned. Although the functioning of firms is analyzed, the word 'entrepreneur' does not even appear in the Index. The typical 'black box' concept of the firm prevails. It is seen as a 'control unit' that maximizes profits using as control variables the rates of use of various inputs and the rates of outputs of the various outputs.

It may be that the circumstances in which the book was written explain some of the omissions. It was written under contract from RAND Corporation, whose main promoter and client was then the U.S. Air Force, and during a period in the Cold War in which 'McCarthyism' prevailed, hundreds of people were arrested and thousands lost their jobs due to investigations often based on anonymous accusations of pro-Communism. In such a context it is understandable that the evident connections between the material and the labor-theory of value were not even hinted at. Notwithstanding such omissions, the book is still today of great value for understanding these linear techniques and their relation to Neoclassical economic theory.

The book gives great importance to the models of Leontief, who was a teacher to Samuelson (and possibly Dorfman) in Harvard during the 1940s. Leontief himself had Bortkiewicz as his thesis director at the University of Berlin and, as mentioned above, part of his 1927 thesis was published in German in 1928, and in English in 1991 with title "The economy as a circular flow." Dorfman, Samuelson and Solow also highlighted the importance of von Neumann's models. According to Leonard (2008) "During World War II, von Neumann was one of the U.S.'s most cherished military advisors, and this was followed by deep postwar involvements in the RAND Corporation and the Atomic Energy Commission."

According to Dorfman et al (1958), "It is a remarkable implication of the Leontief system that even if there were available several different processes for each industry, only one of them would ever be observed" (Dorfman et al, 1958, 224). This assertion is noteworthy since it seems to confuse the simplifying assumptions of the theory with observable reality. Because they assume perfect competition, maximizing behavior, and implicitly entrepreneurial omniscience and absence of impediments to switching rapidly between techniques, they hold that only one technique for each commodity "would ever be observed", i.e., the ones that define the 'consumption possibility frontier' (which is represented in Figure 3).

Samuelson individually took notice and looked into the evident relation between the research in the joint book with Dorfman and Solow and Marx when writing his article "Wages and Interest: A modern dissection of Marxian economic models" (Samuelson 1957). Here, despite some humorous and sometimes extravagant, and even absurd,
assertions\textsuperscript{16}, there is a serious effort to understand Marx’s theory. However, Samuelson tries to analyze this theory within a framework in which it cannot fit, that is, the Neoclassical world of perfect competition, exclusive use of the most profitable technique for each good, firm omniscience, and absence of impediments to immediately switch to the most profitable technique. Nevertheless, he sometimes becomes aware that he is trying to compress Marx’s theory in too small a container. He places his treatment of Marx as “part of a longer study of Ricardo-like systems” (referring to Dorfman \textit{et al} 1958) and apologizes that he “makes no attempt to do justice to the many noneconomic and imperfect-competition aspects of Marx’s thought, but takes seriously his belief that he was baring the inner workings of competitive capitalism” (Ibid., 884). The problem here is that Marx’s conception of ‘competitive capitalism’, as we will see, had very little to do with the neoclassical ‘perfect competition’ that Samuelson was apparently unable to do without.

With respect to multiple techniques Samuelson (1957) writes:

The case of a single fixed-coefficient technique is a very peculiar one indeed... Perhaps Karl Marx really had such a technology in mind. Perhaps not. It may be reasonable to believe that Marx, like Ricardo and other early writers, and unlike modern neoclassicists, never explicitly thought about what properties of the production function (a concept not yet explicitly defined or named) he wished to posit.... On the other hand, he speaks again and again of alternative techniques.\textsuperscript{17} While many of these clearly depict technological change in the production function rather than movement within one function, the fact that the old methods are still known along with the new shows that Marx and Ricardo definitely envisage the existence of more than one technique. Whether or not Marx would resent being interpreted as a believer in a fixed-coefficient single-technique world, I should resent on behalf of the real world any such description (Samuelson 1957, 906-7).

But although Samuelson subscribes to the simultaneous use of more than one technique in the real world and accepts that Ricardo and Marx took it well into account in their analysis, he himself thinks and theorizes in terms of maximizing and omniscient firms that always choose the most profitable technique.\textsuperscript{17} Nowhere does he address Marx’s case of the simultaneous use of different techniques for the production of the same commodity even if some of these are suboptimal (from some point of view).

\textbf{Herbert Simon’s ‘bounded rationality’ and Marx’s multiple techniques} The political scientist and economist Herbert Simon (1916-2001) disagreed with the excessive and unrealistic requirements that economic theory imposed on the modern large

\textsuperscript{16}For example: “Marx can be classified by the modern theorist as ‘Ricardo without diminishing returns’” (Samuelson 1957, 884), or “A minor Post-Ricardian, Marx was an autodidact cut off in his lifetime from competent criticism and stimulus” (Ibid., 911). This last assertion is remarkable, since Marx had an excellent academic training in the best universities of Germany, where Law was often taught alongside Cameralistics (which was akin to what is now called Public Finance). In 1836 Marx’s father signed his consent for him to “enter the University of Berlin next term for the purpose of continuing there his studies of Law and Cameralistics, which he began in Bonn” (MECW 1, 655). On the other hand, the U.S. was a latecomer in the academic training of economists. Harvard University, which was the vanguard, “introduced their first dedicated economics lectures in 1853” (History of Economic Thought website: https://www.hetwebsite.net/het/schools/americanuniv.htm).

\textsuperscript{17}This is what the ‘theorem on substitution’ of Dorfman \textit{et al} 1958, section 9-5 is about, which in section 10-6 was more correctly termed ‘non-substitution theorem’.
corporation such as maximizing profits and having all the necessary information and computational capacity to implement it. When he was awarded the Nobel Prize in 1978, in his conference he synthesized what he considered to be his most important ideas. He said, in particular, “When the goals of an organization cannot be connected operationally with actions (when the production function can’t be formulated in concrete terms), then decisions will be judged against subordinate goals that can be so connected. There is no unique determination of these subordinate goals. Their formulation will depend on the knowledge, experience, and organizational environment of the decision maker” (Simon 1978, 353). He states that the phenomenon of identifying the subordinate goals is only the visible tip of a very large iceberg, the shape of which can be “best appreciated by contrasting it with classical models of rational choice” (Ibid.). The models of rational choice he called ‘classical’ require the complete knowledge of all the possible alternative decisions and their consequences, as well as the ability to make the necessary calculations in order to compare the consequences “in terms of some consistent measure of utility.” For Simon it was imperative to replace such models with one which “would describe how decisions could be (and probably actually were) made when the alternatives of search had to be sought out, the consequences of choosing particular alternatives were only very imperfectly known both because of limited computational power and because of uncertainty in the external world, and the decision maker did not possess a general and consistent utility function for comparing heterogeneous alternatives” (Ibid.).

This alternative way of approaching decision-making in the “elaborate organizations that human beings have constructed in the modern world to carry out the work of production and government can only be understood as machinery for coping with the limits of man’s abilities to comprehend and compute in the face of complexity and uncertainty” (Ibid., 354). The procedures that have been used to cope with these problems fall within what he generically calls ‘bounded rationality’. They include: 1) looking for satisfactory choices instead of optimal ones, 2) replacing abstract global goals with tangible subgoals whose achievement can be observed and measured, and 3) dividing the decision-making task among many specialists whose work is coordinated by means of a structure of communications and authority relations.

The Leontief static model described by Samuelson et al resembles one of the ‘classical models’ described by Simon, with a definite global and abstract goal for producers and perfect knowledge of all the alternative techniques and the income each would generate, for which only one of the techniques, the optimal one, would be chosen by all the producers of the same commodity. In contrast, it can be argued that Marx’s model of SCP is closer to Simon’s notion of ‘bounded rationality’, since on the basis of the observation of empirical reality it admits the coexistence of multiple techniques for the production of each commodity, does not introduce the extreme assumption of full knowledge of all the alternative techniques and their consequences, nor the instant switch to the technique that maximizes income. The same can be said for Marx’s extended model of CCP that we will specify in Chapter 9, where it will be capitalist entrepreneurs that make decisions in firms and seek (not necessarily maximum) profits, and there are multiple techniques for the production of each commodity. As Simon says, “In Administrative Behavior, bounded rationality is largely characterized as a residual category – rationality is bounded when it falls short of omniscience. And the failures of omniscience are largely failures of knowing all the alternatives, uncertainty about relevant exogenous events, and inability to calculate consequences” (Ibid., 356). For these reasons Simon prefers to speak of ‘satisficing’ instead of ‘optimizing’ or ‘maximizing’ behavior.
Matrix techniques for sectoral aggregation  The procedure used in this chapter to obtain Marx’s market values differs from those used by Morishima and Seton (1961), Bródy (1970), and Morishima (1973) for aggregating sectors. They use a sectoral aggregation technique for matrices whose elements represent monetary amounts, i.e., quantity times price. And although Marx’s (Simple and Extended) Reproduction tables in Capital are always of that type, it is known that it is not the best matrix representation for theoretical analysis because the elements of the matrix are dependent on prices (or values). For example, in their cases $A_{ij}$ represents the value of input $j$ directly required for the production of commodity $i$ worth a dollar. These matrices are what mathematicians call stochastic because either all their rows or all their columns (according to how one defines $A_{ij}$) add up to one. The special properties of stochastic matrices are then used in order to aggregate sectors. This matrix form was used by Leontief (1941) to study the American economy (cf. Miller and Blair, 2009) and is very useful to represent the most abundant economic data: those that are expressed in monetary units. However, to represent theoretical matters of economics it is much better to use as starting point matrices that only represent physical quantities, as those we are here using. Such are the matrices used in what is now called the ‘Leontief model’, which are not stochastic because neither their rows nor columns necessarily add up to one.

The aggregation technique that has been used in the last section of this chapter was used by Bródy (1970) to obtain aggregate systems where the elements of the social matrix are ratios of monetary amounts, as explained in the preceding paragraph. We instead use this technique for matrices whose elements are ratios of physical quantities, with the explicit purpose of representing Marx’s ideas of market values vs. individual values (and in Chapter 9 market prices of production vs. individual prices of production). And this is possible because one can always add up quantities of the same commodity, even if they are produced with different techniques. On the other hand, one can always form weighted averages of individual values (or individual prices of production) of such commodities.

Mathematical Appendix to Chapter 6: SCP with multiple techniques

The generalization of the simple case we considered in this chapter is straightforward. Assume there are $n_i$ techniques for producing commodity $i$ and $N$ different commodities. Then the total number of commodities-techniques (or techniques associated to commodities) is:

$$n = \sum_{i=1}^{N} n_i.$$  \hspace{1cm} (6.39)

Assume we have made the permutations necessary to place commodities that are the same but are produced with different techniques adjacent to one another. Then we can write the vectors of produced quantities of commodities and of their individual values, respectively, as:

$$q^Q = \begin{pmatrix} q_{11}^Q & q_{12}^Q & \cdots & q_{1n_1}^Q & q_{21}^Q & q_{22}^Q & \cdots & q_{2n_2}^Q & \cdots & q_{N1}^Q & q_{N2}^Q & \cdots & q_{Nn_N}^Q \end{pmatrix}$$

$$v = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n_1} & v_{21} & v_{22} & \cdots & v_{2n_2} & \cdots & v_{N1} & v_{N2} & \cdots & v_{Nn_N} \end{pmatrix}^T.$$
Adding up the quantities of the same commodity produced by the \( n_i \) adjacent processes yields the aggregate production of good \( i \), \( q_i^{sQ} \):

\[
q_i^{sQ} = \sum_{j=1}^{n_i} q_{ij}^Q.
\]

Hence the vector of aggregate outputs of the \( N \) different goods is

\[
q^{sQ} = \left( q_1^{sQ}, q_2^{sQ}, \ldots, q_N^{sQ} \right).
\]

Let \( Q_{ij}^1 \) be the share of good \( j \) in the aggregate output of commodity \( i \):

\[
Q_{ij}^1 = \frac{q_{ij}^Q}{q_i^{sQ}},
\]

and define the quasi-diagonal matrix of shares in aggregate outputs as:

\[
Q^1 = \begin{bmatrix}
Q_{11}^1 & Q_{12}^1 & \ldots & Q_{1n_i}^1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]

Observe that the effect of postmultiplying the vector of aggregate outputs \( q^{sQ} \) by matrix \( Q^1 \) is to disaggregate, yielding the vector of individual outputs:

\[
q_i^Q Q^1 = q_i^Q.
\]

Now define the value of commodity \( i \) (\( v_i^* \)) as the weighted average of the individual values of that commodity produced by means of different techniques:

\[
v_i^* = \sum_{j=1}^{n_i} Q_{ij}^1 v_{ij},
\]

and the vector of market values of the \( N \) commodities:

\[
v^* = (v_1^*, v_2^*, \ldots, v_N^*)^T.
\]

Notice that premultiplying the vector of individual values \( v \) by matrix \( Q^1 \) yields the vector of market values:

\[
Q^1 v = v^*.
\]

Similarly, assume there are \( m_i \) reproduction processes (or consumption baskets) for labor of skill \( i \), and that there are \( M \) different skills. The total number of consumption baskets is hence:

\[
m = \sum_{i=1}^{M} m_i.
\]

The vectors of reproduced quantities of the different types of labor (or populations of workers with different skills) \( q^L \) and coefficients of reduction of skills to simple labor \( z \), respectively, are the following:

\[
q^L = \left( q_{11}^L, q_{12}^L, \ldots, q_{1m_1}^L, q_{21}^L, q_{22}^L, \ldots, q_{2m_2}^L, \ldots, q_{M1}^L, q_{M2}^L, \ldots, q_{Mm_M}^L \right)
\]

\[
z = \left( z_{11}, z_{12}, \ldots, z_{1m_1}, z_{21}, z_{22}, \ldots, z_{2m_2}, \ldots, z_{M1}, z_{M2}, \ldots, z_{Mm_M} \right)^T.
\]
Let $q_{i}^{*L}$ be the total population of workers with labor type $i$ (that consume various baskets of commodities because they use different techniques):

$$q_{i}^{*L} = \sum_{j=1}^{m_i} q_{ij}^{L},$$

and $q^{*L}$ the vector of aggregate populations of workers of different skills:

$$q^{*L} = (q_{1}^{*L} q_{2}^{*L} \ldots q_{M}^{*L}).$$

The share of $j$ in the aggregate population of workers with skill $i$ is:

$$Q_{ij}^{2} = \frac{q_{ij}^{L}}{q_{i}^{*L}}$$

and the quasi-diagonal matrix of shares in types of labor is:

$$Q^{2} = \begin{bmatrix}
Q_{11}^{2} & Q_{12}^{2} & \cdots & Q_{1m_1}^{2} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Q_{21}^{2} & Q_{22}^{2} & \cdots & Q_{2m_2}^{2} & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \cdots & Q_{M1}^{2} & Q_{M2}^{2} & \cdots & Q_{Mm_M}^{2}
\end{bmatrix}.$$  \hspace{1cm} (6.43)

Then the effect of postmultiplying the vector of populations $q^{*L}$ by matrix $Q^{2}$ is to disaggregate, yielding the vector of individual populations:

$$q^{*L}Q^{2} = q^{*}.$$  \hspace{1cm} (6.44)

We also define the average coefficient of reduction for labor skill $i$ ($z_{i}^{*}$) as the weighted average of the individual reduction coefficients:

$$z_{i}^{*} = \sum_{j=1}^{m_i} Q_{ij}^{2}z_{ij}$$

and the vector of average reduction coefficients of the $M$ labor types:

$$z^{*} = (z_{1}^{*} z_{2}^{*} \ldots z_{M}^{*})^T.$$  \hspace{1cm}

Then premultiplying the vector of individual reduction coefficients by matrix $Q^{2}$ yields the vector of average reduction coefficients:

$$Q^{2} z = z^{*}.$$  

Hence, the averaging effect of matrices $Q^{1}$ and $Q^{1}$ can be expressed as:

$$\begin{bmatrix}
Q^{1} & 0 \\
0 & Q^{2}
\end{bmatrix}\begin{bmatrix}
v \\
z
\end{bmatrix} = \begin{bmatrix}
v^{*} \\
z^{*}
\end{bmatrix}.  \hspace{1cm} (6.45)$$

Now define the ratios $V_{ij}$ between the individual and market values of the various commodities and the ratios $Z_{ij}$ between the individual and average reduction coefficients of the various labor skills

$$V_{ij} = \frac{v_{ij}}{v_{i}^{*}}, \quad Z_{ij} = \frac{z_{ij}}{z_{i}^{*}},$$
as well as the quasi-diagonal matrices (expressed in transposed form to save space)

\[
V = \begin{bmatrix}
V_{11} & V_{12} & \cdots & V_{1n_1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & V_{21} & V_{22} & \cdots & V_{2n_2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & V_{N1} & V_{N2} & \cdots & V_{Nn_N}
\end{bmatrix}^T
\]

\[
Z = \begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1m_1} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z_{21} & Z_{22} & \cdots & Z_{2m_2} & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & Z_{M1} & Z_{M2} & \cdots & Z_{Mm_M}
\end{bmatrix}^T
\]

These are disaggregating matrices, since it can be checked that

\[Vv^* = v, \quad Zz^* = z,
\]

which can be conveniently written as:

\[
\begin{bmatrix}
V & 0 \\
0 & Z
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix} =
\begin{bmatrix}
v \\
z
\end{bmatrix}. \tag{6.48}
\]

Introducing (6.48) in (6.20) we get

\[
\begin{bmatrix}
A & L \\
C_L & 0
\end{bmatrix}
\begin{bmatrix}
V & 0 \\
0 & Z
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix} =
\begin{bmatrix}
v \\
z
\end{bmatrix}. \tag{6.49}
\]

And premultiplying by the aggregating matrix \(Q\), yields:

\[
\begin{bmatrix}
Q^1 & 0 \\
0 & Q^2
\end{bmatrix}
\begin{bmatrix}
A & L \\
C_L & 0
\end{bmatrix}
\begin{bmatrix}
V & 0 \\
0 & Z
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix} =
\begin{bmatrix}
Q^1 & 0 \\
0 & Q^2
\end{bmatrix}
\begin{bmatrix}
v \\
z
\end{bmatrix}. \tag{6.50}
\]

Using (6.45) we obtain the system of market values and average reduction coefficients (if the latter is properly normalized):

\[
\begin{bmatrix}
A^* & L^* \\
C_L^* & 0
\end{bmatrix}
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix} =
\begin{bmatrix}
v^* \\
z^*
\end{bmatrix}, \tag{6.50}
\]

where the aggregate social matrix has been defined as:

\[
M^* = \begin{bmatrix}
A^* & L^* \\
C_L^* & 0
\end{bmatrix} = \begin{bmatrix}
Q^1 & 0 \\
0 & Q^2
\end{bmatrix}
\begin{bmatrix}
A & L \\
C_L & 0
\end{bmatrix}
\begin{bmatrix}
V & 0 \\
0 & Z
\end{bmatrix} =
\begin{bmatrix}
Q^1AV & Q^1LZ \\
Q^2C_LV & 0
\end{bmatrix}.
\]

(6.50) shows that \(M^*\) (as \(M\)) has a dominant eigenvalue equal to one and that the vector \((v^* \ z^*)^T\) is the dominant right eigenvector of \(M^*\). It should be normalized such that the labor of type \(k\) that verifies \(z_k^* < z_i^* \forall i \neq k\) (assuming \(k\) is unique) defines simple abstract labor. Hence, normalize vector \(z^*\) with \(z_k^* = 1\). The remaining elements of \(z^*\) give multiples of simple abstract labor for each labor skill. The two equalities of (6.50) yield

\[
v^* = (I - A^*)^{-1} L^* z^*
\]

\[
C_L^* v^* = z^*.
\]
The first equality expresses the vector of market values as quantities of \textit{abstract labor} socially (directly and indirectly) necessary for the production of the respective commodities. The various requirements of average labors $L^*$ are reduced to simple abstract labor by means of the vector $z^*$. On the other hand, the second equality shows that the normalized vector $z^*$ of coefficients of reduction of skilled labor to simple labor is equal to the vector of market values of the average consumption baskets of the various labor skills $(C_L^* v^*)$.

On the other hand, remember that $q^{*Q}$ and $q^{*L}$ are the vectors of the aggregate quantities of commodities and labor populations by skill. As a complement, we can also obtain the system of aggregate quantities (6.38) using matrix $M^*$. First, from (6.41) and (6.44) we obtain

$$\begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} = \begin{bmatrix} q^Q \\ q^L \end{bmatrix}.$$  

Second, use this expression in (6.19):

$$\begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} \begin{bmatrix} A \\ C_L \\ 0 \end{bmatrix} = \begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix}.$$  

Third, postmultiply by the matrix of (6.48):

$$\begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} \begin{bmatrix} A \\ C_L \\ 0 \end{bmatrix} \begin{bmatrix} V \\ 0 \\ Z \end{bmatrix} = \begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} \begin{bmatrix} V \\ 0 \\ Z \end{bmatrix}.$$  

Fourth, check that

$$Q^1 V = \begin{bmatrix} \sum_{j=1}^{n_1} Q^1_{1j} V_{1j} \\ \ldots \\ \sum_{j=1}^{n_N} Q^1_{Nj} V_{Nj} \end{bmatrix} = I_{n_N \times n_N}$$

$$Q^2 Z = \begin{bmatrix} \sum_{j=1}^{m_1} Q^2_{1j} Z_{1j} \\ \ldots \\ \sum_{j=1}^{m_M} Q^2_{Mj} Z_{Mj} \end{bmatrix} = I_{m_M \times m_M}$$

which implies

$$\begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} \begin{bmatrix} V \\ 0 \\ Z \end{bmatrix} = \begin{bmatrix} Q^1 V \\ 0 \\ Q^2 Z \end{bmatrix} = \begin{bmatrix} I_{n_N \times n_N} \\ 0 \\ I_{m_M \times m_M} \end{bmatrix}.$$  

Finally, taking the definition of $M^*$ into account we have the system of aggregate quantities

$$\begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix} \begin{bmatrix} A^* \\ C_L^* \\ 0 \end{bmatrix} = \begin{bmatrix} q^{*Q} \\ q^{*L} \end{bmatrix},$$

which is the dual equation to (6.50).

\section*{Numerical Exercise \#1}

The second part of this chapter has been somewhat abstract so we here illustrate it with a numerical example. Assuming there are two commodities and two kinds
of labor, matrices $A$, $L$, are $C_L$ are all of dimension $4 \times 4$. As usual, we assume that the two productive processes that produce the same commodity and the two reproduction processes that reproduce labor of the same kind are adjacent. Let the numerical matrices be the following:

$$
A = \begin{bmatrix}
0.05 & 0.2 & 0.3 & 0.1 \\
0.3 & 0.1 & 0.5 & 0.12 \\
0.2 & 0.6 & 0.2 & 0.15 \\
0.3 & 0.2 & 0.068 & 0.05 \\
\end{bmatrix}, \quad L = \begin{bmatrix}
0.04 & 0.1 & 0.07 & 0.2 \\
0.15 & 0.1 & 0.05 & 0.03 \\
0.02 & 0.05 & 0.11 & 0.15 \\
0.05 & 0.04 & 0.2 & 0.05 \\
\end{bmatrix}
$$

$$
C_L = \begin{bmatrix}
0.11 & 0.12 & 0.07 & 0.02 \\
0.12 & 0.1 & 0.05 & 0.03 \\
0.02 & 0.06 & 0.02 & 0.05 \\
0.03 & 0.05 & 0.04 & 0.03 \\
\end{bmatrix}.
$$

The numbers have been chosen (by trial and error) so that the resulting social matrix $M$ has a dominant eigenvalue equal to one. Its left and right dominant eigenvectors, respectively, are shown in the following equalities, where the first is the quantities system and has been written transposed to save space.\(^{18}\)

$$
\begin{bmatrix}
0.05 & 0.3 & 0.2 & 0.3 & 0.11 & 0.12 & 0.02 & 0.03 \\
0.2 & 0.1 & 0.6 & 0.2 & 0.12 & 0.1 & 0.06 & 0.05 \\
0.3 & 0.5 & 0.2 & 0.068 & 0.07 & 0.06 & 0.02 & 0.04 \\
0.1 & 0.12 & 0.15 & 0.05 & 0.02 & 0.03 & 0.05 & 0.03 \\
0.04 & 0.15 & 0.02 & 0.05 & 0 & 0 & 0 & 0 \\
0.1 & 0.1 & 0.05 & 0.04 & 0 & 0 & 0 & 0 \\
0.07 & 0.05 & 0.11 & 0.2 & 0 & 0 & 0 & 0 \\
0.2 & 0.03 & 0.15 & 0.05 & 0 & 0 & 0 & 0 \\
\end{bmatrix} = \begin{bmatrix}
0.42 \\
0.58 \\
0.57 \\
0.23 \\
0.13 \\
0.14 \\
0.17 \\
0.20 \\
\end{bmatrix},
$$

$$
\begin{bmatrix}
0.05 & 0.2 & 0.3 & 0.1 & 0.04 & 0.1 & 0.07 & 0.2 \\
0.3 & 0.1 & 0.5 & 0.12 & 0.15 & 0.1 & 0.05 & 0.03 \\
0.2 & 0.6 & 0.2 & 0.15 & 0.02 & 0.05 & 0.11 & 0.15 \\
0.3 & 0.2 & 0.068 & 0.05 & 0.05 & 0.04 & 0.2 & 0.05 \\
0.11 & 0.12 & 0.07 & 0.02 & 0 & 0 & 0 & 0 \\
0.12 & 0.1 & 0.05 & 0.03 & 0 & 0 & 0 & 0 \\
0.02 & 0.06 & 0.02 & 0.05 & 0 & 0 & 0 & 0 \\
0.03 & 0.05 & 0.04 & 0.03 & 0 & 0 & 0 & 0 \\
\end{bmatrix} = \begin{bmatrix}
0.39 \\
0.56 \\
0.61 \\
0.32 \\
0.16 \\
0.14 \\
0.07 \\
0.07 \\
\end{bmatrix}.
$$

Hence,

$$
q^Q = \begin{bmatrix}
0.42 & 0.58 & 0.57 & 0.23 \\
\end{bmatrix}, \quad v = \begin{bmatrix}
0.39 \\
0.56 \\
0.61 \\
0.32 \\
\end{bmatrix}, \quad z = \begin{bmatrix}
0.16 \\
0.14 \\
0.07 \\
0.07 \\
\end{bmatrix}.
$$

\(^{18}\)To save space we have also rounded off the eigenvectors to two decimals. But the calculations have been done with approximations to five decimals. On the other hand, the eigenvectors are normalized so that the sum of the squares of their elements is one (as the software does automatically). Hence, we leave it to the reader to renormalize the two vectors according to the text as an exercise.
Doing some simple calculations we get

\[
Q^1 = \begin{bmatrix} Q_{11}^1 & Q_{12}^1 & 0 & 0 \\ 0 & 0 & Q_{21}^1 & Q_{22}^1 \end{bmatrix} = \begin{bmatrix} 0.42 & 0.58 & 0 & 0 \\ 0 & 0 & 0.71 & 0.29 \end{bmatrix}, \\
Q^2 = \begin{bmatrix} Q_{11}^2 & Q_{12}^2 & 0 & 0 \\ 0 & 0 & Q_{21}^2 & Q_{22}^2 \end{bmatrix} = \begin{bmatrix} 0.48 & 0.52 & 0 & 0 \\ 0 & 0 & 0.46 & 0.54 \end{bmatrix}, \\
q^{Q*} = \begin{bmatrix} 1.00 \\ 0.27 \end{bmatrix}, \\
v^* = \begin{bmatrix} 0.49 \\ 0.53 \end{bmatrix}, \\
z^* = \begin{bmatrix} 0.15 \\ 0.07 \end{bmatrix}.
\]

\[
V = \begin{bmatrix} 0.79 & 0 \\ 1.15 & 0 \\ 0 & 1.16 \\ 0 & 0.60 \end{bmatrix}, \quad Z = \begin{bmatrix} 1.06 & 0 \\ 0.95 & 0 \\ 0 & 0.97 \\ 0 & 1.03 \end{bmatrix}.
\]

We verify that

\[
Vv^* = \begin{bmatrix} 0.79 & 0 \\ 1.15 & 0 \\ 0 & 1.16 \\ 0 & 0.60 \end{bmatrix} \begin{bmatrix} 0.49 \\ 0.53 \end{bmatrix} = \begin{bmatrix} 0.39 \\ 0.56 \\ 0.61 \\ 0.32 \end{bmatrix} = v.
\]

\[
Zz^* = \begin{bmatrix} 1.06 & 0 \\ 0.95 & 0 \\ 0 & 0.97 \\ 0 & 1.03 \end{bmatrix} \begin{bmatrix} 0.15 \\ 0.14 \\ 0.07 \\ 0.07 \end{bmatrix} = z.
\]

Hence, \(M^\star\) is

\[
M^\star = \begin{bmatrix} A^\star & L^\star \\ C_L^\star & 0 \end{bmatrix} \begin{bmatrix} Q^1 & 0 \\ 0 & Q^2 \end{bmatrix} \begin{bmatrix} A & L \\ C_L & 0 \end{bmatrix} \begin{bmatrix} V & 0 \\ 0 & Z \end{bmatrix} = \begin{bmatrix} 0.32 & 0.55 & 0.20 & 0.16 \\ 0.74 & 0.26 & 0.07 & 0.26 \\ 0.22 & 0.08 & 0 & 0 \\ 0.08 & 0.06 & 0 & 0 \end{bmatrix},
\]

and we can check the following:

\[
\begin{bmatrix} 0.32 & 0.55 & 0.20 & 0.16 \\ 0.74 & 0.26 & 0.07 & 0.26 \\ 0.22 & 0.08 & 0 & 0 \\ 0.08 & 0.06 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.49 \\ 0.53 \\ 0.15 \\ 0.07 \end{bmatrix} = \begin{bmatrix} 0.49 \\ 0.53 \\ 0.15 \\ 0.07 \end{bmatrix},
\]

\[
\begin{bmatrix} 0.32 & 0.74 & 0.22 & 0.08 \\ 0.55 & 0.26 & 0.08 & 0.06 \\ 0.20 & 0.075 & 0 & 0 \\ 0.16 & 0.26 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.80 \\ 0.27 \\ 0.37 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0.80 \\ 0.27 \\ 0.37 \end{bmatrix},
\]

where we have again written the quantities system in transposed form. \(\blacksquare\)
Chapter 7  MONEY AND THE
CIRCULATION OF COMMODITIES

One of the aspects that Marx criticized of the “methods characteristic of apologetic” economics was “the identification of the circulation of commodities with the direct barter of products, by simple abstraction from their points of difference” (B1, 124, footnote 1). In Contribution Marx had elaborated a first version of his theory of commodities and the role of money in the ‘simple circulation of commodities’, the kind of circulation that corresponded to SCP. He there stated that: “The principal difficulty in the analysis of money is surmounted as soon as it is understood that the commodity is the origin of money” (Contribution, MECW 29, 303). In accordance with his historical-genetic method he explained that the book would only focus on “those forms of money which arise directly from the exchange of commodities, but not with forms of money, such as credit money, which belong to a higher stage of production” (Ibid.). When several years later he wrote Book I of Capital he reworked in its first three chapters the main results reached in Contribution (and also eliminated the interesting sections on the historical development of monetary theories). As in Contribution, he states that “Throughout this work, I assume, for the sake of simplicity, gold as the money commodity” (B1, 103). But one should take note that he was referring to Book I, for in Book III he does introduce other forms of money such as credit-money, bank bills, etc. But even in Chapter 3 of Book I there are interesting considerations on silver and bimetallism.

Marx sketches how over the centuries the exchange process gradually segregated one of the commodities exchanged to take the role of ‘general equivalent’. “Money is a crystal formed of necessity in the course of the exchanges” (B1, 97) because in the long run the owners of commodities “cannot bring their commodities into relation as values, and therefore as commodities, except by comparing them with some one other commodity as the universal equivalent... But a particular commodity cannot become the universal equivalent except by a social act. The social action therefore of all other commodities, sets apart the particular commodity in which they all represent their values... To be the universal equivalent, becomes, by this social process, the specific function of the commodity thus excluded by the rest. Thus it becomes money” (Ibid.). And he points out that “Nomad races are the first to develop the money form, because all their worldly goods consist of moveable objects and are therefore directly alienable; and because their mode of life, by continually bringing them into contact with foreign communities, solicits the exchange of products” (B1, 99). And the role of money is attached “either to the most important articles of exchange from outside... or else it attaches itself to the object of utility that forms, like cattle, the chief portion of indigenous alienable wealth.” But the commodities that have historically been selected as money were always such that they could be exchanged between communities; which requires that they must be movable over space. This is why land, for example, could not be made into money, whereas “Man has often made man himself, under the form of slaves, serve as the primitive material of money” (Ibid.).

Marx’s monetary theory
The functions of money

According to Marx, “The commodity that functions as a measure of value, and, either in its own person or by a representative, as the medium of circulation, is money” (B1,
140; italics added). Whereas the exchange of commodities without the intervention of money (i.e., barter) needed multiple expressions of value equivalence (between all the interchangeable pairs of commodities), exchange became much simpler after money appeared:

The expression of the value of a commodity in gold –x commodity A = y money commodity– is its money form or price. A single equation, such as 1 ton of iron = 2 ounces of gold, now suffices to express the value of the iron in a socially valid manner. There is no longer any need for this equation to figure as a link in the chain of equations that express the values of all other commodities, because the equivalent commodity, gold, now has the character of money... But money itself has no price. In order to put it on an equal footing with all other commodities in this respect, we should be obliged to equate it to itself as its own equivalent (B1, 105).

Marx points out that when “money serves as a measure of value, it is employed only as imaginary or ideal money” and that this “has given rise to the wildest theories.” But “price depends entirely upon the actual substance that is money” (B1, 105-6). And since the value of the other commodities is measured in terms of gold, the need arises for establishing a certain quantity of gold as the unit with which prices can be expressed, of establishing a standard of price. Referring to gold, he writes that “As the measure of value it serves to convert the values of all the manifold commodities into prices, into imaginary quantities of gold; as the standard of price it measures those quantities of gold” (B1, 107; italics added). Marx points out that “the establishment of an unvarying unit of measure is all-important. Hence, the less the unit is subject to variation, so much the better does the standard of price fulfill its office”, which requires a certain consensus or norm among market participants. Hence, “the establishment of a standard of prices, is the business of the State” (B1, 135). But he also points out that this should not obscure the fact that “only in so far as it is itself a product of labour, and, therefore, potentially variable in value, can gold serve as a measure of value” (B1, 107-8).1

Marx finds that other fundamental functions of money are its serving as a) a means of hoarding, b) a means of payments, and c) universal money.

Money as a means of hoarding The main function of hoarding “in the economy of the metallic circulation” is to serve as ‘treasure’ or ‘reserve’, since “along with the continual fluctuations in the extent and rapidity of the circulation of commodities and in their prices, the quantity of money current unceasingly ebbs and flows” and “must, therefore, be capable of expansion and contraction” (B1, 144). In terms of the nationwide aggregate, “In order that the mass of money, actually current, may constantly saturate the absorbing power of the circulation, it is necessary that the quantity of gold and silver in a country be greater than the quantity required to function as coin.” And the difference consists of hoarded money, thus allowing for the expansion or contraction of the money in circulation according to the needs of circulation. The need for hoarding arose early in the history of the circulation of commodities. As this history developed, the characteristics of hoarding also developed. And at a certain

1The reader must bear in mind Marx’s assumption in Book I that all commodities exchanged according to their values (even after the introduction of capitalism). That is one of the reasons why he here refers only to labor and not to other factors that also impinge on the equilibrium values in the general case (and are introduced in Book III). The other of course is that in Chapter 3 of Book I he has not yet introduced Capitalism, which he does in Chapter 4.
stage, when money started to be disbursed as capital (in the ‘antediluvian’ forms of capital) it went beyond its function in the ‘metabolism’ of commodities and turned into a “passionate desire” to accumulate wealth in the form of money; hence arising a “greed for gold.” “The money becomes petrified into a hoard, and the seller becomes a hoarder of money” (B1, 141).

**Money as a means of payment** As the circulation of commodities and money developed historically “conditions arise under which the alienation of commodities becomes separated, by an interval of time, from the realisation of their prices” (B1, 145), since the payment for the purchase can be deferred until after the sale and delivery of the commodity by the issuance of a promise of payment. The seller thus becomes creditor to a future payment and the buyer a debtor of a sum of money. The function of money as a means of payments is its performance as a vehicle for the cancellation of debts. This function of money allows commodities to be sold for a promise of payment while the money is only disbursed by the buyer when the time comes to fulfill the payment. In such cases money “serves as an ideal means of purchase. Although existing only in the promise of the buyer to pay, it causes the commodity to change hands. It is not before the day fixed for payment that the means of payment actually steps into circulation, leaves the hand of the buyer for that of the seller” (B1, 146-7).

After a sufficiently developed level of the production of commodities has been reached, “money begins to serve as the means of payment beyond the sphere of the circulation of commodities. It becomes the commodity that is the universal subject-matter of all contracts. Rents, taxes, and such like payments are transformed from payments in kind into money payments” (B1, 150-1). The separation in time between the sale of a commodity and the payment of its price arises from various factors such as a) the different durations of the production processes of different kinds of commodities, b) the association of the production of various commodities with certain seasons of the year, c) the need to transport some commodities over long distances to the market in which they are sold, and d) in some cases (such as houses), what is sold is the use of the commodity and the payment is often in advance while the use ends at the end of a certain period. All such factors lead to some sellers becoming creditors and some buyers debtors, and money performing its function of means of payment.

In Chapter 3 of Book I, Marx especially concentrates on the forms of money that arise from the simple circulation of commodities, such as gold coins or bullion, or “inconvertible paper money issued by the State and having compulsory circulation” (B1, 137). Only in Book III does he introduce more advanced forms of money such as credit money (which is “Money based upon credit”). Credit money “takes root spontaneously in the function of money as the means of payment” (Ibid.) and is intimately linked to sophisticated institutions such as banks. And as capitalist production develops and the circulation process ceases to be simple, hoarding vanishes as a distinct form of accumulating wealth and becomes a reserve for the backing of the means of payments.² In a footnote, Marx illustrates “how little ready money is required in true commercial operations” in modern Capitalism using the 1856 receipts and payments of Morrison Dillon & Co. (“one of the largest London houses”) extracted from a Report from the Select Committee on the Bank Acts. It shows that for each million pound sterling paid, it was only actually necessary for this company to use 28,089 in gold, since the greater part of the payments were in bills payable after date, checks payable on demand, Bank of England notes (as well as a small sum in silver and copper coins). This example

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²We shall see in Chapters 11, 12 and 13, however, that hoarding will also play a vital role in Marx’s theory of the ‘industrial cycle’.
illustrates the importance of what Marx called ‘credit money’ in mid-19th century Capitalism:

Credit money springs directly out of the function of money as a means of payment. Certificates of the debts owing for the purchased commodities circulate for the purpose of transferring those debts to others. On the other hand, to the same extent as the system of credit is extended, so is the function of money as a means of payment. In that character it takes various forms peculiar to itself under which it makes itself at home in the sphere of great commercial transactions. Gold and silver coins, on the other hand, are mostly relegated to the sphere of retail trade (B1, 150).

**Universal money** The development of Capitalism at the national level and its gradual development of international trade tended increasingly to restrict the role of precious metals to the sphere of international payments. In Book III, Marx writes that “metal would indeed be required only for the balancing of international commerce, whenever its equilibrium is momentarily disturbed, if only domestic production were organised” (B3, 514). The fact that the national banks had resorted to the expedient of suspending metallic payments showed that “the domestic market does not need any metal money” (Ibid.). But money still has an important function as *universal money*: “When money leaves the home sphere of circulation, it strips off the local garbs which it there assumes, of a standard of prices, of coin, of tokens3, and of a symbol of value, and returns to its original form of bullion” (B1, 153). In the world market money “adequately corresponds to its ideal concept” of being a commodity among others. Marx also explains that while bimetallism is unsustainable in the national sphere, owing to the problems generated by the changing relative exchange values of gold and silver versus the fixed parities governments establish between them4, in world markets “money acquires to the full extent the character of the commodity”, and “a double measure of value holds sway, gold and silver.” Although money “serves as the universal medium of payment, as the universal means of purchasing, and as the universally recognised embodiment of all wealth”, its predominant function is usually simply that of means of payments. And its function as an international means of purchasing becomes important chiefly “when the customary equilibrium in the interchange of products between different nations is suddenly disturbed” (B1, 154-5).

**Coins and paper money**

According to Marx, the fact that “money takes the shape of coin, springs from its function as the circulating medium”, and “Coining, like the establishment of a standard of prices, is the business of the State” (B1, 135). But since it is very difficult to coin very small quantities of gold, less valuable metals like silver or copper are used for small denominations. The circulation of such coins and their consequent wear generates a “separation between their nominal and their real weight, creating a distinction between

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3By ‘tokens’ Marx refers to the fact that the wearing out of coins makes its gold content of lower (and sometimes much lower) value than its legally mandated payment power. Eventually, there ceases to be any relation at all between these values.

4Marx describes what modern economists refer to as ‘Gresham’s Law’. The English financier Thomas Gresham (1519-1579) described the phenomenon that “bad money drives out good money” when gold and silver circulate with a fixed legal exchange ratio. When the value of gold rises in relation to that of silver, private agents appear who convert gold coins to bullion and sell them for a profit. But this ‘Law’ was not discovered by Gresham. It had already been established by the Frenchman Nicole Oresme (c.1320-1382) and later the Pole Nicolaus Copernicus (1473-1543) in 1519.
them as mere pieces of metal on the one hand, and as coins with a definite function on the other” (B1, 136). This already implies the “possibility of replacing metallic coins by “tokens of some other material, by symbols” that serve the same purpose as the original coins. Likewise, “things that are relatively without value, such as paper notes, can serve as coins in its place” and have a “purely symbolic character” (B1, 137).

Marx makes an interesting historical account that we merely sketch here. The wearing out of coins due to their circulation produced during the Middle Ages a gradual dissociation between their nominal weight and their real weight that continued until the 18th century. Coins of the same nominal value circulated with differences in weight. This process tended to convert coins “into a mere semblance of what they profess to be, into a symbol of the weight of metal they are officially supposed to contain” (B1, 136). And this became recognized by legislation, by fixing what loss of weight was sufficient to demonetize a gold coin, that is, cease to make it legal tender. This process led to the use of paper, of insignificant value in relation to its denomination, as a mere symbol of the commodity money it legally represents. At this stage,

The State puts in circulation bits of paper on which their various denominations, say £1, £5, &c, are printed. In so far as they actually take the place of gold to the same amount, their movement is subject to the laws that regulate the currency of money itself. A law peculiar to the circulation of paper money can spring up only from the proportion in which that paper money represents gold. Such a law exists; stated simply, it is as follows: the issue of paper money must not exceed in amount the gold (or silver as the case may be) which would actually circulate if not replaced by symbols (B1, 137-8).

But if the quantity of paper money issued exceeds “its proper limit, which is the amount in gold coins of the like denomination that can actually be current”, i.e. that are needed in circulation, a greater amount of paper money actually represents the same quantity of gold. The effect would be the same as if an alteration had taken place in the function of gold as a standard of prices. Those values that were previously expressed by the price of £1 would now be expressed by the price of £2.” This of course is what we now call price ‘inflation’: prices rise because the amount of paper money issued rises without the demand for currency having increased in the same proportion on account of the needs of circulation of commodities. Marx point out correctly that it is necessary that the amount of ‘token’ (i.e., paper money) issued “have an objective social validity of its own, and this the paper symbol acquires by its forced currency.” Because the “compulsory action of the State can take effect only within... the territories of the community,” “it is also only within that sphere that money completely responds to its function of being the circulating medium” (B1, 139-40).

When Marx analyzes the function of money as a means of payment he makes a more elaborate estimation of the “sum total of the money current during a given period” (i.e., in circulation): “given the rapidity of currency of the circulating medium and of the means of payment, it is equal to the sum of the prices to be realised, plus the sum of the payments falling due, minus the payments that balance each other, minus finally the number of cycles in which the same piece of coin serves in turn as means of circulation and of payment” (B1, 149-50). On the other hand, in the circulation of commodities there no longer appear “the two equivalents, commodities and money, at the two poles of the process of sale” since it is no longer necessary that the payment take place when the commodity is sold. Money still functions as “a measure of value in the determination of the price of the commodity sold.” But the price fixed by contract
only “measures the obligation of the debtor, or the sum of money that he has to pay at a fixed date.” Now money only “serves as an ideal means of purchase”; not as an actual physical means of purchase. But it still causes the commodity to change hands. It is only on the “day fixed for payment that the means of payment actually steps into circulation, leaves the hand of the buyer for that of the seller” (B1, 146-7). The purchasing debtor only needs money to be able to pay at the fixed day, and if he does not pay the competent authority will step in and may sell his goods to fulfill the payment.

**Money and its wearing out in the matrix representation of SCP**

Marx explains that for gold to play its part as money it must “enter the market at some point or other. This point is to be found at the source of production of the metal, at which place gold is bartered, as the immediate product of labour, for some other product of equal value” (B1, 118-9). In order to reflect this in our systems of equations along with the wear and tear of money in circulation, it is necessary to distinguish gold among our commodities. Assume there are \( n + 1 \) commodities, where the last is gold. Let \( N = \{1, 2, \ldots, n\} \) be the set of non-monetary commodities (where \( N \) represents Non-Gold) and let \( G \) represent gold. We assume in this chapter that all labor is simple and there is only one production technique for each commodity. Let us arrange matrix \( A \) so that the last column and the last row correspond to the production of gold \( G \), that is, the gold mining sector:

\[
A = \begin{bmatrix}
A_{NN} & a_{NG} & 0 \\
a_{GN} & A_{GG} & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]

Here \( A_{NN} \) includes the input-output coefficients of all non-gold commodities, \( a_{NG} \) is the column vector of coefficients that represent the use of gold \( G \) as an intermediate input to the production of the respective commodities of \( N \), and \( a_{GN} \) is the row vector of coefficients that represent the uses of non-gold commodities as inputs to the production of gold. We make the (realistic) assumption that gold is not needed for the production of gold (\( a_{GG} = 0 \)).

We must now take into account that apart from the production and consumption flows of the commodity that plays the role of money there is a stock of money that is necessary both for hoarding and as means of circulation. That stock is normally greater than the production of the model’s time period. Also, due to its circulation a part of the stock of gold wears out in each period and must be replaced by production if there is to be SR. Assume that all the producers of gold are identical and let \( \tilde{q}^G \) represent the stock of gold each producer has in his possession and \( \bar{q}^G \) the part he normally needs to make his purchases during the period. The difference \( a^G \equiv \tilde{q}^G - \bar{q}^G \) is the stock he hoards: his treasure or reserve. An important aspect of Marx’s theory is the fact that \( \bar{q}^G \) varies according to the needs of circulation of commodities, hence producing an (inverse) variation with \( a^G \). Let \( d \) be the percentage of \( \bar{q}^G \) that wears out in the time period owing to money’s function as means of circulation. Hence, if there is to be SR \( dq^G \) must be produced each period.

Hence we can rewrite the systems of quantities (5.2) and values (6.1) as:

\[
\begin{bmatrix}
q^N & q^G & q^L
\end{bmatrix}
\begin{bmatrix}
A_{NN} & a_{NG} & \ell_N \\
a_{GN} & A_{GG} & 0 \\
c_L & 0 & 0
\end{bmatrix}
\begin{bmatrix}
d\bar{q}^G
\end{bmatrix}
= \begin{bmatrix}
q^N & q^G & q^L
\end{bmatrix}, \quad (7.1)
\]
where \( q^G \) is the quantity of gold produced, and \( v^G \) is the value of gold. We have decomposed the vector of direct labor requirement coefficients \( \ell \), separating the one indicating the labor necessary to produce gold \( \ell_G \) from the rest \( \ell_N \), and additionally assuming that the producers do not consume gold for their subsistence (\( c_{LG} = 0 \)) although they do consume it as a medium of circulation. It should be noted, however, that they may consume gold jewelry. But the production process of gold jewels is one of the processes in \( N \), say \( J \), which uses, among other inputs, gold from mining (\( a_{JG} > 0 \)).

The following table shows the individual equations of the quantities and values systems:

\[
\begin{align*}
A & & B \\
1 & q^N A_{NN} + q^G a_{GN} + q^L c_L = q^N & A_{NN} v^N + a_{NG} v^G + \ell_N = v^N \\
2 & q^N a_{NG} + q^L d q^G = q^G & a_{GN} v^N + \ell_G = v^G \\
3 & q^N \ell_N + q^G \ell_G = q^L & c_L v^N + d q^G v^G = 1
\end{align*}
\]

Equation 1A shows that the productions of non-monetary commodities \( q^N \) include the productive (or intermediate) consumption of those commodities in the production of the commodities of \( N \) (\( q^N A_{NN} \)) and in the production of gold \( G \) (\( q^G a_{GN} \)), as well as the producers’ final consumption \( q^L c_L \). Equation 2A shows that the production of gold \( q^G \) includes its use as an input in the productions of commodities \( N \) (\( q^N a_{NG} \)) which, as we have seen, includes the production of gold jewelry \( J \), and also the wear of gold in its function as means of circulation \( q^G d q^G \). This implies that (say at the end of) each period coins are again minted so that at the beginning of the next period they have the same gold content they had initially. 3A shows how total labor \( q^L \) is allocated to the productions of non-gold commodities \( q^N \ell_N \) and gold \( q^G \ell_G \). On the right hand side, 1B decomposes the values vector of non-gold commodities into the values of their non-gold and gold inputs and labor; 2B decomposes the value of gold into the value of its non-gold inputs and labor; and 3B shows that the unit value each producer produces in a unit of time (that is, 1) is equal to the value of his consumption basket plus the value of the gold he consumes in the wear of circulation.

Using the information on the table yields the following equalities:\(^5\)

\[
\begin{align*}
(q^G a_{GN} + q^L c_L) v^N - q^N a_{NG} v^G &= q^N \ell_N \\
(q^N a_{NG} + q^L d q^G) v^G - q^G a_{GN} v^N &= q^G \ell_G
\end{align*}
\]

Equality 1A shows that the net output of non-gold commodities is \( q^N (I - A_{NN}) = q^G a_{GN} + q^L c_L \). Hence, the first of the equalities above shows that the value of the net output of \( N \) minus the value of the gold used as input is equal to the labor of the producers of \( N \). Similarly, the second equality shows that the value of the gold produced minus the value of the inputs \( N \) that are consumed in gold production is equal to the labor of the producers of \( G \).

We can visualize how gold produced is introduced into the economy. When (according to the second equation above) the producers of gold purchase their inputs \( q^G a_{GN} \)

---

\(^5\)Postmultiply 1A by \( v_N \) and premultiply 1B by \( q^N \); the left hand sides of the two resulting equalities are hence equal; simplifying, yields the first of the equalities. Follow the same procedure with 2A and 2B to get the second.
from the producers of non-gold, they give them in return units of gold equivalent to 
$q^G a_{GN} v^N$ units of labor, thus introducing gold produced in the period into the rest of the economy. As in our quotation of Marx above, “gold is bartered, as the immediate product of labour, for some other product of equal value.” The producers of gold retain the units of gold they need in order to replace their share of the wear of gold money in circulation. The producers of non-gold commodities, on the other hand, pay the producers of gold in gold (out of their holdings for circulation) for the $q^N a_{NG}$ units of gold they need as inputs and retain their share of the $q^L d\bar{q}^G$ units of gold necessary to replace the wear of gold in circulation. They take them to the mint along with their stock of worn out coins so that they are re-minted. The producers of gold do the same.

Let us now express the values system in terms of prices (that is, in terms of gold).\textsuperscript{6} As we saw in the preceding chapter, in SCP the prices and incomes system only differs from the values system by the different normalization of its (right) dominant eigenvector. Hence, the prices and incomes system in SCP is:

$$\begin{pmatrix} A_{NN} & a_{NG} & \ell_N \\
0 & a_{GN} & \ell_G \\
c_L & q^L d\bar{q}^G & 0 \end{pmatrix} \begin{pmatrix} p \\
1 \\
w \end{pmatrix} = \begin{pmatrix} p \\
1 \\
w \end{pmatrix}, \tag{7.3}$$

where $p = (p_1 \ldots p_{n-1})^T$ is the monetary prices vector for non-gold commodities and $w$ is the value\textsuperscript{7} of each producer’s consumption (including the wear of money). Since gold has no price, there is a 1 in its place. And since the social matrix is the same in (7.2) and (7.3) and the dominant eigenvector is unique up to a scalar factor, we have the necessary proportionality $(p \; 1 \; w) = \alpha \begin{pmatrix} v^N \\
1 \\
v^G \end{pmatrix}$ for a unique positive $\alpha$, from which we can infer that $\alpha = 1/v^G = w$, and hence $p = v^N/v^G$. Hence, the equilibrium price of each non-gold commodity is its relative value in terms of gold (that is, the ratio between its value and the value of gold). We can also infer that $p/w = v^N$, that is, the price of each non-gold commodity in terms of each producer’s income is equal to its value.

Following the same procedure as above we obtain the following equalities:

$$(q^G a_{GN} + q^L c_L) p - q^N a_{NG} = q^N \ell_N w$$

$$(q^N a_{NG} + q^L d\bar{q}^G) - q^G a_{GN} p = q^G \ell_G w.$$

The first shows that the value of the net output of $N$ minus the payment (in gold) to the producers of $G$ for their inputs of gold are equal to the monetary (i.e., gold) income of the producers of $N$. And the second shows that the output of the producers of $G$ minus the payment they make to the producers of $N$ for their inputs $(q^G a_{GN} p)$ is equal to the monetary income of the producers of $G$.

**The quantity of money necessary for the circulation of commodities**

Marx seeks to find out how much money the sphere of circulation “constantly absorbs”. Other commodities disappear as soon as they are consumed (by an individual or as an input), but since money is the medium of circulation it stays within the sphere of circulation and is constantly reused. In the simplest form of circulation, “money and

\textsuperscript{6}It is now common to refer to these prices as ‘monetary prices’ to distinguish them from the ‘accounting prices’ used when there has been no specification of a commodity or object (as paper money) used as measure of value because only relative prices matter.

\textsuperscript{7}Notice that the word ‘value’ here is not in bold characters since it has the usual meaning.
commodities always come bodily face to face, one at the positive pole of purchase, the other at the negative pole of sale.” Hence, “the amount of the means of circulation required is determined beforehand by the sum of the prices of all these commodities” (B1, 127). For Marx such questions were especially useful for the long run, during which, for example, the value of gold could fall substantially because much more productive mines were found than presently existed, thus lowering the amount of labor that is socially necessary for the production of gold. And this could lead to the erroneous belief that the prices of commodities rose because more gold entered into circulation, instead of the correct one that the discovery of more productive mines made the value of gold fall because less labor is necessary to produce it, hence making the prices of (other) commodities rise. Marx writes: “A one-sided observation of the results that followed upon the discovery of fresh supplies of gold and silver, led some economists in the 17th, and particularly in the 18th century, to the false conclusion, that the prices of commodities had gone up in consequence of the increased quantity of gold and silver serving as means of circulation” (B1, 128).

To find out “how much money this sphere [of circulation] constantly absorbs” in the short run, Marx adopts the simplifying assumption that the value of gold is constant: “On this supposition then, the quantity of the medium of circulation is determined by the sum of the prices that have to be realised. If now we further suppose the price of each commodity to be given, the sum of the prices clearly depends on the mass of commodities in circulation” (B1, 128-9). And if, inversely, it is assumed that “the mass of commodities remain constant, the quantity of circulating money varies with the fluctuations in the prices of those commodities”, for which it is sufficient that there be a “rise or a fall in the prices of a number of leading articles” (B1, 129).

Let us assume that during weekdays the quantities given by (7.1) are produced (and consumed), and that all the transactions (purchases and sales) take place on Sunday at the monetary prices given by (7.3). And let us ask, as Marx does, what quantity of gold is necessary for the realization of all the transactions. The “sum of the prices” of the non-gold commodities produced in the period is $q^Np$. But normally there are purchases and sales of goods or services that are not produced as commodities⁸, and some produced commodities may not be traded (say, because they are used as inputs or directly consumed by their producer). Hence the gold value of the realized transactions can be greater or less than $q^Np$. How much they differ depends on many factors. Also, in reality the sales of the producers are not normally synchronized to happen at the end of the time period of the model. As we have already seen, Marx takes into account that in the ‘currency of money’ the same gold coin can be used sequentially in many transactions by various producers.⁹ In other words, during the period the same unit of money can ‘rotate’ many times; it can be used in many transactions. Hence, Marx finds that for a given interval of time during the process of circulation, we have the following relation: the quantity of money functioning as the circulating medium is equal to the sum of the prices of the commodities divided by

---

⁸“Objects that in themselves are no commodities, such as conscience, honour, &c., are capable of being offered for sale by their holders, and of thus acquiring, through their price, the form of commodities. Hence an object may have a price without having value…; for instance, the price of uncultivated land, which is without value, because no human labour has been incorporated in it” (B1, 112).

⁹“Hence the movement directly imparted to money by the circulation of commodities takes the form of a constant motion away from its starting-point, of a course from the hands of one commodity owner into those of another. This course constitutes its currency (cours de la monnaie)” (B1, 125).
the number of rotations made by coins of the same denomination. This law holds generally... The total number of rotations made by all the circulating coins of one denomination being given, we can arrive at the average number of rotations made by a single coin of that denomination, or at the average velocity of the currency of money. The quantity of money thrown into the circulation at the beginning of each day is of course determined by the sum of the prices of all the commodities circulating simultaneously side by side. But once in circulation, coins are, so to say, made responsible for one another. If the one increase its velocity, the other either retards its own, or altogether falls out of circulation; for the circulation can absorb only such a quantity of gold as, when multiplied by the mean number of rotations made by one single coin or element, is equal to the sum of the prices to be realised. Hence if the number of rotations made by the separate pieces increase, the total number of those pieces in circulation diminishes. If the number of the rotations diminish, the total number of pieces increases (B1, 130)\textsuperscript{10}.

We now try to express these ideas by using our two matrix systems and introducing some \textit{ad hoc} parameters. The “sum of the prices of the commodities” refers to the values of the non-money commodities that circulate, i.e., a sum of ‘price times quantity’ terms. If each unit of the gross production of non-money commodities is sold (and purchased) on average \( s \) times in the period, \((q^N p) \) \( s \) is the “sum of the prices of the commodities” that circulate. Also, let \( r \) be the “number of rotations made by coins of the same denomination” in the model’s period of time. Then, since \( \bar{q}^G \) is the “quantity of money functioning as the circulating medium”, Marx’s formula quoted above is \((q^N p) s/r = \bar{q}^G \). But \( q^N \) and \( p \) depend on the model’s parameters. From (7.1) and (7.3) we have, respectively, \( p = R (a_{NG} + \ell_N d \bar{q}^G) \) and \( q^N = (\bar{q}^G a_{GN} + q^L c_{CL}) S \), where \( R \equiv (I - A_{NN} - \ell_N c_{CL})^{-1} \) and \( S \equiv (I - A_{NN} - a_{NG} a_{GN})^{-1} \). Hence, the quantity of gold necessary for the circulation of commodities is: \( \bar{q}^G = (s/r) (\bar{q}^G a_{GN} + q^L c_{CL}) SR (a_{NG} + \ell_N d \bar{q}^G) \). Notice that \( \bar{q}^G \) appears three times in this expression. Dividing this expression by \( \bar{q}^G \) we obtain the frequency of rotation \( r \) of each unit of money as

\[
r = s \left( da_{GN} + q^L c_{CL} / \bar{q}^G \right) \left( a_{NG} / \bar{q}^G + \ell_N d \right) SR.
\]

We can see that \( r \) varies directly with \( s \) (the average number of times each unit of output is sold in the period) and with \( d \) (the average proportion of the mass of coins in circulation that wear out in the period), and inversely with the stock of money in circulation \( \bar{q}^G \). Also, if \( r \) increases (\textit{ceteris paribus}) \( \bar{q}^G \) must fall, confirming that “if the number of rotations made by the separate pieces increase, the total number of those pieces in circulation diminishes” and vice versa.

Marx sums up his reasoning in the following paragraph:

The total quantity of money functioning during a given period as the circulating medium is determined, on the one hand, by the sum of the prices of the circulating commodities, and on the other hand, by the rapidity with which the antithetical phases of the metamorphoses follow one another. On this rapidity depends what proportion of the sum of the prices can, on the average, be realised by each single coin. But the sum of the prices of the

\textsuperscript{10}We have replaced ‘moves’ by ‘rotations’. Google translates the expression “Umlaufsanzahl ihres einzelnen Elements” that appears in the German version of Book I as “Number of revolutions of a single element”. In the present context ‘rotations’ appears more adequate than ‘revolutions’. 
circulating commodities depends on the quantity, as well as on the prices, of the commodities. These three factors, however, state of prices, quantity of circulating commodities, and velocity of money currency, are all variable. Hence, the sum of the prices to be realised, and consequently the quantity of the circulating medium depending on that sum, will vary with the numerous variations of these three factors in combination (B1, 132).

Additional thoughts on SCP

When considering monetary prices in an economy that lacks the developed monetary institutions of Capitalism it is natural to assume as above that there is a certain commodity, such as gold, that plays the role of money. However, for modeling purposes sometimes it is convenient not to specify the monetary prices (i.e., prices expressed in terms of money) but instead to use some other commodity (or set of commodities) to normalize the price vector and thus play the role of ‘numeraire’ (a term introduced by Walras shortly after the publication of Book I of Capital). For often one is interested in relative prices, especially if one is focusing on non-monetary issues. Marx does this when in Book III of Capital he assumes that the aggregate production of the economy measured in ‘production prices’ is equal to the aggregate production measured in values: \( q^Q p = q^Q v \).\(^{11}\) In other instances he takes as numeraire the workers’ consumption basket, that is \( w = c_L p + d Q^G = 1 \) in the model above (or \( w = c_L p \) if to simplify we assume \( d = 0 \)). In this case the proportionality of the dominant eigenvectors of (7.3) and (7.2) \(( p p^G 1 \) = \( \alpha (v^N v^G 1) \) yields \( \alpha = 1 \) and hence the prices measured in terms of per capita income are equal to their values both for non-monetary commodities as for gold: \( p = v^N \) and \( p^G = v^G \).

How monetary prices are set depends both on institutional and economic factors. Marx assumes that the producers in SCP have no monopoly power in their respective markets. He also assumes that the market price for each commodity is unique, although he does not assume anything resembling ‘perfect competition’ nor does he restrict his analysis to situations of ‘market equilibrium’ in the Neoclassical sense. He does admit variations in quality that may justify different prices, which in the model implies assuming they are different commodities (and hence maintaining the unique market price assumption). As we have already highlighted, in Book III he assumes that there can be various techniques employed for the production of each commodity. Hence, in SCP some producers have lower than average costs and therefore a greater income (and consumption) than average, while others have greater than average costs and greater than average income.\(^{12}\) And as we have already mentioned, in Book I Marx assumed that both in SCP as in CCP commodities sell for prices that are proportional to their market values\(^{13}\), which implies that all producers of the same commodity use the same technique.\(^{14}\) He eliminates these assumptions in Book III. But we are not following Marx’s order of exposition and found it convenient to introduce heterogeneity of techniques already in the context of SCP (in the last section of Chapter 6) as an introduction to the treatment of the matter in the context of CCP (in Chapter 9).

Marx’s SCP is a model (in the modern sense of the word) created to formalize

\(^{11}\text{This can be seen, for example, in the table on of Chapter 9 in Book III (MECW 37, 156), where both the values and the prices of commodities add up to 422.}\)

\(^{12}\text{Similarly, in CCP some capitalists obtain extra profits and others lower than normal profits.}\)

\(^{13}\text{And Marx does not introduce the distinction between market value and individual value until Book III. Hence, in Book I there are only values.}\)

\(^{14}\text{We will see that in the case of CCP this also implies that all branches of industry have the same value composition of capital.}\)
his theory of commodities, an aspect of which is his theory of value.\textsuperscript{15} This theory of commodities includes both their production and their circulation by means of a commodity that adopts the functions of money. The model is based on certain common characteristics of communities (both urban and rural) in which independent producers sell their output in a market. It does not represent urban producers who are members of strongly regimented guilds as existed during much of the Middle Ages in Western Europe. Also, it is a scarcely developed community in which tools are simple but there is a division of labor and each producer specializes in the production of one commodity, with many producers making the same commodity, each working on his own workshop without the assistance of other workers. The market price of each commodity is established by the market and is the same for all. Each producer’s income is dependent on that common price and the cost of raw materials consumed and the wear of tools. Such costs differ among competing producers. The market price can be above, at, or below the equilibrium price (given by the ratio of the commodity’s value and the value of gold), according to whether demand is above, at, or below supply. If there is a persistent excess demand or supply of a commodity, then there is a flow of producers with excess supply towards the branches of production with excess demand. \textit{Ceteris paribus}, the market prices tend towards their equilibrium as supply tends to equal demand for every commodity.

In Marx’s SCP the natural conditions of production (land, rivers, etc.) are of free access and no producer need pay a rent for their use. As we have seen, when Marx introduces money in Chapter 3 of Book I, he does so assuming a community in which there are no financial markets, and hence there is neither money lending (credit) nor interest. Nor does commerce exist as a specialized profession. Hence producers must use a part of their time in the purchase of inputs and the sale of output. Following Marx, we have assumed in this chapter that all such mercantile activities are realized on Sundays (or ‘holidays’), that is, outside the habitual working day:

The metamorphoses \( C - M \) and \( M - C \) are transactions between buyers and sellers; they require time to conclude their bargains, the more so as a struggle goes on in which each seeks to get the best of the other... To effect a change in the form\textsuperscript{16} costs time and labour power, not for the purpose of creating value, however, but in order to accomplish the conversion of value from one form into another. The mutual attempt to appropriate an extra slice of this value... does not create any value... Therefore, if the owners of the commodities are not capitalists but independent direct producers, the time employed in buying and selling is a diminution of their labour time, and for this reason such transactions used to be deferred (in ancient and mediaeval times) to holidays (B2, 133-4).

With this set of assumptions, each producer is not affected if the commodity he produces requires the use of a stock of raw materials and tools that has a higher value

\textsuperscript{15}It should be noted that what we here call Marx’s ‘theory of value’ is specific to his ‘theory of commodities’, which is the core of his ‘theory of capital’. In later economic theory what the Classics called ‘theory of value’ came to be called (usually but not always) ‘theory of prices’. Marx’s theory of value (or prices) for Capitalism, however, is composed of layers that begins with his ‘theory of value’ for SCP, continues with a ‘theory of production prices for CCP’, and a sketch of a theory of ‘modified production prices’ for CCP after the private property of land is introduced. Finally, there are mere indications of the reality of prices that reflect monopoly power in certain branches of industry.

\textsuperscript{16}We have here replaced “state of being” by “form”, taking into account that in the previous paragraph Marx writes: “the conversion of a certain value from one form into another, from the commodity form into the money form or from the money form into the commodity form—a change in the state of being”.
than in those required in the production of other commodities. As long as he can make an adequate accounting of the wear of his durable means of production, and gradually builds up a fund that will enable him to replenish them when they are no longer usable, the reproduction process can continue. Marx’s ‘dialectic method’ required gradually introducing the complications of the concrete, so before introducing financial instruments (and interest) it was desirable to first build a model for SCP in which such complications were absent. Also, the model of CCP he develops in Book I (beginning with Chapter 4) is a model in which merchants’ capital and money lenders’ capital were assumed away so as to focus on industry, where capital is produced: “Circulation, or the exchange of commodities, begets no value. The reason is now therefore plain why, in analysing the standard form of capital, the form under which it determines the economic organisation of modern society, we entirely left out of consideration its most popular, and, so to say, antediluvian forms, merchants’ capital and money lenders’ capital” (B1, 174). In order to focus on the production of capital Marx only needed to unfold the commodity producer/worker of SCP into the capitalist and the wage worker of industrial Capitalism, thus giving priority to the social relations (production relations) established between those participating in the production process within the same workplace (workshop or factory).

Mystified social perceptions of market-mediated social relations

The social relations between the individuals that participate in the economic-social process are always present in Marx’s writings, whether or not they are mediated by markets. And just as the social relations between slave and slave or between slave and master, have certain characteristics, there are specific aspects in the relations between the individual producers of SCP, in the relations between wage workers, and in the relation between wage workers and their capitalist employer. Marx holds that various factors tend to obscure the perception of the existing social relations when markets play an important role. The dependence of the social psychology of individuals on the web of relations they establish in the economic process is a recurrent theme throughout Marx’s work. He was not satisfied with merely searching in the economic literature for the keys to the ‘anatomy’ and ‘physiology’ of capitalist society’ that he could use in his own theory, but also sought to understand and explain the relation between the specific insertion of individuals in the economic process and their mental way or appropriating that reality, their social psychology. In the case of the capitalist mode of production, Marx detected strata of fetishist psychological features that were more difficult to dissect as one proceeded from simple commodity production to the specifically capitalist mode of production of commodities: “The mode of production in which the product takes the form of a commodity, or is produced directly for exchange, is the most general and most embryonic form of bourgeois production. It therefore makes its appearance at an early date in history, though not in the same predominating and characteristic manner as now-a-days. Hence its Fetish character is comparatively easy to be seen through. But when we come to more concrete forms, even this appearance of simplicity vanishes” (B1, 93).

In non-market societies – such as the highly autarchic feudal society of the early European Middle Ages – all individuals lived and produced subject to strong personal ties, without the product of their labor having to be sold as commodities in the market and, hence, without their products seeming to relate to one another through the linkage of exchange value. Therefore, “No matter, then, what we may think of the parts played by the different classes of people themselves in this society, the social relations between

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17 The title of Book I is precisely “The Process of Production of Capital”.
individuals in the performance of their labour appear at all events as their own mutual personal relations, and are not disguised under the shape of social relations between the products of labour” (B1, 88). In the most primitive commodity production, in which the use of money is not yet generalized, the social relations between producers (typically relations between different communities that only exchange marginal parts of their production) was done by barter. But with the development of production for the market and the formation of ‘exchange values’, i.e., the ratios by which producers exchange such commodities, the relations between these producers are mystified, since they are eclipsed by the (fetishist) perception that it is the commodities that establish relations between them.

Already in SCP, the basic nucleus from which capitalist society develops according to Marx, there exist distinctive forms of social psychology that are different from those prevailing in other modes of production. The peculiar social psychology of (simple) commodity producers has the characteristic feature of causing “the social relations of individuals to appear in the perverted form of a social relation between things” (Contribution, MECW 29, 275). “A commodity is therefore a mysterious thing, simply because in it the social character of men’s labour appears to them as an objective character stamped upon the product of that labour; because the relation of the producers to the sum total of their own labour is presented to them as a social relation, existing not between themselves, but between the products of their labour” (B1, 82-3).

In SCP, although the producers of different commodities only meet in the process of exchange that takes place in the marketplace, there is a regulating process of their respective labors that takes place ‘in the background’ and expresses itself in the relation of exchange value that the commodities they produce adopt in the market.

The spontaneous selection in the exchange process of one of the many produced commodities for the adopting the functions of money generates new mystifications (or illusions, or magic) that make it difficult to understand money as a mediator in the relation between people, and tend to make traders perceive it as a riddle, as an object that has magical properties:

These objects, gold and silver, just as they come out of the bowels of the earth, are forthwith the direct incarnation of all human labour. Hence the magic of money. In the form of society now under consideration, the behaviour of men in the social process of production is purely atomic. Hence their relations to each other in production assume a material character independent of their control and conscious individual action. These facts manifest themselves at first by products as a general rule taking the form of commodities. We have seen how the progressive development of a society of commodity producers stamps one privileged commodity with the character of money. Hence the riddle presented by money is but the riddle presented by commodities; only it now strikes us in its most glaring form (B1, 103).

Hence arose “the illusions of the monetary system”, whereby “gold and silver, when serving as money, did not represent a social relation between producers, but were natural objects with strange social properties” (B1, C1).
Chapter 8  CAPITAL AND CAPITALIST COMmodity PROduction

In this chapter we introduce the modifications to the model of commodities and SCP developed in the first sections of Chapter 6 that are necessary to obtain Marx’s basic model of capital and Capitalist Commodity Production (CCP). We maintain the assumption of Simple Reproduction (SR), that is, that the economy’s scale of production and reproduction does not change from one period to another. Extended Reproduction (ER) will be addressed in Chapter 14.

Conceptual framework

Marx was very much aware that the theoretical models he developed were full of simplifying assumptions that kept them far from the complex reality of historical reality. And such is the case of his model of (pure) CCP that we address in this chapter. It deals with a society that is closed in itself, as if it encompassed all of human society, and in which Capitalism is the only existing mode of production: “In order to examine the object of our investigation in its integrity, free from all disturbing subsidiary circumstances, we must treat the whole world as one nation, and assume that capitalist production is everywhere established and has possessed itself of every branch of industry” (B1, 580; footnote 1).

As we have seen in Chapter 4, the ‘cycle of industrial capital’ starts in the ‘sphere of circulation’, that is, in the markets where each capitalist purchases means of production and labor power, continues in the ‘sphere of production’, where commodities are produced, and returns to the ‘sphere of circulation’, where the resulting output is to be sold. If it is sold, capitalists reimburse their capital with profit and after the means of subsistence of wage workers and capitalists have been consumed they again disburse their capital and the cycle starts anew. As in the case of SCP, value is only generated in the ‘sphere of production’. But in CCP not only is value generated but also surplus value.¹ “This circular movement, in which the same phases are continually gone through in succession, forms the circulation of capital” (B1, 564). As in SCP, in CCP the producer has ownership rights on the commodities produced. But in CCP “the laws of appropriation or of private property, laws that are based on the production and circulation of commodities” suffer a radical change. For “the capital which is exchanged for labour power... must not only be replaced by its producer, but replaced together with an added surplus” (B1, 583). Here, of course, the ‘producer’ of capital is the wage worker and the activity of capitalists does not count as ‘labor’. In the production process, the capitalist and the wage workers participate in a process in which there “is only an apparent exchange”, since “what really takes place is this –the capitalist again and again appropriates, without equivalent, a portion of the previously

¹“The conversion of a sum of money into means of production and labour power, is the first step taken by the quantum of value that is going to function as capital. This conversion takes place in the market, within the sphere of circulation. The second step, the process of production, is complete so soon as the means of production have been converted into commodities whose value exceeds that of their component parts, and, therefore, contains the capital originally advanced, plus a surplus value. These commodities must then be thrown into circulation. They must be sold, their value realised in money, this money afresh converted into capital, and so over and over again” (B1, 564).
materialised labour of others” (Ibid.). The labor of wage workers is ‘materialized’ in the produced commodity that the capitalist sells in the market. And a portion of the materialized labor is the surplus **value** appropriated by the capitalist.

Part VII of Book I (“The Accumulation of Capital”) starts with the clarification that accumulation must first be considered “from an abstract point of view.” Consequently, Marx assumes that capitalists not only manage to sell their produced commodities (that is, there is no excess or defect in the production of each individual firm nor hence in the aggregate) but also that they sell them at their **value**. This simplifying assumption has various strands. First and foremost, all commodities sell for what we now call ‘equilibrium prices’, which implies leaving out an important aspect of reality: that there are ‘constant disequilibrium in markets. On the other hand, it implies the special assumption that the ‘equilibrium prices’ considered in the CCP of Book I are proportional to commodity **values**. In Book III Marx shows that the ‘equilibrium prices’ in CCP cannot generally be proportional to **values**. And a part that long volume is devoted to investigating how the ‘equilibrium prices’ of CCP are determined. Marx makes a first approximation by ignoring the existence of landowners and ground rent, as well as the giant firms of modern industrial Capitalism that can avoid competition because they have monopoly power. This first approximation to the equilibrium price is called ‘production price’ by Marx. When he later introduces the private property of land and modern industry these ‘production prices’ will have to be modified to get the equilibrium prices. In Book I Marx also leaves out all the capitalists of the ‘sphere of circulation’ (merchants, bankers, etc.). But when they are introduced (in Book III) industrial capitalists will have to share the surplus **value** they initially obtain when they sell their commodities with the rest of the owning classes (commercial and financial capitalists and landowners).

All these topics are addressed in this and the following chapters. In the present chapter we introduce the simplest model of (pure) CCP.

**Labor power as a commodity**

To model a purely industrial capitalist economy the independent **producer/workers** of SCP (owner of the means of production he uses, and whose aim is to reproduce his existence), is split into two different figures. On the one hand is the **wage worker**, who lacks means of production but is free to sell his **potential labor**, which Marx calls **labor power**. This labor power becomes a commodity which is transacted in a market that did not exist in SCP: the **labor market**. The price of labor power is the **wage**, a category that did not exist in SCP. On the other hand, there is the industrial capitalist, who has the capital that enables him to purchase 1) the means of production that are needed to produce the commodity he wants to sell and 2) the labor power that is to use them in production. There arises a special income for the retribution of the industrial capitalist: profit. By purchasing labor power for a certain time period, the capitalist can consume its use value during that time in the production process. “The capitalist buys labour power in order to use it; and labour power in use is labour itself” (B1, 187). And “The consumption of labour power is at one and the same time the production of commodities and of surplus value” (B1, 185). Marx explains that “In every country in which the capitalist mode of production reigns, it is the custom not to pay for labour power before it has been exercised for the period fixed by the contract, as for example, the end of each week” (B1, 184). But “for a clear comprehension” he finds it useful “to assume provisionally that the possessor of labour power, on the occasion of each sale, immediately receives the price stipulated to be paid for it” (B1, 185), which is
an assumption he modifies when in Book III he addresses the topic of the turnover (or rotation) of capital.

Marx’s concept of ‘labor power’ was meant to reflect the fact that when workers were hired there were no detailed specifications of the tasks that they were to perform. The tasks they did perform were flexible within certain limits imposed by custom, by the results of conflicts between workers and employers (‘class struggle’) and, increasingly, by labor regulations that were beginning to be imposed in England by Parliament. But for Marx what was crucial was that the concept of labor power was key in explaining the generation of surplus value. Labor power was a very special commodity because its consumption in the production process (i.e., the work done) generated an amount of value that exceeded the value of the worker’s consumption basket (during the same time period) and, hence, generated surplus value. Marx writes:

The labour process, turned into the process by which the capitalist consumes labour power, exhibits two characteristic phenomena. First, the labourer works under the control of the capitalist to whom his labour belongs; the capitalist taking good care that the work is done in a proper manner, and that the means of production are used with intelligence, so that there is no unnecessary waste of raw material, and no wear and tear of the implements beyond what is necessarily caused by the work. Secondly, the product is the property of the capitalist and not that of the labourer, its immediate producer. Suppose that a capitalist pays for a day’s labour power at its value; then the right to use that power for a day belongs to him, just as much as the right to use any other commodity, such as a horse that he has hired for the day. To the purchaser of a commodity belongs its use, and the seller of labour power, by giving his labour, does no more, in reality, than part with the use value that he has sold (B1, 195).

Similarly to what we did under SCP, the consumption baskets of wage workers and capitalists $c_L$ and $c_K$ are to be taken as fixed exogenously. It is important to bear in mind that $c_L$ should not be interpreted as a basket of goods that only allows for physical subsistence, since Marx clearly specifies that it is based on customs:

The labourer needs time for satisfying his intellectual and social wants, the extent and number of which are conditioned by the general state of social advancement (B1, 240-1).

The value of labour power is determined by the value of the necessaries of life habitually required by the average labourer. The quantity of these necessaries is known at any given epoch of a given society, and can therefore be treated as a constant magnitude. What changes, is the value of this quantity (B1, 519; italics added).

Hence, $c_L$ represents these quantities of the “necessaries of life habitually required by the average labourer”. And it is important to highlight that $c_{LP}$ does not represent a ‘subsistence wage’, since it is “conditioned by the general state of social advancement.” Marx expressed similar ideas in *Value, Price, and Profit*:

The value of the labouring power is formed by two elements—the one merely physical, the other historical or social. Its ultimate limit is determined

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2This is a report Marx read at the Central Council meetings of the International on June 20 and 27, 1865. It has also been published under the title *Wages, Price and Profit*. 
by the physical element, that is to say, to maintain and reproduce itself, to perpetuate its physical existence, the working class must receive the necessaries absolutely indispensable for living and multiplying. The value of those indispensable necessaries forms, therefore, the ultimate limit of the value of labour... Besides this mere physical element, the value of labour is in every country determined by a traditional standard of life. It is not mere physical life, but it is the satisfaction of certain wants springing from the social conditions in which people are placed and reared up (MECW 20, 144-5).

**Simple Reproduction and Extended Reproduction**

For Marx the concepts of Simple Reproduction and Extended Reproduction are applicable beyond the capitalist mode of production and circulation, but they acquire a special significance in this particular regime, in which the means of production and the means of subsistence confront the wage worker as attributes of capital:

In economic forms of society of the most different kinds, there occurs, not only simple reproduction, but, in varying degrees, reproduction on a progressively increasing scale. By degrees more is produced and more consumed, and consequently more products have to be converted into means of production. This process, however, does not present itself as accumulation of capital, nor as the function of a capitalist, so long as the labourer’s means of production, and with them, his product and means of subsistence, do not confront him in the shape of capital (B1, 594).

In CCP the capitalist receives whatever is left over after the cost of the means of production and the wages are paid. This surplus is profit. But for Marx the concept of *value* he had developed for the study of commodities and SCP is still present at a deeper level in CCP since for him the source of all the incomes of the owning classes (industrial, commercial, or banking profit, interest, and the rent on land) is surplus *value*, that is, the part of the generated *value* that exceeds the *value* of the commodities consumed by wage workers in order to live and reproduce their labor power.\(^3\)

In Marx’s model of CCP the cycle of capital is periodically reproduced. If all the net product in excess of the means of subsistence of wage workers is consumed by capitalists there is Simple Reproduction (SR), and if a part of that surplus is reinvested in order to increase the size of the whole production process there is Extended Reproduction (ER) and capital accumulation. Marx emphasizes the repetitive, cyclical character of the process:

If production be capitalistic in form, so, too, will be reproduction. Just as in the former the labour process figures but as a means towards the self-expansion of capital, so in the latter it figures but as a means of reproducing as capital—i.e., as self-expanding value,— the value advanced... As a periodic increment of the capital advanced, or periodic fruit of capital in process, surplus value acquires the form of a revenue flowing out of capital. If this

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3 “We started with the supposition that labour power is bought and sold at its value. Its value, like that of all other commodities, is determined by the working time necessary to its production” (B1, 239). Evidently, Marx is here referring to the procedure to be used for calculating *value*. It is hardly necessary to clarify that both the cultural component he previously highlighted for the determination of *cL* and the central role that human social relations have in his analyses imply that for Marx labor power as a commodity is very different from any other commodity.
revenue serve the capitalist only as a fund to provide for his consumption, and be spent as periodically as it is gained, then, *caeteris paribus*, simple reproduction will take place (B1, 566).

**Values and the rate of surplus value**

**The quantities system in CCP**

Since there are two social classes in the pure CCP model, we must distinguish the population of capitalists \( q^K \) from that of wage workers \( q^L \), as well as the consumption basket of capitalists \( c_K \geq 0 \) from that of workers \( c_L \), where both are row vectors of given coefficients. The quantities system in the pure CCP model is hence:

\[
\begin{bmatrix}
  q^Q & q^L & q^K \\
  A & \ell & 0 \\
  c_L & 0 & c_K
\end{bmatrix}
= \begin{bmatrix}
  q^Q & q^L
\end{bmatrix}.
\]

We here maintain the *Basic Assumptions* on \( A, \ell, c_L \) (5.24) we used for SCP. The second of these equalities is the same as the second in the SCP model (6.8). It here expresses how wage workers are distributed among the various branches of production according to the technical requirements and levels of production. The first equality however, \( q^Q A + q^L c_L + q^K c_K = q^Q \), crucially differs from \( 6.7 \) in that gross production must be enough to reproduce not only the means of production and the workers but also the capitalists.

Inserting the second equality of (8.1) into the first we get \( q^Q (A + \ell c_L) + q^K c_K = q^Q \). We saw in Chapter 5 (in the subsection on “The Quantities System”) that under our *Basic Assumptions* the matrix \( A + \ell c_L \) is indecomposable and hence its dominant eigenvalue is positive: \( \lambda (A + \ell c_L) > 0 \). We also saw that the consumption basket of producers \( c_L \) could be accommodated so that \( \lambda (A + \ell c_L) = 1 \), which is a requirement for SR in SCP. In CCP, however, we must have \( \lambda^0 \equiv \lambda (A + \ell c_L) < 1 \) (where now \( c_L \) is the consumption basket of wage workers) in order to make room for the consumption of capitalists (and their capital accumulation in the case of ER). By Perron-Frobenius we know that there exists a vector \( x > 0 \) such that \( (A + \ell c_L) x = \lambda^0 x \). To prove that \( \lambda^0 \) must be less than one we now show that \( \lambda^0 \geq 1 \) yields a contradiction. If \( \lambda^0 \geq 1 \) then \( (A + \ell c_L) x \geq x \) and hence \( q^Q (A + \ell c_L) x \geq q^Q x \). But multiplying the first equation of this paragraph by \( x \) yields \( q^Q (A + \ell c_L) x + q^K c_K x = q^Q x \). But \( q^K c_K \geq 0 \) implies \( q^K c_K x > 0 \) and hence \( q^Q (A + \ell c_L) x < q^Q x \), so we have reached a contradiction. Hence, \( \lambda^0 \) must be less than 1.

This highlights the fact that getting from system (5.2) to system (8.1) requires that one or more of the coefficients of \( A, \ell, \) and \( c_L \) diminish in order to make way for \( q^K c_K \). That is, there has to be either technical progress (reductions in some of the coefficients of \( A \) and/or \( \ell \)) or the consumption basket of wage workers in CCP must be lower than that of the workers/producers of SCP. To avoid confusion we could change the notation used for the matrices/vectors of CCP. But we again prefer to avoid excessive notation. From a mathematical point of view the coefficients of \( (A \ell) \) could diminish sufficiently that some or all the coefficients of \( c_L \) increase. In that case the wage workers of CCP would have a better living standard than the producers/workers of SCP. But we have seen in Chapter 3 that for Marx the main channel of Primitive Accumulation in Europe

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4For simplicity we use the same notation here for the population of wage workers that we used for that of producers/workers in the SCP model. But one should bear in mind the differences between them. The same can be said for the use of \( w \) in SCP and CCP.
was the expulsion of peasants from the lands they had traditionally worked on as serfs.\footnote{Marx writes: “these new freedmen became sellers of themselves only after they had been robbed of all their own means of production, and of all the guarantees of existence afforded by the old feudal arrangements” (B1, 706).}
Hence, a reduction in the consumption basket of workers (be they ex-serfs or ex free producers/workers) is more compatible with Marx’s theory.

From \((A + \ell c_L)x = \lambda^0 x\), with \(\lambda^0 < 1\) we can get two alternative systems of equations, both used by Marx to construct his theory of Capitalism. One of them is the system of values and surplus value; the other is the system of production prices, wage, and profit rate. We will address each of these in the following subsections. But for the moment we continue with the quantities system. From the first equality of (8.1) we get
\[
(q^L c_L + q^K c_K) (I - A)^{-1} = q^L. \tag{8.2}
\]
Hence, multiplying by \(\ell\) and using the same expression \(v \equiv (I - A)^{-1} \ell\) for values as we did in SCP (6.4) and the second equation of (8.1) we get an expression that shows that the aggregate consumption of workers and capitalists measured in values is equal to the work generated during the period (as measured by worker population):
\[
(q^L c_L + q^K c_K) v = q^L. \tag{8.3}
\]
Let us define the rate of surplus value \(e\) as the ratio between the values of the consumption baskets of capitalists and workers:
\[
e = \frac{q^K c_K v}{q^L c_L v}. \tag{8.4}
\]
Then (8.3) yields
\[
(1 + e) c_L v = 1. \tag{8.5}
\]
Notice that in CCP, as was the case of SCP, values naturally arise out of the quantities system. What is new in CCP is the appearance of the rate of surplus value \(e\) (and aggregate surplus value \(ec_L v\)).

The values system in CCP

In the Mathematical Appendix to this chapter we prove that under our Basic Assumptions on \(A, \ell, c_L\) (5.24) there exists a unique vector (up to a scalar factor) \(v > 0\) and a unique scalar \(e^0 > 0\) such that \([A + (1 + e^0) \ell c_L] v = v\). Let us normalize \(v\) such that \((1 + e^0) c_L v = 1\). Hence \(Av + \ell (1 + e^0) c_L v = Av + \ell = v\). If we leave out the superindex of \(e^0\) we have the following values system for CCP:
\[
\begin{bmatrix}
A \\
(1 + e) c_L \\
0
\end{bmatrix}
\begin{bmatrix}
v \\
1
\end{bmatrix}
= 
\begin{bmatrix}
v \\
1
\end{bmatrix}. \tag{8.6}
\]
The only difference with the corresponding system in SCP (6.1) is the existence of the positive rate of surplus value \(e\). That rate is a measure of how much the workers produce in excess of their own consumption needs in order to generate the goods consumed by capitalists (in the case of SR which we are dealing with here, since under ER they would also need to produce goods for the expansion of the system). As mentioned, Marx knew that in CCP commodities no longer tend to be transacted according to their values. However, he deemed the values to be fundamental for
understanding both the genesis of CCP starting from SCP and the exploitation of wage labor in CCP once it was established.

From the two equalities of (8.6) we have:

\[ v = (I - A)^{-1} \ell \]
\[ (1 + \varepsilon) c_L v = 1. \]  

(8.7)

(8.8)

The first simply shows that \( v \) is the same vector of \textit{values} that was obtained for SCP (6.4), that is, its components are the quantities of labor that are directly or indirectly necessary to produce a unit of each commodity. In (8.8) \( c_L v \) is the \textit{value} of the labor power of a wage worker in the reference time period. But this \textit{value} is now necessarily less than \textit{one} (the amount of \textit{value} that the worker \textit{generates} in the time period). The rate of surplus \textit{value} \( \varepsilon \) is a measure of this discrepancy.\(^6\) Notice that from (8.8) we obtain an expression for the rate of surplus \textit{value} as the ratio between the surplus of what each worker produces (measured in labor units) above the \textit{value} of his consumption \( (1 - c_L v) \) and the \textit{value} of his consumption \( c_L v \):

\[ \varepsilon = \frac{1 - c_L v}{c_L v}. \]  

(8.9)

\textbf{The constituent parts of the gross product and capital using values}

System (8.6) can be expanded to account for the consumption of capitalists:\(^7\)

\[
\begin{bmatrix}
A \\
(1 + \varepsilon) c_L \\
c_K
\end{bmatrix} \begin{bmatrix}
v \\
1 \\
\varepsilon
\end{bmatrix} = \begin{bmatrix}
v \\
1 \\
\varepsilon
\end{bmatrix},
\]  

(8.10)

where \( \varepsilon \) is the surplus \textit{value} obtained by each capitalist \( (c_K v = \varepsilon) \). Multiplying (8.1) by \((v \ 1 \ \varepsilon)^T\) and premultiplying (8.10) by \( q = (q^Q \ q^L \ q^K) \), equating both left hand sides, and simplifying, yields:

\[ e q^L c_L v = q^K \varepsilon = q^K c_K v. \]

These are three alternative expressions for aggregate surplus \textit{value}: 1) the rate of surplus \textit{value} \( \varepsilon \) multiplied by the \textit{value} of the aggregate consumption of workers \( q^L c_L v \), 2) the population of capitalists multiplied by the surplus \textit{value} obtained by each, and 3) the \textit{value} of the aggregate consumption of capitalists. Notice that 1) and 3) yield the expression (8.4) for the rate of surplus \textit{value}. From (8.3), (6.8), and (8.7), we get:

\[ q^L c_L v + q^K c_K v = q^L = q^Q \ell = q^Q (I - A) v. \]

Hence, the total labor exerted in the period, \( q^Q \ell \), which can be expressed either as the working population \( q^L \) or as the \textit{value} of the net product \( q^Q (I - A) \), is the sum of the \textit{values} of the aggregate consumptions of workers and capitalists.

From the two equations of (8.6) we can decompose the \textit{values} vector as

\[ v = Av + \ell c_L v + eL c_L v. \]

---

\(^6\)Notice that if money is one of the produced commodities we are here making the (implicit) assumption that it does not wear \((d = 0)\).

\(^7\)Notice that this expansion makes the system matrix decomposable. This implies that the third equation does not contribute to the determination of \( v \) and \( \varepsilon \) (already determined by (8.6)) and only serves to determine \( \varepsilon \) given \( v \).
Pre-multiplying by $q^Q$, the value of gross production $q^Q v$ can be expressed as the sum of three terms:

$$q^Q v = C^v + V^v + S^v,$$  \hspace{1cm} (8.11)

where we have defined aggregate ‘constant capital’ $C^v$, ‘variable capital’ $V^v$, and surplus value $S^v$ as:

$$C^v \equiv q^Q Av$$

$$V^v \equiv q^Q (\ell_{CL}) v = q^L_{CL} v$$  \hspace{1cm} (8.12)

$$S^v \equiv eq^Q (\ell_{CL}) v = eq^L_{CL} v = q^K_{CK} v.$$

The last two definitions can be used to obtain the rate of surplus value (already obtained in (8.4)) as the ratio between $S^v$ and $V^v$:

$$e = \frac{S^v}{V^v} = \frac{q^K_{CK} v}{q^L_{CL} v}.$$  \hspace{1cm} (8.13)

$C^v + V^v$ is the aggregate capital disbursed by capitalists expressed in values, the application of which in the productive processes leads to the generation (via the labor process) of a gross production that has value $C^v + V^v + S^v$. Marx defines ‘constant capital’ as the part of capital invested in means of production, which includes buildings, machinery, auxiliary materials, and raw materials. And he defines ‘variable capital’ as the part of capital invested in labor power. The reason for such names is that the elements that compose constant capital do not change in value during the production process: labor transmits the value of the means of production (or the fraction that corresponds to its wearing out) to the produced commodity. In contrast, the labor power in which (variable) capital is invested adds value through the work generated. For labor power generates more value during the production process than is necessary to reproduce the workers that sell it; that is, it generates surplus value $S^v$. Marx expresses this clearly:

That part of capital then, which is represented by the means of production, by the raw material, auxiliary material and the instruments of labour, does not, in the process of production, undergo any quantitative alteration of value. I therefore call it the constant part of capital, or, more shortly, constant capital.

On the other hand, that part of capital, represented by labour power, does, in the process of production, undergo an alteration of value. It both reproduces the equivalent of its own value, and also produces an excess, a surplus value, which may itself vary, may be more or less according to circumstances. This part of capital is continually being transformed from a constant into a variable magnitude. I therefore call it the variable part of capital, or, shortly, variable capital. The same elements of capital which, from the point of view of the labour process, present themselves respectively as the objective and subjective factors, as means of production and labour power, present themselves, from the point of view of the process of creating surplus value, as constant and variable capital (B1, 219).

Notice that when Marx writes that the variable part of capital ‘is continually being transformed from a constant into a variable magnitude’ he is referring to the cyclical process of capital. The money-capital invested in labor power does not change in
magnitude when it is in the circulation process. It does change in magnitude when, during the production process, the worker produces not only the equivalent of his own consumption but also a surplus product the value of which is the surplus value.

Let us now consider Marx’s definition of the ‘composition of capital’:

The composition of capital is to be understood in a twofold sense. On the side of value, it is determined by the proportion in which it is divided into constant capital or value of the means of production, and variable capital or value of labour power, the sum total of wages. On the side of material, as it functions in the process of production, all capital is divided into means of production and living labour power. This latter composition is determined by the relation between the mass of the means of production employed, on the one hand, and the mass of labour necessary for their employment on the other. I call the former the value composition, the latter the technical composition of capital...

The many individual capitals invested in a particular branch of production have, one with another, more or less different compositions. The average of their individual compositions gives us the composition of the total capital in this branch of production. Lastly, the average of these averages, in all branches of production, gives us the composition of the total social capital of a country (B1, 607-8).

Since the technical composition of capital is defined exclusively in terms of quantities, devoid of valuation, in the aggregate it is the relation between the vectors $q^Q A$ and $q^Q \ell c_L (= q^L c_L)$, and in branch of production $i$ it is the relation between row vectors $A_i$ and $(\ell c_L)_i = (\ell c_L)$. In contrast, the value composition of capital is a ratio between two aggregate magnitudes, where the aggregation depends on the valuation used. Since in Book I Marx restricted himself to the use of values in the aggregation, in the quote above ‘value’ refers to our ‘value’. Hence, the (average) ‘value composition’ of capital $\kappa_i$ in industry $i$ is the ratio between the constant and variable capitals of that industry:

$$
\kappa_i = \frac{A_i v}{(\ell c_L)_i v} = \frac{A_i v}{\ell_i \ (c_L v)} = (1 + e) \frac{A_i v}{\ell_i},
$$

where $A_i$ is the $i$-th row of $A$ and for the last equality we used (8.8). And the (average) ‘value composition’ of capital $\kappa$ of the economy (or aggregate ‘value composition’ of capital) is

$$
\kappa = \frac{q^Q A v}{q^Q \ (\ell c_L) v} = \frac{q^Q A v}{q^L c_L v} = \frac{C v}{V v}.
$$

Marx additionally introduced a complementary concept that allowed him to refer to changes in the value composition exclusively due to changes in the technical composition; that is, assuming that there is no change in the values vector. Immediately after the first paragraph of the quote above he writes: “Between the two there is a strict correlation. To express this, I call the value composition of capital, in so far as it is determined by its technical composition and mirrors the changes of the latter, the organic composition of capital.”

It is remarkable that most of Marx’s critics (and even many of his followers) invariably refer to the latter concept, using it almost always erroneously, i.e., ignoring the special assumption it involves. This is especially surprising if we consider the precise manner in which Marx defined these concepts in the only Book of Capital that he...
personally corrected for publication. We will see that some of the confusion originated in some carelessness on the part of Engels when he prepared Book III for publication. We will also see some of the problems derived from that erroneous interpretation when we analyze the ‘Law of the tendency of the rate of profit to fall’ in Chapter 17.

**Graphical representation of CCP using values**

Figures 1 and 2 of Chapter 6 represent SCP under the assumption that only two commodities are produced. They show on the axes both gross outputs \( q^Q = (q_1, q_2) \) and consumptions. These Figures are still valid under CCP if, as in Book I of Capital, it is assumed that commodities are purchased and sold according to their values\(^8\), and the consumption vector is reinterpreted so as to also include the consumption of capitalists. Hence, we have now:

\[
C = q^L c_L + q^K c_K = (q^L c_{B1} + q^K c_{K1}, q^L c_{L2} + q^K c_{K2}) \equiv (C_1, C_2).
\]

The ‘employment line’ \( q^Q \ell = q^L \) is also still valid but now both in \( \ell \) as in \( q^L \) labor refers to wage labor, which has a market (whereas there was no such market in SCP).

Figure 4

Figure 4 focuses on the ‘consumption line’ to highlight certain specific aspects of CCP. First, it shows the vector of aggregate consumption \( C \) as the sum of the vectors of worker consumption \( q^L c_L \) and capitalist consumption \( q^K c_K \). Adding these vectors gives point \( C \) (on the northeast vertex of the parallelogram) that represents aggregate consumption; and it is on the ‘consumption line’ which, as before, is the graphical representation of \( C_1 v_1 + C_2 v_2 = q^L \) when the total amount of work generated \( q^L \) is given, and \( C_1 \) and \( C_2 \) include the consumptions of both classes.

If, hypothetically, capitalist consumption were to disappear and the consumption proportions of the goods workers consume remained the same, the latter could consume according to the dot in Figure 4 named \( e = 0 \), since in this case the rate of surplus

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\(^8\) We see below in Figure 6 that the same type of graph can be made when Marx’s special assumption of Book I is eliminated.
value would be zero. That point, hence, corresponds to a society with SCP. In the other hypothetical extreme, if the consumption of workers tended to zero and the proportions of the goods capitalists consumed remained constant, the latter could get arbitrarily near to the dot \( e = \infty \), since in this limit case the rate of surplus value would be infinite. Between those hypothetical extremes, the true aggregate consumption \( C \) is such that the rate of surplus value is given by (8.4).

Production prices, wage and profit rate
In Book III of Capital Marx first shows that if in CCP commodities were to sell for their values the rates of profit in the different branches of industry would have to differ, which would not be a stable situation since there would be capital flows from the branches in which profit rates are low to those where it is high:

Now, if the commodities are sold at their values, then, as we have shown, very different rates of profit arise in the various spheres of production, depending on the different value composition\(^9\) of the masses of capital invested in them. But capital withdraws from a sphere with a low rate of profit and invades others, which yield a higher profit. Through this incessant outflow and influx, or, briefly, through its distribution among the various spheres, which depends on how the rate of profit falls here and rises there, it creates such a ratio of supply to demand that the average profit [for capitals of equal value] in the various spheres of production becomes the same, and values are, therefore, converted into prices of production...
The incessant equilibration of constant divergences is accomplished so much more quickly, 1) the more mobile the capital, i.e., the more easily it can be shifted from one sphere and from one place to another; 2) the more quickly labour power can be transferred from one sphere to another and from one production locality to another (B3, 194-5).

Marx adds that there was a historical tendency in the development of Capitalism to level the rates of profit as it “develops its own conditions and subordinates to its specific character and its immanent laws all the social prerequisites on which the production process is based” (Ibid.). We will show in Chapter 16 how we can formally represent the coexistence of CCP and SCP, which is an additional component of Marx’s vision of the complexities of commodity producing societies. We will also see in Chapter 9 how extraordinary and infraordinary profits naturally arise when we consider the heterogeneity of techniques in each industrial sector, which is another component of Marx’s theory which is usually ignored. Also, we leave the topic of wage worker unemployment for Chapters 12-14. Here we will show a simple model of ‘pure’ CCP, in which there is only one technique per industry, all workers are wage workers, and there is only simple labor.

The model of pure CCP is one in which a situation of equilibrium prevails in the sense that not only are the supplies and demands of each commodity equal but also the profit rates of all branches are equal \((\text{to } \rho)\), which implies that all flows between branches have ceased. Another crucial assumption of the model is that the capital disbursed in each industry is the same as the cost of production. This last assumption

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\(^9\text{The original says ‘organic.’ Notice that by failing to change to ‘value’ in his edition of Book III Engels was being carelessly inconsistent with the special meaning given by Marx to the ‘organic composition’ of capital starting from the third German edition of Book I, which was prepared by Engels himself using posthumous notes left by Marx. There is more on this in the Appendix to this chapter.}\)
implies that all the means of production last as much as the model’s period, which also coincides with the duration of the production process for each commodity. These assumptions will be eliminated in Chapter 12, which deals with the distinction between fixed and circulating capital and Marx’s theory of the turnover (or rotation) of capital.

**The system of production prices, wage, and profit rate**

We have seen that there exists a vector \( x > 0 \) such that \((A + \ell c_L)x = \lambda^0 x\), where \(\lambda^0 \equiv \lambda (A + \ell c_L) < 1\). If we define \(\rho^0 \equiv 1/\lambda^0 - 1\) and \(p \equiv x\) and drop the superindex in \(\rho^0\) we have

\[
(1 + \rho) (A + \ell c_L) p = p, \quad \rho > 0, \quad p > 0.
\]

(8.16)

And if we define the wage as \(w = c_L p\) we have \((1 + \rho) (Ap + \ell w) = p\), so we can interpret \(p\) as the vector of Marx’s (equilibrium) production prices and \(\rho\) as the global rate of profit. Hence, we can form the following system of production prices, wage, and profit rate that replaces the system (6.9) of SCP:

\[
\begin{bmatrix}
(1 + \rho) A \\
0
\end{bmatrix}
\begin{bmatrix}
(1 + \rho) \ell \\
c_L
\end{bmatrix}
\begin{bmatrix}
p \\
w
\end{bmatrix}
=
\begin{bmatrix}
p \\
w
\end{bmatrix},
\]

(8.17)

where now \(w\) is the wage rate (per worker) in the model’s reference period (instead of the income of the producer/worker of SCP). Vector \((p, w)^T\) is unique up to a scalar. And it can be made unique by choosing the commodity (simple or composite) to be used as a measure of prices and wage rate (i.e., as ‘numeraire’). If we are dealing with monetary production prices and the first commodity is money, we can divide \((p, w)^T\) by \(p_1\) and obtain the monetary production prices and wages \((p^m, w^m)^T\) with \(p_1^m = 1\).

If, alternatively, we consider production prices measured in terms of labor power (i.e., labor power is the numeraire), we divide \((p, w)^T\) by \(w\) and obtain \((p^w, 1)^T\), where \(p^w\) is the vector of production prices expressed in terms of labor power (or wage production prices). The system of production prices differs from the system of values in this respect, since in the latter the values are given in terms of the unit in which labor is measured, which implies that the value of labor power expanded by the rate of surplus value \((1 + e) c_L v\) is one.

But we need not specify a numeraire at this point. From a mathematical viewpoint, just as the only difference between the matrices of (6.1) and (8.6) is that \(e\) becomes positive, the only difference between the matrices of (6.9) and (8.17) is that \(\rho\) becomes positive. The two equations of (8.17) are:

\[
(1 + \rho) (Ap + \ell w) = p
\]

(8.18)

\[
c_L p = w
\]

(8.19)

The first equation shows that the production price \(p_i\) of any industry \(i\) is formed by the capital invested in production (which is here equal to the cost of production) \(A_i p + \ell_i w\) plus profit (or the global rate of profit \(\rho\) multiplied by the capital invested). The second shows that the wage rate is the worker’s reproduction cost, that is, the cost of his consumption basket.

By simple algebraic manipulation of (8.18) we can express the vector of production prices as a function of the rate of profit and the wage rate (as well as the technological coefficients of \(A\) and \(\ell\)):

\[
p = B(\rho, \ell) w,
\]

(8.20)
where \( B(\rho) \) is positive\(^\text{10}\) and can be written in two equivalent ways:

\[
B(\rho) \equiv \left( \frac{1}{1 + \rho} I - A \right)^{-1} = (1 + \rho) [I - (1 + \rho) A]^{-1}.
\]

(8.21)

The second is convenient for appreciating intuitively that \( B(\rho) \) is strictly increasing with \( \rho \) since\(^\text{11}\)

\[
[I - (1 + \rho) A]^{-1} = I + (1 + \rho) A + (1 + \rho)^2 A^2 + ....
\]

is necessarily positive. Premultiplying (8.20) by \( c_L \) and using (8.19) we can eliminate \( w \) and obtain an equation in \( \rho \) that determines the profit rate in terms of the components of the social matrix \((A, \ell, \text{and } c_L)\):

\[
1 = c_L B(\rho) \ell.
\]

(8.22)

Since this equation does not depend on any price, it is independent of any numeraire used to ‘anchor’ prices. And since \( B(\rho) \) is strictly increasing with \( \rho \), so is \( c_L B(\rho) \ell. \) Also, \( c_L B(0) \ell = c_L v < 1 \) and it can be proved that \( c_L B(\rho) \ell \) grows without bound as \( \rho \) tends to \( 1/\lambda(A) - 1 > 0 \) from below (since this is true for every element of \( B(\rho) \)). From this we can infer that there exists\(^\text{12}\) a value of \( \rho \) that satisfies (8.22), that this value is between 0 and \( 1/\lambda(A) - 1 \), and that it is unique.

If the profit rate were zero in (8.17) and we used \( w = 1 \) as numeraire, the system would reduce to that of pure SCP (6.9). But this is purely formal. For in CCP it is necessary that \( \rho \) be sufficiently positive that it can sustain the consumption (and in ER also the reinvestment of profits) of capitalists, just as in the values system (8.6) it is necessary that the rate of surplus value \( \epsilon \) be sufficiently positive. The jump from (6.9) to (8.17) is indicative of the effect that the genesis of CCP starting from SCP has on the exchange values. And this was Marx’s interpretation, though he was only able to formulate a (very good) approximation to the production prices, as we will see in Chapter 11.

A more informative way of expressing the production prices and incomes system is to expand the social matrix of (8.17) by including capitalist consumption:

\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) \ell & 0 \\
c_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix}
= 
\begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix},
\]

(8.23)

where \( \pi \) is the value of the consumption basket of each capitalist.

**The global profit rate** We now check that the global profit rate of the economy that figures in (8.17), (8.16), and (8.23) is the aggregate profit of all capitalists divided by the aggregate capital. The aggregate capital measured in production prices, which we call \( K \), is the sum of the values of all the means of production and labor power

\(^{10}\)Since \( \lambda(A + \ell c_L) = 1/(1 + \rho) < 1 \), \( \ell > 0 \) and \( c_L \geq 0 \), and \( A + \ell c_L \) is indecomposable, 3) of Theorem 2 of the Mathematical Appendix to Chapter 5 implies \( \lambda(A) < 1/(1 + \rho) \). Hence, 1) of Theorem 7 implies \( \lambda(A - 1) \ell A - 1 \) is invertible, and Theorem 5 that its inverse is positive.

\(^{11}\)We apply here Theorem 5 and the Corollary to Theorem 7 of the Mathematical Appendix to Chapter 5, replacing in the latter \( A \) by \( (1 + \rho) A \).

\(^{12}\)For a very small \( \varepsilon > 0 \) we know that the continuous function \( f(\rho) \equiv c_L B(\rho) \ell \) is defined in the interval \([0, 1/\lambda(A) - 1 - \varepsilon] \), that \( f(0) = c_L v < 1 \), and that \( f(1/\lambda(A) - 1 - \varepsilon) \) is (much) greater than 1. The Intermediate Value Theorem ensures that there exists \( \rho^* \epsilon [0, 1/\lambda(A) - 1 - \varepsilon] \) such that \( f(\rho^*) = 1 \) (see Lindstrum (2017, 45)). The uniqueness of \( \rho^* \) is due to \( f(\rho) \) being strictly increasing.
used and consumed in the model’s time period. Hence, $K = q^Q A_p + q^L w$. And since in SR all of capitalist’s profit is spent on consumption, global profit is $q^K c_{K_p}$. Hence, we can write the global profit rate as:

$$\rho = \frac{\text{global profit}}{\text{global capital}} = \frac{q^K c_{K_p}}{q^Q A_p + q^L c_{L_p}}.$$  \hfill (8.24)

This expression can be obtained formally from the quantities and prices and incomes systems. For this pre-multiply (8.18) by $q^Q$ and multiply the first equation of (8.1) by $p$. After equating the left hand sides and simplifying we get (8.24).

**The constituent parts of the gross product and capital using production prices** Using production prices instead of values in the valuation we can decompose the aggregate value of (gross) output the same way we did with (8.11):

$$q^Q p = C^p + V^p + S^p,$$

where we defined aggregate constant capital, variable capital and surplus value\(^{14}\) in terms of production prices:

$$\begin{align*}
C^p &\equiv q^Q A_p \\
V^p &\equiv q^Q (\ell_{CL}) p = q^L c_{L_p} = q^L w \\
S^p &\equiv q^K c_{K_p} = q^K \pi.
\end{align*}$$

Hence we can also write the global profit rate (8.24) as

$$\rho = \frac{S^p}{C^p + V^p} = \frac{e^p}{C^p + V^p} = \frac{e^p}{C^p + V^p} = \frac{S^p}{C^p + V^p} = \frac{e^p}{\kappa_p + 1} \tag{8.25}$$

where for the second equality we defined the ‘rate of surplus value’ and the aggregate ‘value composition of capital’:

$$\begin{align*}
e^p &= \frac{S^p}{V^p} = \frac{q^K c_{K_p}}{q^L c_{L_p}}; \\
\kappa_p &= \frac{V^p}{C^p} = \frac{q^Q A_p}{q^Q (\ell_{CL}) p}.
\end{align*} \hfill (8.26)$$

Notice that the ratios $e^p$, $\kappa_p$ and $\rho$ are all independent of any numeraire.

**The special case of equal value compositions of capital** There is a special case in which the vectors of production prices and values have the same structure. According to whether the components of capital are measured in values or in production prices, the value composition of capital of industry $i$ can be written, respectively, as the ‘value composition’ $\kappa_i$ or the ‘value composition’ $\kappa^p_i$:

$$\begin{align*}
\kappa_i &= \frac{C^v_i}{V_i^v} = \frac{A_i v}{(\ell_{CL})_i v} \\
\kappa^p_i &= \frac{C^p_i}{V_i^p} = \frac{A_i p}{(\ell_{CL})_i p}.
\end{align*}$$

We also define the value and value global compositions of capital, respectively, as

$$\begin{align*}
\kappa &= \frac{C^v}{V^v} = \frac{q^Q A_v}{q^Q (\ell_{CL}) v} \\
\kappa^p &= \frac{C^p}{V^p} = \frac{q^Q A_p}{q^Q (\ell_{CL}) p}.
\end{align*}$$

\(^{13}\)Notice that this ratio is independent of the numeraire used, i.e. $p$ can be multiplied by any scalar without changing $\rho$.

\(^{14}\)Notice that ‘value’ is not in bold. Aggregate ‘surplus value’ is nothing but aggregate profit.
It is the first of these that were defined by Marx in Book I due to his simplifying assumption that all transactions were in values. We define the second versions for clarity and because we do not delay the introduction of production prices. It can be seen easily that the global compositions are weighted averages of the compositions of the individual branches. If for all $i$, $j$ we have $\kappa_i = \kappa_j = \kappa$ we will say that all branches have the same value composition of capital and, similarly, if for all $i$, $j$ we have $\kappa_i^p = \kappa_j^p = \kappa^p$ we will say that all branches have the same value composition of capital.

In the Mathematical Appendix to this chapter we show that the peculiar technological case in which $\ell$ is a dominant eigenvector of $A$ has special economic characteristics. First, all branches have the same value compositions $\kappa$ and the same value compositions $\kappa^p$, and both coincide: $\kappa^p = \kappa$. Also, the rates of surplus value and surplus value also coincide $e^p = e$. Hence, the global profit rate (8.25) can be expressed as

\[ \rho = \frac{e}{\kappa + 1}. \quad (8.27) \]

Also, in this special case $p$, $v$, and $\ell$, are all dominant eigenvectors of $A$ and hence differ only by a positive scalar factor. Finally, the exact relation between $p$ and $v$ is

\[ p = (1 + e) v. \]

Hence, if the consumption basket $c_L$ is used to normalize the price vector (i.e., if we assume $w = c_{LP} = 1$) we get $p = (1 + e) v$. And if we instead normalize prices according to $(1 + e) w = 1$, the production prices of all commodities are exactly equal to their values: $p = v$. But notice that $(1 + e) c_L v = 1$ (one of the equations of our values system (8.8)) and $c_{LP} = c_L v$ imply $(1 + e) w = 1$. Hence, using $(1 + e) c_L$ as numeraire (i.e., assuming $(1 + e) w = 1$) is equivalent to using $(1/(c_L v)) c_L$ (i.e., assuming $(1/c_L v) c_{LP} = 1$, which is the same as $c_L p = c_L v$).

Hence, when Marx writes in several places of Book I that he makes the “assumption, that all commodities, including labour power, are bought and sold at their full value” (B1, 319), he could just as well have assumed that all the technologies used in industry have the same value composition of capital (or, equivalently but very mathematically posed, that they are such that $\ell$ is a dominant eigenvector of the input-output matrix $A$) and that, additionally, he normalizes prices so that the wage is equal to the value of the workers’ consumption basket ($w \equiv c_{LP} = c_L v$). For such assumptions imply that production prices are equal to values.

Notice also that assumptions such as these can help to explain a peculiar phrase of Marx. When he defines the value composition of capital he writes “On the side of value, it is determined by the proportion in which it is divided into constant capital or value of the means of production, and variable capital or value of labour power, the sum total of wages” (B1, 608). In our quote above we suppressed “the sum total of wages” to avoid confusion. But now we know that among the assumptions that make production prices equal to values is $c_{LP} = c_L v$, and hence, for any aggregate (firm, branch, etc.) “the sum total of wages” and the “value of labour power” are equal.

Many of our arguments have depended on the assumption that $A$ is indecomposable. We will show in Chapter 10 that this assumption implies the absence of the ‘luxury’ goods which are absent from the consumption basket of wage workers. We will see that when there are luxury goods $A$ is decomposable. Hence, in that more general case it is necessary to first segregate luxury goods for the preceding arguments to be valid. In

\[ ^{15} \text{We will see in Chapter 11 that this is the formula Marx obtained for the profit rate.} \]
that case, matrix $A$ above is only the square submatrix of a larger matrix that includes luxury goods.

**Graphical representation of CCP using production prices**

Figure 4 above represents the consumption line in CCP when we use values. Figure 5 complements Figure 4 since it uses production prices for valuations and, hence, for the slope of the consumption line. Lines parallel to the consumption line are used to place the components of gross outputs $q^Q$, that is, the means of production $(q^Q A)$ and consumption, both in the aggregate $(C)$ and by social class $(q^L c_L$ and $q^K c_K$). The larger parallelogram shows that (vector) adding consumption and the means of production yields gross output $(q^Q = q^Q A + C)$. And the smaller parallelogram shows that the sum of the aggregate consumptions of wage workers and capitalists (and hence the physical components of the global rate of surplus value $e^p$) yields aggregate consumption: $C = (C_1, C_2) = (q^L c_{L1} + q^K c_{K1}, q^L c_{L2} + q^K c_{K2})$. Since production prices $p$ are used for valuation, the negatively sloped (and parallel) lines that go through each of these components have slopes that differ from the consumption line of Figure 4 (if we exclude the special case in which $p$ and $v$ are proportional since all branches have the same value composition).

**Figure 5**

To determine the level of the production prices we must choose a numeraire. It is here convenient to adopt as numeraire the vector of aggregate consumption per wage worker, that is, to impose $(C/q^L) \bar{p} = 1$, where $(\bar{p}, \bar{w})$ is the vector of prices of production and wage rate when such a numeraire is adopted. With this numeraire, the value of aggregate consumption is equal to the quantity of wage work employed:

$$C \bar{p} = (q^L c_L + q^K c_K) \bar{p} = q^L.$$

(8.28)

From (8.3) we see that the value of aggregate consumption is $Cv = q^L$. Hence, with this numeraire, even though in general production prices are not proportional to
\textit{values}, the value of aggregate consumption is equal to aggregate wage work $q^c$ whether \textit{values} or production prices are used for aggregating. In particular, this implies that the same point $C$ is on the line with slope given by $v$ in Figure 4 as on the line with slope given by $\bar{p}$ in Figure 5. To avoid complicating the latter we have omitted the consumption line with slope given by $v$. On the other hand, from (8.28) and the definition of $e^p$ (in (8.26)) we have

$$(1 + e^p) c_L \bar{p} = (1 + e^p) \bar{w} = 1,$$

which shows that there is an inverse relation between the real wage $\bar{w}$ and the rate of surplus value $e^p$.\textsuperscript{16} Notice the analogy between this relation and the one that arises when \textit{values} are used for valuations (8.8).

In Figure 5, dots named a, b, c and d, have been placed on the ray that parts from the origin and goes through the point that represents the means of production $q^Q A$. Those dots represent, respectively, the consumption of capitalists, of workers, aggregate consumption, and the productive consumption of means of production (all valued in production prices).\textsuperscript{17} Let us call 0$f$ the length of the segment from the origin to some (generic) point $f$. Then we have a graphical representation of the rate of surplus value $e^p = a_0/0b$ (from (8.26)), of the value composition of capital, $\kappa^p = 0d/0b$, and of the profit rate $\rho = a_0/(0d + 0b) = e^p / (\kappa^p + 1)$.

Notice that we could make the same kind of representation in Figure 4 by including point $q^Q A$. Assuming we do, let the corresponding points on the ray be $a'$, $b'$, $c'$ and $d'$. Then we would have $e = a_0'/0b'$, $\kappa = 0d'/0b'$, and $\rho = a_0'/(0d' + 0b') = e / (\kappa + 1)$. Only in the special case of equal value compositions in both branches would the lines in Figures 4 and 5 have the same slopes. As we have seen, in this case production prices are proportional to \textit{values} and hence $e^p = e$ and $\kappa^p = \kappa$.\textsuperscript{18}

\textbf{Some relations between variables}

\textbf{The relation between production prices and values}

The exact relation between the vector of production prices and the vector of \textit{values} obviously depends on the numeraire adopted. But the effect of a change in the numeraire can only imply a proportional change in the price vector. In (8.20) and (8.7) we obtained expressions for $p = B(\rho) \ell w$ and $v = B(0) \ell$. Also, from (8.22) we can determine the profit rate $\rho^*$ independently from any numeraire. Hence, if we use $c_L$ as the numeraire (and hence $w = 1$) we have the following relations between $p^w$ and $v$:

$$p^w = B(\rho^*) (I - A) v$$

$$v = B(0) \left( \frac{1}{1 + \rho^* I - A} \right) p^w. \quad (8.30)$$

These equalities ‘transform’ the \textit{values} into production prices and the production prices into \textit{values}, respectively. The relation generated heated debates towards the end of the 19th century and came to be known as the “Marxian transformation problem.” In particular, when Samuelson (1970) observed that each of these two vectors are given by two different systems of equations he alluded humorously to the transformation as

\textsuperscript{16}On a graph with $\bar{w}$ and $e^p$ on the axes the relation between these variables is an equilateral hyperbola that goes through $\bar{w}$ when $e^p = 0$.

\textsuperscript{17}We could have evidently chosen any other ray that goes through the four lines since we are only interested in proportions.

\textsuperscript{18}As we will see in Chapter 11, in that case $\rho^*$ would be Marx’s approximation to the profit rate and it would be correctly calculated. But this is not the general case.
“a process of rejection and replacement” (subtitle of the article), meaning that one had to abandon the values system and just use the production prices system.

A consequence of measuring production prices in terms of labor power, i.e., normalizing prices according to \( c_L p^w = w = 1 \), is that all the production prices are necessarily greater than the corresponding values, which derives from the fact that \( B(\rho) \) is strictly increasing with \( \rho \). For this implies that if \( \rho > 0 \) then \( p^w = B(\rho) \ell > B(0) \ell = v \). But other normalizations (or numeraires) do not have this characteristic. For example, assume we use as numeraire the vector of aggregate gross output divided by its value: \( q^2/(q^2 v) \). One then obtains the property Marx sometimes uses that the value of aggregate gross output is the same as its value: \( q^2 p = q^2 v \). In this case, if for any commodity \( i \) we have \( p_i > v_i \) then there is necessarily another commodity \( j \) for which \( p_j < v_j \), and vice versa. Hence, unless we have the exceptional case of equal value composition of capital in the production of all commodities, the production prices of some commodities must be above their values and the production prices of others must be below their values. Also, premultiplying (8.20) by \( q^2 \) we get \( q^2 p = q^2 B(\rho) \ell w \), from which (using \( q^2 p = q^2 v \)) we obtain the real wage rate as a (complicated) decreasing function of \( \rho : w = q^2 B(0) \ell /q^2 B(\rho) \ell \) (which has the peculiarity that \( w < 1 \) if and only if \( \rho > 0 \)).

The relation between the rate of profit and the rate of surplus value

Systems (8.17) and (8.6) show that the rate of profit \( \rho \) and the rate of surplus value \( e \) are two alternative ways of measuring the economic surplus that, in the aggregate, the owners of money-capital invested in production obtain through the employment of workers. One uses production prices for valuations and the other uses values. And both are formally correct. Marx had an amazing analytical intuition in seeing this without the help of the mathematical tools that facilitate this. As we will see in Chapter 11, he did have to use an approximation to the vector of production prices. It is easy to obtain an explicit relation between these two alternative ways of measuring the economic surplus. Combining (8.22) with (8.8) gives

\[
1 + e = \frac{c_L B(\rho) \ell}{c_L B(0) \ell}.
\]  

(8.31)

From this relation we can infer that \( e = 0 \) if and only if \( \rho = 0 \), in which case we would be in the world of SCP (with workers as independent producers). We can also infer that \( e > 0 \) if and only if \( \rho > 0 \). Also, since \( B(\rho) \) is strictly increasing with \( \rho \), the relation between \( e \) and \( \rho \) is monotone: the larger \( \rho \) is, the larger \( e \) is. But the relation itself is not too important since, given \( A, \ell \), and \( c_L \), the values of both \( \rho \) and \( e \) are uniquely determined. To see which of these alternative measures of surplus is greater we can expand \( B(\rho) \) and \( B(0) \) in series (using (8.21) and the expression that follows it) to obtain from (8.31):

\[
1 = \frac{1 + \rho c_L [I + (1 + \rho) A + (1 + \rho)^2 A^2 + ...] \ell}{1 + e \cdot c_L [I + A + A^2 + ...] \ell}.
\]  

(8.32)

The second ratio of the right hand side of the equality is necessarily greater than one as long as \( \rho \) is positive. Hence we can conclude that \( e > \rho \) if \( \rho > 0 \), i.e., as if we are in

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19 Morishima (1973) gives the pompous (and mathematical) name of ‘Fundamental Marxian Theorem’ to the proposition that there exist prices and wage rate that determine a positive \( \rho \) if and only if \( e \) is positive.
The reallocation of surplus value by means of the circulation process

With his theory of surplus value, Marx believed he had discovered the particular way in which the dominant class of the capitalist mode of production exploited the working class, a way that was less transparent than in previous modes of production such as the Asiatic mode of production, or those based on slavery or serfdom. As we have seen, values are present in his theory of commodities and, in particular, his model of SCP, where the equilibrium prices of commodities are proportional to their values and if prices are measured in terms of the producers’ income they are equal to values. He formalized his theory of CCP by means of a series of models, the simplest of which differs from the SCP by the presence of two (‘large’) social classes in the production process (instead of one): wage workers and industrial capitalists. The fact that each capitalist pursued higher profits, for which he was willing to switch to another industrial branch if necessary, implied that in general the production prices (or equilibrium prices) were not proportional to values. But Marx chose to ignore this divergence in Book I. And in Book II, on The Process of Circulation of Capital, where he develops his models of SR and ER, it was also highly simplifying to assume equal value compositions of capital, although he did not entirely restrict his analyses to that assumption. It is in Book III, on The Process of Capitalist Production as a Whole, that he addresses the redistribution of the aggregate surplus value generated in the industrial sector as a whole through the circulation of commodities. Part of that redistribution takes place within the industrial sector through the process of reallocation of capitals that tends to equalize the profit rates of the various branches. Another part is a distribution from industrial capitalists to the other owning classes. For the surplus value generated in industry also had to form the profits of the commercial and financial capitalists, the interests of lenders, and the rents of landowners. In the first two Parts of Book III, however, he analyzes the relation between prices of production and values making abstraction of the other owning classes.

For Marx it was necessary to keep both production prices and values within his theory of CCP. While production prices were essential for establishing the equilibrium exchange values in Capitalism (still assuming free access to land), values were essential for giving a foundation to his theory of the exploitation of wage work in Capitalism based on his concept of ‘unpaid labor’, a theory which he deemed essential for understanding the aspects of the social process related to power and hierarchy. Hence, along with the duality between quantities (use values) and production prices (that replaced the values of SCP as regulators of the proportions of exchange), his theory also included the values system which in CCP included the rate of surplus value. We will see in Chapter 11 that Marx had to use certain simplifications in his formulas in order to define production prices with his limited knowledge of algebra (and the limited state of the algebra of his time). But we have seen in this chapter that with the small corrections here introduced (which will be better appreciated after reading Chapter 11) his formulation could be impeccable from a formal point of view. In Chapter 18,

\[\text{[A + (1 + e) eL]} V = V \text{ and } [(1 + p)(A + eL)] p = p.\]

The two matrices within brackets have the same dominant eigenvalue (one): \(\lambda(A + (1 + e) eL) = \lambda((1 + p)(A + eL)).\) From this equality we immediately infer that a) \(e = 0 \iff p = 0 \text{ and } e > 0 \iff p > 0,\) and b) \(e > 0 \Rightarrow p > 0\). The proof of a) is trivial. To prove b), assume that \(p \geq e > 0\). Then \((1 + p)(A + eL) \geq (1 + e)(A + eL) > (A + (1 + e) eL).\) But then (applying Theorem 2 of the Mathematical Appendix to Chapter 5) \(\lambda((1 + p)(A + eL)) > \lambda(A + (1 + e) eL),\) which contradicts the equality we started from.

An alternative way of getting this result is to start from the two reduced systems already obtained: \[\text{[A + (1 + e) eL]} V = V \text{ and } [(1 + p)(A + eL)] p = p.\] The two matrices within brackets have the same dominant eigenvalue (one): \(\lambda(A + (1 + e) eL) = \lambda((1 + p)(A + eL)).\) From this equality we immediately infer that a) \(e = 0 \iff p = 0 \text{ and } e > 0 \iff p > 0,\) and b) \(e > 0 \Rightarrow p > 0\). The proof of a) is trivial. To prove b), assume that \(p \geq e > 0\). Then \((1 + p)(A + eL) \geq (1 + e)(A + eL) > (A + (1 + e) eL).\) But then (applying Theorem 2 of the Mathematical Appendix to Chapter 5) \(\lambda((1 + p)(A + eL)) > \lambda(A + (1 + e) eL),\) which contradicts the equality we started from.
however, we will see that his theory of surplus value had substantial flaws associated with his concept of ‘unpaid labor’.

We have seen that in CCP (which implies \( \rho > 0 \)) the production price vector \( p \) is not in general proportional to the vector of values. Nevertheless, even if the equilibrium exchange values differ from relative values, Marx’s concept of value in CCP still has a precise meaning. And there is nothing to prevent using the values system if it is deemed, as did Marx, that it is theoretically important. As we have already mentioned, although Marx leaned greatly on Ricardo in his investigation on the ‘anatomy’ of capitalist society, he constructed his own theory looking within the sphere of production for the great division between necessary and surplus labor time, and between the value of labor power and the surplus value into which the generated value in any period could be divided. Surplus value was appropriated by the industrial capitalists in the production process itself since it was a constituent part of the value of produced commodities. But, sold at their production prices (assuming equilibrium), the surplus value was redistributed among the industrial capitalists in the different branches of industry through the circulation process. The branches with lower value composition of capital generated more surplus value (per unit of capital) than those with higher composition because they had more variable capital, and hence employed more labor power (per unit of capital). But the surplus value the capitalists there obtained through their profit was lower than the surplus value generated within their firms because profit was proportional (according to \( \rho \)) to total capital (and not just variable capital). And vice versa, branches with lower value composition obtained through their profit a surplus value that was greater than that generated internally. Aside from this redistribution of surplus value between branches of industry in the basic CCP model that did not include other forms of property, in an extended model the surplus value generated in the sphere of industrial production (which for Marx included construction, agrarian and mining output, and transportation and communications services) was also the source of the profits of commercial and banking firms, of interests and of rents. We will look into these extensions in Chapters 13, 15 and 16.

The allocation of capital among branches of industry
The quantities system (8.1) shows how the population of wage workers \( q^L \) is distributed among the various branches of industry but not how the population of capitalists \( q^K \), along with their capital, is distributed. In this section we show how this can be done. Let us assume that all capitalists have the same amount of capital and that, in branch \( i \), \( \eta_i \) is the number of capitalists involved in the production of \( i \) per unit of \( i \) produced. Since \( q_i^Q \) is the number of units produced in branch \( i \), \( q_i^Q \eta_i \) is the number of capitalists in branch \( i \) and the total number of capitalists is:

\[
q^K = q^Q \eta = \sum_{i=1}^{n} q_i^Q \eta_i,
\]  

(8.33)

where \( \eta \) is a column vector with the number of capitalists per unit of output in each of the branches of industry. Since we assume all branches of production need capitalists and capital to function, \( \eta \) is a positive vector. We can hence express the quantities system in the following way, one that is more symmetric than (8.1):

\[
\begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A & \ell & \eta \\
h & 0 & 0 \\
k & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}.
\]  

(8.34)
We can write this system more compactly as $q^+M^+ = q^+$ (where the notation is obvious). Hence, as in (5.2) we have an equation in which the quantities vector $q^+$ is the (left) eigenvector of the social matrix $M^+$ associated with the dominant eigenvalue 1. As the population of capitalists $q^K$ is exogenous, the levels of $q^Q$ and $q^L$ are given and hence $q^+$ needs no further normalization. Notice that we are so far concentrating on the population of employed laborers. In Chapters 12-14 we will distinguish between the population of employed workers $q^L$ and total worker population $q^L$, along with Marx’s concept of the ‘reserve industrial army’ (or mass of unemployed) $u = q^L - q^L$.

The equalities of system (8.34) are:

$$q^Q A + q^L c_L + q^K c_K = q^Q$$
$$q^Q e = q^L$$
$$q^Q \eta = q^K.$$ (8.35)  
(8.36)

As we saw above, the first two yield (8.3), which expresses that the population of employed workers (total labor power transacted) $q^L$ can be decomposed into the sum of the values of the aggregate consumption baskets of workers and capitalists $q^L c_L v$ and $q^K c_K v$. Since each laborer works a certain amount of hours in the model’s period, $q^L$ can also represent the total hours worked. From another point of view, (8.3) expresses the fact that the work done in the period decomposes into the value of labor power and the surplus value. It is therefore natural to define the aggregate rate of surplus value as the ratio between aggregate surplus value and the value of aggregate labor power, as in (8.4). Using this definition, an alternative way of writing (8.3) is (8.8).

That is, the value generated by each worker in the period (which is one) can be written as the value of his labor power $c_L v$ multiplied by one plus the rate of surplus value.

In Marx’s interpretation, $e$ is a measure of the exploitation of wage labor by capitalists. He gives various alternative interpretations to this ratio. One of them is that $e$ measures the ‘unpaid work of labor’, in relation to ‘paid work’. For example, in his posthumous Results of the Direct Production Process he writes:

the commodity as the product of capital contains in part paid, and in part unpaid, labour... a certain total amount of labour is objectified in the commodity. Part of this objetsified labour (leaving aside constant capital, for which an equivalent has been paid) is received in exchange for the equivalent of the wage; another part is appropriated by the capitalist without any equivalent. Both parts are objectified, and are therefore present as parts of the value of the commodity. And to characterise the one as paid, the other as unpaid, labour, serves the purposes of abbreviation (MECW 34, 363).

And in Book I Marx gives the following equivalent formulas for the rate of surplus value (and subsequent comment):

$$\text{Surplus value} \quad \text{Surplus labor} \quad \text{Unpaid labor}$$
$$\frac{\text{Value of labor power}}{\text{Necessary labor}} = \frac{\text{Unpaid labor}}{\text{Paid labor}}$$

... The capitalist pays the value, so far as price coincides with value, of the labour power, and receives in exchange the disposal of the living labour power itself. His usufruct is spread over two periods. During one the labourer produces a value that is only equal to the value of his labour power: he produces its equivalent. Thus the capitalist receives in return for his advance of the price of the labour power, a product of the same
price. It is the same as if he had bought the product ready made in the market. During the other period, the period of surplus labour, the usufruct of the labour power creates a value for the capitalist, that costs him no equivalent. This expenditure of labour power comes to him gratis... All surplus value, whatever particular form (profit, interest, or rent) it may subsequently crystallise into, is in substance the materialisation of unpaid labour (B1, 534).

When in Chapter 18 we focus on our critique of Marx’s theory of surplus value, or theory of the exploitation of wage work in Capitalism, we will argue that it is not the formal validity of Marx’s simplified equations (to be addressed in Chapter 11) nor the more precise ones of the present chapter that should be questioned. Our criticism will instead hinge on the absence of a formal representation of the role that capitalist entrepreneurs play in the (capitalist) economic process. We will also see, however, that this absence also pervades mainstream economic theory and that this is a probable explanation for the paradoxical lack of a clear-cut critique of Marx’s theory despite the superabundance of critics.

The dual system of production prices and incomes

The system of production prices and incomes can now be formulated as the dual of the quantities system (8.34), that is:

\[
\begin{bmatrix}
A & \ell & \eta \\
c_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix}. \tag{8.37}
\]

In compact form, we have \( M^* p^* = p^* \), where \( p^* = (p \ w \ \pi)^T \), \( w \) is the wage rate of each worker and \( \pi \) is the profit of each capitalist. Its individual components are:

\[
Ap + \ell w + \eta \pi = p \tag{8.38}
\]
\[
c_L p = w \tag{8.39}
\]
\[
c_K p = \pi. \tag{8.40}
\]

From the first equation we can obtain the vector of production prices \( p \) in terms of wages \( \ell w \) and profits \( \eta \pi \) of the present period, and those corresponding to the production of the means of production, and those corresponding to the means of production necessary to produce the means of production, etc.:

\[
p = (I - A)^{-1} (\ell w + \eta \pi) = (I + A + A^2 + ...) (\ell w + \eta \pi)
\]

where in the last equality we defined the vector of direct and indirect capital requirements \( y \equiv (I - A)^{-1} \eta \). Hence if we take \( c_L \) as numeraire, the vector of production prices can –in addition to (8.20)– be expressed as the vector of values plus the vector of direct or indirect capital requirements \( y \) multiplied by the profit per capitalist: \( p = v + y \pi \).

Homogeneous or heterogeneous capitalists

To simplify, we often assume in this book that all the capitalists (as all the workers) are homogeneous. When this is the case we can say that we use a ‘representative’ capitalist
(or wage worker). For the assumption of capitalist homogeneity to persist in time it is necessary that they all earn the same profit rate (assuming, as we do here, that each one is in the business of producing one commodity). However, for the treatment of some topics it is necessary to avoid the assumption of capitalist homogeneity. For example, even if the capitalists within each branch are homogeneous there may be differing conditions of competition in the different branches, generating heterogeneous profit rates. In that case, the capitalists of different branches would have to have consumption baskets that correspond to the rate of profit they obtain. Also, as Marx specifies, if there is free competition the existence of different profit rates in different branches generates capital flows between the sectors that tend to equalize them. On the other hand, the average profit rates could be the same in all the branches and nevertheless exist various profit rates within each branch. In different contexts, Marx addresses both kinds of situations. He addresses both in Chapter 10 of Book III on the “Equalization of the general rate of profit through competition.”

There are instances in Marx’s analysis in which the rates of profit are not the same among branches of industry, leading to some capitalists earning higher profits than others (per unit of capital). In some cases these would be situations of transitory ‘dish-equilibrium’ since there would be a dynamical process by which capitalists would shift from one branch with low profit rate to another in which it is higher. And this process would only stop when the profit rates of all branches are equal. But there are other instances in which there are ‘dish-equilibriums’ that are not merely transitory. One such case is found in the process of ‘centralization’. One of the central tendencies highlighted by Marx for ‘modern’ Capitalism, in which many of the firms of a branch of industry are eliminated by the few that manage to substantially increase their productivity and leads to their having monopoly power and rates of profit that are not subject to the process of equalization.

For Marx the dispersion of profit rates within each branch was important, and could be based on the prevalence of equal market price for all firms in the branch but different costs due to the heterogeneity of the techniques used. Such cost dispersion would generate extraordinary and infraordinary profits, with consequent dynamical effects on the firms. For example, starting from a situation of homogeneous techniques in each sector and homogeneous profit rates (within each branch and among branches), a technological or organizational innovation in a firm of a particular branch could generate extraordinary profits for that firm that could lead to the propagation over time of the more profitable technique and/or the disappearance of the firms that persisted with the old technique (an effect that today many call ‘evolutionist’). The higher profit rate in the branch that produced the innovation could also attract capital flows from other branches (unless it was due to monopoly power).

Let us assume for the moment that capitalists are homogeneous and that each one owns and disburses the same amount of money-capital $k$ in order to earn profit. Since the population of capitalists is $q^K$, aggregate capital $K$ can be written in the following alternative ways:

$$K = q^O A p + q^L w = q^K k = q^O \eta k$$  

(8.41)

where for the last equality we used (8.36). Hence, the aggregate (or global) profit rate under SR (see (8.24)) can also be written in the following ways:

$$\rho = \frac{q^K c_{KP}}{K} = \frac{q^K \pi}{q^O \eta k} = \frac{\pi}{k}.$$  

(8.42)

Hence, under the simplifying assumption that all the capitalists are the same and, in particular, own and invest the same amount of capital $k$, they all have the same profit
rate and consume the same basket of commodities \( c_K \). Hence, not only do we have
the same profit rate in each branch of industry but also each capitalist of each branch
earns the same amount of profit. The profit rate of each branch \( i \) can be written as
\[
\rho_i = \frac{\# \text{ of capitalists in } i}{\# \text{ of capitalists in } i} \times \pi \times \frac{\pi}{k}, \quad i = 1, \ldots, n.
\]

On the other hand, the number of capitalists in \( i \) is \( q_i^Q \eta_i \) and the aggregate capital of
the branch is \( q_i^Q (Ap + \ell w)_i \). Therefore, we also have
\[
\rho_i = \frac{q_i^Q \eta_i \pi}{q_i^Q \eta_i k} = \frac{\eta_i \pi}{(Ap + \ell w)_i}.
\]

Since the last equality is valid for all \( i \), if the profit rates are all equal we have:\(^{21}\)
\[
\eta \pi = \rho (Ap + \ell w).
\] (8.43)

It is often convenient to use the consumption basket of capitalists as numéraire,
that is, \( \pi = c_{KP} = 1 \). In that case (8.43) reduces to \( \eta = \rho (Ap + \ell w) \). Using (8.43)
to eliminate \( \eta \pi \) from (8.38), yields (8.18). And (8.18) and (8.39) can be combined
to recover the system of ‘production prices’ formulated above (8.17). Whenever the
convenient square format for the social matrix of (8.34) and (8.37) is used it must be
taken into account that \( \eta \) is a component that is essentially different from the rest.
The rest of the submatrices of the social matrix are composed of coefficients
that are independent of prices, whereas \( \eta \) is a vector that fundamentally depends on
prices (and the wage rate, which is also a price). The relation given by (8.43) implies
an intimate connection between the price systems (8.37) and (8.17), where the latter
omits the vector \( \eta \) that allocates the capitalists to the different branches but relies on
the assumption that the profit rates of all sectors have been equalized. Hence, not
only does \( \eta \) depend on prices and wage: it also depends on the assumption that one
makes on the profit rates of the different branches. The simplest assumption is that
all branches have the same profit rate. And this is often the assumption Marx makes:

The entire process of capitalist production is furthermore regulated by the
prices of the products. But the regulating prices of production are themselves
in turn regulated by the equalisation of the rate of profit and its corresponding
distribution of capital among the various social spheres of production. Profit, then, appears here as the main factor, not of the distribution
of products, but of their production itself, as a factor in the distribution of capitals and labour itself among the various spheres of production
(B3, 868-9).

But although Marx mainly uses the assumption of equal rates of profit in all industrial
branches other assumptions are possible. If the profit rates differ among branches
but are homogeneous within each of them, they can be represented by a diagonal matrix
\[
\hat{\rho} = \begin{bmatrix}
\rho_1 & 0 & \cdots & 0 \\
0 & \rho_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \rho_n
\end{bmatrix},
\]

\(^{21}\) Notice that we have here an equality between two vectors (whereas \( \rho \) and \( \pi \) are scalars).
where $\rho_i$ is the profit rate of branch $i$, and $\rho$ be replaced by $\tilde{\rho}$ in (8.43) and $1 + \rho$ by $I + \tilde{\rho}$ in (8.18) and (8.17). The connection between the two ways of expressing the price system is thus preserved. On the other hand, the heterogeneity of profit rates implies the heterogeneity of capitalists, since those of one branch would have different incomes (and consumptions) than those of others and we assume that each capitalist specializes in a certain branch. What is important for our immediate purpose is that, in the square social matrix postulated above, vector $\eta$ must be consistent with the assumption made with respect to the profit rates in the various branches. This implies, in particular, that in a numerical exercise in which equal profit rates are assumed one cannot start from an arbitrary exogenous $\eta$ (as we can for $A$ and $\ell$ under certain constraints). For that vector depends on the production prices and rate(s) of profit.

**Mystified social perceptions of capitalist social relations**

We have seen in Chapter 7, Marx’s view was that there are various factors that tend to obscure the perception of social relations between individuals immersed in the economic process when markets play a fundamental role, a phenomenon that Marx denominated “fetishism of commodities”. We have seen that this included the perception of money as a thing endowed with magical properties. For Marx such forms of social psychology existed in any commodity producing society but acquired more complex characteristics in Capitalism due to the antagonistic interests of the social classes that must coexist in the production process. Referring to modern political economy, “which looks down with such disdain on the monetary system”, he asks: “does not its fetishism come out as clear as noon-day whenever it treats of capital? How long is it since modern economics discarded the physiocratic illusion that rents grow out of the soil and not out of society?” (B1, 93).

Marx held that in CCP there is the additional illusion that the wage worker is paid for the whole of his labor time. And this became the main anchor of his theory of surplus value. “In the corvee, the labour of the worker for himself and his compulsory labour for his lord, differ in space and time in the closest possible way. In slave labour, even that part of the working day in which the slave is only replacing the value of his own means of existence, in which, therefore, in fact, he works for himself alone, appears as labour for his master. All the slave’s labour appears as unpaid labour.” In contrast, in wage labor “even surplus labour, or unpaid labour, appears as paid. There the property relation conceals the labour of the slave for himself; here the money relation conceals the free labour of the wage labourer” (B1, 539-40).

Marx held that classical political economy had come close to discovering the fact that what the wage worker sells is his labor power during a certain period of time and that, like any purchaser of commodities, the capitalist consumes the use value of that labor power—that is, the work exerted—according to his convenience. And that he did so making sure that the laborer works during a longer period of time than is necessary to reproduce his labor power, thus generating a surplus for the capitalist

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22The original in MECW has ‘superstition’ here. But the correct translation for ‘Fetischismus’ (as it appears in the German edition) is ‘fetishism’.

23The original in MECW has ‘economy’ here instead of ‘modern economics’ for what in the German edition is ‘moderne Ökonomie’. Aside from the strange omission of ‘modern’, we believe that Marx is referring here to the discipline of what is at present called either ‘economics’ or ‘political economy’. Both Google and Microsoft translate ‘political economics’ as ‘Politische Ökonomie’. But the back translation of ‘Ökonomie’ is ‘economics’ in the first and ‘Economy’ in the second. So there appears to be some room for interpretation.

24MECW has ‘unrequited labour’ instead of ‘free labour’. The corresponding word in the German edition is ‘Unsonstarbeiten’, which Microsoft translates as ‘Work for free’. 
that is actually ‘unpaid work’. Marx considered that political economy could not come
to grips with this reality because it tended to see things from the point of view of the
capitalist class: “The classical political economy comes close to the true facts, without
formulating them consciously. She cannot do that as long as she’s in her bourgeois
skin” (B1, 542).25 Marx believed that the great majority of classical economists had
tended to see reality through the tainted glass of the ideology of the dominant class,
an ideology that was intimately linked to its material interests. For him, “reality” is
one and in principle it is possible to advance towards an objective theory; that is, a
theory that is not tainted by material (and subjective) interest of any class or group
of society but takes account of all those conflicting interests – and their perceptions
as they are manifested – as part of the available empirical data. What Marx held is
that the advancement of social science was particularly difficult in periods in which the
relations between the social classes became virulently tense, in which ‘class struggle’
was rife. For “In the domain of political economy, free scientific inquiry meets... the
most violent, mean and malignant passions of the human breast, the Furies of private
interest” (B1, 10). For Marx there is an occult aspect of reality: the appropriation of
the proprietor classes of wage workers’ ‘unpaid labor’ and it is “scientific inquiry” that
must discover it. In Part III of this book we will examine this in detail.

Appendix to Chapter 8

Biblioent Notes

The persistent confusion on an alleged assumption of a ‘subsistence wage’
by Marx Notwithstanding the clarity with which Marx introduces the cultural factor
in workers’ consumption basket, surprisingly many exegesises (both sympathizers and
critics) of Marx have completely ignored his statements. Authors as diverse as Russell
(2009 [1934]) and Samuelson (1971), for example, interpret the consumption basket of
wage workers as a wage so low that it only allows for subsistence. Samuelson refers
to “Marx’s hypothesis of a minimum-subsistence, exploitative wage” (Samuelson 1971,
406). Samuelson’s confusion on this is probably based on Meek (1967), a collection of
essays that he used in his research. For he writes “Especially valuable as well is Meek
[19, 1956], to the later printing of which I shall key my references” (Samuelson 1971,
413, footnote 21). That later printing is one of the essays included in Meek (1967).
Although the essay Samuelson cites is one bearing on the ‘transformation problem’ and
has nothing on a ‘subsistence wage’, in another essay of this collection (“Karl Marx’s
Economic Method”), Meek writes: “The ‘Law of value’ is therefore applied by Marx
to the commodity labour –or rather labour-power– itself, the value of labour-power
being in effect defined as the amount of labour required to produce wage-goods for the
labourers at subsistence level” (Meek 1967, 102; italics added).

Pasinetti is another author that incurs in the same kind of confusion. He writes:

Marx argues... the ruthless logic of market competition also reduces the
price of labor to its cost of production, i.e., to that subsistence wage which
is strictly necessary for the maintenance of the worker and his family
(Pasinetti 1977, 19; italics added).

25MECW has: “Classical political economy nearly touches the true relation of things, without,
however, consciously formulating it. This it cannot so long as it sticks in its bourgeois skin”. We
have here preferred to use the Google translation of the German original: “Die klassische politische
Ökonomie stößt annähernd auf den wahren Sachverhalt, ohne ihn jedoch bewusst zu formulieren. Sie
cann das nicht, solange sie in ihrer bürgerlichen Haut steckt.”
As is well known, Marx (1867) thought that... If then labour is thrown on a market and is traded as any other commodity, then we can only expect the competitive market-price mechanism to perform its job with labour precisely in the same way as it does with any commodity: namely to drive the price of labour toward its cost of production. In the case of labour, the cost of production is the subsistence wage rate; this is what the competitive market-price mechanism would achieve. Therefore the entrepreneurs would reap whatever is above subsistence (‘exploitation’) (Pasinetti 1993, 127; italics added).

Finally, even today the *Stanford Encyclopedia for Philosophy*, for example, states in an article on ‘Exploitation’: “Marx thought that workers under capitalism will therefore be paid just enough to cover the *bare necessities of living*. They will be paid subsistence wages” (Zwolinski and Wertheimer, 2017).

There are hundreds of erroneous interpretations on this subject notwithstanding Marx’s multiple explicit assertions. A part of the confusion may be due to some of the very politicized expressions of Marx and Engels in their youth. For example, in the *Manifesto of the Communist Party* (1848) they write: “The average price of wage-labour is the minimum wage, i.e., that quantum of the means of subsistence, which is absolutely requisite to keep the labourer in bare existence as a labourer” (MECW 6, 449). But, as we have seen, two decades of further studies and theoretical developments substantially modified that initial posture. Marx holds in Book I that “the labourer is often compelled to make his individual consumption a mere incident of production. In such a case, he supplies himself with necessaries in order to maintain his labour power, just as coal and water are supplied to the steam-engine and oil to the wheel.” He adds however that this “appears to be an abuse not essentially appertaining to capitalist production” (B1, 571; italics added). Marx even states that “in periods of prosperity, particularly during the times of bogus prosperity... The working class (now actively reinforced by its entire reserve army) also enjoys momentarily articles of luxury ordinarily beyond its reach, and those articles which at other times constitute for the greater part consumer ‘necessities’ only for the capitalist class” (B2, 409). And aside from the lengthy time period elapsed between the *Manifesto* and *Capital*, it is clearly improper to give the same importance to a pamphlet that had ostensible political aims as to a very carefully constructed work of scientific character.

**Vladimir Dmitriev’s contribution** Vladimir Karpovich Dmitriev (1868-1913) in 1898 published (in Russian) the article “The Theory of Value of David Ricardo: An attempt at a rigorous analysis”\(^\text{26}\), where (probably for the first time) correct algebraic expressions are obtained for the vectors of values and production prices. In the first case, in order to obtain the solution \( v \) to equation \( v = \ell + Av \), he recursively replaces the \( v \) on the right hand side of the equality by the complete right hand side of the equation, that is:

\[
\begin{align*}
v &= \ell + Av \\
v &= \ell + A (\ell + Av) = \ell + A\ell + A^2v \\
& \vdots \\
v &= \ell + A\ell + A^2\ell + A^3\ell \ldots
\end{align*}
\]

The last equality is a correct expression for the values vector, as we have seen in (6.6). To obtain what he called ‘natural prices’, he follows the same procedure. He starts from (8.18) and recursively replaces the $p$ on the right hand side, obtaining an infinite series of wage payments that are ‘inflated’ by the profit rate:

$$
p = (1 + \rho) (\ell w + Ap)$$
$$p = (1 + \rho) \left[ \ell w + A (1 + \rho) (\ell w + Ap) \right]$$
$$= \left[ (1 + \rho) \ell + (1 + \rho)^2 A \ell \right] w + (1 + \rho)^2 A^2 p$$
$$p = \left[ (1 + \rho) \ell w + (1 + \rho)^2 A \ell w + (1 + \rho)^3 A^2 (\ell w + Ap) \right]$$
$$= \left[ (1 + \rho) \ell + (1 + \rho)^2 A \ell + (1 + \rho)^3 A^2 \ell \right] w + (1 + \rho)^3 A^3 p$$
$$p = \ldots$$

The last expression is equivalent to (8.20). Although he did not reach a synthetic formula, Dmitriev did get a formally correct expression for the production prices vector. Dmitriev also indicates that if $c_L$ is the consumption basket of wage workers, the wage is $w = c_{LP}$. Hence, if the last equality is premultiplied by $c_L$, $w$ can be eliminated yielding

$$1 = c_L \left[ (1 + \rho) \ell + (1 + \rho)^2 A \ell + (1 + \rho)^3 A^2 \ell + \ldots \right].$$

Dmitriev makes the point that since everything except $\rho$ are data here and the right hand side grows with $\rho$, this equation determines the (unique) profit rate. From what we have seen, we can add that if $\ell$ is extracted from the brackets the matrix series that remains within the brackets converges to $B(\rho)$ as long as $\lambda (A) < 1/(1 + \rho)$ (Theorem 7 of the Mathematical Appendix to Chapter 5). Hence, Dmitriev’s equation is equivalent to (8.22). For Dmitriev this confirmed the correction of Ricardo’s conclusion that the rate of profit depends exclusively on the conditions of production in the branches of industry whose outputs directly or indirectly contribute to the production of the goods in the consumption basket of workers. We will consider this issue in Chapter 10, where we address luxury commodities.

András Bródy and the interpretation of Marx’s theory in the Soviet bloc

András Bródy’s book Proportions, Prices and Planning: A mathematical restatement of the labor theory of value is a strange mixture of good mathematical modeling and erroneous interpretation of Marx’s thought, notwithstanding the abundance of quotations from his principle works. One cannot but wonder to what point this paradox is a consequence of censorship or self-censorship (since the author wrote in the Communist Hungary of the Soviet bloc around the time of the Soviet invasion of Czechoslovakia) or simply a fundamental lack of understanding of Marx’s theory. What is most remarkable in this book (which in spite of all its flaws is of great interest from a modeling point of view) is that all trace of dominant and ruling classes disappear, just as they do in mainstream economic theory. It uses the term ‘labor theory of value’ because only labor and goods and services produced by labor appear in the book. It is nonsensical to try to represent Marx’s theory of Capitalism without the capitalist-entrepreneur. We understand, of course, that in the Soviet state ideology social classes as such had (supposedly) disappeared and hence all the people involved in the economic process were workers. And among the book’s objectives was that of making recommendations
on economic planning. The *Nomenklatura*, the bureaucratic dominant and ruling class of real Communism could not be represented without defying the power structure. The very concept of a dominant class that receives a surplus had to be archived in the deposit of the old ‘unproductive classes’ that are mentioned in passing (Bródy 1970, 31 and 49).

But Bródy’s book also has disconcerting conceptual confusions on some of the basics of Marx’s thought. For example, he completely confuses the dichotomy SCP/CCP with that of SR/ER. Although Marx limited his model of SCP to SR, from a logical point of view SCP can have both SR and ER. And from a historical point of view there have been periods of non-Capitalist production with growth. But we agree with Marx that both commodity production and accumulation historically received their great impulse from the development of the capitalist mode of production. Another surprising feature of Bródy’s book is his confusion between the dichotomy SR/ER and Marx’s initial simplifying assumption of unitary turnover periods for all the means of production (basically, the absence of fixed capital), an assumption that Marx carries throughout much of Books II and III.

Bródy’s analytical interest was very similar to that of von Neumann: to highlight that different theories or models could be represented by the same mathematical structure. This led him to leave out the central aspect of Marx’s thought (the asymmetric and hierarchical relations that human beings entertain in the economic process), while giving the central stage to the merely formal matters of the mathematical instruments. For this reason he builds a tree of economic models where von Neumann’s is at the top because it has the greatest generality (since it can tackle joint production of two or more commodities). Maybe it was to ingratiate himself with the all-powerful *Nomenklatura* whose *raison d’être* was anchored on a debased ‘Marxism’ that there are so many interesting quotations of Marx. On the other hand, by using conventional mathematical structures in the excess of Marx’s theory he was also building a bridge towards western academia. And the fact that Leontief accepted to write a (very short) preface is an indication of Bródy’s success in this respect.

The model that Bródy builds is actually a model of SCP (under the assumption of SR first and ER later). Social classes are non-existent as is any trace of exploitation and even income distribution. He states in the Introduction that

The purpose of this book is to translate Marx’s original approach into mathematical terms and to indicate the path leading from it to modern quantitative economic reasoning. Once this is done it is possible to prove strict mathematical equivalence of a whole family of theories and models: the labor theory of value, game theory, open and closed static and dynamic Leontief systems, linear programming, the mathematical theory of optimal processes and other general equilibrium models (Bródy 1970, 4).

Both the first and second of these purposes are impossible to achieve if one has a complete misunderstanding of “Marx’s original approach”. In spite of its flaws, Bródy’s book is useful for having displayed with great clarity mathematical instruments that can be fruitfully used for the clarification of various aspects of Marx’s theory, even its fallacies. For this, we are indeed indebted to Bródy.

**Mathematical Appendix to Chapter 8**

**I. A useful result for the construction of the values and prices systems** We have seen that the *Basic Assumptions* on $A$, $\ell$, $c_L$ (5.24) imply that matrix $A + \ell c_L$
is indecomposable and hence that its dominant eigenvalue is positive \( \lambda (A + \ell c_L) > 0 \). We additionally assume that \( \lambda (A + \ell c_L) < 1 \) which, as we have seen in this chapter, is a requirement for Capitalism, since room must be made for the consumption of the capitalist class. By Perron-Frobenius there exists a unique (up to a scalar factor) \( x > 0 \) such that \( (A + \ell c_L) x = \lambda_0 x \), with \( \lambda_0 = \lambda (A + \ell c_L) \). We here prove that 1) there exists a unique (up to a scalar factor) vector \( p > 0 \) and a unique scalar \( \rho_0 > 0 \) such that \( (1 + \rho_0^0) (A + \ell c_L) p = p \), and 2) there exists a unique (up to a scalar factor) \( v > 0 \) and a unique scalar \( e_0 > 0 \) such that \( [A + (1 + e_0^0) \ell c_L] v = v \). The proof of 1) is trivial, since defining \( \rho_0 = 1/\lambda_0^0 - 1 \) and \( p \equiv x \) yields the required equation.

For the proof of 2), we use Theorem 2.6) of the Mathematical Appendix to Chapter 5, which says that for an indecomposable \( n \times n \) matrix \( F \), the dominant eigenvalue is greater or equal to the smallest of its row sums: \( \lambda (F) \geq \min_i \sum_j F_{ij} \). The \( i \)-th row sum of \( F \) can be written as \( F_i u \), where \( u \) is an \( n \times 1 \) column vector of ones. Consider \( F (e) \equiv A + (1 + e) \ell c_L \). Since \( A \) is indecomposable so is \( F (e) \) (for any positive value of \( e \)). Let \( i = f \) be the row of \( \ell c_L \) with smallest row sum. Then \( \lambda (F (e)) \geq (A + (1 + e) \ell c_L) u \geq e (\ell c_L) u \). Since \( \ell > 0 \) and \( c_L \geq 0 \), matrix \( \ell c_L \) cannot have any row of zeros, so in particular \( (\ell c_L) u \geq 0 \). Hence, if \( e \) is made arbitrarily large, the same happens with \( \lambda (F (e)) \). We also know that \( \lambda (F (0)) < 1 \). Hence, by continuity and the fact that \( e (\ell c_L) u \) is strictly increasing with \( e \) there exists a unique value \( e_0 > 0 \) such that \( \lambda (F (e_0)) = 1 \). Therefore, a positive vector exists \( v > 0 \) (unique up to a scalar factor) such that \( [A + (1 + e_0) \ell c_L] v = v \).

II. Two special cases of Figure 4  There are two special cases which can be intuited from Figures 1, 2, and 4.

I) The case of parallel employment and consumption lines.\(^{27}\)

II) The case in which vectors \( C \) and \( g^0 \) are on the same ray that starts from the origin.\(^{28}\)

Let us consider each of these:

Case I)

The slopes of the employment and consumption lines are given by, \( \ell \) and \( v \), respectively. In case I) these vectors are proportional, i.e., there exists a scalar \( \mu > 0 \) such that \( v = \mu \ell \). Hence, \( \ell = (I - A) u = \mu (I - A) \ell \), which implies

\[
A \ell = \left( 1 - \frac{1}{\mu} \right) \ell.
\]  

(8.44)

Hence, we have the very special case in which \( \ell \) is an eigenvector of \( A \). Furthermore, the indecomposability of \( A \) implies that \( \lambda (A) = 1 - 1/\mu \) and hence \( \ell \) must be a dominant eigenvector of \( A \), and the same can be said for \( v \). But this is a mathematical property. Let us interpret it in economic terms. For this consider the ‘value composition’ of capital \( \kappa_i \) of industrial branch \( i \) as in (8.14) and notice that (8.44) implies that all branches have the same value composition\(^{29}\), that is, \( \kappa_i = \kappa \) for all \( i \), which yields (using the last equality in (8.14)) \( A v = (\kappa / (1 + e)) \ell \). Also, since \( A v = v - \ell \) (first equality in (8.6)) we can see that \( v \) and \( \ell \) are proportional:

\[
v = \left( 1 + \frac{\kappa}{1 + e} \right) \ell.
\]

\(^{27}\)The employment line is not shown in Figure 4 to avoid complicating the figure. But as seen in Figures 1 and 2, in general it has a different slope than the consumption line.

\(^{28}\)\(q^0\) is shown in Figure 2 and is not shown in Figure 4 to avoid complicating the graph.

\(^{29}\)This is proved below in III. of this Mathematical Appendix.
Hence, the factor of proportionality assumed above is $\mu = 1 + \kappa / (1 + \varepsilon)$, and is now expressed in terms of magnitudes that have precise meaning and were defined by Marx. Furthermore, (8.44) implies $\lambda (A) = 1 - 1/\mu = \kappa / (1 + \varepsilon + \kappa)$.

Summarizing, in the special case I),

a) the value compositions of the two branches are the same ($\kappa$),

b) the employment and consumption lines of Figures 1 and 2 are parallel,

c) $v$ and $\ell$ are both dominant eigenvectors of $A$, and hence only differ by a scalar factor,

d) that scalar factor is $1 + \kappa / (1 + \varepsilon)$.

We will meet this case again below in III. of this Mathematical Appendix.

Case II)

In case II) vectors $C$ and $q^Q$ are proportional. Hence, there exists a scalar $\alpha > 0$ (where necessarily $\alpha < 1$) such that $C = \alpha q^Q$. Using $C = q^Q (I - A)$ (detailed in (6.14)) we get:

$$q^Q (I - A) = C = \alpha q^Q,$$

(8.45)

that is, $q^Q A = (1 - \alpha) q^Q$, which means that $q^Q$ is the (left) eigenvector of matrix $A$ associated to the dominant eigenvalue $1 - \alpha$.\(^{30}\) We now show that, as in Case I),

$$\lambda (A) = \kappa / (1 + \varepsilon + \kappa).$$

Multiplying (8.45) by $v$ gives (using (8.7)) $q^Q \ell = \alpha q^Q \ell$ and hence (using first (8.7) and finally (8.8))

$$1 - \alpha = 1 - \frac{q^Q \ell}{q^Q v} = \frac{q^Q (v - \ell)}{q^Q v} = \frac{q^Q Av}{q^Q v} = \frac{\alpha q^Q Av}{q^Q \ell} = \alpha \frac{c_L v}{c_L v} \alpha = \frac{\kappa}{1 + \varepsilon},$$

whereby (with a little algebra) we have $1 - \alpha = \lambda (A) = \kappa / (1 + \varepsilon + \kappa)$ and $q^Q = (1 + \kappa / (1 + \varepsilon)) C$.

Summarizing, in the special case II)

a) $q^Q$ and $C$ are proportional, whereby they are on the same ray from the origin,

b) the factor of proportionality between $q^Q$ and $C$ is $1 + \kappa / (1 + \varepsilon)$,

c) $q^Q$ and $C$ are (left) dominant eigenvectors of $A$.

We may note that in the special cases I) and II) the factor of proportionality involved is the same and so is $\lambda (A)$. We will show in Chapter 11 that the two ratios involved in both ($\kappa$ and $\varepsilon$) defines the profit rate $\rho^0 = \varepsilon / (\kappa + 1)$ in Marx’s approximation to the ‘production prices’ in Book III of Capital.

Samuelson’s (1971) case of ‘equal internal compositions of capital’ Figure 1 of Samuelson (1971) reflects a special subcase of case II) of Figure 4 in which $C$ and $q^Q$ are on the same ray that goes through the origin, and he calls ‘equal internal compositions of capital’. He assumes that this condition holds for the consumption basket of wage workers (instead of aggregate consumption $C$) and infers that it must also be valid for the consumption basket of capitalists. The extreme simplification by Samuelson of case II) (which implies that the consumption baskets of workers and capitalists are also proportional) has the effect of making the capitalists ‘disappear’ from the graph or, at least, appear only implicitly. Mainstream economics indulges in this kind of hiding to this day, with the explicit effect of erasing any consideration of social structure, social hierarchy, and matters related to income and power distribution by social class.

---

\(^{30}\) This implies that the special case II) represents the ‘standard system’ that Sraffa (1960) uses to construct a numeraire that gives a simple (and linear under the assumption that wages are paid at the end of the period) between the rate of profit $\rho$ and the wage rate $w$. In the Appendix to Chapter 10 we give our assessment of Sraffa’s book.
III. The case of equal value compositions of capital  Let \( A \) be indecomposable. According to whether the components of capital are measured in values or in production prices, the value composition of capital in branch \( i \) of industry, respectively, can be written in two ways which we will call the ‘value composition’ and the ‘value composition’, respectively:

\[
\begin{align*}
\kappa_i &= \frac{C_i^v}{V_i^v} = \frac{A_i v}{(c_{iL})_i v} = \frac{A_i v}{\ell_i (c_L v)} = (1 + e) \frac{A_i v}{\ell_i} \quad (8.46) \\
\kappa_i^p &= \frac{C_i^p}{V_i^p} = \frac{A_i p}{(c_{iL})_i p} = \frac{A_i p}{w_i} \\
&= \frac{A_i p}{\ell_i (c_L p)} = \frac{1}{w} \frac{A_i p}{\ell_i} . \quad (8.47)
\end{align*}
\]

If for all \( i, j \) we have \( \kappa_i = \kappa_j = \kappa \) we say that there are equal value compositions of capital; similarly, if for all \( i, j \) we have \( \kappa_i^p = \kappa_j^p = \kappa^p \) we say that there are equal value compositions of capital. We now prove the following propositions:

**Proposition 1** There are equal value compositions of capital if and only if \( v \) is a dominant eigenvector of \( A \) associated to the dominant eigenvalue of \( A \), which is \( \lambda(A) = \frac{\kappa}{1 + e + \kappa} \).

**Proof.** To prove this Proposition we decompose it into: a) There are equal value compositions of capital if and only if \( Av = \frac{\kappa}{1 + e} \ell \). b) \( Av = \frac{\kappa}{1 + e} \ell \) if and only if \( Av = \frac{\kappa}{1 + e + \kappa} v \) and hence \( \lambda(A) = \frac{\kappa}{1 + e + \kappa} \) and \( v \) is a dominant eigenvector of \( A \).

Starting with a), from (8.46) we see that if \( \kappa_i = \kappa \) for all \( i \) we have \( Av = \frac{\kappa}{1 + e} \ell \). For the reciprocal assume that \( Av = \frac{\kappa}{1 + e} \ell_i \). Then \( A_i v = \frac{\kappa}{1 + e} \ell_i \) for all \( i \) and hence \( \kappa = (1 + e) \frac{A_i v}{\ell_i} = \kappa_i \). To prove b) we know that \( \ell = (I - A) v \). Hence \( Av = \frac{\kappa}{1 + e} \ell \) if and only if \( Av = \frac{\kappa}{1 + e} (I - A) v \), which is the same as \( Av = \frac{\kappa}{1 + e + \kappa} v \). \( \square \)

**Proposition 2** There are equal value compositions of capital if and only if \( p \) is a dominant eigenvector of \( A \) and \( \lambda(A) = \frac{\kappa^p}{1 + e + \kappa^p} \).

**Proof.** To prove this Proposition we decompose it into: a) There are equal value compositions of capital if and only if \( Ap = \kappa^p w \ell \). b) \( Ap = \kappa^p w \ell \) if and only if \( Ap = \frac{\kappa^p}{1 + e + \kappa^p} p \), and hence \( \lambda(A) = \frac{\kappa^p}{1 + e + \kappa^p} \frac{1}{1 + \rho} \) and \( p \) is a dominant eigenvector of \( A \).  

a) From (8.47) if \( \kappa^p_i = \kappa^p \) for all \( i \) then we have \( Ap = \kappa^p w \ell \). For the reciprocal, assume \( Ap = \kappa^p w \ell \). Then \( A_i p = \kappa^p w \ell_i \) for all \( i \) and hence \( \kappa^p = \frac{A_i p}{\ell_i w} = \kappa^p_i \). To prove b) remember that \( p = B(\rho) \ell w \), which implies \( \ell w = (\frac{1}{1 + \rho} I - A) p \). Hence \( Ap = \kappa^p w \ell \) if and only if \( Ap = \kappa^p \left( \frac{1}{1 + \rho} I - A \right) p \), which is the same as \( Ap = \frac{\kappa^p}{1 + e + \kappa^p} p \). \( \square \)

**Proposition 3** Let \( \lambda \equiv \lambda(A) \), then a) \( Av = \lambda v \) if and only if \( A \ell = \lambda \ell \), and b) \( A \ell = \lambda \ell \) if and only if \( Ap = \lambda p \).

**Proof.** a) Take \( Av = \lambda v \). Since \( v = (I - A)^{-1} \ell \) and \( (I - A)^{-1} A = A(I - A)^{-1} \) (checked by simply expanding the inverse matrix in series), we have \( (I - A)^{-1} A \ell = \lambda (I - A)^{-1} \ell \), which implies \( A \ell = \lambda \ell \). For the converse, take \( A \ell = \lambda \ell \) and premultiply by \( (I - A)^{-1} \) to get \( Av = \lambda v \). b) Take \( A \ell = \lambda \ell \). Since \( p = B(\rho) \ell w \) and \( AB(\rho) = B(\rho) A \), we have \( Ap = AB(\rho) \ell w = B(\rho) A \ell w = B(\rho) \lambda \ell w = \lambda p \). For the reciprocal, assume \( Ap = \lambda p \) Hence \( AB(\rho) \ell = B(\rho) A \ell = \lambda B(\rho) \ell \). Multiplying by \( B(\rho)^{-1} \) we get \( A \ell = \lambda \ell \). \( \square \)

**Proposition 4** If \( \ell \) is a dominant eigenvector of \( A \), then: a) for all \( i \) we have \( \kappa_i^p = \kappa = \kappa_i \), \( \lambda = \frac{e}{1 + e + \kappa^p} \), and \( e^p = e \); b) \( \rho = \frac{e}{1 + \kappa^p} \), and \( p = w (1 + e) v \).
Proof. As a consequence of Proposition 3, if $\ell$ is a dominant eigenvector of $A$ so are $v$ and $p$. Since the dominant eigenvector of an indecomposable matrix is unique up to a scalar factor $\ell$, $v$, and $p$ must all be proportional. a) Since $v$ and $p$ are dominant eigenvectors of $A$ Propositions 1 and 2, respectively, imply that there are equal value and value compositions of capital. Proposition 1 shows that if there are equal value compositions then $\lambda(A) = \frac{\kappa}{1+\epsilon+\kappa}$. Also, since $p$ and $v$ are proportional, the global value and value compositions of capital must be equal: $\kappa^p = q^Q A^p/q^Q (\ell c_L) p = q^Q Av/q^Q (\ell c_L) v = \kappa$; and the aggregate rates of surplus value and surplus value must be equal: $e^p = q^Q c_K p/q^Q (\ell c_L) p = q^Q c_K v/q^Q (\ell c_L) v = e$. b) Given $\kappa^p = \kappa$ and $e^p = e$, $\rho = \frac{e}{\kappa+1}$ follows trivially from (8.25). And since $p$ and $v$ are dominant eigenvectors of $A$ and $p/w = B(\rho) \ell$ and $v = B(0) \ell$ we have $\ell = \left(\frac{1}{1+\rho} I - A\right)p/w = (I - A) v$ and hence $\left(\frac{1}{1+\rho} - \lambda\right) p = (1 - \lambda) w v$. Inserting $\rho = \frac{e}{\kappa+1}$ and $\lambda = \frac{\kappa}{1+\epsilon+\kappa}$, we have $p = (1+e) w v$.

Corollary 5 If $\ell$ is a dominant eigenvector of $A$, then: a) if the numeraire is such that $(1+e) w = 1$ prices of production are equal to values: $p = v$; b) if the numeraire is such that $w = 1$ prices of production are $(1+e)$ times values: $p = (1+e) v$.

Numerical Exercise #2

We here show a numerical example in which two commodities are produced. The systems of quantities, production prices and incomes (in its two variants), and values are the following:

$$
\begin{bmatrix}
545.07 & 444.46 & 350.34 & 100
\end{bmatrix}
\begin{bmatrix}
0.1107 & 0.45 & 0.39 & 0.091944 \\
0.61 & 0.25 & 0.31 & 0.11224 \\
0.39 & 0.22 & 0 & 0 \\
0.77 & 0.11 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
545.07 & 444.46 & 350.34 & 100
\end{bmatrix}
\begin{bmatrix}
0.1107 & 0.45 & 0.39 & 0.091944 \\
0.61 & 0.25 & 0.31 & 0.11224 \\
0.39 & 0.22 & 0 & 0 \\
0.77 & 0.11 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1.106 \\
1.350 \\
0.728
\end{bmatrix}
= \begin{bmatrix}
1.106 \\
1.350 \\
0.728
\end{bmatrix}
= \begin{bmatrix}
1.106 \\
1.350 \\
0.728
\end{bmatrix}
$$

The gross output vector is $q^L = [545.07, 444.46]$ and the populations are $q^K = 350.34$ and $q^K = 100$. We have taken as numeraire the consumption basket of capitalists. The vector of production prices and wages is $[1.106 \ 1.350 \ 0.728]^T$ and the profit rate $\rho = 0.09065$. The values are $v^T = [1.101, 1.308]$ and the rate of surplus value is
\( e = 0.3946 \). In Chapter 10 we will use this example to illustrate how it is affected by a technological innovation. 田
Chapter 9  CCP WITH MULTIPLE LABOR SKILLS AND PRODUCTION TECHNIQUES

In the preceding chapter we constructed a basic model of pure CCP in which there is only homogeneous (simple) labor and one production technique for the production of each commodity. In this chapter we successively lift these restrictive assumptions in order to better represent Marx’s theory of Capitalism.

CCP with multiple labor skills
Marx on the valuation of skilled labor in CCP

In the section of Chapter 6 in which we addressed skilled labor in the SCP model we quoted a paragraph of Chapter 1 of Book I in which Marx refers to the reduction of skilled labor to simple labor. We reproduce the substantial part of this quote because it is equally relevant here:

Simple *average* labour, it is true, varies in character in different countries and at different times, but in a particular society it is given. *Skilled labour* counts only as *simple labour intensified*, or rather, as *multiplied* simple labour, a given quantity of skilled being considered equal to a greater quantity of simple labour. Experience shows that this reduction is constantly being made. A commodity may be the product of the most skilled labour, but its value, by equating it to the product of simple unskilled labour, represents a definite quantity of the latter labour alone (B1, 54).

What we didn’t include in Chapter 6 is the following footnote inserted by Marx: “The reader must note that we are not speaking here of the wages or value that the labourer gets for a given labour time, but of the value of the commodity in which that labour time is materialised. Wages is a category that, as yet, has no existence at the present stage of our investigation.” The clarification was pertinent because in the first phase of his investigation, which was on commodities and SCP no wage category was defined because this belonged to the study of capital and CCP, which began in Chapter 4 of Book I. But it seems possible to interpret that, in Capitalism (in which the ‘wage’ category exists), the relative wages between labor skills are relevant in the reduction of skilled to simple labor.

Marx also refers to skilled labor when in *Theories* he analyzes Samuel Bailey’s objections to Ricardo’s theory:

His last objection is this... A simple working day, for example, is not a measure of value if there are other working days which, compared with *days of simple labour*, have the effect of *composite working days*. Ricardo showed that this *fact* does not prevent the measurement of commodities by labour time if the relation between *simple* and *composite labour* is given. He has indeed not described how this relation develops and is determined. This belongs to the definition of *wages*, and, in the last analysis, can be reduced to the *different values of labour power itself*, that is, its varying production costs (determined by labour time) (B4.32, 350; *Theories*, 891).
Marx defends Ricardo on the grounds that it is possible to measure the values of “commodities by labour-time if the relation between unskilled and skilled labour is given.” But he remarks that Ricardo had not “described how this relation develops and is determined.” And he hints that it is in the terrain of wages that the reduction is to be sought, and that the difference between the wages of complex (or ‘composite’) labor and simple labor is related to the relation between their cost of production in labor time.

When Marx considers production as a process of value creation in Chapter 7 of Book I (where he has already introduced CCP but not production prices which he leaves for Book III) he again refers to the question of skilled labor:

All labour of a higher or more complicated character than average labour is expenditure of labour power of a more costly kind, labour power whose production has cost more time and labour, and which therefore has a higher value, than unskilled or simple labour power. This power being of higher value, its consumption is labour of a higher class, labour that creates in equal times proportionally higher values than unskilled labour does (B1, 208).

Although in the text there is no direct reference to relative wages as given by labor markets, in a footnote inserted at the end of the paragraph, Marx states that the “distinction between skilled and unskilled labour rests in part on pure illusion”, thus referring at least partially to the subjective evaluation of the worker with respect to various kinds of labor, and to customs that have taken hold in a context that has since changed substantially. He also refers to the at times paradoxical accidental changes in the labor market, where circumstances here play so great a part, that these two forms of labour sometimes change places. Sometimes “lower forms of labour, which demand great expenditure of muscle, are in general considered as skilled, compared with much more delicate forms of labour” which “sink down to the level of unskilled labour” (Ibid.). When Marx here refers to “forms of labour” that sometimes change places he seems to be referring to wage levels, e.g. “the labour of a bricklayer, which in England occupies a much higher level than that of a damask-weaver”, to such an extreme that S. Laing (National Distress, &c, London, 1844) included the bricklayers in his definition of the middle class. Aside from referring to the influence of demand on relative wages, Marx was touching on the question of the subjective evaluation by workers of the advantages and disadvantages of different kinds of labor (that have an incidence on their supplies). Both matters were later systematically addressed by the Neoclassical economists by means of the introduction of the subjective aspect of the theory of value. But since Marx lacked the tools that made it feasible to address these issues analytically he left the reduction of skilled labor to simple labor rather vague, as Ricardo also had done previously.

When Marx deals with the equalization of profit rates he explicitly refers to the competition between workers and writes that both the equalization of the rates of surplus values in the various branches as the differences between them respond to “real or imaginary (conventional) grounds of compensation”:

The fact that capitals employing unequal amounts of living labour produce unequal amounts of surplus value, presupposes at least to a certain extent that the degree of exploitation or the rate of surplus value are the same, or that any existing differences in them are equalised by real or imaginary (conventional) grounds of compensation. This would assume competition
among labourers and equalisation through their continual migration from
one sphere of production to another (B3, 173).

Although Marx is not referring here explicitly to skilled labor as distinct from simple
labor, this paragraph seems compatible with the reduction of skilled labor to simple
labor on the basis of relative wages. The assumption that the differences in the rates of
surplus value tend to disappear – “at least to a certain extent” – “by real or imaginary
(conventional) grounds of compensation” seems to leave room for the possibility that
wage differentials between labors of different skills could sustain heterogeneous rates
of surplus value between branches of industry owing to the different combinations of
labor skills involved. The recognition that whatever the stable relation between the
rates of surplus value in the various branches is it requires the “continual migration” of
workers “from one sphere of production to another”, is also compatible with the notion
that the market plays an important role in the formation of the relative values of the
labor powers with different skills. The paragraph continues with: “Such a general rate
of surplus value – viewed as a tendency, like all other economic laws – has been assumed
by us for the sake of theoretical simplification. But in reality it is an actual premise of
the capitalist mode of production, although it is more or less obstructed by practical
frictions causing more or less considerable local differences” (Ibid.; italics added). A
general rate of surplus value can be interpreted as an average of possibly different
rates of surplus value in different branches due to the existence of various skills. But,
as we will see, a unique rate of surplus value in all branches and skills is not generally
compatible with the use of relative wages for the reduction of skilled labor to simple
labor.

The dual systems of quantities and production prices

We have seen in Chapter 6 that the heterogeneity of labor, that is, the existence of
(one or more types of) skilled labor, does not generate significant complications in the
analysis of SCP. It is sufficient to start from distinct consumption baskets for each kind
of labor. We have also seen that in Marx’s representation of CCP there coexist two
systems that have to do with valuations: the system that is dual to the quantities (or
use values) system and determines the exchange values, that is, the system of prices
of production, and the one that deals with values, meant to explain the exploitation
of wage workers through the rates of surplus value. We start here by showing that
there is no difficulty in establishing the dual system in the presence of skilled labor if
we have distinct consumption baskets for each labor type.

The dual systems of quantities and prices in CCP are the following:

\[
\begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A & L & \eta \\
C_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix},
\]  
(9.1)

\[
\begin{bmatrix}
A & L & \eta \\
C_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix},
\]  
(9.2)

where now \(q^L\) is a row vector that contains the populations of workers separated by
skill, each row of submatrix \(C_L\) is a consumption basket for the corresponding worker
skill and each column of submatrix \(L\) is the vector of direct labor requirements for a
certain skill. Also, \(w\) is a vector wages, one for each skill (and one for non-skilled,
simple, labor). To have the vector of prices and wages fully determined we can assume
that the lowest wage (corresponding generally to simple labor) is the numeraire, say
\( w_1 = 1 \) assuming that \( w_i < w_1 \) for \( i = 2, 3, \ldots \). Both systems imply \( \lambda(M^+) = 1 \), where
\( M^+ \) is the social matrix. Let \( M \) be the principal submatrix of \( M^+ \) obtained by the
elimination of the last row and the last column. Assuming the indecomposability of
\( M^+ \), we have \( \lambda(M) < 1. \)

Let us call \( M(\rho) \) the matrix of the following system of prices and wages:
\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) L \\
C_L & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w
\end{bmatrix}
= \begin{bmatrix}
p \\
w
\end{bmatrix}.
\]
(9.3)

Replacing the second equality into the first we obtain \((1 + \rho)(A + LC_L)p = p\) –

analogous to (8.16)– which shows that the dominant eigenvalue of \( A + LC_L \) is \( \lambda(A + LC_L) = 1/(1 + \rho) < 1 \). There is thus a unique profit rate \( \rho > 0 \) and a vector of production
prices \( p \) that satisfy the equation. If we also define the vector of wages as \( w = C_L p \)
we have the vector \((p \ w)^T > 0 \) that solves (9.3), is unique up to a scalar factor, and is
associated to the dominant eigenvalue of \( M(\rho) \), that is, \( \lambda(M(\rho)) = 1 \).

We can also proceed as we did when we only had simple labor to obtain (8.23)
from (8.17), i.e., add a column of zeros and a row with \( c_K \), adding \( \pi \) to the vector
of prices and wages. If we premultiply that expanded system (which we do not write
down to save space) by \( q = (q^Q \ q^L \ q^K) \) and multiply (9.1) by \((p \ w \ \pi)^T \), we again get
three alternative ways of expressing aggregate profit: \( \rho q^Q (A p + L w) = q^K \pi = q^K c_K p \).

Hence we again get the profit rate (8.25) which we repeat for the reader’s convenience:
\[
\rho = \frac{Sp}{Cp + Vp} = \frac{e^p}{\kappa^p + 1},
\]
(9.4)

where the definitions of the aggregate rate of surplus value and aggregate value
composition of capital are
\[
e^p = \frac{Sp}{Vp} = \frac{q^K c_K}{q^L C_L p}, \quad \kappa^p = \frac{Cp}{Vp} = \frac{q^Q A p}{q^K LC_L p}.
\]
(9.5)

These definitions are very similar to (8.26) but, due to the presence of skilled labor,
the definition of variable capital \( V^p \) has been modified to include the aggregate con-
sumption basket of all workers \( q^L C_L \).

The system of values

So far the presence of heterogeneous labor has not presented any difficulties. We will
now show that constructing the system of values so that it reflects Marx’s thought
is not so straightforward. As we have seen, Marx left this matter unsettled. This
may have been due, at least partly, to the algebraic difficulties he encountered when
elaborating numerical exercises. He had no alternative to conforming to the notion
that, given the reduction coefficients of skilled to simple labor, his values and surplus
values remained valid as if all labor were simple. We will see that there at least two
ways of defining the system of values so that it is compatible with what Marx sketched
in this respect. As in other parts of this book, we try to find the formulation that is
both compatible with Marx’s fundamental ideas and mathematically correct. In this
particular case we choose to first exposit the version most found in the literature, and
then the one we prefer on the grounds that it seems closer to Marx’s thought on this
subject. The former postulates that there the same rate of surplus value holds for all

\footnote{We use Theorem 2.4 of the Mathematical Appendix to Chapter 5.}
kinds of labor, and the latter postulates that the reduction of skilled to simple labor is effected according to the values of the consumption baskets measured in production prices (i.e., equilibrium market prices).

1) Equal rates of surplus value for all kinds of labor  We begin with what has been the most usual way of introducing skilled labor in the system of values. Just as profit rates tend to equality in the various branches, the same rate of surplus value tends to prevail for all kinds of labor. Hence, the system that replaces (8.6) is the following:2

\[
\begin{bmatrix}
A & L \\
(1+e)C_L & 0
\end{bmatrix}
\begin{bmatrix}
v \\
z
\end{bmatrix}
= 
\begin{bmatrix}
v \\
z
\end{bmatrix}.
\] (9.6)

As we did with (8.6), eliminating \(z\) yields \([A + (1 + e)LC_L]v = v\). Starting from this equation, since \(\lambda = (A + LC_L) < 1\) and \(A + LC_L\) is indecomposable, by Perron-Frobenius there is a unique rate of global surplus value \(e > 0\) and a unique vector of values \(v\) that satisfy this equation. Defining the vector of coefficients of reduction of skilled to simple labor as \(z = (1 + e)C_Lv\) we again obtain (9.6). And calling \(N(e)\) the matrix of that system we have \(\lambda(N(e)) = 1\). We thus have a unique rate of surplus value \(e > 0\) and a unique positive vector \((v z)^T\) where if \(z_k\) is the smallest of the elements of \(z\) (and assuming it is unique) we take \(z_k = 1\). The two equations of (9.6) yield

\[
v = (I - A)^{-1}Lz \equiv B(0)Lz \quad (9.7)
\]

\[
z = (1 + e)C_Lv. \quad (9.8)
\]

The first is again the vector of values as an expression similar to that in (8.7) with the only difference that instead of \(Lz\), we have \(Lz\), where the direct skilled labor requirements have been reduced to their equivalent in simple labor. The second shows that the vector of skilled labor reductions \(z\) is proportional to the vector of consumption values of the workers with different skills, that is, to the values of their respective labor powers. This is similar to (6.23) of SCP, but differs because the proportionality factor is \(1 + e\) instead of 1. Assuming that labor of type 1 is simple labor \((z_1 = 1)\) and there are \(m\) types of labor, we have

\[
e = 1 - \frac{C_{L,1}v}{C_{L,1}} = \frac{z_2 - C_{L,2}v}{C_{L,2}v} = \ldots = \frac{z_m - C_{L,m}v}{C_{L,m}v}, \quad (9.9)
\]

where \(C_{L,i}\) is the \(i\)-th row of matrix \(C_L\). If we define the rate of surplus value of labor of type \(i\) as \(e_i = (z_i - C_{L,i}v)/C_{L,i}v\), these equalities show that all types of labor have the same rate of surplus value \((e)\), which implies that the rates of surplus value of all branches of industry are the same.

To obtain a formula for \(e\) that helps to interpret its meaning, first eliminate the last equality in (9.1) to obtain the equivalent of (8.6):

\[
\begin{bmatrix}
q & q^L & q^K \\
C_L & 0 \\
c \end{bmatrix}
\begin{bmatrix}
A & L \\
0 & 0
\end{bmatrix}
= 
\begin{bmatrix}
q^Q & q^L \\
0 & 0
\end{bmatrix}.
\] (9.10)

---

2It is unnecessary to repeat here the argument of why when we go from SCP to CCP (in both cases now in the presence of skilled labor) it is necessary that some of the coefficients of \(A\), \(L\), and \(C_L\) diminish.
Multiply this by vector \((v \ z)^T\), premultiply (9.6) by vector \((q^Q \ q^L)\), and equal the respective left hand sides. After simplifying we get:\(^3\)

\[
e = \frac{S^v}{V^v} = \frac{q^K c_K v}{q^L C_L v} = \frac{q^K c_K v}{q^Q L C_L v}.
\]

(9.11)

Hence \(e\) is again the ratio between the values of the aggregate consumption baskets of capitalists and workers, that is, the ratio between the aggregate surplus value and the value of aggregate variable capital.

It should be noticed that since \(v\) is not in general proportional to \(p\), neither is \(C_L v\) to \(C_L p\), which implies, considering (9.8), that the assumption that all the different types of labor have the same rate of surplus value implies that the coefficients of reduction of skilled labor to simple labor \(z\) are not in general proportional to the (equilibrium) market wages \(w = C_L p\). But we have already hinted that it seems reasonable that the reduction of skilled to simple labor should be done in such a way that it respects the market relations between the various skills, that is, relative wages. Hence we now show a method of reduction of skilled to simple labor that can be deemed more solid.

2) Reduction of skilled to simple labor according to relative wages  
We have seen that imposing a homogeneous rate of surplus value for all kinds of labor implies that \(z\) is not in general proportional to \(w = C_L p\). An alternative way of constructing the values system in CCP is to assume from the start that the reduction of skilled to simple labor is done according to the ratios given by the market wages \(w\), that is, to assume that \(z\) and \(w\) are proportional. One concrete possibility is to assume the following relation

\[
z = (1 + e)w,
\]

(9.12)

where \(e\) is the global rate of surplus value defined in (9.11). For this it is necessary to allow for heterogeneity in the rates of surplus value of different kinds of labor. We can define the system of values and rates of surplus values as:

\[
\begin{pmatrix}
A \\
(I + \widehat{e}) C_L
\end{pmatrix}
\begin{bmatrix}
L \\
0
\end{bmatrix}
= 
\begin{bmatrix}
v \\
z
\end{bmatrix},
\]

(9.13)

where \(\widehat{e}\) is a diagonal matrix with elements \(e_i\) on the principal diagonal which are not all equal in general. From the second equation of (9.13) we obtain the following rates of surplus value for the different kinds of labor:

\[
e_1 = \frac{z_1 - C_{L,1} v}{C_{L,1} v}, \quad e_2 = \frac{z_2 - C_{L,2} v}{C_{L,2} v}, \ldots, \quad e_m = \frac{z_m - C_{L,m} v}{C_{L,m} v}.
\]

(9.14)

Proceeding as above, i.e., multiplying (9.10) by \((v \ z)^T\) and premultiplying (9.13) by \((q^Q \ q^L)\), we obtain an equality showing that aggregate surplus value is equal to the aggregate value of the capitalists’ consumption baskets:

\[
q^L \widehat{e} C_L v = q^K c_K v.
\]

(9.15)

Hence (9.11) can be written as a weighted average of the rates of the surplus values \(e_j\) of the different kinds of labor:

\[
e = \frac{q^K c_K v}{q^L C_L v} = \frac{q^L \widehat{e} C_L v}{q^L C_L v} = \sum_j \theta_j e_j.
\]

\(^3\)We get the same result if we premultiply (9.8) by \(L\) and then use the two equations of (9.10).
where the weights are the shares of each kind of labor in the aggregate value of the various kinds of labor:

\[ \theta_j = q_j^L C_{Lj} v \left( \sum_{k=1}^m q_k^L C_{Lk} v \right) . \]

Similarly, \( 1 + \epsilon \) is a weighted average of the factors \( 1 + \epsilon_i \):

\[ 1 + \epsilon = \frac{q^L (I + \epsilon) C_{Lv}}{q^L C_{Lv}} = \sum_j \theta_j (1 + \epsilon_j) . \]  

(9.16)

The convenience of using \( 1 + \epsilon \) as the factor of proportionality in (9.12) is that this way the value and the value (using production prices) of the aggregate consumption basket of workers coincide, i.e., (9.18) holds, as we now verify. The second equations of (9.13) and (9.3) respectively yield

\[ (I + \epsilon) C_{Lv} = z \]

\[ C_{Lp} = w. \]  

(9.17)

Using these in (9.12) we get:

\[ (I + \epsilon) C_{Lv} = z = (1 + \epsilon) w = (1 + \epsilon) C_{Lp}. \]

Finally, premultiplying this expression by \( q^L \) and using the first equality of (9.16) we verify that it is indifferent whether we aggregate the consumption baskets of wage workers using values or production prices:

\[ q^L C_{Lp} = q^L C_{Lv}. \]  

(9.18)

This shows that using (9.12) as we did above to infer (9.27) we have introduced a numeraire; specifically, the price vector has been normalized by means of \( (q^L C_L / q^L C_{Lv}) p = 1 \).

Summarizing, in the two alternative ways of defining the values system when we have skilled labor in CCP, (9.6) and (9.13), \( \epsilon \) is Marx’s global rate of surplus value, that is, the ratio between the values of the aggregate consumption baskets of capitalists and workers, respectively. But whereas using (9.6) that rate is the same for all kinds of labor, using (9.13) we generally have a specific rate of surplus value for each kind of labor that is dependent on the structure of wages according to skill. This heterogeneity in the rates of surplus value makes it possible to achieve the reduction of skilled to simple labor in accordance to (equilibrium) market wages, which we believe is compatible with Marx’s thought.

Some relations between variables

The formal relations between values and production prices and between the aggregate rates of surplus value and profit can be derived the same way as we did when we only had simple labor. We shall only spell it out for the case of heterogeneous rates of surplus value (for different kinds of labor).

The relation between production prices and values

From the first equations of (9.3) and (9.13), respectively, we get:\footnote{\( B(\rho) \) is defined in (8.21).}

\[ p = B(\rho) Lw \]  

(9.19)

\[ v = B(0) Lz. \]  

(9.20)
If we premultiply (9.12) by $L$ and use (9.20), we get
\[ Lw = \frac{1}{1+e} Lz = \frac{1}{1+e} (I - A) (I - A)^{-1} Lz = \frac{1}{1+e} (I - A) v. \] (9.21)

Finally, replacing in (9.19) we have the following relations between $v$ and $p$:
\begin{align*}
p &= \frac{1}{1+e} B(\rho) (I - A) v, \quad (9.22) \\
v &= (1 + e) B(0) \left( \frac{1}{1+\rho} I - A \right) p
\end{align*}

Comparing with (8.29) and (8.30), we observe that the relations only differ by the presence of $e$. This presence, which is merely related to the normalization of the price vector, is due exclusively to its having been inserted in (9.12) with the aim of making the vector of production prices come closer to that of values, since (as we can see in (9.18)) it makes the aggregate consumption basket of wage workers (that now have various skills) the same, whether it is measured using $p$ or $v$. Hence, the ‘transformations’ of values into production prices and vice versa are in essence the same as when labor was homogeneous. And they would be exactly the same if instead of (9.12) we assumed $z = w$, which would imply that the vector of prices of production is normalized such that $q^t C_{LP} = q^t (I + \bar{\epsilon}) C_L v$ (or, equivalently, the vector of wages is normalized such that $q^t w = q^t (I + \bar{\epsilon}) C_L v$).

The relation between the rate of profit and the rate of surplus value

To get the relation between the global rate of surplus value and the global rate of profit when there is skilled labor, premultiply (9.19) by $q^t C_L$ to get $q^t C_{LP} = q^t C_L B(\rho) Lw$. Using (9.18) and (9.20), the left hand side of the equality is equal to $q^t C_L B(0) Lz$. And using (9.12) the right hand side is equal to $(1 + e) q^t C_L B(\rho) Lz$. Hence, we get the following relation:
\[ 1 + e = \frac{q^t C_L B(\rho) Lz}{q^t C_L B(0) Lz}. \] (9.23)

It is analogous to (8.31) in the case of homogeneous labor. As before, expanding $B(\rho)$ and $B(0)$ in series we get an expression analogous to (see (8.32)), with the only differences that $q^t C_L$ replaces $c_L$ and $Lz$ replaces $t$. From this we again infer that $e = 0$ if and only if $\rho = 0$, $e > 0$ if and only if $\rho > 0$, and $e > \rho$ if $\rho > 0$.

The special case of equal value compositions of capital

In Chapter 8 we found that— with homogeneous (or simple) labor— if all sectors have the same value composition of capital the vector of prices of production is proportional to the vector of values. Here we check that this remains valid there is labor heterogeneity. In this case the definitions of the value composition of capital $\kappa_i$ and the value composition of capital $\kappa^p_i$ of branch $i$ only differ from their definitions in the case of homogeneous labor in that matrix $LC_L$ must be substituted for matrix $LC_L$:
\[ \kappa_i = \frac{C_i^v}{V_i^v} = \frac{A_i v}{(LC_L)_i v}, \quad \kappa^p_i = \frac{C^p_i}{V^p_i} = \frac{A_i p}{(LC_L)_i p}, \] (9.24)

Similarly, the global value compositions $\kappa$ and value compositions $\kappa^p$ of capital are
\[ \kappa = \frac{C^v}{V^v} = \frac{q^Q A v}{q^Q (LC_L) v}, \quad \kappa^p = \frac{C^p}{V^p} = \frac{q^Q A p}{q^Q (LC_L) p}. \]
Instead of proving propositions similar to those of the Mathematical Appendix to Chapter 8 we here use more intuitive and less formal reasoning. Assume that all the value compositions of capital are the same ($\kappa_i^{p} = \kappa^{p}$ for all $i$). Then (9.24) and (9.19) yield

$$Ap = \kappa^{p} (LC_L) p = \kappa^{p} Lw = \kappa^{p} \left( \frac{1}{1 + \rho I - A} \right) p. \quad (9.25)$$

Reordering we have:

$$Ap = \frac{\kappa^{p}}{1 + \kappa^{p}} \frac{1}{1 + \rho} p. \quad (9.26)$$

Hence, when all the value compositions of capital are the same, the vector of production prices is a dominant eigenvector of $A$ and the dominant eigenvalue is

$$\lambda (A) = \frac{\kappa^{p}}{1 + \kappa^{p}} \frac{1}{1 + \rho}.$$

Since $Ap$ is proportional to $p$ (from (9.26)) and $Ap$ is proportional to $Lw$ (second equality of (9.25)), transitivity implies that $p$ is proportional to $Lw$. And since $Lw$ is proportional to $Lz$ (from (9.12)), $p$ is also proportional to $Lz$. And the relation is the following:

$$p = \frac{(1 + \rho)(1 + \kappa^{p})}{1 + e} Lz. \quad (9.27)$$

Hence, $Lz$ is also an eigenvector of $A$ associated to $\lambda \equiv \lambda (A)$, that is, $A (Lz) = \lambda Lz$. This implies that for any positive integer $n$, $A^n (Lz) = \lambda^n Lz$. Expanding the inverse matrix of (9.20) in series we get $^5$

$$v = (I - A)^{-1} Lz = (I + A + A^2 + ...) Lz = (1 + \lambda + \lambda^2 + ...) Lz = \frac{1}{1 - \lambda} Lz.$$

Hence, $v$ is also proportional to $Lz$ (and is also a dominant eigenvector of $A$):

$$v = \frac{1}{1 - \frac{\kappa^{p}}{(1 + \rho)(1 + \kappa^{p})}} Lz. \quad (9.28)$$

We have proved that $v$ is proportional to $p$, which implies that (from their definitions) that $\kappa^{p} = \kappa$ and $e^{p} = e$, and hence (in (8.25)) $p = e / (1 + \kappa)$. Finally, using (9.28) to eliminate $Lz$ from (9.27), production prices are equal to values:

$$p = \frac{1 + \rho (1 + \kappa^{p})}{1 + e} v = \left( \frac{1 + \rho (1 + \kappa)}{1 + e} \right) v = v. \quad (9.29)$$

In summary, in the special case of equal value compositions of capital in all the branches of industry $p$ and $v$ are proportional. And with the numeraire we used (9.18), $p$ and $v$ are exactly equal. This is analogous to what we had with SCP when we used as numeraire the per capita income of producers/workers $w$ and obtained as a result that prices were equal to values. Also, as in the case of CCP with homogeneous labor, the case of equal value (or value) composition of capital is due to a technological peculiarity: the vector of direct labor requirements (reduced to simple labor) $Lz$ is an eigenvector of the input-output matrix $A$.

Although the equality between $p$ and $v$ in CCP is only true for the very special case of equal value (or value) compositions of capital in all branches (and an appropriate

5See the Corollary to Theorem 7 of the Mathematical Appendix to Chapter 5.
numeraire), we can say that, at least under SR, Marx was proceeding correctly in a formal sense when he restricted his Book I of Capital to values (even in the case of heterogeneous labor) as long as he made the assumption of equal value compositions of capital in all sectors. In a footnote (that referred to a numerical table) Marx makes explicit the assumption that in Book I production prices coincide with values and also reveals his comprehension that this is only a simplifying assumption: “The calculations given in the text are intended merely as illustrations. We have in fact assumed that price = values. We shall, however, see, in Book III, that even in the case of average prices the assumption cannot be made in this very simple manner” (B1, 229, footnote 2). Since explaining the complex relation between production prices and values would have meant a long detour from the topics that he considered more important in the process of production of capital and also, possibly, due to the fact that he was not entirely satisfied with the relation he had obtained, in another footnote of Book I (already quoted in Chapter 4) Marx explains that “the formation of capital must be possible even though the price and value of a commodity be the same” (B1, 176, footnote 1).

More on Marx’s thoughts on exploitation and skilled labor

One of Marx’s most explicit references to the relation between the relative wages of different kinds of labor and the rates of surplus value is in Chapter 8 of Book III, where he states: “In the present chapter we assume that the intensity of labour exploitation, and therefore the rate of surplus value and the length of the working day, are the same in all the spheres of production into which the social labour of a given country is divided.” He writes:

Adam Smith has already comprehensively shown that the numerous differences in the exploitation of labour in various spheres of production balance one another by means of all kinds of existing compensations, or compensations accepted as such on the basis of current prejudice.... Other differences, for instance those in the wage scale, rest largely on the difference between simple and skilled labour, mentioned in the beginning of Book I (S. 19), and have nothing to do with the intensity of exploitation in the different spheres of production, although they render the lot of the labourer in those spheres very unequal. For instance, if the labour of a goldsmith is better paid than that of a day labourer, the former’s surplus labour in the same proportion also produces more surplus value than the latter’s (B3, 141).

When Marx writes that numerous “differences in the exploitation of labour in various spheres of production” are compensated by factors that do not involve the differences in wages and that largely those that do are due to the distinction between simple and skilled labor, one could interpret that he is suggesting the possibility that the rates of surplus value can differ between labors of different kinds. However, when he adds that such wage differences “have nothing to do with the intensity of exploitation in the different spheres of production” he is apparently indicating the opposite, since if

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6 The original has ‘complicated’ instead of ‘skilled’ labor (which is the term used in Book I).
7 The original has “proportionately more surplus value than the latter’s”. We have changed it in accordance to both the Google and Microsoft online translators from German. The second part of the sentence is “so stellt die Mehrarbeit des Goldschmieds in demselben Verhältnis auch größern Mehrwert her als die des Taglöhners.” Aside from being a more accurate translation, the change preempts the possibility that some readers confuse the expression with “more than proportionately”.

different branches use skilled labor in different proportions, the only way in which wage
differences can not affect the degree of exploitation is that the rate of surplus value
is the same for all kinds of labor. Hence, the relation between the rates of surplus value
and the values of the labor powers endowed with different skills is not clear at all.

Let us focus on the last sentence of the quotation (“For instance, if the labour of
a goldsmith is better paid than that of a day labourer, the former’s surplus labour in
the same proportion also produces more surplus value than the latter’s”8 and see if
any of the two alternative ways of formulating the values system is compatible with
this assertion. The surplus value (which is equivalent to the surplus labor) of skill i is
\(e_i C_{L,i}v\) per unit of labor power. Hence, the surplus value of a goldsmith G in relation
to a laborer J is \(e_G C_{L,G}v/(e_J C_{L,J}v)\) and Marx’s statement would imply:

\[
\frac{w_G}{w_J} = \frac{e_G C_{L,G}v}{e_J C_{L,J}v}.
\]

In case 1) of equal rates of surplus value we have \(e_G = e_J = e\), and hence

\[
\frac{w_G}{w_J} = \frac{C_{L,G}p}{C_{L,J}p} = \frac{C_{L,G}v}{C_{L,J}v}.
\]

But we know that the last equality is in general false, since \(p\) and \(v\) are not proportional.
Hence, Marx’s assertion is not compatible with equal rates of surplus value.

And in case 2) of reduction coefficients proportional to wages \(w\), since \(C_{L,i}v = \frac{z_i}{(1 + e_i)}\), we have

\[
\frac{w_G}{w_J} = \frac{e_G C_{L,G}v}{e_J C_{L,J}v} = \frac{z_G e_G}{z_J e_J} \frac{(1 + e_J)}{(1 + e_G)}.
\]

Since in this case \(w_G/w_J = z_G/z_J\), this implies \(e_J = e_G\). But we have seen that in
general in this case we have \(e_J \neq e_G\). Hence, Marx’s assertion is not compatible with
this formulation of the values system either.

Marx was conscious that he had not elucidated the question of the reduction of
skilled to simple labor. Engels was explicit about this in his Anti-Dühring, where in
Chapter VI (titled “Simple and Skilled9 Labour”) he writes:

The values of the products of skilled labour are expressed by this compar-
ison in definite quantities of simple labour; but this reduction of skilled
labour is established by a social process which goes on behind the backs
of the producers, by a process which at this point, in the development of
the theory of value, can only be stated but not as yet explained (Engels,
Anti-Dühring, 139).

In his Preface to the second edition of Anti-Dühring, dated September 1885, that is
some two years after Marx passed away, Engels writes that he read the whole manuscipt to Marx before it was printed.10 It seems likely that Marx would have objected
to Engels’ frank admission that the “reduction of killed labour” could not be explained

---

8 When in Theories Marx studies “Ricardo’s Theory of Surplus-Value” (title of Chapter XV), he
makes a statement that is almost identical to this one: “If the labour of a goldsmith is dearer
than that of a labourer, then the surplus-time of the goldsmith is proportionately dearer than
that of the labourer” (Theories, 619).

9 We have substituted ‘skilled’ for ‘compound’, here and below.

10 Engels also states that Marx wrote the chapter with the critique of Dühring’s ideas on the
history of Political Economy, and that he himself only shortened it.
“at this point, in the development of the theory of value” if he did not agree. And it seems almost certain that if Marx had disagreed Engels would have omitted it.

In short, Marx’s references to the reduction of skilled to simple labor are imprecise and could be used to justify any of the two variants of the system of values and surplus values. For us the second is more attractive because, just as there is “continual migration from one sphere of production to another” of workers, there is also a (slower) migration of workers of one skill to another skill, thus defining, along with the technical demand for the different skills, a structure of relative wages by skills. Hence it seems reasonable to make the (equilibrium) relative value of the labor powers of different skills compatible with this structure.

CCP with multiple techniques per branch
Preliminaries

The simultaneous use in different firms of various technological processes for the production of the same commodity is a topic that is very present in Capital and plays a fundamental role in Marx’s conception of the dynamics of the accumulation process in Capitalism. This is clearly reflected in the following paragraph in which he is explaining his concept of ‘relative surplus value’ (which will be addressed in detail in Chapter 10). He assumes that a capitalist in a certain branch of industry that produces a commodity included in the consumption basket of workers is able to increase productivity by means of an improvement in the method of production. This makes the ‘individual value’ of his output fall beneath the (average) value of all the producers in this branch (the ‘social value’ or ‘market value’) and hence he makes an ‘extra surplus value’. But this situation cannot be stable since competition gradually forces the rest of the capitalists of the branch to adopt the improved method. Marx writes:

Hence, the capitalist who applies the improved method of production, appropriates to surplus labour a greater portion of the working day, than the other capitalists in the same trade... On the other hand, however, this extra surplus value vanishes so soon as the new method of production has become general, and has consequently caused the difference between the individual value of the cheapened commodity and its social value to vanish. The law of the determination of value by labour time, a law which brings under its sway the individual capitalist who applies the new method of production, by compelling him to sell his goods under their social value, this same law, acting as a coercive law of competition, forces his competitors to adopt the new method (B1, 323-4; italics added).

Since in Book I Marx assumed that the equilibrium prices are proportional to values (thus implicitly assuming equal value compositions of capital in all branches) it is natural that he used here ‘extra surplus value’ instead of ‘extra profit’, and the ‘law of the determination of value by labour time’ instead of the ‘law of the determination of prices of production’. And since in Book I he often explicitly assumed that all transactions are done according to values there was no need to distinguish market values from individual values. The same reasoning for the dynamics of the use of alternative technologies applies when in Book III profits and production prices replace surplus values and values. There, individual production prices are distinguished from market (or average) production prices. This is evident in a paragraph of Book III already quoted partially in Chapter 6 which we repeat in this new context with the clarification then omitted in parentheses: “Our analysis has revealed how the market
value (and everything said concerning it applies with appropriate modifications to the price of production) embraces a surplus profit for those who produce in any particular sphere of production under the most favourable conditions” (B3, 198; italics added).

Marx highlights the importance that the factory system and machinery have had in the development of Capitalism through the extraordinary profits\textsuperscript{11} that they generated when their absence was still predominant in the same industrial branch:

So long as, in a given branch of industry, the factory system extends itself at the expense of the old handicrafts or of manufacture, the result is as sure as is the result of an encounter between an army furnished with breach-loaders, and one armed with bows and arrows. This first period, during which machinery conquers its field of action, is of decisive importance owing to the extraordinary profits that it helps to produce. These profits not only form a source of accelerated accumulation, but also attract into the favoured sphere of production a large part of the additional social capital that is being constantly created, and is ever on the look-out for new investments (B1, 453; italics added).

And the dynamical evolution generated by the introduction of machinery with the aim of making extraordinary profits occurred “so soon as its technical basis, machinery, is itself produced by machinery; so soon as coal-mining and iron mining, the metal industries, and the means of transport have been revolutionised; so soon, in short, as the general conditions requisite for production by the modern industrial system have been established” (B1, 454). For Marx it was fundamental that –for various reasons—many firms were not able to rapidly change their production process. Marx’s framework was much more general than the Neoclassical in this respect, referring constantly to tendencies in industrial branches in which there are heterogeneous techniques. The introduction of an improved technique affects the distribution of the techniques used not only through this disruption but also because certain techniques disappear when the less competitive firms that use them must close down due to the process of ‘centralization’ we described in Chapter 3. He does not limit his analysis to the extreme assumptions of the world of ‘perfect competition’. In his theory competition is far from being so extreme that only one productive technique can exist within each branch and, furthermore, firm profits (net of interests and managerial wages) are reduced to zero as in Neoclassical economic theory. Moreover, in Marx’s theory of modern industry there exist what he calls ‘natural monopolies’, a term which referred to the monopolistic (or oligopolistic) power generated by the dynamics of centralization in certain branches of modern industry. His notion of firm profits is conceptually (though not analytically) related to the topics that, decades later, would be included in the notion of ‘imperfect competition’. In this chapter, however, we leave out these aspects of Marx’s theory (which he only barely sketched anyway) and center on another that he did address conceptually with enough clarity to enable us to express it analytically.

When in Chapter 6 we formalized SCP with heterogeneous techniques for each commodity we saw that for Marx the market value is given by a (weighted) average of the individual values of the various producers of the same commodity. But in Book I, Marx assumed that all transactions were realized in accordance with the values of commodities, even after introducing CCP. This is equivalent to making the double assumption that 1) in each branch of industry all producers use the same technique

\textsuperscript{11}In MECW 37 the term that appears most often is ‘surplus profit’ and to a lesser extent ‘extra profit’. In MECW 39 the term most used is ‘extra profit’.
(making all individual values equal to the market values) and 2) the value compositions of capital are the same in all branches (making production prices proportional to values). Once Marx eliminates these simplifying assumption in Book III, the market production price substitutes for the market value as the center around which market prices fluctuate (according to changes in supply and/or demand), and the individual production prices of the capitalist producers in the same branch substitute for the individual values. This is reflected, for example, in Marx’s assertion that “Competition, on the other hand, shows... the fluctuations of market prices, which reduce the average market price of commodities in a given period of time, not to the market value, but to a very different market price of production, which diverges considerably from this market value” (B3, 206; italics added), and also when he states: “this price of production is not determined by the individual cost price of every single industrial producer, but by the average cost price of the commodity under average conditions of capital in the entire sphere of production. It is, in fact, the market price of production, the average market price as distinct from its oscillations” (B3, 634; italics added).

One must bear in mind that Marx was never able to make corrections and polish his manuscripts of Books II and III. And this explains why there are certain terminological inconsistencies that Engels either did not become aware of or did not want to alter for fear of distorting Marx’s meaning. For example, in some occasions Marx uses the expression ‘general price of production’ in the same sense as ‘market price of production’. This is the case when, dealing with differential rent, he writes: “the surplus profit of the producers who use a natural waterfall as motive power is, to begin with, in the same class with all surplus profit... This surplus profit, then, is likewise equal to the difference between the individual price of production of these favoured producers and the general social price of production regulating the market in this entire production sphere” (B3, 635). Another sample of these small variations in terminology is that on some occasions Marx uses ‘price of production’ when it is evident he is referring to ‘market price of production’. For example: “What has been said here of market value applies to the [market] price of production as soon as it takes the place of market value. The [market] price of production is regulated in each sphere, and likewise regulated by special circumstances. And this price of production is, in its turn, the centre around which the daily market prices fluctuate and tend to equalise one another within definite periods” (B3, 178; text within brackets added).

Analytic formulation

In this subsection we model the double distinction between individual and market values and between individual and market production prices in CCP. We have already seen the distinction between individual and market values in the context of SCP, but it is necessary to adapt it to the context of CCP. As we shall see, the method used is very similar. In this section \( \mathbf{p^*} \) represents the vector of ‘market production prices’ and \( \mathbf{p} \) the vector of ‘individual production prices’. In the context of CCP the market values \( \mathbf{v^*} \) and the individual values \( \mathbf{v} \) are analogous to those in SCP, but now the values system must include either the homogeneous rate of surplus value or possibly (if there is skilled labor) the heterogeneous rates of surplus value. As we did in the case of SCP, we leave the general case for the Appendix and here address the simple case in which there are 2 (non human) commodities and 2 techniques for each commodity. Hence, the total number of production processes is again 4. There are also 2 reproduction processes (or consumption baskets) for each of 2 kinds of labor (skilled and simple), and hence 4 reproduction processes. In CCP it is also necessary to take into account
the reproduction of capitalists. In order to avoid unnecessary complications we assume, as we did in the previous section, that all capitalists are the same and hence consume the same basket of commodities \( c_K \).

As we did for SCP, we assume that the processes have been arranged so that those that produce the same commodity are adjacent, as are the workers that have the same skill. Hence, we can write the vectors of quantities produced \( q^Q \), individual production prices \( p \) and individual values \( v \), as:

\[
q^Q = \begin{bmatrix}
q^Q_{11} & q^Q_{12} & q^Q_{21} & q^Q_{22}
\end{bmatrix},
\]

\[
p = \begin{bmatrix}
p_{11} & p_{12} & p_{21} & p_{22}
\end{bmatrix}^T,
\]

\[
v = \begin{bmatrix}
v_{11} & v_{12} & v_{21} & v_{22}
\end{bmatrix}^T.
\]

For example, \( q^Q_{11} \) and \( q^Q_{12} \) are the quantities of commodity 1 produced with techniques 1 and 2, respectively, and \( p_{11} \) and \( p_{12} \) are their respective individual production prices. Similarly, the following are the vectors of populations of the 2 kinds of workers (non-skilled and skilled) \( q^L \), their respective wages \( w \) and the individual coefficients of reduction of skilled to simple labor \( z \):

\[
q^L = \begin{bmatrix}
q^L_{11} & q^L_{12} & q^L_{21} & q^L_{22}
\end{bmatrix},
\]

\[
w = \begin{bmatrix}
w_{11} & w_{12} & w_{21} & w_{22}
\end{bmatrix}^T,
\]

\[
z = \begin{bmatrix}
z_{11} & z_{12} & z_{21} & z_{22}
\end{bmatrix}^T.
\]

We start with the disaggregated systems that have already been used above in this chapter ((9.1), (9.2) and (9.13)):

\[
\begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A & L & \eta \\
C_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix},
\]

\[
\begin{bmatrix}
A & L & \eta \\
C_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
1
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
1
\end{bmatrix},
\]

\[
\begin{bmatrix}
A \\
(I + \bar{c})C_L \\
0
\end{bmatrix}
\begin{bmatrix}
v \\
z
\end{bmatrix}
= \begin{bmatrix}
v \\
z
\end{bmatrix},
\]

which can be written compactly as:

\[
qM = q, \quad My = y, \quad M(\bar{c})x = x,
\]

where \( q \equiv (q^Q q^L q^K) \), \( y \equiv (p w 1)^T \), and \( x \equiv (v z)^T \). Notice that the prices and incomes vector has already been normalized. We now proceed to make the same kind of aggregation we made for SCP. We must include an extra dimension in order to include the population of capitalists. Define the vector of aggregate quantities and populations:

\[
q^* = \begin{bmatrix}
q^{*Q} & q^{*L} & q^{*K}
\end{bmatrix}
= \begin{bmatrix}
q_{11}^Q + q_{12}^Q & q_{21}^Q + q_{22}^Q & q_{11}^L + q_{12}^L & q_{21}^L + q_{22}^L & q_{11}^K + q_{12}^K & q_{21}^K + q_{22}^K
\end{bmatrix},
\]

\[
(9.34)
\]

where the definitions of \( q^{*Q} \) and \( q^{*L} \) are the same as in Chapter 6 for SCP.

Beginning with the prices and incomes system, we use the same quasi-diagonal matrices \( Q^1 \) and \( Q^2 ((6.24) \text{ and } (6.27)) \) of Chapter 6 to define the market production
prices $p^*$ (or average production prices) for each commodity and the market wages $w^*$ (or average wages) for skilled and unskilled labor:

$$p^* = \begin{bmatrix} p_1^* \\ p_2^* \end{bmatrix} = \begin{bmatrix} Q_{11} p_{11} + Q_{12} p_{12} \\ Q_{21} p_{21} + Q_{22} p_{22} \end{bmatrix} = \begin{bmatrix} q_{11}^{Q} p_{11} + q_{12}^{Q} p_{12} \\ q_{21}^{Q} p_{21} + q_{22}^{Q} p_{22} \end{bmatrix} = \begin{bmatrix} \frac{q_{11}^{Q}}{q_{11}+q_{12}} p_{11} + \frac{q_{12}^{Q}}{q_{11}+q_{12}} p_{12} \\ \frac{q_{21}^{Q}}{q_{21}+q_{22}} p_{21} + \frac{q_{22}^{Q}}{q_{21}+q_{22}} p_{22} \end{bmatrix}.$$  

$$w^* = \begin{bmatrix} w_1^* \\ w_2^* \end{bmatrix} = \begin{bmatrix} Q_{11} w_{11} + Q_{12} w_{12} \\ Q_{21} w_{21} + Q_{22} w_{22} \end{bmatrix} = \begin{bmatrix} \frac{q_{11}^{W}}{q_{11}+q_{12}} w_{11} + \frac{q_{12}^{W}}{q_{11}+q_{12}} w_{12} \\ \frac{q_{21}^{W}}{q_{21}+q_{22}} w_{21} + \frac{q_{22}^{W}}{q_{21}+q_{22}} w_{22} \end{bmatrix}.$$  

These definitions are analogous to those of $v^*$ and $z^*$ for SCP ((6.26) and (6.29), respectively). Using $p^*$ and $w^*$ we define quasi-diagonal matrices $P$ (of ratios between individual and market production prices) and $W$ (of ratios between individual and market wages), which are analogous to matrices $V$ and $Z$ used for SCP:

$$P = \begin{bmatrix} P_{11} & 0 \\ P_{12} & 0 \\ 0 & P_{21} \\ 0 & P_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{Q_{11}^{1} p_{11} + Q_{12}^{1} p_{12}}{Q_{11}^{1} p_{11} + Q_{12}^{1} p_{12}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{Q_{11}^{1} p_{11} + Q_{12}^{1} p_{12}}{Q_{11}^{1} p_{11} + Q_{12}^{1} p_{12}} \end{bmatrix}.$$  

$$W = \begin{bmatrix} W_{11} & 0 \\ W_{12} & 0 \\ 0 & W_{21} \\ 0 & W_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{Q_{11}^{1} w_{11} + Q_{12}^{1} w_{12}}{Q_{11}^{2} w_{11} + Q_{12}^{2} w_{12}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{Q_{11}^{2} w_{11} + Q_{12}^{2} w_{12}}{Q_{11}^{2} w_{11} + Q_{12}^{2} w_{12}} \end{bmatrix}.$$  

We hence have equations that average and disaverage, respectively, the vectors of production prices and wages (analogous to (I') and (II') in SCP):

$$\begin{bmatrix} Q^1 \\ 0 \\ Q^2 \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix} = \begin{bmatrix} p^* \\ w^* \end{bmatrix}. \quad (9.35)$$  

$$\begin{bmatrix} P \\ 0 \\ W \end{bmatrix} \begin{bmatrix} p^* \\ w^* \end{bmatrix} = \begin{bmatrix} p \\ w \end{bmatrix}. \quad (9.36)$$  

Let us now define matrices $Q$ and $Y$ as

$$Q = \begin{bmatrix} Q^1 & 0 & 0 \\ 0 & Q^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Y = \begin{bmatrix} P & 0 & 0 \\ 0 & W & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9.37)$$  

Using (9.35), (9.36), and (9.37), we have 1) $Qy = y^*$, and 2) $Yy^* = y$. Also, using (6.25) and (6.28) we have the following disaggregation of quantities and populations 3) $q^*Q = q^{12}$. Notice that, similarly to (6.37), we have 4) $QY = I$, where in the present case the identity matrix is of dimension $5 \times 5$. We now construct the aggregate systems.

12 Notice that, since quantities of the same commodity (or population) can be added, the aggregation of quantities and populations can be written as $qU = q^*$, where $U$ is a matrix that has the same shape as $Q^T$ (the transpose of $Q$) and ones wherever $Q^T$ can be different from zero. Notice also that $QU = I$. In the present case the identity matrix is of dimension $5 \times 5$. 


To get the system of aggregate quantities and populations, use 3) in \( qM = q \) to eliminate \( q \), which yields \( q^*QM = q^*Q \). Then multiply by \( Y \) and use 4) to obtain \( q^*QMY = q^* \), which is the system of aggregate quantities and populations:

\[
\begin{bmatrix}
  q^*Q & q^*L & q^K \\
  A^* & L^* & \eta^* \\
  C_L^* & 0 & 0 \\
  c_K^* & 0 & 0
\end{bmatrix} = \begin{bmatrix}
  q^*Q & q^*L & q^K \\
  A^* & L^* & \eta^* \\
  C_L^* & 0 & 0 \\
  c_K^* & 0 & 0
\end{bmatrix},
\]

(9.38)

where

\[
M^* \equiv QMY = \begin{bmatrix}
  Q^1AP & Q^1LW & Q^1\eta \\
  Q^2CLP & 0 & 0 \\
  c_KP & 0 & 0
\end{bmatrix} = \begin{bmatrix}
  A^* & L^* & \eta^* \\
  C_L^* & 0 & 0 \\
  c_K^* & 0 & 0
\end{bmatrix}.
\]

(9.39)

To get the system of market production prices, wages and profits, use 2) in \( My = y \), which yields \( MYy^* = Yy^* \). Now premultiply by \( Q \) and use 4) to get \( QMYy^* = y^* \), which is the required system:

\[
\begin{bmatrix}
  A^* & L^* & \eta^* \\
  C_L^* & 0 & 0 \\
  c_K^* & 0 & 0
\end{bmatrix} \begin{bmatrix}
  p^* \\
  w^* \\
  1
\end{bmatrix} = \begin{bmatrix}
  p^* \\
  w^* \\
  1
\end{bmatrix}.
\]

(9.40)

And to get the system of market values and labor reduction coefficients remember that (9.32) can be written compactly as \( M(\bar{c})x = x \). Let \( Q^0 \) and \( X^0 \) be the matrices in (6.30) and (6.33), respectively, and let \( x^* = (v^* z^*)^T \) (where \( v^* \) and \( z^* \) are defined by (6.26) and (6.29), respectively). Then the equations (6.30) and (6.33) can be written as \( Q^0x = x^* \) and \( X^0x^* = x \). Using the latter to eliminate \( x \) from \( M(\bar{c})x = x \) we get \( M(\bar{c})X^0x^* = X^0x^* \). And premultiplying by \( Q^0 \) yields \( Q^0M(\bar{c})X^0x^* = Q^0X^0x^* = x^* \), where the last equality uses (6.37). Hence we have obtained the system of market values and labor reduction coefficients:

\[
\begin{bmatrix}
  A^* & L^* \\
  C_L^* & 0 \\
  c_K^* & 0
\end{bmatrix} \begin{bmatrix}
  v^* \\
  z^*
\end{bmatrix} = \begin{bmatrix}
  v^* \\
  z^*
\end{bmatrix},
\]

(9.41)

where

\[
M^*(\bar{c}) = \begin{bmatrix}
  A^* & L^* \\
  C_L^*(\bar{c}) & 0 \\
  c_K^* & 0
\end{bmatrix} \equiv Q^0M(\bar{c})X^0 = \begin{bmatrix}
  Q^1AV & Q^1LZ \\
  Q^2(I + \bar{c})CLV & 0
\end{bmatrix}.
\]

We can now say that the systems of quantities and populations, production prices and wages, and values that we used in the previous chapter were the systems of aggregate quantities and populations, market production prices and wages, and market values as we have seen here with the additional simplifying assumption that there was only one production technique for each commodity and only one consumption basket for each type of labor. Also, we can proceed as when we derived (8.43) so as to eliminate \( \eta^* \) assuming that that the average rates of profit are the same in all the branches (but, as we shall see, within each branch the profit rates can be heterogeneous). We hence have the following system (analogous to (9.3)):

\[
\begin{bmatrix}
  (1 + \rho^*)A^* & (1 + \rho^*)L^* \\
  C_L^* & 0
\end{bmatrix} \begin{bmatrix}
  p^* \\
  w^*
\end{bmatrix} = \begin{bmatrix}
  p^* \\
  w^*
\end{bmatrix},
\]

(9.42)

the first equation of which yields the system of market production prices under the assumption that the average profit rates are the same in all branches:

\[
(1 + \rho^*) \left(A^*p^* + L^*w^* \right) = p^*.
\]

(9.43)
We can now verify that (under the given assumptions) the profit rate of the system of production prices is the same as the homogeneous profit rate of Chapter 8 (9.3), that is, \( \rho^* = \rho \). Premultiplying (9.43) by \( q^Q^* \) and using the first equalities of (9.40) and (9.38), respectively, yields \( \rho^* = q^K c_K p^*/q^Q^* (A^* p^* + L^* w^*) \). Also using the formulas for \( A^*, L^* \), and \( c_K \) of (9.39), (6.25), and the two equalities of (9.36) we get \( \rho^* = q^K c_K p/q^Q (Ap + Lw) = \rho \), where the last equality comes from (8.24).

Extra profits and infra profits

Marx assumes that market transactions are made using market production prices as long as the aggregate quantities produced correspond to solvent demand. Assuming that this is the case, if there is heterogeneity of production techniques within an industrial branch, there will necessarily be heterogeneity in the profit rates of the producers in this branch. Different firms in the branch will have different costs but the market production price is the same for all. In the simple two branch example we deal with here the two firms in each branch have different technical coefficients by assumption, which makes them have different costs while both sell their output at the same (market) production price. Let us represent the profit rate of producers \( j \) in branch \( i \) as \( \rho_{ij} \) and put the 4 profit rates in a (4 by 4) diagonal matrix \( \tilde{\rho}^0 \). Let \( p^0 \) and \( w^0 \) be the market production prices and wages of the 4 types of firms (two for each branch). In branch \( i \) \((=1,2)\) the market production price for both types of firms is \( p^* \). Therefore, \( p^0 = [p^*_1 \ p^*_1 \ p^*_2 \ p^*_2]^T \). Hence we can write the system of market production prices and wages as:

\[
\begin{bmatrix}
  (I + \tilde{\rho}^0) & 0 \\
  C_L & 0
\end{bmatrix}
\begin{bmatrix}
  p^0 \\
  w^0
\end{bmatrix}
= \begin{bmatrix}
  p^0 \\
  w^0
\end{bmatrix}.
\]

Notice that this system has the same dimensions as the system of individual production prices and wages, but takes into account that all firms in the same branch sell their output at the same production price whereas they use different production techniques, which implies that they will normally have different costs and hence different profit rates.

Eliminating \( w^0 \) yields \( (I + \tilde{\rho}^0) (A + L C_L) p^0 = p^0 \). To have an even more compact notation let us define the expanded input-output matrix that encompasses the requirements for both means of production and the means of subsistence of wage workers: \( N \equiv A + L C_L \). If we partition this matrix into four \( 2 \times 2 \) submatrices we have:

\[
N = \begin{bmatrix}
  N_{11} & N_{12} \\
  N_{21} & N_{22}
\end{bmatrix}
\]

For example, \( N_{12} \) represents the outputs of branch 2 used as inputs by branch 1. Each of these submatrices can also be partitioned to individualize the producers that use each of the two techniques in each branch. For example, \( N_{21} \) represents the outputs that producers that use technique 1 of branch 2 sell to producers that use technique 2 of branch 1. The system of production prices can hence be written as:

\[
\begin{bmatrix}
  1 + \rho_{11}^0 & 0 & 0 & 0 \\
  0 & 1 + \rho_{12}^0 & 0 & 0 \\
  0 & 0 & 1 + \rho_{21}^0 & 0 \\
  0 & 0 & 0 & 1 + \rho_{22}^0
\end{bmatrix}
\begin{bmatrix}
  N_{11} & N_{12} & N_{11}^* & N_{12}^* \\
  N_{21} & N_{22} & N_{11}^* & N_{12}^* \\
  N_{21} & N_{22} & N_{11}^* & N_{12}^* \\
  N_{21} & N_{22} & N_{11}^* & N_{12}^*
\end{bmatrix}
\begin{bmatrix}
  p^*_1 \\
  p^*_1 \\
  p^*_2 \\
  p^*_2
\end{bmatrix}
= \begin{bmatrix}
  p^*_1 \\
  p^*_1 \\
  p^*_2 \\
  p^*_2
\end{bmatrix}.
\]

Let us consider branch 1, where the equations corresponding to the two kinds of producers according to the technique they use are the following:
\[
(1 + \rho_{11}) \left[ (N_{11}^{11} + N_{12}^{11}) p_1^* + (N_{11}^{12} + N_{12}^{12}) p_2^* \right] = p_1^*
\]

\[
(1 + \rho_{12}) \left[ (N_{21}^{11} + N_{22}^{11}) p_1^* + (N_{21}^{12} + N_{22}^{12}) p_2^* \right] = p_1^*.
\]

Dividing term by term yields an inverse relation between relative costs and relative profit rates:

\[
\frac{1 + \rho_{11}^0 (N_{11}^{11} + N_{12}^{11}) p_1^* + (N_{11}^{12} + N_{12}^{12}) p_2^*}{1 + \rho_{12}^0 (N_{21}^{11} + N_{22}^{11}) p_1^* + (N_{21}^{12} + N_{22}^{12}) p_2^*} = 1.
\]

For example, if the producers of branch 1 that use technique 1 have lower costs than those that use technique 2, that is, if the second ratio on the left hand side of the equality is less than one, then their profit rate is necessarily greater: \(\rho_{11}^0 > \rho_{12}^0\). Also, since \(\rho^*\) is the average profit rate (in all branches), we have \(\rho_{11}^0 > \rho^* > \rho_{12}^0\), that is, the producers that use technique 1 have extra (or above average) profits and those that use technique 2 have infra (or below average) profits. The same can be done with the producers of branch 2.

This exercise illustrates the observation Marx makes when in *Theories* he criticizes certain aspects of Ricardo’s theory of cost:

... whatever the circumstances, the capitalists belonging to the first group—whose conditions of production are more favourable than the average—make an excess profit, in other words their [rate of] profit is above the general rate of profit of this sphere. Competition, therefore, does not bring about the market-value or market-price [of production] by the equalisation of [the rates of] profits within a particular sphere of production. (For the purpose of this investigation, this distinction [between market-value and market-price] [of production] is irrelevant since the differences in the conditions of production—hence the different rates of profit for the individual capitalists—in the same sphere, remain, whatever may be the relationship of market-price to market-value.) On the contrary, competition here equalises the different individual values to the same, equal, undifferentiated market-value, [and the different individual production prices to the same market production price] by permitting differences between individual [rates of] profits, [rates of] profits of individual capitalists, and their deviations from the average rate of profit in the sphere (B.4.31, 430; *Theories* 481)\(^{13}\).

In Marx’s view, the market production prices \(p^*\) are the ‘attractors’ for the market prices, that is, the market prices towards which the production prices tend as long as there are no discrepancies between supplies and demands for commodities, in which case there would be a transitory deviation. And the market production prices remain constant if there is no modification in the techniques used in the various branches (and demands remain equal to supplies). Since at any time there is a unique market price whereas the techniques used to produce the same commodity are usually heterogeneous, there will be extra (or surplus) profits for producers that use a technique that leads to lower costs. Inversely, producers that have greater costs due to the technique used will have below average profits. Marx did not attempt to analytically represent the dynamical process by which competition between producers of the same commodity lead to the disappearance of those that systematically lag behind. This is a process that takes time, since it involves disinvestment in some branches and further investment in others, in a constant process of equalization of profit rates, except in the cases of

\(^{13}\)The additions to B.4.2 within square brackets were introduced by us and the one within curly brackets by Progress Publishers in *Theories.*
modern industry in which there exist ‘natural monopolies’. Marx addresses these when he considers the developed system of credit in modern industry. In that case there does not exist a tendency towards the equalization of all profit rates since there are some huge firms that can maintain their profit rates systematically higher than the rest. In his letter to Engels of April 30 1868 Marx writes: “The price thus equalised, which divides up the social surplus value equally among the various masses of capital in proportion to their sizes, is the price of production of commodities, the centre around which the oscillation of the market prices moves. Those branches of production which constitute a natural monopoly are exempted from this equalisation process, even if their rate of profit is higher than the social rate” (MECW 43, 24).

Appendix to Chapter 9
Mathematical Appendix to Chapter 9: CCP with multiple techniques

To model the use of multiple techniques of production for the same commodity and multiple consumption baskets per labor skill we proceed as in the case of SCP. Assume there are \( n_i \) techniques used for the production of \( i \) and \( N \) different commodities. Hence, the total number of production processes is \( n \), which is the sum of the \( n_i \) (as in (6.39)). Also, there are \( m_i \) reproduction processes for labor skill \( i \) and \( M \) different skills (including simple labor). Hence, the total number of consumption baskets is \( m \), which is the sum of all the \( m_i \) (as in (6.42)). The individual quantities and values vectors \((q^Q, v)\), individual labor populations and skill reduction coefficients \((q^L, z)\), aggregate quantities and market values vectors \((q^*Q, v^*)\), aggregate labor populations and market skill reduction coefficients \((q^*L, z^*)\) are as in the Mathematical Appendix to Chapter 6. The same can be said for quasi-diagonal matrices \( V, Z, Q^1 \) and \( Q^2 \). The individual and market production price and wage vectors are

\[
\begin{align*}
\mathbf{p} &= (p_{11} \ p_{12} \ \ldots \ p_{1n_1} \ p_{21} \ p_{22} \ \ldots \ p_{2n_2} \ \ldots \ p_{N1} \ p_{N2} \ \ldots \ p_{NN})^T \\
\mathbf{w} &= (w_{11} \ w_{12} \ \ldots \ w_{1m_1} \ w_{21} \ w_{22} \ \ldots \ w_{2m_2} \ \ldots \ w_{M1} \ w_{M2} \ \ldots \ w_{MM})^T \\
\mathbf{p^*} &= (p_1^* \ p_2^* \ \ldots \ p_N^*)^T, \quad \mathbf{w^*} = (w_1^* \ w_2^* \ \ldots \ w_M^*)^T.
\end{align*}
\]

In CCP we also have to consider the reproduction of capitalists. To simplify, we assume that they all consume the same basket of commodities \( c_K \). Hence, there are \( m + 1 \) reproduction processes. We start from the disaggregated systems (9.30), (9.31) and (9.32). Instead of (9.34), the vector of quantities and populations is:

\[
\mathbf{q}^* = \begin{bmatrix} q^*Q & q^*L & q^K \end{bmatrix}, \quad q_i^Q = \sum_{j=1}^{n_i} q^{Q}_{ij}, \quad q_i^L = \sum_{j=1}^{m_i} q^{L}_{ij}.
\]

We look for a matrix and eigenvectors that satisfy systems (9.38), (9.42), and (9.41) in the text.

Starting with the system of production prices and wages, we use same the quasi-diagonal matrices \( Q^1 \) and \( Q^2 \) ((6.40) (6.43)) that we used for SCP in order to define the average production prices and wages for each type of commodity-technique:

\[
\begin{align*}
\mathbf{p}_i^* &= \sum_{j=1}^{n_i} Q_{ij} p_{ij}, \quad \mathbf{w}_i^* = \sum_{j=1}^{m_i} Q_{ij} w_{ij}.
\end{align*}
\]

Using these we define quasi-diagonal matrices \( P \) and \( W \) (analogous to matrices \( V \) and \( Z \) ((6.46) and (6.47)) defined in the Mathematical Appendix to Chapter 6), with
elements:

\[ P_{ij} = \frac{p_{ij}}{p_i}, \quad W_{ij} = \frac{w_{ij}}{w_i}, \]

respectively. From here on, the argument is exactly the same as in the text, since it does not depend on the number of commodities, techniques per commodity, labor skills, and number of consumption baskets per labor skill.
Chapter 10  PRIMITIVE ACCUMULATION AND ABSOLUTE VS. RELATIVE SURPLUS VALUE

Primitive Accumulation and the System of Quantities

As we have seen in Chapter 3, Marx labeled ‘primitive accumulation of capital’ the “historical process of divorcing the producer from the means of production” (B1, 705-6). In this section we illustrate how the system of quantities (of produced commodities and working time) can be used to represent the social transformation that leads from SCP to (pure) CCP. We use Input-Output analysis to represent by means of a simple jump in the system the long historical process described by Marx. Let us start from the system of quantities in SCP (5.2), which we here repeat for the reader’s convenience:

$$ \begin{bmatrix} q^Q & q^L \\ \ell & 0 \end{bmatrix} \begin{bmatrix} A \\ c_L \end{bmatrix} = \begin{bmatrix} q^Q & q^L \end{bmatrix}. $$

We assume that each worker/producer works during a certain amount of hours that defines the extension of his working day and that $q^L$ represents the total working time of society. In Chapter 5 we illustrated by means of a numerical exercise (the transition from (5.1) to (5.6)) the fact that the extension of the working day in SCP (assuming constant returns to scale) does not change the social matrix coefficients, but only the vector of quantities. Here we develop the same type of exercise to see if it sheds light on the idea of primitive accumulation. Assume that all the independent producers extend their working day by $\mu\%$, thus making the amount of hours worked equal to $(1 + \mu)q^L$ (with no change in the population of producers), which makes it possible to proportionally increase output and consumption (assuming constant returns to scale) with no change in the social matrix. Hence, the quantities system is transformed to

$$ \begin{bmatrix} (1 + \mu)q^Q & (1 + \mu)q^L \\ \ell & 0 \end{bmatrix} \begin{bmatrix} A \\ c_L \end{bmatrix} = \begin{bmatrix} (1 + \mu)q^Q & (1 + \mu)q^L \end{bmatrix}, $$

where we see that gross output and producers’ consumption also increase by $\mu\%$ (going from $q^Q$ and $q^Lc_L$ to $(1 + \mu)q^Q$ and $(1 + \mu)q^Lc_L$, respectively). The latter is a change in per capita consumption with no change in the consumption basket per unit of labor time $c_L$.

Let us now analytically represent ‘primitive accumulation’, where the means of production and gross output become the private property of capitalists (and no longer the private property of producers/workers) and, in correspondence, workers who were once producers have been deprived of the means of production and become wage workers. The quantities system must now generate a surplus above the consumption of wage workers because in CCP, aside from the population of wage workers $q^L$ that consume $q^Lc_L$, there is a population of capitalists $q^K$ that, let us assume, consume $q^Kc_K$. To accommodate the reproduction of this social class we can assume that there is a $\mu\%$ extension of the working day of the simple producers that have been converted to wage labor with no change in their per capita consumption. There is thus a jump from the
original SCP system (5.2), not to (10.1), but to the following CCP system:

$$
\begin{bmatrix}
(1 + \mu) q^Q & (1 + \mu) q^L \\
1 + \mu & q^K \\
\frac{1}{1 + \mu} c_L & 0 \\
c_K & 0
\end{bmatrix} \begin{bmatrix}
A \\
\ell
\end{bmatrix} = \begin{bmatrix}
(1 + \mu) q^Q & (1 + \mu) q^L
\end{bmatrix}.
$$

(10.2)

In this system the square submatrix formed by the first two rows of the social matrix differs from that of (5.2) and (10.1) in that there is a reduction in the consumption basket per hour worked to $c_L / (1 + \mu)$. The equations of (10.1) and (10.2) are rewritten in columns $A$ and $B$, respectively, of the following table:

$$
\begin{array}{ll}
1 & (1 + \mu) q^Q A + q^K c_L + \mu q^L c_L = (1 + \mu) q^Q A + q^K c_K = (1 + \mu) q^Q \\
2 & (1 + \mu) q^Q \ell = (1 + \mu) q^L \ell = (1 + \mu) q^L
\end{array}
$$

Equalities 2A and 2B are identical (and $1 + \mu$ may be eliminated in both). And in 1A and 1B we can see that if the consumption basket of capitalists were the same as that of wage workers $c_K = c_L$ and the population of capitalists were the same as the increase in the working hours of workers expressed as a fraction of the working population, $q^K = \mu q^L$, the net outputs of the two systems $(1 + \mu) q^Q (I - A)$ would be the same. This indicates that the consumption of the (until then inexistgent) class of capitalists exactly replaces that expansion of the consumption of the producers/workers based on the extension of their working day.

This exercise algebraically illustrates what can happen with the system of quantities when ‘primitive accumulation’ takes place if we assume that the population of producers/workers splits into two social classes. The majority become wage workers and the rest become capitalists. The former are hired by the new capitalists and work more hours per day than when they were independent producers. And the capitalists take control of the production process, owning the means of production and the resulting output and receiving the surplus produced after subtracting from the net output the aggregate consumption of wage workers. In Marx’s terminology, the means of production that in SCP were the property of independent producers/workers are not ‘capital’ since, in his theory, for Capital (and Capitalism) to exist the production relation between capitalists and wage workers must exist in the workplace and the former must disburse money-capital with the aim of making a profit (or surplus value). Once there is CCP, the means of production are the elements of constant capital, and the labor forces are the elements of variable capital.

We believe this simple exercise is useful for the intuitive comprehension (in terms of quantities) of what Marx had in mind when he addressed ‘primitive accumulation’ and the ‘expropriation’ of the producer/worker. It is a highly stylized and static representation of the historical process described by Marx. In the latter, instead of a simple split in the class of independent producers there was a long formation process of capitalist classes of merchants and bankers that accumulated wealth and also a long formation process of a proletariat free of feudal and guild bonds (and protection), dispossessed of the conditions of production and in need of making a living. Whereas in the historical process both classes in formation were but a minority within a population in which there still predominated pre-mercantile linkages (self-subsistence economies) and/or pre-capitalist mercantile linkages (such as feudal or slave economies), for the sake of clarity we reduced the population set in the analytic representation to the two classes of pure CCP as if they had developed from the population of pure SCP by means of a simple cleavage.
Considered abstractly, we could alternatively have considered the jump from SCP to CCP by assuming that it is the productive power of labor that increases (without change in the length of the working day). But such an exercise would not represent Marx’s ‘primitive accumulation’ process, which fundamentally seeks to reflect a “historical process of divorcing the producer from the means of production” which is not founded on any technological revolution. And as we have seen in Chapter 3 Marx found abundant evidence for this process in the case of England using the best available historical sources. In Marx’s theory of Capitalism the revolutionary process of technological change takes place only when CCP is not only fully functioning (that is, after ‘primitive accumulation’) but has also surpassed an initial stage based on the generation of ‘absolute surplus value’ and has been replaced by a stage based on the generation of ‘relative surplus value’. We next consider these two stages of Capitalism.

The generation of absolute and relative surplus value
Marx distinguishes two very broad phases in the historical development of Capitalism and endeavors to reflect them in his theory. The ‘primitive accumulation’ that produced the initial polarization between capitalists and wage workers initiated a first phase of Capitalism that Marx characterizes as generating ‘absolute surplus value’. There, the capitalists’ exploitation efforts concentrate on increasing the generation of surplus value by prolonging the length of the working day. In contrast, in the second (and definitive) phase, capitalists fundamentally concentrate their efforts on shortening the part of the working day that is necessary to reproduce workers (the ‘necessary labor time’) by increasing labor productivity. This second phase is characterized by Marx as generating ‘relative surplus value’. The conceptual distinction is very clear in Marx’s words: “The surplus value produced by prolongation of the working day, I call absolute surplus value. On the other hand, the surplus value arising from the curtailment of the necessary labour time, and from the corresponding alteration in the respective lengths of the two components of the working day, I call relative surplus value” (B1, 320).

This conceptual distinction was not exclusively aimed at establishing phases in the development of Capitalism but also to help in the analysis of the functioning of the capitalist mode of production in general, especially in its most advanced stage (in Marx’s time), in which the workers’ struggle for the shortening of the labor day produced a reduction of ‘absolute surplus value’ that could be compensated and even surpassed by generating ‘relative surplus value’, that is, an increase in productivity that reduced the necessary labor time, that is the value of labor power. Here we limit ourselves to formalizing the basic ideas in their purest forms by means of the analytical tools we have developed. But first a caveat is in order. Marx develops these ideas in Book I, in which he makes the simplifying assumption that commodities are bought and sold according to their values. One may wonder whether the basic ideas maintain their coherence without this assumption. To show that they do, in the next section we do a similar analysis of the generation of ‘absolute and relative’ profits, terms we have invented simply to show that Marx’s special assumption in Book I in no way invalidates the formal correction of his arguments.

The generation of absolute surplus value
For Marx ‘absolute surplus value’ was generated by the “prolongation of the working day.” Let us see how we can represent this with our systems. Start from the systems of quantities and values in CCP, (8.1) and (8.6), respectively. Let \( q^t \) represent the hours worked initially in the model’s time period and assume that capitalists manage
to have it expand by $\beta\%$ with no change in the daily consumption basket of workers. This implies a $\beta\%$ reduction in the consumption basket per hour of work. Hence the new systems of quantities and values are the following:

$$
\begin{bmatrix}
q_1^Q & (1 + \beta)q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A
\frac{1}{1+\gamma c_L}
\ell
0
\end{bmatrix}
= \begin{bmatrix}
q_1^Q & (1 + \beta)q^L
\end{bmatrix}

\text{(10.3)}
$$

$$
\begin{bmatrix}
(1 + e_1)
\frac{1}{1+\beta c_L}
\ell
0
\end{bmatrix}
\begin{bmatrix}
v_1
1
\end{bmatrix}
= \begin{bmatrix}
v_1
1
\end{bmatrix}.

\text{(10.4)}
$$

To simplify, we assume that the capitalists’ consumption basket increases proportionally by $\alpha\%$, the value of which we determine below. We use the subindex 1 for the new vectors of quantities and values and the new rate of surplus value. However, from the first equation of (10.4) we get $v_1 = (I - A)^{-1} \ell = v$, which shows that there is no change in the vector of values. From the second equation we get

$$
(1 + e_1) c_L v = 1 + \beta.
$$

And using (8.8) yields

$$
1 + e_1 = (1 + \beta) (1 + e).
$$

Hence, a $\beta\%$ prolongation of the working day produces a $\beta\%$ increase in the surplus value factor.

From the first equation of (10.3) we get

$$
q_1^Q = [q^L c_L + (1 + \alpha) q^K c_K] (I - A)^{-1}.
$$

Hence, multiplying by $\ell$ and using the second equation of (10.3) yields

$$
\beta \ell q^L = \alpha q^K c_K v.
$$

From this we get the percent increase in the consumption basket of capitalists:

$$
\alpha = \beta \frac{q^L}{q^K c_K v} = \beta \frac{q^L c_L v + q^K c_K v}{q^K c_K v} = (1 + e) \beta.
$$

Hence, the rate of increase of the capitalists’ consumption basket is not only positive, but greater than the rate of increase of the working day (by a percentage that is given by the initial rate of surplus value).

Summing up, once there is CCP a prolongation in the working day without an increase in the consumption of wage workers produces an increase in the production of all commodities as well as an increase in the rate of surplus value that allows capitalists to increase their consumption. This is due to the fact that wage workers work more in a day and consume less per hour worked.

**The generation of relative surplus value**

Marx holds that in the beginning of the capitalist mode of production “capital subordinates labour on the basis of the technical conditions in which it historically finds it. It does not, therefore, change immediately the mode of production” (B1, 314), limiting itself to generating surplus value by the prolongation of the working day. In a more advance stage, however, capital needs to generate ‘relative surplus value’ by reducing
the necessary labor time, that is, the portion of the working day that is necessary to produce the commodities consumed by the worker. This is made possible by the transformation of the “technical and social conditions of the process” of production which, by increasing labor productivity, diminishes the value of labor power:

Hitherto in treating of surplus value, arising from a simple prolongation of the working day, we have assumed the mode of production to be given and invariable. But when surplus value has to be produced by the conversion of necessary labour into surplus labour... The technical and social conditions of the process, and consequently the very mode of production must be revolutionised, before the productiveness of labour can be increased. By that means alone can the value of labour power be made to sink, and the portion of the working day necessary for the reproduction of that value, be shortened (B1, 320).

To formally represent this process, we start from systems (8.1) and (8.6) and develop two successive exercises. In both the introduction of a technological or organization innovation makes the labor per unit of output fall, i.e., makes labor productivity increase, which increases the rate of surplus value and hence capitalists’ consumption. Because consumptions baskets are exogenous in the methodology we are dealing with, if there is an exogenous change in labor productivity we can make alternative assumptions on the possible changes in the consumption baskets while preserving a consistent model of SR in CCP.

First exercise: no change in wage workers’ consumption basket. In the first exercise we assume that the innovation that increases labor productivity generates an increase in the consumption of capitalists while the consumption of workers remains constant (per day and per hour, since there is no change in the length of the working day). This exercise is similar to the generation of absolute surplus value in the fact that there is no change in the daily consumption of workers. To make things easy, we assume that the innovation makes the direct labor requirements in all branches of industry diminish proportionally, that is, all the elements of vector \( \ell \) fall by \( \beta \% \). As above, \( q^L \) is the worker population. To simplify, we assume that the consumption basket of capitalists increases proportionally by \( \alpha \% \), the value of which is determined below.

After the innovation, the systems of quantities and values are the following:

\[
\begin{bmatrix}
q_1^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A \\
\frac{1}{1+\beta} \ell \\
0
\end{bmatrix}
= \begin{bmatrix}
q_1^Q \\
q^L
\end{bmatrix},
\]

\[
\begin{bmatrix}
A \\
(1+\alpha) \frac{1}{1+\beta} \ell \\
0
\end{bmatrix}
\begin{bmatrix}
v_1 \\
1
\end{bmatrix}
= \begin{bmatrix}
v_1 \\
1
\end{bmatrix},
\]

Notice that the two individual equations of (10.10) are the same as those of (10.3)—that is, those dealing with the generation of absolute surplus value— notwithstanding the very different assumptions made. Hence, we again obtain (10.7), (10.8), and (10.9). There is a substantial change in system (10.11), however, from which we infer that the new vector of values diminishes proportionally at the same rate as \( \ell \):

\[
v_1 = \frac{1}{1+\beta} (I-A)^{-1} \ell = \frac{1}{1+\beta} v.
\]
Introducing this in the second equality of (10.11) again yields (10.5) and hence (10.6). Therefore, the first equality of (10.10) is identical to the first of (10.3), due to which (10.7) is again obtained. And applying the second equality of (10.10) again yields (10.8) and hence (10.9).

In summary, in this first exercise in ‘relative surplus value’ generation we get the same increase in the consumption basket of capitalists as in the exercise of ‘absolute surplus value’ generation. But for Marx the generation of ‘relative surplus value’ is the revolutionary one since there is a change in the mode of production itself that increases labor productivity.

Before abandoning this exercise, we can use it to check on some the regularities that Marx highlighted on the effects of an increase in the productive force of labor. The value of the net product is the value of labor power plus the surplus value (which is a consequence of the first equation of (8.1) \( q^Q (I - A) = q^L c_L + q^K c_K \) if it is all multiplied by \( v \)). Assuming that the length of the working day, the intensity of labor, and the consumption basket of workers, \( c_L \), are all constant, Marx formulates the following ‘laws’ (under the general assumption of Book I that all commodities are exchanged according to their values), which he attributes to Ricardo:

1. A working day of given length always creates the same amount of value, no matter how the productiveness of labour, and, with it, the mass of the product, and the price of each single commodity produced, may vary.

2. Surplus value and the value of labour power vary in opposite directions. A variation in the productiveness of labour, its increase or diminution, causes a variation in the opposite direction in the value of labour power, and in the same direction in surplus value.

3. Increase or diminution in surplus value is always consequent on, and never the cause of, the corresponding diminution or increase in the value of labour power (B1, 520-22).

If we look at the decomposition of the wage population (and total work realized) before and after the assumed change in the productivity of labor we can verify the veracity of these three ‘laws’:

\[
q^L = q^L c_L v + q^K c_K v
\]
\[
q^L = q^L c_L v_1 + q^K (1 + \alpha) c_K v_1.
\]

Due to the assumption that there is no change in the length of the working day, 1) \( q^L \), that is, the amount of value generated, is the same before and after the increase in the productive force of labor, 2) any reduction (increase) in the value of labor power (from \( q^L c_L v \) to \( q^L c_L v_1 \)) generates an increase (reduction) in the amount of surplus value (from \( q^K c_K v \) to \( (1 + \alpha) q^K c_K v_1 \)), and 3) since the surplus value is the excess of the amount of value generated over the value of labor power, it can only change if the value of labor power changes. Since a basic assumption here is that the consumption basket of workers remains constant (at \( c_L \)), the reduction in the value of labor power is due exclusively to the fact that the increase in the productive force of labor makes the value of that basket fall. In contrast, the increase in surplus value is due to the fact that the expansion in the (physical) consumption basket of capitalists surpasses the reduction in the value of its elements.

---

\(1\) Notice that \((1 + \alpha) v_1 = (1 + \alpha) \frac{1}{1 + \beta} v > v\), where the last inequality is given by (10.9).
Second exercise: both consumption baskets have the same percentage increase  According to Marx, when capitalists introduce technological innovations their objective is to increase the rate of surplus value. Once they are introduced, it is evident that in principle they allow for the per capita increase of all the population, not only that of capitalists as in the preceding exercise. In an alternative exercise we here assume that the consumption baskets of capitalists and workers have the same percentage increase. The following quote shows that this is not a case that Marx ignored, since he specifically mentions the two simple cases we are representing:

The value of labour power is determined by the value of a given quantity of necessaries. It is the value and not the mass of these necessaries that varies with the productiveness of labour. It is, however, possible that, owing to an increase of productiveness, both the labourer and the capitalist may simultaneously be able to appropriate a greater quantity of these necessaries, without any change in the price of labour power or in surplus value... The only result would be that each of them would represent twice as many use values as before; these use values being twice as cheap as before (B1, 523).

In the following exercise both consumption baskets increase in the same proportion as the increase in the productivity of labor. Starting from (8.1) and (8.6), after the increase in labor productivity the new systems of quantities and values are the following:

\[
\begin{bmatrix}
q_1^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A \\
\frac{1}{1+\beta} c_L \\
\frac{1}{1+\beta} c_K
\end{bmatrix}
\begin{bmatrix}
1 + \beta \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
q_1^Q & q^L
\end{bmatrix}.
\]

(10.14)

\[
\begin{bmatrix}
A \\
\frac{1}{1+\beta} c_L \\
\frac{1}{1+\beta} c_K
\end{bmatrix}
\begin{bmatrix}
v_1 \\
0 \\
1
\end{bmatrix}
= \begin{bmatrix}
v_1 \\
1
\end{bmatrix}.
\]

(10.15)

From the first equation in (10.14) we get \(q_1^Q = (1 + \beta) q^Q\), showing that the new technology increases the gross production of all commodities as well as the per capita consumption of capitalists and workers at the same rate as the direct labor requirements fall. In the values system the first equation yields the same reduction in values \(v_1 = v / (1 + \beta)\) we had in the first version of this exercise, and the second yields

\[(1 + e_1) (1 + \beta) c_L v_1 = (1 + e_1) c_L v = 1,
\]

from which, due to (8.8), it follows that there is no change in the rate of surplus value: \(e_1 = e\).

Of course, many other exercises of this type are possible, since the consumption baskets can only be moved exogenously. But with these two we have illustrated the

---

2In chapter 17 ("Changes of magnitude in the price of labour power and in surplus value") Marx explicitly introduces some flexibility into his normal assumption of Book I that all transactions are done according to values, allowing in certain circumstances that the price of labor power be above (but never below) its value. He does this to allow 'class struggle' to have an effect. When he assumes the case in which, due to an increase in the productive force of labor, the value of labor power falls, he allows wages to fall less than the value of labor power, under the allegation that "The amount of this fall... depends on the relative weight, which the pressure of capital on the one side, and the resistance of the labourer on the other, throws into the scale" (B1, 523).

But this flexibility is introduced in an imprecise way, which is evident when he refers to the price of labor power as a complement of surplus value (instead of profit). In the following section we more appropriately develop the same two exercises using the system of production prices, wages and profit rate.
essence of what Marx had in mind when he considered the generation of relative surplus value and tried to make workers aware that their efforts to maintain their share in the pie of the value of net output generated made sense.

In this section we have presented some of the models Marx developed in Book I concerning the efforts of the capitalist class to increase the rate of surplus value by means of a) the prolongation of the working day and b) the introduction of technological or organizational innovations that increase the productive power of labor. And we have found that Marx’s models of absolute and relative surplus value generation have logical and algebraic consistency.

The generation of absolute and relative profits
Marx developed his concepts of absolute and relative surplus value in Book I, in which (almost) all valuation is done in terms of values. But in Parts I and II of Book III (where he continues to abstract away the rent on land) Marx shows that in CCP the equilibrium market prices are the ‘production prices’ and that these in general differ (in their structure) from values. Hence, it is fair to ask whether the models of absolute and relative surplus value developed in the previous section have formal consistency if the valuations are done using production prices, wages and profit rate instead of values and the rate of surplus value. In this section we show that they do.

The generation of ‘absolute profits’
We start from systems (8.1) and (8.17), and introduce a prolongation of the working day. We normalize prices with $w = 1$. After the extension of the working day the new quantities system is (10.3) as before but the production prices system is:\footnote{If we compare this system to (10.4) we see that the only change in the social matrix is that there is a different way of measuring the slack (in the matrix with $c_1 = \partial_1 = 0$) with respect to a social matrix with dominant eigenvalue equal to one. The first uses the production processes to measure that slack and the second uses the reproduction process.}

$$
\begin{align*}
\begin{bmatrix}
\frac{1 + \rho_1}{1 + \beta} & A \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
p_1 \\
1
\end{bmatrix}
= 
\begin{bmatrix}
p_1 \\
1
\end{bmatrix}.
\end{align*}
$$

(10.16)

Instead of (8.20), the vector of production prices is $p_1 = B(\rho_1) \ell$. Premultiplying by $c_L/(1 + \beta)$ and taking into account the second equality of (10.16)) yields:

$$
1 + \beta = c_L B(\rho_1) \ell
$$

(10.17)

(instead of (8.22)). Since $B(\rho)$ is strictly increasing, the global profit rate must have increased ($\rho_1 > \rho$). Hence, all the production prices (measured in terms of wages) increase to $p_1 = B(\rho_1) \ell > B(\rho) \ell = p$. This implies that there is a reduction in the real wage (or the wage’s purchasing power). On the other hand, the value of $\alpha$ that is compatible with the prolongation of the working day is still determined by (10.8) or (10.9), since, as we have seen, the values do not change with this exercise (in contrast to the generalized change in the production prices) and are well defined mathematically.

In summary, the generation of ‘absolute profits’ is the process by means of which the rate of profit increases as a consequence of the prolongation of the working day with no change in workers’ daily consumption basket. The increase in the profit rate affects the production prices. If prices are measured in terms of the wage rate all the production prices increase, which necessarily implies a fall in the real wage.
The generation of ‘relative profits’

Starting again from systems (8.1) and (8.17) we assume that capitalists introduce innovations that proportionally lower the direct labor requirements in all processes. Consider first the case in which there is no change in the consumption basket of workers. The new systems of quantities and production prices and wages are, respectively, (10.10) and

\[
\begin{bmatrix}
(1 + \rho_1) A & (1 + \rho_1) \frac{1}{1+\beta} \\
\ell & 0 
\end{bmatrix}
\begin{bmatrix}
p_1 \\
1
\end{bmatrix} =
\begin{bmatrix}
p_1 \\
1
\end{bmatrix}
\]

(which should be compared with (10.11)). In this case the first equality yields \( p_1 = B (\rho_1) \ell / (1 + \beta) \). Premultiplying by \( c_L \) and using the second equality again yields (10.17), so we can use the same argument as in the case of ‘absolute profits’ to state that the profit rate must rise. Hence, dividing all the direct labor requirements by \( 1 + \beta \) has the same effect on the profit rate as multiplying the working day by \( 1 + \beta \). But the effect on the production prices is different since, although \( B (\rho) \) increases in the same magnitude in both cases, in the generation of ‘relative profits’ \( \ell \) diminishes. Also, due to the chosen numéraire \( c_L p = c_L p_1 = 1 \), so if the production prices of some commodities in the consumption basket of workers fall, others must rise to compensate (and vice versa).

Consider now the case in which the consumption baskets of capitalists and workers both increase proportionally. The quantities system becomes (10.14) and the system of production prices:

\[
\begin{bmatrix}
(1 + \rho_1) A & (1 + \rho_1) \frac{1}{1+\beta} \\
(1 + \beta) c_L & 0 
\end{bmatrix}
\begin{bmatrix}
p_1 \\
1
\end{bmatrix} =
\begin{bmatrix}
p_1 \\
1
\end{bmatrix}.
\]

Since the first equality is the same as when the consumption basket of workers does not change, it again yields \( p_1 = B (\rho_1) \ell / (1 + \beta) \). But using the second equality now gives \( 1 = (1 + \beta) c_L p_1 = c_L B (\rho_1) \ell \), which implies \( p_1 = \rho \), that is, there is no change in the profit rate, just as when we used the values system there was no change in the rate of surplus value.

In summary, the generation of ‘relative profits’ is a process in which an increase in the productive force of labor increases the global profit rate. If workers manage to have capitalists increase the consumption basket they can purchase with their wage, the increase in the profit rate will be less, and if the proportional increase in their consumption basket is sufficiently high the profit rate does not increase at all and can even fall.

The conclusion of this section is that Marx’s models of the generation of absolute and relative surplus value are formally robust to the use of the production prices in the valuation of commodities.

The problem with the exogeneity of consumption baskets

In Chapter 6 we studied the effects of a technological or organizational progress on the systems of quantities and values in the context of SCP assuming that some of the coefficients of \( A \) and/or \( \ell \) diminished. In this section we make a similar, albeit somewhat more specific, analysis in the context of CCP. For simplicity we assume that the change only affects \( A \) and that it happens in only one branch of industry. Assume that, starting from a situation depicted by systems (8.34) and (8.17), there is a technological innovation in branch \( i \) which has the effect of reducing one or more of the coefficients of row \( i \) of matrix \( A \). Hence, we again have \( A' \leq A \). Let us assume
that matrix $A'$ remains indecomposable, and that prices are normalized so that $w = 1$
before and after the change. Before production prices change (through flows of capital
and capitalists between branches) the production price system becomes:

$$
\begin{bmatrix}
(1 + \hat{\rho}) A' & (1 + \hat{\rho}) \ell \\
\frac{p}{c_L} & 0
\end{bmatrix}
\begin{bmatrix}
p' \\
1
\end{bmatrix}
= 
\begin{bmatrix}
p \\
1
\end{bmatrix},
$$

where $\hat{\rho}$ is a diagonal matrix with the profit rates of the various branches on the main
diagonal, and where the profit rate of the branch $i$ in which the innovation took place
is $\rho^*_i > \rho$. The initial increase in $\rho_i$ simply compensates for the fall of some of the
elements of the $A_i$ and reflects the extra profits in that branch. But this situation
cannot last since the greater profit rate in branch $i$ creates an incentive for the flow of
capital and capitalists (on the assumption that all capitalists have the same amount
of capital and specialize in a single branch) towards that branch from other branches,
i.e., that those that still have the profit rate $\rho$. These flows produce a tendency towards
a new equalization of profit rates, i.e., the fall in $\rho^*_i$ and the rise in the other profit rates
towards $\rho'$, which is hence reflected in the final production prices system (where there
are no more intersectoral flows of capital):

$$
\begin{bmatrix}
(1 + \rho') A' & (1 + \rho') \ell \\
\frac{p'}{c_L} & 0
\end{bmatrix}
\begin{bmatrix}
p' \\
1
\end{bmatrix}
= 
\begin{bmatrix}
p' \\
1
\end{bmatrix}.
$$

Let us focus on the effects that the innovation produces in the quantities system.
Since we assumed that the resulting matrix $A'$ is still indecomposable and $A' \leq A$,
the matrix of direct and indirect input requirements necessarily diminishes in all its
coefficients:

$$
0 < (I - A')^{-1} = I + A' + (A')^2 + \ldots < I + A + A^2 + \ldots = (I - A)^{-1}.
$$

This implies that all the values fall: $v' = (I - A')^{-1} \ell < (I - A)^{-1} \ell = v$. We initially
had (8.34), which as we have seen implies that the vector of gross productions
$q^Q = (q^L c_L + q^K c_K) (I - A)^{-1}$ (i.e., (8.2)). Assume for the moment that there is no change in $q^L$, $q^K$, $c_L$ or $c_K$. In that case we have $q^Q' = (q^L c_L + q^K c_K) (I - A')^{-1} < q^Q$.
But this leads to a contradiction, since it implies a fall in the labor force employed:
$q^Q' \ell < q^Q \ell = q^L$. If there is to be no reduction in labor employment it is necessary
that one or both of the consumption baskets $c_L$ and $c_K$ increase. If these become $c'_L$
and $c'_K$, for labor employment to remain at the initial level $q^L$ we must have

$$
q^{Q'} \ell = (q^L c'_L + q^K c'_K) v' = (q^L c_L + q^K c_K) v = q^Q \ell = q^L.
$$

That is, for employment to remain at the initial level it is necessary that the con-
sumption baskets increase sufficiently to compensate for the fall in the values of their
components. Assuming that $c'_L$ and $c'_K$ satisfy this equation, the new system of quan-
tities is:

$$
\begin{bmatrix}
q^{Q'} & q^L & q^K \\
c'_L & 0 & 0
\end{bmatrix}
\begin{bmatrix}
A' \\
c'_L \\
c'_K
\end{bmatrix}
= 
\begin{bmatrix}
q^{Q'} \\
q^L
\end{bmatrix}.
$$

Marx’s analysis sometimes contained assumptions of the ceteris paribus type, which
we used above only to find that it led to an absurd. As we will see, sometimes he
made assumptions in line with, for example, a movement in the real wage towards the
reestablishment of an equilibrium situation, without actually assuming this equilibrium
was ever reached. In Marx’s analysis the real movement always commands: the model is always subordinate to empirical and historical reality. But the exercise above (along with Numerical Exercise #3 in the Appendix to this chapter) makes evident something that can be considered a weakness in Marx’s (and the Classics’) treatment of the demand for goods and the availability (or supply) of labor power (or land). Since these are introduced as exogenous (at least in the models more formally explicit), when there are exogenous changes in other basic data, as in the case of an increase in labor productivity, the compensating changes are not determined by the model and must also be introduced exogenously. We will see in Chapters 11, 12 and 13 that it is easy to make the pertinent modifications in these models in order to represent a great deal of Marx’s analysis on the effects of disequilibria in the real wage, the profit rate, hoarding, and unemployment, by making endogenous the level of workers’ consumption while maintaining the structure of their consumption exogenous. Although we may recognize that Marx’s modeling approach in this is defective, the importance of this deficiency must not be exaggerated since—with the indicated modification—the model may be used to represent the phases of an industrial cycle and its effects on unemployment as based on the hoarding and/or profit reinvestment decisions of capitalists, a stance that several decades later came to be known as ‘Keynesian’.

We will see in Chapter 17 that León Walras found a very ingenious way of introducing the missing determinations through the modeling of individual preferences using what later came to known as utility functions and his theory of ‘general equilibrium’ under ‘perfectly free competition’. But Walras also had to strongly simplify the theoretical representation of reality by focusing (almost) exclusively on situations of general ‘equilibrium’ (along with a highly artificial ‘groping’ process leading to it), which differs greatly from Marx’s methodology, since for the latter the notion of ‘equilibrium’ was a mere auxiliary concept in the analysis. If, for example, an exogenous change initially produces labor unemployment, Walras’ theory (which is as static as the models of Marx we have been considering—although Marx’s theory is essentially dynamic and historical) can show how the endogenous variables must change so that equilibrium again prevails. But if in reality the recovery of full employment can take five or ten years, a theoretical representation that only allows us to consider the final (and hypothetical) new general equilibrium is of limited usefulness. The lack of realism derived from the absence of a theoretical treatment of the dynamics of ‘disequilibrium’ compensates for much of the advantage gained from Walras’ theory in being able to endogenize variables like consumption demand (both its level and its structure) and the supply of labor (or land) through an ingenious theory of decisions based on subjective preferences and objective constraints. More than a century of efforts to surpass Walras’ general framework still shows how difficult it is to achieve this within a rigorous framework, even after having found ways to model the intertemporal dynamics and stochasticity of variables. Keynes in The General Theory of Employment, Interest, and Money chose to cling to the reality of depression with high unemployment and excess capacity of the 1930s even if it meant constructing an extremely ad hoc theory. He was anticipated (and in good measure surpassed) by the Pole Michal Kalecki with his models, his detailed analyses of the Polish economy, and his evident knowledge of Marx’s theory (that he did not always make explicit).

---

[4] The first in this topic was the Prussian Hermann Gossens (in 1854) and a few years later, and independently (and almost simultaneously with Walras) the Englishman Stanley Jevons and the Austrian Karl Menger.

[5] Assuming, evidently, existence and uniqueness of general equilibrium, which became favorite themes for legions of mathematical economists and economically minded mathematicians.

Luxury consumption commodities

Marx correctly observes that the value of labor power \((c_L v)\) falls when there are productivity increases that diminish the value of the consumption basket of workers \((c_L)\):

In order to effect a fall in the value of labour power, the increase in the productiveness of labour must seize upon those branches of industry whose products determine the value of labour power, and consequently either belong to the class of customary means of subsistence, or are capable of supplying the place of those means. But the value of a commodity is determined, not only by the quantity of labour which the labourer directly bestows upon that commodity, but also by the labour contained in the means of production... Hence, a fall in the value of labour power is also brought about by an increase in the productiveness of labour, and by a corresponding cheapening of commodities in those industries which supply the instruments of labour and the raw material that form the material elements of the constant capital required for producing the necessaries of life (B1, 320).

Marx further correctly observes that, in contrast, an increase in productivity “in those branches of industry which supply neither the necessaries of life, nor the means of production for such necessaries, leaves the value of labour power undisturbed” (Ibid.). This points to situations in which, for example, luxury consumption goods exist that are only consumed by capitalists and are not directly or indirectly used for the production of the commodities workers consume. Hence productivity increases in such branches do not affect the value of the consumption basket of workers (i.e., the value of labor power). The same is valid for any means of production that are only used in the production of such luxury goods. Up to now, we have eliminated these possibilities through the simplifying assumption that the input-output matrix is indecomposable (before and after any change in its coefficients). We will see in this section that it is necessary to eliminate this assumption if we want to also represent such luxury goods.

Marx in general distinguished two major departments of industry, each one of which included multiple branches. In his words:

The total product, and therefore the total production, of society may be divided into two major departments:

I. Means of production, commodities having a form in which they must, or at least may, pass into productive consumption.

II. Articles of consumption, commodities having a form in which they pass into the individual consumption of the capitalist and the working class (B2, 394).

Marx also clearly distinguished luxury goods within Department II:

Category II of the annual production of commodities consists of a great variety of branches of production, which may, however, be divided into two great sub-divisions by their products:

a) Articles of consumption, which enter into the consumption of the working class, and, to the extent that they are necessities of life—even if frequently different in quality and value from those of the labourers—also form a
portion of the consumption of the capitalist class. For our purposes we may call this entire sub-division consumer necessities...

b) Articles of luxury, which enter into the consumption of only the capitalist class (B2, 402).

**Luxury goods in the system of values and surplus value**

To show how the existence of ‘luxury’ consumption goods can make the social matrix decomposable let us put the produced commodities in two sets: in set \( M \) are the means of production used to produce all commodities (including ‘consumer necessities’), and in set \( L \) are the ‘luxury’ consumption goods, which are only consumed by capitalists and which we assume are not required as inputs in the production of the commodities in \( M \) although they could be required as inputs for the production of other luxury goods. The system of values is then:

\[
\begin{bmatrix}
A_{MM} & 0 & \ell_M \\
A_{LM} & A_{LL} & \ell_L \\
(1 + \epsilon) c_L & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v_M \\
v_L \\
1
\end{bmatrix}
= 
\begin{bmatrix}
v_M \\
v_L \\
1
\end{bmatrix},
\]  

(10.18)

where \( A_{ML} = 0 \) since luxury goods are not used in the production of the goods in \( M \), and where the workers’ consumption basket \( c_L \) does not include luxury goods.

The social matrix of this system is decomposable, since if the last two rows are permuted and also the last two columns, which merely means putting luxury goods last in the numbering of all commodities (including labor power), the system becomes:

\[
\begin{bmatrix}
A_{MM} & \ell_M & 0 \\
A_{LM} & A_{LL} & \ell_L \\
(1 + \epsilon) c_L & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v_M \\
v_L \\
1
\end{bmatrix}
= 
\begin{bmatrix}
v_M \\
v_L \\
1
\end{bmatrix}.
\]

Comparing with (5.18) we immediately see that the social matrix is decomposable due to the block of zeros above de \( A_{LL} \).\(^7\) This block makes it possible to separate the system into the following two subsystems:

\[
\begin{bmatrix}
A_{MM} & \ell_M \\
(1 + \epsilon) c_L & 0
\end{bmatrix}
\begin{bmatrix}
v_M \\
1
\end{bmatrix}
= 
\begin{bmatrix}
v_M \\
1
\end{bmatrix}
\]

\[
A_{LM} v_M + \ell_L + A_{LL} v_L = v_L.
\]

From the main subsystem, which excludes luxury goods, we obtain, as in (8.7) and (8.8), \( v_M = (I - A_{MM})^{-1} \ell_M \) and \( (1 + \epsilon) c_L v_M = 1 \). This is the subsystem we have previously considered, since we have been implicitly discarding luxury goods by assuming indecomposability. And from the last subsystem we get \( v_L = (I - A_{LL})^{-1} [A_{LM} v_M + \ell_L] \). We have hence verified Marx’s assertion quoted above that “an increase in the productiveness of labour in those branches of industry which supply neither the necessaries of life, nor the means of production for such necessaries, leaves the value of labour power undisturbed.” That is, if some of the coefficients if \( A_{LM} \) or \( \ell_L \) fall, this does not affect \( c_L v_M \), and only affects the values of luxury goods \( v_L \). Moreover, neither is the rate of surplus value affected by changes that only affect the values of luxury goods.

\(^7\) Notice that the matrix could cease to be decomposable if \( A_{ML} \) were different from zero or, alternatively, if the workers’ consumption basket contained luxury goods, since these are precisely the components of the block of zeros above \( A_{LL} \). This is a possibility that Marx took into account since, as we have seen, he held that during the boom in the industrial cycle it was usual for workers (in England) to be able to consume some luxury goods.
Luxury goods in the system of quantities

The system of quantities that corresponds to (10.18) is the following:

\[
\begin{bmatrix}
q^Q_M & q^Q_L & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A_{MM} & 0 & \ell_M \\
A_{LM} & A_{LL} & \ell_L \\
c_M & 0 & 0 \\
c_K & c_{KL} & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q_M & q^Q_L & q^L
\end{bmatrix},
\]

where the vectors of gross output of luxury goods and all the rest are \(q^Q_L\) and \(q^Q_M\) and we assume that capitalists consume both ‘necessities’ \(c_K\) as luxury goods \(c_{KL}\). If we permute columns 2 and 3 and also rows 2 and 3, the system becomes

\[
\begin{bmatrix}
q^Q_M & q^L & q^Q_L & q^K
\end{bmatrix}
\begin{bmatrix}
A_{MM} & \ell_M & 0 \\
A_{LM} & \ell_L & A_{LL} \\
c_M & 0 & 0 \\
c_K & c_{KL} & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q_M & q^L & q^Q_L
\end{bmatrix}.
\]

The block of zeros above \(A_{LL}\) again allows us to solve the system recursively. For this we open the system into two parts:

\[
\begin{bmatrix}
q^Q_M & q^L \\
\frac{1}{c_L} & 0
\end{bmatrix}
\begin{bmatrix}
A_{MM} & \ell_M \\
c_M & 0
\end{bmatrix} + \begin{bmatrix}
q^Q_L & q^K \\
\frac{1}{c_K} & 0
\end{bmatrix}
\begin{bmatrix}
A_{LM} & \ell_L \\
c_K & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q_M & q^L \\
\frac{1}{c_L} & 0
\end{bmatrix} + \begin{bmatrix}
q^Q_L & q^K \\
\frac{1}{c_K} & 0
\end{bmatrix}
\begin{bmatrix}
A_{LL} & \ell_L \\
c_K & 0
\end{bmatrix} = q^Q_L.
\]

From the second part we can obtain the outputs of luxuries autonomously: \(q^Q_L = q^K c_{KL} (I - A_{LL})^{-1}\). And this vector can be then used to solve for the outputs of commodities \(M\) from the two equations of the first part (using the fact that \(q^K\) is exogenous): \(q^Q_M = (q^L c_M + q^K c_{KL} + q^Q_L A_{LM}) (I - A_{MM})^{-1}\) and \(q^Q_M \ell_M + q^Q_L \ell_L = q^L\), where the last equality simply shows how total labor power is allocated between the two large departments. Hence, while the values of the luxury goods (of the special type we have been considering) are obtained using the values of the rest of the commodities, the quantities produced of the rest of the commodities are obtained using the quantities of luxury goods produced.

Luxury goods in the system of production prices and wages

Introducing luxury consumption goods in the system of production prices and wages we get:

\[
\begin{bmatrix}
(1 + \rho) A_{MM} & 0 & (1 + \rho) \ell_M \\
(1 + \rho) A_{LM} & (1 + \rho) A_{LL} & (1 + \rho) \ell_L \\
c_M & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_M \\
p_L \\
w
\end{bmatrix}
= \begin{bmatrix}
p_M \\
p_L \\
w
\end{bmatrix}.
\]

If we permute rows and columns 2 and 3 we again see that the social matrix is decomposable:

\[
\begin{bmatrix}
(1 + \rho) A_{MM} & (1 + \rho) \ell_M & 0 \\
(1 + \rho) A_{LM} & (1 + \rho) \ell_L & (1 + \rho) A_{LL} \\
c_M & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p_M \\
p_L \\
w
\end{bmatrix}
= \begin{bmatrix}
p_M \\
p_L \\
w
\end{bmatrix}.\]
Hence we can again separate the system into the following two subsystems:

\[
\begin{bmatrix}
(1 + \rho) A_{MM} & (1 + \rho) \ell_M \\
\ell_L & 0
\end{bmatrix}
\begin{bmatrix}
p_M \\
p_L
\end{bmatrix}
= \begin{bmatrix}
p_M \\
p_L
\end{bmatrix} = \begin{bmatrix}
p_M \\
p_L
\end{bmatrix} = \begin{bmatrix}
p_M \\
p_L
\end{bmatrix}
\]

(10.20)

\[
(1 + \rho) [A_{LM} p_M + \ell_L w + A_{LL} p_L] = p_L.
\]

(10.21)

From the first equality of (10.20) we get \( p_M = B_{MM} \rho \ell_M w \), where \( B_{MM} \) is defined as in (8.21) substituting \( A_{MM} \) for \( A \). Also, proceeding as we did to get (8.22) yields \( c_L B_{MM} \rho \ell_M = 1 \), which determines the global profit rate \( \rho \) and shows that luxury goods do not contribute to its determination. From (10.21) we get \( p_L = [A_{LM} p_M + \ell_L w] B_{LL} \rho \), where \( B_{LL} \rho \) is defined using \( A_{LL} \) instead of \( A \). Finally, the absolute levels of production prices and wages depend on the choice of numeraire. If we use \( w = 1 \), the levels of production prices and wages are determined. If, alternatively, we use the consumption basket of capitalists as numeraire, we have \( c_K p_M + c_K p_L = 1 \). Hence, using the expressions obtained above for the price vectors, the wage rate is given by the complicated expression

\[
w = 1/[c_K B_{MM} \rho \ell_M + c_K (A_{LM} B_{MM} \rho \ell_M + \ell_L) B_{LL} \rho],
\]

which can be used to get the levels of \( p_M \) and \( p_L \).

In summary, luxury goods—as we have defined them—do not participate at all in the determination of the profit rate or the rate of surplus \textbf{value}. Neither do they participate in the determination of the production prices of the rest of the commodities if the wage rate is the numeraire. Instead, if luxury goods participate in the numeraire they also participate in the determination of the production prices of all the commodities. But this considerably complicates the formulas, and so it is usually convenient to avoid using the luxury goods in the numeraire. Finally, luxury goods do play an important role in the determination of the quantities produced of the rest of the commodities.

**Conclusion**

In this section we have seen how the existence of luxury goods makes the social matrix decomposable and hence leads to a recursive solution for \textbf{values} and the rate of surplus \textbf{value}, for production prices, wages and the profit rate, and also for the quantities produced. One can first get the global rate of surplus \textbf{value} and the \textbf{values} of non-luxury commodities and then, using these, get the \textbf{values} of the luxury commodities. Similarly, one can first get the global profit rate and the production prices of non-luxury goods and using these get the production prices of luxury commodities. Inversely, one can first obtain the quantities produced of luxury goods and then use these get the quantities produced of non-luxury goods. From here on, except otherwise specified, we assume that such special commodities that make the social matrix decomposable either do not exist or have been previously segregated so as to concentrate on the main part of the systems.

**Appendix to Chapter 10**

**Bibliographic Notes**

Ricardo and his search for “an invariable measure of value”  Starting from Section IV of the first chapter of Principles, Ricardo eliminates the assumption of equal cost compositions and admits that different industries can have different proportions of their capital invested in wages. In Section VI, titled “On an invariable measure of value” he focuses on the question of whether there can exist “some invariable standard

---

8 This section was inserted by Ricardo in the Third Edition of the book.
measure of value, which should itself be subject to none of the fluctuations to which other commodities are exposed” (Ricardo 2004, Vol. 1, 43), a matter he had briefly considered in Section III. In Section VI he states that such a standard measure cannot exist. For, even if one assumed that gold is always produced using the same total quantity of labor, its exchange value would be subject to the variations produced by the redistribution of income between workers and capitalists, as any other commodity. If there were an increase in the wage rate, for example, the exchange value of gold would rise with respect to commodities produced by industries that require a smaller proportion of its capital invested in wages. He concludes that “Neither gold then, nor any other commodity, can ever be a perfect measure of value for all things” (Ibid, 45).

At most, he states, gold could be a perfect measure of value for those commodities produced under identical circumstances, but not for the rest.

Nevertheless, Ricardo escapes from this impasse by adopting the additional assumption that gold is produced under the average conditions as the rest of the commodities, that is, with the same proportion of capital invested in wages as there is on average in the economy. He states that, with this assumption, “we shall probably possess as near an approximation to a standard measure of value as can be theoretically conceived” (Ibid.). For, on the one hand, he maintained the assumption that the labor-value of gold is invariable. And on the other, he was convinced that the variations in exchange value based on the redistribution of income were, in the short run, of a second order of importance. Finally, the assumption that gold is produced under the average conditions made the changes in the ‘natural prices’ of commodities due to changes in wages as small as possible.

However, admitting that gold cannot strictly be a perfect or invariable standard of value, Ricardo chose to make the assumption that it is:

To facilitate, then, the object of this enquiry, although I fully allow that money made of gold is subject to most of the variations of other things, I shall suppose it to be invariable, and therefore all alterations in price to be occasioned by some alteration in the value of the commodity of which I may be speaking (Ibid., 46).

Notice that, strictly, this assumption implied going back to his assumption of Section III (which we already mentioned in the third section of Chapter 4 of this book) that the various industries use means of production of equal value and durability and equal quantities of labor (a situation we characterized as having ‘equal cost compositions’ in all sectors). For, as long as the relative natural prices can vary due to changes in the distribution of income, it is not possible to speak with any precision of the invariability of the value of a commodity. In contrast, when there are equal cost compositions in all sectors, since the only causes of variation in the relative values of commodities are the variations in their labor-values, the invariability in the value of gold can have the precise meaning of the invariability in its labor-value. This is the assumption Ricardo made in Section III.

Armed with his theory of ‘natural prices’, that is, his theory of value, and the special assumptions adopted with respect to gold, Ricardo went on to the study of what he considered to be the “principal problem in Political Economy”: determining the laws that regulate the distribution of the social output between the “three classes of the community; namely, the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated” (Ibid., 5).
Piero Sraffa, Georg von Charasoff, and ‘basic’ commodities  With the help of professional mathematicians, Sraffa (1960) followed the path initiated by Ricardo of searching for a standard measure of values endowed with certain invariability features. For this he made use of matrix techniques related to connectivity (that is, matrix decomposability and indecomposability) and applying them to input-output matrices introduced the notion of basic commodities. He defines these as commodities that are directly or indirectly required for the production of all commodities. And he defined non-basic commodities as those that are not basic. Let us leave out for the moment the commodity labor power and concentrate on the rest. From what we have seen in Chapter 5, if A is indecomposable then all commodities are basic. But if A is decomposable as in (5.18) then if A_{11} is indecomposable (that is, cannot be put in the format of (5.18)), the commodities of branch 1 are all basic, while the commodities of branch 2 are all non-basic. A classical example of non-basic commodities is luxury goods. As we have seen in this chapter, to obtain the values of non-luxury commodities (which are basic) the values of luxury commodities (which are non-basic) are not needed, whereas to obtain the values of luxury goods (non-basic) it is necessary to have the values of non-luxury goods (basic).

Writing early in the 20th century on Marx’s theory, the Russian intellectual Georg von Charasoff (1902, 1910, 1912), anticipated by several decades the distinction between basic and non-basic commodities, even using the same terminology, and applying it to the enlarged matrix A + IcL, thus including labor power. Although he did not develop algebraic proofs and worked with numerical exercises, he was apparently the first to use Perron-Frobenius in the field of economics, although it is not known whether he had read the articles of Perron and Frobenius.9

Piero Sraffa and his ‘standard commodity’  The subtitle of Sraffa’s (1960) Production of commodities by means of commodities is “Prelude to a critique of Economic Theory.” In the Preface he states that his “set of propositions” were designed with the purpose of serving as a base for a critique of the “marginal theory of value and distribution” that either himself or someone “younger and better equipped for the task” could eventually attempt. In the book an important role is played by his ‘standard commodity’, inspired in Ricardo’s quest for a “perfect measure of value.” Sraffa recognizes that there can hardly exist an individual commodity whose price would not be affected by a change in the wage rate, since this would require not only that its production process have a cost composition equal to the economy-wide average, but also that the production processes of its inputs have the same property, as well as the production processes of their inputs, etc. Hence he looks for a basket of commodities that might have this property. For this, undoubtedly following the recommendation of his mathematically minded collaborators, Sraffa used the spectral properties of matrix A to formulate his ‘standard commodity’.

In the case in which none of the production processes have joint production of two or more commodities, which is the only case we are here interested in, Sraffa discards non-basic commodities (which do not contribute to the determination of the profit rate nor the –equilibrium– prices of basic commodities) and forms a very special aggregate of the production processes of basic commodities. For the aggregation of these production processes he uses the left eigenvector of A (which is here the part of the input-output matrix that only refers to basic commodities) associated with its dominant eigenvalue, conveniently normalized. Also, he assumes that wages are paid

at the end of the production process, that is, they are not advanced, which is the opposite extreme of much of Marx’s analysis prior to his general treatment when he addresses the turnover of capital (which we shall address in Chapter 12). Hence, since in Marx’s terminology there is no capital invested in wages, instead of (8.18) he has

\[ (1 + \rho) Ap + \ell w = p, \]

from which we get

\[ p = \left[ B(\rho) / (1 + \rho) \right] \ell w \] instead of (8.20). Since non-basic commodities have been left aside, A is indecomposable. It is also necessary that A be productive, i.e., \( \lambda(A) < 1 \), so that a surplus can be produced. Hence, there exists an \( \bar{R} > 0 \) such that \( \lambda(A) = 1 / (1 + \bar{R}) \). By Perron-Frobenius there exists a vector \( \bar{\eta} > 0 \) such that \( (1 + \bar{R}) \bar{\eta} A = \bar{\eta} \). And since \( \bar{\eta} \) is unique up to a scalar factor, it can be made unique by normalization. Sraffa found it convenient to normalize it in such a way that the hypothetical ‘standard system’ he uses to construct his numeraire employs all the available labor force, i.e., \( \bar{\eta} \ell = q^L \), where, if the original system contained non-basic commodities one should bear in mind that \( \ell \) only includes the direct labor requirements of the production processes that produce basic commodities (whereas \( q^L \) includes all workers).\(^{10}\)

All that remains is to normalize the vector of prices, that is, to choose a ‘numeraire’. Sraffa adopts as numeraire the net product of his ‘standard system’ divided by the total labor of the economy, that is \( \bar{\eta}(I - A) / q^L \), so that \( p \) satisfies

\[ \bar{\eta}(I - A) p = q^L. \]

Hence,

\[ q^L = \bar{\eta}(I - A) p = \frac{R}{1 + \bar{R}} \frac{1 + \bar{R}}{R - \rho} \bar{\eta} \ell w = \frac{R}{1 + \bar{R}} \frac{1 + \bar{R}}{R - \rho} q^L w, \]

where the second equality uses

\[ \bar{\eta} B(\rho) / (1 + \rho) = \bar{\eta} \left[ I + (1 + \rho) A + (1 + \rho)^2 A^2 + \ldots \right] = \left[ 1 + \left( \frac{1 + \rho}{1 + \bar{R}} \right) + \left( \frac{1 + \rho}{1 + \bar{R}} \right)^2 + \ldots \right] \bar{q} = \frac{1 + \bar{R}}{R - \rho} \bar{q}. \]

Therefore, (10.22) yields \( \rho = R / (1 + w) \), the inverse and linear relation between \( \rho \) and \( w \) that Sraffa (1960) reaches in his Chapter IV.

What is very problematic about Sraffa’s book is that he insists on working with an incompletely specified system. Except for Chapter 1 (which has 3 pages) on “Production for subsistence”, once he introduces in Chapter 2 the “Production with a surplus” he never again presents a ‘quantities system’ (as we always do).\(^{11}\) And this is especially problematic since he also does not specify the commodities involved in his wage rate \( w \), which is always treated as a parameter. He suggests that the appropriate way to proceed would be, on the one hand, to take a fixed basket that would represent “the goods necessary for the subsistence of workers” (Sraffa 1960, 10) and, on the other, another basket that would be variable and represent the labor’s “share of the surplus product.” But then he decides that he “shall follow the usual practice of treating the whole wage as variable” (Ibid.), which, he writes, involves the drawback of “relegating the necessaries to the limbo of non-basic products.” And he states that hence “improvements in the methods of production of the necessaries of life will no longer directly affect the profit rate and the prices of other products.” Let us see why this is not consistent.

Assume that in (10.19) the commodities of \( M \) are basic and those of \( L \) non-basic. We have \( \beta_{ML} = 0 \) because non-basic commodities are not inputs to basic commodities.

\(^{10}\) We could use a notation such as that of (10.20) but we avoid this to keep the notation as simple as possible.

\(^{11}\) Notice that Sraffa’s Standard System is an artificial construction and not a ‘quantities system’ that includes all quantities and populations in the model’s artificial society.
In (10.19) all the commodities consumed by workers are basic and that implies, as we have seen, that the coefficients of the processes producing non-basics do not contribute to the determination of the value of ρ. But Sraffa believes that this is what would happen if all the commodities consumed by workers were non-basic. And this cannot be so. If the third row of the social matrix of (10.19) were \((0 \ cL \ 0)\) instead of \((cL \ 0 \ 0)\), which is what Sraffa suggests, it can be checked that the social matrix would not be decomposable, and hence all of the commodities would contribute to the determination of ρ. Hence, what is in a ‘limbo’ is Sraffa’s methodology for the construction of a system of prices and wages in which the wage rate or the profit rate can be manipulated as an exogenous variable without having specified the system of quantities. Contrary to what he believed, if the commodities of the consumption basket of workers are all basic, then one can leave out the non-basic commodities and construct his ‘standard system’, with the important difference that one would be taking into account the commodities consumed by workers. The quantities vector that defines the ‘standard commodity’ would then be the left eigenvector associated with the dominant eigenvalue of \(A + \ell cL\) instead of \(A\). Abraham-Frois and Berrebi (1979) do precisely this (in their last chapter) when, assuming like Sraffa that wages are not advanced, they deduce a simple relation between ρ and \(\ell\): \(\rho = \ell / (1 + \pi)\), where \(\pi\) is the global value composition when the ‘standard commodity’ is used as numeraire.\(^{12}\) But if one wants the wage rate to be ‘variable’ (within the context of linear production processes, consumption baskets with an exogenous structure, and no joint-production), the practical and consistent way to proceed is as we do in Chapter 12 below through an exogenous parameter \(\omega\) that multiplies a fixed basket \(\hat{c}_L\). This way changes in \(\omega\) can be explained by capitalist hoarding and can be related to changes in unemployment. The industrial cycle and its relation to the labor market, however, are completely absent in the works of von Neumann and Sraffa, while they are central in the analyses of Capital.

**Numerical Exercise #3**

The increase in the productive power of labor (or productivity) plays a very important role in Marx’s theory. It affects the values and the rate of surplus value as well as the production prices, wages and profit rate. Here we follow up on Numerical Exercise #2 and assume that an innovation in branch 2 reduces its direct requirements of the output of branch 1 from 0.61 to 0.5 (per unit of output). Starting with the system of production prices (with homogeneous profit rates), we here decompose the total effect in two stages, as we did in the text. In the first, the innovation increases the profit rate in branch 2 without any change in prices (nor worker consumption baskets). The price system becomes:

\[
\begin{bmatrix}
(1 + 0.09065)0.1107 & (1 + 0.09065)0.45 & (1 + 0.09065)0.39 \\
(1 + 0.2095)0.5 & (1 + 0.2095)0.25 & (1 + 0.2095)0.31 \\
0.39 & 0.22 & 0
\end{bmatrix}
\begin{bmatrix}
1.106 \\
1.350 \\
0.728
\end{bmatrix}
= \begin{bmatrix}
1.106 \\
1.350 \\
0.728
\end{bmatrix},
\]

where we continue using the consumption basket of capitalists as numeraire. As we can see, the profit rate of branch 2 increases to 20.95% with no modifications in branch 1. But this situation cannot last since the extra profits in branch 2 attract capital from branch 1 until the profit rates are again equal, leading *ceteris paribus* to the following system:

\(^{12}\text{Abraham-Frois and Berrebi, however, are not critical of Marx (except in trivia such as his approximation to the production prices) nor of Sraffa. And they dedicate most of their effort and book to von Neumann’s and Sraffa’s treatment of joint production.}\)
\[
\begin{pmatrix}
(1 + 0.1522) 0.1107 & (1 + 0.1522) 0.45 & (1 + 0.1522) 0.39 \\
(1 + 0.1522) 0.5 & (1 + 0.1522) 0.25 & (1 + 0.1522) 0.31 \\
0.39 & 0.22 & 0
\end{pmatrix}
\begin{pmatrix}
1.118 \\
1.263 \\
0.714
\end{pmatrix}
= 
\begin{pmatrix}
1.118 \\
1.263 \\
0.714
\end{pmatrix}.
\]

Notice that the final profit rate is 15.22% in both branches, quite higher than the original (9.06%), the price of branch 2 fell 6.5%, while the price of branch 1 increased 1.1%, and the wage diminished 2%. The values system (8.50) becomes the following

\[
\begin{pmatrix}
0.1107 & 0.45 & 0.39 \\
0.5 & 0.25 & 0.31 \\
(1 + 0.625) 0.39 & (1 + 0.625) 0.22 & 0
\end{pmatrix}
\begin{pmatrix}
0.977 \\
1.065 \\
1
\end{pmatrix}
= 
\begin{pmatrix}
0.977 \\
1.065 \\
1
\end{pmatrix}.
\]

Both values fall and the rate of surplus value increases to 62.5% (from 39.5%).

So far the exercise has assumed that the consumption baskets \( c_L \) and \( c_K \) do not change. But this is problematic. To confirm this we must consider the quantities system. Assuming we still have SR after the innovation and that all the capitalist population continues investing its capital in the system, we must calculate the new vector \( \eta \). For this we use \( \eta = \rho (Ap + \ell w) \) with the new values of \( \rho, A, p, \) and \( w \), that is:

\[
\eta = 0.1522 \begin{pmatrix}
0.1107 & 0.45 & 0.39 \\
0.5 & 0.25 & 0.31 \\
(1 + 0.625) 0.39 & (1 + 0.625) 0.22 & 0
\end{pmatrix}
\begin{pmatrix}
1.118 \\
1.263 \\
0.71396
\end{pmatrix}
= \begin{pmatrix}
0.14772 \\
0.16685
\end{pmatrix}
\]

The new quantities system is the following (to be compared with (8.48)):

\[
\begin{pmatrix}
348.99 & 290.37 & 226.12 & 100 \\
348.99 & 290.37 & 226.12 & 100
\end{pmatrix}
= \begin{pmatrix}
0.1107 & 0.45 & 0.39 & 0.14772 \\
0.5 & 0.25 & 0.31 & 0.16685 \\
0.39 & 0.22 & 0 & 0 \\
0.77 & 0.11 & 0 & 0
\end{pmatrix}.
\]

The output of both branches falls strongly and the population of employed workers diminishes from 350 to 226 (35%). On the other hand, capitalists continue consuming the same as before, without them benefiting from the innovation in their consumption (nor in their accumulation). To avoid having absurd results such as these it is necessary to assume that there is, aside from the innovation, an increase in consumption. But how much does the consumption of workers increase, and how much that of capitalists? For Marx this depends both on market forces (in the labor and output markets) and ‘class struggle’, that is, the negotiating power of workers and capitalists.

Let us adopt the (arbitrary but simple) assumption that capitalists expand their consumption basket 65% with no change in that of workers. This leads to the following production prices and wage system (with numeraire \( c_K p = 1 \)):

\[
\begin{pmatrix}
(1 + 0.1522) 0.1107 & (1 + 0.1522) 0.45 & (1 + 0.1522) 0.39 \\
(1 + 0.1522) 0.5 & (1 + 0.1522) 0.25 & (1 + 0.1522) 0.31 \\
0.39 & 0.22 & 0
\end{pmatrix}
\begin{pmatrix}
0.678 \\
0.766 \\
0.433
\end{pmatrix}
= 
\begin{pmatrix}
0.678 \\
0.766 \\
0.433
\end{pmatrix}.
\]

Comparing this to (10.23), we see that the matrix is unchanged, and hence there is no change in the profit rate. Prices and wages, however, have had a strong reduction
since the consumption basket of capitalists, used as numeraire, has expanded 65%. Comparing with the situation prior to the innovation (8.49), we observe price reductions of 38.7% in branch 1 and 43.3% in branch 2, and a wage reduction of 40.5%. With these new prices and wages we can again calculate $\eta$ and hence the new quantities system:

$$\begin{bmatrix} 575.89 & 479.16 & 373.12 & 100 \end{bmatrix} \begin{bmatrix} 0.1107 & 0.45 & 0.39 & 0.089527 \\ 0.5 & 0.25 & 0.31 & 0.1012 \\ 0.39 & 0.22 & 0 & 0 \\ 0.77 (1.65) & 0.11 (1.65) & 0 & 0 \end{bmatrix} = \begin{bmatrix} 575.89 & 479.16 & 373.12 & 100 \end{bmatrix}.$$

Comparing this to the original data (8.48), there are increases in the quantities produced (5.6% in branch 1 and 7.8% in branch 2) and in the labor power employed (6.5%). The expansion in output generates the increase in worker employment and in the aggregate consumption of workers with no change in the per capita consumption basket. This obviously requires the previous existence of a sizable ‘industrial reserve army’. And less capital per unit of output is required than in (8.48) because there has been an increase in gross output.

Let us now assume that instead of a proportional increase in the consumption basket of capitalists there is a non-proportional increase in the consumption basket of workers. For example, there is a 37.8% increase in their consumption of the first commodity. Since this increase has been calculated so that the effect of the innovation on the profit rate is compensated, the new price system is:

$$\begin{bmatrix} (1 + 0.09065)0.1107 & (1 + 0.09065)0.45 & (1 + 0.09065)0.39 \\ (1 + 0.09065)0.5 & (1 + 0.09065)0.25 & (1 + 0.09065)0.31 \\ (1 + 0.378)0.39 & 0.22 & 0 \end{bmatrix} \begin{bmatrix} 1.121 \\ 1.248 \\ 0.877 \end{bmatrix} = \begin{bmatrix} 1.121 \\ 1.248 \\ 0.877 \end{bmatrix}.$$

Comparing this to (8.49) we see that there are increases in the wage rate (from 0.728 to 0.877) and in the price of commodity 1 (from 1.106 to 1.121) and a fall in the price of commodity 2 (from 1.350 to 1.248). In contrast, values fall (to the same levels as when it was the consumption of capitalists that expanded, except for rounding off errors), but the rate of surplus value falls (from 0.3946 in (8.50) to 0.3165) instead of increasing (from 0.3946 to 0.625 when it was the consumption of capitalists that increased):

$$\begin{bmatrix} 0.1107 & 0.45 & 0.39 \\ 0.5 & 0.25 & 0.31 \\ (1 + 0.3165) (1 + 0.378)0.39 & (1 + 0.3165)0.22 & 0 \end{bmatrix} \begin{bmatrix} 0.978 \\ 1.065 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.978 \\ 1.065 \\ 1 \end{bmatrix}.$$
Chapter 11 MARX’S CALCULATIONS AND HIS DEFENSE OF WORKERS

In this chapter we address some topics that we postponed because we decided it was important to first formulate Marx’s basic models in a mathematically correct way, one that does away with the approximations he made due to his lack of mathematical training and even the absence in his time of certain analytical instruments that are particularly convenient for expressing much of his theory. First we address the way in which Marx proceeded to calculate the global profit rate and production prices, and the implications these calculations have for his analyses. Then we look into the way Marx segmented the labor time of workers into the parts corresponding to necessary labor and to surplus labor, and his insistence that workers should counter capitalist’s efforts to lengthen the working day without a corresponding increase in their wages and strive towards the reduction of the working day so as to expand their free time, the ‘true realm of freedom’. We also address Marx’s acknowledgement that, because production prices were the equilibrium prices, the ‘law of value’ had little ‘direct’ validity in the capitalist economy. Finally, we show Marx’s specific formulation of his tables of Simple Reproduction and how they may easily be put in the matrix form that has been so convenient in the preceding chapters.

Marx’s calculation of the global rate of profit and production prices
When Marx determines his production prices he uses values for the valuation of both the elements of capital and profits (and hence surplus value). We believe the reason for this is that he lacked the mathematical training necessary to formulate two separate systems of equations: one for the production prices and profit rate and another for values and the rate of surplus value, as we did in the preceding chapters. He defines the global profit rate $\rho^0$ as the ratio between aggregate surplus value and global capital, which yields a simple relation between the rate of profit $\rho^0$ and the rate of surplus value $e$ that depends on the value composition of capital $\kappa$:

$$\rho^0 = \frac{S^v}{C^v + V^v} = \frac{e}{\kappa + 1},$$

(11.1)

where (using definitions (8.12)) $e$ and $\kappa$ are:

$$e = \frac{S^v}{V^v} = \frac{q^K c_K v}{q^L c_{L,v}}, \quad \kappa = \frac{C^v}{V^v} = \frac{q^Q A_v}{q^L c_{L,v}}.$$

Notice that Marx’s formula for the profit rate (11.1) only differs from the one derived from the mathematically precise formulation of his theory (8.25) in that the components of capital and the consumption basket of capitalists are valued in values instead of production prices. And if we choose as numeraire the consumption basket of capitalists deflated by its own value: $c_K/(c_K v)$ (so that $c_K p = c_K v$) we have $S^p = S^v$ and so

---

1In the Appendix to this chapter we show by means of a numerical exercise that his approximate calculations of these variables can be considered the first step of an algorithm that converges to the correctly calculated profit rate and production prices.
the only difference between the two formulas is that Marx values the components of capital using values instead of production prices.

Consistent with his definition of the profit rate, when Marx formulates his production prices he does so starting from the valuation of the components of capital according to values. Hence, his (approximate) production prices are the following:

\[ p^0 = (1 + \rho^0) (A + \ell c_L) v \]  

(11.2)

instead of (the correct ones) those of (8.16). Marx is explicit in that he first calculates the global profit rate in order to calculate production prices:

The prices which obtain as the [profit calculated by multiplying the cost prices by the] average of the various rates of profit in the different spheres of production added to the cost prices of the different spheres of production, constitute the prices of production. They have as their prerequisite the existence of a general rate of profit, and this, again, presupposes that the rates of profit in every individual sphere of production taken by itself have previously been reduced to just as many average rates. These particular rates of profit \( s/(c+v) \) in every sphere of production, and must, as occurs in Part I of this book, be deduced out of the values of the commodities...

Hence, the price of production of a commodity is equal to its cost price plus the profit, added to it in per cent, in accordance with the general rate of profit, or, in other words, to its cost price plus the average profit (B3, 156; text within square brackets added).²

When Marx refers to an “average” he always means a weighted average. From (11.1), (8.12), and (8.33), we verify that the “general rate of profit” is “the average of the various rates of profit in the different spheres of production”:

\[ \rho^0 = \frac{q^K c_K v}{q^2 (A + \ell c_L) v} \frac{\sum_i q_i^Q \eta_i c_i v}{\sum_i q_i^Q (A + \ell c_L)_i v} = \sum_i \rho_i^0 \alpha_i, \]  

(11.3)

where the profit rate in the ‘sphere of production’ \( i \), and this sphere’s share in global capital are, respectively:

\[ \rho_i^0 \equiv \frac{q_i^Q \eta_i c_i v}{q_i^Q (A + \ell c_L)_i v}, \quad \alpha_i \equiv \frac{q_i^Q (A + \ell c_L)_i v}{\sum_i q_i^Q (A + \ell c_L)_i v}. \]  

(11.4)

Hence, Marx’s statement that according to his calculations “the price of production of a commodity is equal to its cost price” \( ((A + \ell c_L)_i v) \) “plus the profit... in accordance with the general rate of profit” \( (\rho^0 (A + \ell c_L)_i v) \) corresponds exactly to (11.2), that is:

\[ p_i^0 = (A + \ell c_L)_i v + \rho^0 (A + \ell c_L)_i v. \]

Marx’s approximate formula has the interesting features that, on the one hand, global profit (II) and surplus value \( (S^v) \) are necessarily equal:

\[ \Pi = \rho^0 q^Q (A + \ell c_L) v = \frac{S^v}{C^v + V^v} (C^v + V^v) = S^v = q^K c_K v, \]  

(11.5)

²We have made a slight change in notation here for capital, since Marx writes for the profit rate \( \ell = s/C \), where \( s \) is surplus value and \( C = c + v \). Our \( C^v, V^v \), a \( S^v \) correspond exactly to Marx’s \( c, v \), and \( s \), since he specifies that they are “deduced out of the values of the commodities”.

Notice that Marx’s slip (overlooked by Engels) that required the added text in square brackets was due to his always considering equal capitals of 100 in all of the spheres, so that if the aggregate profit rate was 22% it was 22 that needed to be added to the cost price.
and, on the other, aggregate gross output is the same whether it is measured in pro-
duction prices or values:

\[ q^Qp^0 = (1 + \rho^0) q^Q (A + \ell c_L) v = q^Q Av + q^Q \ell c_L v + \rho^0 q^Q (A + \ell c_L) v \quad (11.6) \]

\[ = C^v + V^v + S^v = q^Q v. \]

It should be noted that \( q^Qp^0 = q^Qv \) implies that when Marx uses his production
prices he is not using monetary prices (unless one had the very special case that
\((1 + \rho^0) (A + \ell c_L)_j v = 1 \) where \( j \) is gold).

The double approximation Marx uses for his definition of production prices ((11.1)
and (11.2)) is notable for the degree of simplification it meant to him. It implied
that aggregate output was the same whether it was calculated in production prices
or in values and that aggregate profits and aggregate surplus values were the same.
Hence, he could conceptually analyze the formation of profit in the various industrial
branches (or ‘spheres’) as redistributions of the produced surplus value (produced
by means of the circulation of commodities and the reallocation of capital). This
redistribution made the profit obtained by each capitalist in each branch proportional
to capital disbursed (sum of constant and variable capital) while the surplus value
generated in each firm and in each branch was proportional to variable capital. The
industrial branches with greater value composition of capital \( \kappa_i \) (see (8.14)) than
average \( (\kappa) \) having a smaller share in aggregate variable capital had to receive through
the circulation process more surplus value than they generated, and vice versa. In
his reasoning, Marx often assumed that within the set of industrial branches there is
one that has the average value composition and hence had profits exactly equal to the
surplus value generated internally. He writes:

Competition so distributes the social capital among the various spheres of
production that the prices of production in each sphere take shape accord-
ing to the model of the prices of production in these spheres of average
composition, i.e., they = \( k + k\times p' \) (cost price plus the average rate of
profit multiplied by the cost price). This average rate of profit, however,
is the percentage of profit in that sphere of average composition in which
profit, therefore, coincides with surplus value. Hence, the rate of profit is
the same in all spheres of production, for it is equalized on the basis of those
average spheres of production which has the average composition of capital.
Consequently, the sum of the profits in all spheres of production must equal
the sum of the surplus values, and the sum of the prices of production of
the total social product equal the sum of its value (B3, 171-2).

The last two assertions in this quotation correctly express the equalities in (11.5)
and (11.6)\(^3\). Many of Marx’s critics have highlighted that both assertions cannot be
simultaneously true if the correct algebraic formulas for production prices are used.
It is evident that obtaining the exact formula for production prices was an advance.
But emphasizing the approximations and the consequent errors in detail to attack the
theory in its entirety was not the best way to evaluate what can and cannot be deemed
correct or inspiring in Marx’s theory of Capitalism. On the other hand, it should be
highlighted that Marx was quite conscious that he was making an approximation when
he valued costs using values. Since he probably invested much time trying to get more
satisfactory mathematical results, he chose not to get further tangled in this matter.
Both of these aspects are present in the following paragraph:

\(^3\)In the last case, Marx’s terminology for our aggregate gross output \( q^Q \) was ‘total social product’.
We had originally assumed that the cost price of a commodity equaled the value of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost price, and may thus pass as cost price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost price, and to bear in mind that there is always the possibility of an error if the cost price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point (B3, 164; italics added).

And it was certainly a wise decision not to get tangled up in algebraic matters that mathematicians could eventually solve and instead concentrate on the economic, social, institutional, and historical analyses for which Marx was so gifted.

Surplus value and surplus labor time
Necessary and surplus labor time

For Marx, in CCP the labor of wage workers produces value even if, in contrast to SCP, the commodities they produce do not tend to be exchanged in proportion to their values. Since in SCP there is no capital, nor capitalists (nor landowners), all the net output of producers, once sold, is their income. In CCP, in contrast, there exist, in addition to workers, capitalists who tend to allocate their capital to industrial branches in such a way that the rates of profit tend to be equal in all branches. Leaving aside landowners (and ground rent) as well as commercial and banking capitalists (and their profits), produced commodities (capital-commodities) will tend to exchange in proportion to their production prices, which differ generally from their values. But for Marx the latter keep playing a role under the surface, since it underlies all profits. Industrial capitalists redistribute among themselves the aggregate surplus value produced through the circulation of commodities and the reallocation of capital among branches. Just as produced value is measured by the labor time under average conditions of production (as long as society demands and can afford these produced commodities), that is, by the labor time that is socially necessary, the time period of each working day can be decomposed into the necessary labor time in which the workers’ means of subsistence are produced, and the rest of the working day, the surplus labor time, during which they produce surplus value. Marx writes:

We have seen that the labourer, during one portion of the labour process, produces only the value of his labour power, that is, the value of his means of subsistence. Now since his work forms part of a system, based on the social division of labour, he does not directly produce the actual necessaries which he himself consumes; he produces instead a particular commodity, yarn for example, whose value is equal to the value of those necessaries or of the money with which they can be bought. The portion of his day’s labour devoted to this purpose, will be greater or less, in proportion to the value of the necessaries that he daily requires on an average, or, what amounts to the same thing, in proportion to the labour time required on an average to produce them... That portion of the working day, then, during which this
reproduction takes place, I call “necessary” labour time, and the labour expended during that time I call “necessary” labour... (B1, 225-6).

During the second period of the labour process, that in which his labour is no longer necessary labour, the workman, it is true, labours, expends labour power; but his labour, being no longer necessary labour, he creates no value for himself. He creates surplus value which, for the capitalist, has all the charms of a creation out of nothing. This portion of the working day, I name surplus labour time, and to the labour expended during that time, I give the name of surplus labour (B1, 226; italics added).

One must take into account when reading “The portion of his day’s labour devoted to this purpose, will be greater or less, in proportion to the value of the necessaries that he daily requires on an average”, that in Book I Marx was making the simplifying assumption that average prices coincided with values. When he eliminates this assumption in Book III (and before Part VI on ground rent) the “value of the necessaries” that the worker purchases is measured in production prices. But even so, the consumption basket of each worker (which is given by assumption) can be valued both using values and using production prices because they coexist in Marx’s theory of Capitalism, albeit in two different dimensions, the former being the essential one (since it was meant to be the foundation of exploitation in Capitalism) and the latter the phenomonic (since it is in the surface and visible even to ‘vulgar’ economists).

The length of the working day and Marx’s defense of real wages

For Marx it was important to endow workers with theoretical tools that they could use to defend their rights against capitalists’ encroachment. And he understood that the struggle of workers was not only for a better distribution of the income generated, but also for the quality of life in a sense that surpasses the purchasing power of goods and services and hinges on the extension of the non-labor part of the day, where the “true realm of freedom” blossoms, where they could indulge in the “development of human energy which is an end in itself” instead of only a means for survival. He highlights the role of the increase in productivity in extending workers’ necessities as civilization progresses, and also in shortening the working day, the “basic prerequisite” for the development of the realm of freedom:

In fact, the realm of freedom actually begins only where labour which is determined by necessity and mundane considerations ceases; thus in the very nature of things it lies beyond the sphere of actual material production. Just as the savage must wrestle with Nature to satisfy his wants, to maintain and reproduce life, so must civilised man, and he must do so in all social formations and under all possible modes of production. With his development this realm of physical necessity expands as a result of his wants; but, at the same time, the forces of production which satisfy these wants also increase. But it nonetheless still remains a realm of necessity. Beyond it begins that development of human energy which is an end in itself, the true realm of freedom, which, however, can blossom forth only with this realm of necessity as its basis. The shortening of the working day is its basic prerequisite (B3, 87; italics added).
Marx stresses that reasons of a moral and cultural nature impose limitations on the extension of the working day.\(^4\) He shows that the extension of the working day is very elastic, evidenced by the fact that “working days of 8, 10, 12, 14, 16, 18 hours” coexist in various places and branches. The chapter on “The Working Day” of Book I contains a plethora of factual information (with more than 150 footnotes). It extensively describes this matter that for decades had been debated in the British Parliament between those who lobbied for the interests of industrial capital and those that represented feelings of a higher morality (as evidenced by their indignation with the over-exploitation of children). Marx marks his approval of the labor reforms that were gradually reflected in regulations concerning the length of the working day in Britain and that were later introduced in other countries. Marx’s factual description of the social conflicts derived from the duration of the working day probably influenced generations of working class leaders though their reading parts of \textit{Capital} and contributed to the reduction of the working day in countries more advanced in their capitalist development through the propagation of state regulations that constrained the greed of industrial capitalists (as well as that of parents that sent their infant children to work in factories for long hours).

Marx shows that it is not simply the ‘nature’ of markets that determines the duration of the working day (as if labor power were just another commodity), but the result of a social struggle between opposing social forces:

We see then, that, apart from extremely elastic bounds, the nature of the exchange of commodities itself imposes no limit to the working day, no limit to surplus labour. The capitalist maintains his rights as a purchaser when he tries to make the working day as long as possible, and to make, whenever possible, two working days out of one. On the other hand, the peculiar nature of the commodity sold implies a limit to its consumption by the purchaser, and the labourer maintains his right as seller when he wishes to reduce the working day to one of definite normal duration. There is here, therefore, an antinomy, right against right, both equally bearing the seal of the law of exchanges. Between equal rights force decides. Hence is it that in the history of capitalist production, the determination of what is a working day, presents itself as the result of a struggle, a struggle between collective capital, i. e., the class of capitalists, and collective labour, i. e., the working class (B1, 243).

It should be noted that Marx’s assertion that “Between equal rights force decides”, which would now be called ‘negotiating power’, is closely related to the theoretical indetermination (within defined bounds) that results from ‘bilateral monopoly’ in modern Microeconomics. This would be the case, for example, of a large firm that is the only source of labor demand (a monopsonist) in a city in which the supply of labor is monopolized by a labor union. This theoretical indetermination is always solved in reality through a practical determination that is intimately related to the negotiating power of the two parts, including strikes, lock-outs, and the hiring of ‘strikebreakers’, and to the actions of multiple segments of the population that express their opinions through the press, and seek to influence the legislative process, the elections of legislators, and the actions of those in government that write labor regulations.

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\(^4\)“Besides these purely physical limitations, the extension of the working day encounters moral ones. The labourer needs time for satisfying his intellectual and social wants, the extent and number of which are conditioned by the general state of social advancement” (B1, 240-1).
Marx also taught that workers would necessarily lose in any individual claim against their employers and that to succeed they had to associate in trade unions:

On the other hand, the value of labour capacity forms the conscious and explicit basis of the trades’ unions, the importance of which for the English working class can hardly be exaggerated. The aim of the trades’ unions is nothing other than to prevent wages from falling below the level that is traditional in the different branches of industry. Their aim is to prevent the price of labour capacity from being forced down below its value. They are of course aware that a change in the relation of supply and demand brings about a change in the market price... On the one hand, however, the actual occurrence of such a change is something very different from the unilateral assertion of the buyer, in this case the capitalist, that it has occurred. On the other hand, there is “a wide difference between the demand and supply rate of wages, or the rate which the FAIR operation of commodity exchange would give, if the buyer and seller of it were on equal terms, and that which the seller, the worker, is compelled to accept if the employer deals with each man singly, and dictates a reduction, taking advantage of the individual workman’s accidental needs” (which do not depend on the general relation of supply and demand). “The workmen combine to put themselves on something like equality in the bargain for the sale of their labour with the capitalist. This is the rationale” (the logical basis) “of trades’ unions.”5 (MECW 34, 342-3)

But not only did the workers need to form unions to negotiate collectively. They also had to try to influence the government so that it established legal regulations of the working day so that they would prevent individual laborers from underselling their labor:

For “protection” against “the serpent of their agonies”, the labourers must put their heads together, and, as a class, compel the passing of a law, an all-powerful social barrier that shall prevent the very workers from selling, by voluntary contract with capital, themselves and their families into slavery and death. In place of the pompous catalogue of the “inalienable rights of man” comes the modest Magna Charta of a legally limited working day, which shall make clear “when the time which the worker sells is ended, and when his own begins” (B1, 306-7)6.

By means of a highly original type of analysis Marx warned workers that if, for example, the length of the working day increased (say β%) without a proportional increase in the consumption basket their wage could purchase, the rate of exploitation of labor, i.e., the rate of surplus value, would increase. Our analysis of the generation of “absolute surplus value” in Chapter 10 showed that if, starting from (8.34) and (8.6), one assumes a β% prolongation of the working day, the new situation is reflected in (10.3) and (10.4) if there is no change in the consumption basket of workers. The surplus value factor increases β% from 1+e to 1+e’ (as in (10.6)) and the consumption

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5Here Marx quotes J. T. Dunning, (who was Secretary to the London Consolidated Society of Bookbinders), author of Trades’ Unions and Strikes: Their Philosophy and Intention, London, 1860. Notice that Dunning refers to the wage as the “price of labour” while Marx to the “price of labour capacity”, that is, of labor power.

6Marx’s last quotation is from one of the Reports of the Inspectors of Factories of England, and the phrase “the serpent of their agonies” is from a poem by Heinrich Heine.
basket of capitalists increases proportionally more than β%. And if instead of assuming there is no change in the consumption basket of workers one assumes that it increases, but less than β%, there is still an increase, albeit a lower one, in the rate of surplus value. The same arguments Marx makes in Book I using the concepts of value and surplus value can be readily made using the (correctly formulated) production prices, wage, and profit rate, as we also showed above. A β% increase in the working day with no change in worker consumption leads to the price system (10.16) in which the profit rate and all production prices (in terms of the wage rate) increase, hence reducing the real wage.

The non-validity of the ‘law of value’ in CCP
Commodities and Simple Commodity Production (SCP) are the starting point for Marx’s theory of Capitalism, and it is in that context that value is initially defined. We have seen in Chapter 6 ((6.1) and (6.9)) that in SCP the equilibrium prices of commodities measured in terms of the income of producers coincide with their values. Hence, the ‘law of value’ operates directly, since commodities are transacted according to the labor time that is socially necessary for their production. And the market prices fluctuate around the values according to the transitory discrepancies between supply and demand.

The concept of value also had a fundamental theoretical importance in the theory of capital and Capitalist Commodity Production (CCP), since it led to the concept of surplus value, which was the source of the incomes of all propertied social classes and the measure of the exploitation of labor in Capitalism. We have seen in Chapter 8 that in CCP (neglecting the private property of land) the equilibrium prices are the production prices, the structure of which is in general different from that of values. The simplifying assumption of Book I that even in CCP commodities exchanged according to their values made sense for Marx since for him the first and grand distribution between workers and capitalists took place in the production process itself, where workers generated all the produced value but only received a portion through their wages, and industrial capitalists initially appropriated all the surplus value (but had eventually to share it with commercial and financial capitalists, landowners, holders of financial assets, and the State). Since Book I was on the process of production of surplus value (and hence capital, through its accumulation), Marx left the subsequent redistribution of surplus value for a specific and graduated treatment in Book III (as the simplifying assumptions were lifted), which studied “The Process of Capitalist Production as a Whole.” The main simplifying assumptions of Book I were that equilibrium prices were proportional to values (which as we have seen could have been replaced by the assumption of equal value compositions in all industrial branches), and the non-existence of commercial capital, financial capital, credit, ground rent, State, industrial cycles, nor crises.

The ‘law of value’ was still operational in the models of Book I due to the assumption that all commodity exchanges were done according to their values, as in the SCP of the first three chapters. But already in Book I Marx hints at the problem that arises if one lifts the assumption that even in CCP commodities exchange according to their values: “Everyone knows that a cotton spinner, who, reckoning the percentage on the whole of his applied capital, employs much constant and little variable capital, does not, on account of this, pocket less profit or surplus value than a baker, who relatively sets in motion much variable and little constant capital. For the solution of this apparent contradiction, many intermediate terms are as yet wanted” (B1, 311). The missing ‘terms’ were to be gradually introduced in Book III. In Part I of that
book Marx eliminates the special assumption that commodities exchange according to values. He there derives his (approximate) formulas for the profit rate (11.1) and the production prices (11.2).

The particular properties of his formulas led Marx to assert that the ‘law of value’ was still valid “one way or another” in the general case. The ‘law’ held in an indirect way and one more related to the changes in value than to their level: “Since the total value of the commodities regulates the total surplus value, and this in turn regulates the level of average profit and thereby the general rate of profit –as a general law or a law governing fluctuations– it follows that the law of value regulates the prices of production” (B3, 179). Since his formulas were \( p^0 = (1 + \rho^0) (A + \ell c_L) v \) and \( \rho^0 = q^1 c_K v / q^2 (A + \ell c_L) v \), changes in the value \( v_i \) of a particular commodity impinged on the production cost of all commodities that used it as input (directly or through the consumption basket of workers) and also on the global profit rate \( \rho^0 \), thus ‘regulating’ the production prices. But this ‘regulation’ could no longer mean that if the value of a commodity fell, its price of production should necessarily fall. Marx’s approximate formulas implied that \( q^2 p^0 = q^2 v \). If some of the components of \( v \) fell, some of those in \( p^0 \) had to fall. But this is very different from the ‘law of value’ in SCP. Marx knew that increases in the productivity of labor caused reduction in values. But while such increases were exclusive determinants of the reductions in value (assuming the commodity was sold), their effect on production prices was much more complex.

Marx’s understanding that the ‘law of value’ was not directly valid in CCP is well reflected in a letter he sent to Engels on January 8, 1868 in which he criticizes Dühring: “As for Mr. Dühring’s modest objection to the determination of value, he will be astonished when he sees in Volume II how little the determination of value counts for ‘directly’ in bourgeois society” (MECW 42, 515). He adds, “Actually, no form of society can prevent the labour time at the disposal of society from regulating production in one way or another.” But the last qualification is quite vague.

If the correct systems of equations are used things get more complicated for several reasons. First, it is evident that the effect of any exogenous change on the elements of \( A \) and/or \( \ell \) on the absolute level of the production prices depends on the numeraire chosen. Second, given a numeraire, a technological or organizational innovation (that can only reduce the values of commodities) can increase or reduce the production price of any given commodity (unless it is the one that is used as numeraire). Let us assume that prices are measured in terms of the wage rate, i.e., that we have \( w = c_L p = 1 \). Then if an innovation reduces some of the production prices of basket \( c_L \) it must necessarily increase others to compensate. This implies there is no ‘law of value’ in CCP in any sense comparable with that in SCP. While an innovation must necessarily diminish the values of all interconnected commodities and cannot increase the value of any, given a numeraire it can diminish or increase the production price of any commodity (that is not the numeraire), depending on the numeraire used. Finally, and this is the most problematic issue, we saw in Chapter 10 that if SR is to be preserved after an innovation, it is necessary that the consumption basket of workers, of capitalists, or both, increase. And how this comes about is not specified in Marx’s models. However, we will see in the next three chapters that in his dynamic analyses he often treated consumption as endogenous and that the effects of exogenous changes can be consistently modeled as to reflect his analyses.

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7At the time, Marx believed that the second volume of Capital would include Books II and III.
Simple Reproduction in Marx’s analysis

Introduction

When Marx analyzed the production of capital in Book I, he only referred to the circulation process when it plays an important role in the exploitation process of wage labor, the fundamental base for the production of capital in his theory. He there necessarily had to address the capitalist’s purchase of labor power and means of production, which take place in the sphere of circulation of commodities, and the productive consumption of the labor generated by that labor power, which takes place in the sphere of production. In Part I of Book II (titled “The metamorphoses of capital and their cycle”) he examined “the various forms... which capital assumes in its cycle, and the various forms of this cycle itself”; and in Part II he focused on the turnover of capital. But the analysis of the cyclical process of capital had until then centered on the cycle of an individual capital. It is in Part III of Book II (titled “The reproduction and circulation of the aggregate social capital”) that his analysis focuses on aggregate social capital.

We anticipated in Chapter 4 how Marx decomposes the recurring cycle of capital, for the sake of analysis, into three partial cycles. Although both in the ‘cycle of money-capital’ \((M - C...P...C' - M')\) and in the ‘cycle of productive-capital’ \((P...C' - M' - C...P)\) the movement of capital “includes consumption, for the commodity, the product, must be sold” (B2, 391), after its sale “it is immaterial for the movement of the individual capital what becomes of this commodity subsequently” (Ibid.). However, in the ‘cycle of commodity-capital’ \((C' - M' - C...P...C'')\) “the total process of reproduction includes the process of consumption brought about by the circulation quite as much as the process of reproduction of the capital itself” (Ibid.). In the case of SR this cycle is:

\[
C' < \left\{ \frac{C}{c} - M' < \left\{ \frac{M}{m} - \frac{C}{c} < \frac{LP}{MP} \right\} \right\} P...C'' .
\]

Here “the point of departure \(C' = C + c\), the commodity capital, embraces both the constant and variable capital value, and the surplus value. Its movement therefore includes both individual and productive consumption.”

Marx seeks to answer the question: “How is the capital consumed in production replaced in value out of the annual product and how does the movement of this replacement intertwine with the consumption of the surplus value by the capitalists and of the wages by the labourers?” (B2, 392). He makes the simplifying assumptions that “products are exchanged at their values and also that there is no revolution in the values of the component parts of productive capital” (Ibid.).

Recognition for the Physiocrats

When Marx highlights that the interesting aspect of the ‘cycle of commodity-capital’ is that it makes one focus on the global (or aggregate) cycle of capital, he adds that “\(C'...C''\) is the groundwork for Quesnay’s Tableau économique, and it shows great and true discretion on his part that in contrast to \(M...M'\) (the isolatedly and rigidly retained form of the mercantile system) he selected this form and not \(P...P''\) (B2, 105). Marx repeatedly recognized the importance of the Physiocrats in the analysis of the global functioning of capitalist production, especially the role assigned to the tenant farmer as an agrarian capitalist motivated by profit making. This led him to call them “the

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8In the case of Extended Reproduction (ER), at the end of the cycle there are greater quantities of productive-capital and commodity-capital than at the beginning. Hence there appears \(...P'...C''\) instead of \(...P...C'\).
true fathers of modern political economy” (B4.30, 352). With their rupture with the
Mercantilists, “The Physiocrats transferred the inquiry into the origin of surplus value
from the sphere of circulation into the sphere of direct production, and thereby laid
the foundation for the analysis of capitalist production (B4.30, 354). The following
paragraph is clear in this respect:

But as a matter of fact the system of the Physiocrats is the first system-
atic conception of capitalist production. The representative of industrial
capital—the class of farmers—directs the entire economic movement. Agri-
culture is carried on capitalistically, that is to say, it is the enterprise of
a capitalist farmer on a large scale; the direct cultivator of the soil is the
wage labourer. Production creates not only articles of use but also their
value; its compelling motive is the procurement of surplus value, whose
birthplace is the sphere of production, not of circulation. Among the three
classes which figure as the vehicles of the social process of reproduction
brought about by the circulation, the immediate exploiter of “productive”
labour, the producer of surplus value, the capitalist farmer, is distinguished
from those who merely appropriate the surplus value (B2, 358).

Those who “merely appropriate the surplus value” were, of course, the landowners.

Duality in the reproduction process

As we have seen in Chapter 4 Marx stressed the cyclical nature of the production
process in any human society and the fact that in this process the means of production
that were consumed productively had to be reproduced. He also emphasized that in
any human society laborers work cooperatively, respecting certain proportions between
the inputs used and the labor exerted in order to obtain products for final consumption.
For Marx a specific characteristic of mercantile societies was that the relations between
producing individuals or groups of individuals are transformed into (and perceived as)
value relations between commodities in the exchange process. This implied a duality
between the proportions that use values (quantities) must satisfy in the production
and reproduction processes, on the one hand, and the proportions that either values
(in the case of SCP) or production prices (in the case of CCP) must satisfy in the
circulation process.9

When Marx addresses SR in Book II he again tackles the duality theme, this time
between the replacement of value and the material replacement of the elements of the
commodity-capital that is reproduced:

The annual product includes those portions of the social product which re-
place capital, namely social reproduction, as well as those which go to the
consumption fund, those which are consumed by labourers and capitalists,
hence both productive and individual consumption. It comprises also the
reproduction (i.e., maintenance) of the capitalist class and the working
class, and thus the reproduction of the capitalist character of the entire
process of production... For our present purpose this process of reproduc-
ion must be studied from the point of view of the replacement of the value
as well as the substance of the individual component parts of C (B2, 391;
italics added).

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9After the rent of land is introduced, the production prices will have to be appropriately modified, as we will see in Chapter 16.
For Marx SR is in the capitalist mode of production an abstraction, a model. In reality the reproduction process is never repeated exactly each period. Also, the tendency in Capitalism is for capital to produce more capital through the accumulation of a (substantial) part of the surplus value, and hence SR would actually be an anomaly. However, true to his hypothetico-deductive and empirical-historical method (which he called ‘dialectic’), he first analyzes SR because it is important to understand how it can function if one wants to understand the specifics of Extended Reproduction (ER). He states: “Simple reproduction, reproduction on the same scale, appears as an abstraction, inasmuch as on the one hand the absence of all accumulation or reproduction on an extended scale is a strange assumption in capitalist conditions, and on the other hand conditions of production do not remain exactly the same in different years (and this is assumed)... However, as far as accumulation does take place, simple reproduction is always a part of it, and can therefore be studied by itself, and is an actual factor of accumulation” (B2, 393).

The reproduction tables in Capital are expressed as amounts of value, as was the case of Table 10 of Chapter 5, without separating produced quantities (q) from prices (p) nor from technical coefficients aij, a separation that is now easily performed by means of modern matrix methods. Nevertheless, Marx’s analyses of his SR and ER tables are surprisingly detailed and conceptually highlight both the physical flows of commodities, the values of those commodities, and the money used in the transactions. As we have seen, Marx was very conscious of the duality between quantities and prices. Moreover, he explicitly used the notion of duality several decades before it was resuscitated by mathematical economics. For example, before the publication of Book I of Capital, he wrote (sometime between June 1863 and December 1866) in Results:

Capital itself is dual, since it consists of commodities.

*Exchange value* (money), but *self-valorising value*, value which creates value, grows as value, obtains an increment, through the factor that it is value. This can be reduced to the exchange of a given quantity of objectified labour for a greater quantity of living labour.

*Use value*, and here capital appears according to its particular situation in the labour process. But precisely here it does not just remain material of labour, means of labour to which *labour* belongs, and which have incorporated labour, but involves also, along with labour, its *social combinations* and the development of the means of labour which corresponds to these social combinations. Capitalist production first develops the conditions of the labour process on a large scale –first develops them separately from the single independent worker– developing both its objective and its subjective conditions... *(Results, MECW 34, 459).*

### Simple Reproduction as a table

The basic SR table Marx uses in Book and is here reproduced in Table 11 has two departments: means of production are produced in I and means of subsistence in II. The components of capital in each department are: constant capital C, variable capital V, and surplus value S. He makes the explicit assumption that the rate of surplus value is 100% in each department, which implies that the entries in the columns for V and S are the same. Marx also makes the simplifying assumption

---

10 When several decades later Leontief started building input-output tables for national economies, he did the same as Marx because his primary empirical data were expressed in dollar values.
that the two departments have the same value composition of capital, that is, the same ratio between constant and variable capital. As we have seen, that is the correct assumption to make if one wants production prices to be proportional to values (as Marx assumes in Book I and a large part of Book II). Finally, he assumes that the means of subsistence produced in Department II are not used as means of production in any of the two departments and that the means of production produced in Department I are not consumed (except productively). Hence the simplest SR table of Book II is the following:

Table 11

<table>
<thead>
<tr>
<th>Department</th>
<th>( C^v )</th>
<th>( V^v )</th>
<th>( S^v )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>4000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>II</td>
<td>2000</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Marx asserts that the value of the aggregate output of means of subsistence (3000 in Department II) must equal the sum of the variable capitals and the surplus values of both departments (1500+1500);\(^{11}\)

The total social working day is divided into two parts: 1) necessary labour, which creates in the course of the year a value of 1,500 \( v \); 2) surplus labour, which creates an additional value, or surplus value, of 1,500 \( s \). The sum of these values, 3,000, is equal to the value of the annually produced articles of consumption –3,000. The total value of the articles of consumption produced during the year is therefore equal to the total value produced by the total social working day during the year, equal to the value of the social variable capital plus the social surplus value, equal to the total new product of the year (B2, 424).

Just to give an idea of the extensive and detailed analyses of Book II we reproduce the following paragraph, which is also indicative of the fact that behind the simple analysis of amounts of value there are also considerations of flows of quantities (‘bodily form’) of means of production and means of subsistence. Marx here identifies the amounts according to whether they conform constant capital \((c)\), variable capital \((v)\), or surplus value \((s)\), of the respective branches:

We begin with the great exchange between the two classes. \((1,000v + 1,000s)\) I – these values consisting, in the hands of their producers, of means of production in their bodily form, are exchanged for 2,000 IIc, for values consisting of articles of consumption in their bodily form. The capitalist class of II thereby reconverts its constant capital of 2,000 from the form of articles of consumption into that of means of production of articles of consumption, into a form in which it can once more function as a factor of the labour process and for purposes of self-expansion of value as constant capital value. On the other hand the equivalent of the labour power of I \((1,000 v)\) and the surplus value of the capitalists of I \((1,000 s)\) are realised thereby in articles of consumption; both of them are converted from their bodily form of means of production into a bodily form in which they can be

\(^{11}\) The reader must bear in mind that our \( C^v, V^v, \) and \( S^v \), correspond to Marx’s \( c, v, \) and \( s \).
consumed as revenue. Now, this mutual exchange is accomplished by means of a circulation of money, which promotes it just as much as it renders its understanding difficult, but which is of decisive importance... (B2, 397).

Let us now put the information in Marx’s SR table in the formats of systems (8.1) and (8.6), assuming that in these there are only two industrial branches: 1, in which means of production are produced and 2, in which means of subsistence are produced. Notice that the reasoning in the preceding quote implies that means of subsistence are not used as means of production. Hence, matrix $A$ must have the following format:

$$ A = \begin{bmatrix} a_{11} & 0 \\ a_{21} & 0 \end{bmatrix}. $$

Here $a_{12} = a_{22} = 0$ because the means of consumption are not used as means of production in any of the two branches. On the other hand, the preceding quote also implies that the means of production are not consumed by workers nor by capitalists. Hence, the consumption baskets can be written as $(0, c_L)$ and $(0, c_K)$ (since workers and capitalists only consume means of subsistence, which are produced in 2). Therefore, systems (8.1) and (8.6) are the following:

$$ \begin{bmatrix} q^Q_1 & q^Q_2 & q^L & q^K \end{bmatrix} \begin{bmatrix} a_{11} & 0 & \ell_1 \\ a_{21} & 0 & \ell_2 \\ 0 & c_L & 0 \\ 0 & (1+c) c_L & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ 1 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ 1 \end{bmatrix} \tag{11.7} $$

Pre-multiplying (11.8) by the (reduced) vector of quantities of the right hand side of (11.7) and using the third equality of (11.7), that is, $q^Q_1 \ell_1 + q^Q_2 \ell_2 = q^L$, to eliminate $q^L$, we get:

$$ (q^Q_1 a_{11} v_1 + q^Q_2 a_{21} v_2) + (1+c) (q^Q_1 \ell_1 c_L v_2 + q^Q_2 \ell_2 c_L v_2) = q^Q_1 v_1 + q^Q_2 v_2. $$

And putting the components of this decomposition in the format of Table 11 we get:

Table 12

<table>
<thead>
<tr>
<th>Dep.</th>
<th>$C^v$</th>
<th>$V^v$</th>
<th>$S^v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$q^Q_1 a_{11} v_1$</td>
<td>$q^Q_1 \ell_1 c_L v_2 + e * q^Q_1 \ell_1 c_L v_2$</td>
<td>$q^Q_1 v_1$</td>
</tr>
<tr>
<td>II</td>
<td>$q^Q_2 a_{21} v_2$</td>
<td>$q^Q_2 \ell_2 c_L v_2 + e * q^Q_2 \ell_2 c_L v_2$</td>
<td>$q^Q_2 v_2$</td>
</tr>
<tr>
<td>I+II</td>
<td>$q^Q_1 a_{11} v_1 + q^Q_2 a_{21} v_2$</td>
<td>$q^L c_L v_2 + e * q^L c_L v_2$</td>
<td>$q^Q_1 v_1 + q^Q_2 v_2$</td>
</tr>
</tbody>
</table>

First, we know that Marx assumes the rate of surplus value is 100%, i.e., $e = 1$. Second, from (11.7) we can get formulas for $q^Q_1$, $q^Q_2$, and $q^K$ in terms of $q^L$ and other parameters:

$$ q^Q_1 = \frac{1}{\ell_1 + \frac{a_{11}}{a_{21}} \ell_2} q^L, \quad q^Q_2 = \frac{1}{\ell_1 \frac{a_{21}}{1-a_{11}} + \ell_2} q^L \tag{11.9} $$

$$ q^K c_K = \left[ \frac{a_{21}}{\frac{1-a_{11}}{a_{11}} \ell_1 + \ell_2 - c_L} \right] q^L. $$
Also, comparing Table 12 with Table 11 directly yields the numerical values of the non-zero elements of matrix \( A \):

\[
a_{11} = \frac{q_1^Q a_{11} v_1}{q_1^Q v_1} = \frac{4000}{6000} = \frac{2}{3} \quad a_{21} = \frac{q_2^Q a_{21} v_2}{q_2^Q v_2} = \frac{2000}{3000} = \frac{2}{3}
\]

Using these numbers in the first equality of (11.7) yields \( q_1^Q (2/3) + q_2^Q (2/3) = q_1^Q \), and hence \( q_1^Q = 2q_2^Q \). Also, using \( e = 1 \) and the third equality of (11.8) we get the value of the consumption basket of workers:

\[
c_L v_2 = \frac{1}{2} \quad (11.10)
\]

Comparing the aggregate variable capital of the two tables, we have \( q^L c_L v_2 = 1500 \), and hence using (11.10) yields the aggregate of worker labor time used as: \( q^L = 3000 \). Comparing the two tables also yields \( q_1^Q \ell_1 c_L v_2 = 1000 \) and \( q_2^Q \ell_2 c_L v_2 = 500 \), so that (using \( q_1^Q = 2q_2^Q \)) we get \( \ell_1 = \ell_2 \). On the other hand, the first two equalities of (11.8) yield \( a_{11} v_1 + \ell_1 = v_1 \) and \( a_{21} v_2 + \ell_2 = v_2 \). And since \( a_{11} = a_{21} = 2/3 \) and \( \ell_1 = \ell_2 \) we get \( v_1 = 3\ell_1 \) and \( v_2 = (2/3) v_1 + \ell_2 = 2\ell_1 + \ell_2 = 3\ell_1 \), that is, \( v_2 = v_1 = 3\ell_1 \). Hence, (11.10) yields \( c_L = 1/(2v_2) = 1/(6\ell_1) \). And using the values already obtained and the relations in (11.9) we have:

\[
qu_1^Q = \frac{1}{\ell_1 + \frac{1}{2} \ell_2} 3000 = \frac{2000}{\ell_1^2} \quad qu_2^Q = \frac{1}{\ell_1^2 + \ell_2} 3000 = \frac{1000}{\ell_1}
\]

\[
q^K c_K = \left[ \frac{1}{2\ell_1 + \ell_2} - c_L \right] 3000 = \left[ \frac{1}{3} - \frac{1}{6} \right] \frac{1}{\ell_1} 3000 = \frac{500}{\ell_1}
\]

We still have two degrees of freedom, by which we can give \( \ell_1 \) and \( c_K \) exogenous values. For example, if we assume \( \ell_1 = 2/3 \), we have:

\[
\ell_1 = \ell_2 = 2/3, \quad v_1 = v_2 = 2, \quad qu_1^Q = \frac{12000}{4} = 3000
\]

\[
qu_2^Q = \frac{3000}{2} = 1500, \quad q^K c_K = \frac{500}{2/3} = 750.
\]

To get separate values for \( c_K \) and \( q^K \), we can also assume, e.g., \( c_K = 7.5 \), which implies \( q^K = 100 \).

We can check that the numerical values obtained (and assumed) in fact satisfy (11.7) and (11.8):

\[
\begin{bmatrix}
3000 & 1500 & 3000 & 100
\end{bmatrix}
\begin{bmatrix}
2/3 & 0 & 2/3 \\
2/3 & 0 & 2/3 \\
0 & 1/4 & 0 \\
0 & 7.5 & 0
\end{bmatrix}
= \begin{bmatrix}
3000 & 1500 & 3000
\end{bmatrix}
\]

\[
\begin{bmatrix}
2/3 & 0 & 2/3 \\
2/3 & 0 & 2/3 \\
0 & (1 + 1)/4 & 0
\end{bmatrix}
\begin{bmatrix}
2 \\
2 \\
1
\end{bmatrix}
= \begin{bmatrix}
2 \\
2 \\
1
\end{bmatrix}
\]

Hence, we have translated Marx’s numerical table to modern matrix notation. We have also found that the numerical table is compatible with many alternative numerical values for some of the parameters and variables of the matrix model.
Also, since the \textbf{value} compositions of capital are the same in the two departments (as we see in Table 12), the production prices are necessarily proportional to the \textbf{values} of commodities. To check that the \textbf{value} compositions of capital are the same in the two departments let us observe the following matrices and vectors:

\[
A = \begin{bmatrix} 2/3 & 0 \\ 2/3 & 0 \end{bmatrix}, \quad v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \ell_L = \begin{bmatrix} 2/3 & 1/4 \\ 0 & 1/6 \end{bmatrix}.
\]

Since the two rows of both \(A\) and \(\ell_L\) are the same \((A_1 = A_2, (\ell_L)_1 = (\ell_L)_2)\) according to the definition (8.14) we have:

\[
\kappa_1 = \frac{A_1v}{(\ell_L)_1v} = \frac{A_2v}{(\ell_L)_2v} = \kappa_2 = \frac{(2/3) + 2}{(1/6) + 2} = 4.
\]

\textbf{Necessary and luxury consumption goods}

In his analyses Marx also decomposes Department II into two sub-Departments: (IIa) where ‘consumer necessities’ are produced for the consumption of both classes, and (IIb) where ‘luxury’ goods are produced for the consumption of the capitalist class. He again makes the assumption that means of production are not used as means of production.\textsuperscript{12} The \textbf{SR} table Marx uses in this case is the following:

\begin{table}[h]
\centering
\caption{SR table in Capital}
\begin{tabular}{llll}
\hline
\textbf{Sector} & \textbf{\(C^v\)} & \textbf{\(V^v\)} & \textbf{\(S^v\)} \\
\hline
I & 4000 & 1000 & 1000 = 6000 \\
IIa & 1600 & 400 & 400 = 2400 \\
IIb & 400 & 100 & 100 = 600 \\
& 6000 & 1500 & 1500 & 9000 \\
\hline
\end{tabular}
\end{table}

Since 1) none of the consumption goods is used in the production of means of production, 2) the means of production are not consumed, and 3) luxury goods are not consumed by workers, the components of the matrix in (8.34) are the following:

\[
A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{bmatrix}, \quad \ell = \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{bmatrix}, \quad \ell_L = \begin{bmatrix} 0 & c_{LN} & 0 \\ 0 & c_{KN} & c_{KL} \end{bmatrix}
\]

where \(c_{LN}\) and \(c_{KN}\) are the consumptions of necessities by workers and capitalists, respectively, and \(c_{KL}\) is the consumption of luxury goods by capitalists. If we write now the corresponding system of \textbf{values} and permute the last two rows and columns, the system becomes:

\[
\begin{bmatrix} a_{11} & 0 & \ell_1 & 0 \\ a_{21} & 0 & \ell_2 & 0 \\ 0 & (1 + e)c_{LN} & 0 & 0 \\ a_{31} & 0 & \ell_3 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ 1 \\ v_3 \end{bmatrix}.
\]

\textsuperscript{12}See Marx’s classification of the Category II goods quoted immediately before (10.18).
where it is evident that the social matrix is decomposable. Therefore the system can be decomposed into the following two parts:

\[
\begin{bmatrix}
    a_{11} & 0 & \ell_1 \\
    a_{21} & 0 & \ell_2 \\
    0 & (1 + \epsilon) c_{LN} & 0
\end{bmatrix}
\begin{bmatrix}
    v_1 \\
    v_2 \\
    1
\end{bmatrix}
= 
\begin{bmatrix}
    v_1 \\
    v_2 \\
    1
\end{bmatrix}

a_{31} v_1 + \ell_3 = v_3.

The same methods used in the simple two department system can be used to obtain the numerical values of the parameters and endogenous variables.

Two separate sectors for means of production and means of subsistence

We can also formulate a more general system, with many branches that can be grouped in the same two departments: one producing means of production (I), none of which serve as means of consumption, and another that produces means of consumption (II), none of which serve as means of production. In this case \( A_I \) and \( A_{II} \) are matrices, \( c_L \) is a row vector and \( \ell_1, \ell_2, v_1, v_2 \) are column vectors:

\[
\begin{bmatrix}
    A_I & 0 & \ell_1 \\
    A_{II} & 0 & \ell_2 \\
    0 & (1 + \epsilon) c_{L} & 0
\end{bmatrix}
\begin{bmatrix}
    v_1 \\
    v_2 \\
    1
\end{bmatrix}
= 
\begin{bmatrix}
    v_1 \\
    v_2 \\
    1
\end{bmatrix}

\]

Replacing the third equation \(((1 + \epsilon) c_{L} = 1)\) in the first two, we get the reduced matrix system

\[
\begin{bmatrix}
    A_I & (1 + \epsilon) \ell_1 c_{L} \\
    A_{II} & (1 + \epsilon) \ell_2 c_{L}
\end{bmatrix}
\begin{bmatrix}
    v_1 \\
    v_2
\end{bmatrix}
= 
\begin{bmatrix}
    v_1 \\
    v_2
\end{bmatrix}

\]

where one must bear in mind that \( \ell_1 c_{L} \) are \( \ell_2 c_{L} \) are matrices (whereas \( c_{L} \ell_2 \) and \( c_{L} v_2 \) are scalars). Morishima (1973) does most of his analyses using this type of model. We have preferred to work with more general matrices in which, for example, there may be commodities (such as corn) that can be used both as means of production (that produce more corn and also breakfast cereals) and as means of subsistence (along with breakfast cereals). As often happens, greater generality yields simpler equations. On the other hand, Marx was quite conscious that the complete separation between means of production and means of subsistence that he used in his tables was just another simplification meant to make his analysis easier, as this quotation shows: “Commodities are bought either as means of production or means of subsistence to enter productive or individual consumption. It does not alter matters that some commodities may serve both purposes” (B3, 186).13

If we avoid the special assumption of equal value compositions in all branches, values are in general different from production prices and it is necessary to use the production price system (8.17) corresponding to this particular case:

\[
\begin{bmatrix}
    (1 + \rho) A_I & 0 & (1 + \rho) \ell_1 \\
    (1 + \rho) A_{II} & 0 & (1 + \rho) \ell_2 \\
    0 & c_{L} & 0
\end{bmatrix}
\begin{bmatrix}
    p_I \\
    p_{II} \\
    w
\end{bmatrix}
= 
\begin{bmatrix}
    p_I \\
    p_{II} \\
    w
\end{bmatrix}

\]

13Marx also refers to this in Chapter 49 of Book III where he also mentions two concrete examples: “We divided all capital there into two big classes: Class I, producing means of production, and Class II, producing articles of individual consumption. The fact that certain products may serve equally well both for personal consumption and as means of production (a horse, grain, etc.) does not invalidate the absolute correctness of this division in any way” (MECW 37, 823).
If we replace the third equation \((c_L p_{II} = w)\) in the first two we get the following system:

\[
(1 + \rho) \begin{bmatrix}
A_I & \ell_1 c_L \\
A_{II} & \ell_2 c_L
\end{bmatrix} \begin{bmatrix}
p_I \\
p_{II}
\end{bmatrix} = \begin{bmatrix}
p_I \\
p_{II}
\end{bmatrix}.
\]

Appendix to Chapter 11

Bibliographic Note: Tugan-Baranowsky and Bortkiewicz on the ‘transformation problem’

Tugan-Baranowsky (1905) was apparently the first to show why Marx’s procedure for obtaining the global profit rate is not valid. He showed arithmetically how one could go from a SR numerical table with three departments (means of production, workers’ means of consumption and capitalists’ means of consumption) of the type Marx had used but where all valuations are in production prices, to a table of the same type where all valuations are in values. In his arithmetic procedure the dollar amounts were assumed given (that is quantities multiplied by production prices) as well as the profit rate and the amount of labor employed in the department producing means of production. And he obtained the same type of 3 × 3 table but with valuations in values as well as the rate of surplus value (assumed to be the same in all 3 departments). As Bortkiewicz (1907a, 199) writes, it is “the opposite of that which Marx tried to solve.”

Recognizing Tugan Baranowsky’s contribution, Bortkiewicz (1907a) addressed the inverse process of transforming values into production prices. But instead of doing it numerically he did it algebraically. For this he used a table like those of Marx and with the same three departments as Tugan-Baranowsky. Let us take a transactions table like that of Table 10 (which corresponds to Table 9) but with three departments and with values instead of production prices and let us ignore the possible matrix decomposability (and hence the zeros A may have). Using as an illustration the data for our Table 13, for example, Bortkiewicz calls \(C_1\) our \(q_1^Q a_{11} v_1\) (constant capital in branch 1) and \(V_1\) our \(q_1^Q \ell_1 c_L N v_2\) (variable capital in branch 1), etc. Hence, he begins with the following equations:

\[
\begin{align*}
C_1 + V_1 + S_1 &= C_1 + C_2 + C_3 \\
C_2 + V_2 + S_2 &= V_1 + V_2 + V_3 \\
C_3 + V_3 + S_3 &= S_1 + S_2 + S_3
\end{align*}
\]

in which the rates of surplus value are the same in the three departments:

\[
e = \frac{S_1}{V_1} = \frac{S_2}{V_2} = \frac{S_3}{V_3}.
\]

Hence the system is

\[
\begin{align*}
C_1 + (1 + e) V_1 &= C \\
C_2 + (1 + e) V_2 &= V \\
C_3 + (1 + e) V_3 &= eV
\end{align*}
\]

where \(C \equiv \Sigma C_i\), \(V \equiv \Sigma V_i\). This is the starting point and hence \(e\) and the \(C_i\) and \(V_i\) (as well as \(C\) and \(V\)) are data. He then poses the following alternative system for production prices and the profit rate \(\rho\):

\[
\begin{align*}
(1 + \rho) (C_1 x_1 + V_1 x_2) &= C x_1 \\
(1 + \rho) (C_2 x_1 + V_2 x_2) &= V x_2 \\
(1 + \rho) (C_3 x_1 + V_3 x_2) &= eV x_3
\end{align*}
\]
where \( x_i \equiv p_i/v_i \) are the production prices divided by the respective values. Hence the system has three equations with four unknowns \((\rho, x_1, x_2, x_3)\). But if a numeraire is defined, the unknowns are only 3. To make our calculations as easy as possible, let us choose as numeraire \( p_1 = v_1 \), and hence \( x_1 = 1 \). In that case the first two equations can be solved for \( \rho \) and \( x_2 \). Dividing the second by the first we get

\[
\frac{C_2 + V_2 x_2}{C_1 + V_1 x_2} = \frac{V x_2}{C},
\]

which yields the quadratic equation \((VV_1) x_2^2 + (VC_1 - CV_2) x_2 - CC_2\) which has as possible solutions

\[
x_2 = \frac{CV_2 - VC_1 \pm \left( (CV_2 - VC_1)^2 + 4VV_1 CC_2 \right)^{1/2}}{VV_1}.
\]

It is obvious that if we use the subtraction alternative in the numerator we get a negative solution, which has no economic sense. Hence, we use the addition alternative to get the value of \( x_2 \) expressed in terms of the data, which we can call \( x_2^* \). Hence \( p_2^* = x_2^* v_2 \) is the solution for \( p_2 \). And we already have \( p_1^* = v_1 \) due to the numeraire chosen. From the second equation we get the solution for the profit rate

\[
\rho^* = (C_2 + V_2 x_2^*) / V x_2^* - 1.
\]

And from the third,

\[
x_3^* = (1 + \rho^*) \left( \frac{C_3 + V_3 x_2^*}{eV} \right),
\]

and hence \( p_3^* = x_3^* v_3 \). Thus, Bortkiewicz was able to ‘transform’ the values and rate of surplus value into the production prices and rate of profit.\(^{14}\) In the background were the quantities and the input-output coefficients. But this is equally valid for what Marx did with his tables. The explicit use of the dual quantities and prices equations allowed us make the ‘transformations’ in both directions as a trivial matter, as we did in (8.29) and (8.30), only using the two valuation systems. But we know that explicitly posing the corresponding system of quantities is fundamental for representing the complete system. As we showed in the Appendix to Chapter 10, not doing so led Sraffa to some confusion.

\[\Box\] Mathematical Note: Marx’s approximation is the first step of an algorithm that converges to production prices

An interesting question is how good Marx’s approximation to production prices and the profit rate was. In this Note we show that Marx’s approximate formulas are the first step of an algorithm that quickly converges to the true production prices and profit rate, that is, those that are the solution to the production price system of equations correctly formulated. Marx’s approximations to \( \rho \) and \( p \) are the following:

\[
\begin{align*}
\rho^0 &= \frac{e}{\kappa + 1}, \quad e = \frac{q^K c_K v}{q^L c_L v}, \\
p^0 &= (1 + \rho^0) \left( A + \ell c_L \right) v,
\end{align*}
\]

\(^{14}\)Bortkiewicz’ calculations were more complicated because he did not use the most convenient numeraire. But this is not important.
whereas the exact formulas are given by

\[
\begin{align*}
\rho &= \frac{e^p}{\kappa^p + 1}, \quad e^p = \frac{q^K c_K p}{q^L c_L p}, \quad \kappa^p = \frac{q^Q A p}{q^L c_L p} \\
\rho &= (1 + \rho) (A + \ell c_L) p.
\end{align*}
\]

Let us define an algorithm that starts with (11.11) and (11.12), where for the first step we define \( \rho_0 \equiv \rho^0, \ e^{p^0} \equiv e, \ \text{and} \ \kappa^{p^0} \equiv \kappa, \) and for the subsequent steps \( (n = 1, 2, \ldots) \) we have:

\[
\begin{align*}
\rho_n &= \frac{e^{p^n}}{\kappa^{p^n} + 1}, \quad e^{p^n} = \frac{q^K c_K p^{n-1}}{q^L c_L p^{n-1}}, \quad \kappa^{p^n} = \frac{q^Q A p^{n-1}}{q^L c_L p^{n-1}} \\
p^n &= (1 + \rho^{n-1}) (A + \ell c_L) p^{n-1}.
\end{align*}
\]

A formal proof of convergence can be found in Mori (2010), who attributes the algorithm to Georg von Charasoff.\(^{15}\) In the Numerical Exercise \#4 we illustrate how the algorithm converges quickly to values very near the correct ones (to the fourth decimal). \(\blacksquare\)

**Numerical Exercise \#4: Marx’s approximation to production prices and the profit rate**

The data for the exercise are the following:

\[
A = \begin{bmatrix} 0.34 & 0.63 \\ 0.48 & 0.25 \end{bmatrix}, \quad \ell = \begin{bmatrix} 0.19 \\ 0.11 \end{bmatrix}, \quad c_K = \begin{bmatrix} 0.35 \\ 0.31 \end{bmatrix}, \quad q^K = 100.
\]

First we can verify that if the social matrix in (8.6) is to have a dominant eigenvalue of 1 it is necessary that \( e = 0.542. \) In this case the corresponding eigenvector, normalized so that its third element is one is: \( (v \ 1)^T = (1.0996 \ 0.85043 \ 1)^T. \) Second, define

\[
q \equiv \begin{bmatrix} q^Q & q^L \end{bmatrix}, \quad M \equiv \begin{bmatrix} A & \ell \\ c_L & 0 \end{bmatrix}, \quad c_K \equiv \begin{bmatrix} c_K & 0 \end{bmatrix}.
\]

Then (8.1) can be written as \( qM + q^K c_K = q, \) from which we get \( q = q^K c_K (I - M)^{-1}, \) which with the given data allow us to calculate \( q = (835.19 \ 859.56 \ 253.24). \) Third, with the data now available we get

\[
e = \frac{q^K c_K v}{q^L c_L v} = 0.542, \quad \kappa = \frac{q^Q A v}{q^K c_K v} = 8.501, \quad \rho^0 = \frac{e}{\kappa + 1} = 0.057.
\]

Hence, Marx’s approximation to the production prices (11.12) is

\[
p^0 = (1 + \rho^0) (A + \ell c_L) v = (1 + 0.057) \begin{bmatrix} 1.0328 \\ 0.81175 \end{bmatrix} = \begin{bmatrix} 1.0918 \\ 0.85805 \end{bmatrix}
\]

To check that the data are not close to the very special cases we considered above in which either prices or quantities have the structure of a right or left eigenvector of the input-output matrix \( A, \) we observe that 1) the value compositions of the two branches

\(^{15}\)Abraham-Frois and Berrebi (1976) also formulate a similar algorithm but take \( \rho \) as constant, which invalidates their exercise.
are quite different, and hence we are not close to the case of equal value compositions of capital\textsuperscript{16}:

\[
\kappa_1 = \frac{A_1v}{(\ell_cL)_1v} = 7.3826, \quad \kappa_2 = \frac{A_2v}{(\ell_cL)_2v} = 10.380.
\]

and 2) neither are we too close to the other very special case in which the vector of gross outputs is proportional to the consumption (or net output) vector, since

\[
C = q^Lc_L + q^Kc_K = \begin{bmatrix}
138.63 & 118.5
\end{bmatrix}
\]

\[
\frac{q^Q_1}{C_1} = \frac{835.18}{138.63} = 6.0245, \quad \frac{q^Q_2}{C_2} = \frac{859.55}{118.5} = 7.2536.
\]

Taking the data shown above for the start of the algorithm, Table 14 shows the first four iterations.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(e^{\nu^i})</th>
<th>(\kappa^{\nu^i})</th>
<th>(\rho_i)</th>
<th>((p^i)’)</th>
<th>(p^i_1/p^i_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.54192</td>
<td>8.5014</td>
<td>0.057036</td>
<td>(1.0918, 0.85805)’</td>
<td>1.2724</td>
</tr>
<tr>
<td>1</td>
<td>0.54171</td>
<td>8.5075</td>
<td>0.056977</td>
<td>(1.0939, 0.85601)’</td>
<td>1.2779</td>
</tr>
<tr>
<td>2</td>
<td>0.54177</td>
<td>8.5059</td>
<td>0.056993</td>
<td>(1.0933, 0.85657)’</td>
<td>1.2764</td>
</tr>
<tr>
<td>3</td>
<td>0.54175</td>
<td>8.5063</td>
<td>0.056989</td>
<td>(1.0935, 0.8564)’</td>
<td>1.2769</td>
</tr>
<tr>
<td>4</td>
<td>0.54176</td>
<td>8.5062</td>
<td>0.056999</td>
<td>(1.0934, 0.85646)’</td>
<td>1.2767</td>
</tr>
</tbody>
</table>

By the fourth iteration we have reached the exact values to the fourth decimal for the relative prices \(p^i_1/p^i_2\) (since using the ‘exact’ vector of production prices we have \(p_1/p_2 = 1.6869/1.3213 = 1.2767\)). Figure 6 shows that the algorithm produces an oscillating but quick convergence of the relative production price \(p^i_1/p^i_2\) (represented by a solid line) towards the correct value \(p_1/p_2\) (represented by a a dashed line).

Figure 6

\textsuperscript{16}We remind the reader that all the value compositions of capital are equal if and only if all the production price compositions are equal.
Chapter 12  THE TURNOVER OF CAPITAL AND THE INDUSTRIAL CYCLE

Fixed capital and circulating capital
One of the topics Marx addresses in Book II is how the valorization of capital in the production process is influenced by the fact that some of the means of production can be used during several production cycles without having to replace them. In Marx’s words, “a portion of the constant capital retains that definite use form in which it enters into the process of production” (B1, 159). He specifically mentions the buildings in which the labor process takes place and the machinery used, including both in the category of instruments of labor. Since the instruments of labor “never leave the sphere of production, once they have entered it” –given that “Their function holds them there” (B2, 160)–, they are fixed in that sphere once they have been purchased and installed or constructed. Hence it is natural to call the capital thus tied up ‘fixed capital’. On the other hand, Marx calls ‘circulating capital’ the capital invested in the rest of the elements of constant capital (raw materials and auxiliary materials) and in variable capital (labor power).

Table 15 reflects this classification of the elements of productive capital. It is superimposed on the classification of the elements of productive capital into constant and variable capital, which as we have seen was crucial since only labor power generates value, and was also fundamental in the analysis of the effects of changes in wages on the production prices. As the table shows, labor power is a component of both variable capital and circulating capital. But the elements of constant capital can be a part of fixed capital (buildings and machinery) or of circulating capital (raw materials and auxiliary materials).

Table 15

Classification of the Elements of Productive Capital

<table>
<thead>
<tr>
<th>Constant Capital</th>
<th>Variable Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>(according to whether labor transfers existing value or adds value)</td>
<td></td>
</tr>
<tr>
<td>Fixed Capital</td>
<td>Circulating Capital</td>
</tr>
<tr>
<td>(according to whether disbursed capital is recovered in more or less than a year)</td>
<td></td>
</tr>
<tr>
<td>Buildings</td>
<td>Raw Materials</td>
</tr>
<tr>
<td>Machinery</td>
<td>Auxiliary Materials</td>
</tr>
</tbody>
</table>

Marx points out that although “The instruments of labour properly so called, the material vehicles of the fixed capital, are consumed only productively and cannot enter into individual consumption” (B2, 162; italics added) there also exist elements of fixed capital that can, as is the case of means of transportation, since their useful effect is for example a part of the traveler’s consumption. In agriculture, fixed capital is also made up of “the substances added for the improvement of the soil”, which “pass partly into the plants raised and help to form the product. On the other hand their effect is
distributed over a lengthy period, say four or five years” (Ibid.). Marx also remarks that the classification of constant capital as fixed or circulating does not depend on its intrinsic properties but instead on its function in the productive process. And the same commodity can have more than one function. He gives the example of oxen: “As a beast of toil an ox is fixed capital. If he is eaten, he no longer functions as an instrument of labour, nor as fixed capital either” (Ibid.). Hence, “What determines that a portion of the capital value invested in means of production is endowed with the character of fixed capital is exclusively the peculiar manner in which this value circulates.”

The Physiocrats and fixed and circulating capital

We have seen in Chapters 3 and 10 Marx’s enthusiastic acknowledgement of the works of the Physiocrats in having laid the foundation for “the analysis of capital”, establishing the cyclical character of capital, and initiating the study of Simple Reproduction. Marx also highlights their contribution to the distinction between fixed and circulating capital: “In Quesnay the distinction between fixed and circulating capital presents itself as avances primitives and avances annuelles” (B2, 190). Since this economist only considered as productive capital the capital invested in agriculture by a tenant farmer, he only applied this distinction to this branch of production. And he distinguished between the annual advances for the cultivation of land and the original disbursement of capital in any farming enterprise. Later Physiocrats such as Dupont de Nemours, Le Trosne, and Turgot tended to use the term ‘capital’ instead of ‘advances’: “Turgot employs the term capital more regularly for avances, and identifies the avances of the manufacturers still more with those of the farmers” (B2, 191; footnote 23). Hence, capital, whether fixed or circulating, was losing its identification with the money invested in agriculture, and leading to the generalization of Classical economists that capital was the investment of money in any productive branch with the aim of earning a profit. However, in contrast to the Physiocrats, Smith and Ricardo tended to confuse fixed capital with constant capital. Ricardo even held the distinction between fixed and circulating capital to be ‘non essential’ (whereas the Physiocrats believed it was essential indeed) and, instead of concentrating on the distinction between constant and variable capital for the analysis of the effects of changes in the wage rate on prices, Smith and Ricardo pointed to the distinction between fixed and circulating capital.

Referring to Ricardo, Marx writes that “he commits the gravest errors on account of his confusing fixed and circulating with constant and variable capital. Indeed, he starts his analysis on an entirely wrong basis” (B2, 225-6).

The turnover of capital

In Book I and Part I of Book II, Marx assumed for the sake of simplifying that the capital invested (to be used as denominator in the calculation of the profit rate), i.e., the value of the means of production and the labor power used on average in the production process, is identical to the value of the means of production and labor power consumed in the production process. And this is what we have been assuming in the preceding chapters. In Part II of Book II (titled “The Turnover of Capital”), Marx extensively analyzes the turnover of capital and the associated concepts of fixed

---

1Ricardo writes: “According as capital is rapidly perishable, and requires to be frequently reproduced, or is of slow consumption, it is classed under the heads of circulating, or of fixed capital.” In the footnote he adds: “A division not essential, and in which the line of demarcation cannot be accurately drawn” (Principles, Ricardo 2004, Vol. 1, 31).
and circulating capital of Classical political economy with the aim of eliminating this simplifying assumption.

He defines the ‘time of turnover’ of a given capital as the period of time that extends from the moment it is disbursed ‘in a definite form’ to the time of its return in the same form. It is the sum of its ‘time of circulation’ and its ‘time of production’. Within the ‘time of production’ he distinguishes a period of ‘working time’. The latter is always at least a part of the production time, but there are periods in which although “capital is held fast in the sphere of production” (B2, 239) its elements are not engaged in the labor process. He gives several examples in which labor is used “only occasionally during a large portion of the time of production”, such as the grape which after being pressed must ferment, the pottery that must go through a drying process, the winter grain that needs several months to mature, and timber-raising, where after the sowing and other labor activities are performed many years must elapse before the tree is grown and can be further transformed into timber. Hence, the production time of the invested capital can be seen as consisting of two periods. In one the labor process takes place –often intermittently– and surplus value is created. In the rest there is an unfinished product that “is abandoned to the sway of natural processes, without being at that time in the labour process” (B2, 240). The finished product (which is commodity capital) does not exist and hence cannot be converted to money capital until the production period is completed. The length of the turnover period is thus in many cases considerably affected by the length of the production time in which there is no working time.

The fact that the elements of fixed capital can last much longer than the time of production had important consequences from the point of view of the physical production process (in their role as gradually eroding use values), from the point of view of the valorization process (since only a fragment of their value could be passed on to the finished product), and from the point of view of the circulation process (since what circulates each production period is only that fragment of their value). This made it necessary to distinguish the existing stock of use values from the flow of its gradual wearing out and its corresponding value. In the case of an instrument of labor, for example,

so long as it is still effective and need not yet be replaced by a new one of the same kind, a certain amount of constant capital value remains fixed in it, while the other part of the value originally fixed in it is transferred to the product and therefore circulates as a component part of the commodity supply. The longer an instrument of labour lasts, the slower it wears out, the longer will its constant capital value remain fixed in this use form. But whatever may be its durability, the proportion in which it yields value is always inverse to the entire time it functions (B2, 160-1).

Hence, in the case of the elements of fixed capital what circulates during the turnover period is only the value of its wearing out, its amortization, and not its use value since the latter remains fixed within the sphere of production. The value of the elements of fixed capital have ‘a double existence’: a stock of use value that (possibly with some repairing expenditures) can be used during a long period of time, and a flow of value transmitted gradually (as the wearing out takes place) to the finished products (in the form of commodity capital). When the latter are sold for money a fund can be accumulated in such a way that when the element of fixed capital is no longer usable a replacement can be purchased:
The product converts itself by means of its circulation from a commodity into money; hence the same applies to the value part of the instrument of labour circulated by the product, and this value drips down in the form of money from the process of circulation in proportion as this instrument of labour ceases to be a depository of value in the process of production. Its value thus acquires a double existence. One part of it remains attached to its use form or bodily form belonging in the process of production. The other part detaches itself from that form in the shape of money. In the performance of its function that part of the value of an instrument of labour which exists in its bodily form constantly decreases, while that which is transformed into money constantly increases until the instrument of labour is at last exhausted and its entire value, detached from its corpse, is converted into money (B2, 165-6).

As soon as the fixed element, such as buildings, machinery, etc., has been worn out, and can no longer function in the process of production, its value exists alongside it fully replaced by money... This money then serves to replace the fixed capital (or its elements, since its various elements have different durabilities) in natura (B2, 450).

The turnover of capital and the annual profit rate

When a turnover of capital concludes, the money initially disbursed \( M \) is recovered in its monetary form \( M' \) and increased by profit (assuming the finished product is sold). Different branches of production \( j \) have different turnover periods (or times) \( r_j \). For example, if we compare shipbuilding \( j_1 \) with the production of bicycles \( j_2 \), it is obvious that \( r_{j_1} > r_{j_2} \). Marx adopts the year as the unit of measurement for the turnover period (basically because the four seasons of the year are a natural reference in agriculture). He uses the symbol \( T \) to represent that natural unit. Hence, if time is measured in months, \( T = 12 \). He writes:

If we designate the year as the unit of measure of the turnover period [and the latter] by \( T \), the time of turnover of a given capital by \( t \), and the number of its turnovers by \( n \), then \( n = T/t \). If, for instance, the time of turnover \( t \) is 3 months, then \( n \) is equal to 12/3, or 4; capital is turned over four times per year. If \( t = 18 \) months, then \( n = 12/18 = 2/3 \), or capital completes only two-thirds of its turnover in one year. If its time of turnover is several years, it is computed in multiples of one year.

From the point of view of the capitalist, the time of turnover of his capital is the time for which he must advance his capital in order to create surplus value with it and receive it back in its original shape (B2, 159; text within brackets added).

The aggregate turnover of an advanced capital is the average turnover of its various constituent parts (B2, 185).

In Part I of Book III Marx again addresses the turnover of capital, this time in relation to the determination of the profit rate, especially in Chapters 3 ("The relation of the rate of profit to the rate of surplus value") and 4 ("The effect of the turnover on the rate of profit"). The manuscripts of parts of these chapters were some of the least polished left by Marx for Engels' edition. In his Preface to Book III (written in 1894), Engels states that the various difficulties he encountered in preparing the manuscript
for publication made him resort to his friend Samuel Moore, who was a mathematician. He writes:

There was a series of uncompleted mathematical calculations for Chapter III, as well as a whole, almost complete, notebook dating from the seventies, which presents the relation of the rate of surplus value to the rate of profit in the form of equations. My friend Samuel Moore... undertook to edit this notebook for me... It was from his summary, with occasional use of the main manuscript, that I then compiled Chapter III. Nothing but the title was available for Chapter IV. But since its subject-matter, the influence of turnover on the rate of profit, is of vital importance, I have written it myself, for which reason the whole chapter has been placed in brackets. It developed in the course of this work that the formula for the rate of profit given in Chapter III required modification to be generally valid (B3, 8).

It is clear, therefore, that Chapter 4 of Book III on “The Effect of the Turnover on the Rate of Profit” was written by Engels (probably with Moore’s assistance), yet trying to ensure it was in complete accordance with Marx’s ideas and numerical exercises. It contains a very interesting extension of the analysis in the preceding chapters of Book III concerning the calculation of the profit rate when there are different turnover periods in the different industrial branches. And it is totally compatible with what Marx wrote on the turnover of capital in general, and on the turnover of its particular portions, whether they are part of fixed or of circulating capital.

Analytical formulation of capital turnover: stocks and flows

So far we have modeled CCP under the same simplifying assumption Marx extensively used, i.e., that it is not necessary to distinguish between the productive consumption of inputs (means of production or labor power) in the model’s reference time period and the stocks of these inputs that are tied to the production process. That is why we only needed the input-output matrix $[A\ell]$ (or $[A\ L]$ if there is skilled labor in addition to simple labor). In this subsection we show how the price system can be modified so that the various means of production and labor powers can have different turnover periods in general and even in each branch of industry in which they are used. The distinction between the stock that is tied down and the flow that is consumed is important because the rate of profit (and the interest rate when it is relevant) must be calculated with respect to the invested (or disbursed) capital, whereas the cost of production is measured according to the means of production and labor powers consumed in production.

To achieve this, in this subsection we distinguish the productive consumption (or flow) matrix $[A\ L]$ we have been using from the stock matrix $[A^s\ L^s]$ we now introduce. In branch $i$ there is a consumption of $A_{ij}$ units of input $j$ and $L_{ij}$ units of labor power of type $j$ in a year for each unit of $i$ produced. But on average $A^s_{ij}$ units of input $j$ and $L^s_{ij}$ units of labor power $j$ are used (or tied up) for each unit of $i$ produced.

Consider the case of variable capital (which is a part of circulating capital). Assume that in a certain branch of industry wage workers are paid monthly the amount $E$. Then in a year the labor cost is $12E$. But how much capital is tied down as variable capital depends on the length of the turnover of capital in the branch in question. If the output of this branch requires three months to be produced and can be sold immediately (or its production requires two months and its sale requires one), the amount $3E$ turns over

\[\footnote{We are leaving out all financial subtleties here to better focus on Marx’s fundamental ideas.}\]
four times per year, since after every three months the capitalist recovers this capital. Hence, whereas the yearly labor cost is $12E$, the variable capital tied up is only $3E$ on average. This example illustrates why matrices $L$ and $L^S$ are generally quantitatively different. The same can be said of $A$ and $A^S$. But what we shall do is more general, since we let each input to each product have different circulating characteristics, both for the elements of constant capital as for those of variable capital.

This implies that $[A\; L]$ and $[A^S\; L^S]$ not only differ quantitatively, but also dimensionally.\footnote{Brödy (1970) has a rigorous explanation of the dimensional aspect.} If $A_{ij}$ for example, measures gallons of gasoil per ton of iron, $A^S_{ij}$ measures gallons of gasoil-year per ton of iron. If $q^Q_i$ represents the tons of iron produced in a year, $q^Q_i A_{ij}$ represents the gallons of gasoil ($j$) consumed in a year as input in the production of iron ($i$), whereas $q^Q_i A^S_{ij}$ represents the gallons of gasoil that are tied up on average in the production of iron during a year. The relation between the coefficients of these matrices can be formally expressed as $A^S_{ij} = r^C_{ij} A_{ij}$ where $r^C_{ij}$ is the turnover period of element $j$ of constant capital in the production of element $i$ of constant capital (the superindex $C$ refers to ‘constant capital’) and hence $n^C_{ij} = 1/r^C_{ij}$ is the turnover frequency (or number of turnovers) of this element in a year (if the year is used to measure time). If in the production of iron the turnover period of gasoil is three months, that is, a fourth of a year, then $r^C_{ij} = 0.25$ years, and the turnover frequency of gasoil in the production of iron is $n^C_{ij} = 4$ times per year. Then the quantity of gasoil that is tied down on average in the production of iron is $\tilde{q}^Q_j = q^Q_i A^S_{ij} = q^Q_i r^C_{ij} A_{ij} = 0.25 q^Q_i A_{ij}$, that is a fourth of the amount consumed. And if in the production of iron ($i$) the turnover period of an oven ($k$) is 20 years, then $r^C_{ik} = 20$ years, and the quantity of ovens tied down on average in the production of iron is twenty times the annual consumption: $\tilde{q}^Q_k = q^Q_i A^S_{ik} = q^Q_i r^C_{ik} A_{ik} = 20 q^Q_i A_{ik}$. Similarly, we define $L^S_{ij} = r^V_{ij} L_{ij}$ and $n^V_{ij} = 1/r^V_{ij}$ (where the superindex $V$ refers to ‘variable capital’).

We can illustrate the case of fixed capital in a simple way assuming for an instant that all constant capital is made up of machines of different types that all last 3 years ($r^C_{ij} = 3$ for all $i, j$). Hence all constant capital is fixed capital. Then the stock of the different types of machines that are tied down in any given period is three times the annual productive consumption of those machines: $\tilde{q}^Q = q^Q A^S = q^Q 3A = 3q^Q A$. In any given branch and time period, a third of the capitalists use machines (of a given type) that have had one year of use, a third use machines with two years of use, and another third use new machines. Those that are using machines with two years of use, for example, will have already accumulated a fund with two years’ worth of amortization and at the beginning of next year will be in a position to buy new machines. Hence, the number of machines that must be replaced each year is only a third of the total stock $(1/3) \tilde{q}^Q = q^Q A$, and this is also the productive consumption (wearing out) of the machines.

### The production prices and profit rate system

The complications introduced as a consequence of eliminating the special assumption that we have so far been using obviously affects the price system because the profit rates must be calculated on the capital that is tied up (or invested). Let us assume that, as far as the flows of transactions and allocations of labor and capital, the relevant dual equations are (9.1) and (9.2) of Chapter 9. Assuming also that the profit rates are the same in all branches, after lifting the special simplifying assumption used so
far the system, instead of (9.3) is:

\[
\begin{pmatrix}
A + \rho A^S & L + \rho L^S \\
C_L & 0
\end{pmatrix}
\begin{pmatrix}
p \\
w
\end{pmatrix}
=
\begin{pmatrix}
p \\
w
\end{pmatrix}.
\] (12.1)

The global profit rate is still the one that makes the dominant eigenvalue of the social matrix of (12.1) equal to 1. And the vector of production prices and wages is still the right eigenvector associated with this eigenvalue. The only difference is that the social matrix is now more complicated. It is evident that system (12.1) reduces to (9.3) in the special case \(A^S = A\) and \(L^S = L\). On the other hand, the formula that expresses the production prices in terms of the wages and the rate of profit is altered. Instead of (9.19), we now have \(p = \mathcal{B}^S (\rho) L w\), where \(\mathcal{B}^S (\rho) \equiv [I - (A + \rho A^S)]^{-1} (L + \rho L^S)\) again a square matrix that is strictly increasing with \(\rho\). Also, we can verify that if \(\eta \pi = \rho (A^S p + L^S w)\) is used to eliminate \(\eta\) from system (9.2), the first two equations reduce to those of (12.1).

If we disregard Marx’s distinction between labor time and production time (i.e., if we assume they coincide) the values system ((9.6) or (9.13) according to the assumption used) is not affected by the elimination of the simplifying assumption that \(A^S = A\) and \(L^S = L\). This is due to the fact that the values system only depends on the flows of production and reproduction. Moreover, under SR neither is the quantities system (9.10) affected, since when there is no growth the distinction between stocks and flows is not important: the flow matrices are sufficient to describe production and reproduction.4

Following the same procedure used above to obtain an expression for the global profit rate but using (12.1) instead of (9.3) we get:

\[
\rho = \frac{S^p}{C^p + V^p} = \frac{q^K c_{K P}}{q^Q A^S p + q^Q L^S w} = e^p n^V_r \frac{n^V_r}{\kappa^P + 1}
\] (12.2)

where \(C^p \equiv q^Q A^S p\) and \(V^p \equiv q^Q L^S w\) are the two big components of invested (or tied up) capital and we have defined the average frequency of turnover for variable capital \(n^V_r\) (that corresponds to the average turnover period of variable capital \(\tau^V_r\)), and the value composition of invested capital \(\kappa^P\), as:

\[
e^p = \frac{q^K c_{K P}}{q^Q L w} = \frac{q^K c_{K P}}{q^Q C_{L P}}, \quad n^V_r = \frac{1}{\tau^V_r} = \frac{q^Q L C_{L P}}{q^Q L^S C_{L P}}, \quad \kappa^P = \frac{q^Q A^S p}{q^Q L^S w}.
\]

Leaving aside the fact that all valuations are in terms of production prices (modified by turnover), the changes in formulas (9.4) and (9.5) for the global rate of profit are the following: 1) the average composition of capital is defined in terms of capital invested (or tied up) and 2) the rate of surplus value is multiplied by the average frequency of turnover for variable capital. It should be noted that, taking account Marx’s approximation to production prices, 2) is quite well explained in Chapter 4 of Book III (where \(p’\) is the profit rate and \(s’\) is the surplus value rate):

the formula \(p’ = s’ \times v/C = s’ \times v/(c + v)\), is correct only so long as the \(v\) in the numerator is the same as that in the denominator. In the denominator \(v\) stands for the entire portion of the total capital used on

4In Chapter 14 we will see (through systems (14.37), (14.38), and (14.39)) how the dual systems of quantities and prices are modified when, under Extended Reproduction and one type of labor (simple labor), we eliminate the assumption \(A^S = A\).
an average as variable capital for the payment of wages. The $v$ of the numerator is primarily only determined by the fact that a certain quantity of surplus value $s$ is produced and appropriated by it, whose relation to it $s/v$ is $s'$, the rate of surplus value. It is only along these lines that the formula $p' = s/(c + v)$ is transformed into the other: $p' = s' \times v/(c + v)$... (B3, 77).

To make the formula precise for the annual rate of profit, we must substitute the annual rate of surplus value for the simple rate of surplus value, that is, substitute $S'$ or $s' \times n$ for $s'$. In other words, we must multiply the rate of surplus value $s'$, or, what amounts to the same thing, the variable capital $v$ contained in $C$, by $n$, the number of turnovers of this variable capital in one year. Thus we obtain $p' = s' \times n \times v/C$, which is the formula for calculating the annual rate of profit (B3, 78).

Notice that here the assertion “we must multiply the rate of surplus value $s'$... by $n$, the number of turnovers of this variable capital in one year” precisely refers to the multiplication of $c^n$ by $n^c_p$, in expression (12.2) (if we ignore Marx’s approximation that uses values for cost valuations). Also, Marx’s $v/C$ is $v/(c+v) = 1/(c/v+1) = 1/(c+1)$, where $c$ and $v$ are the invested amounts of constant and variable capital. Hence, the only difference with (12.2) is that values are used in the valuation of costs and tied up capital. As we have seen, this is the approximation Marx found for expressing production prices on account of his insufficient mathematical training.

The industrial cycle
Marx wanted to study in Capital “the inner organisation of the capitalist mode of production, in its ideal average, as it were,” i.e., in its tendencies, “because the actual movement of competition belongs beyond our scope” (B3, 818). However, reflections on and analyses of “the periodic cycle, through which modern industry runs, and whose crowning point is the universal crisis” is present in all three Books of Capital. In the Afterword to the Second German edition (written in January 1873) Marx refers to the crisis of 1825 as inaugurating the periodical industrial cycles of the modern era. It was followed by the crises of 1839, 1847, 1857, and 1866 (the last crisis included in the first German Edition of Capital). In the same Afterword Marx refers to an upcoming general crisis: “That crisis is once again approaching, although as yet but in its preliminary stage.” He was anticipating by several months the ‘Panic of 1873’ that began in the Vienna stock market in May of that year and propagated to the New York stock exchange in September, initiating a prolonged depression and great unemployment in Europe and the U.S.A. According to Holbbsawm (1989), it inaugurated ‘The Era of Empire’ (the name he gave to the period 1875-1914) during which the main powers divided among themselves the political and economic domination over a major part of the planet. In most cases these powers self-designated themselves as ‘Empires’ (Great Britain, Russia, Germany, Austria, Japan, China, and Persia). And the voracious dynamics of this process led to World War I (prelude to the even more devastating World War II).

According to Marx “The enormous power, inherent in the factory system, of expanding by jumps, and the dependence of that system on the markets of the world, necessarily beget feverish production, followed by overfilling of the markets, whereupon contraction of the markets brings on crippling of production. The life of modern industry becomes a series of periods of moderate activity, prosperity, overproduction, crisis and stagnation” (B1, 455). The phase of crisis and stagnation generated high unem-
ployment and insecurity in the lives of factory workers, and among capitalists would unleash “the most furious combat for the share of each in the markets”, leading to a general reduction in prices and wages. “Besides the rivalry that this struggle begets in the application of improved machinery for replacing labour power, and of new methods of production, there also comes a time in every industrial cycle, when a forcible reduction of wages beneath the value of labour power, is attempted for the purpose of cheapening commodities” (B1, 455-7). Inversely, in the phase of prosperity “one could only remark that crises are always prepared by precisely a period in which wages rise generally and the working class actually gets a larger share of that part of the annual product which is intended for consumption” (B2, 409-10). For Marx “The crises are always but momentary and forcible solutions of the existing contradictions. They are violent eruptions which for a time restore the disturbed equilibrium” (B3, 248). The main ‘contradiction’ was “that the capitalist mode of production involves a tendency towards absolute development of the productive forces... while, on the other hand, its aim is to preserve the value of the existing capital and promote its self-expansion to the highest limit” (Ibid.). But the dynamics of the industrial cycle leads to innovations that cheapen the elements of capital and hence a “periodical depreciation of existing capital – one of the means immanent in capitalist production to check the fall of the rate of profit and hasten accumulation of capital value through formation of new capital”. This “disturbs the given conditions, within which the process of circulation and reproduction of capital takes place, and is therefore accompanied by sudden stoppages and crises in the production process” (Ibid.).

On the other hand, the industrial cycle took place mounted on certain tendencies, as those we will consider in Chapter 17. One of them, often highlighted by Marx, is that increased competition during the descendent phase of the cycle leads the capitalist to “lower the individual value of his total product below its general value by means of new machines, new and improved working methods, new combinations, i.e., to increase the productive power of a given quantity of labour, to lower the proportion of variable to constant capital, and thereby to release some labourers” (B3, 253). Marx held that this lowering of “the proportion of variable to constant capital”, which as we have seen he called “increase in the value composition of capital”, generated a “tendency of the rate of profit to fall” that was to a large extent counteracted in a complex and irregular way by various factors. And it fundamentally generated massive unemployment, and “overpopulation” of workers: “The circumstances which increased the productive power of labour, augmented the mass of produced commodities, expanded markets, accelerated accumulation of capital both in terms of its mass and its value, and lowered the rate of profit – these same circumstances have also created, and continuously create, a relative overpopulation, an overpopulation of labourers not employed by the surplus capital owing to the low degree of exploitation at which alone they could be employed, or at least owing to the low rate of profit which they would yield at the given degree of exploitation” (B3, 254-5).

In modern industrial capitalism, the phase of prosperity of the industrial cycle culminates in a crisis which is an outcome of an excessive extension of credit to insufficiently prudent and foresighted economic agents: “The credit system appears as the main lever of overproduction and overspeculation in commerce solely because the reproduction process, which is elastic by nature, is here forced to its extreme limits” (B3, 438). Marx highlights that this is due in good measure to the fact that “a large part of the social capital is employed by people who do not own it”, i.e., managers who are much less cautious than “the owner, who anxiously weighs the limitations of his

\[5\text{We correct here an obvious slip in MECW 37, which has “farmer” instead of “power”.}\]
private capital in so far as he handles it himself.” Hence, although “the credit system accelerates the material development of the productive forces and the establishment of the world market”, at the same time it makes the crises more severe. Marx emphasizes that the basis for this complex process lies in the fact that “the self-expansion of capital based on the contradictory nature of capitalist production permits an actual free development only up to a certain point, so that in fact it constitutes an immanent fetter and barrier to production” (B3, 438-9). And though the credit system helps “to develop the incentive of capitalist production”, it also leads to the “most colossal form of gambling and swindling, and to reduce more and more the number of the few who exploit the social wealth” (Ibid.). Marx held that these features showed that the Capitalism of modern industry and advanced credit system was a “form of transition to a new mode of production” in which economic planning would play a major role in a context increasingly free from social antagonisms. And this was a major component of his political agenda, which we address in Part IV.

Hoarding and the industrial reserve army

When dealing with accumulation on an extended scale Marx makes the explicit assumption that “the amount of money in the country in question (the velocity of circulation, etc., being constant) should suffice for both the active circulation and the reserve hoard.” The amount of money had to be larger than in SCP since in CCP “the formation of new money capitals keeps pace with the extension of production, so that the material for corresponding hoard formation must be available (B2, 499). And though “commerce in general, even under precapitalist methods” required “a reserve fund of means of payment and purchase”, this was even more so in Capitalism, where commerce developed increasingly and required ever more money for the circulation of commodities both in the domestic and international markets. But Capitalism also developed economies in the use of money through the development of the banking system. In Capitalism, “outlays of money for purchases, collecting money from sales, making and receiving payments, balancing payments, etc.”, were services that the ‘money dealer’ performed “at first as a simple cashier of the merchants and industrial capitalists” (B3, 317).

Marx points out that “On the basis of capitalist production the formation of a hoard as such is never an end in itself but the result either of a stagnation of the circulation —larger amounts of money than is generally the case assuming the form of a hoard— or of accumulations necessitated by the turnover; or, finally, the hoard is merely the creation of money capital existing temporarily in latent form and intended to function as productive capital” (B2, 347). The first of these motives for hoarding (“stagnation of the circulation”) mainly had to do with the industrial cycle, the second (“accumulations necessitated by the turnover”) responded mainly to the need to form a fund for the eventual replacement of the elements of fixed capital, and the third (money capital ‘in latent form’) consisted in “idle, temporarily unemployed capital in the shape of money, including newly accumulated and not yet invested money capital” (B3, 317), and was mainly related to the accumulation of capital by means of the reinvestment of profits.

On the other hand, the accumulation of capital had the effect of progressively generating a “relative surplus population or industrial reserve army”, for along with “the magnitude of social capital already functioning, and the degree of its increase, with the extension of the scale of production, and the mass of the labourers set in motion, with the development of the productiveness of their labour, with the greater
breadth and fullness of all sources of wealth, there is also an extension of the scale on which greater attraction of labourers by capital is accompanied by their greater repulsion” (B1, 625). Hence, “a surplus labouring population is a necessary product of accumulation or of the development of wealth on a capitalist basis” (B1, 625).

Changes in hoarding and in the worker surplus population play an essential rule in the sketch Marx makes of a theory of the industrial cycles of the central half of the 19th century, that “decennial cycle (interrupted by smaller oscillations), of periods of average activity, production at high pressure, crisis and stagnation.” He distinguishes the ‘capital already functioning’ from the ‘absolute wealth’ of capitalists. The former is only the ‘elastic part’ of the latter, and the other part is the hoard that is kept in reserve. To be able to rapidly increase the disbursement of capital when the opportunity arises, “there must be the possibility of throwing great masses of men suddenly on the decisive points without injury to the scale of production in other spheres. Overpopulation supplies these masses. The course characteristic of modern industry... depends on the constant formation, the greater or less absorption, and the re-formation of the industrial reserve army or surplus population” (B1, 627).

Marx points out that employment and wages grow during the expansive phase of the cycle because there is an intensified accumulation of capital in industry and this impinges on “the varying proportions in which the working class is divided into active and reserve army” (B1, 631). And he states that “as a whole, the general movements of wages are exclusively regulated by the expansion and contraction of the industrial reserve army, and these again correspond to the periodic changes of the industrial cycle. They are, therefore, not determined by the variations of the absolute number of the working population, but by the varying proportions in which the working class is divided into active and reserve army, by the increase or diminution in the relative amount of the surplus population, by the extent to which it is now absorbed, now set free” (B1, 631). And he believed that the common idea (especially, but not exclusively, of Malthus) that linked such changes with changes in the absolute worker population owing to their reproductive customs and their mortality was fundamentally flawed: “It is these absolute movements of the accumulation of capital which are reflected as relative movements of the mass of exploitable labour power, and therefore seem produced by the latter’s own independent movement. To put it mathematically: the rate of accumulation is the independent, not the dependent, variable; the rate of wages, the dependent, not the independent, variable” (B1, 615; italics added). As wages rose resulting from the accumulation of capital, then “Either the price of labour keeps on rising, because its rise does not interfere with the progress of accumulation... Or, on the other hand, accumulation slackens in consequence of the rise in the price of labour, because the stimulus of gain is blunted” (B1, 614). In the latter case “The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labour power... The price of labour falls again to a level corresponding with the needs of the self-expansion of capital” (Ibid.). Marx described a cyclical process of expansion of capital, prosperity, crisis, recession and recovery, in which dishoarding or hoarding of money by capitalists and the resulting fall or rise of the mass of unemployed industrial workers and rise or fall of real wages played fundamental roles. And this process was intimately linked to the accumulation of capital.

In what remains of this chapter we will modify the pure CCP model we have developed with the aim of formally representing a first version of the model of the industrial cycle that Marx described, maintaining the assumption of Simple Reproduction. We will extend this basic model in Chapter 13, where we include financial capitalists and
the rate of interest, and in Chapter 14, where we develop models of Marx’s Extended Reproduction. But it is important to bear in mind that such models would for Marx be but stages in the comprehension of the phenomenon of the industrial cycle because in any actual cycle there can be no balanced reproduction (simple or extended) since the cyclical process takes place in a complex and unique worldwide historical process in which there are certain tendencies (that are the results of various forces) and in which the centralization process increasingly concentrates wealth and capital in a (decreasing) fraction of the population.

**Hoarding and unemployment in a model of the industrial cycle**

So far in this book the models we have used for CCP have focused on systems that deal with invested money capital and work realized by employed labor power. They have been used as a first approximation to Marx’s theory, which is of course much richer. Those models had no way of reflecting the fact that the invested money capital may be (substantially) lower than the capitalist’s wealth or that the contracted labor power may be (substantially) lower than the worker population that needs or wishes to find a job. Hence, the models lacked two of the central concepts in Marx’s theory of Capitalism: the ‘hoarding’ of money by capitalists as ‘latent money capital’ and the ‘industrial reserve army’ of workers. To remedy these deficiencies, in this section we modify one of the models we have developed above for CCP with SR and introduce some additional variables in order to represent in a simple way some of Marx’s ideas with respect to the functioning of the industrial cycle. Since Marx also considered that the phases of the industrial cycles were intimately related to the intensity of the accumulation of capital and that the interest rate was determined in a very different way from the profit rate, the model we build here will be extended in Chapter 13, which deals with financial capital, and in Chapter 14 which deals with Extended Reproduction.

In the model we here construct we assume either that there is a certain idle capacity of the elements of fixed capital at all times, or that all capital is circulating capital. Neither is the labor population an absolute constraint at any moment of the cycle. The true constraint is how much of their wealth the capitalist class prefers to hoard. Assume as previously that the population of capitalists $q^K$ is constant. But we now let the *employed* worker population $q^L$ be variable because industrial capitalists, who jointly own an amount $\hat{K}$ of wealth, only disburse as capital a part $K$ ($<\hat{K}$) of variable magnitude (though $\hat{K}$ is exogenous in the model) that defines the phases of the industrial cycle. The part of wealth that is not disbursed is the global reserve, or hoard, $A = \hat{K} - K$. For the sake simplicity we assume that all capitalists are the same. Each keeps a (gold) reserve $a = A/q^K$ which is variable along the cycle because his disbursed (or invested) capital $k = \hat{K}/q^K$ increases in the expansive phase and falls in the contractive phase (whereas the wealth of each capitalist $\hat{k} = \hat{K}/q^K$ remains constant). We make the additional assumption that capitalists’ consumption is a function of their capital disbursement. Their consumption basket is $\gamma \hat{c}_K$, where $\hat{c}_K$ is a fixed (or ‘basic’) basket and $\gamma$ is the number of basic baskets they consume. We assume that $\gamma$ is a continuous function of $k$ that is increasing for moderate levels of $k$, though it can also become decreasing for higher levels. The reserve of each capitalist is $a = \hat{k} - k$.

Disbursed global capital is $K = q^Q A p + q^L w = q^K k$, where prices and the wage rate are monetary, that is, measured in terms of gold. Assuming that gold is produced commodity 1, we have $p_1 = 1$. Hence it is unnecessary to distinguish between $a$, $\hat{k}$,

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6 Notice that we have returned to the assumption $A^S = A$ and $L^S = L$. 
and $k$ as quantities of gold and their monetary values. Also, since $q^K$ is fixed, there is a direct relation between the global disbursement of capital and the capital of each individual capitalist disburse.

Changes in disbursed capital $k$ make the employed working population $q^L$ vary and hence the consumption basket of employed workers $\hat{c}_L$, where $\hat{c}_L$ is a fixed (or ‘basic’) basket, whereas the number of basic baskets consumed by employed wage workers $\omega$ is variable (and endogenous). Also, there is an ‘industrial reserve army’ (or mass of unemployed workers) $u = \hat{q}^L - q^L$, where $\hat{q}^L$ is the worker population that needs or wants to work and $q^L$ is the employed worker population. The systems of quantities and prices are the following:\footnote{It is unnecessary to write out the complete dual systems of quantities and prices since, as usual, $\eta$ can be eliminated using $\rho$.}

\[
\begin{bmatrix}
q^Q & q^L & q^K \\
\omega \hat{c}_L & 0 & \ell \\
\gamma \hat{c}_K & 0 & 0
\end{bmatrix} = \begin{bmatrix}
q^Q \\
q^L
\end{bmatrix}, \quad (12.3)
\]

\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) \ell \\
\omega \hat{c}_L & 0 \\
\gamma \hat{c}_K & 0
\end{bmatrix} \begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix} = \begin{bmatrix}
p \\
w \\
\pi
\end{bmatrix}, \quad (12.4)
\]

Using $\gamma = \gamma (k)$ in the first equation of (12.3) yields

\[
q^Q = (\omega q^L \hat{c}_L + q^K \gamma (k) \hat{c}_K) B (0). \quad (12.5)
\]

Multiplying by $\ell$ and using the second equation yields $q^L = (\omega q^L \hat{c}_L + q^K \gamma (k) \hat{c}_K) v$, where $v$ is the vector of values (which appears naturally). This gives a direct relation between employment $q^L$ and the level of (employed) worker consumption $\omega$: $q^L = q^K \gamma (k) \hat{c}_K v / (1 - \omega \hat{c}_L v)$.

From the first equation of (12.4) we get $p = B (\rho) \ell w$ (where $B (\rho)$, defined in (8.21), is increasing with $\rho$). And premultiplying by $\omega \hat{c}_L$ and using the second equation we get $w = \omega \hat{c}_L p = \omega \hat{c}_L B (\rho) \ell w$, i.e., $w = 1 / (\hat{c}_L B (\rho) \ell)$, which is an inverse relation between $\omega$ and $\rho$. Inserting this in the expression for employment obtained above we get:

\[
q^L = q^K m (\rho) \gamma (k) \hat{c}_K v, \quad m (\rho) \equiv \left(1 - \frac{\hat{c}_L v}{\hat{c}_L B (\rho) \ell}\right)^{-1}. \quad (12.6)
\]

Notice that the multiplier $m (\rho)$ (which is greater than one since $B (\rho) \ell > B (0) \ell = v$) is decreasing with $\rho$. Hence, the employed worker population $q^L$ varies directly with disbursed capital $k$ and inversely with the profit rate $\rho$.

Because we are dealing with monetary prices, and branch $i = 1$ is gold mining, we have $1 = p_1 = B (\rho)_1 \ell w$ (where $B (\rho)_1$ is the first row of $B (\rho)$). Defining $u_1 \equiv (1, 0, \ldots, 0)$ we can use the (more symmetric) alternative notation $B (\rho)_1 = u_1 B (\rho)$. Hence, the monetary wage rate $w = 1 / (u_1 B (\rho) \ell)$ varies inversely with $\rho$ (as does $\omega$). The profit (and consumption) of each capitalist is

\[
\pi = c_K p = \gamma (k) \hat{c}_K p = \gamma (k) \hat{\pi} (\rho), \quad (12.7)
\]

where to simplify notation we have defined the (monetary) value of capitalists’ `basic’ consumption:

\[
\hat{\pi} (\rho) \equiv \frac{\hat{c}_K B (\rho) \ell}{u_1 B (\rho) \ell}. \quad (12.8)
\]
Hence, using the last equality of (8.42), we have the following relation between capital disbursement $k$ and the profit rate $\rho$:

$$\rho = \frac{\gamma (k) \hat{\pi} (\rho)}{k}.$$  

(12.9)

Since $\rho$ is also on the right hand side, this expression only defines $\rho$ implicitly as a function of $k$.\footnote{This appears with more mathematical detail in the Appendix to this chapter.} Let us write this function as:

$$\rho = \rho (k).$$  

(12.10)

It is not necessarily a monotonous function. In general, it can have decreasing and increasing portions. We assume in what follows that it is decreasing in the relevant segment of the variations in $k$. This is necessarily the case if the effect of $\rho$ on $\hat{\pi}$ is small (the positive effect of an increase of $\rho$ in the numerator de (12.8) largely compensating the positive effect on the denominator). On the other hand, the monetary profit (i.e., in terms of gold) of each capitalist is $\rho (k) k ( = \gamma (k) \hat{\pi} (\rho (k)))$. And $\rho (k) k$ can be increasing even if $\rho (k)$ is decreasing. In the Numerical Exercise in the Appendix to this chapter we show an example in which $\rho (k)$ is increasing and $\rho (k) k$ is decreasing up to a certain point and then becomes increasing.\footnote{For Marx “at a point, therefore, when the increased capital produced just as much, or even less, surplus value than it did before its increase, there would be absolute overproduction of capital; i.e., the increased capital $C + \Delta C$ would produce no more, or even less, profit than capital $C$ before its expansion by $\Delta C$” (B3, 250).} The domain of $k$ has a maximum at $\hat{k}$ (where capitalists’ reserve disappears: $a = 0$). Since $\rho (k)$ is decreasing, it has a minimum $\rho (\hat{k})$ which we assume is positive.

As long as $\rho (k)$ is decreasing, the multiplier $m (k) \equiv m (\rho (k))$ and the wage rate $w (k) \equiv 1/u_1 B (\rho (k)) \ell$ are increasing functions, and so is aggregate employment $q^L (k) \equiv q^K m (k) \gamma (k) \bar{c}_K v$. On the other hand, the profit of each capitalist $\pi (k) \equiv \rho (k) k$ can be decreasing or increasing. Hence, the incomes (and consumptions) of wage workers ($Y^L$) and capitalists ($Y^K$) are the following:

$$Y^L (k) = w (k) q^L (k) = w (k) m (k) q^K \gamma (k) \bar{c}_K v,$$

$$dY^L (k) / dk > 0 (12.11)$$

$$Y^K (k) = q^K \rho (k) k = q^K \rho (k) \hat{\pi} (\rho (k))$$

$$dY^K (k) / dk < 0.$$

On the other hand, the relative incomes of capitalists and workers, which we can call ‘rate of surplus value’, is increasing with $\rho$ and hence decreasing with $k$:

$$\frac{Y^K}{Y^L} \equiv e^p (\rho (k)) = \frac{\hat{c}_K B (\rho (k)) \ell}{m (\rho (k)) \bar{c}_K v} \equiv e^p (k), \quad \frac{de^p (k)}{dk} < 0.$$

Therefore, aggregate income $Y = Y^L + Y^K$ can be expressed as the income of workers multiplied by one plus the rate of surplus value:

$$Y = [1 + e^p (k)] Y^L (k) = [1 + e^p (k)] w (k) m (k) q^K \gamma (k) \bar{c}_K v.$$

Hence, as capital disbursement increases the income of workers increases and the rate of surplus value falls along with the rate of profit. But each capitalist’s profit can be increasing even if the rate of profit is falling. If we assume that at least for low levels of $k$ $\rho (k) k$ is increasing, then in the beginning of the expansionary phase of the industrial cycle all incomes increase, though worker incomes grow faster than
the profits of capitalists. And \( \rho (k) k \) can turn into a decreasing function starting from some level of \( k \).

Let us see how industrial capitalists’ decisions can generate the stylized phases of an industrial cycle. At the beginning of the cycle, presumably after a crisis, disbursed capital \( k \) is low (with respect to its cycle average) and reserves \( a = \hat{k} - k \) high. Hence, the profit rate \( \rho (k) \) is high. Capitalists gradually dare to increase their disbursement of productive capital \( k \) as they observe the consolidation of a recovery. The increase is gradual because nobody wants to get much ahead of the rest before having made sure that they too are disbursing more capital and hence that the expansionary phase of the cycle is firmly in place. As \( k \) increases and \( \rho \) falls, the wage rate \( w \), the real wage \( \omega \), employment \( q^L \), and gross outputs \( q^Q \) all increase. The increase in employment reduces the industrial reserve army \( u = \hat{q}^L - q^L \). The rate of surplus value \( e^p(\rho) \) diminishes along with the rate of profit. Profits may eventually start falling even if the wage rate and employment continue to increase. And at some point the low level of profit can ‘blunt the stimulus of gain’, making capitalists start to reduce their disbursed capital and increase their hoarding since as their profits fall the fear of suffering capital losses from an impending crisis grows. The probability of a crisis would presumably be increased by the risky speculative behavior of financial capitalists (absent in this model) that typically takes place during the peak of prosperity. Hence, a little before or immediately after the beginning of a crisis the reduction in \( k \) accelerates, and hence gross outputs, employment, real wages, and aggregate incomes all fall while unemployment rises, as does the profit rate on the (shrinking) disbursed capital and the rate of surplus value on the (shrinking) variable capital. The contractionary phase of the industrial cycle is typically shorter and more abrupt than the expansionary phase.

Although the simple model we have seen in this section is not inserted in a model of expanded accumulation of capital, does not contain interest and debt, and does not explain what produces the crises, it can nevertheless be used to describe the expansionary and contractive phases of the industrial cycles of the central part of the 19th century in a way that is both interesting and roughly in agreement with Marx’s views and analyses. It highlights that the fundamental determinant of these phases is not necessarily the real investment of capitalists, which does not exist in this model, but a more general mechanism that has to do with hoarding and dishoarding, that is, the disbursement or reimbursement of capital on the part of industrial capitalists. The disbursement would respond to a (prudentially) gradual dishoarding as the confidence of each capitalist that the recovery will continue and allow for attractive profits grows. And the reimbursement would respond to falling profits and cautionary behavior based on the fear of suffering capital losses as there are signs of an upcoming crisis.

In the Appendix to this chapter we show a Numerical Exercise that is based on an economy with three produced commodities (including gold), and we calculate the main model variables for different values of \( k \). Figure 7 is based on this example and graphs the functions \( \rho (k) \) and \( \rho (k) k \). Notice that as \( k \) increases the profit rate falls while the level of profits rises initially and starts to fall from some point on.
Appendix to Chapter 12
Bibliographical Note: joint production in von Neumann (1945) and Sraffa (1960)

Both von Neumann (1945 [1938]) and Sraffa (1960) found that using linear productive processes which can produce more than one good, i.e., joint production of goods, can be useful for the representation of elements of fixed capital (that modern economic theory calls ‘capital goods’). For example, a process that produces hammers using a machine that lasts several years can be represented as a joint production process of hammers and that machine one year older. This can be used to get the value of the depreciation of the machine in the time period between $t$ and $t + 1$ as the difference between the price of the machine at $t + 1$ and the price at $t$. Although one can argue that this procedure is more precise (or sophisticated) than the one Marx (with Engels and Moore) used, the methodology of Chapter 4 of Book III is nevertheless adequate and its conceptual clarity is surprising if one considers it was written more than half a century before von Neumann and Sraffa worked on this topic.

It is not clear whether von Neumann or Sraffa was the first to use joint production to model fixed capital. Von Neumann first presented his paper at Princeton in 1932 and first published it in German in 1938. It was published in English in 1945. Although Sraffa published his book in 1960, he states in the Preface that “Whilst the central propositions had taken shape in the late 1920’s, particular points, such as the Standard commodity, joint products and fixed capital, were worked out in the ’thirties and early ’forties.” Sraffa does not give any clues on whether the idea of introducing multi-product processes was his or he took it from elsewhere. It is surprising that he does not mention von Neumann when he introduces joint production. This fact can make one think that he had no knowledge of his work. However, when Nicholas Kaldor (who was Hungarian like von Neumann) stimulated the publication in English of von Neumann’ article, he asked Champernowne (who was more of a mathematician than he) to write a didactical article on the subject so that it could appear in the same issue of The Review of Economic Studies (see Kurz and Salvatori (1998)). In a footnote, Champernowne thanks Sraffa for his help on the economic issues discussed in the
article. Hence, in 1945 Sraffa was at least familiarized with von Neumann’s treatment of joint production. It may be that because von Neumann was a mathematician (and not an economist), Sraffa gave his contribution the same kind of general consideration he gave to the mathematicicians whose contributions he thanks in his Preface without getting into details. But von Neumann was not even mentioned. It may have been simply a slip. It is also possible that Sraffa had advanced on the path of joint production before von Neumann but, not having published anything on the subject, decided not to say anything about it or about von Neumann’s article.

Details of the industrial cycle model

Let \( \varepsilon_{f,x} \) be the elasticity of function \( f(x) \) with respect to \( x \), i.e., \( \varepsilon_{f,x} \equiv \frac{x}{f(x)} f'(x) \), where \( f'(x) \) is the derivative of \( f(x) \). With \( \gamma(k) \equiv k(0.16 - k) \), we can calculate its elasticity as \( \varepsilon_{\gamma,k} = (0.16 - 2k)/(0.16 - k) \). Hence, \( \varepsilon_{\gamma,k} < 1 \) and also \( \varepsilon_{\gamma,k} < 0 \) if \( k > 0.08 \). Equation (12.9) implicitly defines a continuously differentiable function \( \rho = \rho(k) \) if the conditions of the Implicit Function Theorem are satisfied. We assume that this is the case. Then \( \rho(k)k \) is also a function of \( k \). Let us call it \( \bar{\pi}(k) \equiv \rho(k) \). Its elasticity is \( \varepsilon_{\bar{\pi},k} = 1 + \varepsilon_{\rho,k} \). Totally differentiating the equation \( pk = \gamma(k) \bar{\pi}(\rho) \) we get \( \varepsilon_{\rho,k} = (1 - \varepsilon_{\gamma,k})/(\varepsilon_{\bar{\pi},\rho} - 1) \). Since \( \varepsilon_{\gamma,k} < 1 \), the sign of \( \varepsilon_{\rho,k} \) is the sign of \( \varepsilon_{\bar{\pi},\rho} - 1 \). In particular, \( \varepsilon_{\rho,k} < 0 \) if and only if \( \varepsilon_{\bar{\pi},\rho} < 1 \). Hence \( \varepsilon_{\bar{\pi},k} = (\varepsilon_{\bar{\pi},\rho} - \varepsilon_{\gamma,k})/(\varepsilon_{\bar{\pi},\rho} - 1) \). The behavior of \( \bar{\pi}(\rho) \) is thus key in determining that of \( \bar{\pi}(k) \).

Let us see what the elasticity of \( \bar{\pi}(\rho) \equiv \hat{c}_K B(\rho) \ell /u_1 B(\rho) \ell \) depends on. To calculate \( \varepsilon_{\bar{\pi},\rho} \) we must first obtain the derivative of expressions like \( \hat{c}_K B(\rho) \ell \) with respect to \( \rho \). For this we can expand \( \hat{c}_K B(\rho) \ell \) in series, using the definition of \( B(\rho) \):

\[
\hat{c}_K B(\rho) \ell = \hat{c}_K (1 + \rho) \left[ I - (1 + \rho) A \right]^{-1} \ell \\
= \hat{c}_K (1 + \rho) \left[ I + (1 + \rho) A + (1 + \rho)^2 A^2 + \ldots \right] \ell \\
= \left( 1 + \rho \right) \hat{c}_K \ell + (1 + \rho)^2 \hat{c}_K A \ell + (1 + \rho)^3 \hat{c}_K A^2 \ell + \ldots
\]

and take the derivative term by term:

\[
\frac{d(\hat{c}_K B(\rho) \ell)}{d\rho} = \hat{c}_K \ell + 2(1 + \rho)\hat{c}_K A \ell + 3(1 + \rho)^2 \hat{c}_K A^2 \ell + \ldots
\]

Let us call the matrix series within brackets \( D(\rho) \). Notice that

\[
D(\rho) = I + 2(1 + \rho) A + 3(1 + \rho)^2 A^2 + \ldots
= I + (1 + \rho) A + (1 + \rho)^2 A^2 + \ldots
+ (1 + \rho) A + 2(1 + \rho)^2 A^2 + 3(1 + \rho)^3 A^3 + \ldots
= \frac{B(\rho)}{1 + \rho} + (1 + \rho) A \left[ I + 2(1 + \rho) A + 3(1 + \rho)^2 A^2 + \ldots \right]
= \frac{B(\rho)}{1 + \rho} + (1 + \rho) AD(\rho).
\]

---

\(^{10}\)In his Preface Sraffa writes: “My greatest debt is to Professor A. S. Besicovitch for invaluable mathematical help over many years. I am also indebted for similar help at different periods to the late Mr. Frank Ramsey and to Mr. Alister Watson.”
Hence

\[ [I - (1 + \rho) A] D(\rho) = \frac{B(\rho)}{1 + \rho}, \]

that is,

\[ D(\rho) = \left( \frac{B(\rho)}{1 + \rho} \right)^2. \]

Therefore\(^{11}\)

\[ \frac{d(\hat{c}_K B(\rho) \ell)}{d\rho} = \hat{c}_K \left( \frac{B(\rho)}{1 + \rho} \right)^2 \ell. \quad (12.12) \]

Using the corresponding formula for the derivative of \(u_1 B(\rho) \ell\), and differentiating \(\hat{\pi}(\rho) \equiv \hat{c}_K B(\rho) \ell / (u_1 B(\rho) \ell)\) we get:

\[ \varepsilon_{\hat{\pi},\rho} = \frac{\rho}{(1 + \rho)^2} \left[ \frac{\hat{c}_K B(\rho)^2 \ell}{\hat{c}_K B(\rho) \ell} - \frac{u_1 B(\rho)^2 \ell}{u_1 B(\rho) \ell} \right]. \]

The sign of \(\varepsilon_{\hat{\pi},\rho}\) is the sign of the term within brackets. Each one of the ratios is greater than one and increasing in \(\rho\), since \(B(\rho)^2 = B(\rho) B(\rho) > B(\rho)\) (since \(B(\rho) > I\)) and \(B(\rho)\) is increasing. And within the segment in which \(\rho\) varies there may be a change in the sign of \(\varepsilon_{\hat{\pi},\rho}\), a feature that obviously depends on the characteristics of gold (commodity money) production \(vis\ a\ vis\) that of the basket of commodities consumed by capitalists. Hence, \(\varepsilon_{\hat{\pi},\rho} < 1\) if and only if

\[ \frac{\hat{c}_K B(\rho)^2 \ell}{\hat{c}_K B(\rho) \ell} - \frac{u_1 B(\rho)^2 \ell}{u_1 B(\rho) \ell} < \frac{(1 + \rho)^2}{\rho}. \]

For low levels of \(\rho\) this equality is probably true, since when \(\rho\) tends to 0 (from positive levels) the left hand side tends to a (positive or negative) finite value\(^{12}\) whereas the right hand side tends to infinity. Hence, there is at least a positive neighborhood of 0 in which \(\varepsilon_{\hat{\pi},\rho} < 1\) and hence \(\varepsilon_{\rho,k} < 0\), as we assumed in the text.

**Numerical Exercise #5**

Let us consider an economy with the following data:

\[
A = \begin{bmatrix} 0.1 & 0.2 & 0.5 \\ 0.3 & 0.1 & 0.4 \\ 0.4 & 0.3 & 0.2 \end{bmatrix}, \quad \ell = \begin{bmatrix} 0.1 \\ 0.5 \\ 0.3 \end{bmatrix}, \quad c_K = \begin{bmatrix} 0 & 0.3 & 0.7 \end{bmatrix}.
\]

\(^{11}\)An alternative way of obtaining (12.12) is to use: 1) the fact that if \(B(\rho)\) is a square matrix whose elements depend (continuously and differentiably) on parameter \(\rho\) then the derivative of the bilinear form \(xB(\rho)y\) with respect to \(\rho\) is \(xB'(\rho)y\), and 2) the well-known theorem that says that if \(R(\rho)\) is a square invertible matrix whose elements depend on parameter \(\rho\), then

\[
dR(\rho)^{-1}/d\rho = -R(\rho)^{-1} R'(\rho) R(\rho)^{-1},
\]

where \(R'(\rho)\) is the (element by element) derivative of \(R(\rho)\) with respect to \(\rho\). In our case \(R(\rho) = (I - 1 + \rho A)^{-1}\), that is, \(R(\rho)^{-1} = B(\rho)\) and \(R'(\rho) = -(1 + \rho)^{-2} I\). Hence

\[
B'(\rho) = -B(\rho) \left( \frac{1}{(1 + \rho)^2} I \right) B(\rho) = \left( \frac{B(\rho)}{1 + \rho} \right)^2.
\]

Therefore, using 1) and 2) we get (12.12).

\(^{12}\)This value is \(\hat{c}_K B(0)\) \(v/\hat{c}_K v - u_1 B(0)\) \(v/u_1 v\).
And assume capitalists behave according to $\gamma (k) = k (\bar{k} - k)$ with $\bar{k} = 0.16$. Hence $\gamma (k) / k = 0.16 - k$. From (12.9) we must have $\gamma (k) / k = \rho / \bar{\pi} (\rho)$, that is, $k = 0.16 - \rho / \bar{\pi} (\rho)$. Take $\rho = 0.01$ and calculate the value of $k$. First we calculate $B (\rho)$:

$$\left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \cdot \left( \begin{array}{ccc} 0.1 & 0.2 & 0.5 \\ 0.3 & 0.1 & 0.4 \\ 0.4 & 0.3 & 0.2 \end{array} \right) \frac{1}{1 + 0.01} = \left( \begin{array}{ccc} 1.8318 & 2.3220 & 2.3348 \\ 2.0580 & 1.6011 & 3.3786 \end{array} \right).$$

Hence, using (12.8), $\rho / \bar{\pi} (\rho)$ is

$$\left( \begin{array}{ccc} 1 & 0 & 0 \\ 0.01 & 1.8318 & 2.3220 & 2.3348 \\ 2.0580 & 1.6011 & 3.3786 \end{array} \right) = 0.0084172,$$

and therefore $k = 0.16 - 0.0084172 = 0.15158$, and $\rho k = 0.01 \times 0.15158 = 0.0015158$. We then repeat these calculations for $\rho = 0.03, 0.04, 0.05, 0.07, 0.09, 0.11, 0.13, 0.15, 0.17$, from which we can construct the graph in Figure 7 and the rest of the columns of Table 16. (Using Scientific WorkPlace all this just takes a few minutes.) Notice also that with the given data $\rho (k)$ is decreasing and $\rho (k) k$ increases up to $k = 0.08$ and then starts to fall.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\gamma (k)$</th>
<th>$\rho$</th>
<th>$\rho / \bar{\pi} (\rho)$</th>
<th>$\rho k$</th>
</tr>
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<tbody>
<tr>
<td>0.00113</td>
<td>0.0017952</td>
<td>0.17</td>
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<td>0.0001921</td>
</tr>
<tr>
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<td>0.0029859</td>
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<td>0.0049208</td>
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<td>0.0054002</td>
</tr>
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<td>0.11</td>
<td>0.098955</td>
<td>0.0067150</td>
</tr>
<tr>
<td>0.080081</td>
<td>0.0064000</td>
<td>0.09</td>
<td>0.079919</td>
<td>0.0072073</td>
</tr>
<tr>
<td>0.098652</td>
<td>0.0060521</td>
<td>0.07</td>
<td>0.061348</td>
<td>0.0069056</td>
</tr>
<tr>
<td>0.11676</td>
<td>0.0050488</td>
<td>0.05</td>
<td>0.043241</td>
<td>0.005838</td>
</tr>
<tr>
<td>0.12564</td>
<td>0.0043172</td>
<td>0.04</td>
<td>0.034362</td>
<td>0.0050256</td>
</tr>
<tr>
<td>0.13444</td>
<td>0.0034404</td>
<td>0.03</td>
<td>0.025598</td>
<td>0.004032</td>
</tr>
<tr>
<td>0.15158</td>
<td>0.0012759</td>
<td>0.01</td>
<td>0.0084172</td>
<td>0.0015158</td>
</tr>
</tbody>
</table>
Chapter 13  FINANCIAL CAPITAL

The relation between ‘functioning’ capital and financial capital

In Part V of Book III Marx investigates the “Division of profit into interest and profit of enterprise.” Although in Marx’s theory surplus value was only produced in industry, i.e., not in commerce or banking, the ‘functioning’ capitalist, whether in industry, commerce, or banking, is engaged in the specific functions of capital: reaping profits through the use of wage labor. In much of his analyses he assumes that the profit rate is the same in all sectors of the economy. Hence all ‘functioning’ capitalists tend to share equally in global profits (that is, global surplus value) in proportion to the magnitude of the capital they disbursed. In contrast, non-bank financial capitalists (or simply ‘lenders’) receive an interest merely for being proprietors of money capital that once loaned “becomes a commodity, but a commodity sui generis” (B3, 337). The financial capitalist loans a certain amount of money to the functioning capitalist, who invests it purchasing commodities (means of production and labor power). This is symbolized by Marx by means of the ‘formula’ \( M - M - C - M' - M'' \), where \( M - M \) represents the money loaned by a financial capitalist to a functioning capitalist who in his firm will have it transformed into commodities \( C \) and \( M' - M' \) represents the devolution of the loan with interest (\( M' \)). The loaned capital flows back in two ways. In the process of reproduction it returns to the functioning capitalist, and then its return repeats itself once more as transfer to the lender, the money capitalist, as return payment to the real owner, its legal point of departure” (B3, 343). And “interest expresses the self-expansion of money-capital and thus appears as the price paid for it to the lender” (B3, 353). The (gross) profit the industrial capitalist prima facie reaps unfolds into the interest on the capital loaned and the ‘entrepreneurial profit’: “One appears exclusively as the fruit of owning the capital, the other as the fruit of operating with the capital, the fruit of performing capital, or of the functions performed by the active capitalist” (B3, 372). Marx called ‘profit of enterprise’ the profit made by functioning capitalists once they had repaid the interest on the capital loaned by ‘money capitalists’ (or ‘financial capitalists’); or, as we see below, by banks. The accumulation of capital differed for these two types of capital: “The accumulation of all money-lending capitalists naturally always takes place directly in money form, whereas we have seen that the actual accumulation of industrial capitalists is accomplished, as a rule, by an increase in the elements of reproductive capital itself” (B3, 500).

Time plays a fundamental role in the generation of profit and in the determination of interest. For “the rate of profit is not only determined by the relation of profit made in one single turnover to advanced capital value, but also by the length of this period of turnover, hence determined as profit yielded by industrial capital within definite spans of time.” And similarly in the case of interest-bearing capital “a definite interest is paid to the lender for a definite time span” (B3, 355). But “the circumstances determining the amount of profit to be distributed, of the value produced by unpaid labour, differ widely from those which determine its distribution between these two kinds of capitalists, and frequently produce entirely opposite effects” (B3, 358). Marx observes the characteristics of the phases of industrial cycles (“state of inactivity, mounting revival, prosperity, overproduction, crisis, stagnation, state of inactivity, etc.”) and points out that “a low rate of interest generally corresponds to periods of prosperity or extra profit, a rise in interest separates prosperity and its reverse, and a maximum of interest up to a point of extreme usury corresponds to the period of crisis.” But this does not mean that there is an inverse relation, since it is possible for “low interest to
go along with stagnation, and for moderately rising interest to go along with revived activity” (B3, 359).

Marx stresses that only supply and demand determine the level of the interest rate and that: “In this sphere there is no such thing as a natural rate of interest in the sense in which economists speak of a natural rate of profit and a natural rate of wages” (B3, 360). He holds that there are *structural determinants* for the global profit rate that stem from the complex process of reallocation of capital (and hence means of production and labor power) among the various branches of activities of functioning capitalists that gradually equalizes their profit rates, determinants which are completely absent in the market for ‘interest-bearing capital’. He notes that “the rate of interest itself varies continually in accordance with the different classes of securities offered by borrowers” but that these factors do not explain the average level of the interest rates in any given period of time. Marx holds that there is a fundamental difference between the equilibrium in the market for interest-bearing capital, which is exclusively determined by supply and demand and is achieved quickly, and the slow determination of the production prices and the global profit rate that is based on much more complex structural factors. He calls such structural factors ‘inner laws of capitalist production’:

“It is evident that the real inner laws of capitalist production cannot be explained by the interaction of supply and demand (quite aside from a deeper analysis of these two social motive forces, which would be out of place here), because these laws cannot be observed in their pure state, until supply and demand cease to act, i.e., are equated” (B3, 188). Marx concludes that “The general rate of profit, therefore, derives actually from causes far different and far more complicated than the market rate of interest, which is directly and immediately determined by the proportion between supply and demand.” For Marx supply and demand only explained the deviations of market prices from production prices, not the actual production prices.

**The role of banks in the financial system**

In Part V of Book III Marx shows he has a very concrete view of the functioning of the banking systems of the advanced Capitalism of his time. With the development of capitalist production “the technical operations of receiving and disbursing money, of international payments, and thus of the bullion trade, are concentrated in the hands of the money dealers. The other side of the credit system –the management of interest-bearing capital, or money capital– develops alongside this money-dealing as a special function of the money dealers. Borrowing and lending money becomes their particular business” (B3, 399-400). The bankers are “middlemen between the actual lender and the borrower of money capital.” They concentrate in their hands large masses of loanable money capital and “in place of the individual money lender, the bankers confront the industrial and commercial capitalists as representatives of all money lenders. They become the general managers of money capital.” “A bank represents a centralisation of money capital, of the lenders, on the one hand, and on the other a centralisation of the borrowers. Its profit is generally made by borrowing at a lower rate of interest than it receives in loaning” (B3, 400).

---

1 Also, “If prices of commodities in one sphere are below or above the price of production (...) equalisation occurs through the expansion or curtailment of production... caused by inflow or outflow of capital to and from individual spheres of production. It is by this equalisation of the average market prices of commodities to prices of production that deviations of specific rates of profit from the general, or average, rate of profit are corrected. It cannot be that in this process industrial or mercantile capital as such should ever assume the appearance of commodities vis-à-vis the buyer, as in the case of interest-bearing capital (B3, 364-5).
Marx describes the various sources of loanable capital banks draw from: the reserve funds of industrial and commercial firms, deposits made by capitalists, and, “with the development of the banking system, and particularly as soon as banks come to pay interest on deposits, money savings and the temporarily idle money of all classes” who deposit them in banks. On the other hand, the ways in which bank credit is extended include the “discounting bills of exchange –by converting bills of exchange into money before they come due-”, “loans against securities, such as interest-bearing paper, government paper, stocks of all sorts”, and advances of various kinds (overdrafting of deposits, overdrafts against bills of lading and other titles of ownership of commodities). The concession of credit can also be done by issuing a banknote, which “is nothing but a draft upon a banker, payable at any time to the bearer, and given by the banker in place of private drafts.” Marx points out that “in most countries the principal banks issuing notes, being a peculiar mixture of national and private banks, actually have the national credit to back them, and their notes are more or less legal tender” (Ibid.).

The concentration of loan capital in large banks is for Marx “a consequence of the development of the reproduction process” (B3, 500), and the profit which is the source of this accumulation of money capital is a deduction from the surplus value obtained by industrial capital. Also, “the development of the credit system and the enormous concentration of the money-lending business in the hands of large banks must, by themselves alone, accelerate the accumulation of loanable capital, as a form distinct from actual accumulation” (Ibid.), which is the accumulation of productive capital in industrial firms.

Financial capital in the systems of quantities and prices
Extension of the model of CCP to include financial capital

In the rest of this chapter we leave out commercial capital, which will be addressed in Chapter 15. We also disregard the specificities of banking capital. Hence, functioning capital reduces here to industrial capital and financial capital to loans by money lenders. We adapt the model of the previous chapter so that it can represent the breakdown between entrepreneurial profit and interest on financial capital. Let $q^K$ and $q^F$ be the populations of industrial and financial capitalists, respectively. We assume that all industrial capitalists are identical (as previously) and so are all financial capitalists. Let $i$ be the interest obtained by each member of the financial capitalist class per unit of loan. We assume that each of the latter has a supply function of units of gold $k^F (i)$ that is increasing with $i$. We also assume industrial capitalists disburse an amount of capital of $k^K$ units of gold, which is insufficient for the scale in which they wish to operate (at least without reducing their reserve beyond the level they desire), and so they appeal to financial capitalists. Total productive capital is $K = q^Q Ap + q^L w$, which is higher than the amount industrial capitalists wish to disburse of their own wealth $q^K k^K$. Because we are using monetary prices ($p_1 = 1$), their demand for interest-bearing capital is the difference $K - q^K k^K$. And since, given $i$, the aggregate supply of funds of financial capitalists is $q^F k^F (i)$, the market clearing condition for money capital (i.e., for the ‘money market’) is

$$q^Q Ap + q^L w - q^K k^K = q^F k^F (i).$$

(13.1)

This appears to reflect quite well Marx’s considerations on this topic: “How are demand and supply of money capital determined? It is doubtlessly true that a tacit connection exists between the supply of material capital and the supply of money capital, and,
likewise, that the demand of industrial capitalists for money capital is determined by conditions of actual production" (B3, 416). Also: "Was not this enormous increase of production an increase of capital itself, and if it created a demand, did it not also create the supply, and, simultaneously, an increased supply of money capital? If the interest rate rose very high, then [it is] merely because the demand for money capital increased still more rapidly than its supply, which implies, in other words, that with the expansion of industrial production its operation on a credit basis expanded as well" (B3, 421; text within brackets added in accordance with the Spanish translation).

Let \( \tilde{c}_F \) be the ‘basic’ consumption basket of financial capitalists and \( \zeta \) the number of these baskets that they actually consume. Then the dual systems of quantities and prices and incomes can be expressed as:

\[
\begin{bmatrix}
q^Q & q^L & q^K & q^F
\end{bmatrix}
\begin{bmatrix}
A & \ell & \eta_K & \eta_F \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
A & \ell & \eta_K & \eta_F \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
q^Q & q^L & q^K & q^F
\end{bmatrix},
\tag{13.2}
\]

\[
\begin{bmatrix}
A & \ell & \eta_K & \eta_F \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
\pi \\
i \\
\end{bmatrix}
= 
\begin{bmatrix}
p \\
w \\
\pi \\
i \\
\end{bmatrix}
\tag{13.3}
\]

The following table summarizes the model equations in its first four rows and also includes in A5 the condition for equilibrium of supply and demand in the money capital market (13.1) and in B5 the usual equation for production prices with a homogeneous profit rate.

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((q^Lc_L + q^Kc_K + q^F \tilde{c}_F)B_1(0) = q^Q\eta_K\pi + \eta_F i = p - (Ap + \ell w))</td>
<td>( q^Qc_L = q^L )</td>
</tr>
<tr>
<td>2</td>
<td>( q^Q\ell = q^L )</td>
<td>( c_{LP} = w )</td>
</tr>
<tr>
<td>3</td>
<td>( q^Qc_K = q^K )</td>
<td>( c_{KP} = \pi )</td>
</tr>
<tr>
<td>4</td>
<td>( q^Q\eta_F = q^F )</td>
<td>( \tilde{c}_{FP} = i )</td>
</tr>
<tr>
<td>5</td>
<td>( q^QAp + q^Fw = q^Kk^K + q^Fk^F(i) )</td>
<td>((1 + \rho)(Ap + \ell w) = p )</td>
</tr>
</tbody>
</table>

From B1 and B5 we get the vector of profits as \( \rho (Ap + \ell w) = \eta_K\pi + \eta_F i \). And premultiplying by \( q^Q \) and using A5 and A2 \(-\) A4 we get the global profit rate as a weighted average of the profit rate of each functioning capitalist \( \pi/k^K \) and the interest rate of each financial capitalist \( i/k^F \):

\[
\rho = \frac{q^K\pi + q^Fi}{q^Kk^K + q^Fk^F(i)} = (1 - \alpha(i)) \frac{\pi}{k^K} + \alpha(i) \frac{i}{k^F(i)}, \tag{13.4}
\]

where the weights are a function of the interest \( i \):

\[
\alpha(i) = \frac{q^Fk^F (i)}{q^Kk^K + q^Fk^F (i)}. \tag{13.5}
\]

From the allocations of workers and industrial and financial capitalists in the various production branches \((A2 - A4)\) and the values of the consumption baskets of these three groups \((B2 - B4)\) we get the aggregate incomes of each:

\[
\begin{align*}
q^Q\ell c_{LP} & = q^Lw, \\
q^Q\eta_K c_{KP} & = q^K\pi, \\
q^Q\eta_F \tilde{c}_{FP} & = q^F i.
\end{align*}
\tag{13.6}
\]
The income of the capitalist class (made up of two subclasses) is \( q^K \pi + q^F i = \rho (q^K k^K + q^F k^F (i)) \). Hence, the relative income of the capitalist class and the working class (or rate of surplus value), is:

\[
e^p = \left( \frac{(q^K k^K + q^F k^F (i)) \rho}{q^Q \ell p + q^L w} \right) = \frac{q^Q A p + q^L w}{q^L w} \rho = (\kappa^p + 1) \rho,
\]

where in the second equality we have used (13.1) and in the last we defined the value composition of capital \( \kappa^p \) as in (8.26). Hence, we again have the usual decomposition of the profit rate (8.25):

\[
\rho = \frac{S^p}{C^p + V^p} = \frac{e^p}{\kappa^p + 1},
\]

where in this case profit gross of interest (or surplus value) is \( S^p \equiv q^K c_K p + q^F \zeta c_F p = q^K \pi + q^F i, \) and is the sum of industrial capitalists’ profit and financial capitalists’ interest.

**Solutions for the endogenous variables of the model**

Let us sequentially obtain the values of the endogenous variables that solve the model. From B5 we get the vector of production prices \( p = B (\rho) \ell w \). Premultiplying by \( c_L \) yields the equation \( 1 = c_L B (\rho) \ell \) from which we can (at least numerically) solve for the (equilibrium) value of the profit rate \( \rho^* \). Since prices are monetary and gold is the first commodity we have \( 1 = B (\rho^*) \ell w \). Hence, the wage rate is \( w^* = 1 / (B (\rho^*) \ell) \) and the solution for the vector of production prices is:

\[
p^* = B (\rho^*) \ell.
\]  

(13.7)

Given \( \rho^* \), the first equality in (13.4) allows us to (at least numerically) calculate the interest \( i^* \) that balances supply and demand of money capital:

\[
\rho^* = \frac{q^K + q^F i^*}{q^K k^K + q^F k^F (i^*)}.
\]

Given \( p^*, i^*, \) and \( \rho^* \), from B4 we obtain the number of basic consumption baskets financial capitalists consume: \( \zeta^* = i^* / (\zeta_F p^*) \). And given this value, from A1 we can calculate the vector of gross outputs:

\[
q^{Q^*} = \left( q^L c_L + q^K c_K + q^F \zeta^* \zeta_F \right) B (0) .
\]  

(13.8)

From what we have we can check that the three incomes in (13.6) exhaust the net product. Multiplying (13.8) by \( (I - A) p \) (and using B4) we get: \( q^{Q^*} (I - A) p^* = q^L w^* + q^K \pi^* + q^F i^* \).

To get the values of vectors \( \eta_F \) and \( \eta_K \), first we show that they must necessarily be proportional. The money capital that allows capitalists of branch \( j \) to finance their purchase of means of production and labor power (per unit of output) is \( \eta_{Kj} k^K + \eta_{Fj} k^F (i) = Ap + \ell_j w \). Hence, considering all branches we have

\[
\eta_{Kj} k^K + \eta_{Fj} k^F (i) = Ap + \ell_j w.
\]  

(13.9)

On the other hand, since in each branch \( j \) the financial capitalists contribute the fraction \( \alpha (i) \) of the necessary capital we have\(^2\)

\[
\eta_{Fj} k^F (i) = \alpha (i) \left( \eta_{Kj} k^K + \eta_{Fj} k^F (i) \right) .
\]

(13.10)

\(^2\)Notice that premultiplying by \( q^Q \) we reach the definition of \( \alpha (i) \) in (13.5).
Hence, $\eta_F$ are $\eta_K$ indeed proportional
\[
\eta_F = \frac{\alpha(i) k^K_F}{1 - \alpha(i)} k^F(i) \eta_K = \frac{q^F k^K_F (i)}{q^K k^K_F (i)} k^K = \frac{q^F}{q^K} \eta_K.
\]

On the basis of this proportionality and B1 we can infer the values of $\eta^*_K$ and $\eta^*_F$:
\[
\eta^*_K = \frac{1}{1 + (q^F / q^K) i^*} \left[ (I - A) p^* - \ell w^* \right], \quad \eta^*_F = \left( q^F / q^K \right) \eta^*_K.
\]

We have shown that Marx’s verbal analysis of the relation between the interest of the financial capitalist and the profit of the functioning capitalist (industrial or commercial) made much sense. In particular, it was reasonable to consider interest as the price for the loan of a certain quantity of money capital for a certain period of time, which in the model coincides with the period in which the whole capital turns over. And is was reasonable to point out that there was a fundamental difference between the equilibrium in the market for money capital (determined exclusively and quickly by the supply and demand of funds) and the determination of the production prices and the global profit rate by means of a much more complex and time consuming process which includes the reallocation of means of production and labor power among branches in search for a higher profit rate (as a reaction to any exogenous perturbation –technical or organizational– that could turn the profit rates of the various branches heterogeneous).

In Marx’s theory the system of values and rate of surplus value (8.6) remains valid, and could be extended as in (8.10) to include the consumption of the two capitalist subclasses, that is, by adding an additional row. The rate of surplus value can consistently be defined to include the consumptions of the two proprietor classes: \( e = \left( q^K c_{K}v + q^F \xi^* c_{F}v \right) / \left( q^L c_{L}v \right) \). Multiplying (13.8) by $\ell$ leads to the usual formula $(1 + e) c_{L}v = 1$, that is, the second equation of (8.6).

A model of the cycle with Industrial and Financial Capital

In this section we modify the model of the preceding section in order to transform it into a model of the industrial cycle that expands the one we constructed in Chapter 12. With this new model we can formally represent various textual analyses Marx made on the basis of the abundant empirical information on the English economy he had at hand. It is assumed that both industrial and financial capitalists have at their disposal an invested capital and a reserve of potential money capital, or treasure. But whereas industrial capitalists autonomously decide how much capital they have functioning, which makes their functioning capital an exogenous variable of the model, the financial capitalists are responsive to the interest $i$ they can obtain on their loans, which makes their supply of loans endogenous. Let $\tilde{K}_K$ and $\tilde{K}_F$ be the aggregate wealth of industrial and financial capitalists, respectively, which are composed of their exogenous gold reserves, $A^K$ and $A^F$, respectively, and their respective disbursed capitals, $K^K$ and $K^F$. Then $\tilde{K}_K = A^K + K^K$ and $\tilde{K}_F = A^F + K^F$. Since we use monetary prices ($p_1 = 1$) it is unnecessary to distinguish between quantities of gold and their value. Industrial capitalists appeal to loans from financial capitalists in order to (partially) finance their ‘functioning’ capitals. We assume that the supply of (gold) loans of financial capitalists, $K^F (i)$, is increasing with the interest $i$ they charge. As in the previous section, the populations of industrial and financial capitalists are $q^K$ and $q^F$, respectively, but $q^L$ represents the worker population that is employed. The variables defined above expressed in per capita terms (of their respective populations) are $\tilde{k}_K = a^K + k^K$ and $\tilde{k}_F = a^F + k^F$. 
The systems of quantities (and populations) and prices (and incomes) are now the following:

\[
\begin{bmatrix}
q^Q & q^L & q^K & q^F
\end{bmatrix}
\begin{bmatrix}
A & \ell \\
\omega \tilde{c}_L & 0 \\
\gamma \tilde{c}_K & 0 \\
\zeta \tilde{c}_F & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q \\
q^L
\end{bmatrix},
\]

(13.11)

\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) \ell \\
\omega \tilde{c}_L & 0 \\
\gamma \tilde{c}_K & 0 \\
\zeta \tilde{c}_F & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w
\pi
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
\pi \\
i
\end{bmatrix},
\]

(13.12)

where \(\omega, \gamma, \) and \(\zeta\) represent the number of basic consumption baskets \(\tilde{c}_L, \tilde{c}_K, \) and \(\tilde{c}_F\) that employed workers and industrial and financial capitalists consume. We assume as in Chapter 12 that \(\gamma (k^K)\) is a function of disbursed capital by industrial capitalists.\(^3\)

The vector of gross outputs is \(q^Q = (q^L \omega \tilde{c}_L + q^K \gamma (k^K) \tilde{c}_K + q^F \zeta \tilde{c}_F) B (0)\) and hence the employed working population is

\[
q^Q \ell = q^L = [q^L \omega \tilde{c}_L + (q^K \gamma (k^K) \tilde{c}_K + q^F \zeta \tilde{c}_F)] v,
\]

where \(v = B (0) \ell\) is the vector of values. As in Chapter 12 we get from (13.12) \(p = B (\rho) \ell w, \omega = 1/c_L B (\rho) \ell,\) and \(w = 1/u_1 B (\rho) \ell.\) And from the fourth equation of that system we get \(i = \zeta \tilde{c}_F p = \zeta \tilde{c}_F B (\rho) \ell / u_1 B (\rho) \ell,\) from which we get \(\zeta = i \omega / c_L B (\rho) \ell / \tilde{c}_F B (\rho) \ell = i / \tilde{c}_F (\rho),\) where for brevity we defined the monetary value of the basic consumption basket of financial capitalists \(\tilde{c}_F (\rho) \equiv \tilde{c}_F B (\rho) \ell / u_1 B (\rho) \ell.\)

Hence, we have the employed worker population \(q^L\) and gross outputs \(q^Q\) as functions of \(\rho\) and \(i:4\)

\[
q^L = m (\rho) \left[q^K \gamma (k^K) \tilde{c}_K + q^F i / \tilde{c}_F (\rho) \tilde{c}_F \right] v \equiv q^L (\rho, i; k^K),
\]

\[
q^Q = \left[q^K \gamma (k^K) \tilde{c}_K + q^F i / \tilde{c}_F (\rho) \tilde{c}_F \right] \left[\frac{u \tilde{c}_L}{c_L B (\rho) \ell - u \tilde{c}_L v + I}\right] B (0) \equiv q^Q (\rho, i; k^K),
\]

where \(m (\rho)\) was defined in (12.6) and varies inversely with \(\rho.\) Both are increasing functions of \(i.\) But the effect of \(\rho\) is ambiguous. We make the assumption that in both expressions the effect through \(\tilde{c}_F (\rho),\) which is ambiguous in general and has to do with the consumption of financial capitalists, is negligible in comparison to the effect through \(m (\rho)\) (and hence \(\tilde{c}_L B (\rho) \ell).\) Under this assumption, both are increasing functions of \(\rho.\) Hence, it follows from the first equation that the ‘industrial reserve army’ (or mass of unemployed) \(u = q^L - q^L (\rho, i; k^K)\) is an increasing function of \(\rho\) and a decreasing function of \(i.\)

Aggregate functioning capital can then be written as a function of the same variables: \(K = q^Q A p + q^L w = q^Q (\rho, i; k^K) [AB (\rho) + I] \ell / u_1 B (\rho) \ell.\) In this model we also have the condition for equilibrium in the market for money capital (13.1), which can here be expressed as:

\[
\frac{q^Q (\rho, i; k^K) [AB (\rho) + I] \ell}{u_1 B (\rho) \ell} = q^K k^K + q^F k^F (i).
\]

(13.13)

\(^3\)We make no use here of the extra columns of the social matrix that yield the dual systems of equations.

\(^4\)It is easy to check that multiplying the expression for \(q^Q\) by \(\ell\) yields \(q^L.\) Notice that we separated \(k^K\) from \(\rho\) and \(i\) in these expressions with a semicolon because it is an exogenous variable. See the footnote that precedes (12.6).
Whereas the effect of \( i \) on the left hand side of the equality is positive, the effect of \( \rho \) is ambiguous in general. The effect of an increase in \( \rho \) is decreasing through \( q^Q \) and through the denominator, but it is increasing through \( AB(\rho) \). With no claim to generality, we assume that in the numerator the effect through quantities is greater than through prices. Under this assumption, if there is an increase in \( \rho \) the effects that tend to reduce the left hand side predominate. We also assume that the function \( k^F (i) \) is sufficiently inelastic (or scarcely elastic) so that a variation in \( i \) has more effect on the left hand side than the right hand side. In that case, an increase in \( \rho \) requires an increase in \( i \) in order to restore equilibrium, which implies that in Figure 8 the MC line (for Market for Capital) that represents relation (13.13) has a positive slope, as drawn. Also, an increase in \( k^K \), if \( \rho \) is kept constant, requires an increase in \( i \) to restore equilibrium if \( \gamma (k^K) \) (contained in \( q^Q \)) is sufficiently inelastic, which we also assume. This implies that an increase in \( k^K \) shifts MC to the right, as we show on the graph.

On the other hand, the gross profit rate (which includes the interest industrial capitalists pay to financial capitalists) is gross profit \( q^K \gamma (k^K) \hat{c}_{KP} + q^F \left( i/\hat{\pi}^F (\rho) \right) \hat{c}_{FP} \) divided by aggregate functioning capital:\(^5\)

\[
\rho = \frac{[q^K \gamma (k^K) \hat{c}_{K} + q^F i u_1] B(\rho) \ell}{q^Q (\rho, i; k^K) [AB(\rho) + I] \ell}.
\]

Using (13.13) to simplify the denominator, and defining \( \hat{\pi}(\rho) \) as in (12.7) and (12.8) we get\(^6\)

\[
\rho = \frac{q^K \gamma (k^K) \hat{\pi}(\rho) + q^F i}{q^K k^K + q^F k^F (i)}.
\]

We thus have another (implicit) relation between \( \rho \) and \( i \), which is also affected by \( k^K \). Jointly, (13.13) and (13.14) determine the equilibrium values of these variables, given the value for \( k^K \), the exogenous variable that industrial capitalists handle. Line RHO in Figure 8 represents equation (13.14). In general it can have any slope. In can be proved that if \( k^F (i) \) is sufficiently inelastic and \( \hat{\pi}(\rho) \) either varies inversely with \( \rho \) or is sufficiently inelastic, then the slope of RHO is positive. And this line also depends on the value of \( k^K \). It can also be proved that if \( k^F (i) \) and \( \gamma (k^K) \) are sufficiently inelastic, an increase in \( k^K \) shifts line RHO to the right.

With our assumptions, both lines are positively sloped and shift to the right if \( k^K \) increases. This implies that the effect on \( \rho \) and \( i \) depends on the values of all the parameters and the initial situations. In Figure 8 we have drawn line MC with a greater slope than line RHO and show different possible cases. Starting from point A, if \( k^K \) increases any of cases B, C, or D can result, according to how much the two lines shift to the right. B shows a case in which both \( \rho \) and \( i \) increase, C one in which both fall, and D one in which \( \rho \) falls and \( i \) increases.

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\(^5\)Notice that \( \hat{c}_{FP} B(\rho) \ell/\hat{\pi}^F (\rho) = u_1 B(\rho) \ell. \)

\(^6\)Compare this expression to (12.9).
The aggregate incomes of workers and capitalists, respectively, are the following\(^7\):

\[
Y^L (\rho, i; k^K) = m(\rho) \left[ q^K \gamma(k^K) \frac{\widehat{c}_K}{u_1 B(\rho) \ell} + q^F i \frac{\widehat{c}_F}{c_F B(\rho) \ell} \right],
\]

\[
Y^K (\rho, i) = q^K \gamma(k^K) \widetilde{\pi}(\rho) + q^F i.
\]

They are both increasing with \(i\) (and \(k^K\)) and the aggregate income of workers is decreasing with \(\rho\). In contrast, the effect of \(\rho\) on the income of capitalists is ambiguous. If the effect of \(\rho\) on \(\widetilde{\pi}\) is negative or of low elasticity then the aggregate income is decreasing in \(\rho\). But an increase in \(k^K\) has a direct incidence on profits, individual and aggregate, so that there can be an increase in profits even when the profit rate is declining and the interest is increasing.

Marx holds that a low “interest generally corresponds to periods of prosperity or extra profit, a rise in interest separates prosperity and its reverse, and a maximum of interest up to a point of extreme usury corresponds to the period of crisis” (B3, 359). In Figure 8 the first can be represented by a movement from A to C, where \(k^K\) increases and \(k^F(i)\) falls (due to the fall in \(i\)). But even though this happened “generally” it need not always be the case since “It is possible, however, for low interest to go along with stagnation”, that is, a movement from B to A, and a “moderately rising interest to go along with revived activity”, that is, a movement from A to B. Marx opined that “Not every augmentation of loanable money capital indicates a real accumulation of capital or expansion of the reproduction process” (B3, 484). And he believed that this could be seen most clearly “in the phase of the industrial cycle immediately following a crisis, when loan capital lies idle in great quantities. And such times, when the production process is curtailed (...), when the prices of commodities are at their lowest level, when the spirit of enterprise is paralysed, the rate of interest is low, which in this case indicates nothing more than an increase in loanable capital precisely as a result of contraction and paralysation of industrial capital. It is quite obvious that a smaller quantity of a circulation medium is required when the prices of commodities have fallen,

\(^7\)Notice the resemblance of these formulas with those of (12.11).
the number of transactions decreased, and the capital laid out for wages reduced” (B3, 484). “Hence the demand for loanable money capital, either to act as a medium of circulation or as a means of payment (the investment of new capital is still out of the question), decreases and this capital, therefore, becomes relatively abundant” (Ibid.). Marx’s analysis of the relation between the industrial cycle and the interest rate in Chapter 30 of Book III reflects his keen non-schematic vision of the avatars of loanable money capital and interest during the industrial cycle. He concludes with:

On the whole, then, the movement of loan capital, as expressed in the rate of interest, is in the opposite direction to that of industrial capital. The phase wherein a low rate of interest, but above the minimum, coincides with the “improvement” and growing confidence after a crisis, and particularly the phase wherein the rate of interest reaches its average level, exactly midway between its minimum and maximum, are the only two periods during which an abundance of loan capital is available simultaneously with a great expansion of industrial capital (B3, 488).

Mystified social perceptions related with interest-bearing capital
In Chapters 6 and 7 we briefly addressed Marx’s observations on how social perceptions of the social relations in SCP and CCP are mystified. We here complement them with those Marx directly related with financial capital, characterized by means of the metamorphosis ‘formula’ \( M - M' = M + m \), where \( m \) is the interest perceived by the financial capitalist. He finds that

While interest is only a portion of the profit... it appears now, on the contrary, as though interest were the typical product of capital, the primary matter, and profit, in the shape of profit of enterprise, were a mere accessory and by-product of the process of reproduction. Thus we get the fetish form of capital and the conception of fetish capital. In \( M - M' \) we have the meaningless form of capital, the perversion and materialisation of production relations in their highest degree, the interest-bearing form, the simple form of capital, in which it antecedes its own process of reproduction. It is the capacity of money, or of a commodity, to expand its own value independently of reproduction—which is a mystification of capital in its most flagrant form (B3, 390).

In short: “Capital appears as a mysterious and self-creating source of interest –the source of its own increase” (Ibid.). For Marx in interest-bearing capital “this automatic fetish, self-expanding value, money generating money, is brought out in its pure state and in this form it no longer bears the birthmarks of its origin. The social relation is consummated in the relation of a thing, of money, to itself” (Ibid.; italics added). “It becomes a property of money to generate value and yield interest, much as it is an attribute of pear-trees to bear pears.” And he establishes an interesting parallelism between money and labor power: “As in the case of labour power, the use value of money here is its capacity of creating value—a value greater than it contains” (Ibid.). But for him this is a fetishist perception and, as such, mystified. Because it hides the fact that what is at work here is the participation of interest-bearing capital in the global process of capital by means of a specialization of functions that allows it to appropriate a part of the surplus value that is generated in the production process, which is organized and supervised by industrial capitalists. And this global process involves all the social relations of the capitalist mode of production, including those
that are specific to the circulation of commodities. In the capitalist mode of production, financial capital, as commercial capital, is for Marx subordinated to industrial capital. The specialization of financial capitalists allows them to obtain a share in the global surplus value thanks to the division of labor between these segments of global capital. The relations between human beings in the process of production and circulation of commodities, of money, and of capital lie hidden behind all these mystified perceptions.

Appendix to Chapter 13
Bibliographic Notes

Kalecki and the economic cycle  The Polish economist Michel Kalecki constructed in 1933 (66 years after the publication of the first edition of Book I of *Capital*) a very sophisticated model of an economic cycle that has some features in common with our model of Marx’s theory of the industrial cycle. The model has a consumption good and an investment good (say a machine). Since our treatment with ‘basic’ consumption baskets reduces the dimensions of the consumptions of workers and capitalists, the difference in the dimensional aspect is basically reduced to the inter-sector transactions of the technology through the matrix (A f). On the other hand, although Kalecki’s model initially includes an interest rate it is quickly left aside with the statement: “As everyone knows, over the economic cycle the interest rate rises during the boom and falls during the depression”\(^8\). Although that possibility is captured by Figure 8, we have seen that there are other possible cases in our model and that Marx had a much richer view both of the theoretical aspects and the empirical behavior of the interest rate over the industrial cycle. What is more sophisticated (and mathematical) in Kalecki’s model is that by assuming that ordered machinery takes several periods to be produced and several further periods to be delivered, a very mechanical cycle is generated which is based on such lags and the distinction and relations between orders, productions, and deliveries of machines. The basic assumption of the model is that the aggregate orders for finished machines \(I\) as a ratio of existing machines is an increasing function of the gross profit rate (which he calls gross yield) and a decreasing function of the interest rate, which is itself an increasing function of the profit rate: \(I/K = f (\rho, \iota (\rho))\). He then makes the assumption that the interest rate responds sufficiently slowly to the profit rate that (after having eliminated the interest rate) the resulting function \(I/K = \phi (\rho)\) is increasing with \(\rho\). This is very different from Marx’s view of the interest rate as responding quickly to any difference between supply and demand and the profit rates slowly adjusting as capital flows between branches of industry.

In his 1933 article Kalecki states:

The increase in the consumption of capitalists exerts the same influence as the production of investment goods: the production of capitalists’ consumption goods expands. This leads to an increase in employment, which again increases workers’ demand for consumption goods, which leads to a later increase in production. Aggregate production and profit per unit of output grow ultimately to the point in which the equality between the increase in real profits and the increase in the production of investment and capitalists’ consumption goods is assured (Kalecki 1970, 37).

\(^8\)This statement does not appear in Kalecki (1935), his more technical article published in *Econometrica*. But the assumption remains, albeit implicitly. We have directly translated from the Spanish version of Kalecki’s original essays (in Polish), unable to obtain an English translation.
And in 1935 he added the following remark:

... The conclusion that the increase in the consumption of capitalists increases their own profits contradicts the common belief that the greater consumption is the smaller is savings. What happens is that this reasoning, which is correct for an individual capitalist, cannot be applied to the aggregate of the capitalist class. The money a capitalist spends, whether in consumption or investment goods, is passed on to other capitalist in the form of profits. The investment or consumption of some capitalists creates the profits of others...

This is how capitalists as a whole determine their own profits by means of the size of their investment and personal consumption. In a way they are the “makers of their own destiny” Kalecki (1970, 37-8).

In the industrial cycle model of Chapter 12 and of the present chapter, neither of which has positive net investment (or capital accumulation), it is the simple dishoarding in order to invest in functioning capital (in Marx’s sense) that generates an increase in production and employment. If this also increases profits, capitalist consumption expands and the expansion of production and employment will be even greater. And there is no difference between what each capitalist does and what happens in the aggregate. Moreover, as in all SR models, capitalists do not save. But the more they expand their consumption when they are optimistic and disburse more capital by dishoarding, their profits increase pari passu. This is one of the possible cases of our model (and is reflected in Table 16). As Kalecki writes, “an increase in the consumption of capitalists increases their own profits.” In the Preface, Kalecki highlights the affinity between his theories and those of Rosa Luxemburg (a staunch follower of the political Marx), and it is surprising that he does not mention the resemblance to many of Marx’s analyses. Apparently, he is alluding to Luxemburg’s theory of the deficiency of effective demand and the need for exogenous sources of demand in order to avoid depressions. Since Kalecki wrote during and after the economic depression of the 30s, he placed emphasis in this aspect of his models. But Luxemburg believed there were theoretical flaws in Marx’s models of Extended Reproduction and that balanced growth was impossible. We will see in Chapter 14 that she was wrong.

**Keynes, effective demand, and Eugenics** Keynes states in his *General Theory* that the idea that the ‘aggregate demand function’ could be neglected was fundamental to ‘Ricardian economics’ (in which he included Marshall), and that though Malthus opposed this doctrine by pointing to the data, he had not been able to explain why it was possible for effective demand to be deficient or excessive. Keynes also holds that Ricardo’s view prevailed in England because this matter was not addressed. “The great puzzle of Effective Demand with which Malthus had wrestled vanished from economic literature... It could only live on furtively, below the surface, in the underworlds of Karl Marx, Silvio Gesell or Major Douglas” (Keynes 1936, 32). Keynes says absolutely nothing about Marx’s theory. He dedicates a paragraph to Douglas attributing to him one of the most famous “heretical theories of under-consumption” since the First World War (Ibid., 370), but placing him in a definitely lower rank than Gesell. In contrast, he dedicates several pages to the “unduly neglected prophet Silvio Gesell”9.

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9Silvio Gesell (1862-1930) was born in a town that was then located in Germany and is now part of Belgium. He lived in Switzerland and Germany, and also a number of years in Argentina, where he wrote and published his first works (in German). He was tried for having been (during 7 days)
almost apologizing for having previously “treated his profoundly original strivings as being no better than those of a crank” (Ibid., 353). Referring to the Preface Silvio Gesell wrote for his *The Natural Economic Order* (the English translation of which was published in 1918), he writes “The answer to Marxism is, I think, to be found along the lines of this preface” (Ibid., 355). Keynes explains that the aim of Gesell’s book was “the establishment of an anti-Marxian socialism... on theoretical foundations totally unlike those of Marx in being based on a repudiation instead of on an acceptance of the classical hypotheses, and on an unfettering of competition instead of its abolition” (Ibid.).

Keynes was certainly right in implying that Marx aimed for a (future) abolition of competition. In fact he even aimed for a future abolition of markets. But he reveals his ignorance of Marx’s theory when he writes that he accepted the ‘classical hypotheses’. For Keynes the classical theory of employment was based on two postulates: 1) that the wage is equal to the marginal product of labor and 2) that the supply of labor was based on the disutility of labor. And he states that such postulates can justify the prevalence of ‘frictional’ and ‘voluntary’ unemployment, but not ‘involuntary’ unemployment. For this his *General Theory* was necessary. It is plain, on the one hand, that Marx’s theory had nothing to do with those two ‘postulates’ of Neoclassical economics and, on the other, that his theory was *fundamentally* related to ‘involuntary’ unemployment (which Marx called the ‘industrial reserve army’) and its dynamics over time. Apparently, Keynes was only informed about Marx’s political opinions (or those of his followers) on the future of society. Either he was never acquainted with his theory of Capitalism or when he wrote his *General Theory* he had long forgotten all about it. But Marx’s theory had very much to do with the phenomenon that Keynes wanted to explain theoretically so as to make it possible to ameliorate it. Moreover, he had advanced quite sound theoretical explanations for such phenomena as business cycles and ‘involuntary’ unemployment at least 70 years before Kalecki and Keynes.

It is surely due to such ignorance that Keynes found affinities between his own theory and those of rather mediocre theoreticians such as Douglas or Gesell. When he writes that “the future will learn more from the spirit of Gesell than from that of Marx” one wonders how deep his newfound affinity to Gesell was. He writes enthusiastically that “the preface to *The Natural Economic Order* will indicate to the reader the moral quality of Gesell.” But that preface is a siren song to the benevolent effects of competition on *Eugenics*, that is, the “right evolution” of the human species. “Only through competition, chiefly competition in the economic sphere, is right evolution, eugenics, possible” (Gesell 1958, 1). Gesell considered that it was necessary to eradicate privileges and implement some fundamental reforms so that the success in competition would only be dependent on innate characteristics of individuals. “Only then shall we be justified in hoping that humanity may in time shake off the burden of inferior individuals imposed upon it by thousands of years of unnatural selection” (italics added).

He states that “The best of mankind must be allowed to develop”, thus making “eugenic selection” a necessity. “The Natural Economic Order must, therefore, be founded upon self-interest” because economic life requires strong impulses and the “only impulse of sufficient strength and constancy is egoism.” Of course, his eugenic selection in the economic sphere was “in no way opposed to the higher impulses which preserve the species” (referring to altruistic actions), since it creates not only the opportunities but also the means for altruistic actions. In contrast, under the “opposite form of

Minister of Finance of the very short lived Soviet Republic of Bavaria in 1919. Shortly after his death his son Carlos forested some coastal lands of the Province of Buenos Aires, where he founded a beach resort he named Villa Gesell in tribute to his father.
economic order” (i.e. Socialism) “everyone would send needy friends to an insurance company and sick relatives to a hospital, the State would make all personal assistance superfluous.” Under the Natural Economic Order founded upon egoism Gesell proposes that everyone receive the full proceeds of his own labor and dispose of it as he sees fit. Nobody would require, but nobody would prevent, that a person share his income with the poor. In Gesell’s society, which apparently moved Keynes, “the individual, obeying the impulse of egoism, goes straight for his aim, undisturbed by scruples alien to economic life-scruples” (Ibid., 2). To make all this possible Gesell proposed two fundamental reforms that he called Free-Land and Free-Money. He thought that his Order could also be named “The Manchester System”, in which “the free play of economic forces would rectify the blunders of State-Socialism and short-sighted official meddling” (Ibid., 3).

When Keynes published his book Hitler had already been in power in Germany for almost three years and his government had begun the eugenic policies (recommended by the “German Society for Racial Hygiene”) of forced sterilization that would rapidly lead to the assassination of hundreds of thousands of persons deemed deficient on the ground that they could affect racial health. Eugenics had many sympathizers in Great Britain and, not surprisingly, Keynes was one of the most enthusiastic. He served as treasurer of the University of Cambridge Eugenics Society and also as Director of the British Eugenics Society between 1937 and 1944. A eugenic government policy was never actually implemented in Britain. But forced sterilization policies were implemented in the U.S.A. in some states (especially California). And the American pseudo-scientific racist ideologue and eugenicist Madison Grant published in 1916 the book The Passing of the Great Race: Or, The Racial Basis of European History which so impacted Hitler that he wrote Grant a highly complimentary letter saying that the book was his Bible (Kühl 1994, 85).
Chapter 14  THE ACCUMULATION OF CAPITAL AND EXTENDED REPRODUCTION

As we pointed in Chapter 12, Marx holds that there are three main causes for hoarding in the capitalist mode of production: the industrial cycle, which requires the ability to increase or reduce hoarding, the turnover of capital, which necessitates the periodic replacement of elements of fixed capital of great value and hence requires the gradual accumulation of amortizations in a fund, and the one that specifically interests us in this chapter: the need for “the creation of money capital existing temporarily in latent form and intended to function as productive capital” (B2, 347). To accumulate capital, the industrial capitalist “must first withdraw in money form from circulation a part of the surplus value which he obtained from that circulation, and must hoard it until it has increased sufficiently for the extension of his old business or the opening of a side-line” (B2, 125). Marx calls these two alternative modes of accumulating capital in production ‘intensive’ and ‘extensive’, respectively: “the conversion of surplus value into capital, is essentially a process of reproduction on a progressively increasing scale, whether this expansion is expressed extensively in the form of an addition of new factories to the old, or intensively by the enlargement of the existing scale of operation” (B2, 319). But before investing the surplus value (or profit) accumulated (and not consumed), it must reach the necessary size, meanwhile remaining as ‘latent money capital’. “Surplus value thus congeals into a hoard and in this form constitutes latent money capital—latent because it cannot act as capital so long as it persists in the money form” (B2, 84). And one of the effects of hoarding is that as long as it continues “it does not increase the demand of the capitalist. The money is immobilised. It does not withdraw from the commodity market any equivalent in commodities for the money equivalent withdrawn from it for commodities supplied” (B2, 125). But, as we have seen in the previous chapter, in Book III Marx explains that in modern Capitalism the banking system can recycles those funds so that they do not remain immobilized, while the banks themselves keep reserves in the Bank of England (in the case of Britain) equivalent to a fraction of their liabilities.1

The accumulation of capital is central to Marx’s theory and includes what we currently call economic growth as well as what we call economic development. In this chapter we only address the modeling aspects of the accumulation of capital as they pertain to what Marx did in his schemes of Extended Reproduction. Since they do not consider changes in the structure of industry but only (balanced) growth, they do not deal with development. To model Marx’s ER we use some mathematical instruments that, though elementary, were mostly not available when he wrote (at least for the great majority of economists). We leave for Chapter 17 Marx’s theory of the accumulation of capital as a historical process, with its tendencies and countertendencies.

Although Marx develops his inter-sectoral analysis of Simple and Extended Reproduction in Book II, he had already established its conceptual foundations in Book I, where he writes:

---
1Engels adds backup data showing that in 1892 the 15 largest London banks had 12% of their liabilities as reserves in the Bank of England while they held an additional 1.3% as cash (MECW 37, 473).
Hitherto we have investigated how surplus value emanates from capital; we have now to see how capital arises from surplus value. Employing surplus value as capital, reconverting it into capital, is called accumulation of capital (B1, 578).

In economic forms of society of the most different kinds, there occurs, not only simple reproduction, but, in varying degrees, reproduction on a progressively increasing scale. By degrees more is produced and more consumed, and consequently more products have to be converted into means of production. This process, however, does not present itself as accumulation of capital, nor as the function of a capitalist, so long as the labourer’s means of production, and with them, his product and means of subsistence, do not confront him in the shape of capital (B1, 594).

And in Book II, where Marx develops his inter-sectoral studies, he specifies that SR is merely a stage in the understanding of ER, since only by means of the latter can the ‘compelling motive’ of CCP be adequately represented:

... we had assumed in the analysis of simple reproduction that the entire surplus value of I and II is spent as revenue. As a matter of fact however one portion of the surplus value is spent as revenue, and the other is converted into capital. Actual accumulation can take place only on this assumption. That accumulation should take place at the expense of consumption is, couched in such general terms, an illusion contradicting the nature of capitalist production. For it takes for granted that the aim and compelling motive of capitalist production is consumption, and not the snatching of surplus value and its capitalisation, i.e., accumulation (B2, 503).

This last sentence reflects one of the most individually significant differences between Marx’s theory of Capitalism and mainstream economic theory, in which all individual decisions are taken in the context of a life cycle in which lifelong income ends up being consumed, either by the individual himself or, at most, his descendants when the fiction of ‘dysnastic altruism’ is introduced. For Marx, in contrast, the accumulation of capital fundamentally implies the accumulation of power and the desire for power can even constrain consumption for the sake of accumulating more wealth and power.

In the last chapter of Book II, on “Accumulation and reproduction on an extended scale”, Marx explicitly assumes that “the portion of the newly created money capital capable of being converted into variable capital will always find at hand the labour power into which it is to transform itself” (B2, 501). This implies that the accumulation process there represented takes place in the context of a numerically growing working class. Marx had already highlighted in Book I that inheritance tends to turn the children of capitalists into capitalists, implying that the capitalist class also grows (or can grow) numerically, except when he addresses the process of centralization of capital that can actually (and theoretically) make the aggregate number capitalists diminish, or at least reduce its rate of growth.

Marx also explains that what is reproduced on an extended scale is not only the number of workers and capitalists but the production relation between capitalists and wage workers itself, since in the process of accumulation the children of wage workers will not normally be in condition to work independently by purchasing their own means of production, and will hence remain subservient to capital on an extended scale. Marx holds that ER not only includes the growth of the means of production and the means of subsistence but also de growth of the working class itself, since wages must be
sufficient to allow working families to have sufficient descendents so that there is an
extensive growth of the number of wage workers:

Now in order to allow of these elements actually functioning as capital, the
capitalist class requires additional labour. If the exploitation of the labour-
ers already employed do[es] not increase, either extensively or intensively,
then additional labour power must be found. For this the mechanism of
capitalist production provides beforehand, by converting the working class
into a class dependent on wages, a class whose ordinary wages suffice, not
only for its maintenance, but for its increase. It is only necessary for capital
to incorporate this additional labour power, annually supplied by the work-
ing class in the shape of labourers of all ages, with the surplus means of
production comprised in the annual produce, and the conversion of surplus
value into capital is complete. From a concrete point of view, accumulation
resolves itself into the reproduction of capital on a progressively increasing
scale (B1, 580; text in square brackets added).

Marx’s Extended Reproduction tables
In Chapter 21 of Book II (“Accumulation and Reproduction on an Extended Scale”) Marx gives two quite complete numerical illustrations of ER in which there is popu-
lation growth. In Illustration 1 (B3, 510-4) he experiments with a system with two
departments (I and II as in his SR table) in which there are different value com-
positions of capital. This creates difficulties for him related to his use of values in
valuations and aggregation. However, he manages to make assumptions that, though
arbitrary, constitute an algorithm that produces interesting results. He assumes that
in department I, where the value composition is 4, capitalists reinvest half of the sur-
plus value, which allows them to grow 10% each year. In contrast, in department
II, in which the value composition is 2, capitalists reinvest whatever is necessary to
maintain that composition constant at the initial level, which initially requires a 20%
reinvestment of the surplus value. With such assumptions the scheme quickly con-
verges to a situation in which department II reinvests 30% of its surplus value each
year and grows 10%, the same as in department I. In Illustration 2 (B3, 514-9.) Marx
from the beginning assumes (as is implicit in Book I) that both departments have the
same value composition (equal to 5). His tables span 4 years of growth (a base year
and 3 additional years). They are summarized in Table 17 below. The reader should
bear in mind that Marx’s c, v, and s are our C^v, V^v, and S^v. The profit rate (the cor-
rect formula for which coincides with Marx’s because with equal value compositions
production prices are proportional to values) is the same in both departments and all
time periods. For example, in department I it is \( \rho = 1000/(5000 + 1000) = 0.1616 \)
in the first year (alternatively, \( \rho = e/(1 + \kappa) = 1/(1 + 5) = 0.1616 \)). He also makes
the assumption that 50% of the surplus value of each department is capitalized and
the other 50% consumed. For example, in department I a half (500) of the surplus
value of the first year (1000) is reinvested so that the expansion of constant capital
is 417 and of variable capital 83. Notice that aside from the arithmetical slip in de-
partment II in the second year (in which he has a capitalization of 65% of the surplus
value instead of 50%) and unimportant rounding errors, the growth rate is \( g = 8.3\% \).
This is the ‘balanced’ rate of growth (as it is now called) that Marx introduced in his
numerical exercise. According to Engel’s Preface –of 1893– to Book II, Chapter 21,
where these exercises are, comes from manuscript VIII, written during or after July
1878. It contains what was probably the first two sector balanced growth exercise ever
developed.
Table 17

Condensed table of ER in *Capital*

<table>
<thead>
<tr>
<th>Year</th>
<th>Dep.</th>
<th>$c$</th>
<th>$v$</th>
<th>$s$</th>
<th>$c+v$</th>
<th>$c+v+s$</th>
<th>$\kappa$</th>
<th>$e$</th>
<th>$\rho$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>5000</td>
<td>1000</td>
<td>1000</td>
<td>6000</td>
<td>7000</td>
<td>5</td>
<td>1</td>
<td>16.7%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1430</td>
<td>285</td>
<td>285</td>
<td>1715</td>
<td>2000</td>
<td>5</td>
<td>1</td>
<td>16.6%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>I+II</td>
<td>7715</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>I</td>
<td>5417</td>
<td>1083</td>
<td>1083</td>
<td>6500</td>
<td>7583</td>
<td>5</td>
<td>1</td>
<td>16.7%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1583</td>
<td>316</td>
<td>316</td>
<td>1899</td>
<td>2215</td>
<td>5</td>
<td>1</td>
<td>16.6%</td>
<td>10.7%</td>
</tr>
<tr>
<td></td>
<td>I+II</td>
<td>8399</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9798</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 3</td>
<td>I</td>
<td>5869</td>
<td>1173</td>
<td>1173</td>
<td>7042</td>
<td>8215</td>
<td>5</td>
<td>1</td>
<td>16.7%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1715</td>
<td>342</td>
<td>342</td>
<td>2057</td>
<td>2399</td>
<td>5</td>
<td>1</td>
<td>16.6%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>I+II</td>
<td>9099</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10614</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 4</td>
<td>I</td>
<td>6358</td>
<td>1271</td>
<td>1271</td>
<td>7629</td>
<td>8900</td>
<td>5</td>
<td>1</td>
<td>16.7%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>1858</td>
<td>372</td>
<td>372</td>
<td>2229</td>
<td>2600</td>
<td>5</td>
<td>1</td>
<td>16.6%</td>
<td>8.3%</td>
</tr>
<tr>
<td></td>
<td>I+II</td>
<td>9858</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11500</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Models of Extended Reproduction

Extended Reproduction requires that capitalists save a part of the profits and invest it in the expansion of the productive processes they operate. But this expansion requires either the expansion of the number of wage workers, or the increase in productivity that makes the same number of workers produce more (or a combination of both). In order to not excessively complicate our matrix model, in this chapter we develop several different models of ER instead of a single more general one. First we present two models in which ER is simply based on population growth, as in the numerical examples of Marx in Book II, and then we present two additional models in which the populations of workers and capitalists are constant and ER is based on a constant introduction of technological or organizational innovations that increase the productive power of labor. Although Marx did not include the increase in the productive power of labor in his ER tables, it was an aspect of his theory of Capitalism in its advanced industrial phase (in which relative surplus value is produced) that he persistently highlighted (even as far back as when he wrote the *Manifesto of the Communist Party*). To avoid excessive complications we begin with the simplifying assumption that all constant capital is circulating capital and the *invested* capital (which is used to calculate the profit rate) is identical to the value of the means of production and labor consumed in the productive process. We later present a model that includes fixed capital by distinguishing a stock matrix and a flow matrix. The latter is used in Chapter 17 to analytically express Marx’s view of the tendency for the progressive decline in the profit rate.

Models of ER based on population growth

I. Both populations grow

Marx explicitly includes population growth, especially that of workers, in his theory of the accumulation of capital. He writes: “As simple reproduction constantly reproduces the capital relation itself, i.e., the relation of capitalists on the one hand, and
wage workers on the other, so reproduction on a progressive scale, i.e., accumulation, reproduces the capital relation on a progressive scale, more capitalists or larger capitalists at this pole, more wage workers at that” (B1, 609; italics added). By distinguishing two cases in the capitalist ‘pole’ ("more capitalists or larger capitalists") he is referring to the fact that there can be growth in the population of capitalists but that this can also be prevented by the ‘centralization’ process that can force many capitalists to step aside because they have lost in the competitive process. As we understand it, Marx’s ‘centralization’ process is incompatible with the balanced growth of his ER model. Hence, we do not even try to represent it in this chapter. We first present two variants of population growth. In the first both of the ‘large’ classes of CCP grow at the same pace, and in the second it is only the worker population that grows, generating a progressive concentration of income in the capitalist class, even if there is no ‘centralization’ process involved by which a fraction of the capitalists lose their capitalist-entrepreneurial function each period. This can be considered an extreme way of treating the more realistic case in which workers have more children than capitalists (possibly for cultural reasons).

The static nature of SR made it possible for us to avoid the formal introduction of time in the models, since each period replicates the previous one. But in the context of ER it is necessary to introduce time more explicitly, at least initially. As we will show, the ‘balanced growth’ assumption of Marx’s most successful ER model lets us formulate the model in such a way that an explicit account of time can still be avoided. But one should bear in mind that although Marx made the assumption of balanced growth in his models for the purpose of simplifying the analysis, balanced growth (as ‘equilibrium’ in general) is not a component of his theory of capitalist accumulation but only a convenient reference point to take into account when considering a much more complex reality that is permanently in ‘disequilibrium’. Even in Chapter 21 of Book II he refers to this reality:

The fact that the production of commodities is the general form of capitalist production implies the role which money is playing in it not only as a medium of circulation, but also as money capital, and engenders certain conditions of normal exchange peculiar to this mode of production and therefore of the normal course of reproduction, whether it be on a simple or on an extended scale – conditions which change into so many conditions of abnormal course, into so many possibilities of crises, since a balance is itself an accident owing to the spontaneous nature of this production (B2, 494).

The system of production prices, wage, and rate of profit As we saw in Chapter 8, under SR the CCP can be characterized by the quantities system (8.1) and the system of production prices, wages, and profit rate (8.17). Let \( \rho_0 \) be the profit rate when the balanced growth rate is \( g \). Then the profit rate we represented as \( \rho \) in that chapter is here \( \rho_0 \) since it is the rate that corresponds to \( g = 0 \) (and all profit is used for consumption). With this slight change in notation, let us call \( M(\rho_0) \) the matrix in (8.17). Equation (8.17) shows that \( \lambda(M(\rho_0)) = 1 \) must hold for there to be SR with a homogeneous profit rate \( \rho_0 \). ER in CCP is possible when \( \lambda(M(\rho_0)) < 1 \). In that case, there is a growth rate \( g > 0 \) such that \( \lambda(M(\rho_g)) = 1 \). Hence, the price system with (balanced) growth rate \( g \) is:

\[
\begin{bmatrix}
(1 + \rho_g) A & (1 + \rho_g) \ell \\
0 & \ell
\end{bmatrix}
\begin{bmatrix}
p \\
w
\end{bmatrix}
= \begin{bmatrix}
p \\
w
\end{bmatrix}.
\] (14.1)
This implies that for ER to be possible, the combination of the degree of development of productive power (which is reflected in the coefficients of \([A \ell]\)) and the consumption of the working population (reflected in \(c_L\)) is such that it allows capitalists to obtain profits such that they can not only consume their basket \(c_K\) but also reinvest an additional part of their profits in the expansion of the production process (which includes the employment of more workers), as we see below. Notice that we can obtain the vector of production prices \(p = B(\rho_g) \ell w\) from (14.1), as we did in (8.20). If we use the wage rate as numeraire, the fact that \(B(\rho_g) > B(\rho_0)\) implies that vector \(p\) of (14.1) is necessarily greater than that of (8.20). So we could give it another name such as \(p_g\). But we avoid doing so in order to keep the notation as simple as possible.

The quantities system As an introduction to the ER models, we start by making the assumption that growth is simply based on the expanded reproduction of the population. We also assume here that the populations of workers and capitalists grow at the same rate \(g > 0\) in each period. Assuming that there is no modification in the technology or in the per capita consumption baskets \(c_L\) and \(c_K\), the fact that the technology assumed is linear and there are no resources required in production that have a fixed quantity (such as land), gross outputs will also grow at the rate \(g\). Hence, we have the following relations between the production and population variables in periods \(t + 1\) and \(t\):

\[
q_{t+1}^Q = (1 + g) q_t^Q, \quad q_{t+1}^L = (1 + g) q_t^L, \quad q_{t+1}^K = (1 + g) q_t^K. \tag{14.2}
\]

For ER to exist, gross output in \(t\) must be sufficient to satisfy both the (expanded) consumption of means of production in period \(t + 1\) as the (expanded) consumption of the population: \(q_t^Q = q_{t+1}^Q A + q_{t+1}^L c_L + q_{t+1}^K c_K\). Using (14.2) we have

\[
q_t^Q = (1 + g) \left( q_t^Q A + q_t^L c_L + q_t^K c_K \right). \tag{14.3}
\]

An alternative way of getting (14.3) is to notice that net outputs in \(t\) (in physical quantities), i.e. \(q_t^Q (I - A)\), must be equal to the sum of the corresponding quantities of consumption and investment in \(t\):

\[
C_t \equiv q_t^L c_L + q_t^K c_K, \\
I_t \equiv g \left( q_t^Q A + q_t^L c_L + q_t^K c_K \right). \tag{14.4}
\]

Hence, \(q_t^Q (I - A) = C_t + I_t\), which is simple another way of writing (14.3).

The three variables that are time dependent in (14.3) grow each period. The standard way of treating a system with balanced growth is to transform the equations so that there are only stationary variables (i.e., variables that do not grow nor shrink as time passes). This way it is possible to eliminate subindex \(t\). In this particular ER model it is convenient to define the shares of workers and capitalists in the population, which remain constant under the assumption that they both grow at the same rate \(g\). Let \(\alpha_L \equiv q_t^L / (q_t^L + q_t^K)\) be the share of workers in the population and \(\alpha_K \equiv q_t^K / (q_t^L + q_t^K) = 1 - \alpha_L\) that of capitalists. Also, define per capita output \(\bar{q}_t^Q \equiv q_t^Q / (q_t^L + q_t^K)\). Due to (14.2) these gross outputs \(\bar{q}_t^Q\) are stationary, which allows us to eliminate the subindex \(t\):

\[
\frac{q_{t+1}^Q}{q_{t+1}^L + q_{t+1}^K} = \frac{q_t^Q}{q_t^L + q_t^K} = \bar{q}_t^Q = \bar{q}_Q,
\]
Dividing (14.3) and $q_t^L = q_t^Q \ell$ by the total population $q_t^Q + q_t^K$ we hence obtain
\begin{align*}
\bar{q}^Q &= (1 + g) \left( \bar{q}^Q A + \alpha_L c_L + \alpha_K c_K \right) \\
\alpha_L &= \bar{q}^Q \ell,
\end{align*}
which can also be written as
\begin{equation}
\left[ \begin{array}{ccc}
\bar{q}^Q & \alpha_L & \alpha_K \\
\end{array} \right] \left[ \begin{array}{c}
(1 + g) A \\
(1 + g) c_L \\
(1 + g) c_K \\
\end{array} \right] = \left[ \begin{array}{c}
\bar{q}^Q \\
\alpha_L \\
\end{array} \right].
\end{equation}

This system can be compared with (8.1). Notice that from (14.5) we can get the $\bar{q}^Q = (\alpha_L c_L + \alpha_K c_K) B(g)$, where $B(g)$ has the same definition as $B(\rho)$ in (8.21). Hence, gross per capita quantities are functions of the consumption baskets, the shares in the population, the growth rate, and the technical coefficients.

**The profit rate under ER** If we premultiply (14.1) by $[\bar{q}^Q \alpha_L]$ and multiply (14.6) by $[p \ w]^T$, the resulting left hand sides are equal, and hence
\begin{equation}
\rho_g - g = (1 + g) \frac{\alpha_K c_K p}{\bar{q}^Q A p + \alpha_L c_L p} \equiv (1 + g) \rho_0, \tag{14.7}
\end{equation}
where $\rho_0$ is the part of the profit rate that only reflects the consumption of capitalists (and would be all of the profit rate if there were SR, i.e., if $g = 0$):
\begin{equation*}
\rho_0 \equiv \frac{\alpha_K c_K p}{\bar{q}^Q A p + \alpha_L c_L p} = \frac{q_t^K c_K p}{q_t^Q A p + q_t^L c_L p}.
\end{equation*}
Hence, from (14.7) we get:
\begin{equation}
1 + \rho_g = (1 + g) \left(1 + \rho_0\right). \tag{14.8}
\end{equation}

It is important to stress that while $\pi \equiv c_K p$ was the profit of each capitalist in the context of SR (using the corresponding production prices, which we have seen are different from those of ER), in the context of ER it is only the part of each capitalist’s profit which he uses to purchase consumption goods. For when there is growth, the profit in each branch has to be sufficient to finance not only the consumption of capitalists $q_t^K c_K$ but also the necessary investment $I_t$, so that the needed expansion of the output of means of production and means of subsistence that must be available in the next period takes place. Hence, aggregate profit must be:
\begin{equation}
\Pi_t = q_t^K c_K p + I_t p, \tag{14.9}
\end{equation}
where $I_t$ is defined in (14.4). Both aggregate profit and the quantity variables are increasing period by period. But this is not a problem since what we now need is the global rate of profit, which in the context of balanced growth at rate $g$ we call $\rho_g$. This rate can also be obtained dividing the aggregate profit $\Pi_t$ by the invested capital $K_t$ (which under our assumptions is also the value of the consumed means of production and means of subsistence). Hence, we have
\begin{equation}
\rho_g = \frac{\Pi_t}{K_t} = \frac{q_t^K c_K p + g \left( q_t^Q A + q_t^L c_L + q_t^K c_K \right) p}{\left( q_t^Q A + q_t^L c_L \right) p} = (1 + g) \rho_0 + g. \tag{14.10}
\end{equation}
We again have (14.7), and hence (14.8).

An alternative way of expressing $\rho_0$ is:

$$\rho_0 = \frac{\alpha_{K}c_{KP}}{q^Q A p + \alpha_{L}c_{LP}} = \frac{e^p_0}{\kappa^P + 1},$$

where the following definitions of the rate of surplus value restricted to the part of profits destined for consumption $e^p_0$ and the value composition of $\kappa^P$ were used:

$$e^p_0 = \frac{q^K_{LP} c_{KP}}{q^L_{LP} c_{LP}} = \frac{\alpha_{K} c_{KP}}{\alpha_{L} c_{LP}} \quad \text{and} \quad \kappa^P = \frac{q^Q A p}{q^L_{LP} c_{LP}} = \frac{q^Q A p}{\alpha_{L} c_{LP}}.$$ (14.11)

Also, the aggregate profit is exactly the same thing as the aggregate surplus value (using production prices): $\Pi_t = S^P_t.$ Hence we can make the same type of decomposition of the profit rate as we did in (8.25) for the case of SR, by which

$$\frac{\epsilon^p}{\kappa^P + 1} = \rho_g = (1 + g) \frac{e^p_0}{\kappa^P + 1} + g,$$

which implies $\epsilon^p = e^p_0 + g (\kappa^P + 1 + e^p_0),$ and so $\epsilon^p$ reduces to $e^p_0$ if $g = 0.$

**Expansion of the systems to include the distribution of capital by branch**

The systems of quantities and prices can be expressed more symmetrically if, as in (8.34), we represent the allocation of capital to the various industrial branches by means of $\eta.$ Since $q^K_{LP} = q^Q_{LP} \eta,$ the capitalists’ share in the population is $\alpha_{K} = \bar{q}^Q \eta,$ which allows us to expand the quantities system (14.6) to:

$$\begin{bmatrix} \bar{q}^Q & \alpha_{L} & \alpha_{K} \end{bmatrix} \begin{bmatrix} (1 + g) A \\ (1 + g) c_{L} \\ (1 + g) c_{K} \end{bmatrix} = \begin{bmatrix} \bar{q}^Q \\ \alpha_{L} \\ \alpha_{K} \end{bmatrix}.$$ (14.12)

Hence, the dominant eigenvalue of the system matrix is one and the associated left eigenvector is positive. And it is also unique, since it is normalized so that $\alpha_{L} + \alpha_{K} = 1.$

If we eliminate $\alpha_{L}$ and $\alpha_{K}$ from (14.12) we get

$$\bar{q}^Q = (1 + g) \bar{q}^Q (A + \ell c_{L} + \eta c_{K}).$$ (14.13)

Therefore, $\bar{q}^Q$ is the left eigenvector of matrix $A + \ell c_{L} + \eta c_{K}$ that is associated to the dominant eigenvalue $1/(1 + g) < 1.$ There is a unique (up to a scalar factor) vector of production prices $p > 0$ that satisfies the dual equation to (14.13), that is:

$$p = (1 + g) (A + \ell c_{L} + \eta c_{K}) p.$$ (14.14)

Since the wage rate is $w = c_{LP},$ if we define the profit destined for consumption as $\pi \equiv c_{KP},$ (14.14) is equivalent to $p = (1 + g) (Ap + \ell w + \eta \pi).$ Hence, the last three equalities can be combined in the system of production prices and incomes that corresponds to the present version of ER:

$$\begin{bmatrix} (1 + g) A & (1 + g) \ell \\ c_{L} & 0 \end{bmatrix} \begin{bmatrix} p \\ w \end{bmatrix} = \begin{bmatrix} p \\ w \end{bmatrix}.$$ (14.15)

\(^2\)It is the rates of profit and surplus value that differ due to the fact that, following Marx, they have different denominators (the first capital and the second merely variable capital).
As we did with SR, we can re-express the system in terms of the global profit rate if we eliminate \( \eta \) under the assumption of equalization of profit rates in the various branches. In that case we merely get (14.1), which was our starting point. From the first equations of (14.15) and (14.1), respectively, we get

\[
\eta \pi = \left( (\rho_g - g) / (1 + g) \right) (Ap + \ell w),
\]

and using (14.10) this reduces to

\[
\eta \pi = \rho_0 (Ap + \ell w),
\]

that is, the relation between the vector of direct capital requirements in the various branches \( \eta \) and the part of aggregate profits that only includes capitalists’ consumption (hence excluding their investment).

Also, from the first equality of (14.10) we can calculate what fraction of profit each capitalist has to invest in order to get ER with balanced growth. Since

\[
\rho_g = \frac{\Pi_t}{K_t} = qK_cKp + \frac{I_t p}{K_t} = \rho_0 + \frac{I_t p}{\Pi_t} = \frac{\rho_0 + \frac{I_t p}{\Pi_t}}{\rho_g},
\]

we can see that the capitalists’ saving (and investment) rate \( s_t \equiv I_t p / \Pi_t \) is stationary \( s_t = s \) and equal to

\[
s = s_t = \frac{I_t p}{\Pi_t} = 1 - \frac{\rho_0}{\rho_g}. \tag{14.16}
\]

Hence, \( \rho_0 = (1 - s) \rho_g \) and therefore \( \rho_0 k_t = (1 - s) \rho_g k_t \), which means that \( \rho_0 k_t \) is the non-saved (or consumed) fraction of the profit of each capitalist.

We can conclude that Marx was well oriented when he assumed that each branch invested the same fraction of its profits in his most successful numerical illustration of ER summarized in Table 17. There he had made the assumption of equal value compositions of capital. From what we have seen here, Marx could have avoided the assumption of equal value compositions if only he had been able to use algebraic instruments that either where not available in his time (such as Perron-Frobenius theory) or only an excellent mathematician could handle (such as Cournot, who was seventeen years older than Marx).

Let us try to use the model presented above for the interpretation of the data in Table 17. For this it is convenient to go over the assumptions used there. The profit rate is \( \rho_g = 1000/6000 = 0.1666 \) in both departments in every year. In general, that is, except for the slip in department II in the second year, the rate of growth is \( g = 0.083 \) and the rate of savings (or capitalization) is \( s_t = I_t p / \Pi_t = 50\% \). Then, according to (14.16) we should have \( \rho_0 = (1 - I_t p / \Pi_t) \rho_g = (1 - 50\%) (0.1666) = 0.0833 \). And according to (14.10) we should have

\[
\rho_g = (1 + g) \rho_0 + g = (1 + 0.083) 0.083 + 0.083 = 0.1732 > 0.1666.
\]

Hence, with our formulas obtained above the profit rate would be a little higher than Marx’s. The explanation for this discrepancy is that we assumed that the populations of workers and capitalists grow at the same rate, making it necessary that investment include not only the growth in workers’ consumption but also in capitalists’ consumption. But in Marx’s exercise it is implicit that only the worker population grows. Our assumptions imply that a higher profit rate is necessary so that profits can finance that additional investment.

**The values and surplus value system** As in the case of SR, Marx also needed a system of values and rate of surplus value for the case of ER in order to express his theory of exploitation. Although he was not too successful when he tried to write down a general scheme of ER with balanced growth and differing value compositions, which implied that prices of production should be used in the aggregations, he was right in
trying. The scheme in which he was successful assumed equal value compositions, so production prices coincided in their structure with values. Here, consistent with our goal of building the best possible case for Marx’s theory, we show how a system of values and rate of surplus value for the case of ER can be posed.

Since aggregate surplus value must include the part that capitalists use for consumption as well as the part they reinvest (as in the case of aggregate profit (14.9)) we have:

\[ S_i^v = q_i^K c_K v + g \left( q_i^Q A + q_i^L c_L + q_i^K c_K \right) v. \]

Hence, the global rate of surplus value is:

\[ e_g = \frac{S_i^v}{V_i^v} = \frac{q_i^K c_K v + g \left( q_i^Q A + q_i^L c_L + q_i^K c_K \right) v}{q_i^K c_L v} = (1 + g) e_0 + g (\kappa + 1), \]

where for the last equality we defined the rate of surplus value that is involved in consumption \( e_0 \) and the value composition of capital \( \kappa \), respectively:

\[ e_0 = \frac{q_i^K c_K v}{q_i^K c_L v} = \frac{\alpha_K c_K v}{\alpha_L c_L v}, \quad \kappa = \frac{q_i^Q A v}{q_i^K c_L v} = \frac{\bar{q}^Q A v}{\bar{q}^K c_L v}. \quad (14.17) \]

Hence we have the following relation

\[ 1 + e_g = (1 + g) (1 + e_0) + g \kappa. \quad (14.18) \]

The system of values corresponding to ER is hence:

\[ \begin{bmatrix} A \\ (1 + e_g) c_K \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} v \\ 1 \end{bmatrix}. \]

To check that this equation makes sense, premultiply by \((q^Q \alpha_L)^{-1}\) and multiply (14.6) by \((v 1)^T\). Since the left hand sides of both are equal, simplifying and using (14.17) yields (14.18).

II. Only the working population grows

Here we make a slight modification to the preceding model. We assume that the working population grows at the rate \( g \) and the capitalist population is constant. But the consumption of each capitalist grows at the rate \( g \). Hence (14.2) changes to:

\[ q_{t+1}^Q = (1 + g) q_t^Q, \quad q_{t+1}^L = (1 + g) q_t^L, \quad q_{t+1}^K = q_t^K = q^K, \quad c_{Kt+1} = (1 + g) c_{Kt}. \quad (14.19) \]

The change requires slight changes in the definitions of the elements of aggregate consumption and investment:

\[ C_t \equiv q_t^L c_L + q^K c_{Kt}, \]
\[ I_t \equiv g \left( q_t^Q A + q_t^L c_L + q^K c_{Kt} \right). \]

Hence, the vector of gross output is

\[ q_t^Q = (1 + g) \left( q_t^Q A + q_t^L c_L + q^K c_{Kt} \right). \quad (14.20) \]
Since the working population grows and the capitalist population doesn’t, their shares in total population are no longer constant. In order to transform the non-stationary variables into stationary variables (i.e., eliminate the trend) we can now simply divide all growing variables by \((1 + g)^t\). Hence we have the following stationary variables.\(^3\)

\[
\begin{align*}
\bar{q}^L_t = q^L_t / (1 + g)^t, & \quad \bar{q}^Q_t = q^Q_t / (1 + g)^t, & \quad \bar{c}_K = c_{K,t} / (1 + g)^t.
\end{align*}
\]

(14.21)

As in the preceding models, vector \(\eta\) allocates capitalists to the various industrial branches, and thus in our SR model we had \(q^K_t = q^Q_t \eta\) and in the ER model with both populations growing we had \(q^K_t = q^Q_t \eta_t\). But now \(q^K_t\) is constant while \(q^Q_t\) grows. Since \(\eta\) allocates capitalists (and their capitals) to the various branches per unit of output, now fewer capitalists per unit of output will be needed in each period, so \(\eta_t\) must diminish period by period. Hence, this vector can be made stationary by multiplying it by \((1 + g)^t\):

\[
\bar{\eta} = \eta_t (1 + g)^t.
\]

(14.22)

Hence, the growth of the quantities produced exactly compensates for the fall of capitalists per unit of output, which is consistent with the constancy of the population of capitalists: \(\bar{q}_{t+1}^Q \bar{\eta}_{t+1} = \bar{q}_t^Q \eta_t = \bar{q}_t^Q \bar{\eta} = q^K_t\). We must also make a corresponding transformation in the price system, since the consumption basket of each capitalist \(\pi_t = c_{K,t} p\) now increases. The stationary format for each capitalist’s consumption is then:

\[
\pi = \pi_t (1 + g)^t.
\]

(14.23)

The systems of quantities and prices in stationary form (which allows us to eliminate the subindexes \(t\)) are hence:

\[
\begin{bmatrix}
\bar{q}^Q & \bar{q}^L & q^K
\end{bmatrix}
\begin{bmatrix}
(1 + g) A & \ell & \bar{\eta} \\
(1 + g) c_L & 0 & 0 \\
(1 + g) \bar{c}_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
p / \pi
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
p / \pi
\end{bmatrix}.
\]

We can check that the reduced price system (14.1), the formula for the rate of profit (14.34), and the savings rate of capitalists (14.16) are still valid if we take into account that now, instead of (14.11), we have:

\[
e_0^p \equiv \frac{q^K c_{K,t} p}{q_t^L c_{LP}} = q^K \bar{c}_{KP} = \frac{q^K A p}{q_t^L c_{LP}} = \bar{q}_t^Q A p.
\]

If we compare this model with the preceding (ER) one, we notice that in both the income distribution between the capitalist and working classes is stable in time. The ratio between aggregate profits and wages is in both cases:

\[
\frac{\Pi_t}{q_t^L c_{LP}} = \rho_g c\frac{K_t}{q_t^L c_{LP}} = \rho_g \left( \frac{q_t^Q A p + q_t^L c_{LP}}{q_t^L c_{LP}} \right) = \rho_g (\kappa^p + 1) = e^p.
\]

\(^3\)Taking the vector of gross outputs, for example, we have

\[
\begin{align*}
\frac{q_{t+1}^Q}{q_t^Q} = \frac{q_{t+1}^L / (1 + g)^{t+1}}{q_t^L / (1 + g)^t} = \frac{q_{t+1}^Q / (1 + g)}{q_t^Q} = 1,
\end{align*}

proving that \(\bar{q}_{t+1}^Q = \bar{q}_t^Q\), that is, that \(\bar{q}_t^Q\) is stationary.
But in the present model the ratio between the per capita incomes of capitalists and workers is increasing in time, since the capital per capitalist and the profit per capitalist grow

\[
\frac{\Pi_t}{q^K c_{LP}} = \frac{\rho_g K_t}{q^K c_{LP}} = \frac{\rho_g k_t}{w},
\]

while the wage per worker is constant. Hence, this model has an increasing concentration of income in the hands of a constant number of capitalists simply because they do not reproduce at an extended scale as workers do. And both per capita capital \(k_t\) as aggregate capital \(K_t\) grow in time.

Models of ER based on the increase in productive power

The models of ER so far considered are a good introduction to this topic but are still far from satisfactory from the point of view of what Marx wanted to reflect with his theory of Capitalism: its transforming power, its revolutionary effects on the technical and organizational methods used in production, with the consequent increase in the productive power of labor. Already in 1848 he wrote (with Engels):

The bourgeoisie cannot exist without constantly revolutionising the instruments of production, and thereby the relations of production, and with them the whole relations of society...

The bourgeoisie... has created more massive and more colossal productive forces than have all preceding generations together. Subjection of Nature’s forces to man, machinery, application of chemistry to industry and agriculture, steam-navigation, railways, electric telegraphs, clearing of whole continents for cultivation... what earlier century had even a presentiment that such productive forces slumbered in the lap of social labour? (Manifesto, MECW 6, 489).

Here we present models of ER that reflect in stylized form the revolutionary feature of constantly changing methods of production that Marx highlighted. Since they are still models of pure CCP they cannot reflect the subversive effect of Capitalism on pre-capitalist production relations, that is, the transformation of pre-capitalist and pre-mercantile relations into capitalist relations due to the competitive advantage that the capitalist mode of production has through the constant cheapening of goods in general, eventually making obsolete all pre-capitalist forms of production. Nevertheless, we will see that these models shed light on some of the important matters that Marx analyzed in Capital.

We present below two models that have a constant (balanced) growth of the productive force of labor and in which the populations of workers and capitalists are constant. In the first one, the turnover period of each input is one, as in the preceding models of ER. And in the second we let the various turnover periods be different from one for each input used in each productive process, which requires the introduction of the stock matrix \(A^S\).

I. Unitary turnover periods for all inputs and processes

As before, \(g\) is the rate of balanced growth. But now accumulation will be based on the constant productivity growth. With no pretension to generality, we assume that innovations are constantly introduced by capitalists in their production methods in such a way that they reduce all the direct labor requirements \(\ell_t\) each period at the rate
g, that is \( \ell_{t+1} = \ell_t / (1 + g) \). Since the populations are constant, instead of (14.2) or (14.19) we now have
\[
q^L_{t+1} = q^L_t = q^L, \quad q^K_{t+1} = q^K_t = q^K, \quad q^Q_{t+1} = (1 + g) q^Q_t. \tag{14.25}
\]
Also, since the labor and capital requirements are \( q^L_t = q^Q_t \ell_t \) and \( q^K_t = q^Q_t \eta_t \), the populations are constant, and \( q^Q_t \) grows, vector \( \eta_t \) (representing the number of capitalists in each branch per unit of gross output in that branch) must again be decreasing in time, as in the second of the models in the preceding section, that is, \( \eta_{t+1} = \eta_t / (1 + g) \). Since neither population grows, the output growth allows for a \( g\% \) increase in the per capita consumption of both populations: \( c_{L,t+1} = (1 + g) c_{L,t} \), \( c_{K,t+1} = (1 + g) c_{K,t} \) and we assume this is the case. Aggregate consumption and investment in period \( t \) are hence:
\[
\begin{align*}
C_t & \equiv q^L c_{L,t} + q^K c_{K,t}, \\
I_t & \equiv g \left( q^Q_t A + q^L c_{L,t} + q^K c_{K,t} \right). \tag{14.26}
\end{align*}
\]
Therefore, instead of (14.3) or (14.20) we have:
\[
q^Q_t = (1 + g) \left( q^Q_t A + q^L c_{L,t} + q^K c_{K,t} \right). \tag{14.27}
\]
Vectors \( q^Q_t, c_{L,t}, c_{K,t}, w_t, \) and \( \pi_t \), increase over time whereas \( \ell_t \), and \( \eta_t \) fall. We can express these variables in stationary form as usual, deflating those that grow and inflating those that fall:
\[
\begin{align*}
\bar{q}^Q_t & \equiv q^Q_t / (1 + g), \quad \bar{c}_{L,t} \equiv c_{L,t} / (1 + g), \quad \bar{c}_{K,t} \equiv c_{K,t} / (1 + g). \tag{14.28}
\end{align*}
\]
\[
\begin{align*}
\bar{w}_t & \equiv w_t / (1 + g), \quad \bar{\pi}_t \equiv \pi_t / (1 + g), \\
\bar{\ell}_t & \equiv \ell_t (1 + g), \quad \bar{\eta}_t = \eta_t (1 + g). \tag{14.29}
\end{align*}
\]
Dividing (14.27) by \( 1 + g \) hence yields
\[
\bar{q}^Q_t = (1 + g) \left( \bar{q}^Q_t A + q^L \bar{c}_{L,t} + q^K \bar{c}_{K,t} \right). \tag{14.30}
\]
Since \( \bar{q}^Q_{t+1} \bar{\ell}_{t+1} = \bar{q}^Q_t \bar{\ell}_t = q^L \) and \( \bar{q}^Q_{t+1} \bar{\eta}_{t+1} = \bar{q}^Q_t \bar{\eta}_t = q^K \) are stationary, the quantities system expressed in stationary format is:
\[
\begin{bmatrix}
\bar{q}^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
(1 + g) A & \bar{\ell} & \bar{\eta} \\
(1 + g) \bar{c}_L & 0 & 0 \\
(1 + g) \bar{c}_K & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
\bar{q}^Q & q^L & q^K
\end{bmatrix}.
\tag{14.31}
\]
This can be compared with the corresponding SR system (8.34) and also with the ER system based on the growth of both populations (14.12) and (14.24), respectively.

The price system in stationary format is the following:
\[
\begin{bmatrix}
(1 + g) A & (1 + g) \bar{\ell} & (1 + g) \bar{\eta} \\
\bar{c}_L & 0 & 0 \\
\bar{c}_K & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
\bar{w} \\
\bar{\pi}
\end{bmatrix}
= \begin{bmatrix}
p \\
\bar{w} \\
\bar{\pi}
\end{bmatrix}.
\tag{14.32}
\]
\footnote{We can call this system the \textit{quasi-dual} of the quantities system. In the Mathematical Appendix to this chapter we show that two matrices formed by the product of the same two square matrices but in different order, \( FE \) and \( EF \), have the same eigenvalues. In this case the matrices are the social matrix \( M \) and the diagonal matrix \( G = (1 + g) I \), where \( I \) is the unit matrix of the same dimension as \( M \).}
As we have done previously, we can reduce this system under the assumption of the equalization of profit rates by means of the elimination of \( \bar{\eta} \) and the use of the part of the profit rate that is related only to capitalist consumption, that is:

\[
\rho_0 = \frac{\bar{\eta} q^K \bar{\eta}}{\bar{q}^2 \left( A + \bar{\eta} \bar{w} \right)} = \frac{q^K \bar{\eta}}{\bar{q}^2 A \bar{p} + q^I \bar{w}}. 
\]

(14.33)

We thus have

\[
\begin{bmatrix}
(1 + \rho_g) A \\
\bar{\varepsilon}_L
\end{bmatrix} \begin{bmatrix}
(1 + \rho_g) \bar{\ell} \\
0
\end{bmatrix} \begin{bmatrix}
p \\
\bar{w}
\end{bmatrix} = \begin{bmatrix}
p \\
\bar{w}
\end{bmatrix},
\]

where the profit rate \( \rho_g \) satisfies

\[
1 + \rho_g = (1 + \rho_0) (1 + g).
\]

(14.34)

We can as previously obtain a convenient expression for the vector of production prices (that replaces (8.20)): \( p = B \left( \rho_g \right) \bar{\ell} \bar{w} \). Premultiplying by \( \bar{\varepsilon}_L \) we get an equation that determines the profit rate \( \rho_g \):

\[
1 = \bar{\varepsilon}_L B \left( \rho_g \right) \bar{\ell}.
\]

(14.35)

Also, if we use as numéraire \( \bar{\varepsilon}_K p = \bar{\pi} = 1 \), premultiplying the expression for \( p \) by \( \bar{\varepsilon}_K \) we obtain the stationary real wage as a function of \( \rho_g \): \( \bar{w} = 1/ \left( \bar{\varepsilon}_K B \left( \rho_g \right) \bar{\ell} \right) \). The value composition of capital in this model is again stationary, since the numerator and denominator grow at the same rate:

\[
\kappa_p = \kappa_L^p = \frac{q^Q_A}{q^Q_{c_L,AP}} = \frac{q^Q_A}{q^Q_{c_L,AP}} = \frac{q^Q_A}{q^Q_{c_L,AP}}.
\]

But it is of interest to notice that the technical composition of capital is increasing, since eliminating \( p/w_L \) from the second to last equality leaves a ratio in which the numerator grows and the denominator is constant: \( q^Q_A/q^L \). The constant increase in the real wage (or the constant decrease of the vector of prices in terms of wage rate \( p/w_L \)) is what in this model compensates for the constant growth of the technical composition of capital so as to render stationary the value composition of capital \( \kappa_p \).

In his Book I, written after almost 20 years since he wrote the paragraph of the Manifesto we quoted above, Marx wrote:

Once given the general basis of the capitalistic system, then, in the course of accumulation, a point is reached at which the development of the productivity of social labour becomes the most powerful lever of accumulation... the degree of productivity of labour, in a given society, is expressed in the relative extent of the means of production that one labourer, during a given time, with the same tension of labour power, turns into products. The mass of the means of production which he thus transforms, increases with the productiveness of his labour (B1, 617).

This change in the technical composition of capital, this growth in the mass of means of production, as compared with the mass of the labour power that vivifies them, is reflected again in its value composition, by the increase of the constant constituent of capital at the expense of its variable constituent... The reason is simply that, with the increasing productivity of labour, not only does the mass of the means of production consumed by it increase, but their value compared with their mass diminishes. Their value therefore rises absolutely, but not in proportion to their mass (B1, 618).
The model of ER considered in this subsection reflects these observations of Marx rather well, with the difference that here the reduction in the unit value of the means of production relative to the wage rate \(p/w\) exactly compensates for the constant increase in the technical composition of capital \(q^A/q^L\), so that the value composition of capital \(r^p\) remains constant. This is reasonable (and inevitable) in a model of ‘balanced growth’ like the one we have. But this implies that the model cannot express the tendency that Marx envisioned in reality, according to which despite the reduction in the production prices of the means of production relative to the wage rate, the growth in the technical composition of capital translated into a (smaller) increase in the value composition of capital. Nevertheless, this limitation is the price we must pay for the possibility of studying in a way that is both rigorous and simple the growth of the technical composition of capital based on the increase in the productive force of labor.

It should be mentioned that though we introduced the increase in productivity through the recurrent reduction in the direct labor requirements, there are other possibilities such as the (alternative or complementary) reduction in the coefficients of \(A\). Also, it is obvious that in reality such reductions are never exactly the same in the various branches nor in the various firms in each branch. However, we do not seek maximum generality but only to illustrate the richness of Marx’s thought on these topics.

The rate of profit \(\rho_g\) is determined by (14.35) and, as (14.34) shows, its magnitude depends on \(\rho_0\) and \(g\). But these variables are determined by the data of the model. Given the numeraire \(\pi = 1\), \(\rho_0\) is simply the inverse of the level of the per capita capital (corrected for trend) of the capitalist class \(\bar{k} = K/q^K\):

\[
\rho_0 \equiv \frac{q^K c_{K,t} \bar{P}}{q^A \bar{P} + q^L c_{L,t} \bar{P}} = \frac{q^K c_{K,t} \bar{P}}{K_t} = \frac{\bar{c}_{K,t} \bar{P}}{K_t} = \frac{\bar{c}_K \bar{P}}{k} = \frac{1}{\bar{k}}.
\]

And this is simply the version for ER of the equality (8.42) we had in the context of SR. Notice also that, despite the differences between this model of ER based on technical change and the previous ones based on population growth, we arrive at the same formula for the savings (and investment) rate of capitalists from profits (14.16): \(s = 1 - \rho_0/\rho_g\), and hence \(\rho_0 = (1 - s) \rho_g\).

**II. Arbitrary turnover periods for all the elements of constant capital**

We have seen that Marx associated the development of the productive forces with the growth in the technical composition of capital. The model of ER just seen reflects this idea. But Marx also associated the increases in productivity with the introduction of, for example, machinery in modern industry, and the railroads in transportation, topics that are related to fixed capital, that is, to inputs that have turnover periods much greater than one, something that has not been taken into account in our preceding models of ER. In this section we extend the model of ER that is based on increases in productivity so that it can include arbitrary turnover periods for all non-labor inputs and processes. This way we can include not only fixed capital but also the stocks of the elements of circulating constant capital. For this we distinguish between the _flow_ matrix \(A\) that we have been using and the _stock_ matrix \(A^S\) we already used when we studied the turnover of capital in Chapter 12. In the thus extended model of ER gross output in \(t\) must be sufficient to satisfy not only the (expanded) consumption of means of production and means of subsistence of period \(t + 1\) but also the necessary increase
in the stocks of means of production \( \Delta q^Q_{t+1} A^S \), whether these are elements of fixed or circulating capital:

\[
q_t^Q = q_{t+1}^Q A + \Delta q_{t+1}^Q A^S + q^L c_{L,t+1} + q^K c_{K,t+1}.
\]

The equalities in (14.25) maintain validity in this model, since the populations remain constant whereas output has balanced growth \( \Delta q^Q_{t+1} = q_{t+1}^Q - q^Q_t = g q^Q_t \) with a growth rate \( g \) which is (exogenously) determined by the expansion of the productive forces. Hence, gross output at \( t \) must be:

\[
q_t^Q = (1 + g) \left( q_t^Q A + q^L c_{L,t} + q^K c_{K,t} \right) + g q^Q_t A^S
\]

instead of (14.27). Notice that the setup is quite general, since any element of constant capital could be simultaneously an element of fixed and circulating capital, as in Marx’s example of an ox, which as “a beast of toil” is fixed capital, but if it is to be eaten as food can be an element of circulating capital.

We can get to this same equation considering that net output must be equal to the elements of consumption plus those of investment: \( q^Q_t (I - A) = C_t + I_t \), where here investment also includes the growth in stocks:

\[
\begin{align*}
C_t & \equiv q^L c_{L,t} + q^K c_{K,t}, \\
I_t & \equiv g \left( q_t^Q A^S + q_t^Q A + q^L c_{L,t} + q^K c_{K,t} \right).
\end{align*}
\]

Disbursed (or invested) constant capital now includes both the stocks of the elements of constant capital \( (q^Q_t A^S p) \) as well as the productive consumption of those elements \( (q^Q_t A p) \):

\[
K_t = q_t^Q A^S p + q_t^Q A p + q^L c_{L,t} p.
\tag{14.36}
\]

And since our formulas (14.28), (14.30), and (14.29) for making non-stationary variables stationary still hold, the resulting quantities and prices systems are very similar to those in which all the turnover periods were unitary ((14.31) and (14.32)). It is only necessary to add the component \( g A^S \) to upper left hand block of the social matrix:

\[
\begin{bmatrix}
\bar{Q} & q^L & q^K \\
(1 + g) A + g A^S & 0 & 0 \\
(1 + g) \bar{c}_L & 0 & 0 \\
(1 + g) \bar{c}_K & 0 & 0
\end{bmatrix}
= 
\begin{bmatrix}
\bar{Q} & q^L & q^K \\
(1 + g) A + g A^S & 0 & 0 \\
(1 + g) \bar{c}_L & 0 & 0 \\
(1 + g) \bar{c}_K & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
(1 + g) A + g A^S & (1 + g) \bar{c}_L & (1 + g) \bar{c}_K \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
\bar{p} \\
\bar{p}
\end{bmatrix}
= 
\begin{bmatrix}
p \\
\bar{p} \\
\bar{p}
\end{bmatrix}.
\tag{14.38}
\]

The reduced system of prices with a homogeneous profit rate is hence the following:

\[
\begin{bmatrix}
(1 + \rho_g) A + \rho_g A^S & (1 + \rho_g) \bar{c}_L \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
\bar{p} \\
\bar{p}
\end{bmatrix}
= 
\begin{bmatrix}
p \\
\bar{p} \\
\bar{p}
\end{bmatrix}.
\tag{14.39}
\]

From the first equation of (14.39) we can get the vector of production prices in terms of the wage rate and the profit rate:

\[
p = B^S \left( \rho_g \right) \bar{c}_w,
\tag{14.40}
\]
where we defined \( B^S(\rho_g) \equiv (1 + \rho_g) [I - (1 + \rho_g) A - \rho_g A^S]^{-1} \).

The only change in the definition of the profit rate in this model is that both
investment and capital have an additional component, without in the least altering
the simple relation between the rate of growth and the rate of profit obtained in previous
models:

\[
\rho_g = \frac{\Pi_t}{K_t} = q^K c_{Kt} p + I_t p = q^K \bar{c}_K p + \bar{I}_p = \frac{q^K + g \left( \bar{K} + q^K \right)}{\bar{K}} = (1 + g) \rho_0 + g. \tag{14.41}
\]

Hence, the relation (14.34) is still valid. Also, despite the changes, the formula for
capitalists’ savings rate is the same as in previous models:

\[
s = s_t = \frac{I_t p}{\Pi_t} = \frac{I_t p K_t}{K_t \Pi_t} = g (1 + \rho_0) \frac{1}{\rho_g} = \frac{\bar{c}_K}{\bar{c}_K} \frac{1}{\rho_g} = 1 - \frac{\rho_0}{\rho_g}. \tag{14.40}
\]

Introducing (14.40) in \( \bar{c}_Lp = \bar{w} \), \( \bar{w} \) can be eliminated to obtain an equation that
determines the profit rate:

\[
1 = \bar{c}_L B^S (\rho_g) \bar{L} \tag{14.42}
\]

which is independent of the numeraire used to normalize the price vector, which implies
that the consumption basket of capitalists \( \bar{c}_K \) is not involved in the determination of
the profit rate. It depends exclusively on \( \bar{c}_L, \bar{L}, A, \) and \( A^S \). Also, since matrix \( B^S \)
is increasing with \( \rho_g \), any reduction in the consumption level of workers \( \bar{c}_L \) increases the
profit rate.

Premultiplying (14.40) by \( \bar{c}_K \) and taking the stationary consumption basket of
capitalists as numeraire (\( \bar{\pi} = \bar{c}_K p = 1 \)) we get an inverse relation between the real
wage and the profit rate: \( \bar{w} = 1 / (\bar{c}_K B^S (\rho_g) \bar{L}) \), which we abbreviate as \( \bar{w} (\rho_g) \), with \( \bar{w} (\rho_g) < 0 \). Hence, any exogenous change that has the effect of lowering the profit rate
and any exogenous reduction in the consumption basket of capitalists has the effect of
increasing the real wage. Using (8.26) we also get a direct relation between the rate of
profit \( \rho_g \) and both the rate of surplus value \( e^p \) and the value composition of capital \( \kappa^p \):

\[
e^p = \frac{q^K}{\bar{L} \bar{w} (\rho_g)} = \frac{q^K \bar{c}_K B^S (\rho_g) \bar{L}}{\bar{L}}, \quad \kappa^p = \frac{\bar{c}_L^Q (A^S + A) B^S (\rho_g) \bar{L}}{\bar{L}} \tag{14.43}
\]

As in the preceding model, the technical composition of capital grows period by period,
but that increase is compensated by the increase in the real wage.

From (14.39), (14.38) and (14.41) (and \( \bar{\pi} = \bar{c}_K p = 1 \)), we get

\[
\bar{\eta} = \rho_0 \left[ A + \bar{c}_L + A^S \right] p. \tag{14.44}
\]

And premultiplying by \( \bar{c}_L^Q \) yields:

\[
q^K = \bar{c}_L^Q \bar{\eta} = \rho_0 \bar{c}_L^Q \left[ A + \bar{c}_L + A^S \right] p = \rho_0 \bar{K}, \tag{14.45}
\]

where \( \bar{K} \) is aggregate capital (14.36) in stationary format. This shows that the
consumption component of the rate of profit is \( \rho_0 = q^K \bar{c}_K p / \bar{K} = q^K \bar{c}_L / \bar{K} = 1 / \bar{K}. \)

\(^5\)This is evident if we expand the inverse matrix in series: \((I - J)^{-1} = I + J + J^2 + \ldots, \) where
\( J \equiv (1 + \rho_g) A + \rho_g A^S. \)
The business cycle in a model of ER

The models of hoarding and unemployment presented in Chapters 11 and 12 with the aim of reflecting Marx’s thought on the functioning of the industrial cycle have the limitation of assuming that there is no growth in the reproduction of capital. Capitalists consume all their profits and do not save and thus there is no net investment. For Marx, the vicissitudes of the process of capital accumulation were important for the actual development of the industrial cycle. For him SR was merely a theoretical exercise that served to clear up ideas and he was very much aware that a central characteristic of capitalist production was the reinvestment of a significant part of the profits (and hence surplus value) in the production process by capitalists wishing to expand their wealth and power. The entrepreneurial decisions on reinvestment also had a fundamental role in the generation of the different phases of the industrial cycle. In this dynamic process the wage rate and the labor market played an important role. The following paragraph is illustrative of some of Marx’s analyses of the cycle:

... a rise in the price of labour resulting from accumulation of capital implies the following alternative: Either the price of labour keeps on rising, because its rise does not interfere with the progress of accumulation... Or, on the other hand, accumulation slackens in consequence of the rise in the price of labour, because the stimulus of gain is blunted. The rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labour power. The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates. The price of labour falls again to a level corresponding with the needs of the self-expansion of capital, whether the level be below, the same as, or above the one which was normal before the rise of wages took place... It is these absolute movements of the accumulation of capital which are reflected as relative movements of the mass of exploitable labour power, and therefore seem produced by the latter’s own independent movement (B1, 614-5).

When Marx attributes the cause for the slowing down of accumulation to “the disproportion between capital and exploitable labour power”, he is referring to the changes in the magnitude of the capital disbursed in the productive process. In the context of ER, such changes can have their origin in the hoarding or dishoarding of potential money capital (as in our previous models of the industrial cycle), but also in changes in the proportion of profits that capitalists reinvest in the production process. In our two previous models of the industrial cycle only the first source of variation in the disbursed capital existed. That source is still present in the model of this section. But in the context of capital accumulation it is complemented by another decision variable that functioning capitalists have: the rate of capitalization of profits, that is, the savings (and investment) rate of capitalists from their profits. Marx explains that if the capital accumulation process leads to higher wages and profits decline sufficiently, this can make capitalists capitalize a lower fraction of their profits, thus creating a self-correcting mechanism that guarantees the “reproduction on a progressive scale” of the capitalist system and the fundamental production relation on which it is based:

But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalised, accumulation lags, and the movement of rise in wages receives a check. The rise of wages therefore
is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale. The law of capitalistic accumulation... merely states that the very nature of accumulation excludes every diminution in the degree of exploitation of labour, and every rise in the price of labour, which could seriously imperil the continual reproduction, on an ever-enlarging scale, of the capitalistic relation (B1, 616; italics added).

Furthermore, only by means of models of ER can we account for some of Marx's most penetrating observations on the cyclical dynamics of the accumulation process. For example:

With accumulation, and the development of the productiveness of labour that accompanies it, the power of sudden expansion of capital grows also; it grows, not merely because the elasticity of the capital already functioning increases, not merely because the absolute wealth of society expands, of which capital only forms an elastic part... it grows, also, because the technical conditions of the process of production themselves -machinery, means of transport, &c.- now admit of the rapider transformation of masses of surplus product into additional means of production (B1, 626; italics added).

We see here that for Marx a fundamental source of "the elasticity of the capital already functioning" is the expansion in "the absolute wealth of society", which can only be brought about through Expanded Reproduction.

**An industrial growth cycle model**

We here modify the first of the two models of Extended Reproduction with technical progress developed in this chapter: the one that assumes unitary turnover periods for all inputs and in all production processes. We adapt it so that it can represent an industrial growth cycle. The fact that it has investment and growth can enrich the analysis in the direction of Marx's thoughts on the subject. As all models of ER, it has the limitation of representing balanced growth. And we know that for Marx the introduction of technological and organizational innovations is actually never produced smoothly in time or homogeneously along the various branches or even within each branch. But that simplification lets us generate a model that can be used as an auxiliary to our thought processes. As we did above in this chapter we transform the variables of a dynamical model of balanced growth (in which Marx was pioneer with his ER tables) by making stationary those variables that are non-stationary (i.e., by eliminating trends). The dynamic model is thus made static. But here this balanced growth can be modified each time capitalists change magnitudes that they can change at their discretion and are exogenous to the model. One of them is related to hoarding and dishoarding, as in the two previous models. But here we also have the rate at which capitalists introduce innovations that increase productivity and thus determine the rate of growth. We see below that both variables jointly define capitalists' savings and investment (out of profits) coefficient.

Let us assume that populations of workers and capitalists are constant. And as in the two previous models of the cycle we assume that the employed working population \( q^L \) is an endogenous variable and that whereas capitalists jointly have an aggregate wealth \( \hat{K}_t \) they only disburse as functioning capital a part \( K_t < \hat{K}_t \). Since we have ER it is necessary to assume that not only functioning capital grows but also the 'latent
money capital’ that is hoarded (or kept as a reserve) \( A_t \equiv \hat{K}_t - K_t \) since otherwise with the passage of time the initial reserve would rapidly deplete. We continue with the assumption that all capitalists are the same, so the relations between the aggregate and individual variables are given by: \( K_t = q^K k_t, \ A_t = q^K a_t \) and \( \hat{K}_t = q^K \hat{k}_t \). Since we use monetary prices \( (p_1 = 1) \) it is again unnecessary to distinguish the quantity of hoarded money from its value. Each capitalist’s reserve in units of gold is \( a_t = A_t / q^K \), and we have \( a_t \equiv \hat{k}_t - k_t \).

Aggregate productive (or functioning) capital is \( K_t = q_t^Q A p + q^L c_{L,t} p \), where \( q_t^Q \) and \( c_{L,t} \) are increasing in time, and hence so are \( K_t, \ \hat{K}_t, \ A_t, \ k_t, \ \hat{k}_t, \) and \( a_t \). In the present context transformations (14.28)-(14.30) remain valid for obtaining stationary variables. We now add the following:

\[
\begin{align*}
\hat{K}_t & \equiv \frac{\hat{K}_t}{(1 + g)}, & \hat{\kappa}_t & \equiv \frac{\hat{\kappa}_t}{(1 + g)}, & \hat{\alpha}_t & \equiv \frac{\hat{\alpha}_t}{(1 + g)}.
\end{align*}
\]

By detrending the variables that grow or shrink, if there is balanced growth (as here assumed) the time index can be eliminated; in particular, \( \hat{\kappa} = \hat{k}_t, \ \kappa = k_t, \) and \( \alpha = \alpha_t \). We hence have \( \alpha \equiv \hat{\kappa} - \kappa \).

Each capitalist has a reserve \( A_t \) that normally grows with the trend as the rest of the variables, but he has the ability to change it discretely with respect to the trend, thus producing a change in \( \alpha \) and hence in \( \kappa = \hat{\kappa} - \alpha \), and contributing to the generation of the phases of the industrial (growth) cycle. But in the present model each capitalist can also change his rate of introduction of innovations \( g \) discretely in certain periods if it chooses (and if it is feasible), and hence to change his rate of saving and investment \( s \). Hence, in this model the absence of a time index in the trend-adjusted variables does not necessarily mean that they remain always constant. They only remain constant if capitalists do not make any changes in the variables they control and which can affect the direction of the balanced growth. These variables are: 1) the money capital disbursed in production \( \hat{\kappa} \) and 2) the reinvestment of profits through the magnitude of \( g \) (if the corresponding innovations are available to be implemented in the case of an increase). In order to change their disbursed capital \( \kappa \) they correspondingly change their reserve \( \alpha \) (by hoarding or dishoarding) in such a way that their total capital \( \hat{\kappa} = \hat{k}_t + \alpha \) (disbursed and potential, both detrended) is constant. Changes in \( \hat{\kappa} \) or \( g \) are assumed to be discrete changes that take place at certain moments during the industrial cycle and in between two consecutive time periods. Since non-detrended variables (such as \( k_t \)), cannot jump, it is necessary that when \( g \) changes the detrended variable also change. For example, since \( k_t = \hat{k}_t (1 + g) \) and the capital of the individual capitalist \( k_t \) cannot jump, it is necessary that any increase in \( g \) be accompanied by a corresponding fall in \( \hat{k}_t \). In a graph, what changes discretely (jumps) is the slope of the line representing the rising \( k_t \).

As in the previous models of the cycle there is an ‘industrial reserve army’ (or population of unemployed workers) \( u = \hat{q}^L - q^L \), where \( \hat{q}^L \) is total working population, which is constant. We make the simplifying assumption that all unemployed workers are willing and able to work for the current wage if their labor is demanded. Hence, the unemployed working population \( u \) varies linearly and inversely with the employed working population \( q^L \).

Since we assume that money is one of the produced commodities (gold), any increase in the reserve (hoarding) is a component of the demand for a commodity that is produced in the time period. Hence, to the (detrended) basket of consumption
demand of each capitalist $\tau_K$ must be added his demand for money for hoarding purposes $g\overline{u}$. But this sum must be expressed as a row vector so that it can be added to row vector $\tau_K$. Since gold is assumed to be the first of the $n$ commodities, let $u_1 \equiv (1, 0, \ldots, 0)$. Then we can define the row vector $(A_t, 0, \ldots, 0) = A_t u_1$. Dividing by $q^K$ we have $(a_t, 0, \ldots, 0) = a_t u_1$ and, detrended, $(\overline{a}, 0, \ldots, 0) = \overline{a} u_1$. Hence the detrended demand of each capitalist outside of the ‘production sphere’ is $\tau_K + g u_1 \overline{u}$. In this ER model each capitalist, in order to reproduce himself as such, must consume $\tau_K$, invest $g \tau_K$ as one of the components of his accumulation of constant and variable capital (real investment), and invest $g \overline{u}$ for the expansion of his reserve (financial investment).

As in the previous models of the industrial cycle, when capitalists disburse more capital to expand the productive process, they also increase their consumption to some degree. Hence, their actual (detrended) consumption basket is $\tau_K \equiv \gamma (\overline{k}) \tau_K$, where $\tau_K$ is his ‘basic’ (detrended) consumption and $\gamma (\overline{k})$ is an increasing (and inelastic) function of $\overline{k}$. Hence, the quantity and price systems in the ER model in stationary format are:

$$
\begin{bmatrix}
\bar{q}^Q \\
q^L \\
q^K
\end{bmatrix}
\begin{bmatrix}
(1 + g) A \\
(1 + g) \omega \tau_L \\
(1 + g) \gamma (\overline{k}) \tau_K + g \overline{u} u_1
\end{bmatrix}
= \begin{bmatrix}
\bar{p} \\
\ell
\end{bmatrix}.
$$

Expressions for the endogenous variables

We now consider how changes in two variables controlled by capitalists – (detrended) disbursed money capital $\overline{k}$ and the rate of introduction of innovations $g$ – affect aggregate (detrended) employment, gross output, consumption and income. The wage basket of employed worker is $\omega \tau_L$, where $\tau_L$ is a ‘basic’ (detrended) basket and $\omega$ a measure of how much the wage is above (or below) trend. The second equation of the price system yields $\omega = \overline{w} / \tau_L p$. Notice that this variable is trendless since $w_t$ and $\tau_L t$ both have the same trend and $p$ is trendless. To check this, from the first equation of the price system we get $p = B (\rho) \overline{w}$. Hence $p$ only changes when $\rho$ and $\overline{w}$ change along the cycle. Premultiplying by $\omega \tau_L$ we get $\omega = 1 / \tau_L B (\rho) \overline{w}$, showing that $\omega$ varies inversely with $\rho$. On the other hand, since we have monetary prices (i.e., gold is the numeraire), we have $1 = p_1 = B (\rho) \overline{w} = u_1 B (\rho) \overline{w}$, from which $\overline{w} = 1 / u_1 B (\rho) \overline{w}$ and hence $w_t = 1 / u_1 B (\rho) \ell_t$. The wage grows over time due to the increase in productivity on account of the innovations that reduce $\ell_t$. But this is corrected by the variations of $\rho$, so in any given period it can be above or below its trend.

We must distinguish the quantities involved in real investment (defined by (14.26)) from those involved in financial investment $I^F = g A_t$ that are necessary so that monetary reserves increase with the same trend as the rest of the endogenous variables. Both must be financed by profits, and so these profits must be equal to the sum of capitalists’ consumption, real investment, and financial investment. In stationary format, aggregate profit must hence be:

$$\Pi = q^K \tau_K p + I^F + I^F = q^K \tau_K p + g (\overline{k} + q^K \tau_K p) + g \overline{A} = (1 + g) q^K \tau_K p + g \overline{k}. \quad (14.46)$$

---

6 It is useful to compare these systems with (12.3) and (12.4). In order to avoid more complicated notation we use $\rho$ instead of, say, $\rho_{g, \overline{K}}$.

7 Notice that the last equality uses $\overline{A} \equiv \overline{k} - \overline{K}$, where $\overline{K}$ is always constant.
Then the global profit rate is

\[
\rho = \frac{\Pi}{\bar{K}} = \frac{\Pi / q^K}{\bar{K}/q^K} = \frac{1 + g}{\bar{K} / \bar{q}} \left( \frac{\gamma (\bar{K})}{\bar{K}} \hat{\pi} (\rho) + \frac{g}{\bar{K}} \right),
\]

(14.47)

where

\[
\bar{c}_{KP} = \gamma (\bar{K}) \hat{c}_{KP} = \gamma (\bar{K}) \hat{\pi} (\rho), \quad \hat{\pi} (\rho) = \frac{\hat{c}_{K} B (\rho)}{u_1 B (\rho)},
\]

(14.48)

The effect of changes in \( \rho \) on the value of the capitalists' consumption basket \( \hat{\pi} \) is complex since both the numerator and the denominator vary directly with \( \rho \). The net effect is dependent on the characteristics of the production of these commodities (numerator) and of gold (denominator). We adopt as a general assumption that the effect of \( \rho \) (and hence of \( \bar{K} \) and \( g \)) on \( \hat{\pi} \) resulting from these opposing influences is small in absolute value.

As in (12.9) of Chapter 12, in (14.47) \( \rho \) is on both sides of the equality. We prove in the Appendix to this chapter that if \( \gamma (\bar{K}) \) is inelastic and \( \hat{\pi} (\rho) \) either varies inversely with \( \rho \) or directly but inelastically, then \( \rho \) (as an implicit function of \( \bar{K} \) and \( g \)) varies inversely with \( \bar{K} \) and directly with \( g \):

\[
\rho = \rho (\bar{K}, g), \quad \rho_{\bar{K}} < 0, \rho_g > 0 \quad \text{(14.49)}
\]

In the following we assume that these signs hold. Notice that changes in \( \bar{K} \) and \( g \) also have opposite effects on \( \omega \) since \( \omega = \omega (\bar{K}, g) = 1/\hat{c}_{K} B (\rho (\bar{K}, g)) \bar{\ell} \). Hence, \( \omega_{\bar{K}} > 0, \omega_g < 0 \).

From the first equation of the quantities system we can get the vector of gross outputs as

\[
\bar{q}^Q = \left( q^L \omega (\bar{K}, g) \bar{\pi} + q^K \gamma (\bar{K}) \hat{\pi} (\rho) + \frac{g}{1 + g} (\bar{K} - \bar{K}) u_1 \right) B (g),
\]

(14.50)

where matrix \( B (g) \) is defined as in (8.21) and we used \( \bar{\pi} = \hat{\pi} - \bar{\pi} \). The effect of changes in \( g \) on \( \bar{q}^Q \) is ambiguous, since for example an increase in \( g \) makes \( \omega \) fall while it has the opposite effect on \( B (g) \). It also increases the second of the terms within square brackets, but this only involves the production of gold (through \( u_1 \)). An increase in \( \bar{K} \) has the effect of increasing both \( \omega \) and \( \gamma \), but it has a negative effect on gold production.

Multiplying (14.50) by \( \bar{\ell} \) (and using (14.49) and \( \bar{q}^Q \bar{\ell} = q^L \)) yields the following expression for aggregate employment:

\[
q^L = m (\rho, g) q^K \Psi (\rho, g),
\]

where we have defined a term that is related to the expenditures of capitalists \( \Psi \) and a multiplier \( m \):

\[
\Psi (\bar{K}, g) \equiv \left[ \gamma (\bar{K}) \hat{\pi} + \frac{g}{1 + g} (\bar{K} - \bar{K}) u_1 \right] B (g),
\]

\[
m (\bar{K}, g) \equiv \left( 1 - \frac{\hat{\pi} B (\rho (\bar{K}, g)) \bar{\ell}}{\bar{\ell}} \right)^{-1}.
\]

\[\text{\footnotesize{8}}\text{\footnotesize{Notice the similarities and differences between this and the preceding equation with equations (12.5) and (12.6). If } g = 0 \text{ the former are reduced to the latter.}}\]
We assume that the effects on employment in gold mining is small in relation to the aggregate of the rest of the branches, so that \( \partial \Psi / \partial \kappa > 0 \) and \( \partial \Psi / \partial g > 0 \). On the other hand, it makes sense to call \( m(\kappa, g) \) a multiplier since it is greater than one (being \( g \) is less than \( \rho \)). It varies inversely with \( \rho \) and directly with \( g \). Using (14.49), we then have \( m(\rho(\kappa, g), g) = m(\kappa, g) \), where \( \partial m / \partial \kappa > 0 \) and the sign of \( \partial m / \partial g \) is in general ambiguous. However, observing the formula for \( m(\kappa, g) \) we can see that the effect of \( g \) crucially depends on whether \( \partial \rho / \partial g \) is greater or less than 1. Moreover, \( \partial m / \partial g < 0 \) if and only if \( \partial g / \partial g > 1 \), since in that case a small increase in \( g \) increases \( \rho \) more than it increases \( g \), and hence makes \( \tilde{\tau} L B(\rho(\kappa, g), \kappa) / \tilde{\tau} L B(\rho(\kappa, g), \kappa) \) fall and also \( m \). In the Appendix to this chapter we calculate \( \partial \rho / \partial g \). Using that formula we can see that \( \partial \rho / \partial g > 1 \) if and only if \( \gamma(\kappa) \left[(1 + g) \tilde{\pi}'(\rho) + \hat{\pi}(\rho)\right] + \left(\kappa - \tilde{\kappa}\right) > 0 \), which holds if, as we have assumed, \( \tilde{\pi}'(\rho) \) is small in absolute value, and hence we have the sum of two positive terms. Hence we can conclude that \( \partial m / \partial \kappa > 0 \) and \( \partial m / \partial g < 0 \). As we have seen, \( \partial \Psi / \partial \kappa > 0 \) and \( \partial \Psi / \partial g > 0 \), so our assumptions imply that aggregate employment is a function \( q^L(\kappa, g) \) that is increasing with \( \kappa \) but is only increasing with \( g \) if the positive effect related to capitalists’ expenditures \( \Psi \) predominates over the negative effect on the multiplier \( m \).

On the other hand, since \( \bar{\omega} = 1 / u_1 B(\rho(\kappa, g), \kappa) \), the (detrended) wage is a function \( \bar{\omega}(\kappa, g) \) that is increasing in \( \kappa \) and decreasing in \( g \). The aggregate income of workers is the aggregate of their wages \( \bar{\omega} q^L \) and the aggregate income of capitalists is the aggregate of their profits \( \bar{\omega} q^K = q^K \rho \kappa \). Hence, we have:

\[
\begin{align*}
\bar{\omega} q^L(\kappa, g) &= \bar{\omega}(\kappa, g) q^L(\kappa, g) = \bar{\omega}(\kappa, g) m(\kappa, g) q^K \Psi(\kappa, g) \\
\bar{\omega} q^K(\kappa, g) &= q^K \rho(\kappa, g) \kappa = q^K \left[(1 + g) \gamma(\kappa) \tilde{\pi}(\rho(\kappa, g)) + g\tilde{k}\right].
\end{align*}
\]

Aggregate wages vary directly with \( \kappa \) but respond ambiguously to \( g \) since \( \partial \Psi / \partial g > 0 \), \( \partial m / \partial g < 0 \), and \( \partial \bar{\omega} / \partial g < 0 \). And aggregate profits vary directly with \( g \), since \( \partial \rho / \partial g > 0 \). But as in the case of the industrial cycle model of Chapter 12, \( \rho(\kappa, g) \kappa \) as a function of \( \kappa \) may have both increasing and decreasing segments.

The industrial growth cycle

Let us now consider the effects of changes in \( \kappa \) and \( g \) on the endogenous variables and see if they can describe the phases of an industrial growth cycle in which the growth rate can vary in time (being exogenous in the model) as capitalists make decisions concerning investment. If we first assume \( g \) is kept constant over the cycle we can replicate the exercise of Chapter 12 (which was in the context of SR). At the beginning of the cycle (presumably after a crisis), detrended disbursed capital \( \bar{\omega} \) is low (and reserves \( \bar{\omega} = \bar{\kappa} - \tilde{\kappa} \) high) because industrial capitalists decided to diminish the exposure of their capitals to the turbulences of the preceding crisis. Hence, the profit rate \( \rho \) is high. Along the expansion, capitalists gradually increase \( \kappa \) by dishoarding. This makes the profit rate diminish, while the following variables increase: 1) the detrended wage \( \bar{\omega} = 1 / u_1 B(\rho, \kappa) \) (hence the wage grows faster than trend), 2) the deviation of wage from trend \( \omega = 1 / \tilde{\tau} L B(\rho, \kappa) \), 3) detrended gross outputs \( \bar{q}^D \) (except possibly gold production, which is negatively affected by the reduction of \( \bar{\omega} \)), and 4) industrial employment \( q^K \) (assuming realistically that the reduction of employment in gold mining is more than compensated by the increase in the rest of the industrial branches). Hence, along the expansion there is a reduction in unemployment \( u = \bar{q}^F - q^L \) and in capitalists’ reserves.
\( \sigma = \hat{k} - \tilde{k} \). In this expansionary phase it is possible that in a first subphase profits \( \rho \tilde{k} \)
increase while in a second subphase they fall. The opposite of all this happens in the
contractionary phase of the cycle, in which \( \tilde{k} \) diminishes.

All this is analogous to what we had in the SR cycle model, albeit with some
complications due to the fact that it is a growth cycle. But the fact that capitalists
can also, to some extent, modify the rate of change in the introduction of innovations
\( g \) (that defines the growth rate) lets us represent scenarios in the industrial growth
cycle that are considerably more complex. Let us consider now an increase in \( g \) with
no change in \( \tilde{k} \). By (14.47) and (14.49), if \( g \) increases \( \rho \) must increase (under the
assumptions above on \( \hat{\pi}(\rho) \)) and hence so must aggregate profits \( q^K\rho\tilde{k} \). The base for
employment \( \Psi \) (that is related to capitalist’s expenditures) also rises. But the increase
in \( \rho \) has the effect of diminishing the wage rate \( \bar{w} \) as well as the multiplier \( m \), so that
the effect on aggregate wages is ambiguous and hence so is the effect on aggregate
incomes \( \bar{Y} = \bar{Y}^L + \bar{Y}^K \). All this is evidently symmetrical when \( g \) diminishes.

Since changes in \( g \) and \( \tilde{k} \) in the same direction have opposite effects on \( \rho \), it is
possible that \( g \) and \( \tilde{k} \) jointly increase without having any effect on \( \rho \) and hence neither
on \( \omega \) nor \( \bar{w} \). The necessary relation between the changes in \( g \) and \( \tilde{k} \) that leave \( \rho \) intact
is obtained in the Appendix to this chapter. If capitalists increase \( g \) and \( \tilde{k} \) jointly and
such that they have no effect on \( \rho \), then aggregate profits \( \Sigma^K = q^K\rho\tilde{k} \) increase as well
as aggregate wages \( \bar{Y}^L = \bar{w}(\rho)m(\rho,g)q^K\Psi(\tilde{k},g) \) and hence the aggregate income
of the economy. Let us recall that an increase in \( g \) implies the previous existence of
knowledge on the innovations that increase productivity, so that capitalists are capable
of implementing them in production if they so decide. Hence, an increase in \( g \) may not
be possible. On the other hand, a fall in \( g \) seems to be always possible since it only
requires the implementation of new innovations at a slower rate, just as it is possible
to diminish the functioning capital by hoarding. It seems then that the logic of the
model leads us to not think of the changes in \( \tilde{k} \) and \( g \) as totally unrelated but instead
to think of them as related to the same underlying factor of confidence or caution that
depends on the evaluation of profitability and at the same time the uncertainty and
the fear of suffering severe capital losses.

We can represent as normal behavior of capitalists that in a first part of the ex-
pansionary phase of the cycle they only increase \( \tilde{k} \) and only in a second part they
additionally increase \( g \), whereas the contractionary phase of the cycle is more abrupt,
with an almost simultaneous reduction in both. In the expansionary phase the expect-
cations of greater profits motivate a greater disbursement of capital and sooner or
later a higher rate of investment (as given by an increase in \( g \)). And such expectations
may be confirmed by reality. But eventually the fear of suffering capital losses during
an upcoming crisis makes the industrial capitalists more cautious and they eventually
start to reduce both \( g \) and \( \tilde{k} \), diminishing their exposure to losses but also putting
a break on the economy while the financial capitalists (absent in this model to avoid
excessive complexity) adopt an increasingly risky behavior. This reinforces the cau-
tion of industrial capitalists, making them diminish \( g \) and \( \tilde{k} \) even more as the crisis
approaches or explodes unexpectedly.

In our model of the industrial cycle under ER aggregate saving is necessarily equal
to aggregate investment because this must be true for each capitalist whereas workers
do not save (and it represents a closed economy). But investment is composed of
real and financial investment. Hence capitalists can in principle save and not generate
any real investment. What they save from their profits they can use to finance the
expansion of constant and variable capital as well as reserves. Given \( \tilde{k} \) and \( g \) the
savings rate $s$ of capitalists as a fraction of their profits (i.e., their income) is given by:

$$
s = \frac{I_P + g\bar{u}_t}{\bar{P}} = \frac{\Pi - q^K \bar{v}_K P}{\bar{P}} = 1 - \frac{q^K \bar{v}_K P}{\bar{P}} ^{\frac{1}{\bar{k}}} = 1 - \frac{1}{1 + g \left(1 + \frac{\bar{k}}{\bar{k} \rho(\bar{k}, g)}\right)},
$$

where (14.46) was used for the second equality and (14.47) for the last. Hence, assuming the effects through $\bar{\pi}$ are small, the savings rate $s$ is increasing with $g$ and decreasing with $\bar{K}$. Since this last effect occurs through $\gamma(\bar{K})$, this implies that the more of the disbonding the capitalists dedicate to consumption the less they need to save from profits. This is so because their higher consumption generates an increase in profits that is higher than what is needed to finance it. Notice also that, as in other models, if we call the part of the profit rate that only includes profits destined for consumption $\rho_0$ ($= \gamma(\bar{K}) \bar{\pi}(\rho)/\bar{K}$), the second to last equality shows once more that the capitalists’ savings rate is $s = 1 - \rho_0(\bar{k}, g)/\rho(\bar{k}, g)$.

We have modeled the profit rate as endogenous and as exogenous the capital disbursed and the rate of growth (which defines real—as well as financial—investment, which Marx calls ‘accumulation’). Marx generally considers individual capitalists as motivated by the profit they expect to reap. But in his analyses of the industrial cycle he considers other factors. He writes that accumulation can slacken when the wage rate rises if “the stimulus of gain is blunted.” And he adds “To put it mathematically: the rate of accumulation is the independent, not the dependent, variable; the rate of wages, the dependent, not the independent variable” (B1, 615). And this view is reflected in our model. Marx holds that if the wage rate is too high “The rate of accumulation lessens; but with its lessening... The price of labour falls again to a level corresponding with the needs of the self-expansion of capital” since the demand for labor falls and hence the wage rate also. Marx observes that there is a certain self-equilibrating mechanism: “The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates” (B1, 614). And that is why although crises take place repeatedly, the economy always rebounds with the beginning of a new business cycle. For, “a crisis always forms the starting-point of large new investments. Therefore, from the point of view of society as a whole [it forms], more or less, a new material basis for the next turnover cycle” (B2, 188; text within brackets added).

To avoid complicating the model excessively we have had to leave out one of the aspects that Marx highlighted as relevant for determining the duration of the industrial cycle: the fact that “As the magnitude of the value and the durability of the applied fixed capital develop with the development of the capitalist mode of production, the lifetime of industry and of industrial capital lengthens in each particular field of investment to a period of many years, say of ten years on an average. Whereas the development of fixed capital extends this life on the one hand it is shortened on the other by the continuous revolution in the means of production, which likewise incessantly gains momentum with the development of the capitalist mode of production. This involves a change in the means of production and the necessity of their constant replacement, on account of moral depreciation, long before they expire physically” (B2, 187). Finally, in our model the profit rate of each capitalist is the global profit rate due, among other reasons, to the fact that we have not taken into consideration Marx’s belief that in the advanced Capitalism of his time there were already various branches that could escape the equalization of profit rates. This was due to the process of centralization of capital,
to the development of credit, and to the monopolistic power in the industrial branches that Marx called ‘natural monopolies’. We will address these topics in Chapter 17. It is our understanding that the process of centralization is basically incompatible with the ‘balanced growth’ of the ER models, so we have had to leave it out of this chapter. For the centralization process generates flows of individuals between classes, with the conversion of (failed) capitalists to independent, wage, or unemployed workers.

Appendix to Chapter 14
Bibliographic Notes

Samuelson and Extended Reproduction  Samuelson made good use of his joint research with Dorfman and Solow (Dorfman et al, 1958) to deepen his knowledge on Marx’s theory. In 1957 he published the first of several articles he dedicated to Marx over a period of 15 years, acknowledging that he made “no attempt to do justice to the many noneconomic and imperfect-competition aspects of Marx’s thought” (Samuelson 1957, 884). Nevertheless, Samuelson could not avoid interpreting Marx as someone who deviated from the correct path of perfect competition. And he didn’t miss an opportunity to downplay Marx, as when he writes “A minor Post-Ricardian, Marx was an autodidact cut off in his lifetime from competent criticism and stimulus” (Ibid., 911), or “Although Marx was a learned man, he shows all the signs of a self-taught amateur: overelaboration of trivial points, errors in logic and inference, and a megalomaniac’s belief in the superiority of his own innovations” (Samuelson 1967, 616). But at the same time Samuelson (1967) shows that he has not read Marx directly when he states that much ink and blood would have been spared if Marx had introduced in Book I of Capital the simplifying notion that he did make explicit in his pamphlet Value, Price and Profit that prices are assumed to be proportional to labor-values. One of his recurrent topics is that Book I of Capital was an unnecessary ‘detour’ for the comprehension of competitive Capitalism. In spite of all this he recognized that Marx innovated in his Book II on the subject of reproduction and growth, stating that “Marx’s model of expanded reproduction is perhaps the first example of those golden-age paths of compound interest which Cassel, D. H. Robertson, Von Neumann, Harrod, Domar, and all the rest have made so fashionable in modern economics” (Ibid., 618).

Von Neumann’s model  János Lajos Neumann, the Hungarian mathematician emigrated to the U.S.A. and known as John von Neumann constructed a linear model of balanced growth (von Neumann (1945 [1938])) that is very general from the mathematical point of view. The model admits 1) more productive processes than goods, and 2) productive processes with joint production (of multiple goods). Also, the basic setup has inequalities ($\leq$ and $\geq$), which allow for the inclusion of processes that are not be operated (being unprofitable) and goods that are not be produced (since they would be ‘free’, i.e., have a price of zero). Nevertheless, the model is comparable to the model of Marx we have presented (with ER in CCP) with only one productive technique for each commodity and almost reduces to it if 1) the same number of goods as prices is imposed, 2) joint production is eliminated, and 3) equalities are used instead of inequalities. We say that it almost reduces to our model because there is an important difference: it is poor in reflecting the social structure. Von Neumann includes in his matrices the “necessities of life consumed by workers and employees” (as in our $A+\ell c_L$) but he assumes that “all income in excess of necessities of life will be reinvested.” The obvious questions are: who, or what social class, receives that income? And if they reinvest it all, how do they survive without consumption? Von Neumann’s model is
an early example of the tendency in mainstream economics to sweep under the carpet the social classes that are highest in the social hierarchy of wealth and power, that is, to not represent them explicitly.

In the models of ER considered in this chapter, the assumption of von Neumann (1945 [1938]) that all profit is reinvested would imply \( c_K = 0 \), which makes no sense from the point of view of Marx (nor from our own). Even if capitalist entrepreneurs are eager to accumulate capital, they are no less eager to live well, as most people. Max Weber (2012 [1905]) erroneously identified the ‘spirit of Capitalism’ with the most fanatic asceticism. Marx was closer to the truth when he wrote of the capitalist that “his expenditure grows with his accumulation, without the one necessarily restricting the other. But along with this growth, there is at the same time developed in his breast, a Faustian conflict between the passion for accumulation, and the desire for enjoyment” (B1, 590). From the purely formal point of view, von Neumann’s assumption \( c_K = 0 \) is equivalent to \( \rho_0 = 0 \), which would imply that the rate of profit is equal to the rate of growth \( \rho_g = g \) (see (14.8)), a distinctive characteristic of von Neumann’s model. It is not our intention to diminish the importance of this piece of research of von Neumann. It was the type of contribution that only a great mathematician could make. And it is not surprising that so many Neoclassical (and posterior) economists of the mainstream overlooked the poverty of the social structure implicit in this model since it is the same poverty they show when they do not distinguish between individuals that are great stockholders or top executives of large corporations and those who are not. But it is remarkable that mainstream economists who so admired von Neumann for his mathematical skills tended to ignore the evident family relation of his model with those directly inspired in the Classical economists and Marx (who is a category of his own). Kurz and Salvadori (1998) find it is difficult to hold the opinion of many that von Neumann’ model belongs to the Neoclassical tradition associated with the ‘Walras-Cassel model’

\footnote{This denomination, which Kurz and Salvadori attribute to Dorfman, Samuelson, and Solow, is unfortunate, as Cassel made no personal contribution whatsoever to the sophisticated model Walras developed.}

\footnote{Von Neumann’s article initiated several decades of relentless efforts by mathematical economists to construct a general equilibrium model inspired by the theory of Walras but backed by rigorous proofs of the existence of general equilibrium based on explicit assumptions as those made by von Neumann (1945 [1938]) and using a ‘fixed point theorem’. In fact, one the methods of proof of the Perron-Frobenius theorems use in this book is based on the application of ‘Brouwer’s fixed point theorem’, proved in 1910 by Luitzen Brouwer and independently (and in the same year) by Jaques Hadamard. The theorem says that if a continuous function \( f \) is such that when applied to elements \( x \) of a set \( X \) (that is compact and convex) \( f (x) \) also belongs to \( X \), then there exists an element \( x_0 \) in \( X \) such that \( f (x_0) = x_0 \), that is, such that \( x_0 \) is a ‘fixed point’ of the function. The main application of the theorem to the topics in this book is that, given a social matrix \( M \) (that satisfies some properties as those we have already seen) whose dominant eigenvalue is one (\( \lambda (M) = 1 \)), then there exists a vector \( p \geq 0 \) such that \( Mp = p \) (that is, such that \( p \) is a fixed point of matrix \( M \) considered as a function}
that when applied to a vector yields another vector of the same dimensions). More
generally, if \( \lambda(M) > 0 \) then there exists a vector \( p \geq 0 \) such that \((M/\lambda(M))p = p\)
(that is, such that \(Mp = \lambda(M)p\)). In our case the set \( X \) may be the set of price vectors
\( p \geq 0 \) that satisfy \( Cp = 1 \) (where \( C \) is the numeraire and we can here assume \( C > 0 \)).
In the case of the transpose matrix \( M^T \) the eigenvector \( q^T \) is a vector of productions
and populations and the normalization can be obtained by means of the (or some of
the) populations of the social classes of the model, for example \( q^T \ell = q^T \) when \( q^L \) is
fixed or (if \( q^L \) is variable), say \( q^K = 100 \).

\( \oplus \) Numerical Exercise \#6 on ER with turnover periods different from one

Consider the following data:

\[
A = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.12 \end{bmatrix}, \quad A^S = \begin{bmatrix} 0.3 & 0.6 \\ 0.5 & 0.3 \end{bmatrix}, \quad \ell = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, \quad \tau_L = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, \quad \tau_K = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}.
\]

First we calculate the profit rate \( \rho_g \) that makes the matrix in (14.39) have a dominant
eigenvalue equal to one. We find (by trial and error) that we must have \( \rho_g = 0.2453 \)
in the following matrix:

\[
\begin{bmatrix} (1 + \rho_g) & 0.2 + \rho_g & 0.3 \\ (1 + \rho_g) & 0.3 + \rho_g & 0.5 \\ & & 0.3 \end{bmatrix} \begin{bmatrix} (1 + \rho_g) & 0.4 + \rho_g & 0.6 \\ (1 + \rho_g) & 0.12 + \rho_g & 0.3 \\ & & (1 + \rho_g) \end{bmatrix} = \begin{bmatrix} (1 + \rho_g) & 0.2 \\ (1 + \rho_g) & 0.3 \\ & & 0.2 \end{bmatrix}.
\]

Next we calculate the eigenvector that is associated with the eigenvalue 1 and normalize
it such that \( \pi = \tau_Kp = 1 \). The result is the vector of prices and detrended incomes
(partial in the case of capitalists):

\[
\begin{bmatrix} p_1 \\ p_2 \\ \overline{w} \\ \pi \end{bmatrix} = \begin{bmatrix} 1.1204 \\ 0.87965 \\ 0.51203 \\ 1 \end{bmatrix}.
\]

We check the following:

\[
\overline{w} = \tau_Lp = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix} \begin{bmatrix} 1.1204 \\ 0.87965 \end{bmatrix} = 0.51205
\]

\[
\pi = \tau_Kp = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} 1.1204 \\ 0.87965 \end{bmatrix} = 1.
\]

The vector that allocates capitalists (and their capitals) to the various branches ac-
gerding to (14.44) is:

\[
\overline{\eta} = \frac{0.2453 - g}{1 + g} \begin{bmatrix} 0.59 \\ 0.86 \end{bmatrix} \begin{bmatrix} 1.1204 \\ 0.87965 \end{bmatrix} = \frac{0.2453 - g}{1 + g} \begin{bmatrix} 1.5935 \\ 1.3682 \end{bmatrix},
\]

where we have used the price vector obtained and the matrix

\[
A + \ell \tau_L + A^S = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.12 \end{bmatrix} + \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.5 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.2 & 0.59 \\ 0.3 & 1.06 \end{bmatrix}.
\]
Finally, we can set up the matrix of the price system (14.38) using the available information:

\[
\begin{bmatrix}
(1 + g)0.2 + (g)0.3 & (1 + g)0.4 + (g)0.6 & (1 + g)0.3 & (0.2453 - g)1.5935 \\
(1 + g)0.3 + (g)0.5 & (1 + g)0.12 + (g)0.3 & (1 + g)0.2 & (0.2453 - g)1.3682 \\
0.3 & 0.2 & 0 & 0 \\
0.5 & 0.5 & 0 & 0
\end{bmatrix}
\]

and find out (by trial and error) what the value of g is that makes the dominant eigenvalue equal to one. The result is \( g = 0.13053 \). This implies

\[
\rho_0 = \frac{(0.2453 - 0.13053) / (1 + 0.13053)}{0.10151} = \rho_0.
\]

We now have the data necessary to construct the matrices of the quantities and prices systems (14.37) and (14.38). Since we already have the price vector, we only need to obtain the left eigenvector of the quantities matrix associated with eigenvalue 1 and normalize so that, for example, \( q^K = 100 \). This yields:

\[
\begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix} = \begin{bmatrix}
335.18 & 329.60 & 166.47 & 100
\end{bmatrix}.
\]

With the numbers thus obtained, we can calculate the aggregate detrended capital

\[ K = q^Q [A + \bar{c}_L + A^c] p = 985.1 \]

and hence confirm that

\[
\frac{q^K \bar{c}_K p}{K} = \frac{100 \times 1}{985.1} = 0.10151 = \rho_0.
\]

Also, the savings rate of capitalists is

\[
s = 1 - \frac{\rho_0}{\rho_g} = 1 - \frac{0.10151}{0.2453} = 0.58618.
\]

\[ \blacksquare \]

Mathematical Appendix to Chapter 14

Details of the cyclical growth model

The signs of \( \partial \rho / \partial \bar{k} \) and \( \partial \rho / \partial g \) Totally differentiating the expression for \( \rho \) in (14.47) we get

\[
\frac{\partial \rho}{\partial \bar{k}} = \frac{(1 + g) \gamma' (\bar{k}) \hat{\pi} (\rho) - \rho}{\bar{k} - (1 + g) \gamma (\bar{k}) \hat{\pi}' (\rho)}, \quad \frac{\partial \rho}{\partial g} = \frac{\hat{\pi} (\rho)}{\bar{k} - (1 + g) \gamma (\bar{k}) \hat{\pi}' (\rho)}.
\]

Using (14.47), the denominator of both expressions is

\[
\bar{k} \left[ 1 - \left( 1 - \frac{\bar{k}}{\rho} \right) \varepsilon \hat{\pi} \right]
\]

where we defined the elasticity of \( \hat{\pi} (\rho) \) as \( \varepsilon \hat{\pi} \equiv \hat{\pi}' (\rho) / \hat{\pi} (\rho) \). Also, (14.47) implies that the term in parentheses is positive. Hence the denominator is positive if and only if either \( \varepsilon \hat{\pi} < 0 \) or \( 0 \leq \varepsilon \hat{\pi} < 1 \left( 1 - \frac{\bar{k}}{\rho} \right) \), that is, if and only if either \( \hat{\pi} (\rho) \) is inelastic \( (\varepsilon \hat{\pi} < 1) \) or it is not too elastic. We assume that the denominator is positive.

Since the numerator of \( \partial \rho / \partial g \) is also positive, we have \( \partial \rho / \partial g > 0 \). In the case of \( \partial \rho / \partial \bar{k} \) the numerator can be positive, zero, or negative. It can be written as
\[ \rho \left[ \left( 1 - \frac{g}{\rho \kappa} \right) \varepsilon_\gamma - 1 \right], \] where we defined the elasticity of \( \gamma (\kappa) \) as \( \varepsilon_\gamma \equiv \frac{\kappa \gamma' (\kappa)}{\gamma (\kappa)} \) > 0. Hence the numerator is negative if and only if \( \varepsilon_\gamma < 1 / \left( 1 - \frac{g}{\rho \kappa} \right) \), that is, if and only if the function \( \gamma (\kappa) \) is either inelastic (\( \varepsilon_\gamma < 1 \)) or not too elastic.

Hence, if 1) \( \varepsilon_\gamma < 1 / \left( 1 - \frac{g}{\rho \kappa} \right) \) and 2) either \( \varepsilon_\pi \leq 0 \) or \( 0 < \varepsilon_\pi < 1 / \left( 1 - \frac{g}{\rho \kappa} \right) \) then \( \partial \rho / \partial \kappa < 0 \) and \( \partial \rho / \partial q > 0 \).

From the total differentiation of the expression for \( \rho \) in (14.47) and the fact that \( g \) and \( \kappa \) have opposite effects on \( \rho \) we can calculate what relation is needed between the proportional changes in \( g \) and \( \kappa \) for \( \rho \) not to be affected. That relation is:

\[
\frac{dg}{g} = \frac{1 + g}{1 + \frac{g}{\rho \kappa}} \left[ 1 - \varepsilon_\gamma \left( 1 - \frac{g}{\rho \kappa} \right) \right] \frac{d\kappa}{\kappa}.
\]

**Systems of linear difference equations** We here state two useful theorems on systems of difference equations from Woods (1978).

Let \( A \geq 0 \) be a square matrix. The following are two ‘backward looking’ systems of difference equations, where the first is homogeneous and the second non-homogeneous (since it is ‘forced’ by \( f_{t+1} \)):

\[
x_t = Ax_{t+1},
\]

(14.51)

\[
x_t = Ax_{t+1} + f_{t+1}.
\]

(14.52)

**Theorem 1** Let \( A \) be indecomposable. Then the unique solution of balanced growth of the homogeneous equation (14.51) is \( x^*/\lambda (A)^t \), where \( \lambda (A) \) is the dominant eigenvalue of \( A \) and \( x^* > 0 \) is the unique (up to a scalar factor) eigenvector associated to \( \lambda (A) \). If \( \lambda (A) = 1 \) then the unique balanced growth solution is \( x^* \) (Woods 1978, 157).

For the non-homogeneous equation (14.52) we only deal with the particular case in which \( f_t \) grows at a constant rate: \( f_{t+1} = \alpha^{t+1} f \), where \( \alpha \) and \( f \) are constants.

**Theorem 2** Let \( A \) be indecomposable and \( f_{t+1} = \alpha^{t+1} f \). Then the non-homogeneous equation (14.52) has a meaningful solution if and only if \( \alpha \lambda (A) < 1 \) (Woods 1978, 182).

**Conservation of the spectrum**

1 The matrix of system (14.15) is not the same as that of (14.12), so the systems are not dual. But they are nearly so in a sense. The spectrum of a square matrix is the set of its eigenvalues. A result of linear algebra is that the spectrum of the product of two square matrices \( E \) and \( F \) is invariant to the order in which they are multiplied (\( FE \) or \( EF \)). In the case of semi-positive matrices, if \( \lambda \) is an eigenvalue of \( EF \), by 1) of Theorem 1 of the Mathematical Appendix to Chapter 5 there exists an eigenvector \( x \) such that \( EFx = \lambda x \). Premultiplying by \( F \) we get \( FEFx = \lambda Fx \). Hence, making \( y = Fx \) we have \( FEy = \lambda y \) and hence \( \lambda \) is also an eigenvalue of \( FE \). The same argument is used to prove that if \( \lambda \) is an eigenvalue of \( FE \) it is also one of \( EF \).

On the other hand, by 4) of the same Theorem we also know that a square matrix \( G \) and its transpose \( G^T \) have the same spectrum. Hence, if for two square and semi-positive matrices \( E \) and \( F \) the equation \( q(FE) = q \) has a solution \( q \geq 0 \), then necessarily
\[ \lambda(FE) = 1. \] Since \( 1 = \lambda(FE) = \lambda(EF) = \lambda((EF)^T) \), there also exists a vector \( r \geq 0 \) such that \( r(\EF)^T = r \), that is \( (EF)p = p \) for \( p \equiv r^T \).

Hence, if \( M^+ \) is the matrix corresponding to \( g = 0 \) of (14.12) and (14.15) and we define

\[ \bar{q}^+ = \begin{bmatrix} \theta^0 & \alpha_L & \alpha_K \end{bmatrix}, \quad p^+ = \begin{bmatrix} p & w \pi \end{bmatrix}^T, \quad \Phi = \begin{bmatrix} (1 + g) I_{n \times n} & 0 \\ 0 & I_{2 \times 2} \end{bmatrix}, \]

then (14.12) and (14.15) can be written as \( \bar{q}^+ (M^+ \Phi) = \bar{q}^+ \) and \( (\Phi M^+) p^+ = p^+ \) respectively. Since \( M^+ \Phi, \Phi M^+ \) and \( (\Phi M^+)^T \) have the same spectrum, (14.12) has a solution \( \bar{q}^+ \geq 0 \) if and only if (14.15) has a solution \( p^+ \geq 0 \). Also, if \( M^+ \) is indecomposable, so are \( M^+ \Phi, \Phi M^+ \) and \( (\Phi M^+)^T \), and hence the solutions \( \bar{q}^+ \) and \( p^+ \) are positive and unique up to a scalar factor.

2 The matrices of (14.37) and (14.38) are more complicated than those of (14.12) and (14.15) by the presence of \( A^S \). But they also have the same spectrum due to the simple structure of \( \Phi \) in (14.53). If \( M^+ \) is the matrix of (14.37) and (14.38) corresponding to \( g = 0 \), the matrices of these systems can be written as \( M^+ \Phi + \Omega \) and \( \Phi M^+ + \Omega \), respectively, and the systems can be written as \( \bar{q}^+ (M^+ \Phi + \Omega) = \bar{q}^+ \) and \( (\Phi M^+ + \Omega) p^+ = p^+ \), where

\[ \bar{q}^+ = \begin{bmatrix} \theta^0 & q^L & q^K \end{bmatrix}, \quad p^+ = \begin{bmatrix} p & \bar{w} \pi \end{bmatrix}^T, \quad \Omega = \begin{bmatrix} gA^S & 0 \\ 0 & I_{2 \times 2} \end{bmatrix}. \]

Let \( C = M^+ \Phi + \Omega \) and \( D = \Phi M^+ + \Omega \). Since \( \Phi \Omega = \Omega \Phi \), it can be easily shown that \( C = \Phi^{-1} D \Phi \). This implies that \( C \) and \( D \) have the same spectrum. For, if \( qC = \lambda q \), then \( q\Phi^{-1} D \Phi = \lambda q \), that is, \( (q\Phi^{-1} D) = \lambda (q \Phi^{-1}) \), and hence \( \lambda \) is also an eigenvalue of \( D \). The same argument can be applied for the reciprocal implication that the eigenvalues of \( D \) are also eigenvalues of \( C \). Hence, since \( 1 = \lambda(C) = \lambda(D) = \lambda(D^T) \), if \( qC = q \) has a solution \( q \geq 0 \), there exists an \( r \geq 0 \) such that \( rD^T = r \), that is \( Dp = p \) for \( p \equiv r^T \geq 0 \). The proof of the reciprocal is analogous. Also, if \( M^+ \) is indecomposable, so are \( C, D, \) and \( D^T \), and hence the solutions \( \bar{q}^+ \) and \( p^+ \) are positive and unique up to a scalar factor.

An alternative way of approaching this subject is by eliminating variables in both systems so as to get the ‘dual’ equations \( \bar{q}^2 H = \bar{q}^2 \) and \( Hp = p \), where \( H \equiv (1 + g) (A + \bar{c}_L + \bar{q}_K) + gA^S \) and clearly \( g \) must be such that \( \lambda(H) = 1 \). Assuming that \( A \) is indecomposable, so is \( H \), and hence there exist positive dominant eigenvectors \( \bar{q}^2 \) and \( p \) (which are unique up to a scalar factor). If we also have \( \bar{q}^2 \bar{p} = q^L, \bar{q}^2 \pi = q^K, \bar{c}_L p = \bar{w}, \bar{c}_K p = \pi \), the two systems (14.37) (14.38) can be formulated, proving that their respective matrices also have one as their dominant eigenvalue.

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\(^{10}C = M^+ \Phi + \Omega = \Phi^{-1} \Phi (M^+ \Phi + \Omega) = \Phi^{-1} (\Phi M^+ \Phi + \Phi \Omega) = \Phi^{-1} (\Phi M^+ \Phi + \Omega \Phi) = \Phi^{-1} (\Phi M^+ + \Omega) \Phi = \Phi^{-1} D \Phi.\)
Chapter 15 COMMERCIAL CAPITAL AND UNPRODUCTIVE LABOR

Marx distinguished general categories, applicable to any human society and any period of time, from the categories only valid in capitalist society. Hence when in Chapter 5 of Book I (“The Labour Process and the Valorisation Process”) he considers “The labour process, resolved as above into its simple elementary factors” (B1, 194), he remarks that “from the point of view of its result, the product, it is plain that both the instruments and the subject of labour, are means of production, and that the labour itself is productive labour” (B1, 191). But he warns that this sense of ‘productive labor’ is not adequate for capitalist production. When he begins Chapter 16 (“Absolute and Relative Surplus Value”) he explains that productive labor in CCP is that labor that produces surplus value (and hence profit). In particular, it must be labor exerted in the productive process by labor power that is purchased by means of money that is disbursed as (variable) capital, and hence motivated by the expectation of profit. Therefore, Marx’s use of ‘productive labor’ in the context Capitalism specifically refers to the labor of wage workers in the production process that produces surplus value.

For Marx, not all wage labor in the CCP was ‘productive labor’, not even all the labor exerted by wage workers in capitalist industrial firms, since not all that labor generated value; and if it did not produce value it could not produce surplus value. In particular, in Marx’s theory of Capitalism wage labor dedicated to operations related to the circulation of commodities (sales and purchases) did not produce value. The fact that in Marx’s theory the concept of value represented an absolute magnitude, a determined quantity of abstract labor that was socially necessary for the production of a commodity that was demanded by society, made it necessary to carefully delimit which wage labor produced value and which did not. Wage labor that was paid out of the profits of capitalists (say, house servants), the rents of landowners, or even the wages of workers (say, a house servant employed by a skilled worker), was not productive labor because the disbursement of money to pay such wages was not motivated by profit but by use value. And whereas all the capitalistic firms (industrial, commercial, or banking) had to earn profits to distribute among their owners, only (a majority of) the wage workers of industrial capitalist firms produced surplus value. The profit of these firms was only a part of this surplus value, since the rest had to be used to pay the wages of workers dedicated to commercial or financial activities within these firms, to distribute (through the purchases of inputs) among non-industrial firms to pay for all the incomes related to their activities (wages, profits, interests, or rents), and to pay government taxes.

Thus there were several categories of wage workers that did not produce surplus value and hence were not ‘productive workers’: those working in industrial firms but dedicated to commercial or financial functions, those working in commercial or banking capitalist firms, those working for capitalists, landowners, simple commodity producers, or even wage workers in the sphere of their private consumption (such as house servants), and those employed in the government. None of these wage workers produced value or surplus value and hence were ‘unproductive workers’. Their wages were ultimately paid out of the aggregate surplus value generated in industrial firms by wage workers performing production tasks.
In *Theories* Marx analyzes Adam Smith’s theory of productive and unproductive labor as well as similar theories by his predecessors or successors. He takes the parts of Smith’s theory that suit him and criticizes and discards the parts he considers erroneous, and writes: “Productive labour, in the meaning of capitalist production, is wage labour which, exchanged against the variable part of capital (the part of the capital that is spent on wages), reproduces not only this part of the capital (or the value of its own labour capacity), but in addition produces surplus value for the capitalist. It is only thereby that commodity or money is transformed into capital, is produced as capital. Only that wage labour is productive which produces capital” (B4.31, 8). To ‘produce capital’ it was necessary to produce surplus value and reinvest a part of it as capital.

In Marx’s theory of Simple Commodity Production the time that producers/workers dedicated to purchasing and selling commodities did not generate value. For “these acts constitute merely the conversion of a certain value from one form into another, from the commodity form into the money form or from the money form into the commodity form—a change in the state of being” (B2, 133) that is, a ‘metamorphosis’. It is ‘unproductive labor’, which in the context of SCP means that it did not generate value.1 Similarly, in Marx’s theory of Capitalist Commodity Production the wage labor applied to commercial of financial functions, whether in industrial, commercial, or financial firms is ‘unproductive labor’ since it does not generate value and hence does not produce surplus value.

The same as industrial capital, commercial capital (or capital dedicated to commerce) and financial capital (or capital dedicated to finance or banking) perform fundamental functions in the total process of reproduction of capital. But they constitute forms of capital that function within the sphere of circulation. And “no value is produced in the process of circulation, and, therefore, no surplus value. Only changes of form of the same mass of value take place. In fact, nothing occurs there outside the metamorphosis of commodities, and this has nothing to do as such either with the creation or change of values” (B3, 279). For Marx the ‘functioning capitalist’ can perform in various ‘spheres’ (production, commerce, finance) but only in the sphere of production is surplus value generated, and it is the capitalist in industry (which includes transportation, communications, agriculture and livestock, etc.) who initially appropriates it, retaining only a part since, as he anticipated in Book I, “He has to share it with capitalists, with landowners, &c, who fulfil other functions in the complex of social production. Surplus value, therefore, splits up into various parts. Its fragments fall to various categories of persons, and take various forms, independent the one of the other, such as profit, interest, merchants’ profit, rent, &c” (B1, 564-5). These ‘categories of persons’ with which industrial capitalists share the surplus value constitute segments of the capitalist class that participate in the global division of (entrepreneurial) labor in the cyclical process of capital or else belong to the landowning class, which in modern Capitalism was merely a fraction of the (great) capitalist class. They are in general, owners of the means and conditions of production.

But aside from the wage labor dedicated to functions that belong to the sphere of circulation, the labor of workers whose wages are paid out of incomes of any kind (profits, rents, wages) that are not disbursed as capital (i.e., with the aim of increasing the value of capital), is also unproductive. Such are the cases of various types of domestic servants (butlers, cooks, chauffeurs, etc.), a category of labor that was quantitatively

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1 “So long as small independent producers of commodities spend a part of their own time in buying and selling, this represents nothing but time spent during the intervals between their productive function or diminution of their time of production” (B2, 136).
very important in the England of Marx’s time, as we see below.

Finally, we can point out that the distinction between productive and unproductive labor had particular relevance in Extended Reproduction. For the expansion of the scale of production was based on the reinvestment of surplus value. And only productive labor produced surplus value. The more unproductive workers there were in society, the less were employed in producing surplus value, and hence in the expansion of capital. In Marx’s words: “The difference between productive and unproductive labour is important as regards accumulation, since one of the conditions for the reversion of surplus value into capital is that the exchange should be with productive labour alone” (Results, MECW 34, 452).

In this chapter we adapt our systems of quantities and populations, prices and incomes, and values seen in previous chapters to each of these realities that Marx addresses, which are more complex than what the simplified models we have seen so far can encompass and closer to the complexity of the empirical reality he was always addressing. We restrict our models to the case of Simple Reproduction, since they are sufficient for the purpose of transmitting the essential ideas of Marx in this subject matter.

Unproductive labor in the sphere of production

Marx highlighted that “a part of the selling and buying of commodities always takes place directly between industrial capitalists” (B3, 267) and that “The more developed the scale of production, the greater, even if not proportionately greater, the commercial operations of the industrial capital, and consequently the labour and other costs of circulation involved in realising value and surplus value”, which “necessitates the employment of commercial wage workers who make up the actual office staff. The outlay for these, although made in the form of wages, differs from the variable capital laid out in purchasing productive labour” (B3, 298), since such wage labor does not generate value nor surplus value and is hence ‘unproductive labor’. According to Marx,

The general law is that all costs of circulation which arise only from changes in the forms of commodities do not add to their value. They are merely expenses incurred in the realisation of the value or in its conversion from one form into another. The capital spent to meet those costs (including the labour done under its control)... must be replaced from the surplus product and constitute, as far as the entire capitalist class is concerned, a deduction from the surplus value or surplus product, just as the time a labourer needs for the purchase of his means of subsistence is lost time (B2, 152-3).

In this section we represent “the commercial operations of the industrial capital” as a simple extension of the model developed in Chapter 8. For this we assume that there are two kinds of wage workers in industrial firms: productive labor, that is, labor dedicated to production, and unproductive labor, that is, labor dedicated to circulation (commercial and/or financial). Consequently, in the systems of quantities and prices we separate the population of industrial workers engaged in (industrial) production \( q^{LI} \) from the population dedicated to circulation \( q^{LC} \), and also distinguish their respective consumption baskets, \( c_{LI} \) and \( c_{LC} \), their wages \( w^I \) and \( w^C \), and their direct labor requirement coefficients \( \ell_I \) and \( \ell_C \):

\[
\begin{bmatrix}
q^Q & q^{LI} & q^{LC} & q^K \\
A & \ell_I & \ell_C \\
c_{LI} & 0 & 0 \\
c_{LC} & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^{LI} & q^{LC} \\
\end{bmatrix}
\tag{15.1}
\]
\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) \ell_I & (1 + \rho) \ell_C \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
p w^I \\
w^C \\
\end{bmatrix} =
\begin{bmatrix}
p w^I \\
w^C \\
\end{bmatrix}.
\]

(15.2)

From these equations we obtain the global profit rate as \(^2\):

\[
\rho = \frac{q^K c_K p}{q^Q A p + q^{L I} w^I + q^{L C} w^C}.
\]

As always, the profit rate is the ratio between profits and disbursed capital. But the latter includes the variable capital used in the remuneration of not only productive workers but also unproductive workers. The fact that a part of the workers must be dedicated to operations of circulation seems to reduce the profit rate if we compare with the model in which there was no unproductive labor within the sphere of production. But that was a very stylized model and the present extension adds a dose of empirical richness. Hiring workers to engage in commercial or financial tasks helps the industrial capitalist “reduce the cost of realising surplus value” (B3, 299) and hence increase his profit. And according to Marx “The commercial worker, in the strict sense of the term, belongs to the better-paid class of wage workers – to those whose labour is classed as skilled and stands above average labour” (Ibid.). In our model this would imply that \(w^C > w^I\). But, attentive to empirical data, Marx also points out that the higher “wage tends to fall, even in relation to average labour, with the advance of the capitalist mode of production” (Ibid.). Marx attributes this tendency to various factors such as 1) the deepening of the division of labor within the office, 2) the advancement in “the necessary training, knowledge of commercial practices, languages, etc... with the progress of science and public education", and 3) the fact that “public education makes it possible to recruit such labourers from classes that formerly had no access to such trades and were accustomed to a lower standard of living” (Ibid.), which increases competition and hence tends to lower the wage rate.

Let us now see how the existence of wage workers dedicated to tasks related to the circulation of commodities affects the calculation of value and the rate of surplus value. Since labor affected to circulation is not productive (does not generate value) to formulate the system of values we must omit the row and column that pertain to this kind of labor. Hence, we have:

\[
\begin{bmatrix}
A & \ell_I \\
(1 + e) c_{LI} & 0 \\
\end{bmatrix}
\begin{bmatrix}
v \\
1 \\
\end{bmatrix} =
\begin{bmatrix}
v \\
1 \\
\end{bmatrix}.
\]

(15.3)

This system is exactly the same as (8.6) except for the change in notation (\(\ell_I\) instead of \(\ell\) and \(c_{LI}\) instead of \(c_L\)). There necessarily exists a positive value for the rate of surplus value \(e\) that satisfies this system. To see this, notice that the matrix of (15.2) has a dominant eigenvalue of one. Placing \(p = 0\) and eliminating the last row and column are both operations that lower the dominant eigenvalue of the matrix. Hence, the dominant eigenvalue of the resulting matrix is necessarily less than one and a unique \(e > 0\) can be found that again makes it equal to one. Also, the right eigenvector associated with this matrix is necessarily positive (given the assumption of indecomposability, which implies that any ‘non basic’ commodity has been left out). The two equations of this system yield the vector of values in terms of exogenous data \((A, \ell_I)\) as well as an

\(^2\)Multiply the first equation of (15.1) by \(p\) and pre-multiply the first equation of (15.2) by \(q^Q\); finally simplify after using the rest of the equations in these systems.
expression for $e$ that uses $v$ and exogenous data ($c_{LI}$):

$$v = (I - A)^{-1} \ell_I$$

(15.4)

$$(1 + e) c_{LI} v = 1.$$  

(15.5)

A more convenient formula for $e$ in terms of interpretation can be found by multiplying the first equality of (15.1) by $v$ and using (15.4) and (15.5): $q^L c_{LI} v + q^{LC} c_{LC} v + q^K c_K v = q^Q (I - A) v = q^Q \ell_I = q^L I (1 + e) c_{LI} v$, which yields after simplifying:

$$e = \frac{q^K c_K v + q^{LC} c_{LC} v}{q^L I c_{LI} v} = \frac{S^v}{V^v}.$$  

As in Chapter 8, the global rate of surplus value $e$ is the ratio between the surplus value $S^v$ and the variable capital invested in productive labor $V^v$, but in this case $S^v$ includes not only the value of capitalists’ consumption but also that of the consumption of unproductive laborers. Hence, the value of the labor power of unproductive industrial workers is derived from the surplus value, whereas the labor power that creates value and surplus value $V^v$ is exclusively that of productive industrial workers (those dedicated to production and not circulation).

The comparison of magnitudes expressed in production prices with those expressed in values is only possible if first we choose a numeraire. Suppose we normalize prices so that the net output is the same whether it is measured in production prices or values $(q^Q (I - A) p = q^Q (I - A) v)$. In that case we can check that the global surplus value is: $S^v = q^K \pi + q^{LC} w^C + q^L l (w^l - c_{LI} v)$, i.e., it is the sum of aggregate profits and the wages of unproductive workers corrected by any discrepancy between productive workers’ wages and the value of their consumption baskets.

Unproductive labor in the sphere of circulation

For Marx “it is the tendency of the capitalist mode of production to transform all production as much as possible into commodity production” (B2, 115-6), for which it attracts to its process of circulation, through the participation of commercial enterprises, commodities that have been produced by means of non-capitalist relations of production. The competitive process gradually stimulates the transformation of workers (slaves, serfs, or independent simple commodity producers) into wage workers, and the transformation of non-capitalist modes of production into the capitalist mode of production. According to Marx, “Commercial capital is nothing but a converted form of a part of this capital of circulation constantly to be found in the market, ever in the process of its metamorphosis, and always encompassed by the sphere of circulation” (B3, 267). He distinguishes two forms of commerce: the one that through the division of social labor functions autonomously in the form of commercial firms, and the purchase and sale of commodities that “takes place directly between industrial capitalists” (Ibid.). Since the latter has been dealt with in the preceding section, here we assume that all the labor in the industrial sphere is productive and focus attention (insofar as unproductive labor is concerned) on capitalist commercial firms that use (unproductive) wage labor.

The function of commercial capital is “a necessary element of reproduction” and originates in the specialization of a branch of capital in the process of purchases and sales so that “a smaller part of society’s labour power and labour time is tied up in this unproductive function” (B2, 136). This labor, however, does not generate value nor surplus value, due to which the labor performed in commercial firms in ‘unproductive’. “Commercial capital, therefore –stripped of all heterogeneous functions, such as
storing, expressing, transporting, distributing, retailing, which may be connected with
it, and confined to its true function of buying in order to sell—creates neither value nor
surplus value, but acts as middleman in their realisation and thereby simultaneously in
the actual exchange of commodities, i.e., in their transfer from hand to hand, in the
social metabolism” (B3, 280). But commercial capital “must yield the average annual
profit just as well as capital operating in the various branches of production. Should
merchant’s capital yield a higher percentage of average profit than industrial capital,
then a portion of the latter would transform itself into merchant’s capital. Should it
yield a lower average profit, then the converse would result” (Ibid.). And “No species
of capital changes its purpose, or function, with greater ease than merchant’s capital”
(B3, 281).

**Commercial capital and the systems of quantities, prices, and values**

In order to represent commercial capital by means of our social matrices without ex-
cessive complications we make the following assumptions: 1) industrial firms sell all
their output to commercial firms and commercial firms only sell to industrial firms or
individual consumers, 2) there are as many commercial branches as there are indus-
trial branches, each one specialized in the purchases and sales of the products of one
industrial branch, 3) commercial firms have no other inputs than labor power and the
finished products they purchase from industrial firms, 4) the rate of profit is the same
in all branches, whether they are industrial or commercial, 5) there are two kinds of
labor, industrial and commercial, which can have different wages, and 6) there are no
unproductive laborers in industrial firms. Assumption 1) implies that all consumers,
whether wage workers or capitalists, purchase their means of subsistence from com-
mercial firms. And assumption 6) implies we leave out the case of the preceding section.
With such assumptions the systems of quantities and populations and of prices and
incomes are the following:

\[
\begin{bmatrix}
q^{QI} & q^{QC} & q^{LI} & q^{LC} & q^{KI} & q^{KC}
\end{bmatrix}
\begin{bmatrix}
0 & A & \ell_I & 0 \\
I & 0 & 0 & \ell_C \\
0 & c_{LI} & 0 & 0 \\
0 & c_{LC} & 0 & 0 \\
0 & c_{KI} & 0 & 0 \\
0 & c_{KC} & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^{QI} & q^{QC} & q^{LI} & q^{LC}
\end{bmatrix}
\begin{bmatrix}
p^l \\
p^C \\
w^l \\
w^C \\
p^L \\
p^C
\end{bmatrix}
\]

(15.6)

\[
\begin{bmatrix}
0 & (1 + \rho) A & (1 + \rho) \ell_I & 0 \\
(1 + \rho) I & 0 & 0 & (1 + \rho) \ell_C \\
0 & c_{LI} & 0 & 0 \\
0 & c_{LC} & 0 & 0 \\
0 & c_{KI} & 0 & 0 \\
0 & c_{KC} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p^l \\
p^C \\
w^l \\
w^C \\
p^L \\
p^C
\end{bmatrix}
= \begin{bmatrix}
p^l \\
p^C \\
w^l \\
w^C \\
p^L \\
p^C
\end{bmatrix}
\]

(15.7)

In these systems we distinguish the vector of ‘commercial output’ \(q^{QC}\) from the vector
of industrial output \(q^{QI}\), the commercial working population \(q^{LC}\) from the industrial
working population \(q^{LI}\), and the commercial capitalist population \(q^{KC}\) from the indus-
trial capitalist population \(q^{KI}\). We also distinguish the per capita income of commercial
capitalists \(\pi^C\) from that of industrial capitalists \(\pi^l\), as well as the vector of commercial
prices \(p^C\) from the vector of industrial prices \(p^l\). And, as in the preceding section, we
distinguish the direct labor coefficients \(\ell_I\) and \(\ell_C\), and the wage rates \(w^C\) and \(w^l\). But
here industrial and commercial laborers work in different branches (and hence different
firms). The individual equations of these systems are the following:

1. \[ A \]
2. \[ B \]
3. \[ C \]
4. \[ D \]
5. \[ E \]
6. \[ F \]

\( A \) shows that industrial firms sell all their output to the commercial firms who specialize in selling their output. \( A2 \) shows that commercial firms sell all the goods they purchase from industrial firms to industrial firms as inputs to production \( (q^{QI} A) \), to workers \( (q_{LI}c_{LI} + q_{LC}c_{LC}) \) and to capitalists \( (q^{KI}c_{KI} + q^{KC}c_{KC}) \). Also, \( A3 \) and \( A4 \) show how the industrial \( q^{LI} \) and commercial \( q^{LC} \) workers are allocated to their respective branches. \( B1 \) shows that industrial prices (i.e., 'production prices') are composed of the cost in wages and inputs purchased from commercial firms plus profit. Similarly, \( B2 \) shows the composition of commercial prices as the industrial prices (at which these goods were purchased), wages, and profits. And \( B3 \) – \( B6 \) show that consumers purchase their consumption baskets at the prices of commercial firms.

To get the formula for the global profit rate we can premultiply \( B1 \) by \( q^{QI} \) and \( B2 \) by \( q^{QC} \), and subsequently add the resulting equalities term by term, which yields \( (1 + \rho) \left[ q^{QI} (Ap^C + \ell_I w^I) + q^{QI} (p^I + \ell_C w^C) \right] = q^{QI} p^I + q^{QC} p^I. \) From here we get the global profit rate as the ratio between global profits \( \Pi \) and global invested capital \( K \), where in the numerator both types of profit are included, and in the denominator both types of capital:\(^3\)

\[
\rho = \frac{\frac{q^{QI}c_{KI}p^C + q^{KC}c_{KC}p^C}{q^{QI}(Ap^C + \ell_I w^I) + q^{QC}(p^I + \ell_C w^C)}}{\frac{\Pi}{K}}.
\]

Notice that, due to the assumption that the profit rate is the same in the two large departments (industrial and commercial), the global rate of profit is simply the weighted average of the two equal rates of profit, where the weights are the shares of each large sector in the global capital: \( \rho = \rho * s_I + \rho * (1 - s_I) \), where \( s_I = q^{QI} (Ap^C + \ell_I w^I) / K \).

But if these profit rates were different (possibly because they did not yet reach equality), we would have \( \rho = \rho_I * s_I + \rho_C * (1 - s_I) \) with \( \rho_I \neq \rho_C \).

Notice that \( B2 \) implies that commercial prices are necessarily greater than industrial prices. As Marx writes: “The price of production, or the price at which the industrial capitalist as such sells his commodities, is thus smaller than the actual price of production of the commodity” (B3, 284), where he is calling ‘actual price of production’ what we have called ‘commercial price’. But leaving names aside, what is important is that Marx’s ‘actual price of production’ is greater than the production price because they additionally include costs that are specific to commercial firms as well as their profits. To simplify we only included the wage costs of commercial firms. But Marx was aware that there were also other ‘costs of circulation’:

The costs which we consider here are those of buying and selling. We have already remarked earlier that these resolve themselves into accounting, book-keeping, marketing, correspondence, etc. The constant capital required for this purpose consists of offices, paper, postage, etc. The other

---

\(^3\)A1-A4 and B3-B4 are used here.
costs break up into variable capital advanced for the employment of merchantable wage workers… All these costs are not incurred in producing the use value of commodities, but in realising their value. They are pure costs of circulation. They do not enter into the immediate process of production, but since they are part of the process of circulation they are also part of the total process of reproduction (B3, 287-8).

As he usually did, in this case Marx contrasts the conceptual arrangement of his synchronous theoretical construction with the diachronic evolution of historical reality. He points out that his theoretical construction (the ‘scientific analysis’) had first dispensed with commercial capital and later shown how the profit rate of the simple model (of industrial Capitalism) was affected when commercial capital was also taken into account. However, in the actual historical development the global profit rate had initially been shaped in the sphere of circulation and only with the development of industrial capital was commercial profit reduced to being a slice of the global surplus value in the social process of reproduction:

In the course of scientific analysis, the formation of a general rate of profit appears to result from industrial capitals and their competition, and is only later corrected, supplemented, and modified by the intervention of merchant’s capital. In the course of its historical development, however, the process is really reversed. It is the commercial capital which first determines the prices of commodities more or less in accordance with their values, and it is the sphere of circulation, the sphere that promotes the process of reproduction, in which a general rate of profit initially takes shape. It is originally the commercial profit which determines the industrial profit. Not until the capitalist mode of production has asserted itself and the producer himself has become merchant, is commercial profit reduced to that aliquot part of the total surplus value falling to the share of merchant’s capital as an aliquot part of the total capital engaged in the social process of reproduction (B3, 285-6).

The system of values that corresponds to systems (15.6) and (15.7) is exactly the same as that of the preceding section (15.3), so there is no change in the definition of the vector of values (15.4). What does change is the explicit formula for the rate of surplus value. From A1 – A3 and (15.4)-(15.5), and using the same procedure as in the preceding section we get:

\[
e = \frac{q^{K_I} c_{K_I} v + q^{L_C} c_{L_C} v + q^{R_C} c_{R_C} v}{q^{L_I} c_{L_I} v} = \frac{S^v}{V^v}.
\]

Notice that in the denominator we have the same variable capital (measured in values) as in the preceding section, that is, the capital disbursed in industry for the reproduction of productive laborers. But in the numerator, the surplus value generated by these productive laborers is composed of not only the value of the consumption baskets of industrial capitalists and commercial wage workers, as in the preceding section, but also the consumption basket of commercial capitalists (inexistent in the preceding section).

Unproductive labor in the sphere of consumption
According to Marx’s theory, in CCP productive labor (the labor that valorizes—or increases—capital) can only be done by wage workers. But not all wage labor in CCP
was productive, as we have seen in the case of workers that perform tasks related to the circulation of commodities, whether in industry or commerce. In this section we consider another category of ‘unproductive labor’: that which is done by labor power that is purchased with money that is disbursed ‘as such’, that is, as expenditure out of revenue, and not ‘as capital’, i.e., to be valorized. Such would be the case of house servants, gardeners, physicians, whose services are often purchased out of the part of the surplus value that is spent in consumption by any one of those individuals who receive profits or rents and even by workers out of their wages. In Theories Marx praises Adam Smith for getting “to the very heart of the matter” when he wrote that (one of the forms of) unproductive labor

is labour which is not exchanged with capital, but directly with revenue, that is, with wages or profit (including of course the various categories of those who share as co-partners in the capitalist’s profit, such as interest and rent)... An actor, for example, or even a clown, according to this definition, is a productive labourer if he works in the service of a capitalist (an entrepreneur) to whom he returns more labour than he receives from him in the form of wages; while a jobbing tailor who comes to the capitalist’s house and patches his trousers for him, producing a mere use value for him, is an unproductive labourer. The former’s labour is exchanged with capital, the latter’s with revenue. The former’s labour produces a surplus value; in the latter’s revenue is consumed (B4.31-12-13).

Marx’s example of an actor highlights the fact that his conception of productive labor had nothing to do with the distinction between the production of material goods and the production of services. What matters for Marx is the location of this labor within his view of the structure of society, a structure in which the goal underlying the disbursement of money plays a fundamental role: either purchasing commodities for the personal consumption of its use value, or to valorize the capital disbursed through the productive consumption of its use value. This also shows that unproductive labor need not necessarily be wage labor. But to include non-wage unproductive labor we would need to model SCP and CCP in the same model, as we do in Chapter 16 below. Here we focus exclusively on wage labor.

The systems of quantities, prices, and values

In order to represent the unproductive labor done in the sphere of consumption we again distinguish between two kinds of labor: productive and unproductive. Let $q^{LI}$ and $w_L$ represent, as before, the population of productive workers and their wage rate. And let $q^{LM}$ and $w_M$ be the population of unproductive workers and their wage rate. These two kinds of laborers have consumption baskets $c_{LI}$ and $c_{LM}$, respectively. Let $\ell_M$ be the consumption of unproductive labor of each capitalist. Notice that, in contrast to $\ell_I$ (which is a vector of coefficients), $\ell_M$ is a scalar. The systems of quantities and prices are the following:

$$
\begin{bmatrix}
q^Q & q^{LI} & q^{LM} & q^K
\end{bmatrix}
\begin{bmatrix}
A & \ell_I & 0 & 0 \\
c_{LI} & 0 & 0 & 0 \\
c_{LM} & 0 & 0 & 0 \\
c_K & 0 & \ell_M & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^{LI} & q^{LM}
\end{bmatrix}.
$$

(15.8)
where \( \pi \) is the value of the consumption basket of each capitalist. From these equations we can get \( \rho \) as before:

\[
\rho = \frac{q^K \pi}{q^Q A p + q^{LI} w_I}.
\]

Notice that the profit in the numerator (according to the fourth equality of (15.8)) includes the values of both the consumption basket of produced commodities of capitalists as the consumption basket of unproductive workers whose services are consumed by capitalists and whose incomes derive from profit. And global capital in the denominator only includes the wage cost of productive workers, the only wages for which the money invested in their periodic payment constitutes capital (specifically, variable capital).

The system of values is again (15.3), as in the preceding sections. Hence (15.4) and (15.5) are still valid. But the explicit formula for the rate of surplus value changes. From the equations of (15.8) and (15.4)-(15.5) we obtain:

\[
e = \frac{q^K c_K v + q^{LM} c_{LM} v}{q^{LI} c_{LI} v} = \frac{S_v}{V_y}.
\]

When Marx writes that unproductive labor is revenue, that is, “wages or profit (including of course the various categories of those who share as co-partners in the capitalist’s profit, such as interest and rent)” (B4.31, 12), he asserts that it can also be the case that workers use a part of their wage incomes to purchase unproductive labor. In order to represent this case the necessary modification would be simply to replace the third column of the social matrices of (15.8) and (15.9) by \((0, \ell_{ML}, 0, \ell_{MK})^T\), where \(\ell_{ML}\) and \(\ell_{MK}\) are the direct requirements for unproductive labor in the consumption of productive laborers and capitalists, respectively. This would of course be more realistic for a model of heterogeneous workers where skilled workers earn higher salaries than those who sell simple labor power.

The empirical relevance of this type of unproductive labor

When Marx wrote Capital the ‘unproductive labor’ in the sphere of consumption was of great empirical relevance due to the enormous population of servants of all types that existed in imperial England, the most advanced capitalist country at the time. Marx observed that “the extraordinary productiveness of modern industry... allows of the unproductive employment of a larger and larger part of the working class, and the consequent reproduction, on a constantly extending scale, of the ancient domestic slaves under the name of a servant class, including men-servants, women-servants, lackeys, &c” (B1, 449). This statement was backed by data from the 1861 census for England and Wales that showed that out of a total population slightly greater than 20 million, after subtracting children, people too old to work, women that did not work outside their homes, government officials, priests, lawyers, soldiers, rentiers, paupers, vagabonds, and criminals, there remained around eight million people. Almost half of these (3.9 million) consisted of 5 very numerous categories: workers in agriculture, textile factories, mining, and metallurgy (mostly productive wage workers) and the servant class (unproductive wage workers). As Table 18 shows, the servant class was
the most numerous of these and amounted to 31% of the total, with marked female predominance. There were more servants than agricultural laborers, or textile workers (almost double), or mining workers (more than double), or metallurgical workers (more than triple).

Table 18

1861 Census of England and Wales

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agricultural workers (1)</td>
<td>1,098.261</td>
<td>28%</td>
</tr>
<tr>
<td>Textile workers (2)</td>
<td>642.607</td>
<td>16%</td>
</tr>
<tr>
<td>Mining workers (3)</td>
<td>565.835</td>
<td>14%</td>
</tr>
<tr>
<td>Metallurgical workers (4)</td>
<td>396.998</td>
<td>10%</td>
</tr>
<tr>
<td>House servants (5)</td>
<td>1,208.648</td>
<td>31%</td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td><strong>3,912.349</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

(1) Including shepherds, farm servants, and maidservants living in the houses of farmers.

(2) Employed in cotton, woolen, worsted, flax, hemp, silk, jute, stocking, and lace factories.

(3) Employed in coal mines and metal mines.

(4) Employed in metal works and metal manufactures.

(5) Includes only those serving in private houses. Of these, only 137,447 are male. In the second edition of Book I Marx adds that in the 1870 census the number of male servants had almost doubled to 267,671.

Appendix to Chapter 15

Bibliographical Note: Leontief (1938) on the empirical richness of Capital

The plethora of empirical data that can be found in Marx’s *opera magna* is often ignored by mainstream economics. Only on occasions has it been recognized by renowned researchers, and such was the case of the Russian émigré Wassily Leontief before he made the adjustments (and omissions) that were necessary to survive as an intellectual in the dark American political environment of the McCarthy period. In 1938, Leontief wrote:

The significance of Marx for modern economic theory is that of an inexhaustible source of direct observation. Much of the present-day theorizing is purely derivative, secondhand theorizing... If before attempting any explanation one wants to learn what profits and wages and capitalist enterprises actually are, he can obtain in the three volumes of Capital more realistic and relevant firsthand information than he could possibly hope to find in ten successive issues of the United States Census, a dozen textbooks on contemporary economic institutions, and even, may I dare to say, the collected essays of Thorstein Veblen (Leontief 1938, 9).

The importance of this judgment lies especially in that, beyond his own contributions to Input-Output theory, Leontief mostly focused on the use of this instrument in the empirical analysis of concrete economies (for which he was awarded the Nobel Prize in Economics in 1973).
Chapter 16  GROUND RENT, SCP, AND THE STATE IN CCP

Ground Rent and Landowners
Marx recurrently stressed that the capitalist mode of production eliminates pre-capitalist modes of production by means of its overwhelming advance based on its ability to produce more and at lower cost. The transformation of feudal agrarian and/or small independent farmer production into capitalist agrarian production was one of the achievements of Capitalism that Marx highlighted. Along with the freedom from the personal bonds of workers, such as those of slavery or serfdom, another of the preconditions for the emergence of Capitalism was the concentration of landowning that prevented its free use by a free peasantry, forcing them to convert to wage work in order to survive. Land property had to be transformed so as to make way for the expansion of the capitalist mode of production in rural areas. Marx explains that when rural capitalist production originated, it had to transform the prevailing forms of property: the feudal form of landowning with the use of serf labor, the clan property of land, and the small-peasant property within mark communes: “But the form of landed property with which the incipient capitalist mode of production is confronted does not suit it. It first creates for itself the form required by subordinating agriculture to capital. It thus transforms feudal landed property, clan property, small-peasant property in mark communes –no matter how divergent their juristic forms may be– into the economic form corresponding to the requirements of this mode of production” (B3, 611). These changes made possible one of the major achievements of the capitalist mode of production, i.e., the transformation of agriculture from a mere empirical and mechanical self-perpetuating process employed by the least developed part of society into the conscious scientific application of agronomy” (Ibid.). A major effect of the modification in the form of land property was that it “completely divorces landed property from the relations of dominion and servitude, on the one hand, and, on the other, totally separates land as a condition of labour from landed property and landowner –for whom the land merely represents a certain money assessment which he collects by virtue of his monopoly from the industrial capitalist, the tenant farmer;... Landed property thus receives its purely economic form by discarding all its former political and social embellishments and associations (B3, 611-2). In summary, the process of capitalist development in agriculture transformed land property to the form most convenient for this mode of production, turning it into a simple source of rent, similar to the interest the financial capitalist obtains for lending his money. And this allowed the extended use of scientific knowledge that resulted in huge increases in the productivity of agricultural production.

Marx dedicated much space to the topic of ground rent in Book III of Capital, Part VI of which contains 11 chapters (almost 200 pages) on the subject. In Theories Marx also studied what many previous economists had written on the subject; in particular, Adam Smith, James Anderson, Thomas Malthus, Thomas Hopkins, Johann Rodbertus, Wilhelm Roscher, and of course Ricardo, who had dedicated four short chapters (2, 3, 24 and 32) of his Principles to this topic. In the Preface to this book, Ricardo writes that in 1815 (two years before the First Edition of his Principles) Thomas Malthus¹ and Edward West “presented to the world, nearly at the same moment, the

¹Malthus had written: “Diversities of soil and situation must necessarily exist in all countries. All land cannot be the most fertile: all situations cannot be the nearest to navigable rivers and markets.
true doctrine of rent; without a knowledge of which, it is impossible to understand the
explains that though West and Malthus had put in writing the theory of ‘differential
rent’ before Ricardo, its real inventor had been James Anderson, a Scottish farmer with
no theoretical pretensions who began to write on the nature of rent in a publication
of 1777 (a year after the publication of Smith’s Wealth of Nations), and of whose
writings Ricardo and West were apparently not aware of (B4.31, 344-7)3. However,
what distinguished Ricardo’s exposition was the way he linked rent with his theory of
value.

Ricardo asks whether the creation of rent as a consequence of the private appro-
priation of land occasions any change in the relative values of commodities. And he
argues that if (presumably previously inhabited) fertile lands that are extremely abun-
dant become inhabited there cannot be any rent since nobody would pay for the use
of land for the same reason that nothing is paid for water or air: its great abundance.
If all lands were the same in quality and unlimited in quantity there would be no rent.
Rent could only come about if with population growth lands of inferior quality or less
favorably placed come under cultivation. The farmer that uses land of inferior quality
must obtain the same rate of profit on his capital as the one that uses the best land.
But the latter would obtain a higher yield with the same costs, originating an excess
profit. If this land is owned by someone else, the latter would not yield it to cultivation
unless he receives as rent an equivalent to the excess profit it generates. Ricardo also
considered that it was “of the utmost importance to the science of political economy”
to understand that “rent does not and cannot enter in the least degree as a component
part” of the price of agricultural commodities. The high price of corn was the cause
of rent and not the effect. If the high price of corn were the effect of the existence of
rent, then rent would be a component of price; but Ricardo denied this to be the case.
It was the corn produced under the least favorable conditions that was the “regulator
of the price of corn” (Ricardo 2004, Vol. I, 77), and rent could not be a component
of its price as wages and profits were. “The reason then, why raw produce rises in
comparative value, is because more labour is employed in the production of the last
portion obtained, and not because a rent is paid to the landlord. The value of corn
is regulated by the quantity of labour bestowed on its production on that quality of
land, or with that portion of capital, which pays no rent” (Ibid. 74). This reasoning
was completely incorrect for Marx. In particular, Ricardo’s analysis implied that, in
Marx’s terminology, there could be no ‘absolute rent’, that is, a rent that was paid
even for the least fertile or most unfavorably placed lands, so only ‘differential rent’
could exist, a rent derived from the differences in the yields of different lands.

But the accumulation of capital beyond the means of employing it on land of the greatest natural
fertility, and the greatest advantage of situation, must necessarily lower profits; while the tendency of
population to increase beyond the means of subsistence must, after a certain time, lower the wages of
labour” (Malthus 1815, 21).

Also: “When capital has accumulated, and labour fallen on the most eligible lands of a country,
other lands less favourably circumstanced with respect to fertility or situation, may be occupied with
advantage. The expenses of cultivation, including profits, having fallen, poorer land, or land more
distant from markets, though yielding at first no rent, may fully repay these expenses, and fully answer
to the cultivator” (Ibid., 23).

2In spite of this statement and the fact that Ricardo felt great affection towards Malthus, he felt
the need to dedicate the last chapter (32) of his Principles to the critique of various aspects of his
theory of rent.

3According to Marx, this was not the case of Malthus, whom he accused of often plagiarizing. In
particular, Marx held that the subsequent polemics between Ricardo and Malthus showed that the
latter had not even understood well the theory he had plagiarized from Anderson.
For Marx, in contrast with Malthus and Ricardo, even if all the land under cultivation were the same in fertility and situation, and hence not generating any ‘differential rent’, there could exist an ‘absolute rent’, that is, a rent exclusively based in the fact that, being land necessary for the production of certain goods demanded, the fact that it was the private property of landowners gave them the possibility of demanding a rent for making it available to the farmer. And the existence of absolute rent affected the equilibrium price of the product, in contrast with differential rent which did not. As we will see, this correct appreciation by Marx was not enough to allow him to formalize clearly the determination of the magnitude of absolute rent and the equilibrium price of commodities whose production required the use of lands that were private property. Marx was conscious of this, but considered that what was most important from his point of view was proving the possibility of the existence of absolute rent. In his letter to Engels of August 9, 1862 he writes: “All I have to prove theoretically is the possibility of absolute rent, without infringing the law of value. This is the point round which the theoretical controversy has revolved from the time of the physiocrats until the present day. Ricardo denies that possibility; I maintain it” (MECW 41, 403). But as we shall see, it is important to understand that what he meant here by the ‘law of value’ included the important distinctions between his values, his ‘production prices’, and his ‘modified production prices’ on account of the existence of absolute rents due to the private property of land and other natural resources.

The assumptions in Marx’s theoretical framework

Marx restricts the scope of his theoretical framework for the analysis of ground rent when he writes: “The analysis of landed property in its various historical forms is beyond the scope of this work. We shall be concerned with it only in so far as a portion of the surplus value produced by capital falls to the share of the landowner. We assume, then, that agriculture is dominated by the capitalist mode of production” (B3, 608). And he defends himself from the possible accusation of ignoring other forms of landed property when he writes that such criticism could only be formulated to those who always choose to ignore the historical character of the object of analysis by taking as eternal the institutions of modern Capitalism.4 It is convenient to remember that in 1858 his planned research had 6 Books, of which ‘Capital’ was the only one he was able to develop (in a preliminary version). The second was ‘Landed Property’.5 It is to be presumed that at least part of the material he planned to include in this second Book was finally allocated to various chapters of Capital as his book advanced and he came gradually to understand that his precarious health would make it impossible for him to develop his extended plan.

For his theoretical construction Marx had to restrict his analysis of rent to the form it had reached in the mature state of the capitalist mode of production, in which agriculture (and livestock, etc.) had become just another of the various branches of industry, albeit with some specific characteristics. Hence, he starts from the premise that though “The actual tillers of the soil are wage labourers”, the industrial capitalist was the tenant farmer that employed them and paid the landowner a certain amount

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4 “Thus, for the purpose of our analysis, the objection that other forms of landed property and of agriculture have existed, or still exist, is quite irrelevant. Such an objection can only apply to those economists who treat the capitalist mode of production in agriculture, and the form of landed property corresponding to it, not as historical but rather as eternal categories” (B3, 609).

5 Cfr. his letters to Engels of April 2, 1858 (MECW 40, 298) and to Weydemeyer of February 1, 1859 (MECW 40, 376). The remaining 4 Books were: Wage Labor, the State, Foreign Trade, and the World Market.
of money established by contract called ground rent “no matter whether it is paid for agricultural land, building lots, mines, fishing grounds, or forests, etc.” “Ground rent, therefore, is here that form in which property in land is realised economically, that is, produces value. Here, then, we have all three classes —wage labourers, industrial capitalists, and landowners— constituting together, and in their mutual opposition, the framework of modern society” (B3, 612).

**Differential rent**

To study ground rent Marx starts from the assumption that on average agricultural (or mining, etc.) prices are sold at their prices of production. As we have seen in Chapter 9, for Marx the average price at which commodities are sold when one assumes away the private property of land is the ‘market price of production’ and, owing to heterogeneity in methods of production, the producers with lower costs have extra (or surplus) profits. When he studies ground rent he gives an example of such heterogeneity. He assumes that in a certain branch of production most producers use steam engines, which implies a cost of production of 100, whereas a very small minority use energy produced by natural waterfalls, where the cost of production is 90. Since the price of production in that branch is 115, the profit rate for the vast majority of producers (who use steam engines) is 15% whereas it is 27.8% for those who use waterfalls and consequently have an extra profit of 12.8 percentage points due to their lower costs. The individual price of production for the advantaged sector is only 103.5, that is, the individual cost (90) plus the general profit rate of 15%. Since he assumes there is a minute influence of the advantaged sector in the formation of the market production price, the product sells for 115 and the profit for this sector is 25 (27.8% of 90). As Marx writes, “thus far, the surplus profit of the manufacturer using natural water power instead of steam does not differ in any way from any other surplus profit.” And at this point he makes the assumption that such waterfalls are the ‘private property of landowners, whose extra profits turn into ‘ground rent’, that is, a rent paid to the owners on account of the waterfall being situated within their land:

Now let us assume that the waterfalls, along with the land to which they belong, are held by individuals who are regarded as owners of these portions of the earth, i.e., who are landowners. These owners... can permit or forbid such utilisation. But a waterfall cannot be created by capital out of itself. Therefore, the surplus profit which arises from the employment of this waterfall is not due to capital, but to the utilisation of a natural force which can be monopolised, and has been monopolised, by capital. Under these circumstances, the surplus profit is transformed into ground rent, that is, it falls into possession of the owner of a waterfall (B3, 639).

The case just considered involved the use of two different methods of production, as we considered in Chapter 9. In the following it is assumed that the same methods of production are used in the production of all lands but that the lands differ in certain features that generate different costs per unit of output. Although in the case of differential rent Marx starts from Ricardo’s condensed exercises, he advances in the clarification of some issues. As Malthus and Ricardo, Marx holds that there are two general causes that result in different outputs from capital invested in different lands: their different natural fertilities and their different locations. To simplify, he begins by assuming that the only difference is in natural fertility. “Aside from climatic factors, etc., the difference in natural fertility depends on the difference in the chemical
composition of the top soil, that is, on its different plant nutrition content” (B3, 644). To illustrate he uses numerical tables, the first of which is the following.\(^6\)

<table>
<thead>
<tr>
<th>TS</th>
<th>Acres</th>
<th>PPpQ</th>
<th>Prod.</th>
<th>Cap.</th>
<th>Profit</th>
<th>Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sh.</td>
<td>Sh.</td>
<td>Sh.</td>
<td>Sh.</td>
<td>Q.</td>
<td>Q.</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>60</td>
<td>1</td>
<td>60</td>
<td>50</td>
<td>1/6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 1/6</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 1/6</td>
<td>130</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3 1/6</td>
<td>190</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>600</td>
<td>6</td>
<td>360</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TS: Type of soil; Q.: Quarters; PPpQ: Production Price per Q.
Prod: Product (grain); Cap.: Capital; Sh.: Shillings;

This table is related to a simpler one by Ricardo. But Marx extends it to distinguish the quantities from the amounts of money involved. The fertilities are in increasing order, the price per Quarter of grain is 60 shillings and the profit rate is 20%. Assuming that there is only one acre for each type of land, that only land of types A and B are available and furthermore, that demand (and output) of grain is 3 Quarters, then the two acres of lands A and B can be in cultivation. The land of least fertility (A) does not generate any (differential) rent, since its product of 1 Quarter of grain is sold at the price of production, which consists of the cost (and disbursement) of 50 plus a profit of 10, that is, 60 shillings. In contrast, in land B with the same cost of 50 twice the amount of output is produced (2 Quarters). Since this quantity has a value of 120 shillings, the profit it generates is 70 shillings, of which 10 is the normal profit (of 20%) and 60 is an extra profit that becomes (differential) rent because the landowners would not lease them for a lower amount (assuming they know what the productivities of the different available lands are). The tenant farmer of land B earns a profit of 10, the same as the tenant farmer of land A, and hence they both earn the same profit rate. If instead the demand and production are 6 Quarters, lands of type A, B, and C can be cultivated. The farmers that use land B would have to pay 60 sh. in rent as before, and the farmers of land C would have to pay 120 sh. in rent to the owners of these lands. In the two cases considered the least fertile land (A) pays no rent.

Marx criticizes the opinion of Ricardo and others that differential rents arise only with the transition towards lands increasingly less fertile. For the cultivation of lands of different fertility can actually come about in any order:

The existence of a differential rent and of a graduated differential rent can develop equally well in a descending sequence, which proceeds from better to worse soils, as in an ascending one, which progresses in the opposite direction from worse to better soils; or it may be brought about in checkered fashion by alternating movements...

This takes care of the first false assumption regarding differential rent – still found among West, Malthus, and Ricardo– namely, that it necessarily presupposes a movement toward worse and worse soil, or an ever-decreasing fertility of the soil. It can be formed, as we have seen, with a movement

\(^6\)This table comprises the two different versions of Table I of Chapter 39, Book III.
toward better and better soil; it can be formed when a better soil takes
the lowest position that was formerly occupied by the worst soil; it can be
connected with a progressive improvement in agriculture. The precondition
is merely the inequality of different kinds of soil (B3, 652-3).

Marx considers various examples of the type showed above. Although the latter
assumed a constant price of output (grain), in others he assumes that the price increases
or decreases, which leads him to state that “Depending on its mode of formation,
differential rent may develop along with a stationary, rising or falling price of the
products of the land” (Ibid.). He also writes that though the analysis has been limited
to *fertilities*, a similar analysis can be done for the more or less favorable *locations* of
the different lands. Moreover, the transitions between lands of various fertilities and
locations can be very complex and he gives another sample of the empirical richness of
his analyses by considering the concrete case of different states of the U.S.A.:

Poor soil may be preferred to a relatively better soil on account of location,
which is of decisive importance for every extension of cultivation in young
countries... If inferior soil is surrounded by superior soil, then the latter
gives it the advantage of location in comparison with more fertile soil which
is not yet, or is about to become, part of the cultivated area.

Thus, the State of Michigan was one of the first Western States to become
an exporter of grain. Yet its soil on the whole is poor. But its proximity
to the State of New York and its water-ways via the Lakes and Erie Canal
initially gave it the advantage over the States endowed by Nature with more
fertile soil, but situated farther to the West (B3, 662).

Marx calls the cases so far considered First Form of differential rent, which deal
with the successive application of equal amounts of capital to *different kinds of land*
with varying results. And he calls Second Form of differential rent those cases in which
equal amounts of capital are applied to *land of the same kind* with varying results.
In the edition prepared by Engels of Book III after Marx’s death the analyses of the
Second Form of differential rent occupy Chapters 40 to 44. For our purposes it is
unnecessary to go into this in detail.

**Absolute rent**

Marx points out that although there are “cases in a country with capitalist production,
where the investment of capital in the land can take place without payment of rent”
(B3, 737), they are all cases in which the ‘limitation’ to investment of capital on account
of the private property of land disappears. Such would be the case, for example, of the
farmer who owns the land he cultivates, reason for which he earns an extra profit that
does not become a ‘ground rent’ that must be paid out to a third party. Another case
is that of countries with great abundance of lands, such as the prairies of the U.S.A.,
where “land costs the cultivator nothing, or next to nothing as compared with older
countries” (B3, 664).

Marx’s main criticism of the classical theory of rent is that it does not consider
possible the existence of rent on the least fertile or worst located lands, that is, in
‘marginal’ lands. Although when analyzing differential rent he had made “the assump-
tion that the worst soil does not pay any ground rent”, this was simply to simplify the
analysis, but in reality “landed property as a limitation continues to exist even when
rent in the form of differential rent disappears, i.e., on soil A” (B3, 737). Marx states
that the fact that even for the worst land the payment of rent may be necessary does not in the least invalidate the analysis of differential rent since, for each quality of land, the absolute rent (which is the rent on the land of lowest quality) must simply be added to the differential rent to reach the total rent to be paid to the landowner. And he stresses that the ‘regulating market price’ is no longer the price of production. If the worst land (of class A) must pay a rent \( R^7 \): “The regulating market price of the total output on the market derived from all soils would then not be the price of production which capital generally yields in all spheres of production, i.e., a price equal to costs plus average profit, but rather the price of production plus the rent, \( P + R \), and not \( P \)” (B3, 735).

Agrarian capitalists had to pay landowners an absolute rent because they had the power of withholding the land from being used in agriculture unless they were paid a sufficiently high rent. The expression that Marx, like the other classical economists, used for this power of withholding was that the landowners had the ‘monopoly’ of the lands. Hence, when absolute rent existed it was said that the agricultural output was sold at a ‘monopoly price’ which exceeded the price of production: “When we refer to a monopoly price, we mean in general a price determined only by the purchasers’ eagerness to buy and ability to pay, independent of the price determined by the general price of production, as well as by the value of the products” (B3, 762). The ‘monopoly price’ was the ‘regulating market price’, that is \( P + R \), which is greater than the price of production \( P \).

Marx makes a classification of absolute rent that is related to his double accounting in values and in equilibrium prices and to his theory of prices of production according to which the commodities produced in branches with lower value composition of capital than average tend to have prices of production that are lower than their values (because they use more variable capital in relation to constant capital and hence produce more surplus value than the average, a part of which has to be ‘socialized’ by means of the formation of the price of production). For Marx it was an empirical fact that agrarian production tended to have a lower value composition than the rest of the industrial branches and, hence, than the average of industrial branches. This implied that, given two equal capitals, the variable capital in agriculture would be higher than in other industrial branches. Assuming the same average rate of surplus value in both sectors, more surplus value would be produced in agriculture per unit of capital. And since the rate of profit had to level out through the circulation of produced commodities and the flow of capitals between branches, if we disregard the effect of absolute rent on profits, the profits in non-agricultural industrial branches have to be partially composed of surplus value generated in agriculture. And, vice versa, the profits of agrarian capitalists had to be lower than the surplus value generated in their firms. Therefore, the prices of production of agrarian products tended to be lower than their values, that is, \( P < V \). But in agriculture the equilibrium price (or ‘regulating market price’) was not the price of production. Since the existence of a ‘monopoly price’ generated absolute rent \( R > 0 \), the ‘regulating market price’ was \( P + R \). But \( P + R \) could in general be less than, equal, or greater than the value \( V \) of agrarian output. That is why Marx distinguished the case in which \( P + R \leq V \) (which he also subdivided in \( P + R = V \) and \( P + R < V \)) from the case in which \( P + R > V \). He writes

... absolute rent presupposes either realised excess in product value above its price of production, or a monopoly price exceeding the value of the

\(^7\)We here replace the lower case letter \( r \) Marx uses with the upper case \( R \) to avoid confusion with the \( r \) we use in the model below to represent rent per acre.
product (B3, 791).

However, since the value of the commodities produced by agricultural capital is higher than their price of production, according to our assumption, this rent (save for one case which we shall discuss forthwith) forms the excess of value over the price of production, or a part of it. Whether the rent equals the entire difference between the value and price of production, or only a greater or lesser part of it, will depend wholly on the relation between supply and demand and on the area of land newly taken under cultivation (B3, 749).

The exception was precisely the case in which \( P + R > V \). Marx called ‘normal forms’ of rent the differential rent and the first of the two forms of absolute rent \((P + R \leq V)\). And he called ‘actual monopoly price’ the second form of absolute rent, in which “the rent can be based only upon an actual monopoly price, which is determined neither by price of production nor by value of commodities, but by the buyers’ needs and ability to pay. Its analysis belongs under the theory of competition, where the actual movement of market prices is considered” (B3, 751). The first of these forms of absolute rent would disappear, he states, “If the average composition of agricultural capital were equal to, or higher than, that of the average social capital” (B3, 752). Although there is no intrinsic problem with the classification Marx makes based on his dual accounting, taking \( V \) as the limit for calling the ‘regulating market price’ an ‘actual monopoly price’ doesn’t seem to makes sense.

Marx was conscious that he had not reached a complete integration of his theory of ground rent with his theory of equilibrium prices. Hence, he refers to a “theory of competition, where the actual movement of market prices is considered.” He was not able to develop such a theory, a theory capable of integrating absolute rent with the complete set of ‘regulating’ (or equilibrium, or ‘attracting’) market prices. But he pointed in the right direction when he recognized that the prices of production would no longer be the equilibrium market prices. He was also right in introducing (in the quotation above) the “relation between supply and demand” and the “area of land newly taken under cultivation” in the determination of equilibrium prices, even if he restricted it to only two of the three cases he considered.

Marx could have delineated a general framework analogous to the one he used when addressing the interest as the price for the use of money capital and the determination of that price by the equality of supply and demand. But he didn’t. In that case, the ‘functioning’ capitalists had a demand for money capital and the financial capitalists had a supply and the interest was determined by the balance between supply and demand. Similarly, in the present context he could have assumed farmers had a demand for land and landowners a supply, and that the rent was determined by the balance between supply and demand. This is precisely what we do in the model below, which is an easy procedure that would have been within Marx’s reach and could have allowed him to formulate a more complete theory of absolute rent. As we will see in Chapter 20 (in Part III), a few years after Marx wrote Book III, Léon Walras successfully integrated ground rent into his theory of value when he formulated his theory of ‘general equilibrium’. For this he represented mathematically the subjective decision process of individuals and constructed a theory of equilibrium prices under conditions of ‘perfectly free competition’. Using mathematical methods that Marx (and almost all economists of his time) lacked, Walras was able to make a leap in the integration of several of the theoretical branches that were present in Marx’s work, some of which may be regarded as a culmination of Classical political economy. In particular, he was able to integrate
the supply of natural resources by their private owners as a balance between their enjoyment if they retain them for self-use and the enjoyment of the goods and services purchased with the rent obtained if they are leased out. But certainly Marx would have objected, and rightly so, to some of the fundamental aspects of Walras’ theory, such as the absence of a historical-genetic framework, restricting the theoretical framework to states of equilibrium, and the notion that capitalist firms have neither profits nor losses in equilibrium.

**Ground rent in the matrix model**

**Absolute rent** We assume that there is only one kind land, measured in acres. In order to include landowners and ground rent in the models used above it is sufficient to add a row vector for the consumption basket \( c_T \) of landowners and a column vector with the technical land requirements per unit of output \( t \) in the various branches of production (where for many branches \( j \) we may have \( t_j = 0 \)). We assume that each landowner owns a certain amount of land, which gives him the right to retain it from productive use if he does not receive a satisfactory rent. Hence, let the amount of land available (or supplied) to capitalist farmers \( q^T \) be an increasing function of rent per acre \( r \), that is, \( q^T(r) \). On the other hand, let \( q^T \) represent the amount of lands that are leased (as well as the number of landowners if we assume that they all own the same quantity of land), which is dependent on the structure of output, according to the type of modeling we have been doing. We assume that the consumption basket of landowners has a certain structure \( c_T \) but its level depends on the equilibrium rent that results from the balance between the supply and the demand of lands to be used. Hence, let \( c_T = \xi \hat{c}_T \) be the consumption basket of a landowner, where \( \xi \) is an endogenous positive scalar. The dual systems of quantities and prices are the following:

\[
\begin{bmatrix}
q^Q & q^L & q^K & q^T
\end{bmatrix}
\begin{bmatrix}
A & \ell & \eta & t \\
c_L & 0 & 0 & 0 \\
c_K & 0 & 0 & 0 \\
\xi \hat{c}_T & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K & q^T
\end{bmatrix}.
\]

With these systems we can represent Marx’s definitions of ‘gross output or gross product’ (in quantities and in values), and ‘gross income’:

In order to avoid unnecessary difficulty, one should distinguish gross output and net output from gross income and net income.

The gross output, or gross product, is the total reproduced product. With the exception of the employed but not consumed portion of fixed capital, the value of the gross output, or gross product, equals the value of capital advanced and consumed in production, that is, constant and variable capital plus surplus value, which resolves itself into profit and rent. Or, if we consider the product of the total social capital instead of that of an...
individual capital, the gross output equals the material elements forming the constant and variable capital, plus the material elements of the surplus product in which profit and rent are represented.

The gross income is that portion of value and that portion of the gross product measured by it which remains after deducting that portion of value and that portion of the product of total production measured by it which replaces the constant capital advanced and consumed in production. The gross income, then, is equal to wages (or the portion of the product destined to again become the income of the labourer) + profit + rent. The net income, on the other hand, is the surplus value, and thus the surplus product, which remains after deducting wages, and which, in fact, thus represents the surplus value realised by capital and to be divided with the landlord, and the surplus product measured by it (B3, 826-7).

Hence, aggregate “gross output or gross product” corresponds exactly to the first equation of (16.1): \( q^Q = q^Q A + q^L c_L + q^K c_K + q^T \xi_c T \) (“the material elements forming the constant and variable capital, plus the material elements of the surplus product in which profit and rent are represented”). And the “value of the gross output” corresponds to the decomposition obtained by multiplying the preceding equation by \( p \) and using the rest of the equalities in (16.2): \( q^Q p = q^Q A p + q^L w + q^K \pi + q^T r \) (“the value of capital advanced and consumed in production, that is, constant and variable capital plus surplus value, which resolves itself into profit and rent”). Also, the aggregate “gross income” corresponds to: \( q^Q (I - A) p = q^L w + q^K \pi + q^T r \) (“wages (…) + profit + rent”), whereas aggregate “net income” corresponds to \( q^K \pi + q^T r \) (“the surplus value realised by capital and to be divided with the landlord”).

Assuming that the capitalist farmer pays the rent at the end of the period, it does not imply a disbursement of capital. In that case, the equalization of profit rates on disbursed capital yields the following (reduced) system of prices and incomes:

\[
\begin{bmatrix}
(1 + \rho) A & (1 + \rho) \ell & t \\
\xi_c T & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
r
\end{bmatrix}
= 
\begin{bmatrix}
p \\
w \\
r
\end{bmatrix}.
\]

(16.3)

The following table summarizes the equations of the two dual systems in the first four rows. A5 additionally has the equilibrium condition between available land (supply) for productive use and its demand \( (q^T)^{10} \) and B5 has the first equality of (16.3), that is, the condition that the (equilibrium) prices of commodities are the prices of production (which include costs of production and profits) plus the rents paid (at the end of the period):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( q^L c_L + q^K c_K + q^T \xi_c T ) ( B(0) = q^Q )</td>
<td>( \eta + tr = p - (Ap + \ell w) )</td>
</tr>
<tr>
<td>2</td>
<td>( q^Q \ell = q^L )</td>
<td>( c_{LP} = w )</td>
</tr>
<tr>
<td>3</td>
<td>( q^Q \eta = q^K )</td>
<td>( c_{KP} = 1 )</td>
</tr>
<tr>
<td>4</td>
<td>( q^Q t = q^T )</td>
<td>( \xi_{cT} p = r )</td>
</tr>
<tr>
<td>5</td>
<td>( \widetilde{q}^T(r) = q^T )</td>
<td>( (1 + \rho) (Ap + \ell w) + tr = p )</td>
</tr>
</tbody>
</table>

\(^9\)It is very easy to change what follows by making the alternative assumption that all the rent is advanced.

\(^{10}\)Here, as always, \( B(0) \equiv (I - A)^{-1}.\)
where we used $c_K$ as numeraire. From $B1$ and $B5$ we get \( \eta = \rho (Ap + \ell w) \), and premultiplying by $q^Q$ yields the usual expression for the profit rate for SR:

\[
\rho = \frac{q^K}{K} = \frac{q^K c_K p}{q^Q Ap + q^L c_L p} = \frac{e^p}{k^p + 1}.
\]  

(16.4)

If we multiply $A1$ by $t$ and use $A4$, after rearranging we get an expression that shows that the global demand for land is increasing with the number of basic consumption baskets $\xi$ that landowners can consume: $q^T = (q^L c_L + q^K c_K) B(0) t / (1 - \xi \tilde{c}_T B(0) t)$. Hence, using $A5$ yields

\[
\xi = \frac{1}{\tilde{c}_T B(0) t} \left( 1 - \frac{(q^L c_L + q^K c_K) B(0) t}{\tilde{q}^T (r)} \right) \equiv \xi (r).
\]  

(16.5)

Hence, the level of landowner consumption $\xi$ is a function (which we call $\xi (r)$) of the rent per acre of land $r$; and is increasing because $\tilde{q}^T (r)$ is.

Let us now consider the prices of commodities, the wage rate and the rent on land. $B5$ and $B4$ yield an expression for the vector of prices:

\[
p = B (\rho, \xi) \ell w,
\]  

(16.6)

where we have defined the matrix $B (\rho, \xi) \equiv (1 + \rho) (I - [(1 + \rho) A + \xi \tilde{c}_T])^{-1}$. Using (16.6) in $B2$ gives (after eliminating $w$ and introducing (16.5)) yields $c_L B (\rho, \xi (r)) \ell = 1$. Since $B (\rho, \xi)$ is increasing with respect to its two arguments\(^{11}\) and $\xi$ is increasing with $r$, we have an inverse relation between the profit rate $\rho$ and the rent on land (per acre) $r$ that we can call $\rho (r)$. Hence, the first equality of (16.4) shows that the necessary productive capital $K = q^K / \rho (r)$ is increasing with the rent on land $r$.

Also, using (16.6) in $B2$ we get the real wage as a complicated function of $r$: $w = 1 / [c_K B (\rho (r), \xi (r)) \ell] \equiv w (r)$. Hence, we also get $p = B (\rho (r), \xi (r)) \ell w (r) \equiv p (r)$. And using the functions obtained in $B4$ yields $\xi (r) \tilde{c}_TP (r) = r$, an equality from which we can (at least numerically) determine the equilibrium rent per acre $r^*$.\(^{12}\) Given $r^*$ we can go back to successively get the equilibrium values of the remaining endogenous variables: $\rho^*, \xi^*, w^*, p^*, \tilde{q}^T*, q^Q*, K^*, \eta^*$.

Notice that in Marx’s terminology $p$ would the the vector of ‘regulating market prices’. From $B5$ we can also get:

\[
p = B (\rho) \ell w + \frac{B (\rho) tr}{1 + \rho},
\]

where $B (\rho)$ is defined in (8.21). Using the equilibrium values already obtained in this expression, $B (\rho^*) \ell w^*$ is Marx’s vector of ‘prices of production’. But the vector of equilibrium prices is the sum of this and the vector $[B (\rho^*) / (1 + \rho^*)] tr^*$ representing the part of each equilibrium price reflecting the rent cost corresponding to direct and indirect land requirements (discounted to the beginning of the period by $\rho^*$ since the rent is paid at the end of the period). We thus have an analytical representation of Marx’s ‘regulating market price’, which in the quotations above appeared as $P + R$, where $P$ was the price of production and $R$ an additional positive term for absolute rent. It is also Marx’s ‘monopoly price’, since the second term in the expression for $p$ above only exists because a class of the population ‘monopolizes’ land, that is, is not willing to let producers use it without receiving a satisfactory rent.

\(^{11}\)To see this it is sufficient to expand in series the inverse matrix as we have often done.

\(^{12}\)We here assume that the equation has a solution, and that it is unique.
Differential and absolute rent in a concrete case  Obviously, the above approach could be done for more than one kind of land, whether they differ in fertilities or locations. Different kinds of lands could have different rents for the same reason that they are different. As in the case of absolute rent when there is only one kind of land, we could call the rent paid for the land that (in equilibrium) has the lowest rent per acre ‘absolute rent’ and call the excess of the rent in any of the other lands with respect to the absolute rent ‘differential rent’ and attribute that differential to the various factors that make them command a higher rent than the ‘marginal’ land that only pays absolute rent. Hence, for all the lands used farmers would have to pay the absolute rent to the landowner and for all lands except one (the ‘marginal’ land) they would additionally have to pay the landowner a differential rent.

To avoid complicating the algebra, let us pose an example that transmits the concept of differential rent of the Classical economists and Marx. Assume that in the branch of agriculture there is only one crop, say wheat, and that there are two kinds of lands, one of which is naturally more fertile than the other. Assume also that the rest of the industrial branches do not use land as an input. The coefficients of the first rows of the social matrix are hence the following:

\[
\begin{array}{cccccc}
A_{11} & A_{12} & A_{13} & \ldots & \ell_1 & t_1 & 0 \\
A_{21} & A_{22} & A_{23} & \ldots & \ell_2 & 0 & t_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
\end{array}
\]

The first two rows correspond to the production of wheat. But in the first process wheat is produced using the most fertile land whereas in the second the least fertile land is used. Since \( t_i \) measures the number of acres per bushels of wheat produced when land \( i \) is used, \( 1/t_i \) is the number of bushels of wheat produced per acre on this land, i.e., it is a measure of the productivity of land \( i \). Under our assumption that land 1 is more fertile than land 2, we have \( t_1 < t_2 \). Let \( r_i \) be the rent per acre of land \( i \). Let us focus on the first two production processes, both of which produce wheat. In Marx’s terminology, the price of wheat is the ‘individual price of production’ plus the rent per bushel of output:

\[
\begin{align*}
p_{11} &= (1 + \rho) \left(A_1 p + \ell_1 w\right) + t_1 r_1 \\
p_{12} &= (1 + \rho) \left(A_2 p + \ell_2 w\right) + t_2 r_2,
\end{align*}
\]

where \( A_i \) is the \( i \)-th row of \( A \). But competition implies that there must be only one market price \( p^o_i \) for wheat (as well as the equalization of profit rates). Hence, competition leads to:

\[
\begin{align*}
p^o_1 &= (1 + \rho) \left(A_1 p^* + \ell_1 w\right) + t_1 r_1 \\
p^o_1 &= (1 + \rho) \left(A_2 p^* + \ell_2 w\right) + t_2 r_2,
\end{align*}
\]

where \( p^* = (p^*_1, p^*_2, \ldots)^T \) is the vector of equilibrium prices, which implies

\[
(1 + \rho) \left[(A_2 p^* + \ell_2 w) - (A_1 p^* + \ell_1 w)\right] = t_1 r_1 - t_2 r_2.
\]

Hence, if the unit cost when land 1 is used \((1 + \rho) \left(A_1 p^* + \ell_1 w\right)\) is less than when land 2 is used \((1 + \rho) \left(A_2 p^* + \ell_2 w\right)\), the rent per bushel of wheat must be greater on land 1: \( t_1 r_1 > t_2 r_2 \).

To illustrate, let us consider the special case in which the same production technique is used in any of the two kinds of land. Then \((A_1 \ell_1)\) and \((A_2 \ell_2)\) are proportional.
If land 1, for example, is twice as productive as land 2, then $1/t_1 = 2(1/t_2)$; hence the direct requirement for this kind of land is half the one for land 2: $t_1 = t_2/2$. And since each acre of land 1 yields twice as much wheat, the input requirement per bushel of wheat of each input is one half of what it is in land 2: $(A_1 \ell_1) = (1/2)(A_2 \ell_2)$. Introducing this in (16.7) we have $(1 + \rho)(A_1 p^* + \ell_1 w^*) = t_1 r_1 - t_2 r_2$, and hence $p^*_1 = 2t_1 r_1 - t_2 r_2 = t_2 (r_1 - r_2)$. Since $t_2 > 0$, for the equilibrium price to be positive it is necessary that the rent per acre in the most fertile land be higher: $r_1 > r_2$. In this special case $r_2$ is the absolute rent that the owners of both kinds of land receive, and $r_1 - r_2 = p^*_1/t_2$ is the differential rent that only the owners of the most fertile land receive.

Obviously, this approach can be generalized in different ways: more than two kinds of land, more than one product that requires the use of lands, different production techniques for the production of the same product in different kinds of land, etc. But Marx’s central idea is reflected in this simple exercise. It also illustrates Marx’s idea that when land is the private property of a landowner “the surplus profit is transformed into ground rent.” For, if there was free access to land there would be no rent payment and producers of wheat who have the fortune of being able to use the more productive lands would produce more per acre, and hence have lower costs per bushel of wheat produced. Since competition would nevertheless lead to a unique ‘market price’ for each commodity, including wheat, then we would have:

$$
\begin{align*}
p^*_1 &= (1 + \rho_1)(A_1 p^* + \ell_1 w^*) \\
p^*_2 &= (1 + \rho_2)(A_2 p^* + \ell_2 w^*)
\end{align*}
$$

where $p^* = (p^*_1, p^*_2, p^*_3,...)^T$. And the extra profit per bushel produced of the producers that use the most fertile land would be $[\rho_1 (A_1 p^* + \ell_1 w^*)] - [\rho_2 (A_2 p^* + \ell_2 w^*)]$.

The price of land and the institution of private land ownership Marx took into account the personal activity of the capitalist entrepreneur in his role as ‘orchestra director’ or in vigilance and control. But in his conception, such activities were either only necessary in a regime of exploitation based in the appropriation of ‘unpaid labor’, or, if they were necessary in general, they could be realized by workers that were not capitalist entrepreneurs. And for Marx the case of ground rent showed more clearly that those of industrial or commercial profit that its magnitude did not depend on the personal activity of the recipient: “It is precisely in the economic realisation of landed property, in the development of ground rent, that the following characteristic peculiarity comes to the fore, namely that its amount is by no means determined by the actions of its recipient, but rather by the independent development of social labour in which the recipient takes no part. It may easily happen, therefore, that something is regarded as a peculiarity of rent (and of the product of agriculture in general), which is really a common feature of all branches of production and all their products where the basis is commodity production –and, in particular, capitalist production” (B3, 630). And ground rent was akin to the interest on money loaned in showing transparently that surplus value could be received without the personal effort of the recipient. In the early stages of Capitalism “landed property was still regarded by popular conception as the pristine and respectable form of private property, while interest on capital was decried as usury” (B3, 616). But with the development of Capitalism, ground rent and the interest on money capital were transformed into supplementary forms of Capitalist Commodity Production.

On the other hand, Marx called the ‘capitalization’ of a periodical income ‘imaginary capital’. “Ground rent assumes the form of a certain sum of money, which the
landlord draws annually by leasing a certain plot on our planet. We have seen that every particular sum of money may be capitalised, that is, considered as the interest on an imaginary capital. For instance, if the average rate of interest is 5%, then an annual ground rent of £200 may be regarded as interest on a capital of £4,000. Ground rent so capitalised constitutes the purchase price or value of the land.” Hence, “the price of land may rise or fall inversely as the interest rate rises or falls if we assume ground rent to be a constant magnitude” (B3, 616-7). Marx pointed out that this fact was taken as a justification for the institution of private land ownership by some of its apologists, “since the buyer pays an equivalent for it, the same as for other commodities.” But he believed this to be an invalid justification for this regime arguing that “The same reason in that case would also serve to justify slavery, since the returns from the labour of the slave, whom the slave-holder has bought, merely represent the interest on the capital invested in this purchase” (Ibid.). “But the title itself is simply transferred, and not created by the sale. The title must exist before it can be sold, and a series of sales can no more create this title through continued repetition than a single sale can. What created it in the first place were the production relations” (B3, 762). That is, economic institutions arise “from the process which creates social life” as an answer to social needs related to economic production, and are as transitory and fluid as that life creating social process. When the production relations that are the basis of certain institutions like slavery or private land ownership change, “the material source of the title, justified economically and historically and arising from the process which creates social life, falls by the wayside, along with all transactions based upon it. From the standpoint of a higher economic form of society, private ownership of the globe by single individuals will appear quite as absurd as private ownership of one man by another” (B3, 763). Regardless of what one may think about Marx’s preference for the collective ownership of natural resources, he was certainly right in highlighting the fact that economic institutions change over time. At present in many countries many of the natural resources (especially those underground) are state property. And in particular, in both the most populated country in the world (China) and the most populated country in Africa (Nigeria) land is the exclusive property of the state and can be used productively by private entities by means of long run leases.

Simple Commodity Production in Capitalism

Marx has been criticized ad nauseam by all types of analysts for having supposedly constructed a two class model of society. 13 But anyone who reads Capital can acknowledge how erroneous such critiques are. Along his analyses, Marx takes into account many social classes, sub-classes or social strata, more than we would care to enumerate. And faithful to his ‘dialectical method’, when he builds his main model of the pure capitalist mode of production, he concentrates in the main classes and subclasses that participate in the production and circulation of commodities. However,

13 For example, Morishima (1977, 9) writes: “Walras had a four-class view of society (which I take as more advanced than Marx’s two-class view)...”. It is remarkable that he could have written this after publishing a whole book on “Marx’s Economics” (Morishima 1973).

Even Schumpeter, in whose History of Economic Analysis ‘Marx’ is mentioned 917 times (and is the most mentioned economist – followed by ‘Ricardo’ with 779 and Marshall with 668) and whose thought obsessed him, writes: “Marx, recognizing two classes only, saw class ‘struggle,’ economic and political, only between these two...” (Schumpeter 1954, 528). And just a few lines later he writes: “Marx, as we also know, substituted his two-class schema for this tripartite division of types” (Ibid., 529), referring to the addition of landowners. It should be obvious that building a model of two or three classes does not imply denying the existence of more classes in any concrete society. This is much too evident in Marx’s works to even be worth mentioning were it not for absurd statements such as these from prestigious intellectuals.
the socioeconomic structure that Marx reflects in his analyses is much more detailed, and his observations on the dynamics of the transformation of pre-capitalist modes of production into capitalist ones by the overwhelming force of capital is always present in his work (as it is always absent in mainstream economics). The following paragraph, for example, mentions four different modes of production, three of which (slavery, guild production, independent peasant production) gradually tend to be transformed into the fourth, the capitalist mode of production:

When the peasant who previously produced independently for himself becomes a day labourer working for a farmer; when the hierarchical structure valid for the mode of production of the guild type disappears, to be replaced by the simple antithesis between the capitalist and the handicraftsman who is set to work for him as a wage labourer; when the man who was previously a slaveholder employs his former slaves as wage labourers, etc., production processes with a different social determination are thereby converted into the production process of capital. With this, there occur the changes discussed earlier. The previously independent peasant becomes, as a factor in the production process, dependent on the capitalist, who directs that process; his very employment depends on a contract he has concluded in advance as a commodity owner (an owner of labour power) with the capitalist as a money owner. The slave ceases to be an instrument of production belonging to the employer of that instrument. The relation between master and journeyman vanishes. The master stood towards the journeyman in the relation of master of the craft. Now he relates to him merely as the owner of capital, just as the journeyman now only confronts the master as a seller of labour. Before the production process they all confront each other as owners of commodities, having only a monetary relation in common; within the production process, [they confront each other] as personified agents of the factors of that process. The capitalist functions as ‘capital’, the direct producer as ‘labour’, and their relation is determined by labour, as a mere factor in self-valorising capital (Results, MECW 34, 424-5; the text within square brackets is added for clarity).

When he is not concentrating his analysis in a particular simple model, Marx always takes into account the diversity of the modes of production existing in any concrete society. For example, when he analyses the cyclical process of capital in Book II, he shows how capitalist industrial production enters into contact, through transactions in the world market, with commodities produced by various non-capitalist modes of production:

Within its process of circulation, in which industrial capital functions either as money or as commodities, the circuit of industrial capital, whether as money capital or as commodity capital, crosses the commodity circulation of the most diverse modes of social production, so far as they produce commodities. No matter whether commodities are the output of production based on slavery, of peasants (Chinese, Indian ryots), of communes (Dutch East Indies), of state enterprise (such as existed in former epochs of Russian history on the basis of serfdom) or of half-savage hunting tribes, etc. – as commodities and money they come face to face with the money and commodities in which the industrial capital presents itself... They function as commodities in the market, and as commodities they enter into the
circuit of industrial capital as well as into the circulation of the surplus value incorporated in it. It is therefore the universal character of the origin of the commodities, the existence of the market as world market, which distinguishes the process of circulation of industrial capital (B2, 115).

We have here a glimpse of a worldwide view of the process of gradual capitalist encroachment on the most diverse modes of pre-capitalist modes of production. And central to his theory is a feature that concerns the dynamics of the complex intermingling of heterogeneous production relations that contribute to the production of commodities for their sale in the world market: the transforming role of Capitalism, which effectively tends to convert to its mode of production the productive processes based on production relations that are destined to disappear due to the great competitive advantage that capitalist production achieves through cost-reducing methods of production.

Throughout his works, Marx shows the complexity of his vision of the articulation of classes, sub-classes, and social strata in multiple concrete societies, not only in the capitalist era (and different stages of that era) but also in pre-capitalist eras as in Antiquity and the Middle Ages. As we have seen, in order to construct his models of capitalist production and circulation Marx first focuses on Simple Commodity Production (which is non-capitalist). Taken individually, the models of SCP and CCP are simple models constructed so as to focus on certain crucial aspects of a much more complex reality. But in his works Marx gives ample evidence that, in his vision of his contemporary world, even in countries in which the capitalist mode of production was already predominant, such as Great Britain, it coexisted with pre-capitalist modes of production and, in particular, with Simple Commodity Production. This is reflected in the following quotes in which he addresses the production of craftsmen or independent peasant farmers in a country in which CCP predominates:

But what is the situation with independent handicraftsmen or with peasants who do not employ any workers, hence do not produce as capitalists?... In this relation they meet me as sellers of commodities, not as sellers of labour, and this relation therefore has nothing to do with the exchange between capital and labour... They therefore belong neither to the category of productive workers nor to that of unproductive workers, although they are producers of commodities. Their production is not subsumed under the capitalist mode of production. These producers, who work with their own means of production, may not only reproduce their labour capacity, but also create surplus value, in that their position allows them to appropriate their own surplus labour, or a part of it (for a part is taken away from them in the form of taxes, etc.). And here we meet with a peculiarity characteristic of a society in which a determinate mode of production predominates, although all relations of production have not yet been subjected to it. (B4.34, 141).

Then there is also the law that economic development divides the functions among different persons, so that the handicraftsman or peasant who produces with his own means of production is either turned little by little into a small capitalist who also exploits alien labour, or loses possession of his means of production //...// and is turned into a wage labourer. This is the tendency in the form of society in which the capitalist mode of production predominates. In considering the essential relations of capitalist production, therefore, it can be assumed //...// that the whole world
of commodities, all the spheres of material production—the production of material wealth—have been subjected (either formally or really) to the capitalist mode of production. In this presupposition, which expresses the LIMIT, and therefore approximates ever more closely to exact accuracy, all the workers engaged in the production of commodities are wage labourers and the means of production confront them as capital in all spheres of production (Ibid., 143).

In these paragraphs Marx explains why he chose to focus on the “three big classes of modern society” in the construction of his main models of the capitalist mode of production, that is, why he transits from SCP, where commodity producers such as “independent handicraftsmen and peasants” occupy center stage, to pure CCP, where they are not present. The reason for this is that Marx perceives that this class tends to disappear, and to be substituted by the classes of Capitalism. Hence, if he is to focus in Capital—after Part I of Book I on Commodities and Money—on the “essential relations of capitalist production”, instead of introducing the classes that are specific to Capitalism he had to substitute the one class of simple commodity producers with the two main classes of Capitalism: capitalists and wage workers (and much later show how Capitalism transformed the landowners who were the dominant class in the old feudal mode of production into simple landowners endowed with the right to lease their lands for rent and thus conform what was still one of the three contemporary big social classes).

But though this is true for his construction of a theory that is based on a historical-genetic criterion but uses small and hence manageable models, in his innumerable comments and analyses the class of simple commodity producers that is suppressed in the main theoretical models of CCP is often present. A characteristic example is in his 1965 speech before the Central Council of the General Association of German Workers, which was published posthumously as Wages, Price, and Profit. He refers there to countries like the U.S.A., where wages were high in comparison with those in England because “the law of supply and demand favours the working man.” “Hence the relatively high standard of wages in the United States. Capital... cannot prevent the labour market from being continuously emptied by the continuous conversion of wages labourers into independent, self-sustaining peasants” (MECW 20, 146) due to the great abundance of fertile lands unoccupied (at least by Europeans). The feasible alternative of traveling and settling as an independent producer thus put a lower limit on the wage rate. Another example is his analysis of the (tragic) capitalist accumulation process in Ireland between 1841 and 1861 (census years), during which the population decreased by 33%, from 8.2 to 5.5 million through famine and emigration (more than a million dead during the famine of 1846 and 2.3 million émigrés between 1851 and 1874). Marx observes that notwithstanding the fall in population and agricultural output, aggregate ground rents and rural profits increased. He explains this phenomenon by the concentration of land, the conversion of much of the arable land into pastures with accompanying capital investment, the increase in surplus product due to the fall in consumption, and the increase in the price of meat and wool. A great majority of the dead and emigrated population were independent producers whose means of production were scant (and did not constitute elements of capital under Marx’s definition). Hence, notwithstanding the tragedy there was an increase in the capital invested in Ireland:

The scattered means of production that serve the producers themselves as means of employment and of subsistence, without expanding their own value by the incorporation of the labour of others, are no more capital than
a product consumed by its own producer is a commodity. If, with the mass of the population, that of the means of production employed in agriculture also diminished, the mass of the capital employed in agriculture increased, because a part of the means of production that were formerly scattered, was concentrated and turned into capital (B1, 694).

This paragraph once more highlights how the terminology of economic theory evolved in a very different direction from Marx’s, impoverishing their representation of social relations by stripping them of their direct social character. For Marx “capital is not a thing, but a social relation between persons, established by the instrumentality of things” (B1, 753). Marx’s emphasis on the social relations involved in the economic process pointed to the historical character of these relations, and hence their being susceptible to change. Already in his early “Wage Labour and Capital” (of 1849) Marx had pointed out that: “A negro is a negro. In certain circumstances he becomes a slave. A mule is a machine for spinning cotton. Only under certain circumstances does it become capital. Outside these circumstances, it is no more capital than gold is intrinsically money.... Capital is a social relation of production” (Self-quoted by Marx in B1, 753). Although Marx underestimated Capitalism’s capacity to be reformed and transmuted variously – as is evident from his political revolutionary praxis and so many of his statements – this certainly does not imply that he erred in trying to create an integrated approach to the disciplines that have come to be known as ‘sociology’ and ‘economics’, disciplines that have entrenched themselves behind strong established academic walls (that have the political backing of the elites that govern the most advanced countries of contemporary Capitalism) and thus avoid investing serious efforts in the ‘unification of the fields’ (of economics, sociology, and political science) that is so necessary in social science.

**SCP and CCP in the same model**

As a simple example of a small part of the real world complexity that Marx analyzed, we here show how the production of non-capitalist commodity producers and capitalist production can be articulated within the same model. Since independent artisans or farmers are not wage workers, their productive activity must be included within a sector that is outside of de capitalist mode of production, although both are interconnected through the circulation of commodities. In order to represent the coexistence of SCP and CCP we can decompose matrix $A$, separating the branches of non-capitalist production (in the first rows and columns) from the capitalist branches (in the last rows and columns), which is what we have done in the following dual systems of quantities and prices:

$$\begin{bmatrix}
q_{QS} & q^Q & q_{LS} & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A_{11} & 0 & \ell_S & 0 & 0 \\
A_{21} & A_{22} & 0 & \ell & \eta \\
c_{S1} & c_{S2} & 0 & 0 & 0 \\
c_{L1} & c_{L2} & 0 & 0 & 0 \\
c_{K1} & c_{K2} & 0 & 0 & 0
\end{bmatrix}
=\begin{bmatrix}
q_{QS} & q^K & q_{LS} & q^{LK} & q^K
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
w_S \\
w \\
\pi
\end{bmatrix}
=\begin{bmatrix}
v \\
p \\
w_S \\
w \\
\pi
\end{bmatrix}.
$$

(16.8)
The first and third equations of both systems represent SCP. In (16.8) \( q^{QS} \) and \( q^{LS} \) represent the gross output and the population, respectively, of the simple commodity producers, and \( \ell_S \) represents the vector of direct labor requirements in the branches of SCP. We have made the simplifying assumption that simple commodity producers do not use inputs from the capitalist sector \((A_{12} = 0)\). But they are connected with the capitalist sector by means of the provision of their produced commodities, both as productive inputs for the capitalist sector \((A_{21})\) and as consumption goods \((c_{L1}, c_{K1})\) and by means of the purchase of consumption goods produced in the capitalist sector \((c_{S2})\). By definition, the capitalist sector does not directly use the labor of non-capitalist producers (otherwise the latter would be wage workers) and the non-capitalist sector does not directly use the labor of wage workers (otherwise they would be capitalists). In (16.9) \( v \) and \( p \) are the vectors of the \((\text{equilibrium})\) prices of the outputs of SCP and CCP, respectively, \( w_S \) is the per capita income of the simple commodity producers and \( w \) is the wage rate. Let us assume the per capita income of simple commodity producers is the numeraire \((w_S = 1)\).

Systems (16.8) and (16.9) contain the following equations:

\[
\begin{align*}
1 & \quad q^{QS} (I - A_{11}) - q^{LS} c_{S1} = q^Q A_{21} + q^L c_{L1} + q^K c_{K1} \\
2 & \quad q^Q (I - A_{22}) - q^L c_{L2} - q^K c_{K2} = q^{LS} c_{S2} \\
3 & \quad q^{QS} \ell_S = q^{LS} \\
4 & \quad q^Q \ell = q^L \\
5 & \quad q^Q \eta = q^K \\
\end{align*}
\]

From equation B1 we infer that the \((\text{equilibrium})\) prices of the commodities produced by the simple commodity producers are Marx’s \textbf{values}: \( v = (I - A_{11})^{-1} \ell_S \). B2 shows that the \((\text{equilibrium})\) price vector of the capitalist producers \( p \) is no longer Marx’s vector of prices of production of the model of pure CCP since the \textbf{values} of the inputs purchased from the SCP sector are also involved \((A_{21}v)\). In B3 – B5 we can see the same mixture of \textbf{values} and prices in the consumption baskets of simple producers, wage workers, and capitalists, respectively. Equation A1 shows that the SCP producers sell to the participants in the CCP sector (wage workers and capitalists) the quantities \( q^Q A_{21} + q^L c_{L1} + q^K c_{K1} \) of productive inputs and consumption goods, and A2 shows that capitalist producers sell to the simple commodity producers the quantities \( q^{LS} c_{S2} \) for their consumption. Since it is necessary that there be a \((\text{trade})\) balance between these two sectors, the \textbf{values} and prices must be such that the \textbf{values} of the commodities that simple commodity producers sell to capitalist producers be equal to the value of the commodities they purchase from capitalists: \((q^Q A_{21} + q^L c_{L1} + q^K c_{K1}) v = q^{LS} c_{S2} p\). The following chain of equalities proves this:

\[
q^{LS} c_{S2} p = \frac{q^{LS} (1 - c_{S1} v)}{q^{QS} (I - A_{11})} = q^{QS} \ell_S - q^{LS} c_{S1} v = q^{Q} (I - A_{11}) v - q^{LS} c_{S1} v = (q^Q A_{21} + q^L c_{L1} + q^K c_{K1}) v
\]

where B3 is used in the first equality, A3 in the second, B1 in the third, and A1 in the fourth.

Equation B2 gives the profit per unit of output of each of the capitalist branches as \( \eta \pi = (I - A_{22}) p - A_{21} v - \ell w \). The capital per unit of output of each capitalist branch is given by the elements of vector \( A_{21} v + A_{22} p + \ell w \) (where the first term shows that money capital must be disbursed to purchase commodities from the SCP sector). Hence, the profit per unit of output of each branch can also be written as \( \rho (A_{21} v + A_{22} p + \ell w) \). Equalizing these two expressions yields the vector of equilibrium prices of the capitalist
sector:

\[ p = (1 + \rho) \left( A_{21} v + A_{22}p + \ell w \right). \]  

(16.10)

Using B4 to eliminate \( w \) we get the following ‘transformation’ of values into (equilibrium) capitalist sector prices:

\[ p = \tilde{B} (\rho) \left( A_{21} + \ell c_{L1} \right) v, \]

where \( \tilde{B} (\rho) \equiv (1 + \rho) \left[ I - (1 + \rho) \left( A_{22} + \ell c_{L2} \right) \right]^{-1} \) is increasing in \( \rho \). And using this in B3 yields \( c_{S1} + c_{S2} \tilde{B} (\rho) \left( A_{21} + \ell c_{L1} \right) \) \( v = 1 \), an expression that uniquely determines the equilibrium profit rate \( \rho^* \). Notice that the (equilibrium) prices in the capitalist sector can alternatively be written as \( p = B_{22} (\rho) \ell w + B_{22} (\rho) A_{21} v \), where \( B_{22} (\rho) \) is defined as in (8.21) but replacing \( A \) with \( A_{22} \). Hence, \( p \) is the sum of a term that is equivalent to the prices of production of a pure CCP model \( (B_{22} (\rho) \ell w) \) and another which pertains to the inputs received from the SCP sector.

Using (16.10) we also get a system in which \( \eta \) has been eliminated under the assumption of equal profit rates in all the capitalist branches of production:

\[
\begin{bmatrix}
A_{11} & 0 & \ell_s & 0 \\
(1 + \rho) A_{21} & 0 & 0 & (1 + \rho) \ell \\
c_{S1} & c_{S2} & 0 & 0 \\
c_{L1} & c_{L2} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v \\
p \\
w
\end{bmatrix}
= \begin{bmatrix}
v \\
p \\
w
\end{bmatrix}.
\]

We can get the usual expression for the global profit rate of the capitalist sector by premultiplying (16.10) by \( q^Q \) and using B1, A5, and B5:

\[ \rho = \frac{q^K (c_{K1} v + c_{K2} p)}{q^Q (A_{21} v + A_{22} p + \ell w)}, \]

or also\(^{14}\)

\[ \rho = \frac{\tilde{c}}{\tilde{\kappa} + 1}, \quad \tilde{c} = \frac{q^K (c_{K1} v + c_{K2} p)}{q^L (c_{L1} v + c_{L2} p)}, \quad \tilde{\kappa} = \frac{q^Q (A_{21} v + A_{22} p)}{q^Q (\ell c_{L1} v + \ell c_{L2} p)}. \]

In conclusion, SCP and CCP, neither in pure state, can coexist in one and the same model, as they did in early Capitalism and as they still do in mature present day Capitalism. And this was very present in Marx’s analyses, though not in his models, where the algebra of pure CCP was already sufficiently complicated for his mathematical skills.

The State in Capitalism

The State in Marx’s early works

So far we have only considered ‘civil society’. But a complete picture of Marx’s conception of the capitalist social structure is incomplete if his conception of the State and its relation to ‘civil society’ is not taken into account. Already in The German Ideology he had (with Engels) introduced some synthetic ideas on the State in general. As we have seen in Chapter 2, the authors distinguished a ‘division of labor’ in a hierarchical sense (slave-owner, serf-lord, worker-capitalist) from a horizontal ‘division of labor’ (hunter, fisherman, pastor, etc.). In the latter there was a “contradiction between the

\(^{14}\)We use here a circumflex accent on \( \tilde{c} \) and \( \tilde{\kappa} \) because some of the inputs are valued in values and others in capitalist sector prices.
interest of the separate individual or the individual family and the common interest of all individuals who have intercourse with one another" (Ideology, MECW 5, 46). This common interest assumed “an independent form as the state” which, in the context of antagonistic social classes, appeared to represent the common interests but in reality represented the interests of the dominant class. They wrote: “the practical struggle of these particular interests, which actually constantly run counter to the common and illusory common interests, necessitates practical intervention and restraint by the illusory ‘general’ interest in the form of the state” (Ibid., 47).

The authors had also sketched some ideas on the emergence of the capitalist state, with the formation of cities: “The contradiction between town and country begins with the transition from barbarism to civilisation, from tribe to state, from locality to nation, and runs through the whole history of civilisation... The advent of the town implies, at the same time, the necessity of administration, police, taxes, etc., in short, of the municipality [des Gemeindewesens], and thus of politics in general” (Ideology, MECW 5, 64). A few years later, when Marx was analyzing the period in French history that culminated with the rise of Louis Bonaparte to power, he wrote:

The first French Revolution, with its task of breaking all separate local, territorial, urban and provincial powers in order to create the civil unity of the nation, was bound to develop what the absolute monarchy had begun: the centralisation, but at the same time the extent, the attributes and the agents of governmental power... Bonaparte perfected this state machinery. The Legitimist monarchy and the July monarchy added nothing but a greater division of labour, growing in the same measure as the division of labour within bourgeois society created new groups of interests, and, therefore, new material for state administration. Every common interest was straightway severed from society, counterposed to it as a higher, general interest, snatched from the activity of society’s members themselves and made an object of government activity, whether it was a bridge, a schoolhouse and the communal property of a village community, or the railways, the national wealth and the national university of France... All revolutions perfected this machine instead of breaking it. The parties that contended in turn for domination regarded the possession of this huge state edifice as the principal spoils of the victor (Brumaire, MECW 11, 185-6).

When Marx writes that various heretofore private activities were coming under the authority “of government activity, whether it was a bridge, a schoolhouse and the communal property of a village community, or the railways”, etc., he was correctly introducing the notion of what modern economic theory calls ‘public goods’, an idea that goes back at least to Adam Smith.15 That is, ‘public goods’ are those goods and services that due to their peculiar characteristics cannot possibly, or as conveniently, be produced with the incentive of private profit, i.e., by private firms; in some cases because it is impossible to exclude anyone from consuming them (such is the case of national defense) if they have not paid directly for them; in other cases because certain characteristics of the production process (such as increasing returns to scale) makes it convenient that they be produced by a unique producer who, if private, would impose

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15 Adam Smith had introduced the core of this concept when he included within the three duties of the sovereign: “erecting and maintaining certain public works, and certain public institutions, which it can never be for the interest of any individual, or small number of individuals to erect and maintain; because the profit could never repay the expense to any individual, or small number of individuals, though it may frequently do much more than repay it to a great society” (Smith 2005, 561).
a monopoly price unless firmly regulated (i.e., a toll bridge). In Marx’s terminology, a “higher, general interest” was made to prevail by taking the “common interest” away from the control of individuals or groups in ‘civil society’ and converting it into “material for state administration.” But along with the social end of furthering the ‘common interest’ of civil society, the “parties that contended in turn for domination” in government would try to use this “huge state edifice” for their private advantage, so it was regarded by them as spoils of the ‘war’ between political parties struggling to take over the reigns of government. And these government activities were to be financed by taxation, which could also be used for satisfying the true interests of those in government. “Taxes are the source of life for the bureaucracy, the army, the priests and the court, in short, for the whole apparatus of the executive power. Strong government and heavy taxes are identical” (Ibid., 191).

Marx’s description of the general corruption generated by the ‘strong government’ of Louis Bonaparte, (initially) backed by millions of backward and conservative small farmers and counting on the support of a militia (the Mobile Guard) recruited from the ‘lumpenproletariat’, could basically be used for characterizing innumerable national experiences of the last two centuries:

Industry and trade, hence the business affairs of the middle class, are to prosper in hothouse fashion under the strong government. The grant of innumerable railway concessions. But the Bonapartist lumpenproletariat is to enrich itself. The initiated play tripotage [hanky-panky] on the bourse with the railway concessions... Obligation of the Bank to make advances on railway shares. But, at the same time, the Bank is to be exploited for personal ends and therefore must be caressed. Release of the Bank from the obligation to publish its report weekly. Leonine agreement of the Bank with the government. The people are to be given employment. Initiation of public works. But the public works increase the obligations of the people in respect of taxes. Hence reduction of the taxes by an onslaught on the rentiers... (Brumaire, MECW 11, 195; text in square brackets added).

When Marx writes that “The people are to be given employment. Initiation of public works”, he reminds us that Keynes was not saying anything new when more than eighty years later he advocated the creation of employment by means of public works. What was new (aside from his pragmatic attempt to patch up the prevailing economic theory so that it no longer remained completely blind to its greatest defects) was the depth and scale of the world economic depression of the 1930s. From what we have seen in Chapters 11-13, Marx’s theory of the industrial cycle was only a small step away from the formulation of ‘stabilization policies’. For if the hoarding and abstinence from investment were key causes of recessions and depressions, the government’s actions contrario sensu could compensate them at least to some extent if only they understood the phenomenon and had the political capacity to act in consequence, even if only to avoid mass uprisings and revolutionary attempts. But Marx’s intention was not (as Keynes’ was) to become a source of medicines for the ills of Capitalism but to entirely rid society of the domination of the bourgeoisie, as we will see in Part IV.

**The State in Marx’s tentative plans for Capital**

In his tentative plans for the writing of *Capital* Marx intended to specifically analyze the role of the State in Capitalism. This is manifested in the Preface to *Contribution*, which begins as follows: “I examine the system of bourgeois economy in the following order:
capital, landed property, wage-labour; the State, foreign trade, world market. The economic conditions of existence of the three great classes into which modern bourgeois society is divided are analysed under the first three headings; the interconnection of the other three headings is self-evident” (Contribution, MECW 29, 261). He also mentions that he decided to omit a ‘general introduction’ “since on further consideration it seems to me confusing to anticipate results which still have to be substantiated” (Idem.) The omitted “general introduction” contained a working plan with 5 topics (instead of 6) that are slightly more informative, and of which the first two were later expanded to Books I-III of Capital:

The arrangement has evidently to be made as follows: (1) The general abstract determinations, which therefore appertain more or less to all forms of society, but in the sense set forth above. (2) The categories which constitute the internal structure of bourgeois society and on which the principal classes are based. Capital, wage labour, landed property. Their relation to one another... (3) The State as the epitone of bourgeois society. Analysed in relation to itself. The ‘unproductive’ classes. Taxes. National debt. Public credit. Population. Colonies. Emigration. (4) International character of production. International division of labour. International exchange. Export and import. Rate of exchange. (5) World market and crises (Introduction, MECW 28, 17).16

The first of these items disappeared as such with the later writings of Marx as it was distributed all along Book I. Hence, from the remaining 4 items remain, the first (Capital, wage labor, landed property) conformed Books I-III of Capital, and the second is concerned with the State. When Marx writes that the State is “the epitone of bourgeois society” he is referring to what he perceived as one of the principal functions of the State that is dominated by varying fractions of the capitalist class: its role as referee in the course of making legislation and policy decisions, since all concrete measures necessarily benefit some interests at the cost of others. In Poverty he had written 20 years earlier: “The concentration of the instruments of production and the division of labour are as inseparable one from the other as are, in the political sphere, the concentration of public powers and the division of private interests” (Poverty, MECW 6, 187; italics added). Towards the end of this book, he writes also: “The working class, in the course of its development, will substitute for the old civil society an association which will exclude classes and their antagonism, and there will be no more political power properly so-called, since political power is precisely the official expression of antagonism in civil society” (Ibid., 212; italics added).17 And in a letter to Annenkov (of December 28, 1846, already quoted above) he had written: “If you assume given stages of development in production, commerce or consumption, you will have a corresponding form of social constitution... – in a word, a corresponding civil society. If you assume this or that civil society, you will have this or that political system, which is but the official expression of civil society” (Letters, MECW 38, 96; italics added). Although the State in Capitalism is dominated by representatives of the capitalist class,

16 A year and a half later, in a letter to Weydemeeyer (February 1, 1859) Marx gives an updated plan of this future books: “I divide the whole of political economy into 6 books. Capital; landed property; wage labour; the State; foreign trade; world market” (Letters MECW 40, 376). We have seen that Marx’s Capital includes much about wage labor and landed property in its 3 Books. And the State appeared immediately after these in the working plan.

17 We address the political-prophetic aspect of this statement (and others like it) in Part IV of this book.
its leaders were not considered a social class by Marx but merely a ‘social stratum’ that ‘concentrated’ the interests of the various subclasses of the dominant class. But this stratum had a certain autonomy from the dominant interests, as Marx highlights when he analyzes the political backing that Louis Bonaparte initially had from the numerically most important portion of 1850 France: the peasant small-holders.

References to the role of the State in *Capital*

In *Capital* the role of the State already appears in the theory of commodities and money when Marx writes “Coining, like the establishment of a standard of prices, is the business of the State”, and when he refers to “inconvertible paper money issued by the State and having compulsory circulation” (B1, 136-7). Further down Marx gives concrete examples of how the State often grants subsidies or legal monopolies in order to benefit certain activities that need very large amounts of capital: “Certain spheres of production demand, even at the very outset of capitalist production, a minimum of capital that is not as yet found in the hands of single individuals. This gives rise partly to state subsidies to private persons... partly to the formation of societies with legal monopoly for the exploitation of certain branches of industry and commerce, the forerunners of our modern joint-stock companies” (B1, 313-4). The establishment of import duties and export premiums (which he called “system of protection”) was another form of State intervention that Marx highlighted as an “artificial means of manufacturing manufacturers”: “The European states tore one another to pieces about the patent of this invention, and, once entered into the service of the surplus value makers, did not merely lay under contribution in the pursuit of this purpose their own people, indirectly through protective duties, directly through export premiums. They also forcibly rooted out, in their dependent countries, all industry, as, e. g., England did with the Irish woollen manufacture” (B1, 744-5).

In Book II Marx introduced the notion of state capital after defining social capital as “capital that is newly invested, either as capital newly accumulated in the form of money, or as some old capital which is entirely transformed into money for the purpose of transfer from one branch of industry to another” (B2, 67). It is “equal to the sum of the individual capitals (including the joint-stock capital or the state capital, so far as governments employ productive wage labour in mines, railways, etc., perform the function of industrial capitalists)” (B2, 103). It should be inferred that the government entities that invest ‘state capital’ not only hire wage workers, but sell their product in the market for a profit, even if profit is not necessarily their only or main objective. But Marx also refers to “the less developed stages of capitalist production” in which “undertakings requiring a long working period, and hence a large investment of capital for a long time, such as the building of roads, canals, etc., especially when they can be carried out only on a large scale”, are not carried out by capitalist firms “but rather at communal or state expense.” In contrast with the previous case, the production of such ‘public goods’ is “not carried out on a capitalist basis at all” (B2, 235), but the State involves itself in the construction, financing it out of taxation (or indebtedness) and does not sell a product in a market.

But the activity of the State in capitalist society that Marx most highlighted in *Capital* is its formulations of laws and regulations that impinge on the labor process of factories in different directions, according to circumstances. For example, in the long Chapter 10 of Book I on “The Working Day” Marx writes: “If the Réglement organique of the Danubian provinces was a positive expression of the greed for surplus labour which every paragraph legalised, the English Factory Acts are the negative expression
of the same greed. These acts curb the passion of capital for a limitless draining of labour power, by forcibly limiting the working day by state regulations, made by a state that is ruled by capitalist and landlord” (B1, 247). He highlights how in England the State’s activity was able to sanction factory work regulations that put a limit on the “eagerness for plunder” of capitalists that had “torn up by the roots the living force of the nation” causing periodic epidemics and, in the case of Germany and France (that did not have this restrictive legislation), even reducing the bodily height of their soldiers. Marx writes that the “The establishment of a normal working day is the result of centuries of struggle between capitalist and labourer” in which he perceived “two opposed tendencies”: whereas “the English Labour Statutes from the 14th century to well into the middle of the 18th” tended to prolong the working day, “the modern Factory Acts compulsorily shortened the working day.” And the development of that restrictive regulation was spread haphazardly internationally, with the introduction of this legislation first in the countries most advanced in capitalist development:

It takes centuries ere the ‘free’ labourer, thanks to the development of capitalistic production, agrees, i.e., is compelled by social conditions, to sell the whole of his active life, his very capacity for work, for the price of the necessaries of life, his birthright for a mess of pottage. Hence it is natural that the lengthening of the working day, which capital, from the middle of the 14th to the end of the 17th century, tries to impose by State measures on adult labourers, approximately coincides with the shortening of the working day which, in the second half of the 19th century, has here and there been effected by the State to prevent the coining of children’s blood into capital. That which to-day, e. g., in the State of Massachusetts... has been proclaimed as the statutory limit of the labour of children under 12, was in England, even in the middle of the 17th century, the normal working day of able-bodied artisans, robust labourers, athletic blacksmiths (B1, 276-7).

Marx emphasized that it was necessary that workers actively and jointly, “as a class, compel the passing of a law, an all-powerful social barrier that shall prevent the very workers from selling, by voluntary contract with capital, themselves and their families into slavery and death.” Nevertheless, he also recognized and even highlighted the progressive measures that the most advanced capitalist societies eventually introduced (in their own countries but hardly ever in the societies they colonized and oppressed) while he also condemned the excesses of exploitation wherever they occurred.

The State in the systems of quantities and prices

We here introduce the State in our social matrix by means of simple extensions in the dual systems of quantities and prices we have been using. We start from the simplest model of industrial capital, with neither commercial or financial capital nor landowners. It would be easy to include ‘state capital’ in the model but it would not add much. We prefer to engage in an analytical formulation which includes the ‘social stratum’ of public functionaries who are in charge of government, and who finance their activities and their consumption by collecting taxes. We here call ‘public goods’ the goods and services produced by the State and financed by means of tax collection in order to distinguish them from the ‘private goods’ produced by capitalist firms.

The following dual systems show a three class structure of society (where, for convenience, we call the stratum of functionaries a class) composed of wage workers,
industrial capitalists, and State functionaries (with populations \( q_L \), \( q_K \), and \( q_S \), respectively, and consumption baskets of private goods \( c_L \), \( c_K \), and \( c_S \), respectively). To simplify, we assume that the government produces only one public good, that the latter is not used as an input in production\(^{18} \), and that the quantity produced \( q^G \) is financed by means of a direct income tax on the (net of taxes) income of workers and capitalists with a tax rate \( \tau \) that is paid at the beginning of the period.

\[
\begin{bmatrix}
q^Q & q^G & q^L & q^K & q^S
\end{bmatrix}
\begin{bmatrix}
A & 0 & \ell & \eta & k_r \\
a_G & 0 & \ell_G & 0 & 0 \\
c_L & 0 & 0 & 0 & 0 \\
c_K & 0 & 0 & 0 & 0 \\
c_S & g & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^G & q^L & q^K & q^S
\end{bmatrix},
\]

\[
\begin{bmatrix}
A & 0 & \ell & \eta & k_r \\
a_G & 0 & \ell_G & 0 & 0 \\
c_L & 0 & 0 & 0 & 0 \\
c_K & 0 & 0 & 0 & 0 \\
c_S & g & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p & p \\
p & p \\
p & \frac{w}{1+\tau} \\
p & \frac{w}{1+\tau} \\
\frac{\pi}{1+\tau} & \frac{\pi}{1+\tau}
\end{bmatrix}
= \begin{bmatrix}
p & p \\
p & p \\
p & \frac{w}{1+\tau} \\
p & \frac{w}{1+\tau} \\
\frac{\pi}{1+\tau} & \frac{\pi}{1+\tau}
\end{bmatrix}.
\]

The second row and column of the matrix of these systems represent the production \( q^G \) and the price \( p_G \) of the public good, respectively. And \( a_G \) and \( \ell_G \) represent the requirements of private goods and the requirements of labor, respectively, that are directly necessary to produce a unit of the public good. Also, \( p_G \) is a ‘shadow’ (or accounting) price for the public good, which is valued according to the cost incurred in its production, since the public good is not transacted in a market. Since workers, capitalists, and functionaries have access to the consumption of public goods without paying directly for them, its production must be financed by means of taxation on civil society. We also assume that the State functionaries’ per capita consumption of private goods \( c_E \) are financed through taxation. Notice that here \( w \) and \( \pi \) represent the wage rate and per capita profit gross of taxes. Hence, \( w/(1+\tau) \) and \( \pi/(1+\tau) \) are the respective tax bases and also the incomes that can be spent on their respective consumption baskets (\( c_L \) and \( c_K \)). Finally, the column vector \( k_r \) represents the tax base per unit of output of the various industrial branches on which the tax rate \( \tau \) is applied. In the following we can assume that the numeraire is given by \( \pi = 1 \), that is, \( (1+\tau) c_K p = 1 \), but we will continue using \( \pi \) since it is not necessary to go into all the details of the solution.

The following table shows the equations of these two systems:

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( q^G a_G + q^L c_L + q^K c_K + q^S c_S = q^Q (I - A) )</td>
<td>( (\ell w + \eta \pi)/(1 + \tau) + k_r \tau = (I - A) p )</td>
</tr>
<tr>
<td>2</td>
<td>( q^S g = q^G )</td>
<td>( a_G p + \ell_G w/(1 + \tau) = p_G )</td>
</tr>
<tr>
<td>3</td>
<td>( q^Q \ell + q^G \ell_G = q^L )</td>
<td>( c_L p = w/(1 + \tau) )</td>
</tr>
<tr>
<td>4</td>
<td>( q^Q \eta = q^K )</td>
<td>( c_K p = \pi/(1 + \tau) )</td>
</tr>
<tr>
<td>5</td>
<td>( q^Q k_r = q^S )</td>
<td>( c_S p + g p_G = \tau )</td>
</tr>
</tbody>
</table>

\( A1 \) shows that the net output of private goods \( q^Q (I - A) \) must satisfy the inputs necessary for the production of public goods \( q^G a_G \) as well as the consumption of private goods of workers, capitalists, and functionaries. \( A2 \) shows that \( g = q^G / q^S \) is the production of public goods per functionary. \( A3 \) shows how the total wage labor \( q^L \)

\(^{18}\)If we want the public good to be used as input in the production of (some or all) private goods, we can replace the 0 that is to the right of \( A \) by a \( b_G \).
is allocated to the various branches that produce private and public goods. A4 shows how the population of capitalists \( q^K \) is allocated (along with their capital) among the various branches. And A5 shows that the population of functionaries \( q^S \) that is sustained by this socioeconomic-political system can be theoretically allocated to the different industrial branches according to their respective tax base \( (q^q k_r) \) even if they are not necessarily directly involved in those branches.

\( B1 \) decomposes the vector of private commodity prices \( p \) into a component for the value of the means of production consumed in their production \( (Ap) \), components for the net of tax wages and profits, and finally, a component \( (k_r \tau) \) for the taxes per unit of output that is imputed to the respective branches according to the tax base. \( B2 \) shows that the ‘shadow price’ for the public good \( (p_G) \) is equal to the value of the means of production consumed in their production \( (a_Gp) \) plus the net of tax wages paid in this sector. \( B3 \) and \( B4 \) need no explanation. And \( B5 \) shows that the level of the tax rate must be equal to the value of the consumption basket of private goods of each functionary \( (c_E p) \) plus the value (measured by its shadow price) of the production of public goods per functionary \( (g p_G) \).

Multiplying \( B5 \) by \( q^Q k_r \) and using A5 we get \( q^Q k_r \tau = q^S (c_S p + g p_G) \), which shows that there is fiscal balance, i.e., the aggregate tax collection \( (q^Q k_r \tau) \) is enough to finance the consumption of private goods by public functionaries \( (q^S c_S p) \) and the cost of the public good produced \( (q^S g p_G) \). Also, the tax base in each branch is the (net of tax) income of the workers and capitalists that work there:

\[
k_r = \frac{\ell}{1 + \gamma} + \eta \frac{\pi}{1 + \tau}.
\]

\( (16.11) \)

Hence, eliminating \( k_r \) from \( B1 \) yields a simpler expression for the prices of production:

\[
p = Ap + \ell w + \eta \pi.
\]

\( (16.12) \)

The reduced system of prices and incomes of the private sector obtained assuming equalization of profit rates can be expressed as follows:

\[
\begin{bmatrix}
(1 + \rho) A \\
(1 + \tau) c_L
\end{bmatrix}
\begin{bmatrix}
(1 + \rho) \ell \\
0
\end{bmatrix}
\begin{bmatrix}
p \\
w
\end{bmatrix}
= \begin{bmatrix}
p \\
w
\end{bmatrix}.
\]

\( (16.13) \)

Notice that the workers’ consumption basket must be expanded by the tax factor in order to reflect the fact that it is the net of tax wage that must be able to purchase the basket’s commodities. From (16.12) and the first equation of (16.13) we get expressions for the global gross of income tax profit rate:

\[
\rho = \frac{q^K \pi}{q^QA p + q^Q \ell w} = \frac{q^K (1 + \tau) c_k p}{q^QA p + q^Q \ell (1 + \tau) c_L p}.
\]

Comparing with (8.24), we can see that, due to the insertion of the public sector in the model, the disbursement of variable capital in the denominator and the profits in the numerator are gross of the income tax. And obviously the disbursed variable capital does not take into account the workers employed in the public sector \( (q^Q \ell = q^L - q^G \ell_G) \).

Finally, what has been done in this subsection can be extended to the case of multiple goods produced by the public sector, where some or all of them can be inputs for the production of private or public goods. In this case \( A \) would be partitioned so as to separate the private \( (P) \) and public \( (G) \) sectors:

\[
A = \begin{bmatrix}
A_{PP} & A_{GP} \\
A_{PG} & A_{GG}
\end{bmatrix}.
\]
However, it is unnecessary to pursue further with this matter since the essence of the approach should be sufficiently clear with our simple case.

Appendix to Chapter 16
Bibliographical Note: the ‘contradiction’ found by Böhm-Bawerk and Engels’ challenge

Eugen von Böhm-Bawerk published in 1890 in his “critical history of economic theory”, Capital and Interest, a poignant critique of the “exploitation theory” of profit (which he called interest), focusing his attention on Rodbertus and Marx. However, there was no way he (or anyone else) could make a sensible critique of Marx’s theory since Book III of Capital was only published in 1894 (although it was mostly written in the 1860s). The fact that Marx had chosen to publish Book I with the simplifying assumption that the equilibrium prices in Capitalism were (proportional to) his values thwarted the possibility of understanding the full scope of his theory, since this simplifying assumption was only lifted in Book III. After Engels finally published Book III in 1894, Böhm-Bawerk published in 1896 an essay with strong criticism and his claim that there was a ‘contradiction’ in Marx’s theory:

In the first volume it was maintained, with the greatest emphasis, that all value is based on labor and labor alone, and that values of commodities were in proportion to the working time necessary for their production... And now in the third volume we are told briefly and dryly that what, according to the teaching of the first volume, must be, is not and never can be; that individual commodities do and must exchange with each other in a proportion different from that of the labor incorporated in them, and this not accidentally and temporarily, but of necessity and permanently.

I cannot help myself; I see here no explanation and reconciliation of a contradiction, but the bare contradiction itself. Marx’s third volume contradicts the first (Böhm-Bawerk 1949, 29-30).

As we have seen, there was no contradiction but only the use of simplifying assumptions in Book I in order to treat many issues that Marx wished to publish as soon as possible, giving him time to more satisfactorily elucidate the details of his dual approach to valuation (one for the equilibrium prices of commodities in market transactions and another for his theory of the exploitation of wage labor in Capitalism) as well as many other topics that were already sketched in drafts for Books II and III but required a more prolonged effort. This decision implied postponing the details of the divergence between the prices of production and values, a topic that had the highest priority for economists like Böhm-Bawerk.

But it is also true that Engels had some influence in the matter of the alleged ‘contradiction’ between Books I and III of Capital. For in his Preface (of 1893) to Book II, answering some unjust criticism blaming Marx for plagiarizing Rodbertus, he wrote:

According to the Ricardian law of value, two capitals employing equal quantities of equally paid living labour, all other conditions being equal, produce products of equal value, and likewise surplus value, or profit, of equal

---

19For example: “I think I shall be able to convince my readers...—and this is the most decisive point— that the reasoning is full of the most obvious faults of logic and method which deprive it of all cogency” (Ibid., 65-6).
quantity, in equal periods of time. But if they employ unequal quantities of living labour, they cannot produce equal amounts of surplus value, or, as the Ricardians say, equal amounts of profit. Now in reality the opposite takes place. In actual fact, equal capitals, regardless of how much or how little living labour they employ, produce equal average profits in equal times. Here there is therefore a contradiction of the law of value which had been noticed by Ricardo himself, but which his school also was unable to resolve... Marx had resolved this contradiction already in the manuscript of A Contribution to the Critique... According to the plan of Capital, this solution will be provided in Book III (B2, 23).

When he wrote this, Engels was already editing the manuscripts of Book III, and the readers could only take note of Engels' promise of providing Marx's 'solution' to the 'contradiction' and wait. Hence, when the book was finally published it is natural that careful readers were expecting to find a solution to a 'contradiction' between prices of production and the 'law of value'. To make things worse, Engels continued the Preface with an explicit challenge:

Months will pass before that will be published. Hence those economists who claim to have discovered in Rodbertus the secret source and a superior predecessor of Marx have now an opportunity to demonstrate what Rodbertus' political economy can accomplish. If they can show how an equal average rate of profit can and must come about, not only without a violation of the law of value, but rather on the very basis of it, we are willing to discuss the matter further with them. In the meantime they had better make haste (Ibid.).

Böhm-Bawerk probably took Engels challenge to heart when he was writing his critique, starting a long and useless controversy that could have easily been avoided simply by pointing out that: 1) Marx made a simplifying assumption in Book I that allowed him to concentrate attention on what he considered most important (the production of surplus value in CCP), an assumption that he would eliminate in Book III, and 2) although in his analytics Marx first introduced SCP and then introduced the specific assumptions necessary for CCP, he knew that in capitalist society both modes of production actually coexisted and, furthermore, that they both coexisted with other pre-capitalist modes of production (such as those of servitude or slavery). In this chapter we have shown how the coexistence of SCP and CCP can be formalized.
Chapter 17  THE TENDENCIES, OR ‘LAWS’, OF CAPITAL ACCUMULATION

As we have seen, Marx developed in Book II various bisectoral models of capitalist accumulation (or growth) of which the most successful were those of balanced accumulation with equal value composition of capital both sectors. In contrast, in Chapter 25 of Book I (“The general law of capitalist accumulation”) and in Part III of Book III (“The law of the tendency of the rate of profit to fall”) he stated his theory of capitalist accumulation, based on his vision of how capital accumulation actually proceeded in the most advanced capitalist society of his time. His view was founded on the empirical data that were available to him and also on the observations of “some of the prominent economists of the classical school” such as John Barton, David Ricardo, Richard Jones, and George Ramsay (B1, 625; footnote added to the fourth German edition). It is important to keep in mind that for Marx ‘law’ essentially meant “tendency which necessarily leads to inevitable results”, as he says explicitly in the following statement from the Preface to the first German edition of Capital (1867): “Intrinsically, it is not a question of the higher or lower degree of development of the social antagonisms that result from the natural laws of capitalist production. It is a question of these laws themselves, of these tendencies working with iron necessity towards inevitable results” (B1, 9). Chapter 25 of Book I starts by stating that what will be dealt with is “the influence of the growth of capital on the lot of the labouring class.” Although this is true, since ultimately everything that happens in the process of capital accumulation necessarily affects wage workers, the chapter covers various topics that go far beyond this specific effect and are fundamentally concerned with the industrial and financial organization of the Capitalism of ‘modern’ industry. He formulates certain empirical regularities to which he attributed great importance and for the exposition of which it was not necessary to lift the simplifying assumption (explicit in Book I) that commodities are transacted according to their values. And he relegated to Book III his analysis of the ‘law of the tendency of the rate of profit to fall’ which, though intimately connected to the laws he dealt with in Book I, he preferred to address after having eliminated the simplifying assumptions of Book I. In this chapter we deal with the main tendencies of Capitalism that Marx addressed in various parts of his work.

The effects of capital accumulation on the working class

In his analysis of how the process of capital accumulation affected the working class the most significant factor for Marx was its effect on the demand for labor. Those “prominent economists of the classical school” we listed above had made important contributions but nevertheless “conjectured rather than understood” what he called “The law of progressive diminution of the relative magnitude of variable capital and its effect on the condition of the class of wage workers.” He cites from the main contributions on this matter: John Barton (Observations on the Circumstances which Influence the Condition of the Labouring Classes of Society, 1817), David Ricardo (Principles), Richard Jones (An Introductory Lecture on Political Economy, 1833), and George Ramsay (An Essay on the Distribution of Wealth, 1839). Using his usual ‘hypothesis-deductive’ methodology, he first analyzes what happens if in the course of capital accumulation the composition of capital remains constant, and then investi-
gates how the analysis must be modified if one bears in mind that the composition of capital tends to increase as there is an increasing disbursement of capital in ever more elaborate means of production (in relation to disbursement in variable capital).

The effect on the demand for labor power

Assuming that the composition of capital remains constant, the capital accumulation process implies that “the demand for labour and the subsistence fund of the labourers clearly increase in the same proportion as the capital” (B1, 608), albeit with cyclical growth characteristics. Various factors are at play here that are related to the aim of capitalists of increasing their wealth. The actions of capitalists can generate new stimuli to this desire. For example, “the opening of new markets, or of new spheres for the outlay of capital in consequence of newly developed social wants, &c,” and hence “the scale of accumulation may be suddenly extended, merely by a change in the division of the surplus value or surplus product into capital and revenue” (B1 608-9), i.e. by deciding to invest a higher proportion of profits and hoard and/or consume less (as we have modeled in several of the chapters above). Furthermore, Marx is quite aware of population growth. During the ascendant phase of the industrial cycle, “since in each year more labourers are employed than in its predecessor, sooner or later a point must be reached, at which the requirements of accumulation begin to surpass the customary supply of labour, and, therefore, a rise of wages takes place.” The accumulation process thus makes wage workers receive a “larger part of their own surplus product... so that they can extend the circle of their enjoyments; can make some additions to their consumption fund of clothes, furniture, &c, and can lay by small reserve funds of money” (B1, 613). Eventually however, if the increase in wages is sufficiently large “the stimulus of gain is blunted.” In that case, the “rate of accumulation lessens; but with its lessening, the primary cause of that lessening vanishes, i.e., the disproportion between capital and exploitable labour power. The mechanism of the process of capitalist production removes the very obstacles that it temporarily creates. The price of labour falls again” (B1, 614).

Marx criticized those economists who believed that the fluctuations in labor employment along the industrial cycle were due to the oscillations of the worker population instead of considering that they are the result of the ups and downs of accumulation brought about by changes in the reinvestment of profits. And he called this irregular cyclical growth process ‘The law of capitalistic accumulation’. He synthesized it as follows (where ‘unpaid labor’ refers to both surplus value and profit, which are supposed to be equal in the aggregate):

If the quantity of unpaid labour supplied by the working class, and accumulated by the capitalist class, increases so rapidly that its conversion into capital requires an extraordinary addition of paid labour, then wages rise, and, all other circumstances remaining equal, the unpaid labour diminishes in proportion. But as soon as this diminution touches the point at which the surplus labour that nourishes capital is no longer supplied in normal quantity, a reaction sets in: a smaller part of revenue is capitalised, accumulation lags, and the movement of rise in wages receives a check. The rise of wages therefore is confined within limits that not only leave intact the foundations of the capitalistic system, but also secure its reproduction on a progressive scale (B1, 616).

So far, the analysis has rested on the conditions of capital accumulation most
favorable to workers, since it assumes that in the accumulation process the composition of capital remains constant. But “in the course of accumulation, a point is reached at which the development of the productivity of social labour becomes the most powerful lever of accumulation” (B1, 617). And this adversely affected the demand for labor power during the accumulation process, since the increase in productivity was typically obtained as a result of an increase in the purchase of means of production (the elements of constant capital) that was quantitatively greater than the increase in the employment of labor (the element of variable capital), that is, as a result of an increase in the ‘technical composition’ of capital. However, the increase in productivity also implied a relative cheapening of the means of production. Hence, the ‘value composition’ of capital increased less than the ‘technical composition’ of capital:

the degree of productivity of labour, in a given society, is expressed in the relative extent of the means of production that one labourer, during a given time, with the same tension of labour power, turns into products. The mass of the means of production which he thus transforms increases with the productiveness of his labour.

This change in the technical composition of capital, this growth in the mass of means of production, as compared with the mass of the labour power that vivifies them, is reflected again in its value composition, by the increase of the constant constituent of capital at the expense of its variable constituent.

This diminution in the variable part of capital as compared with the constant, or the altered value composition of the capital, however, only shows approximately the change in the composition of its material constituents... The reason is simply that, with the increasing productivity of labour, not only does the mass of the means of production consumed by it increase, but their value compared with their mass diminishes. Their value therefore rises absolutely, but not in proportion to their mass (B1, 617-8).

The increase in the composition of capital during the capital accumulation process, hence, resulted in a lower increase in the demand for labor power than in the previous case (of constant composition of capital), and hence in a lower increase in real wages during the ascendant phase of the industrial cycle.

**The effect on the ‘industrial reserve army’**

For Marx the existence of a mass of unemployed workers played a fundamental role in the accumulation process: “Capitalist production can by no means content itself with the quantity of disposable labour power which the natural increase of population yields. It requires for its free play an industrial reserve army independent of these natural limits” (B1, 629). The capital accumulation process required, in parallel to an increasing ‘active army’ of employed workers, an increasing ‘reserve army’ whose relative magnitude (to the ‘active army’) shrinks or grows when there is industrial expansion or contraction, respectively, i.e., when the economy is in the ascendant or descendant phase of the (growth) cycle. Hence, “it is capitalistic accumulation itself that constantly produces... a relatively redundant population of labourers, i.e., a population of greater extent than suffices for the average needs of the self-expansion of capital, and therefore a surplus population” (B1, 624).

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1See the definitions of these concepts in (8.15) and the paragraphs that precede it.
Although variable capital grows along with the growth of total capital it does so “in a constantly diminishing proportion” due to the increase in the ‘composition of capital’. Hence, “an accelerated accumulation of total capital... is needed to absorb an additional number of labourers” (B1, 623-4). Although on average the ‘reserve army’ grows in parallel to the growth of the ‘active army’, the former increases at a slower pace than the latter in the ascendant phase of the cycle, and the opposite occurs in the descendent phase. “Independently of the limits of the actual increase of population”, the industrial reserve army “creates, for the changing needs of the self-expansion of capital, a mass of human material always ready for exploitation” (B1, 626).

As we have seen previously, the relative growth or shrinkage of the mass of unemployed plays a decisive role in the determination of the evolution of the real wage, which tends to grow in the expansionary phase, as the ‘reserve army’ diminishes, due to the accumulation of capital, and tends to fall in the contractive phase, as the mass of the unemployed grows. Marx presented verbally an interesting theory of how the real wage evolved along the industrial cycle according to the evolution of the size of the ‘industrial reserve army’, over the industrial cycle. We have seen in Chapters 12, 13, and 14 how some of Marx’s ideas on the functioning of the industrial can be modeled under SR and ER.

The effect on the welfare of wage workers

Even though Marx recognized that in the advanced capitalist countries the real wage increased in the expansionary periods and even as a long run tendency, he pointed out the limitations that this process had on the situation of wage workers. For, under the capitalist mode of production “the labourer exists to satisfy the needs of self-expansion of existing values, instead of, on the contrary, material wealth existing to satisfy the needs of development on the part of the labourer” (B1, 616). And in this system there existed a necessary subordination of workers to the dictates of capital that Marx considered repugnant since, in spite of appearances, it was not too different from the subordination of slaves to their masters. He writes: “just as little as better clothing, food, and treatment, and a larger peculium, do away with the exploitation of the slave, so little do they set aside that of the wage worker. A rise in the price of labour, as a consequence of accumulation of capital, only means, in fact, that the length and weight of the golden chain the wage worker has already forged for himself, allow of a relaxation of the tension of it” (B1, 613). For Marx “the differentia specifica of capitalistic production” was that the aim of the purchase of labor power by the capitalist was the “augmentation of his capital, production of commodities containing more labour than he pays for” (Ibid.). Hence, for him the ‘absolute aim’ of this mode of production was producing profit (or surplus value), not enhancing the well-being of all the population.

For Marx neither a person’s objective situation nor his welfare could be summarized by the size of his real income. The very fact that a worker’s possibilities of self-development had to be subordinated to the decisions of a capitalist (or the dictates of a public sector functionary) implied penury even if the wage worker was handsomely paid. That is why he writes: “in proportion as capital accumulates, the lot of the labourer, be his payment high or low, must grow worse” (B1, 639.). For, within the capitalist system all methods for raising the social productiveness of labour are brought about at the cost of the individual labourer; all means for the development of production transform themselves into means of domination over, and exploitation of, the producers; they mutilate the
labourer into a fragment of a man, degrade him to the level of an appendage of a machine, destroy every remnant of charm in his work and turn it into a hated toil; they estrange from him the intellectual potentialities of the labour process in the same proportion as science is incorporated in it as an independent power; they distort the conditions under which he works, subject him during the labour process to a despotism the more hateful for its meanness; they transform his lifetime into working time, and drag his wife and child beneath the wheels of the Juggernaut of capital (B1, 639).

And the necessary existence of an ‘industrial reserve army’ played an important role in the permanence of workers’ miseries, since “The law, finally, that always equilibrates the relative surplus population, or industrial reserve army, to the extent and energy of accumulation, this law rivets the labourer to capital” (Ibid.). Even if the employed workers’ real wages increased, they were always kept in subordination to the dictates of the capitalist, which Marx considered degrading, and the miseries related to unemployment remained throughout the industrial cycle, albeit with a varying magnitude. As Capitalism propagated and won over more industrial branches in an increasing number of countries, there appeared a growing polarization between workers and capitalists, both at the national and world levels. Marx concludes the above remarks stating: “Accumulation of wealth at one pole is, therefore, at the same time accumulation of misery, agony of toil, slavery, ignorance, brutality, mental degradation, at the opposite pole” (B1, 640).

Another adverse effect of capital accumulation on the welfare of the working class was its tendency to demand an increasing versatility of the labor force, hence tending to eliminate the worker’s interest in his own work. Workers had to be able to switch flexibly from one task to another, and from one branch of production to another, just as capital: “The higher the development of capitalist production in a country, the greater the demand for versatility in labour capacity, the more indifferent the worker is towards the particular content of his labour, and the greater the fluidity of capital’s movement from one sphere of production to another” (Results, MECW 34, 421). Marx highlighted the extent to which the U.S.A. was in the vanguard of this grievous reality:

Nowhere is the fluidity of capital, the versatility of labour and the indifference of the worker towards the content of his labour more clearly apparent than in the United States of North America... On the other hand, people are nowhere... so aware that their labour always delivers the same product, money, and nowhere else do people pass with the same nonchalance through the most disparate branches of labour (Ibid.; footnote 144)).

And this fluidity of wage labor was in sharp contrast with the rigidity of the slave’s work, whose labor power was normally always used in the same way and using only the most primitive instruments of labor.

The forms of ‘relative surplus population’

Marx distinguished various forms of the ‘relative surplus population’ of the working class (or mass of unemployed workers). In order to define them he leaves aside the variations over the industrial cycle and distinguishes three forms of unemployment that always exist: the floating, the latent, and the stagnant forms. In factories that use machinery, “large numbers of boys are employed up to the age of maturity. When this term is once reached, only a very small number continue to find employment in the
same branches of industry, whilst the majority are regularly discharged. This majority forms an element of the floating surplus population, growing with the extension of those branches of industry. Part of them emigrates, following in fact capital that has emigrated” (B1, 635). The movements of workers between country (agriculture, etc.) and town (manufacturing etc.) jobs play a critical role in distinguishing the ‘latent’ from the ‘floating’ forms. The structural factor involved is that as soon as “capitalist production takes possession of agriculture, and in proportion to the extent to which it does so, the demand for an agricultural labouring population falls absolutely.” Hence, a part of the agricultural population is “constantly on the point of passing over into an urban or manufacturing proletariat.” Marx here uses hydrodynamic imagery. A part of the country surplus population “is thus constantly flowing. But the constant flow towards the towns presupposes, in the country itself, a constant latent surplus population, the extent of which becomes evident only when its channels of outlet open to exceptional width” (B1, 636). Finally, the ‘stagnant’ form of working class surplus population is the least favored form of the unemployed, which “furnishes to capital an inexhaustible reservoir of disposable labour power. Its conditions of life sink below the average normal level of the working class.” When these workers are employed they have the lowest working days and the lowest wages. The segment includes those workers in “domestic industry” and ‘pauperism’, which includes those able to work as well as vagabonds, criminals, prostitutes, orphans and pauper children, and “the mutilated, the sickly, the widows, &c.”

Marx held that the ‘industrial reserve army’ grew not only in absolute terms as capital accumulated, but also relatively to the mass of employed laborers. Moreover, he believed that within the ‘industrial reserve army’ the ‘stagnant’ form of unemployment tended to increase its share, being recruited “constantly from the supernumerary forces of modern industry and agriculture, and specially from those decaying branches of industry where handicraft is yielding to manufacture, manufacture to machinery.” And he believed that the misery of this ‘consolidated surplus population’ was “in inverse ratio to its torment of labour”, since when they could not get employment even under the worst of conditions, they led an even more miserable life in order to survive. Such was what Marx called the “absolute law of capitalist accumulation”: a tendency for the increasing polarization of society into an extreme of wealth and an extreme of poverty. “Accumulation of wealth at one pole is, therefore, at the same time accumulation of misery, agony of toil, slavery, ignorance, brutality, mental degradation, at the opposite pole.”

And Marx criticized the folly “of the economic wisdom that preaches to the labourers the accommodation of their number to the requirements of capital”, pointing out that it was based on an inverted perception of reality: “The fact that the means of production, and the productiveness of labour, increase more rapidly than the productive population, expresses itself, therefore, capitalistically in the inverse form that the labouring population always increases more rapidly than the conditions under which capital can employ this increase for its own self-expansion” (B1, 638).

It is as easy as it is false to accuse Marx of having erred, with 150 years of hindsight, by pointing to the increase of superficially measured standards of living of the working classes in the most developed countries. But such would not be the relevant evidence. On the one hand, we should observe the reality of worldwide Capitalism (which was Marx’s modality, though he had to focus particularly on England which had the most reliable empirical data) which shows considerable evidence of a polarization between wealth and misery, where the latter is especially concentrated in the peripheral (or ‘less developed’, or ‘third world’) countries, in which the poorest strata of the population
still suffer all the degradation denounced by Marx as a result of the uneven development of global Capitalism. On the other hand, we should observe that the contemporary reality of many of the more developed countries also shows ample evidence of this polarization in their midst. It suffices to point out that, well into the 21st century, 11.4% (more than one in nine) of the Afro-American male population between 24 and 32 years of age were held in prison (Pew 2010), very many of whom were condemned for petty crimes related to local drug dealing or drug consumption, whereas there is a plethora of evidence of the long standing participation of intelligence organizations in wholesale drug trafficking (McCoy 2003), presumably to finance their covert activities.

Also, the flow of worker ‘surplus population’ from the countryside to the cities that Marx described for 19th century England, is still now a striking characteristic of the ‘less developed’, ‘peripheral’ or ‘third world’ countries, whose shantytowns, ‘villas miseria’ or ‘favelas’ grow unlimtedly in many parts of Africa, Latin America, and Asia. These are receptacles of the drainage of rural workers that seek better opportunities in the cities and only manage to build, often with their bare hands, precarious homes in lots they do not own in overcrowded neighborhoods that have extremely poor (if any) public services. And such shantytowns grow enormously each time the political (and economic) struggles between fractions of the dominant class results in a financial meltdown and an ensuing recession (or depression). The miseries of drug trafficking, addiction, prostitution, and criminality flourish in the fertile terrain generated by unemployment, lack of schooling, and deficient social assistance, hence perpetuating the worse social inequalities in the world.

The concentration and centralization of capital

According to Marx, in capitalist industry the cooperation of many workers in the same organization is a precondition for the development of the productive force of labor, and such was the basis of the manufacturing industries. But in modern industry the production of machinery and its utilization becomes generalized. Only by means of cooperation on a large scale the “division and combination of labour can be organised, and the means of production economised by concentration on a vast scale”, and “instruments of labour which, from their very nature, are only fit for use in common, such as a system of machinery, can be called into being” and the production processes transformed by the “technological application of science” (B1, 619).

Any individual industrial capital presupposes a concentration of means of production and workers. And with the accumulation of capital on an extended scale, the individual capitals grow, generating a concentration of capital with the same owners. Here the concentration of capital occurs in parallel with the accumulation of capital. But at the same time there exist processes of attraction and repulsion of already formed capitals. In the latter, there is a “splitting-up of the total social capital into many individual capitals”, in large measure through “the division of property, within capitalist families” as a consequence of inheritance. But at the same time, there is a contrary tendency of attraction of individual capitals through the “concentration of capitals already formed, destruction of their individual independence” hence producing the “transformation of many small into few large capitals.” But Marx stressed the importance of what he called the centralization of capital, in which there is “expropriation of capitalist by capitalist.” In this case, “Capital grows in one place to a huge mass in a single hand, because it has in another place been lost by many” (B1, 621). Marx points out that times of crises are particularly appropriate for the occurrence of deliberate actions that produce centralization: “Even a cursory examination of competition shows, furthermore, that under certain circumstances, when the greater capitalist wishes to
make room for himself on the market, and to crowd out the smaller ones, as happens in times of crises, he makes practical use of this, i.e., he deliberately lowers his rate of profit in order to drive the smaller ones to the wall” (B3, 223).

The centralization of capital produces “a change in the distribution of capital already to hand, and functioning.” It “shortens and quickens the transformation of separate processes of production into processes socially combined and carried out on a large scale.” Hence, it is “one of the greatest levers” of the accumulation of capital. The following paragraph presents a synthesis of the mechanisms in play in the process of competition between capitals:

The battle of competition is fought by cheapening of commodities. The cheapness of commodities depends, ceteris paribus, on the productiveness of labour, and this again on the scale of production. Therefore, the larger capitals beat the smaller. It will further be remembered that, with the development of the capitalist mode of production, there is an increase in the minimum amount of individual capital necessary to carry on a business under its normal conditions. The smaller capitals, therefore, crowd into spheres of production which modern industry has only sporadically or incompletely got hold of. Here competition rages in direct proportion to the number, and in inverse proportion to the magnitudes of the antagonistic capitals. It always ends in the ruin of many small capitalists, whose capitals partly pass into the hands of their conquerors, partly vanish (B1, 621-2).

It should be noted that when Marx writes that competition acts “in direct proportion to the number, and in inverse proportion to the magnitudes”, he is implying that when a great number of different capitals compete with each other in the same industrial branch competition can be very intense, but as the centralization process advances and the number of competitors diminishes to just a few, the latter compete less, which implies that they tend to turn into what we presently call an ‘oligopoly’ or, in the extreme case in which only one large corporation is left, a ‘monopoly’. Hence, we writes, “in any given branch of industry centralisation would reach its extreme limit if all the individual capitals invested in it were fused into a single capital” (B1*, 435). On the other hand, considering society as a whole “the limit would be reached only when the entire social capital was united in the hands of either a single capitalist or a single capitalist company.”

On the other hand, Marx held that the credit system is a powerful “lever of centralization” which “in its first stages furtively creeps in as the humble assistant of accumulation, drawing into the hands of individual or associated capitalists, by invisible threads, the money resources which lie scattered, over the surface of society, in larger or smaller amounts” but eventually becoming “a new and terrible weapon in the battle of competition and is finally transformed into an enormous social mechanism for the centralisation of capitals” (B1*, 435). The development of the banking system, and its periodic crises, plays a fundamental rule in this process.

Hence the processes of capital accumulation (which for Marx is synonymous to the concentration of capital) and capital centralization are complementary in allowing for the growth in the scale of operations of industrial capitalists, whether individual or associated. “Everywhere the increased scale of industrial establishments is the starting point for a more comprehensive organisation of the collective work of many, for a

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2The few quotes of Book I referenced by B1* have been taken from the English translation published by Progress Publishers, Moscow, because they are not present in MECW 35.
wider development of their material motive forces— in other words, for the progressive transformation of isolated processes of production, carried on by customary methods, into processes of production socially combined and scientifically arranged” (B1*, 436). But a distinctive feature of the centralization of capital is that, in contrast to the simple concentration of capital through accumulation on an extended scale—which operates gradually over time—, it can take place in short periods of time, either through the formation of stock companies, or through bank credit. Both augment “the inducements and facilities to accumulate”, making it feasible to convert “money into capital without becoming an industrial capitalist” (B3, 265). And in particular, “The banking system, so far as its formal organisation and centralisation is concerned, is the most artificial and most developed product turned out by the capitalist mode of production” (B3, 601), since it “places all the available and even potential capital of society that is not yet actively employed at the disposal of the industrial and commercial capitalists so that neither the lenders nor users of this capital are its owners or producers. It thus does away with the private character of capital... By means of the banking system the distribution of capital as a special business, a social function, is taken out of the hands of the private capitalists and usurers. But at the same time, banking and credit thus become the most potent means of driving capitalist production beyond its own limits, and one of the most effective vehicles of crises and swindle” (B3, 602). The latter refers of course to the speculative manoeuvres of financial capitalists in the final phase of the industrial cycle that ends with a financial crisis and economic recession.

The fall in the rate of profit

Introduction

As we have already mentioned, for Marx a ‘law’ was in essence a ‘tendency’, which can be the result of various forces that push in different directions, even opposite ones. On occasions, however, Marx emphasizes this idea by combining both words (‘law’ and ‘tendency’), as when in Part III of Book III he addresses ‘The law of the tendency of the rate of profit to fall’. As we have seen, in Book I (especially Chapter 25) he had dealt with various ‘laws’ that could be formulated without eliminating the (explicit or implicit) assumptions that all branches had the same value compositions of capital, that there was no private property of natural resources, that there was no heterogeneity of productive methods, and that there were no disequilibriums between supply and demand, with the important exception of labor. Marx expressed most of these assumptions with the encompassing assumption that, in Book I, commodities were transacted according to their values, not only in the initial chapters which deal with SCP but also in the rest which deals with CCP. He preferred to leave the tendency of the profit rate to fall to Part III of Book III, that is, after having presented in Part I “The conversion of surplus value into profit and of the rate of surplus value into the rate of profit” and in Part II, the “Conversion of profit into average profit”, but before introducing commercial and financial capital in Part IV, the distinction between interest and profit of enterprise in Part V, and ground rent in Part VI.

In the evaluation of Marx’s law on the tendency of the rate of profit to fall, it is important to bear in mind that the idea and the perception of this ‘tendency’ were prevalent in his times. Adam Smith had written that profits tended to fall in any branch in which the competition between capitals made wages increase, and that the rate of profit did not increase along with the wealth of nations but quite the contrary: it tended to be low in rich countries and high in poor countries. In contrast, Ricardo believed that the rate of profit declined basically as a consequence of the decreasing returns in
agriculture, as increasingly less fertile lands were cultivated, making (aggregate) rents on land rise, and as wages rose on account of the increase in the price of food—which weighed heavily in the expenditure of workers. Although that tendency for profits to fall was sometimes offset by increases in productivity related to the introduction of machinery and discoveries in the ‘science of agriculture’ that made the value of workers’ consumption goods fall and also lowered the demand for labor, in the long run there would be no growth because the rate of profit would be too low (and rents and taxes too high). But even after the time period in which Marx wrote most of his works there were many leading economists that believed that the empirical data showed a diminishing tendency in the rate of profit. W. Stanley Jevons, one of the initiators of the ‘marginalist revolution’, wrote in the second edition of *The Theory of Political Economy* (de 1879):

> It is one of the favourite doctrines of economists since the time of Adam Smith, that as society progresses and capital accumulates, the rate of profit, or more strictly speaking, the rate of interest, tends to fall. The rate will always ultimately sink so low, they think, that the inducements to further accumulation will cease...

Our formula for the rate of interest shows that unless there be constant progress in the arts, the rate must tend to sink towards zero, supposing accumulation of capital to go on. *There are sufficient statistical facts, too, to confirm this conclusion historically.* The only question that can arise is as to the actual cause of this tendency (Jevons 1965, 254; italics added).

Marx had no doubts on the empirical validity of the long run fall of the rate of profit. (And in contrast to Jevons he clearly distinguished the profit rate from the interest rate.) But his argumentation on the factors to consider and his analyses of the tendencies and countertendencies were much more detailed and organic than those of his predecessors and contemporaries. Although he believed it was the predominant long run tendency, which strengthened his belief on the transitory nature of Capitalism and its inevitable collapse, he highlighted various countertendencies that buffered or even reversed it during certain periods of time. Hence, Chapter 13 of Book III is immediately followed by Chapter 14, on the “Counteracting influences.” It is important to emphasize that both the tendency and some of its main countertendencies were considered by Marx as manifestations of the notable progressive increase in labor productivity generated by modern industry.

To understand Marx’s reasoning on this subject, it is important to bear in mind his distinction between the ‘technical composition’ and the ‘value composition’ of capital (already dealt with in Chapter 8). Whereas the former referred exclusively to the *quantities* of means of production in relation to *quantities* of labor forces operating in the production process, the latter referred to their respective *amounts* (valued either in *values* or, more generally, in equilibrium prices3). Hence, this second concept was affected both by the ‘technical composition’ of capital and by the value of each one of the commodities in which capital was invested. For most of his analyses these two concepts were sufficient, but in some cases he came to find useful an intermediate concept he called the ‘*organic composition*’ of capital, which was only used when changes in the ‘value composition’ of capital were considered under the assumption that the relative

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3Remember that the ‘prices of production’ were Marx’s equilibrium prices in CCP only before the introduction of the private property of land and other natural resources, and the ‘natural monopolies’ we deal with below.
prices used in the valuations remained constant and, hence, the changes in the ‘value composition’ were exclusively due to changes the ‘technical composition’. That is why, when he defines the technical and value composition of capital he adds: “Between the two there is a strict correlation. To express this, I call the value composition of capital, in so far as it is determined by its technical composition and mirrors the changes of the latter, the organic composition of capital” (B1, 608).

In particular, he found this concept useful in the analysis of the long run tendency of the rate of profit. Since his formula for the global profit rate (which was approximate in the general case and only exact in the case of equal value compositions of capital in all branches) could be expressed as \( \rho = e/(\kappa + 1) \), if the global rate of surplus value \( e \) remained constant while the global value composition \( \kappa \) increased, the rate of profit necessarily had to fall. But the long run change in \( \kappa \) could be separated into the long run change in the ‘organic composition’ (with constant values) and the changes in values (with constant ‘technical composition’). For Marx the key characteristic of Capitalism –after its initial phase of ‘absolute surplus value’ generation– was the progressive development of the productive forces, i.e., the increase in productivity. And this increase had multiple manifestations. First, there was a reduction in the values and equilibrium prices of produced commodities, since a progressively smaller amount of labor was (directly or indirectly) required. But on the other hand, this increase in productivity manifested itself as an increase in the quantities of means of production that were used by the labor force, that is, an increase in the ‘technical composition’ of capital. Marx’s fundamental explanation of the long run fall in the rate of profit was that the increase in the ‘technical composition’ of capital was in the long run greater than the fall in the unit values of the means of production and the means of subsistence of the wage workers that used them. Since the values of the means of production could fall more than the means of subsistence, the “cheapening of the elements of constant capital” was one of the ‘counteracting influences’ of the ‘law’. But there were also others. He begins Chapter 14 of Book III with the following:

If we consider the enormous development of the productive forces of social labour in the last 30 years alone as compared with all preceding periods; if we consider, in particular, the enormous mass of fixed capital, aside from the actual machinery, which goes into the process of social production as a whole, then the difficulty which has hitherto troubled the economists, namely to explain the falling rate of profit, gives place to its opposite, namely to explain why this fall is not greater and more rapid. There must be some counteracting influences at work, which cross and annul the effect of the general law, and which give it merely the characteristic of a tendency (B3, 230)

In Chapter 5 of Book III, Marx had already addressed the ‘economy in the employment of constant capital’. After considering how capitalists could increase their profit rate by lengthening the working day (thus generating ‘absolute surplus value’), Marx addressed the increases in the profit rate related to the generation of ‘relative surplus value’. If the length of the working day increased without paying higher wages, the rate of profit would increase because the elements of fixed capital would be used more but the disbursement of fixed capital would remain constant. But if the working day remained constant,

Then it is necessary either to increase the number of labourers, and with them to a certain extent the amount of fixed capital... or, if the intensity
and, consequently, the productive power, of labour increase and, generally, more relative surplus value is produced, the magnitude of the circulating portion of constant capital increases in such industrial branches which use raw materials... Hence, an increase in surplus value is accompanied by an increase in constant capital, and the growing exploitation of labour by greater outlays in the conditions of production through which labour is exploited, i. e., by a greater investment of capital. Therefore, the rate of profit is thereby reduced on the one hand while it increases on the other (B3, 81; italics added).

These counteracting influences on the profit rate of the increases in the productivity (or intensity) of labor, which increase the surplus value, on the one hand, but increase the magnitude of the disbursement of constant capital, on the other, are reconsidered by Marx in Part III of Book III when he addresses one of the main ‘counteracting influences’ (to the long run fall in the profit rate). For Marx both the long run increase in the ‘organic composition’ (or ‘technical composition’) of the invested capital and the ‘cheapening’ of the means of production were manifestations of the progressive increase in the productivity of labor, and the two had opposite influences on the global profit rate.

As we have seen, in Chapter 4\textsuperscript{4} of Book III, “The effect of the Turnover on the Rate of Profit” which had previously been studied in Book II, is again addressed. In the latter he had explained that, because the ‘cycle of money capital’ had three phases, in which capital adopts, respectively, the forms of ‘money capital’, ‘productive capital’, and ‘commodity capital’, the turnover of capital extended beyond the ‘time of production’, since there was a ‘time of circulation’ both at the beginning of the cycle (in which inputs are purchased) and at the end (in which the produced commodities are sold) that affected the calculation of the profit rate. He had there made the simplifying assumption—as in Book I that “commodities are produced under normal social conditions and are sold at their values.” But he had also synthetically expressed the core of what he would treat more extensively later, writing that “some of the capital always lies idle, either in the form of money capital, of raw material supplies, of finished but still unsold commodity capital, or of outstanding claims.” Hence, shortening the turnover period was one of the ways in which the rate of profit could be positively affected, since “The shorter the period of turnover, the smaller this idle portion of capital.” And he adds:

The chief means of reducing the time of production is higher labour productivity, which is commonly called industrial progress. If this does not involve a simultaneous considerable increase in the outlay of total capital resulting from the installation of expensive machinery, etc., and thus a reduction of the rate of profit, which is calculated on the total capital, this rate must rise. And this is decidedly true in the case of many of the latest improvements in metallurgy and in the chemical industry. The recently discovered methods of producing iron and steel, such as the processes of Bessemer, Siemens, Gilchrist-Thomas, etc., cut to a minimum at relatively small costs the formerly arduous processes (B3, 74; italics added).

It is noteworthy that in this paragraph (written by Engels) there is an inversion of the ‘tendency and countertendency’ that is specific to this topic as had been pointed out

\textsuperscript{4}As Engels explains in his Preface to Book III, “Nothing but the title was available for Chapter IV. But since its subject-matter, the influence of turnover on the rate of profit, is of vital importance, I have written it myself, for which reason the whole chapter has been placed in brackets.”
by Marx in Part III, since the tendency to the increase of the profit rate due to the increase in labor productivity is mentioned first and then it is noted that the rate of profit may fall if this increase in productivity necessitates a very large investment in expensive machinery, etc. It nevertheless shows that Marx’s (and Engel’s) reasoning was by no means purely abstract or dogmatic, but tried to be anchored in the empirical data that could shed some light on the factors that affected the long run profitability of capital. On the other hand, other factors are also there considered, such as the advances in communications and transportation, which helped to reduce the time of circulation (and hence increase the profit rate).

The ‘law’ according to Marx

Marx begins with a simple numerical example in which \( k \) gradually increases while \( e \) remains constant. And remarks: “This is how the same rate of surplus value would express itself under the same degree of labour exploitation in a falling rate of profit, because the material growth of the constant capital implies also a growth—albeit not in the same proportion—in its value, and consequently in that of the total capital.”

This sentence clearly expresses what we said above about the distinction between the ‘technical composition’ and the ‘value composition’ of capital. Whereas there is a ‘material’ growth of constant capital (which increases the technical as well as the organic composition of capital), its value also increases but “not in the same proportion” due to the cheapening of the elements of constant capital. But Marx did not economize on words:5

This is just another way of saying that owing to the distinctive methods of production developing in the capitalist system the same number of labourers, i.e., the same quantity of labour power set in motion by a variable capital of a given value, operate, work up and productively consume in the same time span an ever-increasing quantity of means of labour, machinery and fixed capital of all sorts, raw and auxiliary materials—and consequently a constant capital of an ever-increasing value. This continual relative decrease of the variable capital vis-à-vis the constant, and consequently the total capital, is identical with the progressively higher organic composition of the social capital in its average. It is likewise just another expression for the progressive development of the social productive power of labour, which is demonstrated precisely by the fact that the same number of labourers, in the same time, i.e., with less labour [per unit of output], convert an ever-increasing quantity of raw and auxiliary materials into products, thanks to the growing application of machinery and fixed capital in general... This mode of production produces a progressive relative decrease of the variable capital as compared to the constant capital, and consequently a continuously rising organic composition of the total capital. The immediate result of this is that the rate of surplus value, at the same, or even a rising, degree of labour exploitation, is represented by a continually falling general rate of profit (B3, 210-1; italics and text in square brackets added).

When in the next chapter Marx addresses the “Cheapening of Elements of Constant Capital” as one of the ‘counteracting influences’, the above interpretation for his argu-

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5Precisely due to the length of many of Marx’s analyses many readers abstain from reading Marx’s Capital. But as we shall see, obtaining information on Marx’s theory through syntheses that do not capture many of the subtleties of his arguments—and sometimes distort them completely—can impair its comprehension.
mentation is confirmed:

... the value of the constant capital does not increase in the same proportion as its material volume. For instance, the quantity of cotton worked up by a single European spinner in a modern factory has grown tremendously compared to the quantity formerly worked up by a European spinner with a spinning-wheel. Yet the value of the worked-up cotton has not grown in the same proportion as its mass. The same applies to machinery and other fixed capital. In short, the same development which increases the mass of the constant capital in relation to the variable reduces the value of its elements as a result of the increased productivity of labour, and therefore prevents the value of constant capital, although it continually increases, from increasing at the same rate as its material volume, i.e., the material volume of the means of production set in motion by the same amount of labour power. In isolated cases the mass of the elements of constant capital may even increase, while its value remains the same, or even falls (B3, 234).

Marx returns to the same subject when in Chapter 15 of Book III he deals with “Excess capital and excess population.” He describes an industrial cycle with “overproduction of capital” that leads to a crisis that will end up preparing the conditions for “a subsequent expansion of production.” For the employment by some capitalists of “new machines, new and improved working methods, new combinations” with the aim of reaping extra profits, increases the composition of capital, on the one hand, but also produces a “depreciation of the elements of constant capital” which tends to raise the rate of profit:

... the fall in prices and the competitive struggle would have driven every capitalist to lower the individual value of his total product below its general value by means of new machines, new and improved working methods, new combinations, i.e., to increase the productive power of a given quantity of labour, to lower the proportion of variable to constant capital... Ultimately, the depreciation of the elements of constant capital would itself tend to raise the rate of profit. The mass of employed constant capital would have increased in relation to variable, but its value could have fallen. The ensuing stagnation of production would have prepared—with capitalistic limits—a subsequent expansion of production (B3, 254; italics added).

Many of Marx’s critics, used to the Neoclassical conception that an adequate model can only show one production method for each commodity at a time (except for anomalous borderline cases), argue that capitalists would not introduce a method of production that could make the global profit rate diminish. However, this argument is incompatible with Marx’s realistic and dynamic perspective, in which there normally coexist various ways of producing each commodity and the capitalist that introduces the innovation is concerned especially with the immediate effect it has in raising his profit rate even if the eventual imitation by his competitors could finally diminish the global profit rate. The Neoclassical argument had already been answered explicitly by Marx:

No capitalist ever voluntarily introduces a new method of production, no matter how much more productive it may be, and how much it may increase

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6 The original has ‘farmer’ instead of ‘power’, an obvious slip.
the rate of surplus value, so long as it reduces the rate of profit. Yet every such new method of production cheapens the commodities. Hence, the capitalist sells them originally above their [individual] prices of production, or, perhaps, above their value. He pockets the difference between their costs of production and the market prices of the same commodities produced at higher costs of production... His method of production stands above the social average. But competition makes it general and subject to the general law. There follows a fall in the rate of profit—perhaps first in this sphere of production, and eventually it achieves a balance with the rest—which is, therefore, wholly independent of the will of the capitalist (B3, 263-4; text in square brackets added).

Other counteracting influences to the ‘law’ of the declining profit rate that Marx considers are the following: 1) the increase in the intensity of labor exploitation in modern industry by means of a) the lengthening of the working day (“this invention of modern industry”), b) the operation of more machines by the same worker, c) increasing the velocity of the machinery that is operated, d) the “widespread introduction of female and child labour into the labor force”; 2) the appearance of new industrial branches with relatively low composition of capital that a) absorb some of the ‘relative overpopulation’ of workers, and b) have lower wages and higher rate of surplus value than the rest, tending to increase the global profit rate, 3) international trade, by cheapening the elements of constant and/or variable capital, 4) the investment of capital abroad, where often higher profit rates are possible because the competition uses more backward production methods, and 5) the increase of the stock capital in “large production enterprises” (like railroads), whose owners obtain dividends that reflect a relatively low profit rate, but since they do not “go into levelling the general rate of profit” they prevent the general rate of profit from falling further.

When Marx elaborates on 4) he asks the question “Is the general rate of profit raised by the higher rate of profit produced by capital invested in foreign, and particularly colonial, trade?” (B3, 236). He answers that “capitals invested in colonies” tended to yield higher profit rates than those invested in England due to the “backward development” prevailing there, including “the use of slaves, coolies, etc.” The competition with local firms using more backward methods of production would tend to increase the English average rate of profit without necessarily equalizing the profit rates that are averaged (at least until the colony reached the English level of development). Hence, the colonial trade would have some counteracting effect on the fall of the average profit rate. But Marx also has a pair of sentence that are difficult to interpret: “It is hard to see why these higher rates of profit, realised by capitals invested in certain lines and sent home by them, should not, unless monopolies stand in the way, enter here into the equalisation of the general rate of profit and thus tend, pro tanto, to raise it. It is hard to see this in particular if these spheres of investment of capital are subject to the laws of free competition (B3, 236-7). The qualification “unless monopolies stand in the way” is especially cryptic (both in the English translation as in the original German). The explanation that we deem most plausible is that Marx was referring to the interference of, for example, a British firm with monopsony power that can negotiate a lower price from the one that makes the overseas investment. In that case Marx would be expressing the same idea that appears further down in Book III where he refers to stock companies: that if the firm making the investment abroad was one of those huge

7In German, the qualification is: “wenn sonst nicht Monopole im Wege stehn”, which the Google translator converts to: “if otherwise monopolies do not stand in the way.”
firms endowed with monopoly power, its rate of profit would not necessarily “enter into the equalisation of the general rate of profit”, raising the average rate but not unleashing mechanisms that tend to equalize them. Indeed, in several places in Book 3 Marx recognizes that there are capitalist branches with profit rates that do not enter into the leveling process for profit rates.

Such is the case of the brief paragraph Marx dedicates to 5), which is difficult to interpret. He states that stock companies do not “go into levelling the general rate of profit.” But he only specifically mentions railways, and says that if these firms did go into the levelling process, the rate of profit “would fall much lower”, “because they yield a lower than average rate of profit.” But then it is difficult to understand why such stock companies were included among the “counteracting influences.” It seems that the only reason is that the presumed fact that they do not enter the equalization process of profit rates prevents the global profit rate from falling further. The only explanation given for the low profit rates in railroads is that they have a very high composition of capital, which is flimsy as an argument. Marx could have here invoked the mechanisms (as J. S. Mill did) that have to do with the capacity to set higher prices due to the failure of the normal competitive process in the presence of a ‘natural monopoly’, and the need for government intervention to protect the public interest, which could have the effect of lowering the rate of profit (even if profits take the form of dividends). But it is not at all clear that this is was he intended to argue.  

**Analytical representation of Marx’s ‘law’**

Marx associated the approximately decennial economic cycle in the England of his time with the generational changes in the stocks of machinery and equipment and also associated the solutions to the crises that marked the end of these cycles with the introduction of a new generation of machinery and equipment that made obsolete a portion of the existing fixed capital. He associated the increase in productivity with the increase in the mass and the value of the constant capital tied up in production. Though the increase in productivity tended to produce a relative cheapening of constant capital, he believed that in the long run the increase in the volume and value of invested constant capital prevailed, thus generating a tendency for the rate of profit to fall that could explain what was widely believed to be an empirical fact. In this section we show the analytics of Marx’s explanation. For this, we abstract from the industrial cycle and focus exclusively on the long run effects. To do this, we make a slight adaptation to the Extended Reproduction model of Chapter 14 with arbitrary turnover periods. A single change is made to that model: in some period there is assumed to be a discrete increase in the stocks of elements of constant capital that are tied up in production (and used). We assume that the populations are stationary and that capitalists introduce recurrent innovations (at the rate $g$) that increase labor productivity.

Analytically, the only modification to that last model of ER models in Chapter 14 that is needed is the introduction of a parameter $\delta$ that multiplies the stock matrix $A^S$

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8 Marx also includes, in passing and as an empirical fact, “The depression of wages below the value of labour power”. He adds that this “has nothing to do with the general analysis of capital, but belongs in an analysis of competition, which is not presented in this work”. Apparently, Marx is here alluding to the double accounting (of values and equilibrium prices) that he sometimes used in his analyses, especially in matters related to rural production.

In early plans for his research (see his letter to Engels of April 2, 1958) Marx had separated his ‘book’ on Capital into 4 ‘sections’: “a) Capital en general. (This is the substance of the first installment), b) Competition, or the interaction of many capitals, c) Credit, where capital, as against individual capitals, is shown to be a universal element, d) Share capital as the most perfected form.” His manuscript for this part of Book III was restricted, in Marx’s mind, mostly to a).
in (14.37), (14.38), and (14.39). The periodic (say, decennial) increases in δ reflect in stylized form the increase in the technical (and ‘organic’) composition of capital that is due to the introduction of a new vintage of constant capital elements that increase the mass of invested constant capital. Such discrete increases in δ at the beginning of a new industrial cycle are additional to the annual increases in the technical composition based on the increase in productivity (and formalized as before by means of the reduction of the direct labor requirements ℓ each period through the parameter g). These decennial increases in δ are assumed to be net of the retirement of elements of fixed capital that may have turned obsolete. Replacing δA^S for A^S in (14.37), (14.38), and (14.39), these systems are valid for various given values of δ. We can represent the transition to a new cycle by the transition from an initial situation in which δ = 1—and for which equations (14.37), (14.38), and (14.39) are valid just as they are—to a new situation in which δ > 1. From the first two equations of (14.37), labor employment is

\[ q^L = (q^L c_L + q^K c_K) B^S (g, 1) \ell \]  

(17.1)

where for any given value of δ we define \(B^S (x, \delta) \equiv (1 + x) [I - (1 + x) A - x\delta A^S]^{-1}\). Notice that \(B^S (g, \delta)\) is increasing in g and in δ. Repeating the procedure used to obtain (14.40) and (14.42), we initially obtain

\[ 1 = \bar{c}_L B^S (\rho_g, 1) \bar{\ell}, \]  

(17.2)

which gives the initial level of \(\rho_g\).

Let us make the (strong) assumption that the transition to the new cycle is neutral in the amount of employment of the labor force \(q^L\). Assume also that the increase in δ does not produce any changes in the levels of \(\ell_t, c_{L,t}\), or \(c_{K,t}\). We now verify that under these conditions it is necessary that the rate of growth g of labor saving innovations diminish. If g falls to \(g' < g\), the assumption on the constancy of \(\ell_t, c_{L,t}\), and \(c_{K,t}\) during the transition to a new cycle implies that the levels of \(\bar{\ell}, \bar{c}_L,\) and \(\bar{c}_K\) must change, say to \(\bar{\ell}', \bar{c}_L',\) and \(\bar{c}_K'\). And observing (14.28) and (14.30), they must be such that

\[ c_{L,t} = (1 + g) \bar{c}_{L,t} = (1 + g') \bar{c}_{L,t}', \]

\[ c_{K,t} = (1 + g) \bar{c}_{K,t} = (1 + g') \bar{c}_{K,t}', \]

\[ \ell_t = \bar{\ell}_t / (1 + g) = \bar{\ell}_t' / (1 + g'). \]

Hence, the new consumption baskets and direct labor requirements corrected for trend (and hence stationary) must have the following relations with the originals:

\[ c'_L = \frac{1 + g}{1 + g'} c_L, \quad c'_K = \frac{1 + g}{1 + g'} c_K, \quad \bar{\ell}' = \frac{1 + g'}{1 + g} \bar{\ell}. \]

Hence, \(\bar{c}_L \bar{\ell}' = \bar{c}_L \bar{\ell} \) and \(\bar{c}_K \bar{\ell}' = \bar{c}_K \bar{\ell}\), i.e., after the increase in δ and the change in g, the increases in \(\bar{c}_L\) and \(\bar{c}_K\) (\(\bar{c}_L' > \bar{c}_L, \bar{c}_K' > \bar{c}_K\)) exactly compensate for the reduction in \(\bar{\ell}\) (\(\bar{\ell}' < \bar{\ell}\)). Therefore, after the increase in δ to a value greater than one the equations that correspond to (17.1) and (17.2) are:\(^9\)

\[ q^L = (q^L c'_L + q^K c'_K) B^S (g', \delta) \bar{\ell}' = (q^L c_L + q^K c_K) B^S (g', \delta) \bar{\ell} \]  

(17.3)

\(^9\)Notice that matrices \(B^S (g, \delta)\) and \(B^S (\rho_g, \delta)\) have identical definitions and only differ in their first parameter \(g\) and \(\rho_g\), respectively. Also, expanding the matrix in series we can easily check that \(B^S (x, \delta)\) is increasing both in \(x\) as well as in \(\delta\).
\[ 1 = \nu_L B^S (\rho_{g'}, \delta) \underline{t} = \nu_L B^S (\rho_{g'}, \delta) \underline{t}, \quad (17.4) \]

Comparing (17.3) to (17.1) we can see that, under the assumption of neutrality in labor employment, the increase in \( \delta \) (from 1) must be compensated by a fall in the rate of growth (from \( g \) to \( g' \)). Similarly, comparing (17.4) to (17.2) we see that the increase in \( \delta \) must be compensated by a fall in the profit rate (from \( \rho_g \) to \( \rho_{g'} \)).

Of course, this fall in the profit rate is based on some assumptions that might not be validated. For example, capitalists could manage to diminish the consumption of workers \( c_{L,t} \) in the transition. That would probably be their preferred option, and it can be associated with some of the counteracting influences pointed out by Marx, such as 1) the increase in the rate of labor exploitation, and possibly 2) the reduction in the wage rate below the value of labor power, which could be interpreted as the beginning of the establishment of a new customary level of worker consumption.

Another assumption that could prove to be invalid is that of the constancy in the amount of labor employed \( q^L \). If it fell, (17.3) shows that an even greater reduction in \( g \) would be needed. Employment would have to increase in order to attenuate the fall in \( g \) or make in increase. How the effect of an increase in \( \delta \) is distributed between falls in \( c_L \) and in \( g \) could be determined by the relative negotiating power of capitalists and workers, i.e., as the result of ‘class struggle’, where the State would also play a role, as Marx often pointed out.

In summary, we have a reasonable analytical representation, in the context of a model of ER, of what Marx called ‘the tendency for a falling rate of profit’ and of some of the possible counteracting influences. The model reasonably reflects the gist of Marx’s arguments as well as the ER model he developed, and can be used to replicate much of his analyses. Using it we can show how a fall in the rate of profit can take place (if it is not compensated by other changes) as a consequence of an increase in the technical (and organic) composition of invested capital that Marx associated to approximately decennial technological revolutions that sustained the process of increasing labor productivity. To simplify, we have generated the increase in the technical composition of capital by means of an even increase in all the stocks of constant capital tied to the production process, not distinguishing between its fixed and circulating elements.

Monopoly, management versus ownership, and State intervention
The concept of monopoly in Marx’s time

In the era of Classical economics and Marx, the term ‘monopoly’ was used with multiple meanings.\(^{10}\) Malthus and Ricardo (in the last chapter of Principles on “Mr. Malthus’s Opinions on Rent”) had distinguished between ‘artificial’ (or ‘legal’) monopolies and ‘natural’ (or ‘necessary’) monopolies. The former were generated by public policy, for example, when exclusive rights were granted to certain enterprises. The term ‘natural monopoly’ was also used by J. S. Mill in his Principles of Political Economy,\(^{11}\) the first edition of which was published in 1848.\(^{12}\) Mill writes that natural monopolies are “those which are created by circumstances, and not by law” (Mill 1871, 410). But the ‘circumstances’ to which Mill referred were quite varied. For example, for him the fact that a goldsmith had a greater wage than other workers was explained by the ‘natural

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\(^{10}\)The notable exception was the French mathematician Cournot, who was pathbreaking in mathematical economics and already in 1838 treated rigorously both monopoly and duopoly in his Researches on the Mathematical Principles of the Theory of Wealth (1838).

\(^{11}\)Marx had read Mills Principles, for he quotes from an 1868 edition (in Chapter 16 of B1).

\(^{12}\)This was the year Rossi (who we encounter below) was murdered in Rome, where he was the Minister of the Interior under Pope Pius IX.
monopoly’ he had on his trustworthiness, that essential characteristic that was needed to be entrusted with highly valuable gold as raw material to work on. However, J. S. Mill also dealt at length with cases that later Neoclassical economics continued calling ‘natural monopolies’. And he argued for State intervention in such cases in order to, either by regulation or by direct State administration, avoid rendering “the liberty of competition almost illusory”:

When, in any employment, the régime of independent small producers has either never been possible, or has been superseded, and the system of many work-people under one management has become fully established, from that time any further enlargement in the scale of production is generally an unqualified benefit. It is obvious, for example, how great an economy of labour would be obtained if London were supplied by a single gas or water company instead of the existing plurality. While there are even as many as two, this implies double establishments of all sorts, when one only, with a small increase, could probably perform the whole operation equally well... Were there only one establishment, it could make lower charges, consistently with obtaining the rate of profit now realized. But would it do so?... Where competitors are so few, they always end by agreeing not to compete... When, therefore, a business of real public importance can only be carried on advantageously upon so large a scale as to render the liberty of competition almost illusory, it is an unthrift dispensation of the public resources that several costly sets of arrangements should be kept up for the purpose of rendering to the community this one service (Mill 1871, 143).

For Mill the logical solution was to “treat it at once as a public function”, either by having the government itself undertake the necessary investment and management or by granting the monopoly to a single firm or association and regulating it “on the best terms for the public.” In the case of railways, for example, only a single line ought to be allowed to connect the same two cities. But “control over that line never ought to be parted with by the State” (Ibid.)

Further down Mill explicitly calls such cases ‘natural monopolies’: “All the natural monopolies (meaning thereby those which are created by circumstances, and not by law) which produce or aggravate the disparities in the remuneration of different kinds of labour, operate similarly between different employments of capital.... A trade may also, from the nature of the case, be confined to so few hands, that profits may admit of being kept up by a combination among the dealers... I have already mentioned the case of the gas and water companies” (Ibid., 410).

**Marx’s uses of the term ‘monopoly’**

Marx refers to artificial monopolies on multiple occasions (usually not calling them ‘artificial’), as when he writes that “The English East India Company, as is well known, obtained, besides the political rule in India, the exclusive monopoly of the tea trade, as well as of the Chinese trade in general, and of the transport of goods to and from Europe” (B1, 740). In his early *Poverty of Philosophy*, Marx referred to Pellegrino Rossi (“whom M. Proudhon quotes several times on the subject of monopoly”) who “distinguishes between artificial monopolies and natural monopolies.”

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13Rossi had given the lectures of his *Cours d’Économie Politique* in the *Collège de France* in 1836-37, and it was published in French in 1840-41. In Book I Marx quotes from a Brussels edition of 1843.
olies, he says, are artificial, that is, arbitrary; bourgeois monopolies are natural, that is, rational” (Poverty, MECW 6, 195). Using Hegelian language, he describes the dialectical relation between monopoly and competition: “In practical life we find not only competition, monopoly and the antagonism between them, but also the synthesis of the two, which is not a formula, but a movement. Monopoly produces competition, competition produces monopoly. Monopolists compete among themselves; competitors become monopolists. If the monopolists restrict their mutual competition by means of partial associations, competition increases among the workers; and the more the mass of the proletarians grows as against the monopolists of one nation, the more desperate competition becomes between the monopolists of different nations. The synthesis is such that monopoly can only maintain itself by continually entering into the struggle of competition” (Ibid.). When he writes that “monopolists restrict their mutual competition by means of partial associations” he is referring to what we now call ‘oligopolistic collusion’.

In Capital Marx often referred to the ‘monopoly’ of capitalists over the capacity to purchase means of production and labor power, and of landowners over land. None of these cases implied that capitalists or landowners colluded to impose what we now call a ‘monopoly price’, i.e., a price that is fixed by a single supplier in order to obtain maximum profit (as in Cournot). On the other hand, in the context of his analysis of differential rent Marx refers to the ‘monopoly’ over “waterfalls, rich mines, waters teeming with fish, or a favourably located building site.” He writes: “Wherever rent exists at all, differential rent appears at all times, and is governed by the same laws, as agricultural differential rent. Wherever natural forces can be monopolised and guarantee a surplus profit to the industrial capitalist using them... there the person who by virtue of title to a portion of the globe has become the proprietor of these natural objects will wrest this surplus profit from functioning capital in the form of rent” (B3, 759). That is, the extra profit obtained by the producer of a commodity by the use of particular natural resource that gives him an advantage is transformed into (differential) rent if his owner ‘monopolizes’ this resource. He also uses in the same context the term ‘monopoly price’ when he writes:

When we refer to a monopoly price, we mean in general a price determined only by the purchasers’ eagerness to buy and ability to pay. A vineyard producing wine of very extraordinary quality which can be produced only in relatively small quantities yields a monopoly price. The wine-grower would realise a considerable surplus profit from this monopoly price, whose excess over the value of the product would be wholly determined by the means and fondness of the discriminating wine-drinker. This surplus profit, which accrues from a monopoly price, is converted into rent (B3, 762).

Here the ‘monopoly’ over a natural resource with special properties, the supply of which cannot be increased, allows whoever exploits it to charge a ‘monopoly price’ that is above the price of production, a price that is higher the higher is the demand for the output.\textsuperscript{14} One of the important matters in this respect for Marx was that such cases transcended his ‘prices of production’, which assumed many competing capitalists under conditions which in the theoretical construction the private property of natural resources had not yet been introduced (in accordance with the sequence of elimination

\textsuperscript{14}This is the sense given by Ricardo when he writes: “When a commodity is at a monopoly price, it is at the very highest price at which the consumers are willing to purchase it. Commodities are only at a monopoly price, when by no possible device their quantity can be augmented; and when therefore, the competition is wholly on one side amongst the buyers” (Ricardo 2004, Vol. 1, 249).
of simplifying assumptions that defined the ‘architecture’ of *Capital*). And in *historical reality* there were many cases in which independent farmers used lands that existed in such abundance that they were of free access; or in which community members made use of communal lands that were freely available to them. We have already seen Marx’s treatment of this matter in Chapter 16.

Although Marx does not quote J. S. Mill when he uses the term ‘natural monopoly’\(^{15}\), when he refers to stock companies and, among them, those involved in railways, he gives the impression of having used some of Mill’s ideas. He also gives his own (also vague) definition of ‘natural monopoly’ when he analyzes the equalization of profit rates. He writes that this process is accomplished “so much more quickly, 1) the more mobile the capital i.e., the more easily it can be shifted from one sphere and from one place to another; 2) the more quickly labour power can be transferred from one sphere to another and from one production locality to another” (B3, 195). And the first condition required “the removal of all monopolies with the exception of the natural ones, those, that is, which naturally arise out of the capitalist mode of production” (Ibid.; italics added). This is quite similar, though with differential hues, to J. S. Mill’s definition that they are those “created by circumstances.” But expressing that ‘natural monopolies’ are those that “naturally arise out of the capitalist mode of production” is still not precise. We will see below that he was probably referring to the effects produced by the process of centralization of capital in some branches and the consequent emergence of huge enterprises (in the legal form of stock companies) that dominated in their respective branches and had certain monopolistic or oligopolistic power. This process was a tendency that naturally arose “out of the capitalist mode of production.”

When in Book II Marx studies the cyclical process of industrial capital he addresses the effect that the tendency to increase the scale of production has on the need firms have for a very large capital. He states that “as the scale of each individual process of production and with it the minimum size of the capital to be advanced increases in the process of capitalist production, we have here another circumstance to be added to those others which transform the function of the industrial capitalist more and more into a monopoly of big money capitalists, who may operate singly or in association” (B2, 112; italics added). Although the word ‘monopoly’ is used casually, it seems to point to the capacity these big firms have to escape from the leveling of the profit rates because they can often avoid the pressures of competition both when they are operated by their owners as when the top management obtains some degree of independence from the stockholders in a stock company. We address below what Marx has to say about this last case.

**Stock companies and huge firms with monopoly power**

Marx again considers the topic of monopoly power when in Part V of Book III he deals with the role of credit. There he highlights not only the processes of ‘centralization’ of capital that tends to eliminate the weakest firms and hence reduce competition, but also the tendency to create *stock companies*. These facilitate the “enormous expansion of the scale of production and of enterprises, that was impossible for individual capitals” (B3, 434) but is feasible for stock companies. “The capital... is here directly endowed with the form of social capital (capital of directly associated individuals)” (Ibid.). According to Marx the huge firms with very high composition of invested capital that are made possible by the formation of stock companies, and whose profits are distributed as

\(^{15}\)One must bear in mind that Marx never prepared Books II and III for publication, and hence they contain much fewer references to other authors than Book I does.
dividends that are akin to the interests on loans, “do not necessarily enter into the equalisation of the general rate of profit”:

Before we go any further, there is still the following economically important fact to be noted: Since profit here assumes the pure form of interest, undertakings of this sort are still possible if they yield bare interest, and this is one of the causes stemming the fall of the general rate of profit, since such undertakings, in which the ratio of constant capital to the variable is so enormous, do not necessarily enter into the equalisation of the general rate of profit (B3, 435; italics added).

From the observation that the existence of such firms “is one of the causes stemming the fall of the general rate of profit” we must infer that their profits, distributed as dividends to stockholders, tend to be higher than average. And the observation that they “do not necessarily enter into the equalisation of the general rate of profit” thus becomes an important structural and tendential phenomenon. Marx was probably referring to such firms when he wrote in Book II (as seen above) on the circumstances that “transform the function of the industrial capitalist more and more into a monopoly of big money capitalists, who may operate singly or in association.” Today we would say that they have (some degree of) monopolistic (or oligopolistic) power.

Marx’s observations when dealing with stock companies that the huge firms that this institution gives rise to tend to have higher profit rates that do not enter the profit rate equalization process are very important. First, they put in perspective his earlier exercises showing that the process of equalization of profit rates explained why the prices of production necessarily had to differ from commodity values (requiring the elimination of the simplifying assumption he used in Book I). Second, it is evidence that Marx did not perceive a general process of equalization of profit rates in reality and that his theoretical construction did not end there (since things change substantially when the mechanisms that gave rise to very large firms are introduced). And finally, it gives support to the conjecture that the real intention of including the propagation of stock companies as one of the counteracting forces to the fall in the general rate of profit was the higher rate of profit they often had sustainably over time, even if he did not express this in so many words.

Hence, even before introducing the rent on land (in Part VI of Book III), Marx showed (in Part V) that the prices of production could not be the true (or unique) attractors to the market prices of modern Capitalism. For there were mechanisms in place that generated monopoly power in certain huge firms (characterized by their high composition of capital), that made their profit rates escape from the process of equalization that other firms were subjected to. If the profit rates of the largest firms was one of the counteracting influences to the fall of the average profit rate it was because they could have higher profit rates than the rest, which was indicative of the monopolistic (or oligopolistic) power that enabled them to set higher prices than those corresponding to a more competitive market structure.

This is how Engels interpreted the matter when he inserted an interesting paragraph updating Marx’s observations (to the year 1894) by adding information on the formation of ‘cartels’ and describing the concrete mechanisms by means of which the effect on the prices could be achieved and the instability they could also have:

Since Marx wrote the above, new forms of industrial enterprises have developed, as we know, representing the second and third degree of stock companies... in short, the old boasted freedom of competition has reached the end
of its tether and must itself announce its obvious, scandalous bankruptcy. And in every country this is taking place through the big industrialists of a certain branch joining in a cartel for the regulation of production. A committee fixes the quantity to be produced by each establishment and is the final authority for distributing the incoming orders. Occasionally even international cartels were established, as between the English and German iron industries. But even this form of association in production did not suffice. The antagonism of interests between the individual firms broke through it only too often, restoring competition. This led in some branches, where the scale of production permitted, to the concentration of the entire production of that branch of industry in one big joint-stock company under single management. This has been repeatedly effected in America; in Europe the biggest example so far is the United Alkali Trust, which has brought all British alkali production into the hands of a single business firm (B3, 435; italics added).

Stock companies and the separation of the capitalist into management and stockholders

Aside from being stock companies an institutional mechanism for achieving an increase in the scale of production and in the size of firms as well as monopoly power, for Marx they had a tendency to separate the stockholders, who received dividends, from the management who exercised the active “function in the actual process of reproduction.” The latter, however, were basically very well paid skilled wage workers. The creation of stock companies, according to Marx, led to the

Transformation of the actually functioning capitalist into a mere manager, administrator of other people’s capital, and of the owner of capital into a mere owner, a mere money capitalist. Even if the dividends which they receive include the interest and the profit of enterprise, i.e., the total profit (for the salary of the manager is, or should be, simply the wage of a specific type of skilled labour, whose price is regulated in the labour market like that of any other labour), this total profit is henceforth received only in the form of interest, i.e., as mere compensation for owning capital that now is entirely divorced from the function in the actual process of reproduction, just as this function in the person of the manager is divorced from ownership of capital (B3, 434-5; italics added).

Even if the dividends contained all of the profits, they were not too different from the interest a lender or bondholder received, in the sense that merely owning the stock gave the stockholder the right to profit without having to actively participate in the production (or commercial) process. Marx here points to the tendency towards a separation between management and stockholders in stock companies, the latter being merely owners of stock that receive dividends. The idea that the management tended to receive wages fell short of a line of analysis that was to become important in the future and had a fascinating precedent in Adam Smith (1776). Smith pointed to the problems derived from the independence (which he condemned) of management from stockholders in the case of colonizing stock companies such as the English East India Company. In a Bibliographic Note in the Appendix to this chapter we address this topic, which was amply expanded a century and a half later by Berle and Means (1933).
Marx also acutely pointed to the core of this topic when he wrote: “On the basis of capitalist production a new swindle develops in stock enterprises with respect to wages of management, in that boards of numerous managers or directors are placed next and above the actual director, for whom supervision and management serve only as a pretext to plunder the stockholders and amass wealth” (B3, 388; italics added). But he did not develop this theme further, and gives the impression of having left it more as a (judicial) moral issue than as a structural feature of increasing importance in the complex web of conflicts of interests (or ‘class struggle’). For Marx “the salary of the manager is, or should be, simply the wage of a specific type of skilled labour, whose price is regulated in the labour market like that of any other labour.” But if those “boards” of managers or directors he mentions were able to “plunder the stockholders”, it was because they had power over the functioning of the firm that allowed them to appropriate for themselves a portion of the profits (not necessarily well accounted for in financial statements) in (latent or explicit) conflict with the stockholders. Marx did emphasize the development of stock companies as a transition towards the socialization of firms, which in his conception would culminate in a future society based on the associated of all the workers. We address this topic in Part IV of this book.

State intervention in the economy

When Marx deals with stock companies he also states that monopolies arise in certain branches of production, which “thereby requires state interference.” For him “the stock company is a transition toward the conversion of all functions in the reproduction process which still remain linked with capitalist property, into mere functions of associated producers, into social functions.” It is a “phase of transition to a new form of production.” The development of stock companies “establishes a monopoly in certain spheres and thereby requires state interference. It reproduces a new financial aristocracy, a new variety of parasites in the shape of promoters, speculators and simply nominal directors; a whole system of swindling and cheating by means of corporation promotion, stock issuance, and stock speculation. It is private production without the control of private property” (B3, 436). When he states that the emergence of monopolies “requires state interference” Marx seems to be referring to the ‘natural monopolies’ treated by J. S. Mill, and the reformist political agenda of those who (like J. S. Mill) held the need to prevent, through regulations or nationalizations, the spontaneous development of market tendencies that impaired the interests of the vast majority. Marx called this process “the abolition of the capitalist mode of production within the capitalist mode of production itself.”

In summary, Marx highlighted certain embryonic developments of the Capitalism of his time that markedly matured in the decades following his death: the generation of huge corporations with very high composition of capital and monopolistic (or oligopolistic) power on the prices of their products, the separation between the corporate control exerted by the higher stratum of directors and the dispersed property of stockholders, and the increasing intervention of the State in the economy.

‘Monopolistic’ sectors in the quantities and prices systems

Both Ricardo and J. S. Mill held that the rates of profit in different branches could be higher or lower according to how agreeable or disagreeable operating there could be (and how risky it was in the case of Mill). Marx did not focus on this aspect but neither did he criticize it. He did take into consideration a more important topic we mentioned above: that there were industrial branches in modern industry that
can have higher profit rates than the ‘general’ rate, that is, can avoid having their profit rate fall to the level of the remaining branches. In this section we show how the systems introduced in previous chapters can be adapted to reflect the coexistence of (and interaction between) ‘monopolistic’ branches (that we would now tend to call ‘oligopolistic’ in most cases) and ‘competitive’ branches. For this we also introduce heterogeneity between capitalists, of whom there are two types, according to whether they operate in ‘monopolistic’ branches or in the ‘competitive’ branches. To keep things simple, we assume Simple Reproduction. But notice that in a model of Extended Reproduction it would be the monopolistic branches that would produce most of the reinvestment of profits for the expansion of capital. Assume that the populations of ‘monopolistic’ and ‘competitive’ capitalists are \( q^M \) and \( q^K \), respectively. To avoid complications, assume that all the monopolistic branches have the same profit rate \( \rho_M \) which is higher than the competitive one: \( \rho_K \). The consumption baskets of these two types of capitalists are \( c_M \) and \( c_K \), respectively. Assume that matrix \( A \) is partitioned into two parts, where the first includes the monopolistic branches. Then the systems of quantities and prices are the following:

\[
\begin{bmatrix}
q^Q & q^L & q^K & q^M
\end{bmatrix}
\begin{bmatrix}
A & \ell & \tilde{\eta}_K & \tilde{\eta}_M \\
c_L & 0 & 0 & 0 \\
c_K & 0 & 0 & 0 \\
c_M & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K & q^M
\end{bmatrix}
\]  

(17.5)

\[
\begin{bmatrix}
A & \ell & \tilde{\eta}_K & \tilde{\eta}_M \\
c_L & 0 & 0 & 0 \\
c_K & 0 & 0 & 0 \\
c_M & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
p & w & \pi^K & \pi^M \\
p & w & \pi^K & \pi^M
\end{bmatrix}
\]  

(17.6)

\[
\begin{bmatrix}
(I + \tilde{\rho}) A & (I + \tilde{\rho}) \ell \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
p & w
\end{bmatrix}
= \begin{bmatrix}
p & w
\end{bmatrix}
\]  

(17.7)

where

\[
A = \begin{bmatrix}
A_{KK} & A_{KM} \\
A_{MK} & A_{MM}
\end{bmatrix}, \ell = \begin{bmatrix}
\ell_K \\
\ell_M
\end{bmatrix}, \tilde{\eta}_K = \begin{bmatrix}
\eta_K \\
0
\end{bmatrix}, \tilde{\eta}_M = \begin{bmatrix}
0 \\
\eta_M
\end{bmatrix},
\]

\[
c_j = \begin{bmatrix}
c_{jK} & c_{jM}
\end{bmatrix}, (j = L, K, M), q^Q = \begin{bmatrix}
q^K & q^M
\end{bmatrix},
\]

\[
p = \begin{bmatrix}
p_K & p_M
\end{bmatrix}, I + \tilde{\rho} = \begin{bmatrix}
(1 + \rho_K) I & 0 \\
0 & (1 + \rho_M) I
\end{bmatrix}
\]

Here \( q^Q \) and \( q^M \) (\( p_K \) and \( p_M \)) represent the vectors of gross output (prices of production) of each of the two large sectors, \( \ell_K \) and \( \ell_M \) are their respective direct labor requirements, \( \eta_K \) and \( \eta_M \) are the vectors that allocate the capitalists of type \( K \) and \( M \) to their respective branches, and \( c_{jK} \) and \( c_{jM} \) are the components of the consumption basket of class \( j \) (for \( j = L, K, M \)). To simplify, we have assumed that the profit rates inside these two sectors are homogeneous, but this could easily be changed. For example, we could let each monopolistic branch have its own profit rate by writing \( I + \tilde{\rho}_M \) instead of \( (1 + \rho_M) I \).

Since the matrix in (17.7) must have a dominant eigenvalue of one (so that a vector of prices and wage that solves the system exists) and \( \tilde{\rho} \) contains two profit rates (\( \rho_K \) and \( \rho_M \)), it is evident that the higher one of them is the lower the other must be. But
using the larger, dual systems we can see that $\rho_K$ and $\rho_M$ have their individual values uniquely determined from the exogenous data. The first equation of (17.7) yields

$$(1 + \rho_K) (A_{KCPK} + A_{KMPM} + \ell_K w) = p_K$$

$$(1 + \rho_M) (A_{MCPK} + A_{MMPM} + \ell_M w) = p_M.$$ 

Premultiplying these by $q^Q_K$ and $q^Q_M$, respectively, and using the equations of (17.5) and (17.6), we get:

$$\rho_K = \frac{q^Q_K [p_K - A_{KCPK} - A_{KMPM} - \ell_K w]}{q^Q_K (A_{KCPK} + A_{KMPM} + \ell_K w)} = \frac{q^Q_K \pi_K}{K_K} = \frac{q^K \pi_K}{K_K} = \frac{q^K c_K P}{K_K}$$

$$\rho_M = \frac{q^Q_M [p_M - A_{MCPK} - A_{MMPM} - \ell_K w]}{q^Q_M (A_{MCPK} + A_{MMPM} + \ell_M w)} = \frac{q^Q_M \pi_M}{K_M} = \frac{q^M \pi_M}{K_M} = \frac{q^M c_M P}{K_M},$$

where in the second of these equalities the aggregate capitals of each sector ($K_K$ and $K_M$, respectively) are defined. It is of course formally correct to calculate an overall average profit rate $\bar{\rho}$:

$$\bar{\rho} = \frac{q^K \pi_K + q^M \pi_M}{K_K + K_M} = \frac{q^K \pi_K}{K_K + K_M} + \frac{q^M \pi_M}{K_M + K_M} = \rho_K \alpha_K + \rho_M \alpha_M,$$

where the shares in global capital are $\alpha_K = K_K / (K_K + K_M)$ and $\alpha_M = 1 - \alpha_K$, respectively. But this would not be very interesting. It would be like averaging the profit rates of small local grocery stores and huge international corporations. This simple exercise, which lacks any ‘microfoundations’, is a much better reflection of an essential aspect of reality than many sophisticated models that overlook the great inequality among capitalist sectors that has characterized Capitalism over history.

To see in more detail how the basic data determine the inequality between the profit rates we can, as in previous chapters, get from (17.7) the vector de prices of production $p = B(\bar{\rho}) \ell w$, where we defined $B(\bar{\rho}) \equiv [I - (I + \bar{\rho}) A]^{-1} (I + \bar{\rho})$. Premultiplying by $c_L$ we get $1 = c_L B(\bar{\rho}) \ell$, which shows that there is an inverse relation between $\rho_M$ and $\rho_K$ that exclusively depends on the elements of $c_L$, $A$, and $\ell$, and where in the present case the partition of these vectors and matrix into sectors $K$ and $M$ is essential. But the (two) elements of $\bar{\rho}$ have already been obtained above, so the novelty is only that the inverse relation between $\rho_M$ and $\rho_K$ exclusively depends of the consumption basket of wage workers and the technological data $A$ and $\ell$. If we use $c_K$ as numeraire ($\pi^K = c_K P = 1$), we get the following expressions for the wage rate and the profit of each capitalist (all in terms of $c_K$)

$$w = \frac{1}{c_K B(\bar{\rho}) \ell}, \quad \pi^K = \frac{c_M B(\bar{\rho}) \ell}{c_K B(\bar{\rho}) \ell}, \quad \pi^K = 1.$$ 

Appendix to Chapter 17

Bibliographic Notes

Adam Smith and the divorce of ownership from control  Adam Smith introduced the modern topic of the “divorce of ownership from control” in firms organized as joint-stock companies. First he introduced the notion of what economists would much later call ‘public goods’. For Smith one of the three duties of the ‘sovereign’ (aside from financing defense and justice) was that of “ereciting and maintaining those public institutions and those public works, which though they may be in the highest
degree advantageous to a great society, are, however, of such a nature, that the profit could never repay the expense to any individual, or small number of individuals; and which it, therefore, cannot be expected that any individual, or small number of individuals, should erect or maintain” (Smith 2005, 590). Smith specifically addressed the institutions “for facilitating the commerce of the society, and those for promoting the instruction of the people” (Ibid.). Commerce “carried on with barbarous and uncivilized nations”, in particular, required special protection, such as fortifications, for the defense of warehouses and embassies. And though the protection of commerce was considered a duty of the commonwealth (or ‘sovereign’) it was not always financed by means of taxes or duties on the particular branch of commerce it protected. In most of the European ‘commercial states’ private trading firms had persuaded the legislature to put them in charge of this task. Smith distinguished three legal forms of enterprises: ‘regulated companies’, ‘private copartnery’, and ‘joint-stock companies’. Regulated companies resembled the Medieval ‘corporations of trades’ in cities and towns. In these companies no citizen could “lawfully carry on any branch of foreign trade, for which a regulated company is established, without first becoming a member of that company” (Ibid., 600). Such companies had a monopoly “more or less strict, according as the terms of admission are more or less difficult, and according as the directors of the company have more or less authority, or have it more or less in their power to manage in such a manner as to confine the greater part of the trade to themselves and their particular friends” (Ibid.). In a private copartnery, on the other hand, no partner could transfer his share to another person, or introduce a new member into the company without the consent of the company. Also, each partner had to respond with all of his wealth to the company debts. Finally, in a joint-stock company any member could by his own decision transfer his share (or stock) by selling it in the market, and the rest of his wealth was not exposed to the company’s debts.

These companies were managed by a “court of directors”, which was formally subject to “the control of a general court of proprietors. But the greater part of these proprietors seldom pretend to understand any thing of the business of the company; and... give themselves no trouble about it, but receive contentedly such half-yearly or yearly dividend as the directors think proper to make to them” (Ibid., 606). This had consequences. First, the fact that the stockholders were exempt “from trouble and from risk, beyond a limited sum” encouraged many people to participate and thus large sums of capital could be raised for such firms as the South Sea company and the Bank of England. Second, since the directors of these companies, “being the managers rather of other people’s money than of their own, it cannot well be expected that they should watch over it with the same anxious vigilance with which the partners in a private copartnery frequently watch over their own.” This very often resulted in negligence. Smith believed that this was the reason that joint-stock companies for foreign trade had seldom been able to compete successfully against “private adventurers” without having an exclusive privilege. But even then they had often failed. “Without an exclusive privilege, they have commonly mismanaged the trade. With an exclusive privilege, they have both mismanaged and confined it” (Ibid., 606-7). The best alternative for establishing “a new trade with some remote and barbarous nation” was to “incorporate them into a joint-stock company, and to grant them, in case of

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16 Smith mentions as subsisting British regulated companies, “the Hamburgh company, the Russia company, the Eastland company, the Turkey company, and the African company.”

17 The English East India company, established in 1600, began as a regulated company with an exclusive privilege for trade with India, becoming a joint stock company in 1612. In 1702 the Crown became one of the owners and in 1708 the name was changed to the “United Company of Merchants trading to the East Indies”.

their success, a monopoly of the trade for a certain number of years.” This could be beneficial if it was limited in time, “like monopoly of a new machine is granted to its inventor, and that of a new book to its author” (Ibid., 617). But after such a period the monopoly had to end, the trade had to be opened to competition and, if the company had found it necessary to build forts and garrisons, these should be handed over to the government with due compensation. A “perpetual monopoly” led to the public being “taxed very absurdly” both by high prices and by their exclusion from a possibly profitable branch of business. And it made the company suffer “the negligence, profusion, and malversation of their own servants, whose disorderly conduct seldom allows the dividend of the company to exceed the ordinary rate of profit in trades which are altogether free.” Smith held the opinion that without a temporary exclusive privilege, such ‘agency problems’ (in modern jargon) as the neglect or corruption of the directors made joint-stock companies only viable in activities in which “all the operations are capable of being reduced to what is called a routine, or to such a uniformity of method as admits of little or no variation.” In his opinion, such were the cases of banking, insurance, making and maintaining navigable canals, and supplying water to a great city.

Adam Smith was an early theorist of the “divorce of ownership from control” in stock companies, a notion that a century and a half later Berle and Means (1933) studied in the case of American corporations, making them famous. Smith showed that this divorce was very important in his own time in the case of trade in colonies exploited by British adventurer capitalists for centuries in remote areas of the world. In such enterprises the bureaucracy in the remote colony had control of the operations and exercised them using a vast repertoire of political maneuvers (both there as in London) and military operations. The stockholders, in contrast, merely received dividends without leaving their home country.

Samuelson, Sweezy, and the ‘organic composition’ of capital  Marx had a great capacity for the conceptual analysis of very complex matters, the subtlety of which many of his critics that were much more versed in mathematics were unable to perceive and often confused completely. Samuelson wrote:

From a tautology relating the profit rate \( r \) to society’s rate of surplus value \( \Sigma S / \Sigma V \) and its organic composition of capital \( \Sigma C / \Sigma V \), Marx deduced the tautology that higher values of the latter, the former being held constant, would necessarily mean that \( r \) falls. Sweezy, Joan Robinson, and most analysts of Marx have rightly, I think, criticized this arbitrary \textit{ceteris paribus} type of argument (Samuelson 1957, 892).

This paragraph shows that Samuelson had not read \textit{Capital} but based his understanding of Marx on expositions such as that of Sweezy (1942) and later Meek (1967) that, like all brief summaries, highlight certain aspects of the original and leave many others aside. The case of Sweezy is of special interest because he is probably the economist that was most successful in making Marx’s work known in the U.S., to a large extent through his 1942 book \textit{The Theory of Capitalist Development. Principles of Marxian Political Economy}, which Samuelson (1957) praised as “by all odds the best book on Marxian economics.”

\footnote{Schumpeter’s \textit{History of Economic Analysis} had been posthumously published in 1954, just three years before Samuelson’s article. There Schumpeter had also referred the reader to Sweezy’s book as the “best introduction to Marxist literature I know” (Schumpeter 1954, Part III, Cap. 1). Samuelson had been a student of Schumpeter in Harvard.}
the term ‘organic composition of capital’ in place of the ‘value composition of capital’ and most of the academic literature continued with this flawed terminology. Hence, it was difficult to capture all the subtleties of Marx’s analyses of the factors that influence the rate of profit (of the competitive sectors of the economy) in the long run.

Sweezy’s confusion with some of the categories used by Marx is surprising. He dedicates a section of Chapter VI of his book to “A Critique of the law”, and there holds that “Marx’s assumption of a constant rate of surplus value might be considered a useful device for focusing attention on the most important element in the situation, and the treatment of changes in the rate of surplus value as a ‘counteracting cause’ could be justified” (Sweezy 1946, 103). But he deems untenable the view of most subsequent Marxian writers who, like Marx, believe that “over any considerable period of time, changes in the organic composition of capital are sure to be enormous”, with the argument that:

In physical terms it is certainly true that the amount of machinery and materials per worker has tended to grow at a very rapid rate for at least the last century and a half. But the organic composition of capital is a value expression; and, because of steadily rising labor productivity, the growth in the volume of machinery and materials per worker must not be regarded as an index of the change in the organic composition of capital. Actually the general impression of the rapidity of growth of the organic composition of capital seems to be considerably exaggerated (Sweezy 1946, 103).

Sweezy here criticizes Marx for (allegedly) not having written what he did write (and with great clarity) but Sweezy overlooked because he was confused about what Marx understood by the ‘organic composition of capital’. That in physical terms “the amount of machinery and materials per worker” tends to “grow at a very rapid rate” is precisely what Marx understood by an increase in the ‘technical composition of capital’, which can also be expressed as an increase in the ‘organic composition of capital’ if one makes the assumption (or it is the case) that relative values of the elements of constant vis a vis variable capital remain constant. For in this case the organic composition evolves exactly as the technical composition. The “growth in the volume of machinery and materials per worker” which Sweezy says “must not be regarded as an index of the change in the organic composition of capital” is exactly that in Marx’s terminology of Book I (at least, as we will see, starting from the third German edition). And the effect of the increasing productivity on the values or the prices of production made the ‘value composition of capital’ increase much less than the ‘organic composition’, or even diminish.

Without this being a justification for Sweezy’s confusion, it is also true that in the preparation of Book III for the press Engels overlooked some instances of this same confusion, leaving ‘organic composition’ of capital when ‘value composition’ was appropriate for consistency with Marx’s later innovation in his own terminology. However, in Chapter 8 of Book III we find:

By composition of capital we mean, as stated in Book I, the proportion of its active and passive components, i.e., of variable and constant capital. Two proportions enter into consideration under this heading. They are not equally important, although they may produce similar effects under certain circumstances. The first proportion rests on a technical basis, and must be regarded as given at a certain stage of development of the productive forces.... This proportion forms the technical composition of capital and is
the real basis of its organic composition.... The difference between the technical composition and the value composition is manifested in each branch of industry in that the value relation of the two portions of capital may vary while the technical composition is constant, and the value relation may remain the same while the technical composition varies. The latter case will, of course, be possible only if the change in the ratio of the employed masses of means of production and labour power is compensated by a reverse change in their values. The value composition of capital, inasmuch as it is determined by, and reflects, its technical composition, is called the organic composition of capital (B3, 144-5).

A footnote reads: "The above has already been briefly developed in the third edition of Book I in the beginning of Kap. XXIII, S. 628." And Engels adds the following: "Since the two first editions do not contain that passage, its repetition here is all the more desirable." Engels’ clarification shows that Marx introduced his subtle distinction between the ‘value composition’ and the ‘organic composition’ after having published the first two (German) editions of Book I. Hence, most of the manuscripts for Books II and III had already been written before the introduction of the distinction.

In the preface to the third German edition of Book I, Engels writes (in November 1883, eight months after Marx’s death): “Marx was not destined to get this, the third, edition ready for the press himself.... Among the books left by Marx there was a German copy which he himself had corrected here and there and provided with references to the French edition; also a French copy in which he had indicated the exact passages to be used. These alterations and additions are confined, with few exceptions, to the last part of the book: ‘The Accumulation of Capital’. It is precisely there where “that passage” that Engels referred to in his footnote is located.

Three years after having written the Preface to the Third German Edition of Book I, Engels wrote the Preface to the English Edition, where he states that he prepared it “with the assistance of notes left by the author.” He also explains that the English edition was based on the third German edition. Hence, Sweezy had easy access to the explanation about the precise meaning of the term ‘organic composition of capital’, which makes his confusion all the more surprising.

Baran and Sweezy (1966) and Samuelson (1967) on Marx’s ‘competitive economy’ When they wrote their book Monopoly Capital: An Essay on the American Economic and Social Order (1968), Paul A. Baran and Paul M. Sweezy must have been very influenced by the perfect competition paradigm then prevailing in Western academia, thus affecting their interpretation of Capital. They rightly state that the “stagnation of Marxian social science, its lagging vitality and fruitfulness, cannot be explained by any simple hypothesis” (Baran and Sweezy 1966, 3). However, this is followed by their erroneous idea that “the Marxian analysis of capitalism still rests in the final analysis on the assumption of a competitive economy” (Ibid., 4). It should be noted that in the language of mainstream economics ‘a competitive economy’ has basically the same meaning as ‘a perfectly competitive economy’, and we have seen in this chapter that this was far removed from Marx’s (and the Classical economists’ in general) conceptual framework, and even more from the tendencies that Marx highlighted with respect to stock companies. And if Baran and Sweezy referred not to Marx’s but to the analyses of many of their contemporary ‘Marxians’, this would im-
ply that these had strayed considerably from Marx’s thought.\textsuperscript{19} To “remedy” this
deficient assumption Baran and Sweezy wrote their *Monopoly Capital*.

Samuelson (1967) soon echoed Baran and Sweezy’s opinion, writing: “I have to
agree with the recent book of Paul Sweezy and Paul Baran which seeks to identify as
an important explanation of the stagnation of Marxian social science the fact that ‘the
Marxian analysis of capitalism still rests in the final analysis on the assumption of a
competitive economy’” (Samuelson 1967, 622). In the case of Samuelson, it should be
noted that there is no doubt that he was referring to ‘perfect competition’. And it is
regrettably that he did not rely directly on a reading of *Capital* instead of expositions
that did not always correctly interpret that work.

● Numerical Exercise \#7 on the tendency for the profit rate to fall

We here start from the Numerical Exercise \#6 (for the ER model with arbitrary
turnover periods) in order to numerically check the basic scenario seen above in which
employment and consumptions are maintained in the transition to the new industrial
cycle. There we had $\delta = 1$, whereas we now assume that in the new cycle $\delta = 1.5$. To
check (17.1), we first calculate matrix $B^S(0.13053, 1)$ (where $g = 0.13053$ was obtained
in Numerical Exercise \#6):

\[
(1 + 0.13053) * \\
\left( \begin{array}{cc}
1 & 0 \\
0 & 1 \\
\end{array} \right) - (1 + 0.13053) \begin{bmatrix}
0.2 & 0.4 \\
0.3 & 0.12 \\
\end{bmatrix} - 0.13053 * 1.5 \begin{bmatrix}
0.3 & 0.6 \\
0.5 & 0.3 \\
\end{bmatrix}^{-1}
\]

\[
= \begin{bmatrix}
2.3815 & 1.5311 \\
1.1672 & 2.1204 \\
\end{bmatrix}
\]

Using this we get (17.1):

\[
1 = \left( \begin{array}{cc}
0.3 & 0.2 \\
\end{array} \right) + \frac{100}{166.47} \begin{bmatrix}
0.5 & 0.5 \\
\end{bmatrix} \begin{bmatrix}
2.3815 & 1.5311 \\
1.1672 & 2.1204 \\
\end{bmatrix} \begin{bmatrix}
0.3 & 0.2 \\
\end{bmatrix}
\]

Also, to check (17.2) we calculate matrix $B^S(0.2453, 1)$ (using $\rho_g = 0.2453$, given by
Numerical Exercise \#6):

\[
(1 + 0.2453) \left( \begin{array}{cc}
1 & 0 \\
0 & 1 \\
\end{array} \right) - (1 + 0.2453) \begin{bmatrix}
0.2 & 0.4 \\
0.3 & 0.12 \\
\end{bmatrix} - 0.2453 \begin{bmatrix}
0.3 & 0.6 \\
0.5 & 0.3 \\
\end{bmatrix}^{-1}
\]

\[
= \begin{bmatrix}
4.6957 & 3.8999 \\
2.9991 & 4.0936 \\
\end{bmatrix}
\]

Hence we have (17.2):

\[
1 = \begin{bmatrix}
0.3 & 0.2 \\
\end{bmatrix} \begin{bmatrix}
4.6957 & 3.8999 \\
2.9991 & 4.0936 \\
\end{bmatrix} \begin{bmatrix}
0.3 & 0.2 \\
\end{bmatrix}
\]

These values for $g$ and $\rho_g$ are the starting point for the present exercise.

\textsuperscript{19}Baran and Sweezy’s statement is: “But there is one important factor that we believe can be
identified and isolated and hence (at least in principle) remedied: the Marxian analysis of capitalism
still rests in the final analysis on the assumption of a competitive economy”.
With the increase of $\delta$ to 1.5, we can check (by trial and error) that $g$ must fall to $g' = 0.1037$. Matrix $B^S (0.1037, 1.5)$ is

$$
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - (1 + 0.1037) \begin{bmatrix}
0.2 & 0.4 \\
0.3 & 0.12
\end{bmatrix} - (0.1037 \times 1.5) \begin{bmatrix}
0.3 & 0.6 \\
0.5 & 0.3
\end{bmatrix}^{-1} =
\begin{bmatrix}
2.3674 & 1.5423 \\
1.1792 & 2.1128
\end{bmatrix},
$$

and we can check (17.3):

$$
1 = \left( \begin{bmatrix}
0.3 & 0.2
\end{bmatrix} + \frac{100}{166.47} \begin{bmatrix}
0.5 & 0.5
\end{bmatrix} \right) \begin{bmatrix}
2.3674 & 1.5423 \\
1.1792 & 2.1128
\end{bmatrix} \begin{bmatrix}
0.3 \\
0.2
\end{bmatrix}.
$$

We can also check by trial and error that $\rho_g$ must fall to $\rho'_g = 0.1909$. Hence matrix $B^S (0.1909, 1.5)$ is

$$
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} - (1 + 0.1909) \begin{bmatrix}
0.2 & 0.4 \\
0.3 & 0.12
\end{bmatrix} - (0.1909 \times 1.5) \begin{bmatrix}
0.3 & 0.6 \\
0.5 & 0.3
\end{bmatrix}^{-1} =
\begin{bmatrix}
4.6646 & 3.9206 \\
3.027 & 4.0885
\end{bmatrix},
$$

And thus we have (17.4):

$$
1 = \begin{bmatrix}
0.3 & 0.2
\end{bmatrix} \begin{bmatrix}
4.6646 & 3.9206 \\
3.027 & 4.0885
\end{bmatrix} \begin{bmatrix}
0.3 \\
0.2
\end{bmatrix}.
$$

Let us see what happens to the consumption part of the profit rate. Using (14.41), we get:

$$
\rho_0 = \frac{\rho_g - g}{1 + g} = \frac{0.2453 - 0.13053}{1 + 0.13053} = 0.10152
$$

$$
\rho'_0 = \frac{\rho'_g - g'}{1 + g'} = \frac{0.1909 - 0.1037}{1 + 0.1037} = 0.079007.
$$

Hence, that part of the profit rate also falls. And the reductions in $\rho_g$, $g$, and $\rho_0$ are:

$-\Delta \rho_g = 0.0544$, $-\Delta g = 0.02683$, and $-\Delta \rho_0 = 0.02513$. □
Part III

Critique of Marx’s theory
Chapter 18  CRITIQUE OF THE THEORY OF SURPLUS VALUE

The inspiring element for the theory of surplus value
Transition to Capitalism and wage labor

As we have seen in Chapter 3, Marx explained that there were two venues through which wage labor emerged as an institution: the direct conversion of the slave or the serf into a wage worker, and the transformation of simple commodity producers into wage workers by means of the “expropriation of the immediate producers”, that is, the independent small farmer or urban craftsman not subject to a guild. Both venues led to Capitalist Commodity Production (CCP). His theoretical construction was based on the latter, since for Marx the functioning of the capitalist mode of production could best be theorized by starting from a more elementary mode of production and circulation of commodities in which worker-producers had control over the conditions of production (land and means of production), such as was reflected in the model for SCP. In his theoretical architecture he had to first build a theory for SCP and then introduce the specificity of CCP, which was the polarization between the capitalists who own capital (and in a first theoretical stage continue using land freely) and workers who are free of the personal bonds of slavery or serfdom but are compelled by the need to make a living to sell their labor power for a wage:

In order that the sale of one’s own labour power (in the form of the sale of one’s own labour or in the form of wages) may constitute not an isolated phenomenon but a socially decisive premise for the production of commodities... historical processes are assumed by which the original connection of the means of production with labour power was dissolved –processes in consequence of which the mass of the people, the labourers, have, as non-owners, come face to face with the non-labourers as the owners of these means of production (B2, 38-9).

In contrast to the cooperation within a combined group of workers that had taken place in various historical circumstances that had in common the prevalence of “relations of dominion and servitude”, large-scale cooperation organized under Capitalism was based on the existence of free wage workers who sold their labor power to capitalists:

The sporadic application of co-operation on a large scale in ancient times, in the Middle Ages, and in modern colonies, reposes on relations of dominion and servitude, principally on slavery. The capitalistic form, on the contrary, pre-supposes from first to last, the free wage labourer, who sells his labour power to capital. Historically, however, this form is developed in opposition to peasant agriculture and to the carrying on of independent handicrafts whether in guilds or not (B1, 340).

And only this large-scale cooperation produced by the capitalist mode of production had revolutionized the technological and organizational methods with speeds never seen before that contrasted with the lack of dynamism of the old modes of production based on slavery or serfdom.
Analytical scheme for the transition to Capitalism

We now approach analytically what we believe was the inspiring element in Marx’s theory of surplus value. For this it is convenient to start from the simplest SCP model in which the ratios between the values given by system (6.1) (with the value of gold in the denominator) are the equilibrium monetary prices, that is, those around which market prices fluctuate (given perturbations to supply or demand). The transition from SCP to CCP—from the point of view of Marx’s theory of value and surplus value—can be represented as the passage from system (6.1) to system (8.6). But in order to represent what we think underlies Marx’s intuition it is convenient to insert an intermediate system in which, starting from SCP (6.1), there has been an exogenous change that can sustain an increase in the consumption of the workers-producers. Let us assume that this exogenous change is a technological improvement that has the effect of reducing some (or all) of the coefficients of \( (A, \ell) \). To avoid complicating the notation we continue calling \( A \) and \( \ell \) the matrix and vector that contains the input-output coefficients after the change. And to simplify we assume that this exogenous increase in productivity allows for a proportional increase in the consumption basket \( c_L \). Then there exists a (unique) rate of expansion of the consumption of the small commodity producers \( \mu > 0 \) such that system (6.11) holds, which is here repeated for the reader’s convenience (but without the apostrophes that indicated the technological change that has taken place):

\[
\begin{bmatrix}
A & \ell \\
(1 + \mu) c_L & 0
\end{bmatrix}
\begin{bmatrix}
v \\
1
\end{bmatrix}
= 
\begin{bmatrix}
v' \\
1
\end{bmatrix}.
\tag{18.1}
\]

The technological improvement has transformed (6.1) into (18.1), where \( \mu \) is the proportional expansion rate of the consumption of the workers-producers. Alternatively, if a new social class of capitalists makes its appearance, we can consider the passage from (6.1) to (8.6), where \( e \) represents the rate of surplus value that lets the capitalists reap a surplus. Assume, for example, a city inhabited by simple commodity producers whose price system is given by (18.1) and in which, through the expropriation of the worker-producers, the means of production end up in the hands of capitalists. Assume also that the expropriated workers have no possibility of going to another city and that, to survive, their only option is to be employed as wage workers by the capitalists who are now the owners of the means of production. Assume also that the consumption basket of these workers diminishes by \( \mu c_L \) when they become wage workers. Hence, their consumption basket is again \( c_L \). In that case, it seems reasonable to interpret \( e \) in (8.6) as a rate that measures, as a flow, the value of the stock of commodities that were taken away from them. And it would also seem reasonable to say that these wage workers are ‘exploited’ at the rate \( e \), since this rate measures, in values, what they cease earning (or consuming) each period as a direct consequence of the illegitimate act of violence. Notice that the fact that in the new mode of production these values no longer represent the equilibrium prices in no way invalidates this argument because these (new) values would represent the equilibrium (relative) prices if the original mode of production (SCP) were preserved.

From system (18.1) we obtain, aside from the vector of values, the equation \((1 + \mu) c_L v = 1\) that indicates that the expanded consumption of the simple com-

---

1 Notice that we also eliminate the apostrophe on \( v \) which indicated that values had fallen.

The social matrix that results from the exogenous change before the expansion in consumption takes place \( N(0) \) now has a dominant eigenvalue that is less than one. Hence (by Perron-Frobenius) the consumption basket can be expanded, and there is a unique consumption expansion coefficient \( \mu > 0 \) such that the dominant eigenvalue of \( N(\mu) \) becomes one.
modity producers is equal to the value of the labor power the reproduction of which this consumption allows. Hence the rate of expansion of the consumption basket that the assumed exogenous change produces is \( \mu = 1/c_L v - 1 \). Comparing with (8.8) we can see that, if we go from SCP to CCP in this hypothetical way, the reduction in the consumption of the independent workers allows for the transformation of a part of the independent producers into capitalists who own the means of production. We thus have, within the economic system represented, a splitting up of the class of simple commodity producers into a class of capitalists who own the means of production, on the one hand, and wage workers, on the other. The increase in labor productivity has generated the value of the consumption basket of the capitalists \( c_N v \), which substitutes for the expansion of the consumption basket of the initial workers-producers \( \mu c_L v \) that could have taken place had they not been expropriated. And the ratio between the value of the consumption basket of capitalists and the value of the consumption basket of wage workers defines the rate of surplus value \( e \) (as in (8.4)). This analogy seems to analytically reflect quite well Marx’s basic idea of the transition to the exploitation of wage labor in Capitalism.\(^2\) For this it focuses on a jump from one system to another instead of the long historical process of the genesis of industrial Capitalism that Marx described with much detail in the case of England and which he called Primitive Accumulation.

When in Book III Marx shows that the equalization of profit rates in the various industrial branches made the prices of production differ from the values of commodities (in the general case of heterogeneous value compositions of capital), he makes an analysis that confirms that source of inspiration. He takes the hypothetical case of workers who are in possession of their means of production and focuses on two workers who work in sectors, I and II, respectively, where I requires means of production of higher value than II:

The punctum saliens will be best brought out if we approach the matter as follows: Suppose, the labourers themselves are in possession of their respective means of production and exchange their commodities with one another. In that case these commodities would not be products of capital... Also suppose the labourers work an equal average length of time, allowing for compensations that arise from the different labour intensities, etc. In such a case, two labourers would, first, both have replaced their outlays, the cost prices of the consumed means of production, in the commodities which make up the product of their day’s work. These outlays would differ, depending on the technical nature of their labour. Secondly, both of them would have created equal amounts of new value, namely the working day added by them to the means of production. This would comprise their wages plus the surplus value, the latter representing surplus labour over and above their necessary wants, the product of which would however belong to them. To put it the capitalist way, both of them receive the same wages plus the same profit, = the value, expressed, say, by the product of a ten-hour working day... The means of subsistence daily consumed by I and II during production, which take the place of wages, here form the part of the invested means of production ordinarily [i.e. under capitalist conditions] called variable capital. But for equal working periods the surplus values

\(^2\)It would reflect Marx’s thought even better if instead of an exogenous increase in productivity we had an increase in the length of the working day. From what we have seen in previous chapters, this alternative can easily be implemented.
would be the same for I and II, or, more precisely, since I and II each receive the value of the product of a day’s work, both of them receive equal values after the value of the invested “constant” elements has been deducted, and one portion of these equal values may be regarded as a substitute for the means of subsistence consumed in production, and the other as surplus value in excess of it (B3, 174-5; text within brackets added).

Although in this analysis Marx wanted to explain why equal rates of surplus value turned into different profit rates we believe it also reflects his fundamental inspiration for his theory of labor exploitation in Capitalism. In CCP surplus value was generated by the worker, along with the value of his own means of subsistence, but is was appropriated by the capitalist, since he was the owner of the capital and of the output of the production process.

It should be noted that the realism he introduced in Book III (different value compositions in the various branches and divergence between prices of production and values) did not change the core of the transformation of SCP into CCP that had been made in Book I by hanging on to values. But he needed to maintain both valuations in parallel: that of values and surplus value for the explanation of global profits on the basis of the exploitation of wage labor, and that of prices of production, wages, and profit rate, for the explanation of the exchange values of transactions.

On the other hand, our analytical argument could be based on an expansion of the working day instead of an increase in productivity, as we did in Chapter 10. We would thus better reflect the argument Marx used in his explanation of Primitive Accumulation. Any one of these two alternatives for the generation of a surplus is compatible with the following statement by Marx:

Productive capital, in performing its functions, consumes its own component parts for the purpose of transforming them into a mass of products of a higher value. Since labour power acts merely as one of its organs, the excess of the product’s value engendered by its surplus labour over and above the value of productive capital’s constituent elements is also the fruit of capital. The surplus labour of labour power is the gratuitous labour performed for capital and thus forms surplus value for the capitalist, a value which costs him no equivalent return (B2, 43; italics added).

We believe this short paragraph contains the fundamental problem of Marx’s theory of surplus value. For when he states that labor power is one of the organs of productive capital he omits considering that, aside from productive capital as property that is materialized in the elements of the money capital disbursed in the purchase of the elements of constant and variable capital, there is the entrepreneur that undertakes the task of achieving that unity of labor and means of production function. The entrepreneur is an ‘organ’ (or ‘agent’ in more modern language) who must receive a compensation somehow commensurate to his contribution to the economic process, since he is a part (and an essential part) of the social structure of Capitalism. Marx recognized this fact in his textual considerations when he distinguished the ‘entrepreneurial profit’ from the interest on a loan from a financial capitalist. But it was absent from his analytical theory of surplus value. In this theory the activity of the capitalist entrepreneur was not reflected theoretically (as it still remains absent in most mainstream economics). It was this absence that allowed him to consider that the “surplus labour of labour power is the gratuitous labour performed for capital”, as if there were an asymmetrical (or
fictitious) exchange in which the industrial capitalist receives something ('profit of enterprise') for nothing, that is, without having made any contribution to the productive process.

The right to appropriate the unpaid labor of others

As already noted, for Marx it was important to distinguish between the historical process that originated the capitalist mode of production and the historical, specifically capitalist, accumulation process. Primitive Accumulation was the process that produced the polarization between a segment of society with exclusive control over the conditions of production (fundamentally the money capital necessary to purchase them) while another segment, deprived of the access to such conditions had no alternative but to offer its labor power for a wage. This process was violent, for it typically implied either the forcible expropriation of communal agricultural lands or the expulsion of (freed) peasants from the land to which they had been attached who were unable to earn a living independently. Marx explains in detail how this occurred in England in Chapter 24 of Book I. In contrast, in his theory of CCP the accumulation of capital was produced by buying and selling commodities at market prices, i.e., without violence, respecting 'property rights'. Marx thus disagreed with the notion of some socialists that profits were based on the exchange of non-equivalents in the circulation process. The explanation he found was that productivity had to be sufficiently high to leave a surplus over the production of the means of subsistence of the labor force (with habitual levels of consumption) in the hands of capitalist entrepreneurs that enabled them to finance, not only their consumption but, fundamentally, the reinvestment on an extended scale (i.e., capital accumulation). Although wage workers received (on average) the market price of their labor power, the capitalist made sure that they worked longer hours than those strictly necessary to cover all costs, thus leaving a profit. This was Marx's conception of the phase of Capitalism based on the generation of absolute surplus value. In the later phase based on the generation of relative surplus value, the emphasis was on the innovations that increased productivity and thus diminished the part of the working day necessary for the reproduction of the work force. Moreover, he admitted that the standard of living of the workers could increase over time and that the working day tended to diminish after a certain phase, as was happening in England.

Marx's basic argument was that labor power was a very special commodity since labor, aside from transmitting to the product the value of the means of production consumed in the production process, created value in the labor process, and fundamentally, produced surplus value for the capitalist because the latter, as any other buyer, could consume his purchased commodity (i.e., make it work) during a greater number of hours than the equivalent to the value of its labor power (i.e., the value of its consumption during that period of time):

The value of the new product further includes: the equivalent of the value of the labour-power together with a surplus value. This is so because the value of the labour-power –sold for a definite length of time, say a day, a week, etc.– is less than the value created by its use during that time. But the worker has received payment for the exchange-value of his labour-power and by so doing has alienated its use value –this being the case in every sale and purchase.

The fact that this particular commodity, labour-power, possesses the peculiar use value of supplying labour, and therefore of creating value, cannot
affect the general law of commodity production. If, therefore, the magnitude of value advanced in wages is not merely found again in the product, but is found there augmented by a surplus value, this is not because the seller has been defrauded, for he has really received the value of his commodity; it is due solely to the fact that this commodity has been used up by the buyer (B1*, 407).

The surplus labor, the labor time that exceed that necessary to reproduce the worker and his dependents and conformed the surplus value, was ‘unpaid labor’ and had to be clearly distinguished from any capital originated in the savings from outside the capitalist production process. This is clearly expressed in the following paragraph:

The original capital of £10,000 brings in a surplus value of £2,000, which is capitalised. The new capital of £2,000 brings in a surplus value of £400, and this, too, is capitalised, converted into a second additional capital, which, in its turn, produces a further surplus value of £80. And so the ball rolls on... The original capital was formed by the advance of £10,000. How did the owner become possessed of it? “By his own labour and that of his forefathers”, answer unanimously the spokesmen of political economy... But it is quite otherwise with regard to the additional capital of £2,000... There is not one single atom of its value that does not owe its existence to unpaid labour (B1, 581).

Hence, the exchange of equivalents in the circulation process was only a partial aspect of the relation between workers and the industrial capitalist. To have a complete representation it was necessary to understand that it was in production process where the recurrent appropriation of ‘unpaid labor’ by the capitalist took place. And in Marx’s view it was there were the essential unfairness of “the laws of capitalist appropriation” was to be found:

The exchange of equivalents, the original operation with which we started, has now become turned round in such a way that there is only an apparent exchange. This is owing to the fact, first, that the capital which is exchanged for labour power is itself but a portion of the product of others’ labour appropriated without an equivalent; and, secondly, that this capital must not only be replaced by its producer, but replaced together with an added surplus... At first the rights of property seemed to us to be based on a man’s own labour. At least, some such assumption was necessary since only commodity owners with equal rights confronted each other, and the sole means by which a man could become possessed of the commodities of others, was by alienating his own commodities; and these could be replaced by labour alone. Now, however, property turns out to be the right, on the part of the capitalist, to appropriate the unpaid labour of others or its product, and to be the impossibility, on the part of the labourer, of appropriating his own product (B1, 582-3; italics added).

It is evident that here ‘At first’ refers to SCP and ‘Now’ to CCP. In a footnote, Marx adds that “Just as at a given stage in its development, commodity production necessarily passes into capitalistic commodity production (...), so the laws of property that are based on commodity production necessarily turn into the laws of capitalist appropriation.” And just before the quoted text he had written in Hegelian language: “the
laws of appropriation or of private property, laws that are based on the production and circulation of commodities, become by their own inner and inexorable dialectic changed into their very opposite” (Ibid.). That opposite was the unfair capitalist appropriation of ‘unpaid labor’. When Marx dealt with matters he deemed of great importance he “coquetted with the modes of expression peculiar to” Hegel, as he admits in the Afterword to the second German edition of Capital referring to the “chapter on the theory of value.” And we clearly see this also in this chapter on “The conversion of surplus value into capital” that we are dealing with, in which he explains that ‘unpaid labor’ is the basis for the exploitation of wage labor in CCP.

Why Marx’s theory of surplus value is invalid
The transition to Capitalism and the ‘first phase of Communism’

Just as Marx visualized the origin of Capitalism in the appropriation of the conditions of production by a segment of society, he also visualized the demise of Capitalism as its transformation into a society he deemed more advanced in which full rationality would prevail in a globally planned production process lacking the severe problems generated by markets and private ownership of the means of production. This progress would only be possible thanks to the enormous achievements of Capitalism in the advancement of the productive force of human labor. And it could probably only come about by means of a revolutionary transformation in which “The expropriators are expropriated” (B1, 750) and their entrepreneurial role is substituted by managers that work for and with all the associated workers. Because the capitalists and their State would do everything they could to prevent this transformation, in Marx’s perspective putting an end to the capitalist organization of society would require a political revolution that would expropriate capitalists, who by then would only be a small fragment of society due to the advanced process of ‘centralization’ of capital.3

Let us see how we can represent this expropriation using the analytical tools we developed in Part II of this book. Let us assume that, starting from CCP (with quantities system (8.34)), a revolution takes place by which the capitalists’ means of production are expropriated and become the property of the firms in which they are used, firms which become the property of the workers (collectively). Also, let us assume that the ex-capitalists do not run away to exile during the revolution but start to work to make a living, as the rest of society. Under such conditions the dimension that corresponds to capitalists \(q^K\) disappears from the quantities system, and the working population is increased from \(q^L\) to \(q^L + q^K\). The workers operate the firms in association and distribute the net revenues the firms generate in proportion to the work done. Finally, let us assume for simplicity that they all consume a basket of goods with the same structure as the global average that prevailed in the forgone capitalist era. Hence, the consumption basket for all workers is the weighted average of the two previous baskets: \(c_L\) and \(c_K\), i.e., \((q^L c_L + q^K c_K) / (q^L + q^K)\). Since there has been no technological change, the fact that there are more workers than there were in the capitalist era generates an increase in production in the same proportion as the working population has grown, that is, \((q^K / q^L)^\%\). Hence, the new quantities system is transformed from (8.34), which for the reader’s convenience is repeated here as (18.2), to (18.3), where \(\bar{q}^Q\) is the new gross output vector:

3It was no coincidence that the second Russian Revolution of 1917 (the October Revolution) did precisely this. Lenin, who led the process, and without whom it probably wouldn’t have taken place, was highly knowledgeable of Marx’s works, especially its political aspects.
\[
\begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}
\begin{bmatrix}
A & \ell & \eta \\
c_L & 0 & 0 \\
c_K & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K
\end{bmatrix}, \quad (18.2)
\]
\[
\begin{bmatrix}
\bar{q}^Q & q^L + q^K
\end{bmatrix}
\begin{bmatrix}
A \left(1 + \frac{q^K}{q^L}\right) \\
\frac{q^L c_L + q^K c_K}{q^L + q^K}
\end{bmatrix}
\begin{bmatrix}
\ell \\
0
\end{bmatrix}
= \begin{bmatrix}
\bar{q}^Q & q^L + q^K
\end{bmatrix}. \quad (18.3)
\]

The passage from one system to the other is completely compatible with Marx’s vision of the expropriation of capitalists as the first step in the generation of Socialism, first phase of Communism. For Marx the capitalist entrepreneurs were redundant in the production process since their active role (such as being ‘orchestra directors’) could be done by specialized workers (and their ‘vigilance’ role would be unnecessary in the new society). Hence, they could and should be expropriated. From the first equalities of these two systems we get through simple algebra an equality that shows an increase in the gross outputs as a result of the revolution:
\[
\bar{q}^Q = \left(1 + \frac{q^K}{q^L}\right) (q^L c_L + q^K c_K) B (0) = \left(1 + \frac{q^K}{q^L}\right) q^Q. \quad (18.4)
\]

The Gordian knot of the invalidity of the theory of surplus value

As a first step in the elucidation of why Marx’s theory of surplus value cannot be valid we pose the following numerical exercise.

The Gordian knot exposed numerically (Take a pure CCP society in which only one commodity is produced. A numerical example of the quantities (18.2), prices (8.23), and values (8.10) systems is the following:
\[
\begin{bmatrix}
333.33 & 200 & 100 \\
0.6 & 0 & 0 \\
0.8 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
333.33 & 200 & 100 \\
0.6 & 0 & 0 \\
0.8 & 0 & 0
\end{bmatrix}, \quad (18.5)
\]
\[
\begin{bmatrix}
(1 + 0.3158) & 0.4 \\
0.6 & 0 \\
0.8 & 0
\end{bmatrix}
\begin{bmatrix}
(1 + 0.3158) & 0.6 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
1.6667 \\
1.0 \\
1.3333
\end{bmatrix}
= \begin{bmatrix}
1.6667 \\
1.0 \\
1.3333
\end{bmatrix}, \quad (18.6)
\]
\[
\begin{bmatrix}
0.4 & 0.6 & 0 \\
(1 + 0.6666) & 0.6 & 0 \\
0.8 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
1.0 \\
1.0 \\
0.8
\end{bmatrix}
= \begin{bmatrix}
1.0 \\
1.0 \\
0.8
\end{bmatrix}. \quad (18.7)
\]

We initially have a population of wage workers \(q^L = 200\) and capitalists \(q^K = 100\). Gross output is \(q^Q = 333.33\) units of the only output. We have normalized vector \((p w \pi)’\) so that \(w = 1\). The profit rate is \(\rho = 0.3158\) and the surplus value rate \(e = 0.6666\); the profit per capitalist is \(\pi = 1.3333\) and the surplus value per capitalist \(\varepsilon = 0.8\).

After the revolution, the population of associated workers is \(q^L + q^K = 300\), a 50% increase. Hence there is also a 50% increase in gross output \(\bar{q}^Q = 500\). As we can see in (18.8), with the socialized means of production (and no ER) it would no longer be necessary to include a profit rate in the price system. And since there are no longer capitalists that consume the surplus output, workers (who are no longer wage workers) can increase their consumption by 50%. Hence, the system can again be represented
by a single values system (18.9) (as we had under SCP) and the prices of production disappear. The dual quantities and values systems are the following:

\[
\begin{bmatrix}
500 \\
300
\end{bmatrix}
\begin{bmatrix}
(1 + 0.5) & \frac{0.4}{200 + 100 + 0.8} & 0.6 \\
(1 + 0.5) & \frac{0.4}{200 + 100 + 0.8} & 0
\end{bmatrix}
\begin{bmatrix}
200 + 100 + 0.8 \\
0
\end{bmatrix}
= \begin{bmatrix}
500 \\
300
\end{bmatrix}
\]

(18.8)

Comparing (18.9) to (18.7), we can see that 1) the value of each unit produced is the same before and after the revolution, 2) there is a 50% increase in per capita physical consumption, i.e., the new per capita consumption of the ex-wage workers and ex-capitalists is the old per capita consumption of wage workers in the capitalist era increased by the (old) surplus value rate that was the basis for the consumption of capitalists. Hence, the workers that are now ex-wage workers have an increase in their income from 0.6 to 1, i.e., a 66.666% increase in their physical and value consumption (0.66666 being the rate of surplus value of the capitalist era, as seen in (18.7)). Finally, we have the paradoxical loose thread that should make anyone suspicious that not all is well here: workers who are ex-capitalists also have an increase in their consumption, in this case by 25%, their consumption increasing from 0.8 to 1.0! Hence, if the initial theoretical setup was valid and capitalists were intelligent and interested in their own material welfare, they would have wanted to increase their consumption... by encouraging the revolution!

**The Gordian knot exposed algebraically** After showing this numerical exercise that reveals certain absurd consequences of Marx’s theory of exploitation in the capitalist mode of production and its dissolution by means of the ‘expropriation of the expropriators’, we now focus on the algebra. For this it is sufficient to compare systems (18.2) and (18.3). The former takes into account the allocation of capital to the various industrial branches. And in the latter this is no longer necessary since the capital of the ex-capitalists is now common property of the associated workers. Also, in the latter equation we have the effect that results from the addition of workers that before only appropriated surplus value without contributing to the generation of value in any sense that is represented analytically. Since the technology is linear, the fact that the ex-capitalists now work increases output and per capita consumption by \(q^K / q^L\)% i.e., as much as the working population has increased, as seen in (18.4). To see this in greater detail, let \(C = (q^L c_L + q^K c_K)\) be global consumption in CCP, and \(c\) the corresponding per capita consumption:

\[
c = \frac{C}{q^L + q^K} = \frac{q^L c_L + q^K c_K}{q^L + q^K}.
\]

Gross output in CCP is \(q^Q = C (I - A)^{-1} = CB (0)\). Let \(\overline{q}^Q\), \(\overline{C}\), and \(\overline{\sigma}\) be the gross output, aggregate consumption, and per capita consumption after the revolution. The increase in available labor makes gross output increase (from \(q^Q\) to \(\overline{q}^Q\), as seen in (18.4)) as well as global consumption (from \(C = q^Q (I - A)\) to \(\overline{C} = \overline{q}^Q (I - A)\)) by \(q^K / q^L\)% . The value of consumption also increases (from \(Cv\) to \(\overline{C}v\)), as the following shows:

\[
q^L = q^Q \ell = C (I - A)^{-1} \ell = Cv
\]

\[
q^L + q^K = \overline{q}^Q \ell = \overline{C} (I - A)^{-1} \ell = \overline{C}v.
\]

(18.10)
In brief, due to the assumptions that 1) the conservation in the aggregate structure of consumption and 2) the expansion of the labor force with the work of the ex-capitalists, there is a \( q^K/q^L \% \) increase in per capita consumption:

\[
\bar{\tau} = \frac{C}{q^L + q^K} = \left(1 + \frac{q^K}{q^L}\right) \frac{q^L c_L + q^K c_K}{q^L + q^K} = \left(1 + \frac{q^K}{q^L}\right) c.
\]

On the other hand, the consumption of the ex-capitalists does not necessarily increase. It increases if the initial situation in the value of capitalist consumption is not excessively high in relation to that of wage workers. More precisely, the consumption of ex-capitalists increases if and only if the ratio between the consumption of wage workers and capitalists is higher than one minus the ratio between the populations of capitalists and wage workers. Alternatively, the consumption of the ex-capitalists increases if and only if the rate of surplus value is lower than the ratio between the capitalist population and the difference between the worker and capitalist populations. These statements are proved by the following sequence:\(^4\)

\[
c_K v < \bar{\tau} v \iff \frac{c_L v}{c_K v} > 1 - \frac{q^K}{q^L} \iff e \equiv \frac{q^K c_K v}{q^L c_L v} < \frac{q^K}{q^L - q^K}.
\]

We have come to the core of the fallacy that underlies Marx’s theory of exploitation in Capitalism: in his representation of the production process his ‘functioning’ capitalists only appear as passive recipients of a part of the value generated, without the equations reflecting any activity on their part. Hence, there is nothing ‘functioning’ about them other than the name Marx gives them with the intention of differentiating them from the loan capitalists, who can legitimately be considered ‘passive’ except in the case of banking entrepreneurs, in which case their activity would also need to be represented in the system. In Marx’s analytical representation the ‘functioning’ capitalists do not contribute to the production process except by their disbursement of capital, even though in Marx’s textual explanations they had various functions, including that of ‘orchestra director’. This is why per capita output increases when they become workers, even generating the absurd possibility that their expropriation leads to the increase in their consumption if the initial disparity was not excessive (that is, if the rate of surplus value was not excessive).

It is quite surprising that, there having been so many critiques of Marx’s theory of surplus value, it was never analyzed (to the best of our knowledge) in the (integral) way in which we have. The analytical representations of Marx’s theory have consisted in incomplete systems in which quantities and populations (especially that of capitalists) do not appear explicitly, thus obscuring the overall view. This deficiency is related to the similar deficiency of mainstream Economics. During the last 150 years Economics has tended to ignore the specific role of the entrepreneur (whether or not he owns a part of the capital) in the theoretical representation of the economy. The firm appears as a black box into which inputs (including labor) go and out of which comes output, without a specific agent actively engaged in making sure the integral process develops smoothly, in spite of a plethora of changing circumstances and uncertainties. This modality prevailed in almost all Neoclassical economists and had its paradigmatic format in Walras’ Principles, where although the entrepreneur exists as a concept, he is an almost fictitious character since he can only consume, and therefore live, if he contributes either work (for which he receives a wage), capital (for which he receives interest), or land (for which he receives rent). There is an absence of a ‘profit of enterprise’ in the theory. We will see this with some detail in Chapter 19.

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\(^4\)The last inequality holds in our numerical exercise, since \(0.66666 < 100/(200 - 100) = 1\).
The theoretical representation of the reality of Capitalism

Marx believed he had exposed how illusory was the notion that there was no exploitation in the capitalist mode of production. In slavery or feudal serfdom it was evident that at least a part of the worker’s time was appropriated by his exploiter for his own benefit through his appropriation of the net product and the severe restrictions on the workers’ freedom. Marx believed that something very similar occurred in Capitalism with wage work, and that it was an illusion that the worker was compensated for all his labor time. Behind this illusion was for Marx the reality that the origin of capitalists’ consumption and his capital accumulation was the unpaid labor of wage workers, i.e., the work they did for ‘free’. Marx expresses this clearly when he writes: “There the property relation conceals the labour of the slave for himself; here the money relation conceals the free labour of the wage labourer” (B1, 539-40).5

Wanting to destroy the illusion that the worker was paid for all his labor time, Marx’s theory of exploitation distinguished between the working day’s labor time, which created value, and the interpretation of the wage as the price capitalists paid for a day of labor power which once purchased could be consumed by them during a period of time sufficiently greater than the number of hours necessary, in the aggregate, to produce the workers’ consumption baskets. Hence, there was a surplus product, the value of which formed the surplus value, the aggregate of which was the source of all profits and rents. The value of the aggregate net output was thus necessarily equal to the value of workers’ consumption goods plus the value of the propertied classes’ consumption goods (plus the value of aggregate investment goods in the case of ER). Hence, the explanation of the exploitation of wage workers was the unequal (and fictitious) ‘exchange’ between workers and capitalists in the sphere of production (and not the exchanges in the sphere of circulation). In this ‘exchange’ the capitalist received and the worker contributed ‘unpaid’ labor.

Marx held that there was a great difference between the wage as the price of work, which he considered “the phenomenal form” that political economy exclusively looked at, and the price of labor power, which was “the essential relation manifested therein.” In reference to the outward appearances (phenomena) and the underlying reality, he held that “The former appear directly and spontaneously as current modes of thought; the latter must first be discovered by science. Classical political economy nearly touches the true relation of things, without, however, consciously formulating it. This it cannot so long as it sticks in its bourgeois skin” (B1, 542). Marx believed he had penetrated the “true relation of things” by means of his scientific analysis, having thus left aside economists’ point of view, whose vision was blurred by their tendency to identify with the perspective and the interests of the classes that benefited from worker exploitation. As we have seen, however, his theory of surplus value was not sufficiently solid to achieve his aims, despite its sophistication.

Nevertheless, it must be recognized that Marx tried (during most of his life, and with great sacrifice) to establish a theoretical framework that could represent a reality that he perceived and synthesized with assertions such as: “Capital further developed into a coercive relation, which compels the working class to do more work than the narrow range6 of its own life wants prescribes. As a producer of the activity of others, as a pumper-out of surplus labour and exploiter of labour power, it surpasses in energy, disregard of bounds, recklessness and efficiency, all earlier systems of production based on directly compulsory labour” (B1, 314). We hold that the theoretical framework he

5Near the end of Chapter 8 we quoted the paragraph that contains this sentence.
6MECW 35 has ‘round’ instead of ‘range’, which is clearly a slip.
developed was unable to truly represent a reality that we could today synthesize by means the following alternative statement: “As producers of activity (their own and that of the workers they employ), as planners, organizers, commanders, and controllers of firms engaged in production, commerce, and banking, as introducers of technological and organizational innovations, by doing their utmost to thwart the force of competition through formal or informal agreements, and by exerting their influence on political power and the State, big corporate capital surpasses in energy, disregard of bounds, recklessness and efficiency, all earlier modes of production, commerce, and banking.” Notwithstanding the deficiencies of Marx’s theoretical framework (visible with the benefit of an additional 150 years of hindsight) Marx was trying to represent by means of a new theory (as always, constructed on the shoulders of a multitude of previous theoretical developments) a historical and contemporary reality. His perception of that reality was broadly based on empirical data that he meticulously gathered and analyzed. And it was explicitly based on a point of view that intended to further the interests of the exploited and oppressed taking into consideration the possibilities that the very effectiveness of Capitalism allowed insofar as the achievement of greater human liberty and fulfillment. His historically endorsed position was that all previous socioeconomic-political organizations of society had been eventually replaced by others. And his highly original conception was that the common thread for the comprehension of the historical dynamics was the functionality of the existing human relations for the reproduction and progress of society as it was perturbed by the factors that tended to increase the productivity of labor (the ‘productive forces’).

Marx’s conception of the capitalist entrepreneur

The aim of Marx’s theory of value (in contrast to his theory of value, i.e., of equilibrium prices) was to build a foundation for his theory of the exploitation of wage work in Capitalism. It made it possible to strictly differentiate the (abstract and socially necessary) labor time required for the reproduction of the ‘productive’ working class (i.e., those workers that produced surplus value) from that required for the reproduction of the classes that owned the conditions of production (means of production and land) and other ‘unproductive’ classes. Marx called ‘unpaid’ labor whatever labor ‘productive’ workers performed in excess of that necessary for their own reproduction. Since the equilibrium (or ‘regulating’) prices differed in their structure from that of values, it was necessary to ‘transform’ the ‘paid’ and ‘unpaid’ components of productive labor into the corresponding incomes of the different social classes: the wages of workers (productive or not), the profits of entrepreneurs, the interests of financial capitalists, and the rent of landowners.

The function of the capitalist entrepreneur (or ‘active’ capitalist) was fundamentally to ensure the production and appropriation of surplus value, i.e., the ‘unpaid’ labor of productive wage workers. In Book I Marx warns against the idea that the creation of value can be partly composed of the labor of the capitalist entrepreneur. After analyzing the process of valorization, he paints a lively picture of a capitalist who, knowing intimately that the secret to reaping surplus value from the production process is having workers work during a working day that is longer than that necessary to reproduce their means of subsistence, amuses himself by explaining that his work “of superintendence and of overlooking” also creates value (while mocking the economists who make a living out of proving it):

Our friend, up to this time so purse-proud, suddenly assumes the modest demeanour of his own workman, and exclaims: “Have I myself not worked?
Have I not performed the labour of superintendence and of overlooking the spinner? And does not this labour, too, create value?” His overlooker and his manager try to hide their smiles. Meanwhile, after a hearty laugh, he re-assumes his usual mien. Though he chanted to us the whole creed of the economists, in reality, he says, he would not give a brass farthing for it. He leaves this and all such like subterfuges and juggling tricks to the professors of political economy, who are paid for it. He himself is a practical man; and though he does not always consider what he says outside his business, yet in his business he knows what he is about (B1, 203).

Marx dedicated quite a few pages to what the functions of the entrepreneur in the production process are and to the nature of his income, knowing it was crucial for his theory of surplus value – based on the ‘unpaid’ work of wage labor. He had destroyed the idea that the foundation of surplus value was to be found in the entrepreneur not paying the worker for the full value of his labor power when he purchased it, an idea that was grounded on the circulation of commodities (market transactions). And he had replaced it with the idea that surplus value originated in the production process, by consuming the purchased labor power during a sufficiently long working day. He held that the key idea was that labor power was a unique commodity: it not only generated value but was also able to generate a value greater than its own value, that is, than the value of the consumption basket that the worker needed for his reproduction (and those of his dependents). However, this argument was very weak. For it was based on a very strange ‘contract’: the capitalist offers and the worker accepts an agreement in which the former pays the latter a remuneration that allows for his maintenance but where the amount of time that he will work per day is left to the will of the capitalist (with the argument that every buyer has the right to consume the use value of what he has purchased – in this case labor power – as he sees fit). To base his theory of capitalist exploitation on the distinction between work and labor power was ingenious but fragile. Marx well understood that it was erroneous to say that the work exerted by the laborer had a value, since (abstract socially necessary) labor was the ‘substance’ of value and could not be a measure of itself. Instead, the value of labor power was that of the consumption basket the worker needed. But a certain quantity of work of a certain intensity exercised over a working day of specified duration could have a price and could be used for a contract. The question here is whether anything more than this is necessary. Marx was convinced of the affirmative, that the ‘esoteric’ part of his theory (the theory of value and surplus value) was indispensable as a foundation for its ‘exoteric’ part (the theory of the ‘regulating prices’, wages, profits, interests, and rents). We are here using terms that Marx himself used to criticize Adam Smith for his recurrent ‘confusion’ between these two levels: “Adam Smith’s successors... can always regard Adam Smith as their base, whether they follow the esoteric or the exoteric part of his work or whether, as is almost always the case, they jumble up the two” (B4.31, 391). Marx held that Ricardo’s great contribution had been to establish that “the starting-point for the physiology of the bourgeois system... is the determination of value by labour time” (Ibid.). This idea would inspire his construction of the ‘esoteric’ part of his own theory.

It was crucial for Marx’s theory to conserve the values (which he first developed for SCP, as the ‘regulating prices’) even after introducing CCP and the heterogeneity of value compositions since this was the foundation for the definition of surplus value. He knew, and so he stated, that in Capitalism the equilibrium prices differed from the values of commodities, but he held that these still played a central part at a deeper level than that of the superficial phenomena from which the ‘vulgar’ political
economists could not escape. The values in Capitalism enabled the measurement of the exploitation of wage work by means of the concept of surplus value, source of all the incomes of the propertied classes. And, as we have seen in Part II of this book, this can be consistently done if only one corrects the approximation Marx used for the prices of production and recognizes that although he only sketched the basic ideas of a theory of absolute rent he was on the right track (within the limitations of the Classical theoretical framework, lacking in the Neoclassical ‘subjective’ concepts useful for the endogenization of demand and the supplies of non-produced resources).

The ‘command’ of the industrial capitalist over a multitude of laborers working in cooperation in a factory had turned into a “requisite for carrying on the labour process itself, into a real requisite of production”, “as indispensable as that a general should command on the field of battle” (B1, 335); since “combined labour on a large scale requires, more or less, a directing authority, in order to secure the harmonious working of the individual activities, and to perform the general functions that have their origin in the action of the combined organism.” And Marx gave a musical analogy: “A single violin player is his own conductor; an orchestra requires a separate one” (B1, 336). So too in the capitalist firm: “The work of directing, superintending, and adjusting, becomes one of the functions of capital” (Ibid.). But the capitalist’s control over the functioning of the production process had a dual character. Aside from its role as “a special function, due to the nature of the social labour process, and peculiar to that process”, there was “at the same time a function of the exploitation of a social labour process”, which was not inherent to the labor process itself but “rooted in the unavoidable antagonism between the exploiter and the living and labouring raw material he exploits”; since, “in proportion to the increasing mass of the means of production, now no longer the property of the labourer, but of the capitalist, the necessity increases for some effective control over the proper application of those means” (Ibid.). The wage work of free laborers had been a great institutional advance over the work of slaves, where it was virtually impossible for them to handle tools of any sophistication without damaging them. That was why under slave labor it was mandatory “only to employ the rudest and heaviest implements and such as are difficult to damage owing to their sheer clumsiness” (B1, 207, footnote 1). In Capitalism it was still necessary to ensure by vigilance the proper use of increasingly complex machinery as well as the compliance with other norms that lowered costs (such as a minimum of labor intensity). This commanding function was more related to the creation of surplus value than to the production of use values, and for Marx it would be unnecessary in a society in which the antagonistic character of the production process disappeared. This antagonistic character made capitalist control “despotic” (B1, 337). As large scale cooperation developed in Capitalism, first “the capitalist is relieved from actual labour”; then when his capital has grown “he hands over the work of direct and constant supervision of the individual workmen, and groups of workmen, to a special kind of wage labourer”; and finally the work of supervision becomes the “established and exclusive function” of managers, foremen, overlookers, who “while the work is being done, command in the name of the capitalist” (Ibid.). And Marx reproaches ‘political economists’ for treating “the work of control made necessary by the co-operative character of the labour process as identical with the different work of control, necessitated by the capitalist character of that process and the antagonism of interests between capitalist and labourer” (Ibid.).

In Chapter 23 of Book III (“Interest and profit of enterprise”) Marx returns to the theme of the entrepreneur and the profit of enterprise when he separates ‘gross profit’ into two components: the interest on loan capital, which is based on the mere
ownership of capital—a form of property he calls ‘inert’—and the profit of enterprise, which is based on the “function of capital in the reproduction process”:

The functioning capitalist is here assumed as a non-owner of capital. Ownership of the capital is represented in relation to him by the money capitalist, the lender. The interest he pays to the latter thus appears as that portion of gross profit which is due to the ownership of capital as such. As distinct from this, that portion of profit which falls to the active capitalist appears now as profit of enterprise, deriving solely from the operations, or functions, which he performs with the capital in the process of reproduction, hence particularly those functions which he performs as entrepreneur in industry or commerce (B3, 371; italics added).

To simplify, he here assumed a complete separation between the ownership of capital (that belongs to “money capitalist, the lender”) and “the operations, or functions” performed by the entrepreneur (the ‘active’ capitalist) in industry or commerce. The latter’s income, the ‘profit of enterprise’, is not a simple ‘sinecure’, as is the case of the interest on loan capital, but is a special income derived from the active role of the entrepreneur, a role that “entails exertion”:

The profit of enterprise springs from the function of capital in the reproduction process, hence as a result of the operations, the acts by which the functioning capitalist promotes these functions of industrial and commercial capital. But to represent functioning capital is not a sinecure, like representing interest-bearing capital. On the basis of capitalist production, the capitalist directs the processes of production and circulation. Exploiting productive labour entails exertion, whether he exploits it himself or has it exploited by someone else on his behalf. Therefore, as distinct from interest, his profit of enterprise appears to him as independent of the ownership of capital, but rather as the result of his functions as a non-proprietor—a labourer” (B3, 378; italics added).

The entrepreneur’s perception that the ‘profit of enterprise’ is a result of his own labor reappears in one of the last chapters of Capital: “One portion of profit, as opposed to the other, separates itself entirely from the relationship of capital as such and appears as arising not out of the function of exploiting wage labour, but out of the wage labour of the capitalist himself. In contrast thereto, interest then seems to be independent both of the labourer’s wage labour and the capitalist’s own labour, and to arise from capital as its own independent source” (B3, 816; italics added).

Marx struggled to find a way of representing the role of the entrepreneur that would not subvert the essence of his theory of surplus value based on ‘unpaid labor’. Although he recognized the entrepreneur’s exertion (i.e., his labor), he separated a legitimate and necessary part that was not inherent to Capitalism but responded to the complexity of cooperative work and the need for its coordination from another part that was inherent to ensuring the production of surplus value. Marx considered illegitimate, although in accordance with ‘bourgeois law’, the appropriation of surplus labor, as is reflected in the expression ‘unpaid labor’ (and in the political proposal of ending with Capitalism). For Marx the entrepreneur merely perceived that “his profit of enterprise, far from being counterposed to wage labour and far from being the unpaid labour of others, is itself rather a wage or wages of superintendence of labour, higher than a common labourer’s, 1) because the work is far more complicated, and
2) because he pays them to himself” (B3, 378). Thus, the entrepreneur “entirely lost sight of” the “fact that his function as a capitalist consists in creating surplus value, i.e., unpaid labour, and creating it under the most economical conditions.”

Although Marx recognized that the capitalist entrepreneur actively commands everything related to his enterprise and that it implies an exertion (i.e., work), he believed that in a more rational society endowed with production relations exempt from asymmetries, hierarchies, and despotism (but not necessarily income inequality), the legitimate tasks of the capitalist entrepreneur would be performed by a manager contracted by a worker cooperative or by the State in exchange for a retribution that would no longer be a ‘wage’ (but much less a ‘profit’). The ‘labor of management and superintendence’ of the capitalist entrepreneur had thus a dual character: on one hand the work of commanding was like that of an “orchestra conductor” and was necessary to “coordinate and unify” the “total activity of the workshop”:

On the one hand, all labour in which many individuals cooperate necessarily requires a commanding will to coordinate and unify the process and functions which apply not to partial operations but to the total activity of the workshop, much as that of an orchestra conductor. This is a productive job, which must be performed in every combined mode of production (3, 382; italics added).

This labor responded to “a special function determined by the nature of all combined social labour.” It was not inherent to the entrepreneur as a capitalist. But on the other hand, there was a labor of supervision that was intimately related to the antagonistic character of the relation between the worker and owner of the means of production, and which had existed in pre-capitalist modes of production as well as the specifically capitalist:

this supervision work necessarily arises in all modes of production based on the antithesis between the labourer, as the direct producer, and the owner of the means of production. The greater this antithesis, the greater the role played by supervision. Hence it reaches its peak in the slave system. But it is indispensable also in the capitalist mode of production, since the production process in it is simultaneously a process by which the capitalist consumes labour power (Ibid.).

The labor of coordinating and unifying was positive and productive, for “the capitalist’s labour... inasmuch as it does not confine itself solely to the function of exploiting the labour of others; inasmuch as it therefore originates from the social form of the labour process, from combination and cooperation of many in pursuance of a common result, it is just as independent of capital as that form itself as soon as it has burst its capitalistic shell” (B3, 385). In other words, the entrepreneur’s role as ‘orchestra conductor’ was inherent to the complexity of the productive process and did not respond to Capitalism as such, with its antagonistic essence. In contrast, the role of ‘supervision’, necessary to ensure the generation and appropriation of surplus value, would be superfluous in a society devoid of class antagonisms. Those who exercised this role in capitalist society were increasingly “a numerous class of industrial and commercial managers” whose “wages of superintendence, like any other wage, found their definite level and definite market price” (B3, 387).

Marx continued to search for a way of interpreting the role of the capitalist entrepreneur that would not undermine his theory of surplus value up to the end of his life.
He made some of his last annotations on economic matters sometime in 1881 or 1882, with his notes on the margins of the German economist Adolph Wagner’s *Textbook on Political Economy*, who criticized Marx’s theory. In particular, he wrote that Marx proceeded arbitrarily when he reduced costs “solely to what is termed labour output in the narrowest sense of the term. That always presupposes proof which is hitherto lacking, namely that the production process is possible entirely without the mediation of the activity of private capitalists in amassing and employing capital” (MECW 24, 535). And “As long as such proof has not been furnished... Then profit on capital is also in fact... a ‘constitutive’ element of value, not, as in the socialist view, simply a deduction from, or ‘robbery’ of, the worker” (Ibid.). The substance of Marx’s comment is the following:

... in my presentation even, “profit on capital” is in actual fact not “a deduction from, or robbery of, the worker.” On the contrary, I depict the capitalist as the necessary functionary of capitalist production and demonstrate at great length that he not only “deducts” or “robs” but enforces the production of surplus value, thus first helping to create what is to be deducted; what is more, I demonstrate in detail that even if only equivalents were exchanged in the exchange of commodities, the capitalist –...– would have every right, i.e. such right as corresponds to this mode of production, to surplus-value. But all this does not make “profit on capital” the “constitutive” element of value but only proves that the value not “constituted” by the labour of the capitalist conceals a portion which he can appropriate “legally”, i.e. without infringing the law corresponding to the exchange of commodities (Ibid., 535-6).

Hence, Marx explains that he considers the capitalist entrepreneur as a “necessary functionary of capitalist production” who enforces the production of surplus value by “helping to create what is to be deducted” (i.e., surplus value), and that he does this “without infringing the law corresponding to the exchange of commodities.” On the one hand, he ensures that wage workers produce surplus value and, on the other, in conformity with the laws of circulation of commodities, ‘deducts’ it, i.e., keeps the profit of enterprise (after deducting interests –whether or not the capital is loaned– and rents –whether or not the premises of the factory or the farmland is leased). And, as we saw above, a part of the ‘profit of enterprise’ was considered by Marx as legitimate since it corresponded to a type of directing work that was inherent to complex production processes that required coordination and direction. But what is problematic from a theoretical point of view is that the directing labor of the entrepreneur did not explicitly appear in his analytical framework, neither in the ‘esoteric’ nor in the ‘exoteric’ part.

**Entrepreneurial activity in the systems of quantities and prices**

We believe that from a scientific point of view the capitalist production process can only be adequately represented if the entrepreneur’s activity is explicitly reflected in the equations we use, even if it is in done only in highly stylized form. Marx was correct in holding that the capitalist organization of production was born historically as a consequence of the formation of great masses of free people who had no possibility of earning a living without selling their labor power, and on the other hand the concentration of money capital in the hands of entrepreneurs that could invest in the conditions of production and the hiring of wage workers. But the capitalist organization of firms required from its beginnings the active participation of the entrepreneur in tasks that were very difficult to delegate in the hands of wage workers for various reasons. Among
these are the incentive that can induce going through an intense and highly specialized effort focused not only on the organization and supervision of the labor process within the firm but also on the adaptation of the production and circulation process to the changing context in which the firm operates, a context that includes the markets in which it operates directly and also the macroeconomic and political situation and its likely future development. This adaptive process is crucial for the firm’s survival and cannot be achieved successfully without a special comprehension of that complex and changing context. Of course, in some branches this entrepreneurial ‘talent’ often includes a complete moral disassociation both from the consequences of the undertaking and from the means employed for reaping profits. But let us leave this aspect of the matter aside for now. A crucial aspect is the uncertainty under which the firms operate and the possibility that an erroneous evaluation of circumstances can lead to the loss of the owner’s capital and the entrepreneur’s power (whether or not they are the same person or people). And this is an aspect that cannot adequately be represented by means of the instruments we have been using.

Restricting ourselves to the same kind of techniques we have been using to interpret Marx’s theory, we can take a first step in correcting one of the main problems with Marx’s formulation: the non representation of the entrepreneur’s exertion in his formal apparatus. As we have seen, although Marx recognized in his texts the role of ‘orchestra conductor’ of the entrepreneur, he did not include it in his equations. According to our alternative view, the capitalist entrepreneurs (industrial, commercial, or financial), or the specific agents that may act in their name (directors, managers, executives, etc.) actively participate in the firm they work in by Planning, Organizing, Commanding and Controlling (POCC), henceforth ‘POCC labor’, and this labor must be represented in the theory and, in particular, in the models employed. Hence, in the rest of this section we modify the principal dual systems of Part II in order to explicitly reflect this POCC labor in the systems of quantities and prices.

We here dispense with the system of values and surplus value because it is deemed unsuitable. This alone implies that we need not make a radical distinction between the industrial, commercial, and banking (or financial) spheres, since the firms in these spheres all equally need to reap profit to function in Capitalism. And we make a small expansion to the systems we have been using to represent the ‘exoteric’ part of Marx’s theory. We assume that the capitalist entrepreneurs (whether they are industrial, commercial, or financial) own all the capital they use and that they impute as cost their own POCC labor and also receive a profit that is proportional to the disbursed capital. To simplify, we assume that the entrepreneurs’ POCC labor is homogeneous, that the amount required in branch $i$ is $\ell_{K,i}$ per unit of output, and that each capitalist exerts a unit of this work and owns an amount of capital of given magnitude (which is common to all). Hence, $q^K$ represents not only the capitalist population but also the total amount of POCC labor that is available. Instead of (8.34), (8.37), and (8.23), we have the following systems:

\[
\begin{bmatrix}
q^Q & q^L & q^K & q^K
\end{bmatrix}
\begin{bmatrix}
A & \ell_L & \ell_K & \eta_K \\
\ell_L & 0 & 0 & 0 \\
\ell_K & 0 & 0 & 0 \\
\eta_K & 0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^L & q^K & q^K
\end{bmatrix},
\]

\[
\begin{bmatrix}
A & \ell_L & \ell_K & \eta_K \\
\ell_L & 0 & 0 & 0 \\
\ell_K & 0 & 0 & 0 \\
\eta_K & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
p \\
w \\
w_K \\
\pi
\end{bmatrix}
= \begin{bmatrix}
p \\
w \\
w_K \\
\pi
\end{bmatrix},
\]

(18.11)  
(18.12)
\[
\begin{pmatrix}
(1 + \rho) A & (1 + \rho) \ell_L & \ell_K \\
-c_L & 0 & 0 \\
-c_K \ell & 0 & 0
\end{pmatrix}
\begin{pmatrix}
p \\
w \\
w_K
\end{pmatrix} =
\begin{pmatrix}
p \\
w \\
w_K
\end{pmatrix}.
\] (18.13)

These systems of equations contain additional information with respect to Marx’s since they reflect the active participation of the capitalist-entrepreneurs in the social process, a participation which is of fundamental importance both in the production as in the circulation of commodities. The last two equalities of (18.11) show that the capitalists are allocated in two different ways to the various branches: according to the POCC work they exert \( \ell_K \) and according to the capital they disburse \( \eta_K \). We thus have \( q^2 \ell_K = q^2 \eta_K = q^K \). Some of the branches may require more capital (per unit of output) than POCC labor (per unit of output) and others less. Hence, \( \ell_K \) are \( \eta_K \) not proportional in general. In (18.13) we assume that the profit rates are all the same in the various branches. Since the entrepreneur’s POCC labor is not disbursed as capital, \( \ell_K \) is not multiplied by \( 1 + \rho \). The last two equalities of (18.12) show that the capitalist entrepreneurs have two different consumption baskets: one \( (c_K \ell p = w_K) \) that corresponds to the part of their income that comes from their POCC work, and another \( (c_K \eta p = \pi) \) that corresponds to the part that comes from the profit on capital disbursed. It is important to note that \( w_K \) does not represent a wage (since the entrepreneur’s work is not wage work). It is the retribution he imputes for his POCC work and is accounted for in his costs. According to whether we use (18.12) or (18.13) the equilibrium price can be written in two ways:

\[
p = (1 + \rho) (Ap + \ell_L w) + \ell_K w_K = Ap + \ell_L w + \ell_K w_K + \eta_K \pi.
\]

Premultiplying by \( q^Q \) we obtain from the second equality the global profit rate:

\[
\rho = \frac{q^K \pi}{q^Q Ap + q^L w} \frac{e^P}{\kappa^P + 1}, \quad e^P = \frac{q^K \pi}{q^L w}, \quad \kappa^P = \frac{q^Q Ap}{q^Q \ell_L w}.
\]

Notice that each capitalist’s consumption is \( w_K + \pi = (c_K \ell + c_K \eta) p \) but only \( \pi \) comes from his profit. The other part is imputed as a cost, and hence is a part of the price, but is not a component of capital. Hence, \( w_K \) does not appear in the expression for \( \rho \). Although Marx’s analytical apparatus does not reflect this distinction, he did mention it in his texts, as for example:

The specific functions which the capitalist as such has to perform, and which fall to him as distinct from and opposed to the labourer, are presented as mere functions of labour. He creates surplus value not because he works as a capitalist, but because he also works, regardless of his capacity of capitalist. This portion of surplus value is thus no longer surplus value, but its opposite, an equivalent for labour performed (B3, 380-1; italics added).

If we change ‘surplus value’ for ‘profit’, we have an approximate verbal description of the model above.

If we 1) eliminate the ‘esoteric’ part of Marx’s theory, and 2) in the ‘esoteric’ part explicitly represent the labor of entrepreneurs (capitalists and executives, or merely executives if none of the stockholders participate in the firm’s operations), we get a simple but reasonable approximation to the basic functioning of Capitalism (before or after the domination of corporate capital). Such changes do away with the need Marx had of strictly separating the sphere of production from the sphere of circulation. This separation was due, on the one hand, to the primacy Marx gave to production
for historical-philosophical reasons (his Historical Materialism): production existed in human society for thousands of years before the production of commodities, that is, production of goods and services for their sale in a market. And, on the other hand, since Marx’s \textit{values} were absolute magnitudes it was fundamental to strictly separate what kind of labor created \textit{value} and what kind of wage labor created surplus \textit{value}. Marx held that only labor in the sphere of production could be ‘productive’ (of \textit{value} and surplus \textit{value}). In the case of SCP, the work dedicated to buying and selling did not produce value. And in CCP neither wage labor involved in commercial or financial transactions within industrial firms, nor any form of wage labor in commercial or financial firms could be ‘productive’ of surplus \textit{value}. But once we dispense with the theory of surplus \textit{value} such delimitations are irrelevant.

In the resultant structure there is a simple set of dual equations of quantities and populations and prices and incomes by means of which we can represent the class structure of Capitalism with various degrees of detail, as long as the entrepreneur’s work appears explicitly. This framework is far from the distortion of Capitalism given by the economic theory based on perfect competition, since the ‘profit of enterprise’ is present (even in equilibrium). And the oligopolistic sectors of modern corporate Capitalism (which we sketched in Chapter 17) can also be represented. But it is also far from the distorted representation of Capitalism Marx generated with his theory of surplus \textit{value} as ‘unpaid labor’, which implied the outright elimination of entrepreneurial activity (from the equations even though they were present in the texts).

In any acceptable model of the capitalist economy the P OCC labor of capitalist entrepreneurs is essential and, hence, if capitalists were expropriated and displaced they would have to be substituted by others who perform the same functions if a catastrophic fall in output is to be avoided. Cassel saw this clearly when he criticized some socialist proposals, as we show in a Bibliographic Note in the Appendix to this chapter. And that is (part of) what actually happened in the first stage of the October Revolution in Russia with agricultural output, before Lenin implemented the New Economic Policy that reintroduced market incentives in rural areas. Instead of the production functions (5.19) we introduced in Chapter 5 when we dealt with SCP (for the simple case of two produced commodities) we now have for any branch \( i \) a production function in which entrepreneurial POCC labor appears explicitly. In the case of a fixed coefficients technology (which we do not abandon), we have:

\[
q_i^Q = \min \left( \frac{q_{1i}^Q}{a_{i1}}, \frac{q_{2i}^Q}{a_{i2}}, \ldots, \frac{q_{ni}^Q}{a_{in}}, \frac{q_{Li}^L}{\ell_{Li}}, \frac{q_{Ki}^K}{\ell_{Ki}} \right), \quad i = 1, \ldots, n, \tag{18.14}
\]

where \( q_i^K \) is the POCC labor used in branch \( i \). With this formulation, if \( q_i^K = 0 \) in branch \( i \) output is necessarily null (\( q_i^Q = 0 \)). Thus, here entrepreneurial activity is essential in all branches, just as wage labor is.

\textbf{More evidence on the invalidity of the theory of surplus value}

If we define the (total) consumption basket of the entrepreneur as \( c_K = c_{KL} + c_{KQ} \) we again get (8.2) from the first equation of (18.11). Multiplying (8.2) by \( \ell_L \) yields

\[
q_L = q^Q \ell_L = (q^L c_L + q^K c_K) v_L \tag{18.15}
\]

where we have defined vector \( v_L \equiv (I - A)^{-1} \ell_L \), whose components we can call ‘L-values’. This vector is identical to Marx’s \textit{values}. Hence, if we define the ‘rate of
surplus L-value’ as

\[ \epsilon_L = \frac{q^K c_K v_L}{q^L c_L v_L}, \]

we can rewrite (18.15) as:

\[ (1 + \epsilon_L) c_L v_L = 1. \tag{18.16} \]

This expression is identical to (8.8) (except for the change in notation): the L-value of the consumption basket of wage workers, when expanded by the rate of surplus L-value, is one.

Do we again have an expression that reflects the exploitation of wage workers? To answer this question it is convenient to also multiply (8.2) by \( \ell_K \) and get

\[ q^K = q^Q \ell_K = (q^L c_L + q^K c_K) v_K \]

where we have defined the vector of ‘K-values’ as \( v_K \equiv (I - A)^{-1} \ell_K \). If we now define the ‘rate of surplus K-value’ as

\[ \epsilon_K = \frac{q^L c_L v_K}{q^K c_K v_K}, \]

we get

\[ (1 + \epsilon_K) c_K v_K = 1. \]

This expression is symmetrical to that of (18.16). These formulations are mathematically correct. But how can we interpret them? Evidently, it makes no sense to say that capitalists exploit wage workers and that the latter also exploit the former. Once we formally recognize that capitalist entrepreneurs make a (fundamental) contribution to the productive process through their exertion, the idea that they can be dispensed with is destroyed, as well as the conception that the income of entrepreneurs and owners is based on an ‘unpaid’ portion of the labor of wage workers.

Let us observe in passing that the capitalist entrepreneurs that handle firms endowed with monopoly power or monopsony power can have a higher rate of ‘profit of enterprise’ than average at the expense of the sectors devoid of monopoly or monopsony power. But also, in advanced corporate Capitalism the executives that exert control over the corporation (and supposedly represent the interests of stockholders) can skim the cream off the profits and distribute diluted dividends to (in many cases millions of dispersed) stockholders. We have seen in Chapter 17 that Marx referred to this process in Book III, when he addresses stock companies. We will resume this line of thought in Chapter 19 when we deal with the “corporate revolution” of Berle and Means. It is a direction that the historical development of Capitalism has followed, especially in the United States, and which was also present in Marx’s works: the increasing role of the ‘executives’ of the largest stock companies (or corporations). Their role was increasingly independent from the control of stockholders, the vast majority of which were not involved in company decisions because they were too many and very fragmented. We have seen above Marx’s division of gross profit into interest on loaned capital and ‘profit of enterprise': “One appears exclusively as the fruit of owning the capital, the other as the fruit of operating with the capital, the fruit of performing capital, or of the functions performed by the active capitalist” (B3, 372). And the latter part could be received by stockholders (“Even if the dividends which they receive include the interest and the profit of enterprise, i. e., the total profit...” (B3, 434)) but could also be partially appropriated by those who actually controlled the firm (“... a new swindle develops in stock enterprises with respect to wages of management, in
that boards of numerous managers or directors are placed next and above the actual director, for whom supervision and management serve only as a pretext to plunder the stockholders and amass wealth” (B3, 388)). This last variant was directly related to the ‘corporate revolution’ that Berle and Means (1933) detected in the evolution of American Capitalism half a century after Marx’s death and was based on the “divorce of ownership from control.”

Marx tells us that “Stock companies in general—developed with the credit system—have an increasing tendency to separate this work of management as a function from the ownership of capital, be it self-owned or borrowed” (B3, 386). On the one hand, “the mere owner of capital, the money capitalist, has to face the functioning capitalist.” On the other, “money capital itself assumes a social character with the advance of credit, being concentrated in banks and loaned out by them instead of by its direct owners” (who in this case are the bank depositors). Finally, “the mere manager who has no title whatever to the capital, whether through borrowing it or otherwise, performs all the real functions pertaining to the functioning capitalist as such” (Ibid.). Hence, for Marx in the most advanced form of the Capitalism of his time, “only the functionary remains and the capitalist disappears as superfluous from the production process” (Ibid.), since the capital that finances these entrepreneurial functionaries is funded either by stockholders that merely receive dividends or by loans from banks that receive interest and also pay dividends to their own stockholders. Marx thus roughly anticipated characteristics that would be of increasing importance in the advanced Capitalism of the following 150 years, a process in which the high corporate bureaucracy (especially in the U.S.A. and in lesser degree in Europe) obtains control over the operations of large industrial organizations and the immense majority of stockholders are passive recipients of dividends that have often been previously skimmed of sometimes enormous, arbitrary, self-allocated portions of the real profits by those who exert power and control.

Appendix to Chapter 18
Bibliographical Notes

Cassel’s critique of Marx The Swedish economist Gustav Cassel (1866-1945) was a great disseminator of Neoclassical theory, especially that of León Walras. It is thus surprising that in his most important book, The Theory of Social Economy, published in German in 1919 (and the fifth edition in 1932), he does not even mention Walras. Since he studied mathematics before economics, the reason cannot be found in Walras’ use of mathematics. In the present context the greatest interest in Cassel’s book is that 1) it exemplifies the most typical lack of understanding of Marx’s theory of value, and 2) he expressed with some clarity why the ‘socialist’ doctrine on “the worker’s right to the whole product of his labor” could not be accepted.

1) is evidenced when he writes: “The thesis of the socialist theory of value is that the value of a commodity is equal to the amount of labour which is necessary under normal conditions to produce it. This assumption is quite arbitrary, and in contradiction to the actual facts... A science which on this point makes concessions to Marxist scholasticism does not know what it owes itself” (Cassel 1967 [1932], 189). The statement simply shows that Cassel ignored the distinction between Marx’s theory of value and his ‘theory of value’ in the sense we now usually give to the theory of prices, or of equilibrium prices. He confuses Marx’s values with his ‘regulating prices’ in CCP, which necessarily had to be different from the former (though in Book I he eliminates the distinction in order to simplify). And 2) is reflected in the section on
“The Problem of Imputation and the Social Problem of Distribution.” An alternative name he gives to the ‘problem of imputation’ is the ‘problem of attribution’. He argues that when two people collaborate in the production of a product the question arises “as to how much each of them has contributed” (Ibid., 177). He states that it is necessary to look at “the more complex one of the general distribution of social production”, which is fundamental “in an age of widespread division of labour and sharp stratification of classes in the community.” And “if the activities required to make the product are of very different kinds, it is impossible to reduce them to a common measure, and there can be no ‘correct’ distribution in the objective sense.” He first states that there can be no ‘common measure’ for “the work of the thinker, the artist, the manager of a business, and the manual worker. Their common product can never be shared according to the work done by each.” We have seen in our extension of Marx’s theory to the heterogeneity of labor power that we could circumvent this question by inverting the terms and using the exogenous consumption baskets (given by customs, etc.) to obtain the relative values (and values) of the different kinds of labor. But if, as in neoclassical theory, consumption baskets are endogenous, then it is the general equilibrium in all markets (including those for labor services and natural resources) that must solve this problem, as we will see in Chapter 20. Cassel continues as follows:

The impossibility of this kind of imputation is made still clearer when other factors of production of various kinds are added to ‘labor’—capital, natural materials, or land. It is, however, precisely in this case that... the most bitter controversy as to who is to have the proceeds of production arises. Each party is naturally inclined to emphasise the importance of its own share of the work, to claim as large a portion as possible of the returns, and, consequently, to denounce the actual apportionment as unfair. As a rule, the so called proof of this is to imagine one’s own share of the work withdrawn and then ask what the other factors of production would do without it. Unfortunately, this argument suffers from the weakness that it can be used with the same striking effect with regard to each factor of production that is indispensable. Taking away that particular factor of production always reduces the result of the activity of the others to zero (Cassel 1967, 178-9).

This is precisely the argument we have given in this chapter with respect to entrepreneurial (or POCC) labor: its elimination reduces output to zero. Cassel also links the argument of the necessity of labor to the “fundamental tenet of Socialism” of the worker’s right to the “full product of his labour”:

With the support of this argument, it has actually been claimed that the whole proceeds of the productive process belongs to labour; that is, “the right of the worker to the full product of his labour.” We need not linger over the advocacy of this programme. The programme itself is interesting as a fundamental tenet of Socialism, representing the positive expression of the denial in principle of the justice of an income based upon private property. Theoretical economics is obliged to make it perfectly clear that, and why, an imputation along the lines of this programme is economically impossible (Cassel 1967, 179).

Marx implicitly negates that entrepreneurial labor is an “indispensable” “factor of production” when he omits its formal representation. In his textual analyses he not
only recognizes this type of labor but even considers it important. And Marx’s lack of consistency between his texts and his analytical formulation is at the core of the fallacy of his theory of surplus value. Cassel expressed this with some clarity. What is paradoxical is that mainstream economics has also tended to omit entrepreneurial labor from production functions and costs as well as the retribution of the entrepreneur (or executive) in its general equilibrium theory. We will see more on this in the next chapter.
Chapter 19 THE ENTREPRENEUR AND PROFIT IN ECONOMIC THEORY

It is noteworthy that, as was the case of Marx, almost all economic theory after Walras has left out the entrepreneur’s work when analytically representing the functioning of an economy. In this chapter we address a significant sample of the diversity of ideas that economists have had on the nature of the entrepreneur. We begin with those of three important economists (Cantillon, Turgot, and Ramsay) that were well known to Marx and whose main ideas on this topic he either ignored or to some extent misinterpreted, probably because they strongly contradicted his theory of surplus value. Then we address the ideas expressed by the two most famous representatives of Classical economics, Smith and Ricardo, on the entrepreneur. In the next stage we approach Walras’ conception of the entrepreneur. Walras’ theory decisively influenced mainstream economic theory, and we will focus on it Chapter 20. Finally, in the last section we take a look at a sample of the multiple ideas that economists have had on the entrepreneur after Walras.

The entrepreneur’s work before Marx

One of the characteristics of the entrepreneur whose work involves Planning, Organizing, Commanding, and Controlling (POCC) is that it is subject to uncertainty on the retribution he will get for his activity, i.e., his profit, and that he faces the possibility of losing not only his expected profit but his wealth. Such uncertainties make profit a very different form of income from those received by the suppliers of the services the entrepreneur purchases at (usually certain) market prices. This way of visualizing the work of the entrepreneur and his income has a long history that gradually became blurred as economic theory came to be more mathematical in its language and substance under the great influence of Walras. It was very present in the works of several economists that preceded Marx and whose works he cited abundantly in his Theories: Cantillon, Turgot, and Ramsay. However, Marx tended to avoid these economists’ ideas on the entrepreneur and his profits. The reason for this is that the purpose of Theories was not to write a critical history of political economy but to seek in the works of economists elements related to the perspective that Marx was in the process of elaborating: the theory of surplus value as (‘esoteric’) foundation of (the ‘exoteric’ part of) his theory of Capitalism. This explains why he did not stop to analyze the views these economists had on the role of the entrepreneur, views that anticipated that of Frank Knight (1964 [1921]) many decades later.

Richard Cantillon

Apparently the first to define the work of the entrepreneur was the notable Irish economist Richard Cantillon (circa 1680-1734), who was also a banker and financier who got rich by speculating during the famous Mississippi Company stock price bubble. The Company monopolized the French lands in North America and was managed on behalf of the French crown by the Scottish economist John Law. And the experience led to the great financial crisis of 1720. Cantillon’s Essay on the Nature of Commerce in General, written around 1730, was one of the first general treaties of economics. He
died in London (presumably murdered by instigation of his debtors) in 1734 and his *Essay* was only published in 1755 in France with the proviso ‘Translated from the English language’. There was much confusion over this book since there was also a quite adulterated version published in English in 1759. Marx had read both in the British Museum and quoted from them in Book I when addressing ‘Piece Wages’ (in Chapter 21). In a footnote he wrote:

Cantillon, from whom Quesnay, Sir James Steuart & A. Smith have largely drawn, already here represents piece wage as simply a modified form of time wage. The French edition of Cantillon professes in its title to be a translation from the English, but the English edition, *The Analysis of Trade, Commerce, etc. by Philip Cantillon, late of the city of London, Merchant*, is not only of later date (1759), but proves by its contents that it is a later and revised edition; e.g., in the French edition, Hume is not yet mentioned, whilst in the English, on the other hand, Petty hardly figures any longer (B1, 555).

Marx also mentions Cantillon along with Petty when in Book III he studies the “Genesis of capitalist ground rent.” But he is far from adequately characterizing Cantillon insofar as his view of entrepreneurial profits is concerned when he writes that “Petty, Cantillon, and in general those writers who are closer to feudal times, assume ground rent to be the normal form of surplus value in general, whereas profit to them is still amorphously combined with wages, or at best appears to be a portion of surplus value extorted by the capitalist from the landlord” (B3, 770). Cantillon clearly separated profit from wage. He even repeatedly invoked the contrast between the certainty of the wage that is contracted and the uncertainty of the entrepreneur’s profit and the possibility of failure. Marx was especially interested in tracking aspects of economists’ theories that he could relate to his own theory of surplus value, and clearly avoided Cantillon’s theory of the profit of enterprise as expressed in Chapter 13 of his *Essay*.1 Cantillon there explains that “The farmer is an entrepreneur who promises to pay the property owner, for his farm or land, a fixed sum of money (generally assumed to be equal in value to a third of the production) without assurance of the profit he will derive from this enterprise” (Cantillon 2010, 73). The whole chapter emphasizes the idea that the entrepreneur “conducts the enterprise of his farm with uncertainty” (Ibid., 74), either because he buys inputs at a certain price and has no certainty on the price at which he will later sell his output, or because of the uncertainty related to the weather, or to many other factors. And not only did he consider tenant farmers to be entrepreneurs. So were the merchants, and the manufacturers, who equally operated under uncertainty: “These entrepreneurs never know how great the demand will be in their city, nor how long their customers will buy from them since their rivals will try, by all sorts of means, to attract their customers. All this causes so much uncertainty among these entrepreneurs that every day one sees some of them go bankrupt” (Ibid.). Moreover, “the entrepreneurs of their own labor who need no capital to establish themselves, like journeymen artisans, coppersmiths, seamstresses, chimney sweeps, water transporters, live with uncertainty.” Hence, Marx’s simple commodity producers would also be subject to uncertain incomes and for Cantillon would qualify as entrepreneurs, even though they did not employ wage laborers.

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1It is noteworthy that in *Theories* Marx also mentions Adam Smith’s only quote from Cantillon, which refers to his estimation that a common worker must earn a wage at least twice the cost of his consumption basket in order to raise two children. The object of Marx’s quote was plainly to show that Smith was conscious that a worker had to work during more hours than was necessary for his and his family’s maintenance, i.e., the idea underlying his theory of surplus value.
Cantillon divides society into three big social classes: landowners (headed by ‘the prince’), entrepreneurs, and wage workers, where the distinction between the last two classes is based upon the worker’s certain wage and the entrepreneur’s uncertain income (or profit) whether or not he owns capital. He writes:

it may be established that, except for the prince and the property owners, all the inhabitants of a state are dependent. They can be divided into two classes, entrepreneurs and hired workers. *The entrepreneurs are on unfixed wages while the others are on fixed wages* as long as there is work, although their functions and ranks may be very unequal. The general who has his pay, the courtier his pension and the domestic servant who has wages, all fall into this last class. *All the others are entrepreneurs, whether they are set up with capital to conduct their enterprise, or are entrepreneurs of their own labor without capital, and they may be regarded as living under uncertainty; even the beggars and the robbers are entrepreneurs of this class* (Cantillon 2010, 76; italics added).

What Cantillon had in common with “those writers who are closer to feudal times” was the predominance in his theoretical framework of the landowning class: “There are none but the prince and the property owners who live independent” and “all the inhabitants of a state derive their living and their advantages from the property of the landowners and are dependent” (Ibid.). And Cantillon’s argument was based on the power of the “prince and property owners,” for if they “close their estates and will not allow them to be cultivated, it is clear that there would neither be food nor clothing for any of the habitants of the state.” Because “it is the property owners who have the control and direction of the landed capital, to give the most advantageous turn and movement to the whole” “everything in a state depends mainly on the moods, modes and ways of life of the property owners” (Ibid., 71).

Cantillon refers to the ‘monopoly’ of land that for Marx explained the existence of absolute rent. In Cantillon’s time, and especially in continental Europe, where he traveled extensively during the 1720s, the interest of landowners was still predominant in the actions of the State, which is why Cantillon places the ‘prince’ as the head of the ‘property owners’. However, in the times of Classical economics (which is posterior to those of the Mercantilists and the Physiocrats) the power relations had shifted and the interests of capitalist entrepreneurs tended to predominate in the political arena.

**Anne Robert Jacques Turgot**

Less than forty years after Cantillon wrote his *Essay*, the French thinker and public functionary Anne Robert Jacques Turgot (1727-1781) also wrote on the labor of capitalist entrepreneurs who directed the operations of their enterprises, made capital disbursements, and received “the subsistence and the profits... which are the rewards for his labour.” Marx’s commentary on Turgot in *Theories* has a common feature with that on Cantillon. Since he was mainly interested in the relations between previous economist’s theories with the one he was in the process of perfecting, he tended to interpret their thought in terms of his own. And some topics we can consider central were largely ignored, as is the case of the entrepreneur’s labor.

Ever since he was young Turgot held positions in the public sector, first as councilor to the Parliament of Paris, then as mayor of Limoges, and finally as minister, first of the navy, and later as General Comptroller (a position similar to that of a Minister of
Finance). In his last positions he opposed France’s participation in the War of Independence of the British colonies in North America for fiscal reasons. But he was defeated in this, and the effect this participation had on the economic and financial stability of the monarchy contributed to the extreme social conditions that gestated the French Revolution. Turgot became initiated in the economic discipline in the school of the Physiocrats, and he was acquainted with several of their most important representatives, such as Quesnay, Dupont de Nemours, and Mirabeau. He wrote in 1766 (and published in 1769-70) his formidable *Reflections on the Formation and Distribution of Wealth*.

Turgot had a highly stylized historical vision of the evolution of social institutions and social classes. Like the Physiocrats that preceded him, he gave great importance to rural production. He tentatively begins with the assumption that land was equally possessed by individuals and each had enough for his own consumption, but immediately clarifies that such a state of affairs never existed, for “the earth has been cultivated before it has been divided” (Turgot 2011, 5). It was only after the communal land was divided and many subsequently lost their original lands that society formulated “that law which secures to every one his property.” Producing agricultural goods required a great exertion of labor. And so did the production of goods that used them as inputs: “The crops which the earth yield to satisfy the different wants of man... must undergo various changes and be prepared by art. Wheat must be converted into flour, then into bread; hides must be tanned or dressed; wool and cotton must be spun” (Ibid., 6). Furthermore, since different soils were more adequate for different uses, farmers would tend to specialize in a single output, hence emerging the division of labor and the exchange of goods.

So far, Turgot essentially describes in stylized form the transition from a society of communal producers with very low productivity to one in which the division of labor has increased productivity and generated exchanges. What emerges is a society very similar to Marx’s Simple Commodity Production (SCP). The producers were all ‘laboureurs’, ‘cultivateurs’, and ‘ouvriers’2. But eventually the farmer found that he could produce more by also exchanging a part of the output for labor: “The same motive which has established the exchange of various crops between the cultivators of different types of soil, must also have brought about the exchange of crops for labor, between the cultivators and another part of society which came to prefer the occupation of preparing and working up the produce of the land to that of growing it” (Ibid., 7). In Turgot’s jargon, farmers (or ‘cultivators’) ‘produce’ and artisans ‘prepare’. At this point, Turgot seems to present the genesis of the wage worker as the result of a decision based on preferences, saying nothing about the conditions under which such decisions were made: the lack of alternative ways of making a living which Marx (rightly) so emphasized. However, a few pages down he explains how many independent farmers lost their lands, putting them in that dire situation.

For Turgot the *labor of the farmer*, which “causes the land to produce beyond his personal wants”, generates “the sole fund for the payment of wages which the other members of society receive in exchange for their labor” (Ibid., 8). Wage workers “only return to him precisely what they have received” by the advance of their wages. And Turgot emphasizes the importance of this “very basic difference between these two kinds of labor”, a difference that has “innumerable consequences which flow from it” (Ibid.). Turgot’s *laboureur* has become a *capitalist* farmer, whose work is complemented by that of wage workers, for “he can, with the surplus which nature accords him

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as a pure gift above the wages of his toil, purchase the labor of other members of society.” These two classes are “both industrious.” The class of wage workers “sells its labor to the first class, and receives its subsistence in exchange.” And the class of capitalist farmers “by its labor, produces or rather, draws from the soil, wealth which is continually regenerated, and which supplies the whole society with subsistence and with materials for all its needs.” The farmer is “the unique source of the wealth which, by its circulation, animates all the industry of society, because he is the only one whose labor produces more than the wages of his labor.” It should be clear that here by ‘wages of his labor’ Turgot refers to a remuneration the farmer can set aside for his own consumption and savings and not to a wage that is the market price of an entrepreneurial form of labor.

So far in Turgot’s exposition the land belongs to the farmers, who at first had only used their own labor and later also the labor of wage workers. Only then does he explain how things got to that point, how wage labor emerged. Initially all land was held in common, but at some point those who cultivated the fields “enclosed them to secure their harvest... against all invasion from without.” In the early times, land was abundant and, “as every industrious man would find as much land as he wished, he could not be tempted to till the soil for another.” Hence there were no wage workers. But along with the occupation and cultivation of all the land, there was also a process through which many people lost their lands. Turgot explains that various sources of inequality among the cultivators led to the loss of the lands of those who for various reasons had become indebted and were unable to repay. Among such sources of inequality were the differences in ‘strength’ (which enabled the strongest to occupy more lands), in ‘industriousness’, in the number of children and hence family working hands, in the fertility of their soil, and in thriftiness.

It was only after “societies were consolidated” and “public power, or the law, becoming predominant over individual power was able to guarantee to each man the tranquil possession of his property against all invasion from without”, that wage work came to being. Before this it would not have been safe to employ wage labor since the latter, “having taken all the trouble would not easily have understood that the whole harvest did not belong to him” (Ibid., 10). Interestingly, for Turgot the importance of the power and laws of the State was that it avoided what so many Socialists (including pre-Marxians, Marx, and post-Marxians) believed was fair: to claim for the workers employed by the capitalists the whole of the net output (in this case the harvest), implicitly denying the legitimacy of the profits attributable to the special labor of the entrepreneurs and (active) capitalists.

Turgot then introduces another institutional development. The landowning farmers could if they pleased lease the land so that other farmers could do the entrepreneurial work (along with that of the wage workers they might employ). “We now behold society divided into three classes: the class of Husbandmen, for whom the name of productive class may be reserved; the class of artisans and others receiving stipends⁴ from the produce of the land; and the class of proprietors” (Ibid., 12). Since the latter were not “confined by the need of subsistence to a particular species of labor,” they could be “employed for the general needs of the society, such as war, and the administration of justice.” The landowner’s right to his land was protected by laws, but these “can secure the man who does not work only that part of the produce which the land gives over and above the return due to the cultivators” (Ibid., 13).

The agricultural entrepreneur is thus “the prime mover in the whole machinery of

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⁴Stipends is a word Turgot often uses instead of (and with the same meaning as) ‘wage’ (salaire in French).
society.” His own subsistence “as well as the wealth of the proprietor and the wages of all the other labors” depend on his labor (Ibid., 14). The landowner could thus enjoy a certain and even rent over many years, while it was the capitalist farmer who risked losing his ‘advances’, since his livestock and other implements were used as guarantee for his rent payments: “The proprietor, on the other hand, gained thereby a more tranquil enjoyment of his revenue, being freed from the care of making the advances and keeping an account of the product; a more steady enjoyment, since he received every year the same price of his lease; and a more certain enjoyment, because he never ran the risk of losing his advances, and the cattle and other effects with which the tenants had stocked his farm [and] became a security for his payment” (Ibid., 18; italics and text within brackets added). This eminently capitalist organization of production was, according to Turgot, “established everywhere where there are wealthy cultivators, capable of making the advances of cultivation.” And, “as wealthy cultivators are able to bestow much more labor and manure upon the land, there results from it a prodigious increase in the productions and the revenue from estates” (Ibid.). Hence, Turgot introduced, as Cantillon, the uncertainty factor; the risks run by the entrepreneur, albeit in a less systematic way.

After thus treating agrarian production, Turgot expands to other sectors of the economy. All enterprises, including manufactures, required disbursements (avances des entreprises de fabrication et d’industrie). As in the farming activity, there were also wealthy manufacturing or trading entrepreneurs “all owners of large capitals, which they invest profitably as advances for setting men at work” (Ibid., 37). The latter are “simple artisans, who have no other property than their arms, who advance only their daily labor, and receive no profit but their wages” (Ibid., 37). In all these enterprises, the entrepreneurs who are owners of large capitals must receive

apart from the return of their capital, i.e., their original and annual advances, firstly, a profit equal to the revenue they would be able to acquire with their capital without any labor; secondly, the wages and the price of their labor, of their risk and their industry; thirdly, the wherewithal to replace annually the wear and tear of their property, the cattle that die and the tools that wear out, etc. (Ibid., 38)

In the case of agricultural enterprises, after these items have been deducted from the price of the good “the surplus serves the cultivator to pay the proprietor for the permission he has given to use his field for establishing his enterprise”, i.e., the rent. For Turgot “another way of being rich, without laboring and without possessing lands” was “living by what is called the revenue of one’s money, or on the interest drawn from money placed on loan”, which is why this was one of portions of the entrepreneur’s profits if he was in fact the owner of the capital he advanced.

Marx held that the Physiocrats “present the landowner as the true capitalist, that is, the appropriator of surplus labour,” and that they treat agriculture “as the branch of production in which capitalist production —...— exclusively appears” (B4.30, 358); and that this confuses Feudalism with Capitalism. “While feudalism is thus made bourgeois, bourgeois society is given a feudal semblance.” However, he takes exception with Turgot, by whom “the Physiocratic system is presented as the new capitalist society... in the epoch when the latter breaks its way out of the feudal order” (B4.30, 359). But Marx can’t help but interpret Turgot’s theory in terms of his own. For example, when Turgot explains the fundamental difference between the labor of the farmer and that of the wage worker, Marx sees there his surplus value:
How then does surplus value arise? It does not arise from circulation, but it is realised in circulation. The product is sold at its value, not above its value. There is no excess of price over value. But because it is sold at its value, the seller realises a surplus value. This is only possible because he has not himself paid in full for the value which he sells, that is, because the product contains a portion of value which has not been paid for by the seller, which he has not offset by an equivalent” (B4.30, 363).

Marx is clearly not transcribing Turgot’s theory, in which the combination of Nature, the ‘laboriousness’ of the entrepreneur, and his ‘wisdom’ in the use of adequate methods, generates a value that exceeds both his needs and those of wage workers he employs. For Turgot, what Nature grants “is the physical result of the fertility of the soil, and of the wisdom, far more than the laboriousness, of the means which he [the farmer] has employed to render it fruitful” (Turgot 2011, 9; text in brackets and italics added). Marx was reading his own theory into Turgot’s when he wrote that “the seller realises a surplus value. This is only possible because he has not himself paid in full for the value which he sells.” In contrast, Turgot stated that “The artisan, on the contrary, receives his wages either from the proprietor or from the cultivator, and gives them by the exchange of his labor, only the equivalent of these wages, and nothing more” (Turgot 2011, 14). Also, when explaining Turgot’s introduction of the separation between the farmer and the landowner, Marx interpreted that “surplus value is explicitly stated to be the part of the cultivator’s labour which the proprietor appropriates to himself without giving any equivalent” whereas for Turgot the only ‘surplus value’ is the landowner’s rent, which he receives in exchange for an equivalent: the use of land that the landowner yields to the farmer. Furthermore, when Marx reads in Turgot’s text that the “proprietor has nothing but by the labor of the cultivator. He receives from him his subsistence, and the wherewithal to pay for the labor of the other stipendiaries”, he again misinterprets him. He writes: “We see thus how, within the limits of agricultural labour, the Physiocrats have a correct grasp of surplus value; they see it as a product of the wage labourer’s labour, although they in turn conceive this labour in the concrete form in which it appears in use values” (B4.30, 365-6; italics added). But Turgot was referring to the cultivator as a capitalist farmer who also works in a very special way and not as a wage laborer! For him the capitalist farmer was “the unique source of the wealth which, by its circulation, animates all the industry of society, because he is the only one whose labor produces more than the wages of his labor”, while the wage worker received the equivalent of the value of his work (his wage). And that is why he called the former the “productive class” and the latter the “sterile class.” When the farmer did not own the land, the landowner received the equivalent of the price for the use of his land (rent). And when he did not own the capital he advanced, the money capitalist received the equivalent of the price for the use of his money (interest).

Finally, when Marx comments on Turgot’s definition of ‘capital’ as an accumulation of ‘moveable values’, he writes: “As industry develops, larger advances and continuity of the process of production are necessary. This is then undertaken by the possessor of capital. In the price of his products he must recover all his advances and a profit equal to ‘what his money would have been worth to him if he had employed it in the purchase of an estate’, besides his wages” (B4.30, 367). But here Marx is simply putting ‘wages’ in place of what for Turgot are the entrepreneurs’ incomes: “the wages and the price of their labor, of their risk and their industry” (Turgot 2011, 38).
Adam Smith and David Ricardo

As the English economist Ramsay clearly stated in 1836, even in his time British economists did not yet use the French word entrepreneur. Adam Smith (1723-1790), who wrote half a century before Ramsay, used the word ‘employer’ or ‘undertaker’.\footnote{With the evolution of the English language the word ‘undertaker’ became more limited in its scope, and one of its meanings is that of an entrepreneur in the business of funeral services.} It was only much later that the French entrepreneur used by Turgot was adopted.\footnote{Although some English versions of the Cantillon’s book use the word entrepreneur, one must bear in mind that the original version (which was probably in English) was lost and that the several later published versions in English are based on translations from the French. Since by then the word entrepreneur had been widely adopted this explains its use there.} Adam Smith wrote: “As soon as stock has accumulated in the hands of particular persons, some of them will naturally employ it in setting to work industrious people, whom they will supply with materials and subsistence, in order to make a profit by the sale of their work, or by what their labour adds to the value of the materials. In exchanging the complete manufacture either for money, for labour, or for other goods, and above what may be sufficient to pay the price of the materials, and the wages of the workmen, something must be given for the profits of the undertaker of the work, who hazards his stock in this adventure” (Smith 2005, 45; italics added). Smith refers to the ‘undertaker of the work’ without explicitly saying whose work it is. It is evident that it includes the work of the ‘workmen’. But the profits of the undertaker included both the interest on the capital (whether loaned or owned) and a compensation for his ‘troubles’ and for putting the capital at risk, which is quite close to the spirit of Cantillon:\footnote{Cantillon is one of the few economists that Smith mentions, and only on one occasion, as explained above.}

The revenue derived from labour is called wages; that derived from stock, by the person who manages or employs it, is called profit; that derived from it by the person who does not employ it himself, but lends it to another, is called the interest or the use of money... Part of that profit naturally belongs to the borrower, who runs the risk and takes the trouble of employing it, and part to the lender, who affords him the opportunity of making this profit (Ibid., 49; italics added).

Smith observes that because usually at least part of the capital was loaned, the ‘ordinary rate of clear profit’ in Great Britain was approximately twice the usual market interest rate. For “In a country where the ordinary rate of clear profit is eight or ten per cent, it may be reasonable that one half of it should go to interest, wherever business is carried on with borrowed money” (Ibid., 85). It was also normal for the entrepreneur to assume the repayment risk, since the material elements of his business would serve as guarantee for the loan. Hence, “four or five per cent may, in the greater part of trades, be both a sufficient profit upon the risk of this insurance, and a sufficient recompence for the trouble of employing the stock” (Ibid.). This ‘trouble’ obviously referred to the entrepreneur’s various exertions in the enterprise. Smith also holds that “The profits of stock, it may perhaps be thought, are only a different name for the wages of a particular sort of labour, the labour of inspection and direction” (Ibid., 46), but they are regulated by different principles. Profits must keep a certain proportionality with the capital invested. In contrast, the labor of inspection and direction can be more or less the same for enterprises that require capitals of very different sizes, and hence can be “committed to some principal clerk” whose wage is related “not only to his
labour and skill, but to the trust which is reposed in him” (Ibid.). Hence, according to Smith the price of commodities includes both the labor cost of inspection (which may actually be wages or imputed as such) and a profit that is proportional to the capital disbursed and includes the interest on the capital that is used.

For Smith there were two reasons that could make the rate of profit systematically lower or higher in different branches of economic activity: “the agreeableness or disagreeableness of the business, and the risk or security with which it is attended” (Ibid., 96). As to the latter, he held that “the ordinary rate of profit varies more or less with the certainty or uncertainty of the returns... The ordinary rate of profit always rises more or less with the risk. It does not, however, seem to rise in proportion to it, or so as to compensate it completely. Bankruptcies are most frequent in the most hazardous trades (Ibid.; italics added).

Ricardo used the same two words as Smith for the entrepreneur, though by far the one he used most often was ‘employer’, rarely using ‘undertaker’. His entrepreneur employed both workers and capital, of which he was usually the owner, though he could also get it through interest paying loans. Ricardo did not elaborate much on the work of the entrepreneur. He writes: “A capitalist, in seeking profitable employment for his funds, will naturally take into consideration all the advantages which one occupation possesses over another. He may therefore be willing to forego a part of his money profit, in consideration of the security, cleanliness, ease, or any other real or fancied advantage which one employment may possess over another.” And he writes that if the profit rate in three alternative employments were 20%, 25%, and 30%, respectively, “they would probably continue permanently with that relative difference, and with that difference only; for if any cause should elevate the profits of one of these trades 10 per cent, either these profits would be temporary, and would soon again fall back to their usual station, or the profits of the others would be elevated in the same proportion” (Ricardo 2004, Vol. 1, 90).

George Ramsay

Around seventy years after Turgot’s Reflections and almost 20 after the first edition of Ricardo’s Principles, the Scottish economist George Ramsay (1800-1871) wrote and published (in 1836, when he was 30 years old) his first book: An Essay on the Distribution of Wealth, often quoted by Marx in Capital, and extensively analyzed in Theories. For Ramsay Political Economy is the science “which treats of the Production, Distribution, Exchange, and Consumption of wealth” (Ramsay 1836, 15), where wealth is defined as “Those material objects necessary, useful, and agreeable to man, which are not provided spontaneously by Nature in unlimited abundance” (Ibid., 12-3).

For Ramsay the sources of wealth were either original or derived. The original sources were land and waters, from where agricultural products, minerals and fish were extracted, and other agents that generate motion, such as wind, running water, steam, and also “Man, whose labour is indispensable to production” (Ibid., 17). The original sources generated wealth, which can either be directly consumed or used as a derived source of wealth in order to facilitate production. This derived source of wealth he calls ‘capital’, of which there are two types: fixed capital and circulating capital. From the way Ramsay defines these concepts it is seen that they basically correspond to Marx’s concepts of constant and variable capital. In Ramsay’s theory, in “the earliest stage of society” people would make their living only hunting and fishing, and “there being but one class, the whole produce must belong to it” so that there was no distribution among classes. But as soon as capital began to be
generated and its owners conformed a class, “society comes to consist of Capitalists and Labourers.” And if these classes cooperate in any activity, the output must be distributed between them. In a third stage, there emerged a third class, formed by the “heads of establishments, agricultural, manufacturing or commercial” (Ibid., 78), who were not necessarily owners of the capital they used, although they could be. Ramsay states that the English language does not have a specific word for this class of men, which the French call entrepreneurs and he calls masters. We, however, will continue using the word ‘entrepreneurs’ that the English language later adopted from the French. For Ramsay, when a country has little population and abundant lands that have not been appropriated, nothing will be paid for its use. But when all the land, at least the most fertile and best positioned, has been made private property, those owners that do not want to use their land themselves in production can lease them to others, as long as they receive a rent.

Hence, for Ramsay there were four fundamental classes in his times: “We have Labouurers, Masters, Capitalists, and Landlords, possessed respectively of the three sources of wealth, Labour, Capital, and Land, the first of these being common to the two former sets of people” (Ibid., 79-80). This last assertion, that Labor is a source of wealth that is common to Laborers and Entrepreneurs (Masters), is a fundamental statement of Ramsay that Marx systematically ignores. The total product is thus distributed among these four classes. And Ramsay explains this distribution following a certain sequence. First he leaves out the landowners, as well as the distinction between Entrepreneurs (Masters) and Capitalists, which he fuses into an Entrepreneur-capitalist (‘Master-capitalist’). Hence, he begins by explaining the distribution between Workers and Entrepreneur-capitalists, where the real wages (or remuneration) of a worker “consist in the amount of necessaries, of comforts, and luxuries, which his exertions enable him to command” (Ibid., 84). Although before any capital had accumulated the whole output of industry belonged to the class of workers, with time there emerged a class of capitalists that owned sufficient fixed capital so as to employ workers who, aside from working for themselves, worked for an owner of capital, distributing the output between them “according to some previous arrangement.” It is clear that such workers were not wage workers but sharecroppers who, for example, get half of the net output.

Then Ramsay introduces a later phase of development in which wage payments arise: if the workers do not have the means of subsistence to survive during the working period, the capitalists can “supply them with food and other necessaries on condition of their abandoning all claim to a share in the finished commodity” (Ibid., 85). In this phase then, the capitalist owns both fixed and circulating capital. The former consists of means of production and the latter of the means of subsistence they ‘advance’ as the (real) wages of workers. Ramsay makes an interesting analysis of what determines the real wage. The “immediate cause” in the determination of the wage rate is the “proportion existing between the supply of labour and the demand.” But that ‘proportion’ itself depends on “ultimate causes”: the productivity in those branches of industry that produce subsistence goods and “the style of living rendered necessary by the nature of the climate, or considered by opinion as necessary to the existence of the labouring population” (Ibid., 86).

The main differences between the conception of Ramsay and that of Marx are to found in the following statements by Ramsay, where we have added italics in order to highlight the most important aspects of these differences:

In commencing the inquiry into the subject of Profits, I mentioned that I took this term, as is usual with English writers, to signify the entire surplus which remains to the master-capitalist after replacing all the capital, fixed
and circulating, expended in production. At the same time, I observed, that this whole surplus was not always the property of one individual, for that one person might be the owner of the capital, another might take upon himself the trouble and risk of employing it; and thus that *profits really comprehended two very different species of income, one being a compensation for the use of capital, the other for the trouble and risk incurred, and the skill exerted in the business of direction and superintendence. Gross-profits then may be properly divided into two parts, the *net-profits of Capital, and the profits of Enterprise*⁷ (Ramsay 2011, 193).

Whatever remains to the master of any establishment, agricultural, manufacturing, or commercial who puts in motion borrowed capital, after paying these profits or interest to the owners of the stock, constitutes the *profits of enterprise, the remuneration of the skill exerted and the trouble and risk which necessarily accompany all productive occupations* (Ibid., 197).

Now this *profit* consists, in the case of a nation, of many commodities, in that of an individual, of perhaps only one, and is *an effect of the beneficence of nature, aided and directed by art*. To understand the theory of profits, it is above all things necessary to be thoroughly convinced of the truth of this fundamental principle (Ibid., 205).

... the French economist Say, or the Russian, Storch. By them the *Profits de Entrepreneur* are carefully distinguished from the *Profits des Capitaurs*. And in truth, such a separation is not a matter of philosophical accuracy merely, but really enters into the view of all persons at the head of productive establishments. They consider nothing as their gain or profit, but what they make over and above the current interest of money, and very justly; for supposing them possessed of capital, they have no occasion to put themselves to any trouble, to give up their leisure, to exert their intellectual faculties, and to incur the risk of failure, in order to enjoy an income equal to this interest... It is therefore quite essential to consider this surplus by itself. And here, in commencing, I must remark the compound nature of this kind of revenue. *It is not, like the income of the labourer, derived from labour alone* (Ibid., 208-9).

What essentially constitutes the character of master, is the possession of all the qualifications necessary for carrying on any business, in conjunction with a power of commanding capital, whether his own or not. The peculiar advantages attending this state are, that whatever profits he may make by his industry, over and above interest, all belong to himself; the disadvantages, that he is constantly liable to lose, not only his income but the capital also, for more or less of uncertainty attends all productive occupations. It is the risk and trouble to be incurred, the variety of talents and knowledge required for carrying on any business, which, together with the necessity for presenting good security, always limit the number of persons who create an effective demand for capital to be employed productively (Ibid., 215-6).

... though the master does not labour with his hands, yet his head must be constantly at work, his time and pains must be given principally to the management of the concern, which, without his superintendence, would

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⁷In a footnote, Ramsay observes that “Net-profits of capital, when estimated and paid in money, are called Interest”. 

soon go to ruin; and he must frequently be liable to mental anxiety (Ramsay 2011, 217).

We thus see that there was much in common in the theories of the economists we have considered on the function of the entrepreneur in capitalist society. And they were all incompatible with Marx’s theory of surplus value, based on the ‘unpaid labor’ of wage workers, and excluding the representation of the work of entrepreneurs in its analytical framework. For all these economists, entrepreneurs not only worked but their type of labor was fundamental in the organization of the capitalist economy. As in Marx’s theory, their exertions were motivated by the expectation of profit, and their role in the economic process required having authority within the firm and being able to hire and fire wage workers. But their profit was not simply based on not paying workers their due and thus appropriating ‘unpaid labor’. And Marx in his comments on his predecessors very clearly tended to avoid the paragraphs that most clearly addressed these fundamental issues and in some cases misinterpreted them.

The entrepreneur in Walras’ theory
Walras’ general theoretical framework and his conception of the entrepreneur

Léon Walras (1834-1910) expressed his fundamental works in a trilogy that included a treatise on ‘pure economics’, and two volumes that did not have the same organic character: one on ‘applied economics’ and another on ‘social economics’. This helps to place in context the restricted definition he gave of ‘pure economics’ in his magnum opus: Elements of Pure Economics or the theory of social wealth (the first edition of which appeared in 1874 and the fourth in 1900). In his view, “Pure economics is, in essence, the theory of the determination of prices under a hypothetical régime of perfectly free competition” (Walras 1954, 40). He viewed the economy “as a vast general market made up of diverse special markets where social wealth is bought and sold.” Walras calls social wealth all things, whether material or not, that are both useful and limited in quantity, a combination that defined them as ‘scarce’.8 He excludes from the analysis things so abundant that everyone can be satiated by its use (like air), the rest being scarce. And he set upon the task of discovering “the laws to which these purchases and sales tend to conform automatically. To this end, we shall suppose, to start with, that the market is perfectly competitive, just as in pure mechanics we suppose, to start with, that machines are perfectly frictionless” (Ibid., 84). In the case of production,

free competition consists in, on the one hand, in allowing entrepreneurs the freedom to expand output in the case of profit and to restrict output in the case of a loss; and, on the other hand, in allowing landowners, workers, and capitalists, as well as entrepreneurs, freedom to buy or sell services and products by bidding against one another (Walras 1954, 255).

If the entrepreneur has profit or loss, he can either change the quantity of output produced or switch to the production of another, more profitable, commodity (Ibid., 225). This was analogous to Marx’s assumptions on the process of profit rate equalization.

8He exemplifies the breadth of his concept of ‘usefulness’ when he writes “we need not concern ourselves with the morality or immorality of any desire which a useful thing answers or serves to satisfy. From other points of view the question of whether a drug is wanted by a doctor to cure a patient, or by a murderer to kill his family is a very serious matter, but from our point of view, it is totally irrelevant. So far as we are concerned, the drug is useful in both cases” (Walras 1954, 65).
But in the case of Walras, in the state of general equilibrium each firm necessarily has zero profit (neither profit nor loss).

Walras created a very consistent and general framework for expressing the equilibrium prices of produced goods and each of the productive services used by firms, given the existing distribution of resources between individuals. And although the questions of how the distribution of resources among a country’s individuals in a given period of time came about and how in could be modified looking forward through state intervention were matters of great importance to him, he understood that they were of no concern to ‘pure economics’. Hence, he left for ‘applied economics’ and ‘social economics’ the questions of how best to organize production and what state intervention could best influence the distribution of wealth (and hence incomes) to make it more equitable. Walras considered himself a ‘socialist’ (though not in Marx’s sense) and was in favor of nationalizing lands so that the State could finance its activities without having to collect taxes.

The contrast Walras made between ‘applied economics’ and ‘social economics’ was based on what he considered a ‘fundamental distinction’ in the realm of human affairs: in the field of Economics, he called ‘theory of industry’ (or ‘applied economics’) the study of the “relations between persons and things”, and he called ‘theory of institutions’ (or ‘social economics’) the study of the “relations between persons and persons.” These distinctions clearly show that philosophical matters were not among Walras’ strengths. If Marx had read his book he would surely have pointed out that the ‘relations between persons and things’ cannot be separated from the ‘relations between persons and persons’, and that the power relations in any given society have always determined how economic resources were distributed in the past and how they are permanently redistributed (or prevented from being redistributed) by means of state intervention (or without it). Walras’ great contribution, however, was in obtaining a set of equations based on the decisions of individuals and under certain idealized conditions that were clearly specified, the solution of which yields the equilibrium prices and corresponding quantities (a ‘general equilibrium’). This feat is not diminished by his weakness in philosophical matters. It is indubitable that the advancement of economic theory required the incorporation of mathematical instruments that could facilitate the formal representation of the economic interrelations generated by production processes and market transactions. We have seen that the lack of such instruments affected Marx’s development of his theory and that in the last stage of his life he turned to studying mathematics. However, Walras’ mathematics turned out to be excessive for the economists of his time and, furthermore, he wrote in French, which was resisted by English speaking economists. Hence, it was only after World War II that his influence grew strongly among economists.

For Walras social wealth is composed of different types of ‘capital’. Among them he distinguished between ‘fixed capital’, which consist in goods not entirely used up in a period of time (durable goods), and ‘circulating capital’, which are non-durable goods. The use of ‘fixed capital’ generates a flow of services. He defines three

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9In this section we use the words ‘capital’, ‘capitalist’, etc., in quotation marks when we refer to their use by Walras in order to distinguish them from their use by Marx.

10For Walras ‘circulating capital’ is a form of ‘income’. Other forms of ‘income’ are the services generated by ‘fixed capital’. This is an annoying aspect of Walras’ nomenclature. For example, he writes: “Income comprises not only articles of private consumption but also the raw materials of agriculture and industry” (Ibid., 212); “In order to bring out the distinction between capital and income we shall designate all those incomes which consist in the uses made of capital by the name of services” (Ibid., 213); “land, persons and capital proper constitute capital; while the services of land (i.e. land-services), the services of persons (i.e. labour) and the services of capital proper (i.e. capital-
categories of ‘fixed capital’, which when used by entrepreneurs in production constitute the ‘elementary factors of production’: ‘persons’ (‘personal capital’), ‘land’ (‘landed capital’), and ‘capital proper’, which includes all the remaining ‘capital assets’. The last of these are produced, and include dwelling houses, public buildings, factories, machinery, etc. He specifies that by durability he means economic durability and not physical durability, and that the materiality or immateriality of fixed or circulating capital is indifferent.

Hence, Walras’ ‘capital’ differs from Marx’s concept of capital. While for the latter capital is fundamentally a social relation between persons in the economic process, a ‘social relation of production’ that appeared in a certain historical period (and not before) between wage workers and capitalists and can only be understood as the culmination of a commodity producing society, for Walras the different forms of ‘capital’ are the elements that constitute ‘social wealth’, regardless of any concrete historical era. Walras’ categories thus have that a-historicity that Marx criticized of political economy (although he himself did specify some very general categories to which he attributed that character).

For Walras each one of the three types of ‘fixed capital’ can generate a service that can be used (either in consumption or in production) and which has a market price. And wage is the price of labor, i.e., the service produced by ‘personal capital’, rent is the price of the service of ‘land capital’, and interest is the price of the service of ‘capital proper’. He calls the owners of these three types of ‘capital’ workers, landowners, and capitalists, respectively. We may notice that Walras’ definition of ‘personal capital’ is independent of whether it refers to a free person or a slave. In the former case, the person ‘owns’ his ‘personal capital’ and in the latter the slave owner owns the slave’s ‘personal capital’. But Walras, as did the rest of the economists of his time and later, was concerned mostly implicitly with an exclusively capitalist system in which persons are free. Hence, the coexistence of different modes of production, a central topic for Marx, was not a topic of theoretical interest for Walras.

The entrepreneur has a very peculiar role in Walras’ theory. First, his role is central in the operations of the firm: organizing the productive process, purchasing the productive services and raw materials—the combination of which yields the production of final goods—and selling these in order to gain a profit. Second, in a regime of ‘free competition’ and in a situation of general equilibrium, firms (and entrepreneurs) have neither profits nor losses. This implies that no entrepreneur can live exclusively out of his entrepreneurial activity. He can only have an income to live from if he is also a worker, a landowner, or a ‘capitalist’. Hence, in Walras’ theory the entrepreneur as such cannot have an independent existence: he can only incarnate a complementary aspect of someone who makes a living by selling in the market at least some of the services of his ‘fixed capital’.

Walras criticized the theory of interest of the ‘English School’ (which included Ricardo and J. S. Mill) for not distinguishing between the role of the ‘capitalist’ and that of the entrepreneur, and “a certain number of French economists” who considered the entrepreneur as a “worker charged with the special task of managing the firm” (Ibid., 222). He admits that in reality it is “difficult but not impossible to be an entrepreneur without being a capitalist”, but points out that frequently “men, who have no capital of their own, but whose intelligence, honesty, and experience are known, obtain loans for agricultural, industrial, commercial, or financial enterprises” (Walras 1952, 423). He states that it is important to distinguish between these two roles

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services) constitute income” (Ibid., 215). According to W. Jaffé (the translator), this terminology had its origins in Walras’ father. We will only use the word ‘income’ with its usual sense.
and that the English School did not understand that the ‘profit of enterprise’ “is the correlative of possible loss, that it is subject to risk, that it depends on exceptional and not normal circumstances, and that theoretically it ought to be left to one side” (Ibid.; italics added), leaving only the interest of the ‘capitalist’. Walras assumed that there could be no equilibrium in a situation in which an entrepreneur had profit or loss, for either of these would lead him to make changes. Hence, in general equilibrium no entrepreneur would have profit or loss, and the price of each commodity would be equal to its cost of production.

Walras’ ‘profit of enterprise’ is thus similar to Marx’s ‘extra profits’ (which tended to be compensated by other firms’ ‘infra profits’). But while ‘profits of enterprise’ are positive on average for Marx, they are null for each and every entrepreneur for Walras in general equilibrium. Although these are important conceptual differences, both in Marx’s and in Walras’ works (but for different reasons) the role of the entrepreneur is not well reflected in the theory. Whereas in Marx’s theory the entrepreneur can live out of the ‘profit of enterprise’ (except in the extreme case in which it is entirely received by the stockholders of a stock company and the entrepreneur is replaced by a worker with specialized skill), in Walras’ theory he cannot since his equations only reflect the state of general equilibrium in which they disappear. It is possible to interpret that Walras’ entrepreneur was necessarily a person and hence owner of his ‘personal capital’. If he used his personal services in the firm he operated (instead of selling them to other entrepreneurs in the market for such services), he had to impute his ‘income’ as a cost, although Walras was not at all explicit on this point. If this would require the existence of a market for his type of ‘entrepreneurship’ then he was very close to those French economists he criticized.

In his texts Marx did something similar with the role entrepreneur as administrator. But, as we have seen, he also considered the entrepreneur’s role as an ‘orchestra conductor’. However, neither of these roles appeared in his equations. And in Walras’ analytical framework the entrepreneur is represented in a qualitatively different form from the owners of ‘fixed capital’. For, as we show in detail in the next chapter, the latter made decisions in which the utility (or disutility of effort in the case of ‘personal capital’) of the personal consumption of the service of his ‘fixed capital’ was taken into account in order to determine how much of the service of his resource to offer in the market and how much to reserve for his own enjoyment. In contrast, this did not happen in the case of the entrepreneur, who in a regime of ‘free competition’ ended up setting the price of his produced commodity equal to its cost of production, without any utility (or disutility) considerations coming into play.

**Walras’ polemic with Edgeworth and later economic theory**

Although Walras preserved the figure of the entrepreneur that seeks profit (he nowhere said that he maximized them), his was a very abstract agent since he could only have an income by also being a proprietor of some of the forms of ‘capital’ and selling their ‘services’ in the market. This ‘capital’ could be money, land, or his very person (which he necessarily owned if he was not a slave). Walras held that “From the scientific point of view, we must keep these roles separate and avoid both the error of the English economists who identify the entrepreneur with the capitalist and the error of a certain number of French economists who look upon the entrepreneur as a worker charged with the special task of managing a firm” (Walras 1954, 222). Avoiding these ‘errors’, Walras invented an entrepreneur fictitiously separated from his entrepreneurial work and from any possibility of satisfying his search for profit in a state of general equilibrium, the
only state of reality that was modeled in his *Elements of Pure Economics*.

His treatment of the entrepreneur was not accepted without controversy. There was one that involved the Irish economist Francis Edgeworth (1845-1926) and took place between 1889 and 1891 (Marchionatti 2003). Edgeworth had a good mathematical training but reflected the opinion of many of his contemporaries when he said that Walras sinned by an excess of mathematics. And there is no doubt that *Elements* had a proportion of equations to text that was until then unknown in the economic literature. Edgeworth also had a critical view of Walras’ invention of a process of crying out prices as in an auction, and of his conception of the entrepreneur having neither gains or losses, which is the one of interest here. For he held that, “surely he goes too far in the way of abstraction when he insists that the ideal entrepreneur should be regarded as ‘making neither gain nor loss’... Perhaps his views on this and other points would have been more exact if he had considered the part which the ‘disutility’ of labour—to use Jevons’s phrase—plays as a factor of economic equilibrium, instead of confining his attention to ‘final utility”’ (Edgeworth 1889, 2). Walras asked Bortkievicz (who had left behind his early Ricardian postures and was much younger) to write an article answering Edgeworth’s criticism. Bortkievicz accepted and Walras sent the resulting article to the *Revue d’économie politique* with his endorsement. When next year Edgeworth published in the same journal a long article on “The Mathematical theory of supply and demand and the cost of production”, Bortkievicz refused to continue with the polemic, recognizing that some of Edgeworth’s arguments made sense. Edgeworth considered that Walras’ entrepreneur was an ‘extreme abstraction’ (Marchionatti 2003, 5), and did not approve that in Walras’ market equilibrium there was no cost of production involving sacrifice and effort, that is, ‘disutility’.

Mainstream economic theory gradually accepted Walras’ paradigmatic theory of general economic equilibrium, adopting the assumption of ‘perfect competition’: a competition that was so strong that prices are driven down so as to eliminate all profit. Hence, all prices are equal, in equilibrium, to costs, including the rewards to the ‘productive factors’, among which there is seldom an ‘entrepreneurial factor’. Neither is a tradeoff represented for the entrepreneur who heads a firm and in real life must put a (subjective) limit to his personal effort in the search for the highest possible profits. Thus, Walras’ asymmetric treatment was carried over to the modern competitive general equilibrium paradigm. From the beginnings of Neoclassical economics the consumer-worker’s decision has been represented as a tradeoff between working for an income and enjoying leisure time. But heads of firms were not modeled; only the ‘firm’ that (somehow) acts to maximize profit. This eliminated the human figure of the entrepreneur (or the top executives of corporations), along with Marx’s most important social class in Capitalism as far as decisions in economic processes is concerned. In equilibrium only the incomes corresponding to the ‘factors’ of production hired by ‘firms’ exist. Here risk bearing is often included (and its reward through a risk premium) but what is left out is the idea of a (stable) profit of enterprise (or direction) as such: a self-stipulated reward for a *sui generis* type of work which is crucial for the operation of economic activities in firms and (when successful) is the positive bottom-line of the firm’s financial statement. On the other hand, different variants of ‘imperfect competition’ are also a part of the economist’s ‘toolkit’. But there is no unified theory that covers all the variants (or at least the main ones), including the unrealistic perfectly competitive extreme. It is well accepted that the development of large corporations has generated oligopolistic power that is heterogeneous in the various sectors according to “market conditions”. And this often leads to hiding significant portions of the true profits in the forms of ‘bonuses’ (or ‘stock options’, etc.) for the main executives,
in detriment of the stockholders who often have no say in this. Hence, what mainly counts in the real-life appropriation of true profits is not property as such but control, and especially the power that grants the controllers the possibility of using their position to their benefit. And this goes far beyond the explanation of high incomes within the big corporations, also extending to the possibility of significantly influencing the course of events in the political life of the countries in which the firms operate. And though competition is less imperfect than for large corporations for the vast majority of firms in any capitalist country, the figure of the entrepreneur (along with his profit of enterprise) is fundamental and should be represented in any acceptable theory of the functioning of society in general and the economy in particular.

The entrepreneur after Walras

Gustav Cassel and his entrepreneur

Gustav Cassel, who was a great disseminator of the non-Microeconomic aspects of Walras’ theory\(^\text{11}\), followed the latter’s criterion when, 20 years after Edgeworth’s article, he published the first edition of *The Theory of Social Economy*. He wrote: “The part played by entrepreneurs can be best explained by saying that they bring together other factors of production than their own labour for productive purposes, and set them to work... The director of a large business in which all the factors of production are used is a typical entrepreneur” (Cassel 1967, 172). Although he normally owns a capital that is used in the enterprise, “In this respect, however, he is a capitalist, not an entrepreneur. He also does work of one kind or other on behalf of the business. To that extent, he is a worker.” As Walras, Cassel stated that “we must, as it were, distinguish the different aspects of his personality in order that we may be able to grasp the nature of the work which he performs in his specialised capacity as an entrepreneur” (Ibid.).

Cassel made slight variations to Walras’ entrepreneurs when, for example, he stated that the profit of enterprise is often made to include some “elements which have an approximately definite price”, such as “the personal work of the entrepreneur in his capacity of employer, in organising and directing the undertaking” (Ibid., 173). Hence, they should be included among the costs of production and not the profit. “There is a price for each of these functions of enterprise in every economically advanced country. This price follows the ordinary rules of pricing: the relative scarcity of competent individuals in proportion to the demand for them is the primary determining factor of the price of the entrepreneur’s work.” Hence, “The work of an entrepreneur can, if so desired, be regarded as a special factor of production, in addition to labour in the usual sense of the term” (Ibid., 174, footnote). The same could be said of “any risk against which it is possible to insure, such as fire, or shipwreck, and so forth. Generally, insurance premiums are paid for these risks and included in the costs” (Ibid., 174). Leaving aside these two items, “we may regard whatever remains of the net proceeds of the business as the entrepreneur’s profit in the strict sense of the word –as a pure profit of enterprise” (Ibid., 175). Cassel writes that “As a rule, it is not possible to insure against the actual business risks of an undertaking”, and that there is no doubt that entrepreneurs bear this in mind when, planning a certain business, they (subjectively) add a risk premium to the interest rate on the safest securities to obtain the discount factor. Hence, this ‘pure profit of enterprise’, defined “as the surplus of gross proceeds over costs” certainly exists: “There can be no doubt that in real life

\(^{11}\)By this we mean that though Cassel summarizes some of the main systems of equations in Walras’ *Elements*, he does not get into individual decision processes based on preferences and budget constraints.
there is an entrepreneur’s profit of this kind” (Ibid.).

But then he makes a series of qualifications that downplay this correct assertion and tend to Walras’ view that the profits of enterprise “ought to be left to one side” as far as theory is concerned. For example: “only in comparatively few cases is the pure entrepreneur’s profit very large”; “Probably, on the average, the returns are barely sufficient to cover all the costs, as we understand them here. A relatively large number of entrepreneurs can never cover these costs, but work at some loss”; “Pure profit is not a normal thing, but a specific element of the individual business. It is often the outcome of sheer accident”; referring to “the business”, he writes: “Perhaps, being fully developed and well-organised, it enjoys an advantage over smaller competitors; or it possibly has at its disposal so much capital that it can crush competition.” And then he drifts into the crucial ‘market structure’ question when he states that the ‘fully developed and well-organised’ firm “may have a legal monopoly, or perhaps has secured a monopoly by forming a trust.” Also: “If, on the other hand, the business in question is in a position of monopoly, and succeeds in using it to make a pure profit, this can only be done by raising the price of the product above the cost, and the entrepreneur’s profit must, in that case, be regarded distinctly as a part of the price of the product” (Ibid., 177). Since Cassel does not formalize any of this, he can keep the ‘pure profit of enterprise’ and ‘market power’ issues in this state of general assertions, pretty much as Marx did. And the only place where he does summarize some of Walras’ main systems of equations (without even mentioning him), the pure profit of enterprise disappears (Ibid., 139-44).^12

**Joseph Schumpeter and his innovating entrepreneurial leadership**

Joseph Schumpeter (1883-1950) had his own theory of the entrepreneur, most of which he formulated early in his career in *The Theory of Economic Development* (1949), first published in German in 1911 and somewhat modified in the edition of 1926. An English translation of the latter was published in 1934. In order to constrain his enquiry, Schumpeter first distinguishes economic conduct as that particular human conduct directed to the acquisition of goods, and then further specifies that he will deal only with “that economic conduct which is directed towards the acquisition of goods through exchange or production”, thus leaving out, for example, the acquisition of goods through violent means. And based on the division of labor and specialization, he distinguishes “classes of people whose chief activity is economic conduct or business, from other classes in which the economic aspect of conduct is overshadowed by other aspects.” Before focusing on his special topic of economic development, he sets the stage in a long chapter titled “The circular flow^13 of economic life as conditioned by given circumstances.” He concentrates here on “a commercially organised state, one in which private property, division of labor, and free competition prevail” (Schumpeter 1949, 5). For Schumpeter “to produce means to combine the things and forces”, and different methods of production are distinguished by how these are combined. He

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^12 Actually, he only summarizes the special case in which there are given endowments of the factors of production, whereas Walras directly treats the general case in which there are supplies of these services that are dependent on the prices of both produced goods and factor services. In Chapter 20 we will see in detail both Cassel’s particular case and Walras’ general case. There we will start from the decisions of individuals, as Walras did (but not Cassel).

^13 The German word that was translated as ‘circular flow’ is ‘Kreislauf’. The translator (Redvers Opie) writes: “After considerable reflection I decided upon ‘circular flow’ for ‘Kreislauf,’ for reasons which it would take too long to relate”. ‘Kreislauf’ can be translated as either ‘circuit’ or ‘cycle’. In the English version of Marx’s *Das Kapital* it has been translated as ‘circuit’. The Google translator has ‘cycle’ in the first place. The Spanish version of *Capital* uses ‘ciclo’, which means ‘cycle’.
views an “enterprise as such and even the productive conditions of the whole economic system” as combinations (Ibid., 14). The ‘forces’ that are combined with objects include both land and labor. The services of labor and land are for Schumpeter the “ultimate elements in production”, and both are indispensable in production. He makes the distinction between directing and directed labor: directing labor “stands higher in the hierarchy of the productive organism. This direction and supervision of the ‘executing’ labor appears to lift the directing labor out of the class of other labor... the directing labor has something creative in that it sets itself its own ends” (Ibid., 19-20). However, “the characteristic of being in a higher rank, the function of superintendence in itself, constitutes no essential economic distinction... Much more importance seems to attach to the other element, which lies in the decision about the direction, method, and quantity of production” (Ibid., 20).

Having been trained in the University of Vienna under the influence of the Austrian neoclassical economist Böhm-Bawerk, he based the functioning of his ‘circular flow’ on John Bates Clark’s Distribution of Wealth. Hence, “rent and wages, are determined by the marginal productivity of land and labor”, and therefore “under free competition landlord and laborer receive the product of their means of production” (Ibid., 25). Furthermore, the entire net product is accounted for by these incomes, so “production must flow on essentially profitless.” Following Böhm-Bawerk, he writes that there are two main sources of profits and losses: various ‘frictions’, and mainly, the “changes in the data with which the individual is accustomed to reckon”, which requires an adaptation that can only be achieved over time. Schumpeter also mentions risk, including both the risk of technical failure as of commercial failure. But he states that “in so far as these dangers are foreseen they operate immediately upon economic plans” and thus businessmen either include a risk premium in their cost accounting or simply avoid “the more risky branches until the consequent increase of prices in the latter offers a compensation” (Ibid., 32). Hence, “The risk-premium is no source of gain for the producer.” But “The matter is different, of course, if the risks are not foreseen or at any rate are not taken account of in the economic plan. Then they become on the one hand sources of temporary loss and on the other hand sources of temporary gain.”

However, for Schumpeter the function of combining the services of land and labor in the “economic cycle” “is performed in every period mechanically as it were, of its own accord, without requiring a personal element distinguishable from superintendence and similar things.” Therefore, “If we choose to call the manager or owner of a business ‘entrepreneur,’ then he would be an entrepreneur faisant ni benefic e ni perte without special function and without income of a special kind.” What is crucial for Schumpeter is that it is necessary to go beyond the “economic cycle” by introducing “the fact of historical change” (Ibid., 58). And because the “economic aspect of things” depends fundamentally on the non-economic aspects of society, it is strictly “not possible to explain economic change by previous economic conditions alone.” He finds, however, that this limitation is “very much diminished, practically if not in principle, by the facts which form the basis of economic interpretation of history”, by which he seems to be referring to Marx’s Historical Materialism. And though he does not feel “compelled to take a stand for or against this view”, he states that “the economic world is relatively autonomous because it takes up such a great part of a nation’s life, and forms or conditions a great part of the remainder” (Ibid.).

Schumpeter finds that economic life experiences changes that “cannot be under-

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14 This is supported by the fact that just a few lines below he inserts a very long footnote that culminates with his assertion that his statement of the problem is “more nearly parallel to that of Marx”. 
stood by means of any analysis of the circular flow, although they are purely economic
and although their explanation is obviously among the tasks of pure theory” (Ibid., 61).
The process of development is “spontaneous and discontinuous change in the channels
of the flow, disturbance of equilibrium, which forever alters and displaces the equilib-
rium state previously existing” (Ibid., 64). It receives its impulses from innovations “in
the sphere of industrial and commercial life”; and in the development process “materi-
als and forces” are combined differently due to innovations. Development is defined as
the “carrying out of new combinations”, that is, introducing new goods, new methods
of production, opening new markets, or creating a new economic organization, whether
it is a firm or a trust, or the destruction of an existing position of monopoly. Schum-
peter adds that the increasing destruction of competition by huge ‘combines’ is a fact,
and that this transformation “is great enough to serve as the water-shed between two
epochs in the social history of capitalism” (Ibid., 67).

Up to here one could say that Schumpeter is expressing with his own words a series
of concepts and reasoning that Marx developed in some cases and only sketched in
others. But in the next step he clearly departs from Marx (and other economists) with
his unconventional concept of enterprise and entrepreneur: “The carrying out of new
combinations we call ‘enterprise’, the individuals whose function it is to carry them out
we call ‘entrepreneurs’ ” (Ibid., 74). This implies that, as Schumpeter highlights, his
‘entrepreneur’ can be a firm employee, a manager, or a member of a board of directors,
as long as he conceives and realizes new combinations. On the other hand, for Schum-
peter ‘capitalists’ are the people who provide credit, which is what stockholders do.
And “entrepreneurs do not form a social class in the technical sense, as, for example,
landowners or capitalists or workmen do”, but success will lead him and his family
to “certain class positions.” Finally, the specificity of entrepreneurs is not that they
‘create’ or ‘invent’ but that they implement what others may have created or invented
through their enterprising leadership in the establishment of new combinations, a sig-
nificant part of which is obtaining the necessary financing by convincing bankers of
the potential profitability of the new combination. He states that his characterization
also “disposes of the conception of the entrepreneur as risk bearer”, for “Risk obvi-
ously always falls on the owner of the means of production or of the money-capital
which was paid for them, hence never on the entrepreneur as such.” Although share-
holders can also be entrepreneurs, as shareholders they are “merely capitalists, who in
consideration of their submitting to certain risks participate in profits” (Ibid., 75).

It can be said that in essence Schumpeter’s entrepreneurial profits are simply Marx’s
‘extra profits’ with another name. And Schumpeter’s own contribution of the innovat-
ing entrepreneur that does not necessarily have a capital of his own nor directs any firm
in the conventional meaning of the word is not very convincing. When Schumpeter
wrote his book Business Cycles (1939), more than a decade after the second edition
of his Theory of Economic Development, he drew heavily on the latter. He wrote: “It
is not always easy to tell who the entrepreneur is in a given case... Nobody ever is an
entrepreneur all the time, and nobody can ever be only an entrepreneur” (Business
Cycles 1939, 101). It was easy to identify entrepreneurship in the times of competitive
Capitalism: “The entrepreneur will there be found among the heads of firms, mostly
among the owners.” But “In the times of giant concerns the question is often as difficult
to answer as, in the case of a modern army, the question who is the leading man or
who really won a given battle. The leading man may, but need not, hold or acquire
the position that is officially the leading one. He may be the manager or some other
salaried employee. Sometimes, he is the owner of a controlling parcel of shares without
appearing on the list of responsible executives at all” (Ibid.).
Schumpeter’s entrepreneurs are not individuals that weigh the utility of consumption against the disutility of effort, like the normal consumer. They “retire from the arena only when and because their strength is spent and they feel no longer equal to their task” (Schumpeter 1949, 92), as if they were warriors or gladiators. Their motivations are often based on “the dream and the will to found a private kingdom”, “the impulse to fight, to prove oneself superior to others, to succeed for the sake, not of the fruits of success, but of success itself”, and “the joy of creating, of getting things done, or simply of exercising one’s energy and ingenuity” (Ibid., 93) Whereas amassing private wealth only plays a role in the first of these motivations, for Schumpeter the others “may in principle be taken care of by other social arrangements not involving private gain from economic innovation” (Ibid., 94). He seems to suggest that his concept of the entrepreneur could be valid in a socialist country. Instead of Marx’s capitalist entrepreneur who basically wants to obtain profit for accumulating wealth and power, Schumpeter’s entrepreneur has something of Nietzsche’s Superman, relentlessly leading others in the introduction of innovations in the economic process. He emphasizes the role of exceptional individuals more than sets of heterogeneous individuals working in a hierarchical organization that jointly seek certain objectives that include the economic profits they can individually appropriate and the collective power over the course of society that can ensure the future enhancement of such profits.

Frank Knight, the entrepreneur, and uncertainty

Frank Knight published in 1921 Risk, Uncertainty and Profit, where he endeavored “to isolate and define the essential characteristics of free enterprise as a system or method of securing and directing cooperative effort in a social group” (Knight 1964, viii). For this he examined “the role of the entrepreneur or enterpriser, the recognized ‘central figure’ of the system, and of the forces which fix the remuneration of his special function.” Writing less than four years after the Russian Revolution, he adds that the “final verdict on questions of social policy depends upon a similar study of other possible systems of organization and a comparison of these with free enterprise in relation to the tasks to be accomplished.” And he prudently hazarded the conclusion that “no one mode of organization is adequate or tolerable for all purposes in all fields.”

As all of Schumpeter’s books, Knight’s is completely verbal (i.e., non-mathematical) and he contemplates economists as forming a spectrum that goes from “mathematical economists and pure theorists” to those who “repudiate abstraction and deduction altogether, and insist upon a purely objective, descriptive science.” And he held that mathematical economics was “likely to remain little more than a cult, a closed book to all except a few of the ‘initiated’ ” (Ibid., 14). As Marx would have found proper, Knight restricts the scope of Economics to Capitalism, or the “free enterprise or the competitive system”, which he defined as “a particular form of organization of human want-satisfying activity which has become prevalent in Western nations” (Ibid., 9)15. Although the system is “not at all completely or perfectly competitive”, “the study,

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15 In the preface to the 1948 reprint, Knight updated his view, stressing “the fallacy of the view so commonly expressed with respect to the classical or price-mechanics type of economics, that it is descriptively or practically relevant only to societies economically organised on the pattern of modern capitalism or free enterprise” (Knight 1965, xlvi-xlviii). For “no socialistic or authoritarian movement tries or seriously proposes to do away with the purchase and sale of goods and services for money, in markets (more or less free), as the main feature of economic organisation in the concrete... for, the allocation of resources, technical conduct of production and rationing of product would present an insuperable administrative problem”. Hence, Economics was still relevant for the type of society that had emerged in the Soviet Union, where private firms were banned but markets and money transactions still operated.
as a first approximation, of a perfectly competitive system, in which the multitudinous degrees and kinds of divergences are eliminated by abstraction, is clearly indicated." Knight restricted his study to an economic system in which there is no monopoly power in price-setting and was especially interested in the kind of “imperfect competition” that is due to the unquantifiable uncertainty that exists in the world in which human decisions are made.

He set himself the task of explaining ‘pure profit’ from the “standpoint of the problem of profit in distributive theory.” While in ‘perfect competition’ pure profit is zero, “in actual society, cost and value only ‘tend to equality;... they are usually separated by a margin of ‘profit,’ positive or negative” (Ibid., 19). Hence, Knight contrasted the pure profit that disappears in the theoretical case of perfect competition with the pure profit that may be positive, null, or negative in actual competition. But this required “a thorough examination and criticism of the concept of Uncertainty, and its bearings upon economic processes.” The crucial point was that “Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated” (Ibid.).

In times of the English Classical school, “in the dominant form of industry men used their own capital” and this “hid the fundamental difference between the total income of the capitalist manager and contract interest.” Adam Smith and his immediate followers “recognized that profits even normally contain an element which is not interest on capital”, including a remuneration for “work and care of supervising the business” (Ibid., 24). J. S. Mill noted that profits include “a payment for risk, as well as wages of management (and interest).” The early French economists clearly separated profit from interest but defined the former as a wage. However, in the fourth edition of his Traité, J. B. Say also “included in profit the reward for risk-taking.” And although the “older German economists varied widely in their treatment of profits” there was a group that “contended that profit should be recognized as a unique form of income”, most notably Thünen and Mangoldt. Knight did not include the “so-called ‘scientific’ socialists” in this group (referring to Marx and others) because he apparently ignored that Marx had an ‘exoteric’ theory, believing that the value and surplus value theory was all he had to offer.

The gist of Knight’s contribution to the theory of profit is that “There is a fundamental distinction between the reward for taking a known risk and that for assuming a risk whose value itself is not known. It is so fundamental, indeed, that, as we shall see, a known risk will not lead to any reward or special payment at all” (Ibid., 43-4). “If risk were exclusively of the nature of a known chance or mathematical probability, there could be no reward of risk-taking; the fact of risk could exert no considerable influence on the distribution of income in any way. For if the actuarial chance of gain or loss in any transaction is ascertainable, either by calculation a priori or by the application of statistical methods to past experience, the burden of bearing the risk can be avoided by the payment of a small fixed cost limited to the administrative expense of providing insurance” (Ibid., 46). Hence, measurable uncertainty gives rise to an insurance cost and has nothing to do with profits. For Knight the crucial issue was that “profit arises from the fact that entrepreneurs contract for productive services in advance at fixed rates, and realize upon their use by the sale of the product in the market after it is made.” Hence the decisions are made with a certain anticipation of what the selling price will be when the product is finally sold in the market. But because in real life there is not only change but, more importantly, an imperfect knowledge of the future, profits and losses arise. Knight stressed the “theoretical difference between the probability connected with an estimate and that involved in such phenomena as
are dealt with by insurance” (Ibid., 226). In the latter case, “an uncertainty which can by any method be reduced to an objective, quantitatively determinate probability, can be reduced to complete certainty by grouping cases” (Ibid., 232). Hence “measurable uncertainties do not introduce into business any uncertainty whatever.” In brief, it “is this true uncertainty which by preventing the theoretically perfect outworking of the tendencies of competition gives the characteristic form of ‘enterprise’ to economic organization as a whole and accounts for the peculiar income of the entrepreneur” (Ibid., 232).

When this true uncertainty is present, “the actual execution of activity, becomes in a real sense a secondary part of life; the primary problem or function is deciding what to do and how to do it... the work of forecasting and at the same time a large part of the technological direction and control of production are still further concentrated upon a very narrow class of the producers, and we meet with a new economic functionary, the entrepreneur.” “Centralization of this deciding and controlling function is imperative, a process of ‘cephalization,’ such as has taken place in the evolution of organic life, is inevitable, and for the same reasons as in the case of biological evolution” (Ibid., 268).

Many years later, in the Preface to the reprint of 1957, Knight summarized his view as follows:

Universal foreknowledge would leave no place for an ‘entrepreneur.’ His role is to improve knowledge, especially foresight, and bear the incidence of its limitations. Thus an essay on the theory of profit becomes an analysis of the price economy, with especial reference to the entrepreneurial function and income –positive or negative, profit or loss. The word ‘uncertainty’ seemed best for distinguishing the defects of managerial knowledge from the ordinary ‘risks’ of business activity, which can feasibly be reduced if not eliminated by applying the insurance principle through some organization for grouping cases. Thus uncertainty explains profit and loss; but profit, when it occurs, is not properly a ‘reward for risk-taking,’ though the expectation of gain is the incentive for assuming the entrepreneurial role. Nor is entrepreneurship to be treated as a ‘factor of production’ on a par with others, since it is not in at all the same sense measurable or subject to varying proportions and marginal imputation. Profit (when positive) is not the price of the service of its recipient, but a ‘residual,’ the one true residual in distribution (Knight 1964, lix).

In this Preface, Knight also gave an interesting example of how the differentiation of profit and wage can naturally arise. He posed the case of two workers that wish to “carry out a project together, with no other ‘factors’ involved.” They have two distinct possibilities as how to establish a contract: either negotiate a detailed agreement in advance with respect to what each of them is to do; or, more simply, have one of them “take charge and assure to the other a more or less definite return, his own ‘share’ (positive or negative) to depend on the outcome.” Knight holds that this hypothetical case “exemplifies all the theoretical essentials of entrepreneurship and profit” (Ibid.). Of course, this contractual view does not seem to have much to do with the actual historical emergence of entrepreneurship, which was more related to the existing asymmetries

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16It is remarkable that Knight does not even mention Cantillon in his original book nor, more surprisingly, in his much later prefaces. Probably neither he nor his thesis directors were acquainted with Cantillon’s book, even though Jevons had written a lengthy article in 1881 praising it as “more than any other book I know, the first treatise on economics” (Jevons 1881).
between those who either own or can obtain capital to invest in a (commercial or industrial) enterprise and those who do not and cannot. Neither does this contractual view have much to do with the dynamics where the successful capitalist entrepreneurs continue performing their role while those that fail do not (al least eventually). But Knight’s overall thesis is at least potentially related to such dynamics.

However, it is remarkable that after these very interesting considerations (and many more we have no space for) Knight ends up opining that on average businesses suffer losses: “It may then well be that entrepreneurs lose more than they make, the difference coming out of the returns due them in some capacity other than that of entrepreneur. The question of fact is thus whether entrepreneurs as a class receive on the average more or less than the normal competitive rate of return on the productive services of person or property which they furnish to business” (Ibid., 364). Although he admits that the evidence is not conclusive, he states that he is “strongly of the opinion that business as a whole suffers a loss.” For Knight the “prestige of entrepreneurship and the satisfaction of being one’s own boss” plays such a vital role that it makes the average entrepreneur have a gambling attitude in which, though he should expect losses, the possibility of having big profits induces him to engage in entrepreneurship. Hence, the “social theory of private property rests... on the belief that there will be a greater stimulus to progress through inducing men to take the risks of action” (Ibid., 370).

One of the greatest defects of Knight’s theory is that it seems applicable only to the large mass of smaller entrepreneurs, not those that are most relevant in the conformation of the structure of (economic and political) power in capitalist society. He leaves aside completely the road that Marx (with Engels) sketched decades earlier. Instead of completely ignoring monopoly power and then averaging mice and elephants he could have shown that there is a tendency towards increased concentration and centralization in vital sectors of the economy, and that this naturally leads to monopoly power in price setting in such sectors. Hence, more than a gambling attitude, it is the desire and ability to get rich and then always richer, which is only possible for a small fraction of the capitalist-entrepreneurial class by successfully engaging in oligopolistic behavior. This leads to an increasing power over others, including the government, which is the ultimate goal of a very significant, though minute, fraction of the entrepreneurial capitalist class.

The “corporate revolution” of Berle and Means

As we have seen in the Appendix to Chapter 17, Adam Smith had written on the deficiencies in the functioning of the great colonizing enterprises such as the East Indies Company due to the fact that their directors in remote places could in great measure escape from the control of the stockholders in England. With the development of Capitalism this feature of divorce between the stock owners and the directors that actually control the operations gradually propagated to other branches of the economy as joint stock companies multiplied and stock markets developed, although the characteristics of this process varied widely in the various countries of advanced Capitalism. We have seen that Marx referred to this process when he focused on the role of stock companies in the modern Capitalism of his times, which allowed the “Transformation of the actually functioning capitalist into a mere manager, administrator of other people’s capital, and of the owner of capital into a mere owner, a mere money capitalist.” This strand of thought was also followed by various economists of the 20th century. The topic is pertinent in this book because it was intimately related to what Marx wanted to do at the theoretical level as far as representing his vision of a hierarchical structure of
classes involved in the economic process and in which this hierarchy is based on power relations. ¹⁷

A landmark in the development of the theory of the capitalist firm was the contribution of Berle and Means (1933) with their theory of the divorce between the control exerted by the main executives over the functioning of large corporations and the property of the stockholders, who tended to become mere creditors with a right to dividends but lacking in control over the corporations they nominally (and legally) own. Berle and Means called this process the ‘corporate revolution’ and they were conscious that they were building on the initial contribution of Adam Smith on this topic. They believed that they were in the midst of this revolution but that it remained unrecognized as such because it was as silent as the ‘industrial revolution’ had been until it became well advanced. The fact that one of them (Adolf Berle) was a lawyer and the other (Gardiner Means) an economist gave their research a wealth of perspectives that research done exclusively by economists lacked (ever since ‘economics’ adopted a theoretical foundation that kept it separate from the rest of the social sciences).

For these authors, just as there had been a ‘feudal system’ in the Middle Ages, a ‘corporate system’ had already developed in the United States during the first decades of the 20th century and had become a fundamental socioeconomic institution. ‘Corporations’ had become means by which the wealth of thousands of individuals could be concentrated in huge firms controlled by a group of individuals that exerted a unified command over all its activities. “The concentration of economic power separate from ownership has, in fact, created economic empires, and has delivered these empires into the hands of a new form of absolutism, delegating ‘owners’ to the position of those who supply the means whereby the new princes may exercise their power” (Berle and Means 1933, 124). Just as the basis of the industrial revolution had been the concentration of great numbers of workers in a factory in which a capitalist entrepreneur exerted control, with the new revolution the holder of wealth, by purchasing stocks or bonds, handed over their wealth to those who controlled the corporation, managers that usually owned an insignificant fraction of the stocks.

Berle and Means perceived this as an ongoing development process in which there existed a wide dispersion in the fraction of the firm’s net worth owned by those who controlled their operations. They made a list of the gross assets of the 200 non-financial firms in the U.S.A. (in 1929) with greatest gross assets. These represented 49.2% of the gross assets of all the non-financial firms. And these 200 firms, owning almost half of all the assets, represented only 0.0007% of the approximately 300 thousand existing non-financial companies. On the other hand, (in 1929) the greatest individual stockholders of the largest railroad company (Pennsylvania Railroad), the largest public services firm (AT&T) and the largest steel company (United States Steel Corporation) owned respectively 0.34%, 0.70%, and 0.90% of the stocks of these companies, where the total number of stockholders were 500 thousand, 196 thousand, and 182 thousand, respectively. “Under such conditions control may be held by the directors or titular managers who can employ the proxy machinery to become a self-perpetuating body, even though as a group they own but a small fraction of the stock outstanding” (Berle and Means 1933, 5).

But even in less extreme cases a great mass of stockholders had been generated who

¹⁷ As Marx and Engels wrote in Ideology: “If power is taken as the basis of right, as Hobbes, etc., do, then right, law, etc., are merely the symptom, the expression of other relations upon which state power rests” (Ideology, MECW 5, 329). For Marx and Engels power was the foundation of jurisprudence, but economic and political power were intimately related and the power of the State was closely linked to the hierarchical power relations of the economic sphere.
virtually exerted no control whatsoever over the corporate activities and simply earned dividends and bore risk on the size of future dividend flows. Berle and Means called such corporations ‘quasi-public’, to differentiate them from “private, or closely held,” corporations managed by their main owners, and held that the economic functioning of these two types of corporations were different. In particular, in the ‘quasi-public’ ones the property rights were no longer intimately linked to the privileges resulting from the firm’s operations. And power was concentrated in a few hands. The divorce between control and ownership generated a situation in which “the interests of owner and of ultimate manager may, and often do, diverge, and where many of the checks which formerly operated to limit the use of power disappear” (Ibid., 6). The unity of the concept of property that included control, which had been the foundation for the economic order of the past three centuries, was in the process of dissolution. In the largest firms, that unity was separating into control, on the one hand, and ownership as a right to distributed profits, on the other. As property was being diluted, power was concentrating in the hands of a few corporate directors. Stockholders and holders of other financial assets supplied capital to the corporation and bore risk while a group of directors made all decisions, including those regarding their own compensation and the selection of the people that would eventually replace them at the highest positions. Since it was only necessary to keep stockholders satisfied with their dividends, the effort to obtain profits above such satisfactory dividends could benefit the directors in different ways. For Berle and Means this new reality required a revision of the traditional concepts of private property, wealth, private firm, private initiative, profit as motivation, and competition.

Berle and Means wrote during the world economic depression of the 1930s and before the worst excesses of the Soviet Stalinist dictatorship. They correctly viewed the corporations within the context of the overall economic organization of a country and the consequent bids for the use of power in society. For the desire of some to exert power for their own benefit coexisted with that of others who wanted to ensure that this power was used for the benefit of the majority: “Absolute power is useful in building the organization. More slow, but equally sure is the development of social pressure demanding that the power shall be used for the benefit of all concerned” (Ibid., 353). For Berle and Means the ‘communist movement’ in its purest form insisted “that all of the powers and privileges of property, shall be used only in the common interest” (Ibid.). They did not seem very well informed about the first 15 years of Soviet history.\(^{18}\) They complemented this statement with: “In less extreme forms of socialist dogma, transfer of economic powers to the state for public service is demanded. In the strictly capitalist countries, and particularly in time of depression, demands are constantly put forward that the men controlling the great economic organisms be made to accept responsibility for the wellbeing of those who are subject to the organization, whether workers, investors, or consumers.” Berle and Means thus held that the community demanded that the corporation be managed in such a way that it benefits not only the directors, the stockholders, and other people involved in its operations, but all of society. “It is conceivable, –indeed it seems almost essential if the corporate system is to survive, – that the ‘control’ of the great corporations should develop into a purely neutral technocracy, balancing a variety of claims by various

\(^{18}\)It is probable that when they wrote the dreadful famine of 1933 had not yet begun and almost certainly they did not know about it. But there had already been a great famine in 1921 as a consequence of, not only of a severe drought, but also rural economic policies that for the sake of rapid industrialization completely disregarded the welfare of the peasant masses that were the vast majority of the soviet people (Nove 1969, 83-6, 176-81).
groups in the community and assigning to each a portion of the income stream on the basis of public policy rather than private cupidity.” But who will take on this task? For the authors, “the controlling groups, by means of the extension of their corporate powers, have in their own interest broken the bars of tradition which require that the corporation be operated solely for the benefit of the owners of passive property” and have thus cleared the way for the ‘community’ “to demand that the modern corporation serve not alone the owners or the control but all society” (Ibid., 355-6).

For Berle and Means “The divorce of ownership from control consequent on that process almost necessarily involves a new form of economic organization of society... To us there is much to indicate that the process will go a great deal further than it has now gone” (Berle and Means 1933, viii). The authors wrote when the Soviet Union was in full process of accelerated industrialization. This made them aware of the “problem of the relation the corporation will ultimately bear to the state –whether it will dominate the state or be regulated by the state or whether the two will coexist with relatively little connection. In other words, as between a political organization of society and an economic organization of society which will be the dominant form?” (Ibid., viii). Undoubtedly, Berle and Means more than eighty years ago asked some very important questions regarding the future evolution of capitalist society and human society in general. Although they did not give answers, formulating the questions often goes a long way towards finding the answers.\(^\text{19}\)

Ronald Coase and the nature of the firm

In an early article titled *The Nature of the Firm* (Coase 1937), the English economist Ronald Coase (1910-2013) addressed one of Marx’s favorite topics: the fact that within firms there was a detailed planning of activities whereas without the ‘chaotic’ market reigned supreme. He did so in an era in which economists emphasized the rationality of resource allocation that markets make possible instead of the limitations this process has (or ‘market failures’, as they would come to be called). First he tried to give a precise definition of what a firm is ‘in the real world’. To those that questioned economic planning (which at the time alluded to the Soviet economy) saying that in the capitalist system the allocation of the factors of production between various uses is determined by means of the price mechanism, he replied by saying that in Capitalism (“our economic system”) economic planning *exists*, since within firms “markets transactions are eliminated and in place of the complicated market structure with exchange transactions is substituted the entrepreneur co-ordinator, who directs production” (Coase 1937, 2). He mentioned as a precedent to this idea Marshall’s ‘organization’ as a fourth factor of production, J. B. Clark’s ‘entrepreneur’ who has the function of coordinating, and Knight’s coordinating ‘managers’. And he asked why such ‘organization’, ‘entrepreneurs’, or ‘managers’ should be needed if the resource allocation problem was solved by the market mechanism.

In particular, he quoted from the British Marxist Maurice Dobb when he described Adam Smith’s idea of the capitalist as an entrepreneur who consciously plans and organizes the division of labor within the firm taking into account that his firm is merely a unit within a much wider economic system, “as a single cell in a larger organism” (Coase 1937, 3). Coase thus proposed to enquire why in one case it is

\(^{19}\)In the case of the U.S.A., the issue would be completely settled 30 years after the authors published their book, when the coup d’état that ended President Kennedy’s life consolidated the already existing domination of the industrial-military-intelligence complex. The large corporations linked to the military and to oil dominate foreign policy and much of domestic policy in a way that surely does not benefit the great majority of the population.
the entrepreneur that coordinates and in the other it is the market price mechanism. His most general answer is that in Capitalism there exists an “‘optimum’ amount of planning” (Coase 1937, 15). The vertical integration of productive processes involves the replacement of the price mechanism by the entrepreneurial coordinating mechanism and varies much from one industry to another and from one firm to another. For Coase what is essential is the existence of costs related to market transactions: the costs of discovering the relevant market prices, the costs of negotiating and making contracts for each market transaction, etc. Instead of making a costly highly detailed contract for all the motions of a worker in a firm in each hour of the day it is much more economical to make a generic contract in which the worker, within certain limits, accepts to obey the entrepreneur’s directives. Also, bearing in mind people’s risk preferences, it is often more feasible to make a contract for a long period of time than for a shorter one.

All this is obviously much related to Marx’s research on the origins of the capitalist mode of production. Marx meticulously searched for the available historical records on the different stages of Capitalism. But he also made abstract (theoretical) arguments of the type Coase made but with important differences (aside from the obvious ones stemming from the fact he wrote 70 years earlier). His main interest was in the relations of domination and subordination in the ‘antagonistic’ (asymmetrical and hierarchical) relations of production but he was also extremely interested in explaining how the capitalist mode of production had achieved wonders in the increase of the productive power of human labor. The ‘monopoly’ of a fraction of society over wealth and the enterprising capitalists’ profit motivation that made them willing to invest a substantial part of their wealth in productive enterprises that could take advantage of the more adaptable elements of the great mass of paupers lacking in means of subsistence, led to the propagation of industrial firms that over the centuries began to seek greater profits by means of productivity enhancing machinery and more generally the application of science to production.

Coase’s arguments can be seen as substantial complements to those of Marx that can help in explaining that complex historical process. They can help explain, in particular, why in certain cases there are mergers of previously coexisting firms that either produced the same product or were such that one of them produced inputs for the other. While Coase emphasizes the reduction of transaction costs, the consequence of those lower costs is the (temporary) increase in profits, that is, the achievement of the objective of further enrichment that Marx emphasized for capitalists. But Marx had also highlighted other related mechanisms, in particular the ‘centralization’ process that may also generate the expansion of existing firms (by means of the increase in the scale or scope of production or the production of previously purchased inputs) and the failure of other, less successful, firms. Hence, certain aspects of Coase’s contribution may be seen as a complement to the ‘exoteric’ part of Marx’s theory. Coase states, for example: “A firm, therefore, consists of the system of relationships which comes into existence when the direction of resources is dependent on an entrepreneur” (Ibid., 6), which is completely compatible with Marx’s view of the relations of production of industrial Capitalism. And Coase’s definition of the firm starts from inquiring into “the reasons why organizations such as firms exist in a specialized exchange economy in which it is generally assumed that the distribution of resources is ‘organized’ by the price mechanism” (Ibid.). This certainly is pertinent to Marx’s transition from Simple Commodity Production to Capitalist Commodity Production, where independent (symmetrical and a-hierarchical) commodity producers that participate in markets in which they buy inputs and sell outputs are transformed into much larger but fewer (asymmetrical and hierarchical) firms in which an industrial capitalist hires a multi-
tude of wage workers who were previously independent producers. It is also pertinent to the subsequent process of ‘centralization’ in which the allocation of the roles of entrepreneur and worker are further redistributed.

It is evident that something similar can be said about Knight’s emphasis on the uncertainty of profits and the convenience of having the type of labor contracts underlying Cantillon’s framework: the dominant part to the wage contract offers a certain wage to the subordinate part, reserving for himself an uncertain prospective profit the magnitude of which will critically depend on his ability to organize, plan, command and control what happens within the firm while he is constantly adapting it to a changing environment and risking the loss of his net worth. Knight’s framework can also be seen as a valuable complement to the ‘exoteric’ part of Marx’s theory, but in this case a complement that Marx surely preferred to ignore when he read Cantillon because he had already decided to base his overall theory on the ‘esoteric’ portion: his theory of surplus value.

**Tibor Scitovsky and the puritan entrepreneur**

Another landmark in economic theory’s explanations of the nature of the entrepreneur and his profits is Tibor Scitovsky’s article “A note on profit maximization and its implications.” Although in our view it is of little interest, it turned out to be the typical justification of mainstream economics for its ultra simplified treatment of the entrepreneurs or firm executives in its basic framework (of profit maximization). Scitovsky (1943) first points out that it is somewhat contradictory that, in contrast with the representation of the consumer worker, for whom the tradeoff between consumption and leisure is fundamental, in the theory of the firm an analogous tradeoff is completely absent. This critique is in line with Ricardo’s observation that the rates of profit are not completely equalized since if a certain activity is more secure, cleaner, easier, or has “any other real or fancied advantage” over another, it will tend to have a lower rate of profit. It is also in accordance with Edgeworth’s polemic with Walras on the same subject.

But Scitovsky ends his disquisition arguing that the entrepreneur is actually an atypical person who, like the Puritans who arrived in the Mayflower, considers leisure sinful. He thus concludes that the simplifying assumption that the entrepreneur earns as much as possible without measuring effort is an acceptable approximation. It is evident however that if this argument were valid for entrepreneurs it would also be valid for other economic agents. Workers would also work during all their available time (at least piece workers) without measuring foregone leisure and thus labor supply would not respond to the wage rate. But this is not done because it is admitted that it would not generate an adequate representation of the labor market. The economics discipline has found it convenient to dehumanize the figures of precisely those who are leaders in society, whether they are empowered to lead a private firm or to perform government activities (including the managing of public firms). This has the effect of distorting the representation of the functioning of the social classes in much of the economic analysis. In essence, the topic of the social classes and of the functioning of the mechanisms of control in society and in firms is left to sociologists, political scientists, and business management theoreticians; or alternatively to highly specialized economic models (such as those in ‘agency theory’). The problem with such procedures is that they constitute fetters to the emergence of a minimally unified social science.
Appendix to Chapter 19
Bibliographic Note: Joseph Schumpeter and Marx’s theory

Marx’s works had great influence on Joseph Schumpeter, either positively or negatively as he differentiated his own theoretical output. And this is as true for his early essays on social classes and imperialism as it is to his popular *Capitalism, Socialism and Democracy* and his posthumous *History of Economic Analysis*, edited by his widow from his manuscripts. His early *Theory of Economic Development*, where he presented his theory of the innovating entrepreneur, is considered by many his greatest contribution to economic theory. He there recognized the influence Marx had on his theory only in the last two sentences of a page long footnote. He there expressed his disagreement with the way in which economic theory, from Adam Smith to J. S. Mill, had introduced change as exogenous to the theory itself, which was basically static. Although J. B. Clark clearly separated ‘statics’ from ‘dynamics’, he basically “saw in the dynamic elements a disturbance of the static equilibrium” (Schumpeter 1949, 60). And though Schumpeter recognized the need for this view insofar as some of the changes in non-economic factors influence an economy, he felt the need for a different approach in the case of “changes in technique and in productive organisation”, which required a different type of analysis and, furthermore, “a new conception of the economic process, which overcomes a series of fundamental difficulties and thus justifies the new statement of the problem in the text. This statement of the problem is quite analogous to that of Marx. For according to him there is an internal economic development and no mere adaptation of economic life to changing data.” With some humility, which was not typical of him, he added: “But my structure covers only a small part of his ground.”

In his *Business Cycles, A Theoretical, Historical and Statistical Analysis of the Capitalist Process*, published in 1939, Schumpeter retrieved much of the basic content of his early theoretical book and also explicitly recognized that some of the central elements of his conception coincided with Marx’s. When he criticizes “a high authority in our field” that held that it was not “capitalistic enterprise” that accounted for the great increase in output of the 19th century but “technological progress (invention, machinery)”, he argued, “entirely agreeing with Marx”, that “technological progress was of the very essence of capitalistic enterprise and hence cannot be divorced from it” (Schumpeter 1939, 16). In this book Schumpeter linked the notion of ‘economic evolution’ to Darwin’s biological concept of ‘evolution’, saying that the ‘stationary flow’ is useful for contrasting with the ‘economic evolution’ he is interested in. “The commonsense of this tool of analysis may be formulated as follows: first, if we deal with, say, the organism of a dog... We may be interested in the processes of life going on in the dog, such as the circulation of the blood, its relation to the digestive mechanism, and so on. But however completely we master all its details, and however satisfactorily we succeed in linking them up with each other, this will not help us to describe or understand how such things as dogs have come to exist at all. Obviously, we have here a different process before us, involving different facts and concepts such as selection or mutation or, generally, evolution” (Ibid., 28-29). Further down Schumpeter states that “The changes in the economic process brought about by innovation, together with all their effects, and the response to them by the economic system, we shall designate by the term Economic Evolution... This terminological decision is but the expression of an analytic intention, namely, the intention to make the facts of innovation the basis of

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20As we have seen, the book was first published in German in 1911 and a second edition with various changes was published in 1926, which was used for the English edition of 1934.
our model of the process of economic change” (Ibid., 83). And he identifies innovations with changes in the production functions. More specifically, he defines an innovation as “the setting up of a new production function. This covers the case of a new commodity, as well as those of a new form of organization such as a merger, of the opening up of new markets, and so on” (Ibid., 84).\(^{21}\)

Many of the less exotic elements in Schumpeter’s theory of economic development originates in *Capital*. To a great extent, he tried to build on the foundations of Marx’s theory, differentiating his own product. But a substantial part of that differentiation consists of his admiration for those individuals blessed with the attributes of initiative and leadership he called ‘entrepreneurs’. It is difficult to see a substantial progress. Especially after the 2nd World War, the community of economists tended increasingly towards the mathematical modeling of economic phenomena, and that possibly explains why his lengthy verbal treatment was hardly attractive for most. But some of the ideas he transmitted or developed inspired economists somewhat peripheral to the mainstream.

\(^{21}\)Notice that in Part I of this book we represented analytically the increase in the productive power of labor by means of reductions in technical coefficients. Such reductions are precisely changes in the constant coefficients production functions we used in order to represent Marx’s thought. Also, since these coefficients were taken in Chapters 6 and 9 as averages of the corresponding coefficients of all the productive processes that produce the same commodity, the setup can include the special case of the creation of a new technical or organizational method as long as it is also of the fixed coefficients type. A slight extension would be necessary to also allow for the introduction of a new product that defines a new industrial branch.
Chapter 20  WALRAS’ THEORY
AND ITS RELATION TO MARX’S

Walras’ theory
As we have seen in the preceding chapter, Walras’ framework is very general. Any person (excluding children, retirees, the incapacitated, etc.) can own any subset of the ‘elementary factors of production’ (except the null subset, for he would not survive). He can be simultaneously a worker, a ‘capitalist’, and a landowner, or a worker and ‘capitalist’, or a worker and a landowner, etc., according to the 7 possible possibilities¹. And he can also be an entrepreneur and owner of at least one type of ‘fixed capital’ (but not simply an entrepreneur). Hence there are 14 possibilities. Because the context implies that they are free persons, these definitions have the strange peculiarity that any (free) person is a ‘worker’ for the simple fact that he owns his ‘personal capital’, even if he has never worked or received any other income than, say, rent on land, in his life. For in the theory of Walras if a person does not sell his labor services it is simply because he chooses to retain them for his personal enjoyment, or, as appears in modern Microeconomics textbooks, he chooses to enjoy them as ‘leisure’ instead of selling them for a wage. For this to be possible of course he must own some other form of ‘fixed capital’, the services of which he can sell so as to finance his consumption. Something similar happens with land, for if an individual is a landowner, he is free to lease it all or retain it (or part of it) for his personal enjoyment. Hence, he is a landowner even if he only owns the ground on which his house is. Walras put money in a special category (apart from ‘fixed capital’ and ‘circulating capital’) because of what he called its ‘mixed’ role. “From the social point of view” it was ‘fixed capital’, since it could be used more than once for making payments, and “from the individual point of view” it was like ‘circulating capital’, “for no individual can use it more than once, since he no longer has it after making a payment” (Walras 1954, 219).

Walras’ system in matrix form

In this chapter we seek to contrast Marx’s ‘exoteric’ analytical framework, with that of Walras suitably particularized. And for the purposes of this book, the clearest way to do this is to begin by modifying the dual equations of quantities and prices that we have been using so that they can reflect the specifics of Walras’ theory. This implies introducing some of the innovations produced by the Neoclassical economists (Gossens, Jevons, Menger, and especially Walras) in the analytical representation of the subjective element in decision-making. But we abandon the generality with which Walras treats the subject and follow a ‘class’ division of society as in Marx. Hence, within Walras’ framework we consider the particular case in which workers do not own any other ‘capital’ except their ‘personal capital’, the landowners only own their land, and the ‘capitalists’ only own the money capital they can loan to entrepreneurs and thus enables these to purchase means of production and productive services. This is closer to the spirit of Marx (and the Classical economists in general) of portraying the ‘anatomy of society’ (in order to explain its ‘physiology’) in terms of social classes. Walras’ framework is more general but also in a sense less informative. To make the model compatible with Walras’ theory we must also abstain from representing aspects that

¹As in Chapter 19, here we use the words ‘capital’, ‘capitalist’, etc., in quotation marks when we refer to their use by Walras in order to distinguish them from their use by Marx.
were important for Marx’s theory, such as the existence of profits beyond interests, the normal existence of unemployment, the heterogeneity of techniques for the production of each commodity, various dynamical concepts for the accumulation of capital, the progressive centralization process, and the formation of huge firms with (some degree) of monopoly power. We thus constrain the model to the Walrasian ‘perfectly free competition’ but maintaining the distinction between (pure) social classes. To facilitate the comparison with the systems of Marx we have been dealing with, we proceed in two stages. In the first, the owners of the three types of ‘fixed capital’ or ‘productive factors’ supply the totality of the productive services these can generate. In the second, these owners may decide to keep a part of these services for their personal consumption and only offer to sell the surplus.

Fixed supplies of ‘productive factors’

According to Walras, “Capital goods, proper, apart from buildings and a few special kinds of furnishings and machines, are hired out not in kind, but in money. The capitalist accumulates his capital by successive savings and lends money to the entrepreneur for a given period; the entrepreneur converts this money into capital proper and at the expiration of the contract he returns the money to the capitalist” (Ibid., 228). This is notably similar to Marx’s money capital, which is loaned by the financial (or ‘passive’) capitalist to the ‘active’ capitalist. Hence, the system of fixed production coefficients (we now call ‘input-output’ coefficients) that Walras initially set up in the first editions of his Elements and also maintained in the fourth edition as an introduction to the more sophisticated version with variable coefficients, is very similar to Marx’s systems. But it has some important differences. First, the consumption baskets are functionally dependent on the prices of the commodities produced and the prices of the services of the productive factor owned. Second, these consumption demand functions are obtained from the decisions of each of the individuals of each social class (which signified an important advance with respect to Cournot (1897 [1838]), who had introduced these demand functions as an assumption).

The (dual) equations of quantities and prices ‘à la Walras’ are the following:

\[ \begin{bmatrix} q^q & q^F \\ A & C(p, \omega) \end{bmatrix} \begin{bmatrix} L \\ 0 \end{bmatrix} = \begin{bmatrix} q^Q & q^E \end{bmatrix} \]  

(20.1)

\[ \begin{bmatrix} A & L \\ C(p, \omega) & 0 \end{bmatrix} \begin{bmatrix} p \\ \omega \end{bmatrix} = \begin{bmatrix} p \\ \omega \end{bmatrix}. \]  

(20.2)

They have the following characteristics: 1) there is only one type of worker, one type of land, and one type of ‘capital proper’ (in Walras’ terminology) which is the money that the entrepreneur receives in loan from the ‘capitalist’, 2) the per capita consumption baskets of each of the three social classes (workers, landowners, and ‘capitalists’) are given by \( C = [C^L C^T C^K]^T \), a vector that contains the consumption baskets per unit of ‘fixed capital’ owned by each type of owner, 3) the vector \( \omega = (w r i)^T \) represents the equilibrium prices of the respective productive services, where \( w \) is the wage (per worker), \( r \) is the rent (per unit of land), and \( i \) is the interest (per unit of money ‘capital’), 4) \( q^F = [q^L q^T q^K] \) is the vector of populations of each type of owner (workers, workers, and capitalists), respectively.  

\(^2\)Although Walras expanded his theory in order to include accumulation, we prefer to stay within the simpler framework of Simple Reproduction. Walras also departed from his framework of “free competition” in his ‘applied economics’ and considered the cases of monopoly and ‘natural monopoly’ (and even makes some interesting considerations on this topic in Elements). But we cannot go into these matters here.
landowners, and ‘capitalists’, respectively). 5) since each individual owns a unit of his type of ‘fixed capital’, the populations are also the quantities available of their respective ‘fixed capitals’, 6) the technological submatrix \( L \) of the social matrix is formed by the column vectors that represent, respectively, the direct requirement coefficients of the three types of productive services \( L = [\ell t \eta] \) that are used in production, and finally, 7) the three components of \( C \) are continuous functions of the prices of produced commodities \( p \) and the prices of productive services \( \omega \), that is,

\[
C = C(p, \omega) \equiv [C^L(p, \omega) \ C^T(p, \omega) \ C^K(p, \omega)]^T.
\]

We will see below how Walras derives these functions from the decision process of each individual.

Since here we are considering the simplest case in which the owners of the three types of ‘fixed capital’ offer the services of all their stock, here \( q^F = [q^L \ q^T \ q^K] \) is an exogenous vector. In particular, \( q^L \) is all the working population. The components of these systems are the following (vector) equations:

\[
\begin{align*}
q^Q & = q^F C(p, \omega) B(0) \quad (20.3) \\
qu^F & = q^Q L \quad (20.4) \\
p & = V \omega \quad (20.5) \\
\omega & = C(p, \omega)p \quad (20.6)
\end{align*}
\]

where \( V \) is the matrix of direct and indirect requirements of the services of each one of the production factors in each one of the production processes:

\[
V = (I - A)^{-1} L = B(0) \begin{bmatrix} \ell & t & \eta \end{bmatrix} \equiv \begin{bmatrix} V^L & V^T & V^K \end{bmatrix}.
\]

We note that \( V^L \) is simply Marx’s vector of values. But now we additionally have the analogous vectors for land services \( V^T \) and money capital services \( V^K \).

(20.3) gives the gross outputs \( q^Q \) necessary to satisfy final demands \( q^F C \). (20.4) is the condition that the (exogenous) supplies of the three productive services \( q^F = [q^L q^T q^K] \) are equal to the respective demands derived from the gross outputs \( q^Q L = q^Q \begin{bmatrix} \ell & t & \eta \end{bmatrix} = [q^Q \ell \quad q^Q t \quad q^Q \eta] \). (20.5) is the condition that (due to ‘perfectly free competition’) the price of each commodity produced is equal to its cost of production, and hence profit is null for each firm in each branch of production:

\[
p = V \omega = V^L w + V^T r + V^K i = B(0) \ell w + B(0) tr + B(0) \eta i.
\]

Another way of expressing this equality is that the price of each commodity is the sum of the value of intermediate produced inputs, of wages, of rents, and of interests:

\[
p = Ap + \ell w + tr + \eta i. \tag*{Finally, (20.6) shows that the equilibrium price of a unit of each productive service must be equal to the value of the consumption basket of each class:}
\]

\[
\omega = [w \ r \ i]^T = [C^L(p, \omega)p \ C^T(p, \omega)p \ C^K(p, \omega)p] .
\]

Each of (20.3) and (20.5) contains \( n \) equations, one for each commodity produced, and each of (20.4) and (20.6) contains 3 equations, one for each social class. Hence, we have \( 2n + 6 \) equations. But the number of endogenous variables are the number of components of \( q^Q, p, \) and \( \omega \), that is, \( 2n + 3 \) (and \( 2n + 2 \) if we adopt a numeraire). Hence, we can choose either (20.4) or (20.6) and leave out the other. For example, if we leave out (20.6), it is easy to reduce the remaining system to one of three equations
with three unknowns: the components of \( \omega \). Concretely, introducing (20.5) in (20.3) and the resulting equation in (20.4) yields the following vector equation that represents the equilibrium in each one of the markets of productive services:

\[
q^F C (V \omega, \omega) V = q^F.
\]  

(20.7)

To the left of the equality are the demands for each one of the three productive services formed from the sum of the consumption demands of the three classes \( q^F C \) and expanded (by multiplying by the matrix of multipliers \( V \)) to take the intermediate consumption of commodities into account. The three equations of (20.7) are:

\[
\begin{align*}
q^L \hat{C}^L (w, r, i) + q^T \hat{C}^T (w, r, i) + q^K \hat{C}^K (w, r, i) & \quad V^L = q^L \\
q^L \hat{C}^L (w, r, i) + q^T \hat{C}^T (w, r, i) + q^K \hat{C}^K (w, r, i) & \quad V^T = q^T \\
q^L \hat{C}^L (w, r, i) + q^T \hat{C}^T (w, r, i) + q^K \hat{C}^K (w, r, i) & \quad V^K = q^K,
\end{align*}
\]  

(20.8)

where to save space we have defined \( \hat{C}^j (w, r, i) \equiv C^j \left(T^j (w r i)^T, (w, r, i)^T\right) \). \( j = L, T, K \). We hence have what Walras lucidly wrote was needed: “Strictly speaking, we have to formulate a system of equations of which rent, wages and interest charges are the roots” (Walras 1954, 217).

‘Walras’ Law’

Walras started from the decision problem of each individual, formalized as the maximization of a utility function subject to the (budget) restriction that expenditure on consumption is equal to income.\(^4\) In this the individual takes as given the market prices of produced commodities and productive services and decides how much he demands of each produced commodity and how much he supplies of the services of each type of ‘fixed capital’ he owns. As we have mentioned, his framework was very general, and we here restrict it to a society strictly divided into social classes. Hence, the budget constraints of a ‘representative individual’ of each one of the classes (where \( h \) represents a worker, \( i \) a landowner, and \( j \) a ‘capitalist’) are the following:

\[
\sum_s^h C^L_{h,s} P_s = w, \quad \sum_s^T C^T_{i,s} P_s = r, \quad \sum_s^K C^K_{j,s} P_s = i.
\]  

(20.9)

Let us take the case of a worker \( h \), who decides how much to demand of each commodity according to the maximization of a utility function \( U^{Lh} \left(C^L_{h,1}, \ldots, C^L_{h,n}\right) \) that represents his preferences. We assume it is increasing in the consumption of each good \( C^L_{h,s} \) (\( s = 1, \ldots, n \)) but with declining increases as the consumption of the good grows.\(^5\) The

\(^3\)There is evidently more than one way of obtaining a system of three equations with the prices of the three productive services as unknowns. An alternative is to insert (20.5) in (20.6), yielding \( C (V \omega, \omega) V \omega = \omega \).

\(^4\)Neoclassical economics gradually improved the theories of its initiators. We are not here interested in the specific way in which Walras introduced his utility function because it is not relevant for the purposes of this book. We will freely use the format it reached later. The Italian economist Wilfredo Pareto (1848-1923), who in 1893 succeeded Walras in his chair of Political Economy at the University of Lausanne, and the Irish economist Francis Edgeworth (1845-1926) at Oxford University both made substantial contributions.

\(^5\)In mathematical terms, we assume \( U^{Lh}_{C^s_s} > 0 \) (positive marginal utility) and \( U^{Lh}_{C^s_s, C^s_s} < 0 \). (decreasing marginal utility) for \( s = 1, \ldots, n \). We here introduce concepts and notation that to some extent are posterior to Walras but nevertheless naturally arise out of his works. We treat the subject informally since it can be seen in any intermediate level Microeconomics textbook.
worker makes his fundamental decision by maximizing his utility \( U^{Lh} (C_{h1}^L, \ldots, C_{hn}^L) \) subject to his budget constraint \( \sum_s C_{hs}^L p_s = w \). As a result he obtains his demand functions for each of the \( n \) goods produced: \( C_{h1}^L (p, w), \ldots, C_{hn}^L (p, w) \). These depend on the prices of the goods and his income, which is here his wage. These functions that correspond to worker \( h \) can be grouped in the vector: \( C_h^L (p, w) \equiv (C_{h1}^L (p, w), \ldots, C_{hn}^L (p, w)) \). Similarly, each landowner \( i \) and ‘capitalist’ \( j \) maximizes a utility function \( U^T_i (C_i^T, \ldots, C_i^L) \) and \( U^K_j (C_j^K, \ldots, C_j^L) \), respectively, subject to his respective budget constraint (of (20.9)), and yielding his demand for goods \( C_i^T (p, r) \) and \( C_j^K (p, i) \) (already expressed in vector form). Taking these functional dependencies into account, the budget constraints (in (20.9)) can be written as:

\[
C_h^L (p, w) p = w, \quad C_i^T (p, r) p = r, \quad C_j^K (p, i) p = i. \quad (20.10)
\]

Since there are \( q^u \) \( (u = L, T, K) \) individuals of each class, adding the budget constraints over the individuals of each class yields:

\[
\sum_{h=1}^L C_h^L (p, w) p = q^L w, \quad \sum_{i=1}^T C_i^T (p, r) p = q^T r, \quad \sum_{j=1}^K C_j^K (p, i) p = q^K i,
\]

which can also be written as in (20.6):

\[
C^L (p, \omega) p = w, \quad C^T (p, \omega) p = r, \quad C^K (p, \omega) p = i, \quad (20.11)
\]

if we define the average consumption basket of each class: \( C^u (p, \omega) \equiv \left[ \sum_{s=1}^u C_s^k (p, \omega) \right] / q^u \) for \( u = L, T, K \).\(^6\) If we add term by term the three equations of (20.11) after multiplying each by the respective population \( q^u \) we get the so-called ‘Walras’ Law’, which is simply the sum of the budget constraints of all the individuals in the economy:\(^7\)

\[
q^L C^L (p, \omega) p + q^T C^T (p, \omega) p + q^K C^K (p, \omega) p = q^L w + q^T r + q^K i.
\]

‘Walras’ Law’ can be written more compactly as: \( q^F C (p, \omega) p = q^F \omega \).

Notice that multiplying (20.7) by \( \omega \) (and taking (20.5) into account) yields ‘Walras’ Law’. Since the latter is implied directly by the budget constraints of the individuals, i.e., is contained in the data of the decision process of individuals, the three equations in (20.7) (i.e., those of (20.8)) cannot be independent. Only two of them can exist. But each of these has three endogenous variables. In order to have the same number of variables as there is equations it must be recognized that the system can only determine relative prices and a numeraire must be used to determine in terms of which commodity the rest of the prices are measured. For example, the prices are monetary if the first commodity is money and we use \( p_1 = 1 \).\(^8\) Alternatively, we can take \( w = 1 \), i.e., express all prices in terms of the wage. In this case we can use two of the three equations of (20.8) to

---

\(^6\) Due to the simplifying assumption that each individual owns a unit of his type of ‘fixed capital’ the elements of \( q^F = (q^L q^T q^K) \) represent both the populations of the classes as the endowments of their respective resources.

\(^7\) According to Arrow (1972), the name ‘Walras’ Law’ first appeared in Oskar Lange (1942), a Polish ‘Marxist’ economist of the Soviet bloc who managed to participate in the academic environment of the Capitalist bloc.

\(^8\) Notice that this clears any doubt that could arise for having associated the population of ‘capitalists’ (a quantity) with the per capita ‘capital’ (an amount of money). If we have a commodity money, e.g. gold, this is analogous to what we have seen in Part I of this book. If, however, we are dealing with paper money there are new complications into which we will not get into here. Suffice it to say that it is necessary to specify the utility of keeping paper money: for example, it reduces the costs of transactions and hence enhances the consumption of utility yielding commodities.
obtain the equilibrium prices $r^*$ and $i^*$ (measured in terms of work). If, for example, we discard the first of the three equations we get:

$$
\left[ q^L \hat{C}_L (1, r, i) + q^T \hat{C}_T (1, r, i) + q^K \hat{C}_K (1, r, i) \right] V^T = q^T,
$$

$$
\left[ q^L \hat{C}_L (1, r, i) + q^T \hat{C}_T (1, r, i) + q^K \hat{C}_K (1, r, i) \right] V^K = q^K,
$$

which in principle yields the equilibrium values $r^*$ and $i^*$. Given these prices of productive services, we have $\omega^* = (1, r^* i^*)'$ and, from (20.5) and (20.3), respectively, the equilibrium prices of the produced commodities $p^* = V \omega^*$ (expressed in terms of work) and the gross outputs $q^Q^* = q^FC (p^*, \omega^*) B (0)$ necessary to satisfy final demand. Finally, if we prefer to use monetary prices, $(p_1 = 1)$, from (20.5) we get $1 = B (0) \left( \ell w + tr + \eta i \right)$. Hence this equation and any two of (20.8) determine the three elements of $\omega^*$.

We have seen that the equations of (20.11) arise directly from the budget constraints of the persons that conform the three social classes. These are precisely Marx’s “three big classes of modern society” as we see in the beginning sentence of the very short and truncated Chapter 52 of Book III (“Classes”):

The owners merely of labour power, owners of capital, and landowners, whose respective sources of income are wages, profit and ground rent, in other words, wage labourers, capitalists and landowners, constitute the three big classes of modern society based upon the capitalist mode of production (B3, 870).

But Marx’s profit, which included a ‘profit of enterprise’ as well as an interest on money loaned by the financial capitalist, is reduced in the Walrasian system to the interest of the ‘capitalist’ who loans money to the entrepreneur, since for Walras in general equilibrium there are no profits (nor losses).

Variable supplies of ‘productive factors’

We now advance another step towards Walras’ contribution in his Elements. It is a step we are particularly interested in because it is intimately linked to the portion of his ‘exoteric’ theory that Marx left in the least satisfactory shape: his theory of absolute rent. Although Marx believed (and manifested) that for his purposes it was sufficient to prove that except for cases in which there was great abundance of unoccupied lands there had to exist an absolute rent, the fact is that his theory remained unfinished on this topic. Walras’ big step forward in this respect was that instead of assuming that every individual offers in the market the totality of the services of the factor he ‘owans’, as we assumed above, he assumed that each of them decides if he wants to ‘consume’ directly a part of it and only offer the rest in the market.\(^9\) For example, if each worker $h$ has a unit of time available (the period of time that is relevant for the model) he can decide to reserve $\hat{C}_h^L$ for his personal consumption\(^10\) and only offer

\(^9\) Walras’ framework was even more general, since he assumes that any individual can demand the services of the resource owned by any other individual. But this does not concern us here. Marx’s framework also included this generality when, for instance, he deals with the ‘unproductive labor’ (of servants, etc.) paid out of the incomes of both the owning and non-owning classes. But Marx always assumed consumption baskets that are exogenously given (at least in their structure), whereas Walras starts from the decision-making process of each individual and obtains these baskets as a result.

\(^10\) Modern microeconomics calls it ‘leisure’. For Marx it was the ‘reign of freedom’, in which the individual could unfold all his personal faculties freely without being constantly bogged down by the need to make a living.
1 − \overline{C}_h^L \text{ in the market. His decision will depend on the prices of the commodities he consumes and the wage he gets for the time he works (and abstains from getting during the time he does not work). Of course, this freedom of choice is more realistic in the case of the independent worker than in the case of a wage worker, who must typically accept the pre-existing length of the working day (or together with other members of his class struggle for its reduction). But we leave this complication aside. The other two \textit{‘capitals’} are more important in this respect for the purpose of this book. On the one hand, in the Walrasian framework each landowner can decide how much of his land to offer and how much to reserve for his personal enjoyment (say, hunting deer, horseback riding, or simply for going out for a stroll). This way of approaching the subject is important for getting the equilibrium rent on land (more precisely, on each type of land). On the other hand, each \textit{‘capitalist’} can decide how much of his money to lend and how much to hoard. We have seen that for Marx this was a crucial aspect of his (informal) theory of the industrial cycle, in which the decisions on hoarding and discharging of the capitalist class played a central role. Not having these decisions as endogenized as Walras, he was able to sketch a theory of the industrial cycle based on decisions that were exogenous to the rest of the general model. In Walras’ theory, being the supply of money capital so determined by given preferences, it would have been more difficult (though not impossible) to relate these with the phases of an industrial cycle.

By maximizing his utility function subject to his budget constraint, each individual \( s \) of class \( u (= L, T, \text{ or } K) \) now defines not only his consumption demand of produced commodities \( C_s^u \) but also his consumption demand for the service of the \textit{‘fixed capital’} he owns \( \overline{C}_s^u \), and hence his supply of its service \( F_s^u \equiv 1 − \overline{C}_s^u \) (since we assumed each was endowed with one unit of his type of \textit{‘fixed capital’}). Let us take the case of a worker \( h \), who now decides how much of each produced commodity to demand and also how much free time he wants to consume by maximizing his utility function

\[
U^{Lh}\left(C_{h1}^L, \ldots, C_{hn}^L, \overline{C}_h^L \right),
\]

which now includes as an addition argument the consumption of non-labor time \( \overline{C}_h^L \). His budget constraint must now take into account that, given the wage, the more non-labor time he consumes the less will be the amount of commodities he can purchase: \( \sum_{s=1}^n C_{hs}^L p_s + \overline{C}_h^L w = w \). The non-labor time is valued by the wage because this is what he loses from earning, thus reducing his capacity for consumption of produced commodities. As a result of this decision process he obtains his demand functions for the \( n \) produced commodities: \( C_{h1}^L(p, w), \ldots, C_{hn}^L(p, w) \) and additionally the demand function for non-labor time \( \overline{C}_h^L(p, w) \). The latter determines his supply function of labor services, i.e., \( F_h^L(p, w) \equiv 1 − \overline{C}_h^L(p, w) \). We can similarly get each landowner’s supply function for the service of his land \( F_i^T(p, r) \equiv 1 − \overline{C}_i^T(p, r) \) and each \textit{‘capitalist’s’} supply function for funds \( F_j^K(p, i) \equiv 1 − \overline{C}_j^K(p, i) \).

For each social class \( u \) we hence get an aggregate demand function for the services of their type of \textit{‘fixed capital’} for personal consumption that can generically be written as \( \overline{C}_u^u(p, \omega) \) (even if only one of the three elements of \( \omega \) is relevant in each case). In this more general case of variable supplies of productive services we have, instead of (20.1) and (20.2), the following system:

\[
\begin{bmatrix}
q^Q & q^F \\
A & C(p, \omega) & L \\
C(p, \omega) & \overline{C}(p, \omega)
\end{bmatrix}
= \begin{bmatrix}
q^Q & q^F \\
A & C(p, \omega) & L \\
C(p, \omega) & \overline{C}(p, \omega) & p \\
\omega & \omega & \omega
\end{bmatrix},
\]
where

\[ C(p, \omega) \equiv \begin{bmatrix} C_L(p, \omega) \\ C_T(p, \omega) \\ C_K(p, \omega) \end{bmatrix}, \quad \overline{C}(p, \omega) \equiv \begin{bmatrix} \overline{C}_L(p, \omega) & 0 & 0 \\ 0 & \overline{C}_T(p, \omega) & 0 \\ 0 & 0 & \overline{C}_K(p, \omega) \end{bmatrix}. \]

\( \overline{C}(p, \omega) \) is a diagonal matrix with the consumption demand functions of the three social classes for the services of their respective ‘fixed capital’ on the main diagonal (available time, available land, available money or hoard). The equations obtained, instead of (20.3)-(20.6) are:

\[ q^Q = q^F C(p, \omega) B(0) \quad (20.12) \]
\[ q^F = q^Q L + q^F \overline{C}(p, \omega) \quad (20.13) \]
\[ p = V\omega \quad (20.14) \]
\[ \omega = C(p, \omega) p + \overline{C}(p, \omega) \omega. \quad (20.15) \]

Notice that only (20.13) and (20.15) differ from the system where all resource services were supplied to the markets. Now each individual can consume part of the services of his own resource, which must be taken into account in his budget constraint, valuing that consumption according to its price. We have, instead of (20.10) the following budget constraints:

\[ C_h^L(p, w) p + \overline{C}_h^L(p, w) w = w \]
\[ C_i^T(p, r) p + \overline{C}_i^T(p, r) r = r \]
\[ C_j^K(p, i) p + \overline{C}_j^K(p, i) i = i. \]

Since there are \( q^u \) \((u = L, T, K)\) individuals in each class, if we add up the budget constraints of each one of these classes we get the following:

\[ \Sigma_{h=1}^{q^L} \left[ C_h^L(p, w) p + \overline{C}_h^L(p, w) w \right] = q^L w \quad (20.16) \]
\[ \Sigma_{i=1}^{q^T} \left[ C_i^T(p, r) p + \overline{C}_i^T(p, r) r \right] = q^T r \quad (20.17) \]
\[ \Sigma_{j=1}^{q^K} \left[ C_j^K(p, i) p + \overline{C}_j^K(p, i) i \right] = q^K i, \quad (20.18) \]

And since \( F_h^L(p, w) = 1 - \overline{C}_h^L(p, w), \) \( F_i^T(p, r) = 1 - \overline{C}_i^T(p, r), \) and \( F_j^K(p, i) = 1 - \overline{C}_j^K(p, i), \) adding over \( h, i, \) and \( j, \) respectively, we have for each social class \( u \) an aggregate supply function for the service of their respective ‘fixed capital’, which can generically be written as \( F^u(p, \omega) \) (even if only one of the three elements of \( \omega \) is relevant in each case). Hence, we have a diagonal matrix \( F(p, \omega) \equiv I - \overline{C}(p, \omega) \) which in the main diagonal has the average per capita supply functions of the three productive services. Hence, (20.16)-(20.18) can be written as

\[ C^L(p, \omega) p = F^L(p, \omega) w, \quad C^T(p, \omega) p = F^T(p, \omega) r, \quad C^K(p, \omega) p = F^K(p, \omega) i, \]

\[ \text{Walras’ framework is more general, since the non-diagonal elements of this matrix can also be functions. For example, a worker can hire a plot of land, or a ‘capitalist’ hire the services of a gardener, or a landowner take a personal loan. But Marx also admitted such possibilities, and clearly separated the ‘spheres’ of consumption and production. What is new here is the functional dependence of the consumption demands on the prices and the way they are obtained.} \]

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11 Walras’ framework is more general, since the non-diagonal elements of this matrix can also be functions. For example, a worker can hire a plot of land, or a ‘capitalist’ hire the services of a gardener, or a landowner take a personal loan. But Marx also admitted such possibilities, and clearly separated the ‘spheres’ of consumption and production. What is new here is the functional dependence of the consumption demands on the prices and the way they are obtained.
if (aside from the already defined average consumption baskets of produced goods) we define the respective personal consumption demand functions of each one of the classes: 
\[ \overline{C}^u(p, \omega) \equiv \left[ \sum_{s=1}^{n} \overline{C}_s^u(p, \omega) \right] / q^u \text{ for } u = L, T, K. \]

Hence, (20.13) and (20.15) can be rewritten as

\[ q^Q L = q^F F(p, \omega) \tag{20.20} \]
\[ C(p, \omega) p = F(p, \omega) \omega. \tag{20.21} \]

If we add term by term the three equations of (20.19) after multiplying each by the respective population we get ‘Walras’ Law’:

\[ q^L C^L(p, \omega) p + q^T C^T(p, \omega) p + q^K C^K(p, \omega) p = q^F F^L(p, \omega) w + q^T F^T(p, \omega) r + q^K F^K(p, \omega) i. \]

It can be written more compactly as: \[ q^F C(p, \omega) p = q^F F(p, \omega) \omega. \] This shows that only three of the equations of (20.19) (or of (20.21)) can be independent. Taking two of them and \( w = 1 \) (or else \( p_1 = 1 = V_1 \omega \) as numeraire, we can in principle determine \( \omega^* = (w^* \times r^* \times \omega)^T, \ p^* = V \omega^* \), and \( q^{Q*} = q^F C(p^*, \omega^*) B (0) \). And in this case we additionally have the (endogenous) supply of productive services: \[ F(p^*, \omega^*) = I - \overline{C} (p^*, \omega^*). \]

As Walras writes:

Equilibrium in production, which implies equilibrium in exchange, can now be easily defined. First, it is a state in which the effective demand and offer of productive services are equal and there is a stationary current price in the market for these services. Secondly, it is a state in which the effective demand and supply of products are also equal and there is a stationary current price in the products market. Finally, it is a state in which the selling prices of products equal the costs of the productive services that enter into them (Walras 1954, 224).

This well describes equations (20.20), (20.12), and (20.14) (in that order), with the proviso that there Walras had not yet introduced the intermediate consumption of produced goods, which implies subtracting \( q^Q A \) from \( q^Q \). But he did this a few pages below. Let us also point out that Walras was satisfied with showing that the number of equations was equal to the number of unknowns, taking for granted that the equations had a solution. This was very reasonable, given the state of mathematics when he wrote this. Several decades passed before Wald (1936) for the first time gave sufficient conditions for the existence of a solution.

**Walras as complementary to the ‘exoteric’ Marx**

In several senses Marx’s theory was more ambitious than that of Walras. First, he wanted to explain not simply the functioning of a pure ‘capitalist mode of production’ but that of the capitalist society of his time, with all the complexity of the superposition of various modes of production, its social classes, and the State. Also, he wanted to study this process from a historical-genetic point of view, which implied understanding how the capitalist regime had developed from previous modes of production, what the specific differences with pre-capitalist regimes were, how the dynamics of its devastating effect on previous modes of production functioned, and what vestiges already existed of a future and surpassing mode of production lacking wage worker exploitation (such as worker cooperatives and the decreasing power of stockholders for the benefit
of those that actually controlled the stock companies). We have developed our critique of the ‘esoteric’ portion of Marx’s theory, that is, his theory of surplus value, the invalidity of which becomes evident as soon as we explicitly account for the activity of entrepreneurs. But the ‘esoteric’ part of his theory, that is, what is left once expunged of its ‘esoteric’ part, is still a formidable structure. From the point of view of the static interdependence of multiple markets, and the joint solution to the equilibrium prices under conditions of extreme competition, Walras’ theory was able to contribute valuable elements. Such advancements could only be elaborated by means of mathematical methods that Marx did not handle (such as differential calculus) and that Walras was able to use with such dexterity that it irritated the economists who tried to read it (to the point that his most important book was not published in English until 1954). It would have been possible to integrate some of Walras’ contributions to the more lasting aspects of Marx’s formal theoretical framework, which is undoubtedly in the ‘esoteric’ part of his theory. The discovery of how one could model people’s most elementary decision-making (such as, how much am I willing to work in order to consume more?) through the use of a theory of utility (or, more generally, a theory of preferences) would have made it possible to construct a better analytical framework in which to reflect many of the historical-genetic elements of Marx’s theory; aspects that were to a large extent absent from Neoclassical economic theory. But as is well known, in the following decades, and especially after the Russian Revolution, there emerged an increasing segmentation in socioeconomic-political thought between a dogmatic, schematic, and sterile ‘Marxism’—which became the official ideology of a new Soviet dominant class—and the Economics discipline. Furthermore, there was an increasing tendency in the Capitalist bloc towards highly fragmented ‘social sciences’ (‘economics’, ‘sociology’, ‘political science’, ‘anthropology’, etc.) that had important deficiencies in their specificities owing to a great extent to their artificial delimitation. In mainstream Economics there predominated an a-historical, a-sociological, a-political, and apologetic approach in constant search for ingenious ways to prove that political democracy can uphold reforms that raise the general welfare while concomitantly ignoring the unequal distribution of wealth and power among people belonging to different social classes that prevent such reforms from being little more than make-up.

We may validly ask what aspects of Walras’ theory Marx could have used if he had had the mathematical training necessary to formulate a system of equations. First, he would have been able to formulate the system of ‘regulating’ (or ‘equilibrium’) prices and wages, profit rate, and rents, as completely independent from his system of values and surplus value. This is precisely what we did in Part II of this book. Second, a more substantial weakness of Marx’s systems was taking as given (or exogenous) the per capita consumption baskets of the individuals of the various classes, at least in their structure (for we have seen that many of Marx’s analyses do reflect the cyclical exogenous changes related to hoarding or dishoarding, and to greater or less investment of money capital by the industrial capitalist class, on the levels of consumption). Marx (or his followers) could have taken advantage of the theory developed by Walras to link in a coherent way the consumption or investment demands with the prices of produced commodities, the wage, the profit rate, and the rent on land. Instead, there was a sterile polemic around the ‘objective’ versus ‘subjective’ ‘theories of value’.

Let us remember that when Marx developed his theory of the ‘prices of production’ he had not introduced the private ownership of land, in consonance with the architecture of Capital (that recognized that in the advanced Capitalism of the 19th century the landowners had become merely a fraction of the capitalist class). Once he did introduce it, along with the landowners, he found that differential rent did not present
important problems and was quite compatible with his conception of ‘extra’ profits (which were transformed into differential rents when there was private ownership of land). But he faced insurmountable difficulties with the treatment of absolute rent. He found that the “The regulating market price... would then not be the price of production... but rather the price of production plus the rent, $P + r$” (B3, 735). In the presence of ‘absolute rent’, the agricultural output would sell at a ‘monopoly price’, in the colloquial sense that absolute rent could only exist because the owners of land were empowered to prevent industrial firms (agricultural firms included) from using them if not adequately compensated. But Marx was unable to formalize a theory that could integrate such natural resources as land with the rest of his theory of (equilibrium) prices. He had to be content with the general statement that absolute rent could only be determined “by the buyers’ needs and ability to pay. Its analysis belongs under the theory of competition, where the actual movement of market prices is considered” (B3, 751).

Walras was the first to achieve a mathematically rigorous construction of a system of very general equations, the solution of which were the equilibrium prices of produced commodities along with wages, interests, and rents (along with the corresponding quantities), all in ‘general equilibrium’, by making the explicit assumption that there prevailed conditions of ‘perfectly free competition’ (or ‘unlimited competition’) due to which the commodity prices were equal to their cost of production, that is, the sum of the costs of produced inputs and productive services that collaborate in their production. Walras’ concept of equilibrium, like Marx’s, was that of an ‘attractor’, and the reasoning behind the explanation of why prices and quantities tended towards equilibrium was similar to Marx’s.

Equilibrium in production, like equilibrium in exchange, is an ideal and not a real state. It never happens in the real world that the selling price of any given product is absolutely equal to the cost of the productive services that enter into that product, or that the effective demand and supply of services or products are absolutely equal. Yet equilibrium is the normal state, in the sense that it is the state towards which things spontaneously tend under a régime of free competition in exchange and in production. In fact, under free competition, if the selling price of a product exceeds the cost of the productive services for certain firms and a profit results, entrepreneurs will flow towards this branch of production or expand their output, so that the quantity of the product [in the market] will increase, its price will fall, and the difference between price and cost will be reduced (Walras 1954, 224-5; text in square brackets is in the original; italics added).

Due to his vision of the real world, in which there was a prevalence of historical trends, for Marx the ‘equilibrium’ was merely an auxiliary concept that was useful in studying certain dynamical processes but was always subordinate to the study of a reality for which “this constant tendency to equilibrium, of the various spheres of production, is exercised, only in the shape of a reaction against the constant upsetting of this equilibrium” (B1, 361). And this disequilibrium always predominated in the essentially irreversible historical process. What kept the theory together (with the help of various models) was the knowledge of historical reality. Walras, on the other hand, preferred to keep his theoretical framework (of pure economics) separate from the specific applications that were to be placed elsewhere. But even in his Elements he gave an interesting verbal account of a theoretical dynamics in which equilibrium is never reached because there are basic data that are continually changing and perturbing the
equilibrium towards which the economy gropes, and sometimes the perturbation is so strong that it produces a crisis:

Finally, in order to come still more closely to reality... we pass from the static to the dynamic state. For this purpose, we shall now suppose that the annual production and consumption, which we had hitherto represented as a constant magnitude for every moment of the year under consideration, change from instant to instant along with the basic data of the problem... Every hour, nay, every minute, portions of these different classes of circulating capital are disappearing and reappearing. Personal capital, capital goods proper and money also disappear and reappear, in a similar manner, but much more slowly. Only landed capital escapes the process of renewal. Such is the continuous market, which is perpetually tending towards equilibrium without ever actually attaining it because the market has no other way of approaching equilibrium except by groping, and before the goal is reached, it has to renew its efforts and start over again... Viewed in this way, the market is like a lake agitated by the wind, where the water is incessantly seeking its level without ever reaching it. But whereas there are days when the surface of a lake is almost smooth, there never is a day when the effective demand for products and services equals their effective supply and when the selling price of products equals the cost of the productive services used in making them... It can happen and frequently does happen in the real world, that under some circumstances a selling price will remain for long periods of time above cost of production and continue to rise in spite of increases in output, while under other circumstances, a fall in price, following upon this rise, will suddenly bring the selling price below cost of production and force entrepreneurs to reverse their production policies. For, just as a lake is, at times, stirred to its very depths by a storm, so also the market is sometimes thrown into violent confusion by crises, which are sudden and general disturbances of equilibrium. The more we know of the ideal conditions of equilibrium, the better we shall be able to control or prevent these crises (Walras 1954, 380-1).

Walras’ conception of ‘general equilibrium’ or “normal state... towards which things spontaneously tend” had consequences that Marx would not have accepted, and quite rightly. In particular, in Walras’ general equilibrium there was no unemployment, that is, Marx’s ‘industrial reserve army’ was reduced to zero (for not even the frictions that generate transitory unemployment could exist, much less what Keynes much later called ‘involuntary unemployment’). If the theory could only model a state of equilibrium then it was defective except for small perturbations and was not suitable for explaining the periodically generated recessions (or depressions) with high unemployment. One must bear in mind, however, that in his two complementary works Walras tried to put his ‘pure economics’ within a much wider context; and also, that he was in favor of government intervention to remedy the problems the private sector could not solve, being an enemy of laisserz faire. But Marx no doubt gave a more central place to the phenomenon of unemployment and its influence on the poverty of a large portion of the ‘proletariat’.

Several decades had to pass before it became possible for Economics to (crudely) model the phenomenon of unemployment. And we can say that there has never been a satisfactory theory of some topics as important as the depression of the 1930s\textsuperscript{12}.

\textsuperscript{12}We should remember that this depression was a causal factor in the seizure of political power by
Marx’s theory reflected the fact that the industrial cycle generated reductions in unemployment in the expansive phase and increases in the contractionary phase. His theory was not limited to one or several models but used models to clear up the relations between important variables, always trying to explain the historical process and infer tendencies. In his view of the industrial cycle the capitalist entrepreneur’s decisions on hoarding and dishoarding and on increasing or reducing the reinvestment of profits played fundamental roles (as we have seen in Chapters 12-14). There was a short step from that broadly correct vision of the nature of the phases of the cycle and the ‘discovery’ in the 1930s that government economic policies could play a strong role in leaving early a looming depression or altogether avoiding it. In Chapter 16 we mentioned that when Marx (in *Brumaire*) analyzed the situation in France in 1850 he succinctly wrote “The people are to be given employment. Initiation of public works.” In the U.S. the equivalent of German rearment was the set of highly interventionist government policies called the *New Deal*. Keynes, who detested anything smacking of ‘Marxism’, well could have found inspiration in many passages of *Capital* when he wrote his *General Theory of Employment, Interest and Money* instead of identifying with the much less solid theory of Silvio Gesell (and his ideology of “eugenic selection” of the fittest).

On the other hand, the core of Walras’ theory (his ‘pure economics’) assumed a degree of competition that was very far from the notion of competition in Marx, for whom there was no model that was a central core of his theory but a series of models, all auxiliary, in the construction of his theoretical representation of a reality which already in his times (in the most advanced capitalist countries) showed a process of ‘centralization’ of capitals in certain industrial branches and even in whole sets of industrial branches (as Engels inserted) that tended to monopoly or oligopoly. That said, it is also true that although Walras used his notion of ‘free or unlimited competition’ for theoretical purposes he was conscious that the theory he developed in *Elements* could not be directly applied to aspects of reality he preferred to address in his complementary works on *Applied Economics* (or ‘theory of industry’) and *Social Economics* (or ‘theory of institutions’, or ‘science of the distribution of social wealth’).

As we have seen, Walras’ systems had in common with those of the ‘exoteric’ Marx the fact that the role of the entrepreneur was not analytically represented in a convincing way. Walras’ theory was deficient in only being capable of containing ‘profits of enterprise’ out of general equilibrium. But only general equilibrium was depicted in his system of equations. Hence, no individual could specialize in the complex entrepreneurial tasks of the real world because he would not have had an income as such except in non-equilibrium situations, aside from a remuneration for specialized work (if we accept the assumption that he imputes himself a ‘wage’ that reflects the equilibrium price in a market for this type of specialized work) or the interests he could receive if he was also a ‘capitalist’. Moreover, Walras’ entrepreneurial decisions were

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militarist factions both in Germany and in Japan, the two countries that most propelled the military expansionism that triggered the devastating Second World War.

13 Cfr. the Bibliographic Note on Keynes in the Appendix to Chapter 13.

14 Notice that the mechanics of price searching he invented *ätionnement*– did not occur in real time.

15 One must recognize, however, that even in his *Elements*, Walras introduced a Lesson entitled “Price fixing and monopoly”, in which he explains that things change radically when instead of ‘unlimited competition’ there is monopoly: “In the case of unlimited competition, the entrepreneur is an intermediary whom we may disregard... In the case of monopoly, entrepreneurs intervene not only to combine the productive services and convert them into products, but to levy a certain portion of the wealth exchanged for their own benefit” (Walras 1954, 438-9). And under monopoly, entrepreneurs did maximize profits.
characterized by a marked asymmetry with the decision processes of the owners of ‘personal capital’. The latter had to balance the disutility of the effort needed to earn a wage, with the utility of the goods such an effort would enable them to consume, whereas the entrepreneur merely sought “to avoid losses and to make profits” (Walras 1954, 225), as if this activity did not generate any disutility. On the other hand, we have seen that though Marx assumes a positive average profit rate that allows the entrepreneur that works with loan capital to make a living, he also lacked the representation of such activity in his equations. This would have contradicted the ‘esoteric’ part of his theory (based on wage workers’ ‘unpaid labor’). And the latter was the main anchor of Marx’s theoretical representation of his worldview (which was also the guide for his political activity, as we will see in Part IV).

Post-Walras economic theory maintained the essence of the Walrasian form of representing the entrepreneur’s role (converting him into a ‘firm’ that maximizes profit in ‘perfect competition’ as well as in monopoly) as well as the concept of an extreme degree of competition that eliminates all profit in equilibrium. Such were the foundations of the theoretical framework that culminated (insofar as static economics is concerned) in the theory of Arrow and Debreu (Debreu 1959, Arrow and Hahn 1971). Multiple factors, among which we can highlight the apologetic-ideological ones, led to the Walrasian ideas becoming the ‘general equilibrium under perfect competition’ paradigm (direct descendent of Walras’ general equilibrium under perfectly free or unlimited competition), especially after World War II. The mathematics was greatly sophisticated with proofs of existence of general equilibrium. And there were important advances in the intertemporal dynamic theory of growth, the introduction of stochastic components, the study of various variants of ‘imperfect competition’, ‘externalities’, and ‘public goods’, etc. But there was little progress in the development of a general theory of capitalist society that could integrate the economic, sociological, and political elements that were studied by several artificially segregated disciplines with the corporate interests of the respective academics entrenched behind jealously guarded borders.

Appendix to Chapter 20
Bibliographical Note: Walras and the French school of mathematical economics

The French school of ‘mathematical economics’ begins with Achilles Nicolas Isnard (1748–1803), who published his Traité des Richesses in 1781 (Van den Berg 2006), continues with Antoine Augustin Cournot (1801–1877), whose Recherches sur les principes mathématiques de la théorie des Richesses was published in 1838, and his contemporary Auguste Walras (1801–1866). It culminated in the extensive works of the latter’s son, Marie-Esprit-Léon Walras (1834–1910). Although Gossens, Jevons and Menger had written before Walras on the embryonic concepts of ‘marginal utility’, when Walras wrote the first edition of his work he did not know these writings existed; he discovered them afterwards. In the preface to the fourth edition of Elements he wrote: “I readily acknowledge Gossen’s priority with respect to the utility curve and Jevons’s priority with respect to the equation of maximum utility in exchange, but these economists were not the source of my ideas. I am indebted to my father, Auguste Walras, for the fundamental principles of my economic doctrine; and to Augustin Cournot for the idea of using the calculus of functions in the elaboration of the doctrine” (Walras 1954, 37). Responding to the insufficient recognition by Jevons in the second edition of The
Theory of Political Economy of the importance of his Elements and its innovations, Walras writes this significant paragraph:

From the moment the principle of the theory of exchange found a place in the science, it was inevitable that the principle of the theory of production would soon follow, which it most effectively did. In the second edition of his Theory of Political Economy Jevons became aware of a point he had missed in the first edition, namely, that if the Final Degree of Utility determines the prices of products, it must also determine the prices of the productive services, i.e., the rent, wages, and interest, because the selling prices of products and the costs of the services employed in producing them tend towards equality under a régime of free competition (Walras 1954, 44-5).

Walras was here reminding readers that it was his work that integrated in a unified conceptual framework the general equilibrium theory of production and the theory of exchange of goods and services, that is, the simultaneous determination of the equilibrium prices of produced goods and the productive services by means of which they are produced. The notable disinterest of the Anglo-Saxon public for French works led to the unavailability of an English version of Elements until 1954 and even more in the case of Studies in Applied Economics and Studies in Social Economics.¹⁶

¹⁶In his preface as translator of Elements, William Jaffée wrote that “Walras himself urged his contemporaries in England and America to translate some of his works, but he was told in 1885 by an English publisher of note that ‘it is very hard to persuade the English public to read foreign books at all, however good they may be’ ”, and in 1906 an American “economist of high scientific standing” wrote to him he doubted the wisdom of translating an abridged version that Walras had prepared (Walras 1954, 7).
Part IV

Marx’s utopia and his political praxis
Although Marx believed that capitalist entrepreneurs played an important role as ‘orchestra conductors’, his ‘esoteric’ analytical framework did not reflect it. The theory of surplus value was constructed by means of an analytical framework in which capitalist entrepreneurs did not contribute anything that could not be done by a (skilled) worker with a market determined income. And the ‘esoteric’ part of his theory was based on these foundations. Produced surplus value was distributed among the owners of means of production and land and formed their incomes (profit, interest, and rent). Marx’s political praxis used this analytical scheme as foundation for his project for the achievement of the ‘emancipation’ of the working class by replacing the capitalist mode of production by a mode of production devoid of exploitation and hence without the roles of wage labor and capital. A requisite for achieving the ‘emancipation’ of the working class was the formation of a political party that would propagate among the working class the scientific explanation of how Capitalism functioned and, through their organization –especially political–, generate sufficient revolutionary momentum so as to eventually make the leap to power that would enable the transformation of capitalist society into the “first phase of communist society as it is when it has just emerged after prolonged birth pangs from capitalist society” (Gotha, MECW 24, 87).

In Chapter 18 we showed the absurd consequences of Marx’s formal representation of the capitalist entrepreneur as appropriator of the ‘unpaid labor’ of wage workers. We showed that it generates the paradox that if the capitalist entrepreneurs became workers (presumably working for a cooperative or for State factories run by the ex-wage workers) after the suppression of their class by means of the expropriation of their firms, output would increase proportionally to the resulting increase in the working population.\(^\text{17}\) Since in Marx’s analytical theory the capitalists did not make a contribution to the generation of value, and global surplus value was the foundation for the incomes of the owning classes, the entrepreneurs were parasitic and would not be necessary in a ‘superior’ mode of production in which capital was abolished and all production comes to be generated in firms run by their own associated workers responding to their own State’s central planning.

Marx’s huge intellectual efforts in the construction of a scientific theory of Capitalism –achieved at the cost of immense personal and family hardships– always had the objective of modifying a reality that he saw as an impediment to the ‘emancipation’ of the exploited classes. Significantly advancing towards that ‘emancipation’ became the principle aim of his life project, in which he was accompanied at least since around 1844 by Engels. And Marx was absolutely consequential in the realization of that project. Because it was so important –and because he trusted his own capacity to give form to that project– he was constantly alert to the weaknesses of alternative projects of more or less similar root. That is why he left behind many that inspired or accompanied him at some stage, not before submitting them to devastating critiques.

In this chapter we review the evolution of Marx’s political thought and praxis. First we review Marx’s early vision as it is reflected in his writings of the 1843-48 period (some written jointly with Engels). Then we review his works mainly related to the European revolutions of 1848-49 and his political praxis in this period. Finally, we focus on his mature political thought, after his intense studies in England, as they are reflected in Capital and in his political writings of the last stage of his life.

\(^{17}\) The increase would be proportional only if we maintain the assumption of constant returns to scale.
Chapter 21  CONCEPT AND PRAXIS OF COMMUNISM IN THE YOUNG MARX

From ‘human emancipation’ to Communism in the young Marx

The gestation of Marx’s political project took place during the 1843-1848 period, that is, when he was from 24 to 30 years old. The period precedes his systematic studies of Political Economy, though he had read quite a few of the more significant books of that discipline. His early philosophical-political thought has on occasions almost mystical, ‘millenarian’, overtones evolving towards the conclusion that the political and social struggles of the working class would culminate in a revolution that would place it as a dominant class that would be able, from that position of power, to eradicate the exploitation of some classes by others, generate a planned economy, and gradually extinguish all class distinctions. This viewpoint was of fundamental importance for Marx throughout his life; and though it suffered some minor changes as he developed his studies and his political and organizational activity, it remained almost intact in its essence up to the end of his life. Since his objectives were mainly political, social, and economic, it is logical that they could not be minutely detailed. But even bearing this in mind, their schematism, naivety, and utopia strongly contrast with the realism (except for the crucial matter of the role of entrepreneurs), organicity, and empirically founded character of his magnum opus.

French-German Yearbook (1843-44)

As we have seen in Chapter 1, Marx contributed two articles to the French-German Yearbook (published in February 1844), which we will examine successively.

1. Contribution to the Critique of Hegel’s Philosophy of Law. Introduction This article had been conceived as an introduction to a very ambitious critique of Hegel’s Philosophy of Law, of which he only made an extensive draft (which was published posthumously in 1927) of a reduced number of the paragraphs of this book. Marx’s published article only deals tangentially with Hegel, focusing instead on the prospects for the triumph of a revolution in Germany. He starts with the statement that in Germany the critique of religion has reached its conclusion and that the basis of that critique is that religion is the product of man. We there find the notable and personal characterization of religious distress (which became universally famous in a condensed form that lends itself to misinterpretation): “Religious distress is at the same time the expression of real distress and also the protest against real distress. Religion is the sigh of the oppressed creature, the heart of a heartless world, just as it is the spirit of spiritless conditions. It is the opium of the people” (MECW Vol. 3, 175). He concluded that in order to overcome religion “as the illusory happiness of the people” it was necessary to “fight against the world of which religion is the spiritual aroma” (Ibid.).

For Marx it was evident that “The weapon of criticism cannot, of course, replace criticism by weapons, material force must be overthrown by material force” (Ibid., 182). However, he stated that “theory also becomes a material force as soon as it has gripped the masses... as soon as it becomes radical”, that is, when it attacks the
root of the matter. “But for man the root is man himself.” Hence, the critique of
religion leads to “the teaching that man is the highest being for man”, which we may
call ‘humanism’. And he derived from this humanism “the categorical imperative to
overthrow all relations in which man is a debased, enslaved, forsaken, despicable being”
(Ibid., 182). And this could only be achieved by means of a revolution. Marx recognized
that religion was a legitimate manifestation of the human being and that it was an
expression of suffering within a context of generalized injustice. And he was extremely
ambitious. At 25 years of age Marx felt and expressed the moral responsibility of
radically eliminating the conditions underlying the suffering of the less advantaged.

In his article Marx focused on the situation in Germany, which was far behind
England and France, both of which had undergone great transformations in the 17th
and 18th centuries, respectively, whereas the most radical revolution in Germany had
been the peasant uprising of 1525, crushed by the princes (with the strong endorsement
of Martin Luther) and producing the martyrdom of Thomas Müntzer. Marx writes
that “The relation of industry, of the world of wealth generally, to the political world is
one of the major problems of modern times” (Ibid., 179). But the fact that in Germany
there were protectionist tariffs was evidence that “People are thus about to begin in
Germany with what people in France and England are about to end”, for the latter
were already leaving protectionism behind.

Marx recognized that in Germany there were special difficulties for the occurrence
of a radical revolution, which required conditions that were absent there. Nevertheless,
he concluded that a merely political revolution in Germany could be discarded, i.e. one
“which leaves the pillars of the house standing” (Ibid., 184). For a radical revolution
to be possible, it was necessary that a class be formed “which has a universal character
by its universal suffering” (Ibid., 186). And for Marx this class was the ‘proletariat’
with which he identified to the point of projecting onto it his own thoughts strongly
tinged with messianism: “By proclaiming the dissolution of the hitherto existing world
order the proletariat merely states the secret of its own existence, for it is in fact the
dissolution of that world order” (Ibid., 187). And the world order to dissolve was pri-
vate property and, hence, the very proletariat. He concluded that “The only practically
possible liberation of Germany is liberation that proceeds from the standpoint of the
theory which proclaims man to be the highest being for man”, that in “Germany no
kind of bondage can be broken without breaking every kind of bondage”, and that
though the “head of this emancipation is philosophy, its heart is the proletariat.” It is
reasonable to interpret that Marx saw himself contributing the Philosophy and, hence,
the rational part –the head– of the project in consideration. The article ended with
a prophetic forecast that the future revolution would be international, that it would
begin in France, more evolved, and that it would extend to Germany: “When all inner
requisites are fulfilled the day of German resurrection will be proclaimed by the ringing
call of the Gallic cock” (Ibid., 187).

2. On the Jewish Question  In On the Jewish Question Marx criticized two articles
by Bruno Bauer related to the Jewish claim for political equality in Germany: “On the
Jewish question”, and “The ability of today’s Jews and Christians to become free.”
We are not here particularly interested in what Bauer wrote but in distilling from
Marx’s critical writings at this early stage of his development what were his ideas on
‘human emancipation’ and religious ideologies (as well as their relation to the State
and ‘civil society’). Marx here contrasted the German States of his time with other
states, and especially with some of the States that formed the U.S.A. According to
Marx the German State of his time was ‘imperfect’ (or less developed), since it was a
State with privileges and restrictions that did not correspond to an advanced industrial country. The German State was considered a ‘Christian State’ by its leaders, which implied that Christians had rights that were denied to minorities such as that of the Jews. It granted the latter the ‘privilege’ of practicing its own religion, but the German Jews wanted ‘political emancipation’, that is, the same political and social rights as the Christians. Bauer criticized the idea of a unilateral ‘political emancipation’ of the Jews because he claimed that both Jews and Christians should emancipate from religion in general. Marx criticized Bauer for limiting his critique to the ‘Christian State’ and not extending it to the State in general, and for not enquiring on the relation between ‘political emancipation’ and ‘human emancipation’. He criticized Bauer for remaining in the theological sphere.

Marx explained that one could see clearly in some of the more developed states of the U.S.A. the relation between religion and the State towards which countries with less developed capitalist institutions such as Germany tended. Although the people of these American states were highly religious, their constitutions did not impose any specific requirement of religious belief or practice in order to have access to political rights. For Marx, instead of focusing on the relation between political emancipation and religion one should focus on the relation between political emancipation and human emancipation. It was necessary to address the question of ‘political emancipation’ from the viewpoint of the establishment of a secular State, one with no official religion, one that no longer discriminated between the religions of its citizens. And this by no means required that the members of civil society should cease being religious, regardless of their religion. But for Marx there was a huge difference between this transformation of the State towards secularity and the achievement of ‘human emancipation’. When the modern State abolishes all distinctions of birth, social rank, education, and occupation in order to make all members of the community equal insofar as political rights such as suffrage, the State still “allows private property, education, occupation, to act in their way... Far from abolishing these real distinctions, the state only exists on the presupposition of their existence” (MECW 3, 153). “Where the political state has attained its true development, man—not only in thought, in consciousness, but in reality, in life—leads a twofold life, a heavenly and an earthly life: life in the political community, in which he considers himself a communal being, and life in civil society, in which he acts as a private individual, regards other men as a means, degrades himself into a means, and becomes the plaything of alien powers” (Ibid., 154). These ‘alien powers’ were those of the division of labor, private property, markets, and money. Marx thus introduced in this early stage of his intellectual development the essential nexus between his initial philosophical and legal studies and the economic studies on which he would focus in the later stage of his exile in England.

Bauer merely argued against the religious expression of such ‘antitheses’ as “the conflict between the general interest and private interest, the schism between the political state and civil society”, instead of facing the antitheses themselves. Nevertheless, Marx recognized that though “political emancipation” was not the “final form of human emancipation in general,” it was a great step forward. But the way to politically achieve emancipation from religion was to take religion out of the sphere of public law and place it instead in the sphere of private law.

Religion is no longer the spirit of the state, in which man behaves —although

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1 One should bear in mind that Napoleon’s defeat led to the loss of many of the rights that the German Jewish community had gained by imposition of the French invaders. Only with the Revolutions of 1848 would there be an important reversal towards equality of rights.
in a limited way...— as a species-being, in community with other men. Religion has become the spirit of civil society, of the sphere of egoism, of bellum omnium contra omnes. It is no longer the essence of community, but the essence of difference. It has become the expression of man’s separation from his community, from himself and from other men (Ibid., 155).

Hence, the displacement of religion from the State to civil society was the culmination of political emancipation, not simply one of its stages. It “neither abolishes the real religiousness of man, nor strives to do so.” Since for Marx religion was—we quote again—“the sigh of the oppressed creature, the heart of a heartless world, just as it is the spirit of spiritless conditions”, individuals would only be able to gradually overcome it after the overthrow of “all relations in which man is a debased, enslaved, forsaken, despicable being” (Ibid., 182).

For Marx the religious sphere only has a weak correspondence with the economic sphere of society in which it flourishes; because human societies’ cultural expressions only change slowly. But he saw that the presence of religion at the State level in Germany did not correspond to what occurred in the typical advanced democratic State (he uses the expression “perfect democracy”). The latter “does not need religion for its political completion” (Ibid., 157). In the democratic State (in general) individuals “are religious owing to the dualism between individual life and species-life, between the life of civil society and political life... They are religious insofar as religion here is the spirit of civil society, expressing the separation and remoteness of man from man.” But in “the perfect democracy... Christianity attains here the practical expression of its universal-religious significance in that the most diverse world outlooks are grouped alongside one another in the form of Christianity and still more because it does not require other people to profess Christianity, but only religion in general, any kind of religion” (Ibid., 159).

Hence, for Marx “political emancipation from religion leaves religion in existence, although not a privileged religion” as Christian religion was (and especially Luther’s Reformed Christian religion) in the relatively backward Germany. But the “emancipation of the state from religion is not the emancipation of the real man from religion” (Ibid., 160). In contrast with Bauer, he said to the Jews: “Because you can be emancipated politically without renouncing Judaism completely and incontrovertibly, political emancipation itself is not human emancipation.” And if the Jews of Germany sought political emancipation without having achieved human emancipation, that “half-hearted approach” was not specifically theirs since it was underlying in the very concept of political emancipation. Christian Germans had political emancipation but, like Jewish Germans, lacked human emancipation since the latter could only be achieved by overthrowing the conditions that prevented individuals from living as a community—from having a ‘species-life’,— by abolishing the conditions that were an impediment to overcoming this ‘alienation’.

Bauer had written that “man has to sacrifice the ‘privilege of faith’ to be able to receive the universal rights of man.” In order to disprove him, Marx examined “the rights of man in their authentic form, in the form which they have among those who discovered them, the North Americans and the French” (Ibid.). He classifies them as 1) ‘political rights’, or ‘rights of the citizen’, and 2) the remaining ‘rights of man’. The former “can only be exercised in a community with others.” They are the right to participate “in the community, and specifically in the political community, in the life of the state.” These rights do not presuppose the abolition of religion, whether Jewish or Christian. The rights in 2) are the ‘rights of man’ that go beyond the ‘rights of the citizen’. They include “freedom of conscience, the right to practise any religion one
chooses” and also the rights related to the economic life in civil society. Marx quoted articles of two versions of the “Declaration of the rights of man and the citizen” (that of 1789 and the more radical one of 1793, when the Jacobins were in power), as well as articles from the Constitutions of the American states of Pennsylvania and New Hampshire. The first three of these stipulated the right to practice one’s own religion and the fourth the right to freedom of conscience. It is very significant that Marx characterized the rights related to economic life as “the rights of a member of civil society, i.e., the rights of egoistic man, of man separated from other men and from the community” (Ibid., 162).

The Declaration of 1793 begins by stating that Government is instituted in order to guarantee men their natural rights, which are equality, liberty, security, and property. Liberty is defined as the power that belongs to man to do whatever is not injurious to the rights of others. And for Marx the practical application of this right is the right to private property. Article 16 says “The right of property is that which belongs to every citizen to enjoy, and to dispose at his pleasure of his goods, income, and of the fruits of his labor and his skill.” For Marx this implied the right to enjoy and dispose of one’s property “without regard to other men, independently of society,” and was hence the right to egoism. “This individual liberty and its application form the basis of civil society. It makes every man see in other men not the realisation of his own freedom, but the barrier to it” (Ibid., 163). And Article 8 said that “Security consists in the protection afforded by society to each of its members for the preservation of his person, his rights, and his property.” Marx notes that this is the “concept of police” and that hence “The concept of security does not raise civil society above its egoism. On the contrary, security is the insurance of its egoism.” He concludes then that such rights of man do not conceive of man as a species-being and hence society appears “as a framework external to the individuals, as a restriction of their original independence. The sole bond holding them together is natural necessity, need and private interest, the preservation of their property and their egoistic selves” (Ibid., 164).

For Marx the ‘political emancipation’ achieved by the bourgeois revolutions had not resulted in an increased integration between man’s economic role in society, characterized by unrestrained egoism, and man’s moral duty towards his brethren. On the contrary, it had led to a dual mankind, to “the reduction of man, on the one hand, to a member of civil society, to an egoistic, independent individual, and, on the other hand, to a citizen, a juridical person.” For “Throwing off the political yoke meant at the same time throwing off the bonds which restrained the egoistic spirit of civil society” (Ibid., 167). In civil society there was an industrial and commercial life in which there was no room for solidarity with others, in which each seeks his own benefit. The moral, species-being, communitarian, man that to some extent existed in the guilds became restricted to the sphere of the State, where public policies seeking to diminish the disparities generated by economic life may or may not exist. For Marx ‘human emancipation’ required putting back together what had been separated, and would only be possible when “individual man re-absorbs in himself the abstract citizen, and as an individual human being has become a species-being in his everyday life, in his particular work, and in his particular situation.” When the bourgeois political revolution got rid of feudal bonds and medieval corporations, it dissolved them into a set of independent and egoistic individuals, it separated the man from the citizen. And hence, this revolution “resolves civil life into its component parts, without revolution-

\[2\] Notice that Marx is clearly alluding to Rousseau’s notion that man in society loses the original independence of ‘natural’ man. Rousseau’s writings had been an important source of inspiration for the French revolutionaries, and especially the Jacobins.
ising these components themselves or subjecting them to criticism... man as a member of civil society is held to be man *in the proper sense, homme* as distinct from the *citoyen*, because he is man in his sensuous, individual, *immediate* existence, whereas *political* man is only abstract, artificial man, man as an *allegorical, juridical* person” (Ibid., 167). And at this point Marx explicitly quotes Rousseau’s correct description of “the abstract idea of political man”, when in his *Contrat Social* he praises the great legislator that establishes the basic rules of a society:

> He who dares undertake to give institutions to a people ought to feel himself capable of changing, so to speak, human nature; of transforming every individual, who in himself is a perfect and solitary whole, into part of a greater whole, from which he receives in some manner his life and his being; of altering man’s constitution in order to strengthen it; of substituting a partial and moral existence for the independent and physical existence which we have all received from nature. In a word, it is necessary to deprive man of his own powers in order to give him others which are alien to him, and of which he cannot make use without the aid of other people.³

In Rousseau, the ‘natural state’ of man (a fictitious state he had invented in which all men are completely independent of one another) reflected the charm of innocence which man in society had lost. Since men had very few relations between them (aside from those within the family) there were allegedly no wars nor slavery in this natural state. But even though “progress” had brought about all kinds of defects, some societies were better organized than others. A wise legislator was one who established institutions more adequate for the particular characteristics of the territory and persons living in it. A ‘social contract’ was established by which the individual (“a perfect and solitary whole”) gave up his primitive independence in order to become a component of society (“part of a greater whole”), having thus a “partial and moral existence.” In contrast, for Marx, whereas the *political emancipation* achieved by the French revolutionaries –inspired by Rousseau– had *separated* the individual into an independent egoist that operated in the economy and a moral or political person that acted as a citizen of a State, *human emancipation* would only be achieved by *reunifying* the economic and the political individual, the *man* and the *citizen*:

> Only when the real, individual man re-absorbs in himself the abstract citizen, and as an individual human being has become a *species-being* in his everyday life, in his particular work, and in his particular situation, only when man has recognised and organised his ‘forces propres’ as social forces, and consequently no longer separates social power from himself in the shape of *political* power, only then will human emancipation have been accomplished (Ibid., 168).

Marx viewed as pre-conditions for human emancipation the suppression of commerce, money, and private property, which for him entailed anti-social aspects of contemporary society. And with respect to the concrete theme of Bauer’s articles, he admitted that since Jewish traders and bankers had significantly contributed to the historical development of commerce and money –which gave them great economic power– they had played a role in the gestation of these anti-social aspects. He quotes

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³We have here made slight changes to the text in Rousseau (2002, 181) to bring it closer to the original French text in Rousseau (2012, §232).
Bauer when he writes: “The Jew, who in Vienna, for example, is only tolerated, determines the fate of the whole Empire by his financial power” (Ibid., 170). But Marx also held that Christianity had absorbed these anti-social aspects and, even more, had developed them. Hence, he could say that although “The Jew has emancipated himself in a Jewish manner” by acquiring financial power, “The Jews have emancipated themselves insofar as the Christians have become Jews.” For “Only under the dominance of Christianity, which makes all national, natural, moral, and theoretical conditions extrinsic to man, could civil society separate itself completely from the life of the state, sever all the species-ties of man, put egoism and selfish need in the place of these species-ties, and dissolve the human world into a world of atomistic individuals who are inimically opposed to one another” (MECW 3, 173).

**Comments** Marx’s thoughts on the secular character of advanced capitalist society and his critique of Bauer’s narrow viewpoint are very inspiring. So is his view that in a desirable society it should be possible to ensure that the most disadvantaged and defenseless be protected by the rest of the community. But his posture on the egoism of civil society and ‘human emancipation’ aimed far beyond the need for public policies aimed at leveling the most extreme inequalities generated in civil society through economic mechanisms. It is very significant in this sense that he did not mention Article 21 of the 1793 Declaration (the one he most quoted), which says that “Public relief is a sacred debt. Society owes maintenance to unfortunate citizens, either procuring work for them or in providing the means of existence for those who are unable to labor;” nor did he mention Article 22 that says that society should “put education at the door of every citizen.” These omissions show that Marx had in mind something much more revolutionary than the most radical formulations of the Jacobins, the most radical politicians of the French Revolution.

But his stance is also more vague, abstract, and somewhat messianic. What concrete meaning can we find in the conversion of each “individual human being” into a “species-being in his everyday life, in his particular work, and in his particular situation”? If there is a dichotomy between the mechanical, chaotic, random and amoral ‘industrial and commercial’ life, and the “species-being, in community with other men” (i.e., moral man), that remains constrained to the State sphere (where policies such as those of Articles 21 and 22 can be implemented), we can infer nothing about the type of society distinct from Capitalism that would prevent the existence of such a dichotomy, or, even more ambitious, transform the individual man into a “species-being” in his daily life. Moreover, we can argue that such a dichotomy does not exist, since both in the State sphere as in industrial and commercial life individuals interact with various degrees and types of morality (or amorality, or immorality). Marx’s stance was that the achievement of ‘human emancipation’ required the elimination of private economic life based on the functioning of markets, money, and private property, leaving only public economic life, where universal human solidarity between individuals could supposedly be achieved. But he gives neither evidence nor reasoning as to why such an idyllic life could be achieved by such a change in economic organization.

The idea of (the Eden) of a communitarian life or ‘species-life’ that was lost with the evolution of society has a resemblance to Rousseau’s idea of a lost ‘state of nature’. We will see that in time Marx would leave behind the nostalgic idea of a lost ‘species-life’. But inverting its place in the arrow of time, he would transform it into the final goal of full Communism, where in some sense the communitarian ‘species-life’ could be achieved. But it would only be possible on the basis of the great advance in the productive forces achieved by Capitalism, and after a period in which the prole-
tariat in power eliminates the private property of the means of production, markets, money, and even the division of labor. However, it is extremely difficult to think in concrete terms on a complex and populated society that can adequately function without those auxiliary tools that have been gestated over thousands of years of economic and social development. And this is possibly the greatest contradiction in all of Marx’s ideological-scientific stance, as we have seen, he always emphasized the increasing complexity of economic life in the development of Capitalism, brought about in parallel to the increasing division of labor. From a profoundly moral, but abstract, schematic, and in the last instance false conception, even after deepening his economic studies and elaborating Capital, he seemed to believe that the mode of production of future society would have laws so different from those of the past that the institutions there established would bring about a radical change in ‘human nature’ (as in the above quoted paragraph of Rousseau).

In On the Jewish Question Marx inserts an interesting reflection on the attempted abolition of religion and private property in the most radical phase of the French Revolution. He writes: “in periods when the political state as such is born violently out of civil society, when political liberation is the form in which men strive to achieve their liberation, the state can and must go as far as the abolition of religion” as it also “proceeds to the abolition of private property” (Ibid., 156). For at “times of special self-confidence, political life seeks to suppress its prerequisite, civil society and the elements composing this society, and to constitute itself as the real species-life of man devoid of contradictions.” But this can only be achieved by means of “violent contradiction with its own conditions of life” and hence “the political drama necessarily ends with the re-establishment of religion, private property, and all elements of civil society.” Marx’s reflection is that at times of revolution its leaders tend to overshoot in their aspirations in comparison with the changes that the economic base of society justify and allow. But when the revolutionary foam disappears, the non-sustainable changes must be reversed. With the benefit of hindsight, we can say that Marx’s keen observations have been verified after Marx’s death. In the two large countries in which the type of revolution Marx later proposed triumphed – Russia and China– the initial elimination of religion eventually had to be reversed. However, in dissonance with Marx’s political project, the same happened with the abolition of the private property of the means of production, which finally had to be reinstated, at least in terms of the legal existence of private firms, and after huge and indescribably painful disruptions in the socioeconomic lives of these societies.

Economic and Philosophical Manuscripts (1844)

While in Paris, between April and August of 1844 Marx wrote some Manuscripts on Economics and Philosophy, of which only three partially survived and were published almost half a century after his death. They were written during the period in which Marx was strongly impacted by Feuerbach. Some parts exhibit a prophetic and almost religious vision of the beneficial effects that Communism could have through its “positive transcendence of private property as human self-estrangement” (Manuscripts, 296). In a draft for a Preface Marx writes that his “results have been attained by means of a wholly empirical analysis based on a conscientious critical study of political economy” and that he has used French, English, and German socialist sources (Ibid, 231-2). He mentions in particular some of his German sources: the article Outlines of a Critique of Political Economy that Engels had published in the Yearbook, “Weitling’s writings”, and three articles Moses Hess had published in a Swiss journal: “Socialism
and Communism”, “The One and All Freedom”, and “Philosophy of Action.” As is evident from last title, at this stage Hess was strongly influenced by Feuerbach, as Marx was. Among his philosophical sources, aside from Hegel’s Phenomenology and Logic, Marx specifically mentioned Feuerbach’s Philosophy of the Future and Thesis for the reform of Philosophy. He states: “It is only with Feuerbach that positive, humanistic and naturalistic criticism begins.”

After linking Political Economy with alienated labor in the First Manuscript, in the short Second Manuscript Marx briefly addressed one of his life-long concerns: the misery of unemployment. The worker’s life “is looked upon as a supply of a commodity like any other.” “As soon, therefore, as it occurs to capital (whether from necessity or caprice) no longer to be for the worker... he has no work, hence no wages, and since he has no existence as a human being but only as a worker he can go and bury himself, starve to death... Political economy, therefore, does not recognise the unemployed worker, the workingman, insofar as he happens to be outside this labour relationship. The rascal, swindler, beggar, the unemployed, the starving, wretched and criminal workingman—these are figures who do not exist for political economy... For it, therefore, the worker’s needs are but the one need—to maintain him whilst he is working and insofar as may be necessary to prevent the race of labourers from [dying] out” (Ibid., 283-4).

And in the Third Manuscript Marx holds that Communism is “the positive transcendence of private property human self-estrangement” (Ibid., 296). “It takes actual communist action to abolish actual private property. History will lead to it; and this movement, which in theory we already know to be a self-transcending movement, will constitute in actual fact a very rough and protracted process. But we must regard it as a real advance to have at the outset gained a consciousness of the limited character as well as of the goal of this historical movement—and a consciousness which reaches out beyond it” (Ibid., 313). For Communism already had a history (Babeuf, Saint-Simon, Fourier, Cabot, etc.) and it had a future. Marx distinguished different forms of Communism. The first of these was a “completely crude and thoughtless communism” (Ibid., 296), and was “merely a manifestation of the vileness of private property” (Ibid., 295). Here, “the dominion of material property bulks so large that it wants to destroy everything which is not capable of being possessed by all as private property. It wants to disregard talent, etc., in an arbitrary manner” (Ibid., 294). Instead of doing away with the ‘category of worker’, it seeks to extend it to all men. Marx was probably referring here to the type of Communism proposed by Babeuf during the French Revolution and other even more embryonic forms known in history. Just a few months after Marx’s writing of the Manuscripts, Engels published (in The New Moral World No. 19, November 4, 1843) his article Progress of Social Reform on the Continent, where he says that Babeuf’s revolutionary plot did not succeed because “Communism itself was of a very rough and superficial kind” (MECW 3, 394). The second form of Communism Marx mentions is still of political nature, whether democratic or despotic. Although the State is abolished (as a mechanism of oppression), society is “still affected by private property, i.e., by the estrangement of man” (Ibid., 296).

Finally, there was “Communism as the positive transcendence of private property human self-estrangement, and therefore as the real appropriation of the human essence by and for man.” This form would bring forth a “complete return of man to himself as a social (i.e., human) being... embracing the entire wealth of previous development.” And this form of Communism was “the riddle of history solved” (Ibid., 297). In the prophetic and romantic vision of young Marx this form of Communism was “the genuine resolution of the conflict between man and nature and between man and man... between
the individual and the species” (Ibid., 296). For Marx “communism, as the supersession of private property, is the vindication of real human life as man’s possession and thus the advent of practical humanism... communism is humanism mediated with itself through the supersession of private property. Only through the supersession of this mediation –which is itself, however, a necessary premise– does positively self-deriving humanism, positive humanism, come into being” (Ibid., 341-2).

Money was another mediation to be surpassed. “By possessing the property of buying everything... money is thus the object of eminent possession” (Ibid., 296-7). “That which I am unable to do as a man, and of which therefore all my individual essential powers are incapable, I am able to do by means of money” (Ibid., 325). But money “confounds and confuses all things, it is... the confounding and confusing of all natural and human qualities. He who can buy bravery is brave, though he be a coward” (Ibid., 326). Contrasting with the purely mercantile human relations and the values they tend to impose, Marx invoked a ‘humanist’ vision of the non-mercantile relations between human beings: “Assume man to be man and his relationship to the world to be a human one: then you can exchange love only for love, trust for trust, etc... If you love without evoking love in return –that is, if your loving as loving does not produce reciprocal love; if through a living expression of yourself as a loving person you do not make yourself a beloved one, then your love is impotent –a misfortune” (Ibid.). With very romantic phrases Marx expressed his admiration for the ‘communist artisans’, and especially ‘French socialist workers’, who when they associated with one another for theory and propaganda “acquire a new need –the need for society– and what appears as a means becomes an end... the brotherhood of man is no mere phrase with them, but a fact of life, and the nobility of man shines upon us from their work-hardened bodies” (Ibid., 165; italics added).

Marx wanted to overcome the mediations that impede the development of fraternal human relations by means of a political praxis that first transforms the existing communist movement. And, in contrast with his romantic phrases, Marx came to the materialistic conclusion that the revolutionary movement had to have its foundation on the understanding of the functioning of the economy: “It is easy to see that the entire revolutionary movement necessarily finds both its empirical and its theoretical basis in the movement of private property –more precisely, in that of the economy” (Ibid., 297). We have there, condensed in two lines, the substance of Marx’s life project when he was 25 or 26 years old. Based on the conviction that an adequate reconstruction of Political Economy could contribute to the scientific understanding of what is necessary to accomplish such ambitious and laudable aims as achieving the “complete return of man to himself as a social (i.e., human) being”, Marx embarked on his lifelong journey of studying Political Economy from a scientific viewpoint but with a very ambitious practical aim, that is, with a ‘political’ intention in the noble sense of the word: the moral sense. And Marx’s view of the social and moral sense of scientific activity is also to be found in these Manuscripts: “But also when I am active scientifically, etc. –an activity which I can seldom perform in direct community with others– then my activity is social, because I perform it as a man. Not only is the material of my activity given to me as a social product (as is even the language in which the thinker is active): my own existence is social activity, and therefore that which I make of myself, I make of myself for society and with the consciousness of myself as a social being” (Ibid., 298). The combination of scientific and political activity, both linked to the moral element but inevitably exercised by means of completely different procedures, was the common thread of all of Marx’s efforts after completely abandoning Philosophy as a specific activity.
The German Ideology (1845-1846)

We have already seen the importance this long book had in the gestation of Historical Materialism. One of ideas that Marx and Engels sketch here has to do with the objectives of their political activity, and concerns the negative aspects of the division of labor as it developed along with commerce and markets. It implied “the unequal distribution, both quantitative and qualitative, of labour and its products, hence property”, which is initially to be found in the family, “where wife and children are the slaves of the husband” (Ideology, 46). The division of labor also implied a “contradiction between the interest of the separate individual or the individual family and the common interest of all individuals who have intercourse with one another.” For “as long as man remains in naturally evolved society, that is, as long as a cleavage exists between the particular and the common interest, as long, therefore, as activity is not voluntarily, but naturally, divided, man’s own deed becomes an alien power opposed to him, which enslaves him instead of being controlled by him.” Each individual “has a particular, exclusive sphere of activity, which is forced upon him and from which he cannot escape. He is a hunter, a fisherman, a shepherd, or a critical critic, and must remain so if he does not want to lose his means of livelihood” (Ibid., 47). In contrast to “historical development up till now”, in which the production of individuals had consolidated into “a material power above us, growing out of our control”, in communist society—the long run objective—each individual “can become accomplished in any branch he wishes, society regulates the general production and thus makes it possible for me to do one thing today and another tomorrow... without ever becoming hunter, fisherman, shepherd or critic” (Ibid.).

In past historical development, owing to the “contradiction between the particular and the common interests, the common interest assumes an independent form as the state, which is divorced from the real individual and collective interests, and at the same time as an illusory community, always based, however, on the real ties existing in every family conglomeration and tribal conglomeration... and especially, as we shall show later, on the classes, already implied by the division of labour, which in every such mass of men separate out, and one of which dominates all the others” (Ibid., 46). Hence, “all struggles within the state, the struggle between democracy, aristocracy, and monarchy, the struggle for the franchise, etc., etc., are merely the illusory forms—altogether the general interest is the illusory form of common interests— in which the real struggles of the different classes are fought out among one another” (Ibid., 46-7). We interpret that it is the owning classes that are (more or less) represented in the State and, as they only constitute a fraction of society, the State is only an “illusory community.” Although the struggles that take place “within the state” are concerned with conflicts of interests that actually exist in society, they are “illusory forms” because the State “is divorced from the real individual and collective interests.” The State is located in a sphere that is different from the economic sphere in which the divergence of interests arise. In the sphere of the State ideologies play an important role. The “general interest” is an “illusory form” of certain “common interests.” Political struggles have a deceitful (“illusory”) appearance and it is necessary to see beneath the appearance to understand the underlying processes. And these are the overpowering markets, in which “The social power, i.e., the multiplied productive force, which arises through the co-operation of different individuals as it is caused by the division of labour, appears to these individuals... as an alien force existing outside them, of the origin and goal of which they are ignorant, which they thus are no longer able to control, which on the contrary passes through a peculiar series of phases and stages independent of the will
and the action of man, nay even being the prime governor of these” (Ibid., 48).4

This “alien force” can only be abolished “by the individuals again subjecting these material powers to themselves and abolishing the division of labour” (Ibid., 78). And this can only be achieved within the ‘community’: “Only within the community has each individual the means of cultivating his gifts in all directions; hence personal freedom becomes possible only within the community. In the previous substitutes for the community, in the state, etc., personal freedom has existed only for the individuals who developed under the conditions of the ruling class, and only insofar as they were individuals of this class” (Ibid.). The State was merely a substitute for the true community, and only the latter was capable of guaranteeing the personal freedom of all individuals and not merely those of the dominant class. The class State was actually only a ‘community’ for the dominant class, and “since it was the combination of one class over against another, it was at the same time for the oppressed class not only a completely illusory community, but a new fetter as well.” In contrast, in “the real community the individuals obtain their freedom in and through their association” (Ibid.). Marx abstractly makes a correspondence between “the real community” in which the individuals overcome the division of labor and the ‘association’ between free individuals. We will see that this abstraction, which has some similarity to a ‘return’ to a lost Eden –though under the assumption that there has previously been a huge development of the productive force of labor–, was maintained by Marx to the end of his intellectual and political life, albeit relegated to a ‘second phase’ of Communist society, after a ‘first phase’ of undetermined duration that from 1850 on he called the “dictatorship of the proletariat.”

A conclusion of Ideology is that the proletarians must abolish (alienated) labor “if they are to assert themselves as individuals”; that this must abolish “the hitherto prevailing condition of their existence (which has, moreover, been that of all society up to then)”; that they must overthrow the State, “the form in which, hitherto, the individuals, of which society consists, have given themselves collective expression” (Ibid., 80). But how could this be materialized? For Marx it was essential to transform the consciousness of the masses, but this could only be done within a process in which the revolution itself is a transforming agent of beliefs and forms of association: “Both for the production on a mass scale of this communist consciousness and for the success of the cause itself, the alteration of men on a mass scale is necessary, an alteration which can only take place in a practical movement, a revolution” (Ibid., 52-3). This revolution would be instrumental in transforming the proletariat’s consciousness (“ridding itself of all the muck of ages”) and making it fit “to found society anew”.

For Marx “the real intellectual wealth of the individual depends entirely on the wealth of his real connections” (Ibid., 51) with other individuals. Hence, the fact that class membership is such a strong determinant of the relations between individuals was problematic and had to be surpassed. Based on this belief he concluded that the first step in this overcoming had to take place within the “community of revolutionary proletarians”, vanguard of the future superior community, since this association would “take their conditions of existence and those of all members of society under their control” (Ibid., 80). Up to the present, “the communal relation into which the individuals of a class entered, and which was determined by their common interests as against a third party, was always... a relation in which they participated not as individuals

4Marx adds: “Or how does it happen that trade... rules the whole world through the relation of supply and demand”; and further below: “In history up to the present... individuals have... become more and more enslaved under a power alien to them... and, in the last instance, turns out to be the world market (Ideology, 51).
but as members of a class.” In contrast, the community of revolutionary proletarians would “take their conditions of existence and those of all members of society under their control” as participating individuals. And, always assuming a sufficiently high level of development of the productive forces, this “is the association of individuals... which puts the conditions of the free development and movement of individuals under their control—conditions which were previously left to chance and had acquired an independent existence over against the separate individuals” (Ibid.). By the expression “left to chance” the authors referred to the markets, which would be substituted by centralized economic planning by the revolutionary proletarians once in power, for “in communist society... society regulates the general production” (Ibid., 47).

We may remark that it is quite obvious that any particular group can “take their conditions of existence” under their control as long as it can finance its activities. But to think that such a group can take under their control the conditions of existence “of all members of society” only seems possible under conditions of a dictatorship of that group over at least a great part of society. With hindsight, we can interpret that this is precisely what happened in actual fact when the Bolsheviks came to power in Russia in the last months of 1917 and also when the Communists came to power in China in 1949. Hence, it is convenient to remember that the reasoning that in Ideology seems very abstract actually had very important practical realizations several decades later in these countries (and several other smaller countries), no matter what one’s evaluation of the later evolutions of those experiences may be. One crucial question of course is whether the dictatorship of a revolutionary party necessarily evolves into a new class society with a new dominant class or, eventually, the same old one.

For Marx “every class which is aiming at domination... must first conquer political power in order to represent its interest in turn as the general interest, which in the first moment it is forced to do” (Ibid., 47). And this was also true for the very special case of the working class. The “estrangement” of social power brought about by markets could only be abolished if there was a marked polarization between a great propertyless mass and a “world of wealth and culture.” And it required “a great increase in productive power” because “without it privation, want is merely made general” (Ibid., 48-9). But this increase in productive power had to be on a world scale, and interlinked by world markets, so that the polarization between the propertyless and the wealthy would also take place on a world scale, leading to an interlinking of revolutions in different nations. “Without this, 1) communism could only exist as a local phenomenon; 2) the forces of intercourse themselves could not have developed as universal, hence unendurable powers: they would have remained home-bred ‘conditions’ surrounded by superstition; and 3) each extension of intercourse would abolish local communism” (Ibid., 49). Then, instead of the ‘invisible hand’ of the relation between supply and demand that “allots fortune and misfortune to men, sets up empires and wrecks empires”, there would be a “communistic regulation of production (and, implicit in this, the abolition of the alien attitude of men to their own product), the power of the relation of supply and demand is dissolved into nothing, and men once more gain control of exchange, production and the way they behave to one another” (Ibid., 48).

In Marx and Engel’s 1845-46 vision of a future communist society, the rational and fraternal production and distribution of output—which would replace the blind, random, and insensitive markets—would liberate humanity from the ‘alienated’ labor generated by the division of labor in mercantile capitalist society. The prophetic character of Marx and Engels’ Communism is manifested in particular in the following comment: “Communism is for us not a state of affairs which is to be established, an ideal to which reality [will] have to adjust itself. We call communism the real movement
which abolishes the present state of things” (Ibid., 49). If we take this literally, the political project that Marx and Engels would months later contribute to specify through the Communist League would merely try to clarify the actual tendency of events and advance it in time. But, can we take their prophecy as more than an article of faith? Events of great magnitude in human societies are the outcome of the actions of very many wills with different and often antagonistic views and interests. Nobody can anticipate with any degree of precision the result of so many wills with various degrees of coincidence or divergence. Having said this, it is necessary to recognize that the decision that triggers the action of any individual makes it necessary for him to neglect the fact that the effects of this action and of future events cannot be predicted accurately but nevertheless requires the formulation of a subjective image of how the events that one wants to influence will (probably) unfold. Otherwise one would never go beyond the contemplative attitude Marx had criticized in his ‘theses on Feuerbach’. The vision of the future of any potential leader who can formulate his project with information and intelligence, and express it convincingly, is much more capable of engaging more followers than alternative visions and would be leaders. Marx was particularly gifted in the art of giving his vision a rational, informed, and convincing expression. And like so many other founders of great movements, he struggled with determination against similar views that diverged in some aspects he considered vital, and thus he formulated implacable critiques of various thinkers he departed from, such as Hegel, Bauer, Feuerbach, Saint-Simon, Proudhon, and Lasalle. It is remarkable that he was able to advance so much in his political project before having focused his colossal efforts on Political Economy. But it is even more remarkable that he modified the teleological aspects of his worldview so little after some 15 years of painstaking and systematic construction of his magnum opus.

The Poverty of Philosophy (1847)

Shortly after The German Ideology, Marx had the opportunity to again address some of its topics when he wrote and published his critique of Proudhon in The Poverty of Philosophy. We are not interested here in what Proudhon said in the book subject to critique, but in extracting from the non-critical parts of Marx’s book his thoughts about the future society he was striving for and the means to be used to attain it. He contrasted the role of ‘economists’ with that of ‘socialists’ and ‘Communists’ from a class struggle perspective projected onto social science, stating: “Just as the economists are the scientific representatives of the bourgeois class, so the socialists and the Communists are the theoreticians of the proletarian class” (Poverty, 177). Different schools of economists had emerged, reflecting different ideological views. There were fatalists (both Classics and Romantics) and humanitarians. The latter at least wanted to palliate the worst effects of capitalism on workers, advising them “to be sober, to work hard and to have few children” (Ibid.). There was a ‘philanthropic’ version of the ‘humanitarian school’, that denied the necessity of class antagonisms and tended to convert theory into an idealization of reality. Marx held that so long as a) Capi-

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5On a religious level, the case of Martin Luther comes to mind. For, apart from his struggle against the Church in Rome (and his various pathological hatreds), he permanently struggled against other reformists like Zwingli and Müntzer, and even against some who emerged from his own womb such as Karlstadt and Melanchthon (Roper 2017). However, we should not stretch the analogy, since Luther and other religious reformists never indulged in scientific activity to build a foundation for their doctrines. Marx, in contrast, made enormous efforts to understand the functioning of the capitalism he wanted to abolish. In the process, he generated some authentic scientific progress, even if it was framed within a political project inspired by his underlying millenarian vision of the future.
talism has not yet sufficiently developed the productive forces, and b) the struggles of the working class against capitalists have not yet assumed a political character (which he also refers to as the proletariat not yet being sufficiently developed “to constitute itself as a class”), “these theoreticians are merely Utopians who, to meet the wants of the oppressed classes, improvise systems and go in search of a regenerating science,” not noticing that the misery of the proletariat has a “revolutionary, subversive side, which will overthrow the old society” (Ibid., 178). But once theoreticians understand this, “science, which is produced by the historical movement and associating itself consciously with it, has ceased to be doctrinaire and has become revolutionary” (Ibid.). We thus have here, in compressed synthesis, the amalgamation that Marx upheld between the science of human society, that seeks to understand the historical processes, and the political activity, that using its results, assumes the representation of the revolutionary force, in this case the working class, in order to radically transform society, but only if the necessary conditions for this are given (i.e., a) and b) have been surpassed).

Marx observes that “While inside the modern workshop the division of labour is meticulously regulated by the authority of the employer, modern society has no other rule, no other authority for the distribution of labour than free competition” (Ibid., 184). And “If one took as a model the division of labour in a modern workshop, in order to apply it to a whole society, the society best organised for the production of wealth would undoubtedly be that which had a single chief employer, distributing tasks to the different members of the community according to a previously fixed rule” (Ibid.). This inference was based on his perception that “authority in the workshop and authority in society, in relation to the division of labour, are in inverse ratio to each other”, since “It can even be laid down as a general rule that the less authority presides over the division of labour inside society, the more the division of labour develops inside the workshop, and the more it is subjected there to the authority of a single person” (Ibid., 185). The transformation of society realized by the proletariat endowed with political power would modify the balance between authority in the workshop and authority in society. Market competition would be replaced by a central authority, and this would allow the relaxation of authority in the factory, transforming it into an ‘association’ of workers. In the new society the concern of industrial production would be the product itself and not the profit to be obtained by selling the product. Whereas Proudhon believed that those socialists who wanted to eliminate competition because it was a form of egoism were mistaken, Marx held that “Socialists know well enough that present-day society is founded on competition,” and that a positive aspect of competition was that it “always becomes the more destructive for bourgeois relations in proportion as it urges on a feverish creation of new productive forces, that is, of the material conditions of a new society” (Ibid., 193-4) in which competition would no longer be needed. As “competition was engendered by feudal monopoly”, in modern industry competition leads to monopoly and to increased polarization between workers and capitalists. “If the monopolists restrict their mutual competition by means of partial associations, competition increases among the workers; and the more the mass of the proletarians grows as against the monopolists of one nation, the more desperate competition becomes between the monopolists of different nations... monopoly can only maintain itself by continually entering into the struggle of competition” (Ibid., 195-6).

Another fact of the development of industrial Capitalism was the continual growth of workers’ combinations. These were so characteristic of industrialization that “the degree to which combination has developed in any country clearly marks the rank it occupies in the hierarchy of the world market” (Ibid., 210), so that permanent
combinations had been formed England in the form of ‘trades unions’. The mass of wage workers clearly had common interests to struggle for and was hence “already a class as against capital.” But it was not yet “a class for itself”, which required going beyond union action by also embracing political action. The bourgeoisie had “constituted itself as a class under the regime of feudalism and absolute monarchy” and in a second phase of development had been able to overthrow both. Similar phases were to be expected for the working class, the oppressed class. But, “For the oppressed class to be able to emancipate itself it is necessary that the productive powers already acquired and the existing social relations should no longer be capable of existing side by side” (Ibid., 211).

Marx asked whether “after the fall of the old society there will be a new class domination culminating in a new political power?” (Ibid.). Before answering, Marx poses a strange analogy. He writes that the “condition for the emancipation of the working class is the abolition of all classes, just as the condition for the emancipation of the third estate, of the bourgeois order, was the abolition of all estates and all orders.” In the German edition of 1885 Engels explains in a footnote that “Estates here in the historical sense of the estates of feudalism, estates with definite and limited privileges. The revolution of the bourgeoisie abolished the estates and their privileges. Bourgeois society knows only classes.” The bourgeoisie had eliminated the estates of the feudal State (the clergy and the nobility in the case of France) but society was still divided into social classes –defined in terms of the economic functioning of civil society– and what Marx posed as an objective was the elimination of classes, so as to reach a society that is in a sense simpler (since it is not stratified): “The working class, in the course of its development, will substitute for the old civil society an association which will exclude classes and their antagonism, and there will be no more political power properly so-called, since political power is precisely the official expression of antagonism in civil society” (Ibid.). Marx posed as the objective of the working class’s transformation of society the elimination of the capitalist class, which not only dominated over the workers in firms but also dominated society in general through their control over the State. The latter exerted the political power that settled the disagreements between sectors of the dominant class and ensured the domination over the working class. Since in the transformed society there would be no capitalists and hence no conflicts between workers and capitalists, “political power properly so-called” would disappear. And Marx assessed that once class distinctions were eliminated there would be a completely different dynamics of social change, since it would no longer require political revolutions: “It is only in an order of things in which there are no more classes and class antagonisms that social evolutions will cease to be political revolutions” (Ibid.).

In light of the experience in the time elapsed since Marx wrote, we can here remark that it was highly adventurous to state that the society best organized for the production of wealth would be one with a “single chief employer, distributing tasks to the different members of the community according to a previously fixed rule.” Marx understood this as the elimination of private firms, each with its own direction, and their replacement by firms operated in its fundamental decisions from a single central regulator. Today we know that, concerning production, decentralization has advantages in efficiency (to a large extent based on the quick gathering of new information and adjusting operations in consequence); and also that state bureaucracy has efficiency disadvantages. Also, there is no doubt that there have been important advances towards macroeconomic regulations from Government agencies that allow for a better coordination between private firms and a dampening of economic fluctuations through
adequate adjustments in monetary, fiscal, and exchange policies to exogenous shocks. Furthermore, at least in the developed world there have been important advances (though not nearly enough) in the generation of mechanisms that reduce unemployment and palliatives that reduce the suffering it causes. And even Russia and China returned to the widespread functioning of private firms after having prohibited them during several decades.

Marx would leave floating too many questions that begged for meditated answers before the launch of his radical proposals. How would the central allocating agency function? What would be the criteria for the establishment of his “previously fixed rule” for the allocation of labor? ¿How would the laborers’ works be coordinated within each producing unit without some kind of hierarchical authority? ¿How would the workers’ representatives come to an agreement on the decisions to be made? And last but primordial, how would it be possible to avoid that the very function of holding and using power in the new workers’ state generate a new dominant social class with its own interests, ‘antagonistic’ with those of the actual workers? To believe that replacing the markets in toto could be achieved without enormous catastrophes seems highly naive, and trying to do it before answering all these questions (and many more) seems daring. Apparently, Marx simply brushed aside the possibility that the elimination of markets have adverse repercussions on the economy’s productivity, that this could generate conflicts within the very body of workers’ representatives, and that hence the elimination of capitalists and markets could generate the conditions for the emergence of a new dominant bureaucratic class, which would imply the persistence of social classes and their ‘antagonism’, possibly aggravated by the worsening of the economic situation, and hence the need for a harshly repressive state.

If we are to call ‘utopias’ all those projects of social and economic reform based on good intentions but with no possibility of coming to life, at least in the manner posed, Marx’s political project of 1847 can certainly be called utopian. The undeniable (and important) scientific contributions of Marx’s later works allowed him and Engels, and later their epigones, to disguise the utopian character of some of the main components of their political project during several decades. And the erroneous part of Marx’s theory of Capitalism—the theory of surplus value—became the link between their utopian political project and the ‘exoteric’ part of Marx’s theory, the part containing true and notable scientific advances.

Manifesto of the Communist Party (1848)

Marx lived in Brussels between February of 1845 and March of 1848, where he devoted himself to intense writing and political activities. In mid-1845 he travelled with Engels for a little more than a month to England, where they contacted leaders of the Chartist movement and of the London community of the League of the Just. According to Engels in his “On the history of the Communist League” (1885), the League of the Just was “half propaganda association, half conspiracy,” and, “as Paris remained the decisive battleground, the League was at that time actually not much more than the German branch of the French secret societies” (MECW 26, 314). In particular, it was intimately related to the Société des Saisons, led by Blanqui and Barbés. The League was composed of ‘communes’ and was especially strong in Switzerland, where Wilhelm Weitling was active, but also in Paris. The German Workers’ Educational Society “served the League as a recruiting ground for new members.” It was largely formed by tailors, a craft in which the German language was predominant. Weitling himself was a tailor. The members of the League of working class extraction typically had skills like
tailoring, and naturally aspired to become “small masters” like the ones they worked for. They generally lacked even elementary knowledge of Economics, and this greatly distanced them from Marx and Engels, who had already begun to apply Historical Materialism and to seek in Economics the foundation for the struggles between the social classes. According to Engels, “It was our duty to provide a scientific substantiation for our view, but it was equally important for us to win over the European, and in the first place the German, proletariat to our conviction” (Ibid., 318-9). Hence, they founded in Brussels the German Workers’ Society. Early in 1846 they also organized a communist correspondence committee in Brussels with the aim of communicating with socialists of various countries.

Early in 1847 the London committee of the League of the Just sent one of its leading members (the German watchmaker Joseph Moll) to Brussels to invite Marx and Engels to join the League, for “they were as much convinced of the general correctness of our views as of the need to free the League from the old conspiratorial traditions and forms. Should we join, we would be given an opportunity of expounding our critical communism before a congress of the League in a manifesto, which would then be published as the manifesto of the League” (Ibid., 321). Hence, Marx and Engels joined the League, which had its first congress in London during the summer of 1847, in which its name was changed to ‘Communist League.’ The first article of its statute read: “The aim of the League is the overthrow of the bourgeoisie, the rule of the proletariat, the abolition of the old bourgeois society which rests on the antagonism of classes, and the foundation of a new society without classes and without private property” (Ibid.). According to Engels, the organization “was thoroughly democratic, with elective and removable authorities. This alone barred all hankering after conspiracy, which requires dictatorship” (Ibid., 321-2).

In its second congress, towards the end of the year, Marx defended in a long debate the new theory that would be the foundation of the organization, and Marx and Engels were then charged with writing a manifesto with the principles of the League. Engels had already written between the end of October and November of 1847 a draft of a program named Principles of Communism. He and Marx decided to transform it, in particular to “abandon the catechetical form” it had (MECW 38, 149). This resulted in the Manifesto of the Communist Party, published early in 1848. It contained much of what Marx had previously written, with slight variations, and also ideas from Engels’ Principles. Whereas the leitmotif of the League of the Just was “All Men Are Brothers”, the Communist League’s leitmotif displayed its class and internationalist orientation: “Working Men of All Countries, Unite!”

The Manifesto states that with the deterioration of the situation of modern workers “it becomes evident that the bourgeoisie is unfit any longer to be the ruling class in society... Society can no longer live under this bourgeoisie” (Manifesto, 495-6). In capitalist society “those of its members who work, acquire nothing, and those who acquire anything, do not work” (Ibid., 500). We have seen throughout this book that the idea that capitalists do not work was maintained intact in the analytical construction of Marx’s theory of Capitalism, being the mainstay of the theory of surplus value, though not in many of the textual expositions on capitalist entrepreneurs.

The Manifesto announces a fundamental historical break in the way in which the working class would access its future role of dominant class to make the desired transformations. Whereas in the past all the classes that became dominant “sought to fortify their already acquired status by subjecting society at large to their conditions of appropriation”, the “proletarians cannot become masters of the productive forces of society, except by abolishing their own previous mode of appropriation, and thereby
also every other previous mode of appropriation” (Ibid., 495). The proletarian class, being “the lowest stratum of our present society, cannot stir, cannot raise itself up, without the whole superincumbent strata of official society being sprung into the air” (Ibid.). Whereas previous historical revolutions had always established minorities in power to ensure their interests as a dominant class, the workingmen’s revolutionary movement proposed to lift to power “the immense majority, in the interest of the immense majority.” And since the bourgeoisie was not going to voluntarily lose its grip on power, “the violent overthrow of the bourgeoisie lays the foundation for the sway of the proletariat” (Ibid., 495).

According to the Manifesto, capital—a term which at this stage still referred to the means of production under Capitalism—is a collective output that can only be set in motion “by the united action of many members, nay, in the last resort, only by the united action of all members of society” (Ibid., 499). A transformation of capital “into common property, into the property of all members of society” is proposed. This personal property of capitalists would become social property. “It is only the social character of the property that is changed. It loses its class character” (Ibid.). And this implied the abolition of “bourgeois freedom,” by which is understood “the present bourgeois conditions of production, free trade, free selling and buying” (Ibid.). This implied radical action that was not in the least founded on argument and evaluation: the elimination of market transactions, of supply and demand, of markets.

The Manifesto established a distinction between the proletarians and the communists. The latter “do not form a separate party opposed to other working-class parties”, since “They have no interests separate and apart from those of the proletariat as a whole” (Ibid., 497). Although “They do not set up any separate principles of their own, by which to shape and mould the proletarian movement,” they are distinguished in that “they always and everywhere represent the interests of the movement as a whole” (Ibid.). “In the national struggles of the proletarians of the different countries, they point out and bring to the front the common interests of the entire proletariat, independently of all nationality.” And “they have over the great mass of the proletariat the advantage of clearly understanding the line of march, the conditions, and the ultimate general results of the proletarian movement.” As “all the other proletarian parties,” their immediate aim is the “formation of the proletariat into a class” by the overthrow of the capitalist class and the “conquest of political power by the proletariat” (Ibid., 498). It absurdly declared that the “theoretical conclusions of the Communists are in no way based on ideas or principles that have been invented, or discovered by this or that would-be universal reformer,” as if the ideas and principles there proclaimed had not emerged from the brains of the (revolutionary) reformers Marx and Engels.

The Manifesto established as the first step in the working class revolution “to raise the proletariat to the position of ruling class, to win the battle of democracy” (Ibid., 504). This equivalence between the conversion of the working class into the dominant class and winning “the battle of democracy” is not very clear in the Manifesto. In his Principles of Communism, Engels had stated that it would not be possible to “abolish private property at one stroke,” that the revolution would first “inaugurate a democratic constitution and thereby, directly or indirectly, the political rule of the proletariat. Directly in England, where the proletariat already constitutes the majority of the people. Indirectly in France and in Germany, where the majority of the people consists not only of proletarians but also of small peasants and urban petty bourgeois, who... are becoming more and more dependent on the proletariat and therefore soon will have to conform to the demands of the proletariat” (MECW 6, 350). Engels believed that this conformity would “perhaps involve a second fight, but one that can
end only in the victory of the proletariat.” For him, “Democracy would be quite useless to the proletariat if it were not immediately used as a means of carrying through further measures directly attacking private ownership and securing the means of subsistence of the proletariat” (Ibid.).

Using Engels’ strategic conception we can interpret the equivalence in the Manifesto between getting to power and winning the battle of democracy as the notion that, being the working class the majority of the nation (at least in England, and also in France and Germany if the peasantry and urban petty-bourgeoisie are included), even if the expropriation is an act of despotism against the affected classes, there would be democratic procedures within the camp of the majority. However, Engels’ statement that in the case of France and Germany the alliance between the working class and the petty-bourgeoisie could lead to a new struggle between these two classes if the latter did not accept the working class actions opens the door to the possibility that the aspirations of the ample peasant and petty-bourgeoisie majority become frustrated by the greater organizational power of the proletariat; in which case the latter might also expropriate the petty-bourgeoisie by violent means. And in that case it would be pure demagoguery to talk about democracy. Also, to say that “Democracy would be quite useless to the proletariat if it were not immediately used as a means... ” shows that the democratic procedures referred to would be an instrument to use if they proved to be useful for the aim sought (the expropriation of the owning classes and the collectivization of the economy) but could be dumped if they proved to be useless. It is evident that there is only a step between this recognition of power as the last instance foundation of political action and the total elimination of democracy by a political party, or a fraction of a party, or a small group surrounding a leader bent on perpetuating their rule and becoming the core of a new dominant class, separate from, and exercising its power on, the working class and the rest of civil society.

The Manifesto states that the “proletariat will use its political supremacy to wrest, by degrees, all capital from the bourgeoisie, to centralise all instruments of production in the hands of the State, i.e., of the proletariat organised as the ruling class; and to increase the total of productive forces as rapidly as possible” (Manifesto, 504). The authors do not present any argument that would support the possibility that the productive forces could increase without the intervention of the capitalist class. They also state that the centralization of the means of production in the hands of the State “cannot be effected except by means of despotic inroads on the rights of property, and on the conditions of bourgeois production; by means of measures, therefore, which appear economically insufficient and untenable, but which, in the course of the movement, outstrip themselves, necessitate further inroads upon the old social order, and are unavoidable as a means of entirely revolutionising the mode of production” (Ibid.). Implicit is the message that it is impossible to forecast all the difficulties that might arise in the attainment of the goal but that solutions would somehow be found. However, the radical aim of suppressing not only the largest private firms, but even the median and the small, and of suppressing not only private capital and wage labor but even commodities, i.e., markets, commerce, and production for sale in markets, leads one think that in practice these measures could only generate problems of such magnitude that they would only be possible to attempt through an increasing despotism on the mass of the population suffering the consequences of such policies.

The Manifesto states that the initial measures could be different in different countries, but that in the majority of the most advanced they would include: the expropriation of lands and its lease, using rents as a source of State revenue; the expropriation of the properties of the emigrated and seditious; the centralization in the State of all
credit (by means of a State Bank with exclusive monopoly) and of the means of transportation and communication; fiscal measures as a heavily progressive income tax and the abolition of all right of inheritance. Other measures would include the “Extension of factories and instruments of production owned by the State; the bringing into cultivation of waste-lands, and the improvement of the soil generally in accordance with a common plan” (Ibid., 505). Thus, although the immediate expropriation of lands is called for, the same is not proposed with respect to industrial firms, which explains the wrestling “by degrees, all capital from the bourgeoisie,” as long as the owners had not emigrated or showed a seditious behavior. Hence, a certain dose of prudence was exhibited for the immediate measures, only advancing towards the goal if the correlation of political forces so allowed. Other measures to be adopted included the abolition of child labor, free education for all children in public schools, and the “gradual abolition of the distinction between town and country, by a more equitable distribution of the population over the country.” There is no indication, however, of how the latter would be achieved, there being a wide range of possibilities from economic incentives to forced transportation of communities.

According to the Manifesto, with the passage of time class differences would gradually disappear and all production would be “concentrated in the hands of a vast association of the whole nation” (Ibid.). But the character of this association remains nebulous. The only feature specified is that the direction over the economy would be centralized by a State, the proletarian character of which is presupposed. It is said that gradually “public power will lose its political character” since it “is merely the organised power of one class for oppressing another,” and it is assumed that, once the working class holds the reins of power, it gradually “sweeps away by force the old conditions of production... the conditions for the existence of class antagonisms and of classes generally, and will thereby have abolished its own supremacy as a class”, at which point there is no longer a State nor social classes. Such a transformation would finally replace capitalist society by “an association, in which the free development of each is the condition for the free development of all” (Ibid., 506). With this phrase Marx proved once more his great capacity for formulating sentences that impact by their beauty and nobility of feelings but on second thought make one wonder if they only encapsulate wishful thinking.

In his Principles, Engels had proposed very similar initial measures, with slight differences in the implementation. He stated that they could not all be carried out at once, but that “one will always lead on to the other. Once the first radical onslaught upon private ownership has been made, the proletariat will see itself compelled to go always further, to concentrate all capital, all agriculture, all industry, all transport, and all exchange more and more in the hands of the State. All these measures work towards such results; and they will become realisable and will develop their centralising consequences in the same proportion in which the productive forces of the country will be multiplied by the labour of the proletariat” (MECW 6, 351). Once again we have the assertion (almost a wish) that the measures would increase the productive forces without arguing how or why. He added that once the process of concentration of “all production, and all exchange... in the hands of the nation,” is completed “private ownership will automatically have ceased to exist, money will have become superfluous, and production will have so increased and men will be so much changed that the last forms of the old social relations will also be able to fall away” (Ibid.; italics added). We have thus a pious and naive wish that the elimination of the private property of the means of production will produce a very favorable change on human beings.

Reflecting the expectations of the League’s leaders with respect to a coming revolu-
tionary situation, in *Principles* Engels is very clear with respect to the foreseen tactic for underdeveloped Germany when he writes that there “the decisive struggle between the bourgeoisie and the absolute monarchy is still to come. Since, however, the Communists cannot count on the decisive struggle between themselves and the bourgeoisie until the bourgeoisie rules, it is in the interests of the Communists to help bring the bourgeoisie to power as soon as possible in order as soon as possible to overthrow them again” (Ibid., 356).

The *Manifesto* also has an evaluation of the different types of Socialism. First there are the different variants of “reactionary socialism”: “feudal”, “petty-bourgeois”, and “German or ‘True’ ” Socialism. Sismondi is considered a leading exponent of petty-bourgeois Socialism. ‘True’ Socialism had been amply criticized in *Ideology*, and for the authors had the importance of being German, as they. Among the names they use to refer to it is the “Philosophy of Action,” which alluded to the ideas of Moses Hess, who had written an article with that title. Although Hess had collaborated in writing the part of *The German Ideology* that centers on ‘True’ Socialism and was furthermore a member of the Communist League and had contributed his own draft for the document on principles that the League wanted to publish, Marx and Engels were sure that they had to differentiate themselves from that movement which, for them, had not surpassed the humanist ideas of Feuerbach, lacking in foundations based on the struggles of the proletariat.\(^6\) Second, there was “conservative or bourgeois” Socialism, in which Proudhon was expressly included. “To this section belong economists, philanthropists, humanitarians, improvers of the condition of the working class” (Manifesto, 513). It was a form of Socialism that assimilated the bourgeoisie’s benign view of Capitalism. “In requiring the proletariat to carry out such a system, and thereby to march straightway into the social New Jerusalem, it but requires in reality, that the proletariat should remain within the bounds of existing society” (Ibid.).

Finally, there were the “critical-utopian” versions of Socialism and Communism, that is, the ‘systems’ “of Saint-Simon, Fourier, Owen and others.” But to these reformers “the proletariat, as yet in its infancy, offers to them the spectacle of a class without any historical initiative or any independent political movement,” and hence they did not propose that the proletariat become organized as a class in order to emancipate itself and all of society along with it. What’s more, they repudiated revolutionary action and proposed to “attain their ends by peaceful means” (Ibid., 515). Their publications were to be commended for attacking “every principle of existing society. Hence they are full of the most valuable materials for the enlightenment of the working class.” But their practical proposals pointed “solely to the disappearance of class antagonisms” and hence had “a purely Utopian character” (Ibid., 123-516). Instead of generating a revolutionary program for the working class, they sought to “deaden the class struggle and to reconcile the class antagonisms.” With hindsight, we can say that the extremely radical nature of the objectives sought by Marx and Engels made their own political program much more utopian than the “critical-utopian” versions of Socialism and Communism they criticized.

In the Preface to the 1883 German edition of the *Manifesto*, written shortly after Marx’s death, Engels felt obligated to recognize that the “basic thought running through the Manifesto... belongs solely and exclusively to Marx” (MECW 26., 118). This basic thought was that ever since the dissolution of the primeval regime of communal ownership of land all of history had been a history of class struggles “between

\(^6\)Despite Hess’ ascription to ‘True’ socialism, of which he had been a leading member all along, and despite Marx’s strong criticisms of his ideas and political stance, they continued to have a cordial relation throughout the rest of Marx’s life; and Hess even collaborated with Marx on some occasions.
exploited and exploiting, between dominated and dominating classes at various stages of social development,” and that this struggle had reached a stage in which “the exploited and oppressed class (the proletariat) can no longer emancipate itself from the class which exploits and oppresses it (the bourgeoisie), without at the same time forever freeing the whole of society from exploitation, oppression and class struggles” (Ibid.). Engels thus conceded to Marx primacy in the forging of the ‘millenarian’ political project that would forever emancipate humanity from the evils of exploitation and oppression.

The European revolutions of 1848-49 and the gestation of the Second French Empire

The European revolutionary process of 1848-1849

During 1848 and 1849 a series of revolutionary uprisings took place throughout continental Europe that enabled Marx and Engels to put to test and further develop their sociopolitical theory to some degree. In France, although feudalism had been eliminated by the Revolution, Bonaparte’s defeat in 1814 had led to the restoration of the Bourbon monarchy. In most of the rest of continental Europe the states in which feudal political forces still predominated were being increasingly questioned by democratic forces disgusted with the subsisting feudal privileges. They claimed increased freedom for the development of industrial Capitalism and in many cases also claimed for national unifications of societies that were politically highly segmented, like the various Kingdoms and Principalities in Germany that were not a part of the Kingdom of Prussia. Also, the multilingual Austrian Empire of the Habsburg dynasty dominated over a series of peoples (Italians, Hungarians, Slovanes, Poles, Czechs, Slovaks, Ukrainians, Rumanians, Serbs and Croats), many of whom had nationalist sectors anxious to obtain national independence. Aside from the political discontent of liberal forces against the monarchies, on the one hand, and nationalist forces against the imperial dominance of Austria, Russia and Prussia, on the other, a critical economic situation deepened the discontent. An agricultural crisis produced by poor harvests and the contractionary effects that radiated from the economic crisis in the highly industrialized England generalized the unemployment and hardships for the masses.

The revolutionary political process began in January of 1848 with an insurrection in Palermo (Sicily) against the absolute monarchy of the (Bourbon) Kingdom of the Two Sicilies that stretched over the southern half of present day Italy. King Ferdinand II caved in to the popular pressure by resuscitating the 1812 (Napoleonic) Constitution. In February a Constitution was also introduced in Florence (capital of the Great Duchy of Tuscany) and rebellions began within the Austrian Empire in Milan, so a state of siege was declared in Lombardy. Just days later there was an insurrection in Paris of such gravity that King Louis Phillippe7 abdicated. A provisional government was formed which under popular pressure created National Workshops to alleviate unemployment and the widespread misery. It also declared the establishment of a Republic. In March a Republic was also proclaimed in Venice, and there was also an insurrection in Vienna (capital of the Austrian Empire) that made Chancellor Metternich, who had been in charge of foreign policy during four decades, flee along with the Emperor. In the same month there were also Polish insurrections in the Great Duchy of Posen against the Prussian domination, and of Hungarians and Italians (of Milan) against

7Louis Phillippe was of the House of Orléans, a minor branch of the Bourbon kings who governed during the Restoration under a constitutional monarchy but were displaced from power by the July Revolution of 1830.
the Austrian domination. In Florence a Democratic government was formed.

After a series of defeats, the Austrian Empire seemed on the verge of collapse. But in June, the Austrians were able to crush an insurrection in Prague after a Slavic Congress that had gathered representatives of all the Slavic populations of Europe (Poles, Ukrainians, South Slavs, and Czechoslovaks). And in October there was a popular insurrection in Vienna that was contained by the ciriclement of the city by troops, and later a bloody repression. In December Francis Joseph I succeeded his granduncle Ferdinand I as Emperor. Invoking the Holy Alliance (between Russia, Austria and Prussia, established in 1815), he asked the Czar Nicholas I for help, who responded by sending an army of more than 200,000 soldiers. The two forces were jointly able to defeat the Hungarian revolutionaries in August of 1849 and imposed a brutal martial law. The Austrian Empire thus began to stabilize. The democratic forces were also defeated in Piedmont and Sardinia, the Monarchy was reestablished in Florence, and the Republic of Venice capitulated.

In the Great Duchy of Baden and in the Kingdom of Baviera (both in Southwest Germany) there were democratic assemblies starting in February of 1848 that led to incipient revolutionary processes. In March there were great demonstrations with barricades in Berlin. In May, a German National Assembly began to operate in Frankfurt, covering Prussia, Austria, and many other German states. But already in early June the counter-revolutionary forces gained momentum. The Republican experiment in Baden and Baviera failed. And in December of 1848 the King of Prussia dissolved the National Assembly and gave a Constitution that suited him.

In June of 1848 the provisional French government repressed a popular rebellion in Paris sparked by the elimination of the National Workshops. There were national elections in April of that year for a Constitutional Assembly with universal masculine suffrage, and the 10th of November Louis Bonaparte (nephew of Napoleon) was elected President of the Second French Republic thanks to the massive support of the peasant masses. In November of that year the murder in Rome of Pellegrino Rossi (economist and Interior Minister of Pope Pius IX) led to the Pope fleeing from Rome, and in February 1949 a short lived Roman Republic was inaugurated, led by Mazzini and guarded by Garibaldi. But in April Louis Bonaparte’s military intervention in favor of the Pope (who was sacred to French Catholics) led shortly after to the demise of the Republic. In June an insurrection led by the Socialist-Democratic party in Paris also failed and its leaders fled to exile. Finally, President Louis Bonaparte restricted suffrage in France and, because the National Assembly curtailed his power, he unconstitutionally closed it in December of 1952 by means of a coup, proclaiming himself Emperor shortly after. The Second French Empire would last until 1870, when Louis Bonaparte made the blunder of attacking Prussia and suffered a rapid and ignominious defeat that led to an insurrection in Paris and the establishment of a short-lived Paris Commune.

**Marx and Engels during the European revolutions of 1848-49**

When the European revolutions began, Marx had been living in Brussels for more than three years. In early March of 1848 he was detained and expelled from Belgium, but the new Provisional Government of France authorized his stay in Paris. The Brussels branch of the Communist League instructed him to form there of a new central Committee, which he began to preside. But with the development of revolutionary events in Germany, in April Marx and Engels settled in Cologne (in the Rhineland, then part of Prussia) and dedicated their efforts to the publication of a *New Rhenish*
Newspaper (NRN) to try to exert some influence on the course of the events. There being no need for clandestine activity for the moment, the activity of the League in Germany was suspended, leaving its members free to place themselves at the left of the democratic forces. Between June of 1848 and May of 1850 several issues of the NRN were published, containing many articles by Marx and Engels on the European events. The newspaper’s subtitle was *Organ of Democracy*. It was used to campaign for a unified German state and for the support of the various struggles of national liberation then existing. In July Marx was elected to represent the Democratic Association of Cologne in various forums (one of which was the German Workers’ Association), and in August Marx and Engels participated in the first congress of the Rhenish democrats, in Cologne. Marx traveled to Berlin and Vienna in search of funds for the NRN and to establish relations with democratic leaders. He was very disappointed with the cautious behavior of the German bourgeoisie during the events of 1848. In his December 15 NRN article “The bourgeoisie and the counter-revolution”, Marx contrasted the “European” revolutions of 1648 (in England) and 1789 (in France) to the March revolution in Prussia: “Far from being a European revolution it was merely a stunted after-effect of a European revolution in a backward country”; “The German bourgeoisie developed so sluggishly, timidly and slowly that at the moment when it menacingly confronted feudalism and absolutism, it saw menacingly confronting it the proletariat and all sections of the middle class whose interests and ideas were related to those of the proletariat” (MECW 8, 161-2).

The failure of the German insurrections led to the immediate closure of the NRN by the Prussian authorities and to Marx’s expulsion. After a brief stay in Paris, he finally settled in London. There he wrote several articles on the recent political events in France, which he published in Hamburg in a *New Rhenish Newspaper: Politico-Economic Review* along 1850 (and many years later were gathered by Engels in book form as *The Class Struggles in France 1848 to 1850*). According to the introduction Engels wrote (a few months before dying in 1895), this work was “Marx’s first attempt to explain a piece of contemporary history by means of his materialist conception, on the basis of the prevailing economic situation... to demonstrate the inner causal connection in the course of a development which extended over some years... to trace political events back to effects of what were, in the final analysis, economic causes” (MECW 27, 506). After another year of French events, Marx wrote in 1852 a sequence of articles that were published as the first issue of a German magazine (*Die Revolution*) edited in the U.S.A. by Weydemeyer (member of the Communist League emigrated to New York to make a living). It was titled *The Eighteenth Brumaire of Louis Bonaparte*. The title was based on the date according to the calendar of the French Revolution (18 Brumaire of Year VIII, i.e., November 9, 1799) in which Bonaparte redirected an ongoing coup against the then governing five man Directory and became First Council, which led to his coronation four years later as Emperor (of the First French Empire). But Marx applied this title to similar events led by Napoleon’s nephew Louis Bonaparte. Together, Marx’s two works on the subject vividly describe the short and tumultuous period of the Second French Republic that sprang from the ‘February Revolution’ of 1848 and culminated with the coup by Louis Bonaparte that made him Emperor.

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8It is not certain that he actually was his biological nephew. Napoleon made his brother Louis marry his own stepdaughter Hortense Beauharnais, daughter of his wife Josephine with her first husband (who died in the guillotine) in order to ensure a dynastic descent. The doubts on Louis’ father are based on the fact that it was an imposed marriage, the two avoided all contact, and Hortense had lovers. Nevertheless, Louis Bonaparte always gave great importance to his imperial ascendancy and in his youth twice engaged in failed coups against King Louis Philippe.
The revolution in France and the gestation of the Second Empire

Since Napoleon’s defeat in 1814 (leaving aside the 100 days in which he took arms again to be defeated in Waterloo), France had been governed successively by the Restored Bourbon kings Louis XVIII and Charles X (brothers of the guillotined Louis XVI), and then (between the revolutions of 1830 and 1848) by Louis Philippe of Orléans. Marx explains that under the reign of Louis Philippe it was the great bourgeoisie that ruled: “bankers, stock-exchange kings, railway kings, owners of coal and iron mines and forests, a part of the landed proprietors associated with them—the so-called finance aristocracy” (Class Struggles, 48). And the “industrial bourgeoisie proper” was only represented in the Chambers as a minority, in increasing opposition to the King’s absolutism, while the urban petty-bourgeoisie and the peasantry were completely excluded from political power. The high public expenditures led to an enormous corruption. And the monarchy was financially dependent on the ‘financial aristocracy’ for the finance of the public deficit, which gave bankers great power. There had been a potato blight and crop failures in 1845 and 1846, and the industrial and commercial crisis that struck England in 1847 turned the Parisians’ discontent into the uprisings of February 1848. These made Louis Philippe abdicate, and a Provisional Government was formed that reflected in its composition the various parties of the opposition. According to Marx, it was due to the pressure imposed by the workers that a Republic was quickly proclaimed, not waiting until the elections demanded by prominent members of the Provisional Government.

Marx assigned to most of the political leaders and groupings the political representation of the interests of various classes or subclasses, or of strata (or estates) such as the Bourbon ‘legitimists’, or the ‘Orléanists’ who in 1830 had replaced them in power. However, this assignment was by no means mechanical. For example, the poet and Provisional President Lamartine “essentially represented no real interest, no definite class; for such was the February Revolution, the general uprising with its illusions, its poetry, its imaginary content and its rhetoric,” even though he himself, “according to both his position and his views, belonged to the bourgeoisie” (Ibid., 53). The working class only had two representatives in the Provisional Government (Louis Blanc and Alexandre Martin—known as Albert) but had a great capacity for mobilization. The Provisional Government named a special committee charged with finding a way to improve the situation of the working class, presided by Blanc and Albert. For its sessions it was assigned the Luxembourg palace, which in fact kept it at arm’s-length from the seat of Government.

In the Brumaire (written with a longer time perspective than Class Struggles) Marx divides the events of the 44 months of the Second Republic into three time periods: 1) a brief (2 months) initial period which was actually a prologue to the Republic and that Marx characterized as a “Universal brotherhood swindle;” 2) a (13 month) period during which the Constituent National Assembly was in session, a proletarian uprising took place that was harshly repressed, and Louis Bonaparte was elected President; and 3) a longer (29 month) period during which the Legislative National Assembly functioned and culminated with Louis Bonaparte’s coup (against the Assembly) and his turning into Napoleon III (inaugurating the French Second Empire).

In the beginning of the second period (in May 15 1848) there was an aborted attempt by the group led by Louis Blanqui to cope the Assembly and proclaim a new Provisional Government. Their imprisonment resulted in “removing Blanqui and his comrades, that is, the real leaders of the proletarian party, from the public stage for the entire duration of the cycle” (Ibid., 110). Although Blanqui was condemned
to 10 years imprisonment, he and his group apparently became nevertheless involved with the Communist League. The Address of the Central Authority to the League of June 1850 (written by Marx and Engels) informed its members on the situation of the League circles after “The defeats of the revolutionary party last summer brought for a moment the League to the point of almost total disorganisation.” It describes the situation of the League in Belgium, Germany, Switzerland, France, and England. The League was strongest in London, where the Central Committee was located. The Address informed that, “Of the French revolutionaries the really proletarian party, led by Blanqui, has joined forces with us. The delegates of the Blanquist secret societies are in regular and official contact with the League delegates whom they have entrusted with important tasks in preparation for the next French revolution” (MECW 10, 377).

The Provisional Government had established National Workshops in order to alleviate the misery of unemployment. When their high fiscal cost became apparent, the Assembly began to place obstacles to their functioning, and when it announced that bachelor men had to abandon them the armed uprising erupted (in June 22, 1848). After some four days of struggle the insurgents were harshly repressed and hundreds were killed⁹. Marx saw this as a confirmation of his views on the eventual successful victory of the proletariat against Capitalism. This had been “the tremendous insurrection in which the first great battle was fought between the two classes that split modern society. It was a fight for the preservation or annihilation of the bourgeois order” (Class Struggles, 67).

The universal masculine suffrage established by the Provisional Government gave political access to the great majority of the population, the ‘peasant class’ that, evoking how much they had benefitted from Napoleon with the legalization of their smallholdings –expropriated from the landowning aristocracy– proved to be very much in favor of his nephew, giving him an overwhelming victory in the presidential election. However, the Constitution adopted on November 4, 1848, gave him only a four year non-renewable mandate. Hence, the second Sunday of May, 1852, became a key date for Louis Bonaparte when he decided to engage in a coup d’état.

During Marx’s long third period there grew a polarization between the ‘Party of Order’ and the “so-called Social-Democratic party.” The former amalgamated the two monarchical factions, those that “exercised more unrestricted and sterner domination over the other classes of society than ever previously under the Restoration or under the July monarchy, a domination which, in general, was only possible under the form of the parliamentary republic, for only under this form could the two great divisions of the French bourgeoisie unite, and thus put the rule of their class instead of the regime of a privileged faction of it on the order of the day” (Brumaire, 129). The two monarchical factions, Legitimists and Orléanists, represented “the two great interests into which the bourgeoisie is split –landed property and capital–,” since land ownership in France was no longer feudal since the 1789 Revolution and there was no longer a landowning nobility as a class. On the other hand, the Social-Democratic Party was formed by a coalition of petty-bourgeois and workers, and was represented in the Assembly by a block of representatives called the ‘Mountain’. The leaders of the petty-bourgeoisie, after the June repression, had come closer to the workers in order to increase their numerical importance. But for Marx this was dangerous situation, since “The revolutionary point was broken off from the social demands of the proletariat and a democratic turn given to them; the purely political form was stripped from the democratic claims of the petty bourgeoisie and their socialist point turned outward” (Ibid., 130). For Marx “The peculiar character of Social-Democracy is epitomised

⁹According to Marx, “over 3000 prisoners” were massacred.
in the fact that democratic-republican institutions are demanded as a means, not of superseding two extremes, capital and wage labour, but of weakening their antagonism and transforming it into harmony”, that is, of reforming “society in a democratic way, but a reformation within the bounds of the petty bourgeoisie” (Ibid.).

The concrete measures vindicated by the Manifesto, and especially by the “Demands of the Communist Party in Germany” that Marx and Engels prepared shortly after for the case of Germany, included various democratic demands (such as free public education for all children) that were perfectly compatible with the interests of the farmers and the urban petty-bourgeoisie. But Marx held that economic development would eventually increase the numerical importance of the working class, whose interests, in his view, were rooted in the collectivization of the means of production, including lands. On the other hand, he did not establish a mechanical relation between the objective class situation of an individual and his political stance. Hence, he wrote that “Just as little must one imagine that the democratic representatives are indeed all shopkeepers or enthusiastic supporters of shopkeepers. In their education and individual position they may be as far apart from them as heaven from earth.

What makes them representatives of the petty bourgeoisie is the fact that in their minds they do not get beyond the limits which the latter do not get beyond in life, that they are consequently driven, theoretically, to the same problems and solutions to which material interest and social position drive the latter in practice. This is, in general, the relationship between the political and literary representatives of a class and the class they represent” (Brunaire, 130-1). Similarly of course, the representatives of the working class would not all be workers, and “in their education and individual position” could be as far apart as Marx and Engels were.

When in April 1849 the Government sent French forces to Rome to put an end to the incipient Roman Republic, the Mountain accused the President and his cabinet of not complying with the Constitution, since the latter required that any military intervention abroad be previously approved by the Legislative Assembly. This sparked a popular uprising that was quickly repressed and there followed the exile of some of the members of the Mountain while others were imprisoned. The political power of the Social-Democrats hence disappeared and, henceforth (until Louis Bonaparte’s coup) the political struggles in France were reduced to those between the Party of Order, the predominant force in the Legislative Assembly, and the Executive power of Louis Bonaparte.

Marx had a markedly negative opinion of the then French President and later Emperor. He had created a ‘Society of December 10’ by organizing the Parisian ‘lumpen-proletariat’ “into secret sections, each section being led by Bonapartist agents, with a Bonapartist general at the head of the whole” (Ibid., 149) to be used as a shock force whenever necessary. Marx highlighted the immense power the President derived from his control over the machinery of the State, which had grown greatly during the period of absolute monarchy “in the same measure as the division of labour within bourgeois society created new groups of interests, and, therefore, new material for state administration” (Ibid., 186). According to Marx, “under the absolute monarchy, during the first revolution, under Bonaparte, bureaucracy was only the means of preparing the class rule of the bourgeoisie. Under the Restoration, under Louis Philippe, under the parliamentary republic, it was the instrument of the ruling class, however much it strove for self-rule10”. (Ibid.)

10 MECW 11 has here “power of its own” instead of “self-rule”. In the German original the phrase is “so sehr sie auch nach Eigenmacht strebte”. And the Google translator yields “however much it strove for self-rule.”
The last phrase is interesting since, not imagining it, Marx was here pointing to the fundamental problem of his conception of working class emancipation through its taking over State power by means of the organization that presumably represented it. After taking power, the political party that self-identifies itself with the working class would have to take charge of the State, and even if it completely eliminated the existing bureaucracy it would have to create another one, which would have its own interests as soon as it consolidated, even if the revolution changed the class (origin) composition of that bureaucracy. With the political power derived from the handling of the State machinery, nobody would be able to prevent it from acting as a new self-ruling social class, from organizing itself according to its own interests in order to ensure its domination over the rest of society (and benefit from special privileges), as has happened all along the history of stratified society. Even if the capitalist class disappeared (by it expropriation and the prohibition of private firms) there would still be a class society and the dominance of one class over others. And the bureaucratic dominant class would essentially pursue its own interests. Eventually, a dominant fraction could come to evaluate that it suited its interests to resurrect Capitalism by allowing and encouraging private firms. And this is what actually happened in both Russia (with the dissolution of the Soviet Union and ‘Communism’) and China (with the ‘reforms’ that reintroduced private firms and allowed the emergence of billionaires while preserving the exclusive political domination of the Communist Party).

Marx explains that under Louis Bonaparte, although the State seemed “to have made itself completely independent... state power is not suspended in mid air. Bonaparte represents a class, and the most numerous class of French society at that, the small-holding peasantry. Just as the Bourbons were the dynasty of big landed property and just as the Orleans were the dynasty of money, so the Bonapartes are the dynasty of the peasants, that is, the mass of the French people. Not the Bonaparte who submitted to the bourgeois parliament, but the Bonaparte who dispersed the bourgeois parliament is the chosen man of the peasantry” (Brumaire, 186-7). Marx held that French farmers were impoverished. After having benefited from the ownership of their small-holdings under the first Bonaparte, their population growth, their being unable to make sufficient productivity increasing investments, their getting indebted under usurious interest rates, and the high taxes they paid, had led to a situation in which “Sixteen million peasants (including women and children) dwell in hovels, a large number of which have but one opening” (Ibid., 190). And for Marx the peasant mass that was the political support of Louis Bonaparte was, despite its poverty, predominantly conservative. For “The small-holding peasants form a vast mass, the members of which live in similar conditions but without entering into manifold relations with one another. Their mode of production isolates them from one another instead of bringing them into mutual intercourse. The isolation is increased by France’s bad means of communication and by the poverty of the peasants” (Ibid., 187). Since there was only a local articulation between the small-holding peasants, “the identity of their interests begets no community, no national bond and no political organisation among them, they do not form a class. They are consequently incapable of enforcing their class interests in their own name, whether through a parliament or through a convention. They cannot represent themselves, they must be represented. Their representative must at the same time appear as their master, as an authority over them, as an unlimited governmental power that protects them against the other classes and sends them rain and sunshine from above” (Ibid., 187-8). In Brumaire Marx thus uses his distinction between the concept of a social class to which individuals belong due to their role in the economy and the idea that such individuals “form a class” only when “the identity of their
interests” leads them to have conscience of this identity and thus form a “community” that organizes itself politically to struggle for its common interests.

Marx also upheld the thesis that at the time of the 1848 Revolution “The interests of the peasants, therefore, are no longer, as under Bonaparte, in accord with, but in opposition to the interests of the bourgeoisie, to capital” (Ibid., 191). And hence, “the peasants find their natural ally and leader in the urban proletariat, whose task is the overthrow of the bourgeoisie order” (Ibid.). Marx and Engels had already upheld this idea that the working class has the historical ‘task’ of overthrowing Capitalism in earlier writings, and they would uphold it to the end. It constituted one of the pillars of the international communist movement they organized, and which endured long after their deaths. Enlightened communists would aim to guide the working class so that it ‘form a class’ and they, as the vanguard of the revolutionary forces, would overthrow the existing ‘material’ order. But in the case of mid-19th century France the workers “could not take a step forward, could not touch a hair of the bourgeois order, until the course of the revolution had aroused the mass of the nation, the peasants and petty bourgeois, standing between the proletariat and the bourgeoisie, against this order, against the rule of capital, and had forced them to attach themselves to the proletarians as their protagonists” (Class Struggles, 57).

Marx was well aware that the development of the industrial working class was conditioned by the development of the French industrial capitalist class, and that the latter was more developed than those in the rest of continental Europe and less developed than that of England. He also held the hypothesis that the more developed a country’s industry was the greater was the possibility that its working class become revolutionary, i.e., be capable of organizing itself politically to realize its ‘task’. But “the industrial bourgeoisie did not rule France. The industrial bourgeoisie can rule only where modern industry shapes all property relations to suit itself, and industry can win this power only where it has conquered the world market, for national bounds are inadequate for its development” (Ibid., 56). And that was not the case of French Capitalism, which “to a great extent, maintains its command even of the national market only through a more or less modified system of prohibitive tariffs” (Ibid.). Due to its insufficient industrial development (and with the exception of the great urban centers) the French industrial working class was insignificant in comparison to the “superior numbers of peasants and petty bourgeois” (Ibid., 57). Hence, the French working class was still unable to make its own revolution and it was only logical then for the Paris proletariat to seek “to assert its own interests side by side with the interests of the bourgeoisie, instead of enforcing them as the revolutionary interests of society itself” (Ibid.).

But even though in 1852 Marx understood that there were reasons for a proletarian revolution not taking place in France, he still thought that in a proximate future a revolution with petty-bourgeois features would take place due to the necessary disappointment of the peasantry. Marx’s forecast in the original (1852) edition of Brumaire was that “When he is disappointed in the Napoleonic Restoration, the French peasant will part with his belief in his smallholding, the entire state edifice erected on this smallholding will fall to the ground and the proletarian revolution will obtain that chorus without which its solo becomes a swan song in all peasant countries” (Brumaire, 193, footnote). In the 1869 edition of his book (17 years later) Marx naturally left out this forecast of an upcoming revolution, since the events had marched along a completely different road. Napoleon III had managed to consolidate his power and govern over all of France, including the working class, the peasantry, and the urban
petty-bourgeoisie. Also, the accumulation of capital had successfully advanced (even in the countryside), though the expansion of the Empire had had its successes (the colonization of Indochina) and its failures (the tragic attempt to establish an Emperor -Maximilian- in Mexico). There were also other paragraphs of Brumaire that had become outdated, though Marx decided to leave them unchanged. In the preface to the 1869 edition he admitted that the book “was written under the immediate pressure of events” and that “A revision of the present work would have robbed it of its specific colouring” (MECW 21, 56-7). With the benefit of hindsight one must admit that two characteristics that Marx remained unaware of was his (and Engels’) posing an impossible ‘task’ for the working class –that of eliminating class society– and his tendency to see a beginning of the realization of this ‘task’ every time there was a significant popular uprising.

Class Struggles contains interesting manifestations of Marx’s conception of the proletarian revolution at this stage of his thought. He considered that the ‘right to work’ that had been demanded by the workers’ representatives in the initial phase of the February Revolution, leading to the creation of the National Workshops, was “in the bourgeois sense, an absurdity, a miserable, pious wish.” For it posed a menace to capital, since “behind the right to work stands the power over capital; behind the power over capital, the appropriation of the means of production, their subjection to the associated working class and, therefore, the abolition of wage labour, of capital and of their mutual relations” (Class Struggles, 78). Additionally, he held that the proletarian revolution had to have an international scope (related to the most industrially advanced countries) and that the organization of the working class consequently had to have a supranational scope in order to confront with a coalition of capitalist states (as the European monarchies had confronted with the French Republic in the last decade of the 18th century). The ‘task’ of the working class could not be realized in France in isolation, given its relative backwardness. It could only be accomplished when the proletarian revolution burst in England, where the capitalist and working classes were most developed, on account of the greater development of English Capitalism. But the international scope of the frustrated revolutions of 1848-49 had convinced him that the realization of the ‘task’ of the working class could not be “accomplished anywhere within the national walls; the class war within French society turns into a world war, in which the nations confront one another. Accomplishment begins only when, through the world war, the proletariat is pushed to the fore in the nation which dominates the world market, to the forefront in England. The revolution, which finds here not its end, but its organisational beginning, is no short-lived revolution” (Ibid., 117). He forecasted a long lasting ‘world war’ between the national working classes of the most industrialized countries in coalition with the national bourgeoisies similarly in coalition.

Marx continued to develop ideas he had already expressed in previous works. In particular, that workers’ emancipation ideas would be “the abolition of bourgeois production and its order” (Ibid., 59), which implied also the “abolition of bourgeois credit” since “private credit rests on confidence that... the bourgeois order, will not be touched, will remain inviolate.” In the dynamics of revolution, “the proletariat increasingly organises itself around revolutionary Socialism, around Communism, for which the bourgeoisie itself has invented the name of Blanqui. This Socialism is the declaration of the permanence of the revolution, the class dictatorship of the proletariat as the

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11In 1871, Marx recognized that under Louis Bonaparte “bourgeois society, freed from political cares, attained a development unexpected even by itself. Its industry and commerce expanded to colossal dimensions” (Civil War, MECW 22, 330). That development was certainly not expected by Marx in 1852.
necessary transit point to the *abolition of class distinctions generally*, to the abolition of all the relations of production on which they rest, to the abolition of all the social relations that correspond to these relations of production, to the revolutionising of all the ideas that result from these social relations" (Ibid., 127).

It was apparently in *Class Struggles* where Marx first used the expression “dictatorship of the proletariat” (in the clearer form of “class dictatorship of the proletariat”). In his letter to Weydemeyer of March 5, 1852, Marx explained that he did not “claim to have discovered either the existence of classes in modern society or the struggle between them”, since the subject had already been described by bourgeois historians and economists. In contrast, he did claim that his contribution was to have shown that “1... the existence of classes is merely bound up with certain historical phases in the development of production; 2. that the class struggle necessarily leads to the dictatorship of the proletariat; 3. that this dictatorship itself constitutes no more than a transition to the abolition of all classes and to a classless society” (MECW 39, 62-5).

These thoughts were certainly his, but in the light of history it is impossible to defend their substance. The first item is not problematic if it refers to the remote “primitive communism” in which social stratification had not yet emerged. But the next two items show that he was also referring to a *future* society devoid of classes. He claimed to have ‘shown’ that there would be a future classless society, and that it would emerge after a political party representative of the interests of the working class takes the reigns of the State and proceeds to eliminate all social classes because this is in the interest of the working class. It seems obvious that one cannot show something that has not occurred; one can only argue about the probability of its occurrence. But to be fair, let us avoid stressing the absurdity of this claim and take into account that it occurred in a personal letter to a political comrade. What we do want to highlight is that its substance faithfully reflected Marx’s vision as of 1852 of the great revolutionary events that the future held in store, that his vision varied very little after he dedicated many years of efforts to the construction of his theory of the emergence and functioning of the capitalist mode of production, and that his vision had severe flaws which Marx was never able to surpass. Among these flaws were the beliefs that the elimination of the capitalist class could be beneficial to the working class, that an increase in the welfare of workers within Capitalism was either infeasible or a mere deception, and that a dictatorship of the (representatives of) the working class could lead to the disappearance of the social classes.
Chapter 22  THE CONCEPT OF COMMUNISM IN THE MATURE MARX

In his maturity, Marx continued to base his practical-political but ‘millenarian’ conception on the idea that the objective (‘task’) of the organized working class in the most advanced capitalist countries was taking over the reins of political power, and thus inaugurating the ‘dictatorship of the proletariat’. He barely added some ingredients to the ideas he had already developed as of 1852. At the head of the machinery of State, a political party representative of the working class would implement the transformation of society, that is, the expropriation and socialization of the means of production and the replacement of capital, markets, and money, by the workers’ state production and distribution coordinated by centralized planning. Workers would receive incomes derived from their individual labor by means of a certificate that would reflect how much he has worked in the period, and that he could exchange for goods and services. After this ‘first phase of Communism’, society would eventually reach a stage in which there would no longer be a division of labor imposed on individuals by circumstances, there would be no antagonistic social classes, and solidarity and peace would predominate, both in each community as among communities.

Marx’s ideas on Communist society
The relation between the working man and the means of production as a dialectical process

In broad strokes, we can say that in his maturity Marx conceived humanity’s evolution in the last centuries of history and in a future of uncertain duration as a (dialectical) process in which the ‘primitive accumulation’ of capital, having destroyed “the Original Union existing between the Labouring Man and his Instruments of Labour” (negation), led to several centuries of intense capitalist accumulation, and was in the process of setting the foundation for a future revolution that would restore the original unity between the working man and his means of labor “in a new historical form” (negation of the negation). Possibly the most synthetic paragraph in which Marx expresses his conception in this respect is one from the conference he gave in June 1865 before the Central Council of the International Working Men’ Association (First International). Referring to the purchase of labor power by capitalists and its corresponding sale by wage workers, he asks how the strange phenomenon appeared:

That the one set buys continually in order to make a profit and enrich themselves, while the other set continually sells in order to earn their livelihood? The inquiry into this question would be an inquiry into what the economists call ‘Previous, or Original Accumulation’, but which ought to be called Original Expropriation. We should find that this so-called Original Accumulation means nothing but a series of historical processes, resulting in a Decomposition of the Original Union existing between the Labouring Man and his Instruments of Labour... The Separation between the Man of Labour and the Instruments of Labour once established, such a state of things will maintain itself and reproduce itself upon a constantly increasing
scale, until a new and fundamental revolution in the mode of production should again overturn it, and restore the original union in a new historical form (*Value, Price and Profit*, MECW 20, 128-9).

At that time Marx was intensely working on *Capital*, where he expresses the same ideas with much more detail. He describes an apparent vicious circle in the accumulation of capital, since capitalists turn money into capital, and the latter generates surplus value, which is transformed into money and a part of this money is reinvested to produce more money, a part of which is turned into capital. “The whole movement, therefore, seems to turn in a vicious circle, out of which we can only get by supposing a primitive accumulation (previous accumulation of Adam Smith) preceding capitalistic accumulation; an accumulation not the result of the capitalist mode of production, but its starting point” (B1, 704). He compares the ‘primitive accumulation’ of capital with the biblical story of Adam and Eve, according to which when Adam bites the apple “thereupon sin fell on the human race”, thus associating the primitive accumulation of Capitalism with original sin, and Capitalism with sin. The substantial part is the following:

The capitalist mode of appropriation resulting from the capitalist mode of production, hence the capitalist private property, is the first negation of individual private property based on own work. But capitalist production begets, with the inexorability of a law of Nature, its own negation. It is the negation of negation. This does not re-establish private property for the producer, but gives him individual property based on the acquisitions of the capitalist era, i.e., on co-operation, and the possession in common of the land and of the means of production (B1, 751).1

Whereas the transformation of the private property based on dispersed personal labor (of Small Commodity Producers) into capitalist private property was realized during a long historical period Marx called ‘primitive accumulation’ (or ‘original accumulation’), the second transformation (‘negation of the negation’) would be much shorter. The fact that modern Capitalism already used ‘socialized’ production methods—like the cooperation of great numbers of workers in large firms (many of them stock companies) and in worker cooperatives—, led Marx to believe that the transformation of capitalist property into socialized property would require a much shorter period. And, whereas the first transformation entailed “the expropriation of the mass of the people by a few usurpers”, the second required “the expropriation of a few usurpers by the mass of the people.” Millenarian justice would be achieved, the inner foundation of all of Marx’s social cosmology.

Furthermore, individual freedom would expand with the common control of the production process by the associated producers. For it would bring about an increase in productivity that would enable the shortening of the working day, and thus expand “the realm of freedom” that “actually begins only where labour which is determined by necessity and mundane considerations ceases.”

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1 Slight changes have been introduced to the first and last sentences of this paragraph of the English version in MECW on the basis of the original German text and the Google translator. The first sentence is the Google translation of the German text. The last sentence has been slightly changed to reflect that the “possession in common...” is not an acquisition of the capitalist era. The last sentence in the German original is: “Diese stellt nicht das Privaetigentum wieder her, wohl aber das individuelle Eigentum auf Grundlage der Errungenschaft der kapitalistischen Ära: der Kooperation und des Gemeinbesitzes der Erde und der durch die Arbeit selbst produzierten Produktionsmittel.”
Just as the savage must wrestle with Nature to satisfy his wants, to maintain and reproduce life, so must civilised man, and he must do so in all social formations and under all possible modes of production. With his development this realm of physical necessity expands as a result of his wants; but, at the same time, the forces of production which satisfy these wants also increase. Freedom in this field can only consist in socialised man, the associated producers, rationally regulating their interchange with Nature, bringing it under their common control, instead of being ruled by it as by the blind forces of Nature; and achieving this with the least expenditure of energy and under conditions most favourable to, and worthy of, their human nature (B3, 807).

It is impossible to disagree with this beautiful paragraph, very worthy of the creative genius of its author. But we can and must disagree with the project Marx designed to implement a rational regulation of the economy by ‘socialized man’, as we have already suggested in the preceding pages.

**The two phases of a future communist society**

In his analyses of the capitalist mode of production Marx perceived vestigial signs of the future society such as the increasing internal planning in large modern firms, where owners (stockholders) tended to lose control of the firm’s operations; where the State increasingly intervened in the economy, establishing limits to the working day and to child labor, and regulating ‘natural monopolies’ and foreign trade. In his view, though this process tended to reduce the blind and chaotic market mechanisms, it was not nearly enough to solve the social misery arising from the unemployment that market mechanisms generated. The concerted activity of the working class was needed to give the *coup de grâce* to the capitalist class and the markets. We have seen that he had the firm conviction that society was advancing in this direction long before he wrote *Capital*. On this conviction was based his (and Engels’) effort to construct an organized working class party and movement under the banner of Communism. But it was fundamental to conceptually separate the immediate objective of the ‘dictatorship of the proletariat’ that would take charge of a society that was until then capitalist, from the *final* objective of a fully Communist society in which there would be no more social classes, no more State (as a means of domination of one class over others), no more markets, no more money, and not even ‘division of labor’ other than the voluntary one, since each would labor in whatever activity he enjoyed and labor would be “life’s prime want.” In the fully Communist society there would be genuine solidarity. He expressed these ideas quite poetically in 1875, in his critical notes to the draft program of the future united Social-Democratic party, *Critique of the Gotha Program*:

In a higher phase of communist society, after the enslaving subordination of the individual to the division of labour, and thereby also the antithesis between mental and physical labour, has vanished; after labour has become not only a means of life but life’s prime want; after the productive forces have also increased with the all-round development of the individual, and all the springs of common wealth flow more abundantly –only then can the narrow horizon of bourgeois right be crossed in its entirety and society inscribe on its banners: From each according to his abilities, to each according to his needs! (*Gotha*, MECW 24, 87).
The State in the “first phase of communist society as it is when it has just emerged after prolonged birth pangs from capitalist society” would be a ‘dictatorship’ of the vast majority over a minority probably still interested in recovering its lost dominating position. It would be a State of associated workers in which there would still be division of labor dictated by the limitations of the productive forces, with workers more or less qualified, as in the big capitalist firms. But such distinctions would take place among workers with the same type of income, an income based on labor time and intensity (and possibly productivity and qualification) and net of various subtractions necessary to fund several ‘social funds’ as well as real investment. And the central planning of production and distribution would ensure the absence of unemployment.

In the legal sense this transition society would still be ‘bourgeois’, since incomes would ultimately be based on the quantity of labor exerted. It is not clear whether Marx held that incomes would be differentiated by labor complexity or skill. He does not say anything about this in *Gotha*, but we can presume that there would be different levels of remuneration, although certainly none related to commercial or industrial profits. In his comments (of 1874-75) on the margin of Bakunin’s (1873) book *Statism and Anarchy*, where the latter warned Marx that in his project these ‘rulers or representatives’ would cease to represent the workers and would come to represent themselves, Marx writes that if Bakunin “were familiar even with the position of a manager in a workers’ co-operative factory, all his fantasies about domination would go to the devil. He should have asked himself: what forms could management functions assume within such a workers’ state, if he wants to call it that?” (MECW 24, 520) Since surely the manager in a workers’ co-operative factory earned more than a non-skilled worker, this seems to back our presumption. Only in “a higher phase of communist society” would there be full solidarity as a distributive principle, and the ‘bourgeois right’ that associated incomes with quantities of labor would disappear.

**Communism as the overcoming of mercantile society**

When in Chapter 1 of Book I Marx addressed ‘commodity fetishism’, he displays an intellectual exercise that—in the light of other works of his—has undeniable links to his vision of the future of human society. For him the commodity is a “mysterious thing,” since “in it the social character of men’s labour appears to them as an objective character stamped upon the product of that labour” (B1, 82-3). The fact that independent producers only enter into social contact (as far as their role in the economy is concerned) when they interchange their products hides the social nature of their labors behind the intricacies of the markets, behind the exchange values of their respective commodities. In their minds this inverts what is essentially a social relation between individuals (subjects) participating in the economy into a social relation between objects: an exchange relation between commodities, a market phenomenon. And this gives commodities that mysterious character that Marx characterizes as fetishism. Marx finds that only when the production of commodities (i.e., of goods and services for their sale in markets) is fully developed, that is, in the capitalist mode of production,

the scientific conviction springs up, that all the different kinds of private labour, which are carried on independently of each other, and yet as spontaneously developed branches of the social division of labour, are continually being reduced to the quantitative proportions in which society requires them. And why? Because, in the midst of all the accidental and ever fluctuating exchange relations between the products, the labour time socially
necessary for their production forcibly asserts itself like an over-riding law of Nature (B1, 86).

At this point Marx inserts a footnote in which he quotes from Engels’ early contribution to the *Yearbook* ("Outlines of a Critique of Political Economy"), where he describes the “law of competition” as demand and supply always striving to complement each other but never succeeding and periodically producing “trade crises, which reappear as regularly as the comets, and of which we have now on the average one every five to seven years” and bringing “in their train more misery and more immorality” than the great plagues. “If the producers as such knew how much the consumers required, if they were to organise production, if they were to share it out amongst themselves, then the fluctuations of competition and its tendency to crisis would be impossible. Carry on production consciously as human beings—not as dispersed atoms without consciousness of your species—and you have overcome all these artificial and untenable antitheses” (MECW 3, 433-4). This early 1843 article by Engels had impressed Marx very favorably and probably helped motivate him to further his studies of political economy (and enhance his association with Engels). For the mature Marx, the “scientific discovery, that the products of labour, so far as they are values, are but material expressions of the human labour spent in their production, marks, indeed, an epoch in the history of the development of the human race” (B1, 85). And the fact that in capitalist commodity production the equilibrium exchange proportions were no longer relative values did not impair that discovery but merely modified the equilibrium (or ‘regulating’) exchange proportions in a much more complex society; one in which despite the meticulous planning within each firm there was an absence of a global planning that could make the production of each good coincide with social needs and, furthermore, eliminate the recurring crises that generated mass unemployment and human misery.

Marx reviews in very condensed form in the first chapter of Book I some historical modes of production of human society, since “The whole mystery of commodities, all the magic and necromancy that surrounds the products of labour as long as they take the form of commodities, vanishes therefore, so soon as we come to other forms of production” (B1, 87). He begins with an imaginary Robinson Crusoe in his island (“a favourite theme with political economists”), who, in order to survive must distribute his limited time between a great quantity of different tasks, for which he keeps a detailed accounting of the time spent in each task. He continues with the European Middle Ages, where “we find everyone dependent, serfs and lords, vassals and suzerains, laymen and clergy.” This was a society in which personal dependence “characterises the social relations of production just as much as it does the other spheres of life.” Hence, “for the very reason that personal dependence forms the groundwork of society, there is no necessity for labour and its products to assume a fantastic form different from their reality. They take the shape, in the transactions of society, of services in kind and payments in kind” (B1, 88). In this society in which labor is compulsory, it “is just as properly measured by time, as commodity-producing labour; but every serf knows that what he expends in the service of his lord, is a definite quantity of his own personal labour power.” Hence, “the social relations between individuals in the performance of their labour, appear at all events as their own mutual personal relations, and are not disguised under the shape of social relations between the products of labour” (Ibid.). He also addresses the case of “the patriarchal industries of a peasant family” that produce various goods to satisfy their own needs. In this society the various labors are “functions of the family, which, just as much as a society based on the production of commodities, possesses a spontaneously developed system of division of labour” (Ibid., 89). In all three cases (Robinson, feudalism, patriarchal family industries), the
allocation of labor time is socially determined, in contrast to a society that produces commodities, in which this allocation assumes that particular form in which “the labour of the individual asserts itself as a part of the labour of society, only by means of the relations which the act of exchange establishes directly between all the different kinds of private labour, which are carried on independently of each other” (B1, 83-4). And that is why in the mercantile world “the relations connecting the labour of one individual with that of the rest appear... as what they really are, material relations between persons and social relations between things.”

Finally, Marx culminates with a hypothetical non-mercantile society in which production is carried out by “a community of free individuals” who work with means of production held in common and in which “the share of each individual producer in the means of subsistence is determined by his labour time.” He imagines

a community of free individuals, carrying on their work with the means of production in common, in which the labour power of all the different individuals is consciously applied as the combined labour power of the community. All the characteristics of Robinson’s labour are here repeated, but with this difference, that they are social, instead of individual. Everything produced by him was exclusively the result of his own personal labour, and therefore simply an object of use for himself. The total product of our community is a social product. One portion serves as fresh means of production and remains social. But another portion is consumed by the members as means of subsistence. A distribution of this portion amongst them is consequently necessary. The mode of this distribution will vary with the productive organisation of the community, and the degree of historical development attained by the producers. We will assume, but merely for the sake of a parallel with the production of commodities, that the share of each individual producer in the means of subsistence is determined by his labour time. Labour time would, in that case, play a double part. Its apportionment in accordance with a definite social plan maintains the proper proportion between the different kinds of work to be done and the various wants of the community. On the other hand, it also serves as a measure of the portion of the common labour borne by each individual, and of his share in the part of the total product destined for individual consumption (B1, 89-90.).

For Marx, in such a society the “social relations of the individual producers, with regard both to their labour and to its products” would be “perfectly simple and intelligible, and that with regard not only to production but also to distribution.” But it would no longer be the case that “the process of production has the mastery over man, instead of being controlled by him.” In modern industry the industrial capitalist was in the process of decomposing into a simple manager on the one hand, an administrator of others’ capital, and on the other the mere owners of capital, the stockholders. The owners tended to receive dividends quite as passively as a bondholder received interest, whereas the firm’s management became increasingly autonomous. And for Marx this process was a “phase of transition to a new form of production” (B3, 436), since “the stock company is a transition toward the conversion of all functions in the reproduction process which still remain linked with capitalist property, into mere functions of associated producers, into social functions” (B3, 436). This transition also had another expression in the “cooperative factories of the labourers themselves ”, in which “the antithesis between capital and labour is overcome within them, if at first only by way
of making the associated labourers into their own capitalist, i.e., by enabling them
to use the means of production for the employment of their own labour" (B3, 438).
The new mode of production towards which there was a transition taking place was
one in which there would be a “reconversion of capital into the property of producers,
although no longer as the private property of the individual producers, but rather as
the property of associated producers, as direct social property” (B3, 434). In such a
society there would be “no class distinctions, because everyone is only a worker like
everyone else” (Gotha, MECW 24, 86). And “nothing can pass to the ownership of
individuals except individual means of consumption” (Ibid.).

Paradoxes and flaws in Marx’s political project
Although these observations are extremely interesting, they have a high level of ab-
straction and leave gaps that with the benefit of hindsight are evident. First, it is
difficult to make Marx’s vision square with the empirical observation that over many
millennia human society has been getting increasingly complex, and that this is clearly
evident in the economic sphere. The hierarchical structures and networks between the
individuals that participate in industrial and commercial organizations have become
denser, and a reduction in the number of hierarchical levels and in the division of la-
bor is not in sight. Hence, even if historically obsolete functions (like the ones behind
feudal privileges) have been disappearing, as well as their corresponding social classes
(or ‘estates’), those specific to the capitalist organization of the economy have become
increasingly complex. This includes the increasing number of markets, not only for
goods and services but also for financial assets, and the increasing complexity of mar-
et transactions (e.g., financial derivatives markets). And the interrelations between
direct planning, market operations, and regulations aiming to correct ‘market failures’,
have also become increasingly complex. It seems plausible that these tendencies, so
clear today, were also perceptible in the 1860s, when Marx was active writing Capital.
Having Marx intelligently detected various tendencies of the capitalist society of his
time (some of which he gathered from various sources and skillfully integrated into his
theory), it is difficult to find an explanation for his incorrect (and implicit) belief in
a future simplification of the functioning of human society. Simplification is implicit
in the prediction of having fewer hierarchical functions in economic organizations and
less division of labor to the point that social classes tend to disappear, in the prediction
of having fewer markets—even if strongly regulated—to the point that all markets
tend to disappear, and the prediction of having fewer labor specializations to the point
that the technologically imposed division of labor tends to disappear. Only deeply in-
grained psychological mechanisms arising from childhood and adolescence can explain
the contrast between Marx’s capacity to build a theoretically consistent and sophisti-
cated theory of capitalist society (even redundant in its ‘esoteric’ portion), which was
basically correct in many important aspects, and his persistence in sustaining final
objectives contradictory to some of the fundamental tendencies of the society of his
time, at least as we can visualize them a century and a half later.

As to the immediate political objective of the ‘dictatorship of the proletariat, how
could the ‘community’ of producers be organized without falling into the kind of prob-
lems that Adam Smith had already pointed out with respect to the control in joint-stock
companies of his time such as the East Indies Company? That is, what would pre-
vent those in control of the economic processes (production, distribution, etc.) from
using that power for their own benefit, possibly hindering the interests of the rest of
society? How could a network of controls with various hierarchical levels function so
as to coordinate the productions of the large firms in this new society and avoid the
generation of antagonistic interests related to the inevitable division of labor between those who work in various levels of direction and subordination? With what degree of centralization would the planning and most fundamental decisions related to it be done? How could the supply in practice be made to adjust to demand without the help of markets without imposing all sorts of rationing?

It is obvious that Marx cannot be blamed for not having designed in detail how the "collective society based on common ownership of the means of production" (Gotha, MECW 24, 85) would function. But it was feasible to at least consider two extreme cases and analyze what factors could make the actual process go in one direction or the other (or an intermediate one). On the one extreme is the ‘libertarian’ form of associated producers in which collective decisions are taken by means of some form of ‘democratic’ mechanism that is different from that of ‘bourgeois democracy’ (in which the great disparity of wealth enables the rich to ‘purchase’ by different means—such as publicity, campaign finance, deficient education, etc.—the favor of voters and thus actually have a ‘plutocracy’). On the other extreme is the formation of a new dominant class that handles the state machinery and benefits from the control it exerts over the collective means of production. In this extreme there would be no private firms, and hence no countervailing power on the part of powerful entrepreneurs and capitalists to restrict the freedom of action of the dominant governing class. The latter could thus avoid the democratic procedures in the decision processes, and would thus be incompatible with the project of forming a classless society, one with no class antagonisms.

There is sufficient evidence to indicate that what Marx had in mind was the ‘libertarian’ and democratic variant, with the important exception that the (minority of) ex-capitalist entrepreneurs would be excluded from this democratic process, at least in the first phase of the new society. We can say that Marx was sympathetic to the anarchists’ aim of doing away with authority both at the political and the economic levels insofar as the final objective of the ‘second phase’ of communist society was concerned, but not at all regarding the political process that would lead to the ‘dictatorship of the proletariat’ and the ‘first phase’ of communist society in which the proletarian state would ensure the disappearance of all vestiges of Capitalism. On the other hand, Marx never actually made an estimate of how long that first phase could last, so the second phase could in practice be postponed to a remote future and hence relegated to the category of a ‘dream’ or wishful thinking.2

When the Paris Commune ended tragically in 1871 Marx wrote (in his role as member of the General Council of the International Working Men’s Association, or First International) that the Commune wanted to transform the means of production into “instruments of free and associated labour” and in ‘Communism’ “united cooperative societies... regulate national production upon a common plan, thus taking it under their own control, and putting an end to the constant anarchy and periodical convulsions which are the fatality of Capitalist production” (Civil War, 335). After a century and a half of historical experience we now know that the simple mandate of regulating national production upon a common plan hides gigantic complexities if attempted without the aid of functioning markets, and that even with their help no country has been able to avoid the appearance of deep financial crises and protracted

2Such a ‘dream’ brings to memory that notable sentence of Rousseau (quoted above): “He who dares undertake to give institutions to a people ought to feel himself capable of changing, so to speak, human nature.” It also brings to mind the fact that throughout history great popular movements, including religions, have been motivated by such ‘dreams’ of a future better life, even if it was an after-life in ‘heaven.’ Many people learned and took to heart ‘Marxism’ in the 19th and 20th centuries as if it were a (secular) religion.
recessions now and then. Hence, there was a great dose of naive willfulness in what Marx proposed, even in 1871.

The revolutionary transition that Marx foresaw and proposed was supposed to take place almost simultaneously in advanced capitalist nations in which there were already many large firms in which control was exerted either by its private owners or by a management structure. How would it be possible to ensure that the transition State, after expropriating all the owners and ensuring the loyalty of the management structure, not come to represent the interests of the managers of large firms and the functionaries of other government agencies? If such were the case, the transition State would become a permanent State controlled by a new and very powerful dominant class. The complexity of the historical process tends to belie all medium and long run prognostications. It is now a fact that not one of the countries in which Communist-inspired revolutions were successful in the 20th century was advanced in its Capitalism, in stark contrast with Marx’s vision. Russia was scarcely industrialized in 1917 and 80% of its population were peasants that were mostly illiterate. And China was even less developed in 1949, when the Communist Party finally took control, after 12 years of wars, first against the Japanese imperial invaders and then against the Nationalist forces. In both Russia and China communist parties were able to take over political power due to catastrophic situations triggered by successive world wars fought between blocks led by imperialist powers fighting for supremacy. It is also a fact that in both the Soviet Union and the People’s Republic of China dominant and extremely centralized bureaucratic classes were formed that exerted total power over civil society, and that a few decades later they both allowed and encouraged the emergence of privately owned and privately managed firms.

Marx prophesied that “the credit system will serve as a powerful lever during the transition from the capitalist mode of production to the mode of production of associated labour; but only as one element in connection with other great organic revolutions of the mode of production itself” (B3, 602). However, he also believed that “As soon as the means of production cease being transformed into capital (...), credit as such no longer has any meaning”, possibly on account of the fact that at the time almost all credit was for industrial or commercial firms that would cease being private under the ‘dictatorship of the proletariat’. Apparently, Marx believed that it would even be convenient to eliminate financial assets markets. Workers would presumably simply save by not spending a portion of the purchasing power materialized in a certificate they would receive for their hours of work, and which could be spent in future periods. He may have had something analogous in mind for firms, although he never put it in writing. But all these cases seem to indicate that he intuitively simplifyed a simplification of economic life that was hardly compatible with the further increase in the productive forces that he claimed would be achieved by the new society.

He proposed nothing less than the elimination of all markets, that is, the elimination of commodities, commodity-money and paper money, on top of the elimination of wage labor and capital in all its forms (money-capital, commodity-capital, productive-capital). He wrote: “If we conceive society as being not capitalist but communist, there will be no money capital at all in the first place, nor the disguises cloaking the transactions arising on account of it. The question then comes down to the need of society to calculate beforehand how much labour, means of production, and means of subsistence it can invest, without detriment, in such lines of business as for instance the building of railways, which do not furnish any means of production or subsistence, nor produce any useful effect for a long time, a year or more, while they extract labour, means of production and means of subsistence from the total annual production” (B2, 314).
In his view this would prevent the great disturbances that “may and must constantly occur” in capitalist society, “where social reason always asserts itself only post festum” (Ibid.). Furthermore, “after the abolition of the capitalist mode of production, but still retaining social production, the determination of value continues to prevail in the sense that the regulation of labour time and the distribution of social labour among the various production groups, ultimately the bookkeeping encompassing all this, become more essential than ever” (B3, 838). But the ‘value’ in consideration here would be fundamentally different from the value in commodity production (whether simple or capitalist) since it would merely be an accounting concept, as he clearly states. There would no longer be a monetary price fluctuating according to supply and demand around an equilibrium monetary price. There would no longer be a (relative) value as an ‘attractor’ for (relative) prices as in Simple Commodity Production, nor a (relative) ‘regulating price’ as an ‘attractor’ for (relative) prices as in Capitalist Commodity Production (‘production price’ or ‘modified production price’).

This is clearly manifested in Gotha where – in reference to the first phase of communist society – he writes: “Within the collective society based on common ownership of the means of production, the producers do not exchange their products; just as little does the labour employed on the products appear here as the value of these products, as a material quality possessed by them, since now, in contrast to capitalist society, individual labour no longer exists in an indirect fashion but directly as a component part of the total labour... the individual producer receives back from society –after the deductions have been made– exactly what he gives to it. What he has given to it is his individual quantum of labour” (Gotha, 85-6). There would be an “exchange of equivalents” as in Capitalism, but here this equivalence would hold for each individual exchange, whereas in Capitalism “the exchange of equivalents in commodity exchange only exists on the average and not in the individual case”, because in this case the (relative) regulating value is merely the attractor of (relative) market prices, that is, (relative) market prices fluctuate around equilibrium (relative) prices according to the vagaries of supply and demand, which rarely coincide. Operationally, each worker in the future society would receive “a certificate from society that he has furnished such and such an amount of labour (after deducting his labour for the common funds), and with this certificate he draws from the social stock of means of consumption as much as the same amount of labour costs” (Gotha, 86).3

In the first phase of communist society there would be “no class distinctions, because everyone is only a worker like everyone else.” However, since the level of the productive forces would still be a conditioning factor it would be necessary to accept “the unequal individual endowment and thus productive capacity of the workers as natural privileges” and hence not all workers would have the same income. Also, “one worker is married, another not; one has more children than another, etc., etc. Thus, given an equal amount of work done, and hence an equal share in the social consumption fund, one will in fact receive more than another, one will be richer than another,

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3Some of the concepts Marx developed for his first phase of communist society gathered ideas that the British-American reformer John Francis Bray (1809-1897) developed in his early book Labour’s Wrongs and Labour’s Remedy. Marx quoted extensively from it in Poverty and also in Theories. Bray proposed a revolutionary transformation that eliminated privately owned firms and instituted community ownership of the means of production, including land. He also proposed a two-stage process quite similar in many regards to Marx and Engels’ project. Marx’s misinterpretation of some of Bray’s proposals in Poverty is surprising and may have been due to his not yet fully commanding the English language. The only instance Marx quoted from Bray in Capital is when (in a footnote) he criticizes Proudhon for having developed a petty-bourgeois socialism that was long before “attempted with much better success by Gray, Bray, and others” (B1, 79, footnote 1)).
etc... But these defects are inevitable in the first phase of communist society” (Gotha, 86). Here, “the same principle prevails as in the exchange of commodity-equivalents: a given amount of labour in one form is exchanged for an equal amount of labour in another form.” Hence, in Marx’s conception of the first phase of communist society there would be both a reunion of the associated workers with the means of production—which would be owned collectively by the workers—and a return to ‘exchange’ proportions based on the labor apportioned. But there would no longer be a market-based concept of value as a foundation of transaction proportions and merely an accounting concept of value calculated by a central planning agency.

For Marx the expropriation of the owners of the means of production by the worker-dominated State would simply be the culmination of the process already commenced by the process of “centralization”, or “expropriation of many capitalists by few” (B1, 750). To the end, he perceived an increasing polarization between very few very rich capitalists and the exploited and poor great majority: “Along with the constantly diminishing number of the magnates of capital, who usurp and monopolise all advantages of this process of transformation, grows the mass of misery, oppression, slavery, degradation, exploitation” (Ibid.). But this polarization would also increase “the revolt of the working class, a class always increasing in numbers, and disciplined, united, organised by the very mechanism of the process of capitalist production itself” (Ibid.), and reaching a situation in which the “Centralisation of the means of production and socialisation of labour at last reach a point where they become incompatible with their capitalist integument. Thus integument is burst asunder.” Although Marx’s project-prognostication was impeccable from a logical point of view, and although some of the central tendencies he visualized were correct, a century and a half of evolution of Capitalism in the most advanced capitalist countries of his time belied his belief that Capitalism was approaching its demise. Marx was unable to perceive that each developed capitalist country would tend to be transformed gradually, adopting an increasing amount of state intervention in the markets and anti-cyclical policies that would alleviate a great part of the unemployment problem, with ever more firms of all sizes, including an infinity of small and very small firms that would gradually absorb the growth of the labor force despite the great increase in productivity that continuously expels labor from many productive processes and despite the fact that very many firms go bankrupt every year. Thus there was in these countries a significant increase in the real incomes of most sectors of the working class, alongside that of capitalist entrepreneurs, even taking into account the incredible waste of resources accomplished by two world wars, the cold war, and the huge investment in unproductive military hardware. There has also subsisted an important sector of simple commodity producers (independent workers who are not wage workers and do not employ wage workers) that produce all types of services.

We can say in Marx’s favor that the economic process of the last century and a half has tended to concentrate economic and political power in each country in the hands of an elite formed not only by the greatest capitalist entrepreneurs but also the most powerful executives of the largest corporations and national state bureaucrats. We can also say in his favor that the evolution of Capitalism has tended to polarize at the international level the power and wealth of the elites of the (more or less advanced) capitalist countries and the vast majority of the population of peripheral Capitalism. But Marx’s vision of the tendency towards Communism in the most advanced capitalist countries was belied by historical reality, having the dominant classes learned various control techniques of the subordinate population that have been especially effective since the second post-war, and which have irradiated mainly from the intelligence
agencies of the U.S.A. Hence, there has been an increasing mismatch between the doctrinaire vision of Marx’s epigones –Engels included– and reality. As we will see in Part V, this mismatch would eventually generate tensions in the socialist movement, particularly in Germany (where it achieved its greatest penetration in the working class in the last decades of the 19th century). Owing to the doctrinaire character that Marx and Engels themselves tended to impose on the project they had jointly created –and also to the imposing theoretical structure that Marx had built– a long period elapsed before these tensions became explicit. Their politically motivated ‘scientific socialism’ confused Marx’s scientific inquiry into the functioning of Capitalism with his political project, one that combined apparently feasible, rational, and just revolutionary changes with utopian and millenarian final objectives.
Part V

Conclusion
Chapter 23  FINAL THOUGHTS

The entrepreneur and his work in Capitalism

Marx failed in not making his theory reflect the positive and important contribution of entrepreneurs in the private firms of Capitalism to the end result of the production process and hence to the value of output, even if he was correct in his demand that legislation place a limit to their greed by prohibiting child labor and reducing the working day and the intensity of labor. Although Marx accepted as a fact the work of the entrepreneur and elaborated on it in his texts, it did not appear reflected in his formal representation of the economic process. If it had been reflected, he would not have been able to claim that property incomes are based on the ‘unpaid’ labor of industrial wage workers. And this was the basis for the absurd implication of the ‘esoteric’ part of his theory: that if the capitalist entrepreneurs were expropriated and transformed into (normal) workers then output would increase.

Marx recognized the entrepreneur’s work as a sort of ‘orchestra director’, which he even characterized as ‘productive’, and also as performing ‘vigilance’). The latter he associated to production regimes in which prevailed ‘antagonistic’ relations in the production process. One can justifiably argue that it is possible to delink the work of vigilance from the entrepreneur and represent it as a kind of complex wage labor. But the many other aspects of the entrepreneurial work that are summed up in the ‘orchestra director’ imagery were completely absent from his analytical theory. Let us mention the work of planning the investment or disinvestment of capital, the selection of production techniques and the workers that are to use them, the bearing of unquantifiable risk of capital losses --if he is an owner or stockholder-- or loss of control --if he is not--, the bearing of uncertainty in his retribution as a director or owner, the work of organizing the production and circulation (sales) processes in their totality and adapting them to changing circumstances. All these tasks, some of which are notoriously complex, were absent from his theory of surplus value. Only non-entrepreneurial work was explicitly reflected in his analytical framework, only non-entrepreneurial work was considered ‘labor’. Hence, all output (net of intermediate inputs) that was not destined to the consumption of wage workers was a surplus product, a product produced by wage workers and (illegitimately) appropriated by capitalist entrepreneurs. And the wage labor that had (solely) produced it was surplus labor, the source of surplus value. And since aggregate profits, interests, and rents were a reallocation of aggregate surplus value, the entrepreneurial work was similarly not taken into account in the ‘exoteric’ part of his theory. There, the incomes of owners (profits, interests, rents) resulted from the excess of sales receipts over the payment of wages and other inputs in each firm, and was linked to the aggregate surplus value generated in the industrial sector through the circulation process and the process of profit rate equalization among sectors (inter-sector flows of capital).

Entrepreneurial work includes many of the most delicate aspects of the productive process and its exertion is motivated by the expectation of profit (the ‘bottom line’ in financial accounts) and the avoidance of losses. It requires searching for inputs of lower costs for a given output level. And entrepreneurial work is important both when the firm is (at least partially) owned by the entrepreneur as when ownership is dispersed among thousands of stockholders and we are in the case of corporate Capitalism that Berle and Means documented in the case of the U.S.A. Marx constantly mentions the profit motive of capitalists, their desire to valorize capital, etc. but he does not consider the practical difficulties to making a wage income (or a ‘certificate’) play the same role
as the expectation of profits in encouraging the exertion of the huge personal effort that is required for the organization and management of the whole production and circulation process, not overlooking a single detail that may hamper it and ensuring that it is carried out so that the end product can be sold with a positive profit that compensates both the efforts and the unquantifiable risk of losing wealth (or prospective wealth in the case of executives that are autonomous with respect to stockholders).

Marx highlighted the paradox that in Capitalism there was planning within firms whereas outside the firms reigned anarchy due to the caprice of markets. But that within-firm planning (as an activity) was absent from his formal representations, and although he was right in finding flaws in the ‘invisible hand’ argument for laissez faire he clearly underestimated the positive role of markets as well as the possibility of improving their functioning through appropriate regulations and anti-cyclical policies. He was very conscious of market failures, manifested especially in the periodic crises of the industrial cycle, but he was deeply utopian concerning the practical feasibility of completely replacing markets by centralized planning of state production and concerning the convenience (for workers) of replacing the (to some degree existing) decentralization of power in Capitalism by a centralized State power that, as we have argued, is inevitably controlled by a new bureaucratic dominant social class whose material interests are antagonistic with those of workers and who is even more empowered than the capitalist State in subduing their claims.

Marx’s equations did not reflect the entrepreneur’s role in planning and organizing, nor in commanding and controlling the production process (which we have called POCC entrepreneurial work), with all that it implies insofar as using the available means for introducing rationality in the production processes and technological and organizational innovations because it is motivated by the expectation of profit. Such were the organizational forms that in the development of Capitalism resulted in increasing the living standards of successful capitalist entrepreneurs prima facie, but also in the long run of wage workers, to the extent that capital investments increase and entrepreneurs compete for (a limited number of) workers. The POCC labor of entrepreneurs (or their ‘orchestra director’ role in Marx’s terminology) in capitalist society constitutes a fundamental constituent of the prices of outputs through profits. And this work has a central aspect (that Marx completely omitted from his analyses) which is related to the firm’s adaptation to a changing economic, social, and political environment; a process in which the firm’s survival (along with the net worth of its owners and the position of power of its executives) can crucially depend on the speed of adjustment and the absence of ‘cognitive dissonance’ in each conjuncture in adapting the firm’s activity in order to avoid losses. Although Marx accepted to a certain extent the importance of the entrepreneur’s specific activity in his verbal argumentation, the foundation of his concept of surplus value on the unpaid work of wage workers and its role as source of all incomes based on ownership (and control) are clear evidence that he never integrated this entrepreneurial work into his theoretical structure. The ‘orchestra director’ role of entrepreneurs to which he only paid lip service has the effect of increasing not only profits but also production and its value, the firm’s competitive advantages and its probability of survival and expansion. The historical process shows that through introducing changes in organization, adapting it to the changing environment, and introducing innovations in the productive and commercial processes, the POCC labor of the entrepreneur has also made possible increases in real wages and improvements in the conditions of labor, including the reduction of the work day. Though this was all recognized by Marx to some extent in his texts, it was not integrated into his analytical theory.
On the other hand, one must recognize that the POCC labor of entrepreneurs often has a negative, predator, social character, when it seeks to increase profit by adversely affecting other firms or consumers, or the populations of other countries (sometimes even with the collaboration of their governments). One of the socially negative ingredients of POCC labor in the largest firms consists is seeking to annul competition by means of (usually informal) oligopolistic agreements with firms that produce the same or similar goods so that they can share the joint oligopolistic profits—which are higher than what they would get if they competed— and are actually snatched from the incomes of the rest of the population. Other such socially negative practices are lobbying to deviate public expenditure towards their own products in detriment of the interests of the rest of the population, generating corruption in connivance with politicians and/or government officials, lobbying to avoid, annul, or blunt anti-monopoly or pro-worker regulations or to stimulate enterprises (e.g. wars, drug trafficking, etc.) that constitute ‘public bads’ for the immense majority. Hence, the recognition of the positive aspects of POCC labor from the point of view of society as a whole by no means implies leaving unnoticed the many and varied negative aspects, as well as the need to combat them. But this leads to objectives that are very different from the complete substitution of private firms by centrally planned state production proposed by Marx. The historical experience proves that the latter objective, instead of improving the prospects for greater social justice and greater welfare for the population in general and the poorest in particular, often tends to the very opposite.

One of the objectives that can and should be sought for is social mobility based on individual innate capacities. There has always been some degree of social mobility, but in modern societies there is more than in the past. Hence, it is quite possible today that in the course of his lifetime an individual belong to different social classes or strata from the point of view of his insertion in the economy (Marx’s main criterion, a secondary one being that of ‘class consciousness’). Among these we can mention being a student, an unemployed worker, an independent worker, a wage worker (in a private or public firm), an entrepreneur (in a small, medium, or large firm), an executive (in a small, medium, or large firm), a politician, a public servant, and a retiree. But just as some individuals change their economic insertion in a direction that favors their income and influence over others, there are others who at the same time do so in the opposite direction. Sometimes such mobility is due to a personal choice, but often (especially when they imply a fall in incomes) it is completely involuntary and may be highly traumatic. The position of a person’s parents is often one of the most important factors in explaining the individual’s insertion in the economic social structure and his prospects for an ascendant mobility because of the advantages or disadvantages it implies in such factors as access to quality education, better health attention, information, assistance from people favorably located in firms, universities, and government, etc. We believe it is necessary to focus on and disable the mechanisms that typically prevent potentially more apt individuals (in their youth) from having the opportunity of distinguishing themselves in social classes of greater economic or political power than that of their family of origin. It is necessary to propitiate equality of access to good education, to good medical care, to information, etc. Let us recall that some of these objectives were already present (albeit modestly) in the ‘Declaration of the rights of man and the citizen’ of 1793, which in Article 22 said that public education was a right for every citizen. But giving access to better education to individuals born in lower income households implies fiscal costs that those who have political power and enjoy the privileges of the existing situation try to avoid because it negatively affects them economically (and often socially as well, because relative advantage is often
highly valued by the higher strata of society). And it is the influences of individuals of the privileged classes on public policies that perpetuate avoidable inequalities (or even increase them). It is obvious that this structural phenomenon has adverse consequences on the welfare of the disfavored as well as on the efficiency of the economic system, since there is a (dynamic) misuse of the existing human resources.

Fighting against the obstacles imposed by the existing power structure in capitalist society is a difficult task. Marx and Engels were very pessimistic on the possibility of doing so successfully within capitalist society, and believed that it was necessary to aim for the destruction of the existing power structure by means of the elimination of the private firms on which the economic power of the capitalist class and political power in capitalist society are based. The historical experience has shown that the objective of achieving better public education and health assistance was feasible in a society in which private firms were banned. In fact, in both Russia and in China the literacy rate grew quickly after their respective communist revolutions. However, the disappearance of bourgeois power brought about the appearance of an absolute state power that in time consolidated into a dominant bureaucratic class endowed with its own interests that were antagonistic with those of the rest of society. And the management of all economic processes by the state (and the Communist Party) bureaucracy proved to be much less efficient then when the private management of private firms is combined with state management and regulation (as occurred increasingly in all the countries that continued being capitalist). This relative inefficiency was increasingly understood by the governing elite, but there predominated the fear of losing the power and privileges over the need to better harness the energies of its potentially entrepreneurial citizens, a need that floundered against a deep-rooted ideology that turned into a device for detecting and punishing heretics. We can express this conflict using Marx’s concepts: there was an increasing contradiction between the fetters of the bureaucratic production relations based on Marx and Engels’ political project, on the one hand, and on the other the need to increase the productive forces in order to face the increasing pressure from the imperial hegemon of the capitalist world.1 And the latter need could only be satisfied by reintroducing Capitalism. It is this contradiction that in the last instance led to the predominance within the leadership of the Soviet Union of those who aimed to increase their own power by dissolving the Union into its constituent parts (Russia, Belarus, Ukraine, Poland, etc.) and reintroducing private firms and Capitalism. An analogous (but much more controlled and gradual) process took place in China when towards the end of the 1970s private capital and private firms were reintroduced without the loss of political control by the Communist Party.

It is in the periphery of the capitalist world where political groupings still vindicating Marx and Engel’s political project survive, thus deviating political resources that could potentially be giving impulse to institutional reforms towards a more just and efficient capitalist society. Such groups appear to combine a magical, willful, and doctrinaire thought that ignores the historical experience with the search for power for its own sake (and the privileges it confers). They often look towards countries like Cuba, which having achieved a revolution inspired by Marx and Engels and having progressed in public health and education (as Russia and China did), are still run by a dominant bureaucratic class that obstinately keeps prohibiting the existence of private firms that could otherwise substantially increase the economy’s productivity, which is a requisite for increasing the living standards of the vast majority. As was the case of the Soviet Union, the elite’s fear of a diminishing internal political power makes its

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1 We specifically refer here to the passage from Nixon and Kissinger’s policy of detente in the early 1970s to the policy of confrontation of Reagan and Bush (Sr.) in the early 1980s.
economic power constantly fall in relation to that of the capitalist countries’ elites.

**Marx and the integration of the dispersed modern social sciences**

Aside from his contributing to an intersectoral form of analyzing the socioeconomic structure of a society which, as we have seen, can be easily represented by systems of dual equations, Marx supplied a huge mass of historical-empirical data and was able to highlight some of the fundamental tendencies of the capitalist society of the 19th century that still today seem surprisingly correct in many respects. Even having failed to foresee the future possibilities of Capitalism, and to comprehend the intrinsic difficulties in implementing the centrally planned economy that he proposed, his achievements are impressive. In particular, he gathered a plethora of historical, statistical, and institutional data, carefully analyzed the preceding theories of political economy, and used various social and economic aspects of the genesis and development of the approximately 500 years of Capitalism through its various phases (until, say, 1875) in a way that no previous thinker had done or, much less, organized into a comprehensive theoretical structure. As Leontief wrote before completely taking the ‘politically correct’ course that his circumstances demanded (i.e., not even mentioning Marx unless it was to vilify him): “Neither his analytical accomplishments nor the purported methodological superiority can explain the Marxian record of correct prognostications. His strength lies in realistic, empirical knowledge of the capitalist system” (Leontief 1938, 8). Furthermore:

However important these technical contributions to the progress of economic theory, in the present-day appraisal of Marxian achievements they are overshadowed by his brilliant analysis of the long-run tendencies of the capitalistic system. The record is indeed impressive: increasing concentration of wealth, rapid elimination of small and medium sized enterprise, progressive limitation of competition, incessant technological progress accompanied by the ever growing importance of fixed capital, and, last but not least, the undiminished amplitude of recurrent business cycles—an unsurpassed series of prognostications fulfilled, against which modern economic theory with all its refinements has little to show indeed (Leontief 1938, 5).

The theoretical structure Marx developed had great originality even if it linked magnificently with the classical Political Economy he had studied in depth, as we have seen, albeit with some biases that went in the direction of his ideological postulates.² This theoretical structure had important failures, principally in its ‘esoteric’ portion that with his values and surplus values ran parallel to the ‘exoteric’ theory of equilibrium prices, profits, rents, and wages. If we eliminate the ‘esoteric’ portion of his theory there is left a very rich theoretical structure that should be reevaluated objectively and that has inspired various social thinkers (economists, sociologists, political scientists, anthropologists, etc.) that distanced themselves from the sterile and dogmatic ‘Marxism’ of most of Marx’s epigones. Some famous economists (like Schumpeter) drew inspiration from elements of the ‘exoteric’ portion of Marx’s theory, avoiding an explicit reference to the source because in the most dynamical capitalist countries Marx and ‘Marxism’ had become taboo, especially after the Russian Revolution and even more in the post-World War II era, when the U.S.A. reached the undisputed hegemony in the capitalist universe.

²I especially allude here to Marx’s choosing to ignore certain crucial aspects of the works of economists like Cantillon, Turgot, and Ramsay, all of which he had read.
Marx’s theoretical framework did not include an analytically clear formal representation of the relation between changes in the real wage and changes in the supply and demand of labor power. However, it must be admitted that this absence did not prevent Marx from using common sense in the conceptual analysis of these phenomena, as we have seen in several chapters of Part II of this book. The simplification of assuming a given structure for workers’ consumption basket which was based on habits and not formally related to prices and incomes was a deficiency of his general framework that Marx shared with all classical economists and was only remedied by the ‘neoclassical’ (or ‘marginalist’) theories by means of the use of mathematics for the representation of the decisions of economic agents based on their preferences and their constraints. As we have seen in Chapter 20, this led to the construction of demand functions for commodities and supply functions for productive services. We there showed how Walras’ theory can be expressed within the dual equations of quantities and prices we have used for a formal exposition of Marx’s theory, as long as we restrict its generality so that we can visualize the class structure that Marx and the Classical economists always had in mind, a framework that is still very useful for the representation of the structure of society in a given period of time. However, those methodological advances of the last decades of the 19th century were unable to model some of the most serious problems of Capitalism, in particular, that of the industrial cycles, the contractionary phase of which sometimes becomes a depression that generates massive and persistent unemployment. This led in the late 1930s to the invention of Keynesian Macroeconomics as a way to address aggregate economic phenomena in a more ad hoc way, a way that was less constrained by the (to some degree apparent) methodological advances of the neoclassical economists.

Various mainstream economists did all they could to place obstacles on the attempts to build a foundation to economic theory by means of some variant of ‘imperfect’ competition. They backed instead a theory of ‘perfect’ competition that was imposed as an unnecessary corollary to the important and valid contribution of the principal neoclassical economists that were nevertheless defective in certain important facets of reality that were crucial to the welfare of most people. Walras had developed a theory that linked the demands of goods and the supplies of productive services to their prices, and it started with the individual decisions of each and every individual in a very general framework. The simplification that the concept of ‘perfectly free competition’ introduced allowed for the derivation of a consistent model of how the complex interactions of economic agents by means of market transactions can result in the allocation of resources in such a way that all the demands of all social agents are satisfied if the equilibrium prices hold. This type of model satisfied the practical needs of various public institutions of the post-World War II world. Even though it was erroneous as a theory of the complex economic interactions of the modern industrial world, this model was useful on ideological grounds both for the capitalist world and for the ‘real Communism’ of the Soviet bloc, albeit for different reasons (and to very different extents). In the capitalist world, it was useful for hiding the hierarchical nature of the relations between the social classes and the inequalities in incomes, power, and political representation behind the trivial idea that the set of all consumers are necessarily the owners of all the productive factors and firms. Hence, the various forms of incomes would be allocated to individuals in some very complex way that was too difficult to reflect in the general theoretical model, especially considering that any wage worker can simultaneously be a stockholder or owner of land that he leases out, etc. And in the USSR a similar technocratic theory very similar to the one prevalent in the

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capitalist world also had some utility for the dominant bureaucratic class in deviating attention from their huge privileges and their absolute control over all the levers of power while millions of citizens spent hours in queues every day in order to purchase scarce consumption goods. For it is a theory that altogether does away with the notion of social class because all the agents represented are worker-consumers-owners. Hence, there is no dominant class, capitalist or bureaucratic.

In our opinion, the only scientific way of surpassing both Marx’s theory and that of mainstream economics is to construct an integrated framework for the social sciences, now basically disconnected and entrenched behind solid academic-corporate barriers. Such an integration is possible by means of the development of a theoretical framework that goes far beyond human interactions through market transactions. Such a theoretical framework must centrally include the interactions produced by what Microeconomic theory calls ‘externalities’. The very name of this concept, which encompasses the direct influence (that is, not market mediated) of individuals’ or firms’ actions on the welfare of others, highlights the narrow atomistic worldview that prevails in the mainstream, where (except where it is too evident that they are not, as in the case of ‘pure public goods’) individuals are atoms that jointly compose a social system and exclusively interact through the links established by market transactions. The so-called ‘public goods’ and ‘public bads’ (variants of ‘externalities’), where what is a ‘good’ for some may be a ‘bad’ for others, should play a central role in any theory that is capable of coherently representing the empirical thread of human social history. The latter includes the prominent role of the ruling (or governing) classes (ever since they emerged several thousand years ago) in organizing the production of large public works (like dykes, canals, irrigation systems, roads, etc.) and armed forces. In contrast, in mainstream economic theory such concepts are usually limited to playing a marginal role in general, and a central one only when special and partial topics are addressed, such as public parks, traffic congestion, pollution, etc.

One way of modeling positive entrepreneurial profit is the theory of monopolistic competition, where each entrepreneur-capitalist is a monopolist of a particular variety of a product and earns monopoly profits. But for this model to represent (in a stylized way) a reality that is much more complex it is at least necessary to leave aside the contrived and irrelevant ‘long run equilibrium’ in which the massive and instantaneous entry of new monopolists–each with its exclusive variety–again eliminates profits. Entrepreneurial profits can also be modeled using the theory of general oligopolistic equilibrium. Marx wrote Capital more than half a century before Edward Chamberlin, towards the end of the 1920s, developed his theory of monopolistic competition, and a century before Dixit and Stiglitz (1977) gave it a rigorous mathematical formulation based on a convenient theory of index numbers. Something similar can be said of general oligopolistic equilibrium theory which, though it is a simple extension of the duopoly theory elaborated by Cournot before Marx’s time, also needed the passing of time before it could be rigorously re-elaborated ⁴. But even though such models are useful for understanding certain important aspects of reality, they must be considered partial components of a theory of the society in which we live, with its irreversible evolution and its uncertain future.

The modern models of monopolistic and oligopolistic competition in a context of general equilibrium can be used for the extension and deepening of much of the analysis of Capitalism initiated by Marx. They have realism in allowing for price setting in a framework in which there are elements of both competition and monopoly, a matter on which Marx reflected upon in Poverty. The main convenience of the standard model

of monopolistic competition is that it can reflect in a very stylized way extremely complex mechanisms related to the strategic behavior of firms that compete to some extent. Among such mechanisms is R&D, the introduction of technological innovations, the creation of new designs that draw the attention of partially captive customers, advertising, etc. On the other hand, admitting that the monopolistic competition model is a convenient way of modeling a much more complex reality in which many firms and industries interact and do so with more or less oligopolistic behaviors, and in which the government’s activity plays an often essential role, there is a truly vast field of potential ways to pursue the analysis of reality initiated by Marx in the ‘exoteric’ part of his theory. This could lead to an interesting integration between economics and sociology, since the concept of social class would not be cast aside, as in mainstream economics; and neither would the economic mechanisms that are behind much of the hierarchical interaction between individuals and human groups be disregarded, as they often are in sociology.

Through the introduction of ‘public goods and bads’ produced by the government into the monopolistic competition model it would also be possible to combine much of economics and sociology with political science, since the structure of power has been intimately related to the State’s production of these special ‘goods’ in every human society with a minimum of complexity. In the chapter that models Capitalism of Escudé (2017), for example, a model of monopolistic competition expanded by a government sector producer of ‘public goods’ is used. Each capitalist-entrepreneur exerts POCC labor that, at least beyond some degree of exertion, affects his welfare adversely (as in the modeling of the consumer/worker in economics). This leads to a trade-off between the amount of entrepreneurial work exerted and the size of the monopolistic profit obtained. The politicians that control the government exert a tax-gathering POCC work that also adversely affects their welfare and use the proceeds to finance the production of ‘public goods’. The resulting system of equations reflects a hierarchical structure of capitalist society with full employment, with wage workers at the bottom, capitalist-entrepreneurs that decide over the production and circulation processes of private goods, and governing-politicians that decide on the State production of public goods (and ‘bads’). The politicians’ decisions, taken with full knowledge of how the private sector functions, condition the decisions of entrepreneurs, and both the politicians’ and entrepreneurs’ decisions condition the decisions of wage workers. Thus, the asymmetrical and hierarchical power relations between the individuals of these three big classes are reflected. Also, there is a fundamental role for the production of public goods and bads, i.e., goods or services that affect all individuals (some positively and others possibly negatively) but are not apt for being produced through market incentives by private firms and can only be produced in the government sector. Among such public goods (or ‘bads’, for individuals or groups that are negatively affected) are, for example, laws and regulations, police control, and the functioning of a military establishment. The members of each class (normally) make their decisions taking fundamentally into account their own interests. For example, a military establishment can defend against external aggression, but it can also oppress the internal population if this is convenient to the interests of those at the top of the hierarchy. This simple static framework can of course be extended in many different directions. It could also be extended to a much more complicated dynamical framework or to an international framework where there are hierarchies of national (or imperial) powers. Planning, Organizing, Command, and Control: all these ingredients play a crucial role in the functioning of any stratified/hierarchical society, whether or not market mechanisms are in place. When they are, as in Capitalism, they have complex and subtle
interactions with the market mechanisms.

**Marx’s methodology and worldview**

A distinctive characteristic of Marx’s theoretical construction is that he developed in parallel: 1) *models* that through explicit hypotheses simplify the matter studied so as to be able to focus on the interrelations between a limited number of relevant variables, and 2) a *historical-genetic theory* of Capitalism. In this dual construction, models are always auxiliary to the theoretical construction and hence past and contemporary empirical reality always has primacy. For Marx wanted to explain the development of human society in its various dimensions, and tried to integrate anthropological, economic, sociological, political and ideological aspects that the later development of the social sciences has tended to keep strictly separate. The exegesis of Marx’s theory (especially the critical exegesis) has tended to focus on certain aspects of his models (often highlighting its defects) and to ignore the important contributions Marx made to the development of socioeconomic theory.

Marx built a coherent theory of the genesis and development through time of the capitalist ‘mode of production’, including its interaction with the then existing pre-capitalist modes of production. He also made very significant contributions to the development of models. Such was the case of his models of Simple and Extended Reproduction of capital, which meant a great leap forward with respect to the Physiocratic scheme that he took as starting point. Marx’s models of Extended Reproduction were a fundamental precedent to what is today known as multisectoral economic growth theory. Unfortunately, the millenarian-revolutionary aspect of Marx’s thought— to some extent simplistic, naive, and erroneous— was one of the causes of the explicit repudiation of valuable aspects of his theory by many social thinkers of the last decades of the 19th century and the 20th century who could have been more open to his more lasting achievements had they not been so impregnated by the radicalism and millenarianism of his political project. But it is also true that the mainstream of international academic research was highly-influenced by the powerful conservative and reactionary forces for which any search for progressive change (in the most positive sense of ‘progressive’) is dangerously ‘radical’. Marx characterized as ‘vulgar’ those economists who became standard bearers and servants of the predominant interests. And he denounced those who became enemies of free scientific research on account of the fact that the “peculiar nature of the material it deals with” (B1, 9) affected their particular interests or the interests of those that supported or inspired them. In this sense, a good portion of mainstream economic theory is certainly ‘apologetic’ (or ‘vulgar’) when it continues to place at its core concepts such as ‘perfect competition’, ‘Pareto optimum’, and ‘benevolent’— or ‘altruistic’— government, while it relegates to a second level concepts such as social class, ‘externalities’, ‘public goods’ (or ‘public goods’), ‘monopoly’, and ‘oligopoly’.

In contrasting to what Marx did, economic theory has tended to construct a gigantic collection of models and leave to historians the socioeconomic-political specificity of the various societies in which man has lived and, in particular, the specificities of Capitalism. Models distinguish variables dichotomously: some are exogenous and others endogenous. Given the values of the exogenous variables (or their path through time) the equilibrium values (or paths) of the endogenous variables are obtained. And it is well that it should be so, since this is how the language of mathematics works. But in the reality of social development (in all its aspects) that is the object of study of social science it is not possible to make this distinction. The flow of this reality is the result of the interaction of millions of individuals (in the context of changing natural
phenomena) and one can only make the relevant distinction (that is fundamentally different) between the aspects least susceptible to modification by human beings (like the climate and the tides) and those that can be modified to some extent. When the theoretician builds a model in which certain ‘variables’ are considered exogenous, he makes a drastic simplification of that reality with the purpose of getting a better representation of the interrelations between a few (but important) aspects of the material studied. The construction of a worldview that guides the adoption of stances with respect to socioeconomic-political reality so that one can act upon it can be aided by such models, if they are pertinent. The use of such models and much additional information on the past and the present leads to a mental construction of an image of the social process in its complexity and the subject’s insertion in it (as an individual and/or as a member of a collective). And this construction inevitably contains various ideological components that almost always go beyond the pure rationality usually recommended for scientific research. Marx’s theoretic-practical conception should be understood in terms of the predominance of a praxis that was oriented by his worldview. The weakest aspects of that conception was the rejection of the possibility of substantial reforms of Capitalism (which Marx considered either a deception or self-deception), the invention of an alleged ‘historical task’ for the working class of becoming the dominant class in order to eliminate the foundations of Capitalism, the personal and group objective of building a political party that claims the representation of the working class and seeks to perform this ‘historical task’, the absence of analysis and evaluation of how feasible and desirable it was to substitute markets and private firms by centrally planned state production, and the millenarianism of his vision of the ‘Communism’ to be reached in the long run.

One of Marx’s achievements was his ‘materialist conception of history’, as long as we are willing to segregate it from his ‘millenarian conception of the future’, as we have done in Parts I and IV of this book. Another is his theory of the existence of a correlation between the worldview of (the majority of) social subjects and their insertion in the class structure that defines their material interests. This correlation could be higher or lower according to concrete circumstances and, in particular, according to the greater or lower intensity of conflict (or ‘class struggle’ in Marx’s language) in the situation. And Marx considered that even scientific praxis was negatively affected by political conflicts based on the antagonism of economic interests. Many of his correct intuitions in this field were, however, tainted by his erroneous belief that it was the ‘task’ of the working class to eliminate Capitalism and his erroneous conviction that the end of the capitalist mode of production was approaching. He would have been closer to what was possible (and beneficial for the working class) if, instead of seeking the elimination of Capitalism, he had strived for the implementation of reforms that corrected its main defects without eliminating its achievements (that he so highly praised). But beyond these defects, Marx’s position on the virtual impossibility of reaching objectivity in the scientific treatment of social matters, especially when social and political conflicts are increasing in intensity, made much sense and was based on empirical data. Well known are the obstacles early free-thinkers had to surpass (or were unable to, like Giordano Bruno who was burned at the stake in 1600) in order to advance science, and the book-burning and expulsion to exile of scientists or intellectuals in so many countries and epochs (including our own). Darwin and Wallace’s evolutionary theory of the species faced enormous resistance during Marx’s lifetime in the most advanced country in the world. And there is ample evidence that, long after Marx’s death, in the wake of the Cold War the predominant interests in the main imperial powers (U.S.A. and the USSR) had an enormous influence on the way the social sciences were
developed in their respective spheres of influence during most of the second half of the 20th century. It is necessary to understand the development of the social sciences in the capitalist world within this context, since to a great extent they took shape in the U.S.A. during the Cold War. It was also within this context that Marx’s creative thought was frozen into the sterile and dogmatic ‘Marxism-Leninism-Stalinism’ of the official Soviet ideology. This ideology was so sclerotic that when Nixon and Kissinger’s détente turned into Reagan and G.H.W. Bush’s confrontation, the Soviet gerontocracy was only able to muster Gorbachev’s timid reforms. And the pressures on the central structure of Soviet power rapidly led to the most brutal predation of state assets under Yeltsin’s leadership by a small band of gangster ‘oligarchs’ who pillaged the Russian people’s savings that had forcefully been accumulated with great sacrifice.

The theory of surplus value was not a good foundation for the theory of exploitation in Capitalism. But there was a need for a theory that explained the asymmetrical and hierarchical relations that in Capitalism replaced the more transparent ones of societies based on slavery or serfdom. Marx saw in his re-structuring of Ricardo’s labor-value theory the venue for his development of the theory of surplus value, but found that the equilibrium (or ‘regulating’ or ‘attracting’) prices could not be the values (as Ricardo also knew). To explain labor exploitation in Capitalism he turned to the idea that industrial capitalists made sure that wage workers would work during a work day substantially longer than that strictly necessary to produce the means of subsistence that they could purchase with their wages. It was a witty idea. It countered the notion that profits were due to a mark-up that took place in the sphere of circulation of commodities, replacing it with the notion that, like any buyer of a commodity, the capitalist had the right to consume the use value of labor power after purchasing it. The part of the working day in which this consumption exceeded the part necessary to produce the means of subsistence of a wage worker, i.e. surplus labor (in excess of necessary labor) was the time during surplus value was produced. However, that notion was built on the erroneous idea that the capitalist-entrepreneur did not make any positive contribution to the production process, that he was a simple parasite that could be eliminated from the process without any adverse effect on the end product. For it implicitly reflected the idea that the capitalist’s role as ‘orchestra director’ could (like his role as supervisor) be replaced by a skilled worker that responds to the directions of a planning agency without any adverse effect on aggregate production. There was a very naive underlying notion of the possibility and desirability of replacing the functioning of markets and private firms by the collectivization of the means of production and the central planning of the whole economy. One must admit, however, that there had never been any historical experience with this way of organizing the economy, and that there was more realism in trying to use the State to make great institutional changes than in the vision of anarchists that the State must first be eliminated so that the desirable changes can take place in some spontaneous manner.

The erroneous prediction of Marx and Engels and their epigones of a not very distant and almost simultaneous socialist-communist revolution in the most advanced capitalist countries of Europe floundered upon the reality that it was in the weakest links of the chain of social formations with (some degree of) capitalist production that revolutions inspired by their ideas triumphed, societies in which the vast majority of the population were illiterate peasants and, at the state level, the landowning aristocratic interests prevailed. This led to the ‘bourgeois revolutions’ of Russia (in February of 1917, in the midst of World War I) and China (in 1911-1912, shortly before that war) creating republics that were so weak that they did not resist the offensive of communist parties when international wars put them to test. Their respective tests
were the continuation of Russia’s participation in World War I (due to the obtuse insistence of the Menshevik government), and in China the long national war against the Japanese invaders and, once they were defeated, the Communist Party’s war against the bourgeois-nationalist forces of Chiang Kai-Shek. The Russian Revolution of October 1917 quickly showed the weak spots in the program of the Bolsheviks led by Lenin. The latter was based on the idea that even if there was no Communist revolution in the advanced capitalist countries, Marx’s notion of banning private firms and having centrally planned state production could still be applied with benefit to countries of the Capitalist periphery. Lenin himself, in spite of his convictions, had to retreat to more market friendly mechanisms with his New Economic Policy, shortly before the accelerated deterioration of his health. His less pragmatic successors, led by Stalin, imposed a reversion towards the drastic changes suggested by the inherited doctrine (such as rural collectivization and accelerated industrialization) by means of large-scale brute force.\(^5\)

The simple truth is that the market-oriented actions of millions of small and medium peasants and of a smaller number of entrepreneurs in small, medium or large firms could not be eliminated and replaced by state production without adverse (and even catastrophic) consequences for the economy. It was naive not to see that the factional disputes that would inevitably arise within a State that almost completely controlled production and distribution would have very adverse effects on economic efficiency both at the macro level as within each firm due to the lack of decentralization in decision making and adequate incentives for the people in charge of these processes. Without denying the sometimes predatory activities of capitalist firms (including lobbying and exercising various corrupt influences on public policy in order to profit at the expense of others), one of the advantages these firms have is that their managers typically concentrate most of their energy on increasing efficiency and reducing costs in order to reap satisfactory profits.

This said, one cannot deny that Marx’s achievements were enormous. He built a global vision of social phenomena that was founded on a profound knowledge of historical facts in their social, economic, and political dimensions. His vision of the evolution of societies stratified into classes had strong empirical foundations that anthropology generally upholds.\(^6\) His representation of a hierarchical interlinkage between the members of the principal social classes in the production of goods and services in which the dominant class has special influence on the State and uses it in order to profit, was well oriented, even if he did not get to develop this level of his initial research plans. His vision of a progressive accumulation of capital by a minority that gradually concentrates the dominion over the means of production while on the other hand the decomposition of the pre-capitalists modes of production such as slavery or serfdom gradually liberated the potential wage workers was supported by abundant historical and contemporary evidence. The same may be said about his vision of a ‘centralization’ of capital based on the closure of firms that lose in the competition process; of the role in this process of the expansion of credit by means of market instruments (stocks and bonds) or bank loans that can finance huge enterprises that surpass the possibilities of any particular firm; of his insistence on the power of capital to increase the productive power of labor (i.e., its productivity) through organizational changes and technological

\(^5\)One may validly speculate, nevertheless, on how historical reality would have evolved had an insufficient Soviet industrialization paved the way to Nazi domination over the USSR. For alongside the millions of soldiers who lost their lives, it was their use of Soviet tanks, cannons, and airplanes that defeated the German forces.

\(^6\)Cfr. Harris (2001 [1979] and 1979 [1968]).
innovations; and of his theory of the industrial cycle based on the decisions of capitalists to either hoard or disburse money as capital; and on their decisions to reinvest more or less of their profits. Marx gave a great impulse to the intersectoral vision of the economy. His analyses form the very base of modern national accounts as well as the ‘Input-Output analysis’ that Leontief successfully developed and is widely used today in the construction of intersectoral transaction tables. Marx clarified important aspects of the capital accumulation process and, in particular, its intersectoral aspects, which he materialized in his theory of Extended Reproduction.

We have seen in Part II of this book that the finality of human action plays an important role in Marx’s theoretical framework and that, according to him, the characteristic aim of capitalists is to increase their capital, i.e., expand their wealth. This implies that what motivates the largest ‘capitalists’ or ‘entrepreneurs’ (or corporate directors) is not merely the greater consumption that enrichment allows but, fundamentally, the power that a great fortune or top positions in a great corporation commands over others. Mainstream economic theory chooses to ignore these issues in its general framework. In economics the finality of the actions of individuals is always their present or future consumption, or that of their descendents (in ‘dynastic’ versions in which the individual is interested not only on his own welfare but also in that of his descendents). Marx’s assumptions are more realistic, for although consumption is always a part of the interest of the great capitalist entrepreneur or corporate director, what is most characteristic is his accumulation of wealth oriented towards the accumulation of power (actual or imaginary) over others and, in particular, over the political process that encompasses them. After 150 years of additional history, this aspect of Marx’s framework has a solid empirical base showing that a few hundred individuals own a very high percentage of world wealth. According to Oxfam (2016), in 2015 only 62 persons in the world owned the same wealth as the poorest half of humanity (3600 millions) and the richest 1% owned more wealth then the other 99%. Also: “Since the turn of the century, the poorest half of the world’s population has received just 1% of the total increase in global wealth, while half of that increase has gone to the top 1%.”7

Huge fortunes create an immense influence on political events through the financing of political parties, candidates, and publicity campaigns (and consequent manipulation of the electorate) as well as through the corruption of legislators and judges. To a great extent, it is the ambition of belonging to the elite of big decision-making that makes the objective of attaining a great fortune so attractive. And the entrepreneurial venue to attaining great political power is complemented by the political venue that is linked to State power (and to the corruption that positions of power within the State facilitate).

Marx underestimated the survival possibility of the large spectrum of firms and fortunes according to their sizes that capitalist societies display. Only a minute percentage of capitalists attain or inherit a very large fortune, and they coexist with millions of medium, small, and very small, capitalist-entrepreneurs. Most capitalist entrepreneurs probably only have the objective of attaining a more or less modest fortune that will guarantee a comfortable future retirement and a certain inheritance to their descendents. Marx’s revolutionary political vision erred in its underestimation of the beneficial social role of entrepreneurs in the production process itself, and also in his underestimation of the advantages that spring from the decentralization of power when there are multiple poles of corporate power vying to influence political decisions and curtailing the power of entrenched government elites. He was not conscious of the harm that the absolute centralization of power in the hands of whoever monopolizes

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7 See also the abundant empirical data in Piketty (2014).
the reins of the State can inflict on all the classes of civil society when private firms are banned. Marx also had a notable lack of realism in his view of the feasibility of replacing private firms and markets by central planning and state production, and even eliminating money, without adverse and widespread economic consequences.

On the other hand, we should not ignore that many of the regulations imposed and policies practiced in the most advanced modern societies (with more or less success, with more or less conflict between those who decide to implement them and those that resist them because they harm their interests) respond to the need pointed out by Marx of introducing an economic rationality different from the pure market mechanism of supply and demand, different from a policy of laissez faire. In 1864, in the inaugural conference of the Working Men’s International Association (WIA), Marx pointed to “the great contest between the blind rule of the supply and demand laws which form the political economy of the middle class, and social production controlled by social foresight, which forms the political economy of the working class” (MECW 20, 11). Though he erred in believing that to surpass the “blind rule of the supply and demand laws” it was necessary to first eliminate the owning classes, it cannot be denied that all the anti-cyclical policies and economic regulations that at present are so familiar and widely accepted are precisely mechanisms designed to surpass that “blind rule”, that is, respond to the defects of the market mechanisms of the laissez faire Capitalism that Marx wanted to change, and gained momentum when the absence of correcting mechanisms led to the crisis of Capitalism in the 1930s, with widespread unemployment, social unrest, and in many countries to fascist and militaristic political processes that led to the 2nd World War. Also, many of Marx’s claims for the rights of society’s less advantaged were highly justified in his epoch and are still valid today, as is the need to eradicate the exploitation and oppression of the victims of various forms of modern slavery.

The need to generate organizations that strive to prevent bloody wars motivated by the greed of great power elites in their striving for universal hegemony is still present today. In the 1864 inaugural conference of the WIA mentioned above, Marx held that it was the duty of the working classes to “master themselves the mysteries of international politics” in order “to watch the diplomatic acts of their respective Governments; to counteract them, if necessary, by all means in their power... and to vindicate the simple laws of morals and justice, which ought to govern the relations of private individuals, as the rules paramount of the intercourse of nations” (MECW 20, 13) And in the July 1870 conference of the WIA, only days before the beginning of the Franco-Prussian war, Marx stated that “in contrast to old society, with its economical miseries and its political delirium, a new society is springing up, whose International rule will be Peace, because its national ruler will be everywhere the same –Labour” (MECW 22, 7). Although Marx evidently erred in his objective and prognostication of a future world government of an association of workers, his call for Peace as a new international principle is since Hiroshima more imperative than ever, for elites of just a few hundred individuals play with the survival of the human species through military policies that in their quest for the resources of the weakest regions of the planet threaten each other with global extermination (Eellsberg 2017).

Marx’s millenarianism before and after his death

Marx’s life project in a nutshell

There is a great consistency in Marx’s life, a notable continuity between the project for intellectual production and political practice that he forged in his youth and the
actual intellectual and political activity that he practiced until his death. In Part I of this book we concentrated on the gestation of the materialist conception of history and only touched upon the simultaneous gestation of his ethical-political project. As we have seen in Chapter 21, this second track of his juvenile thought took shape when he came to the conclusion that he should not simply try to understand the world (as philosophers did) but should simultaneously try to transform it, a conclusion he expressed synthetically in his *Theses on Feuerbach*. But to change human society he had to understand how it functioned, he had to understand the interrelations between its economic, social, political, and ideological aspects as they had evolved in history. His youthful quasi-religious posture and his empathy with human suffering led him to seek to decipher “the riddle of history” in order to find a radical solution to the plight of the destitute, one that would get to the roots of the matter. His very early rupture with the religious stance and his studies on Philosophy, Law, and History, led him to the construction of his ‘materialist conception of history’, one that stressed the importance of socioeconomic matters (such as the relations persons entered into in the production process and the productive force of their labor) instead of the ideas people had about themselves and the world. And the combination of this conception with his millenarian attitude towards the definitive solution of the main social problems (poverty, exploitation, oppression, injustice, inequality) led him to the transformation of pre-existing socialist and communist ("critical-utopian") projects in a direction that he deemed compatible with the actual tendencies of capitalist society as he perceived them in the late 1840s. The European revolutions of 1848-49 made him adopt concrete political stances. He joined the League of the Just and (with Engels) transformed it into the Communist League. And he acted politically in the very scene of the events in the case of Germany (mainly through journalism), and also interpreted and analyzed in detail the events in France. The latter gave him a concrete opportunity to apply his ‘materialist conception’ of the interrelation between the social classes defined according to economic criteria and the contemporary political phenomena. He was able to confirm that the political stances of the majority of individuals (but not all) was directly correlated with the economic interests of their class membership, and that those who held power at the State level in the various phases of the revolutions and counterrevolutions acted so as to balance the concrete claims of the different sectors of society according to their economic and political power while hanging on to the reins of power. His millenarian conception of the ‘task’ of the working class as taking over political power through its revolutionary communist vanguard played a fundamental role both in his political actions and in his analyses.

Later, in his London exile, he concentrated his efforts on the construction of his great integrating theory of the functioning of Capitalism, starting from the critique of Political Economy, of the social analyses of many writers, a the assessment of a great mass of empirical information. For this the abundant material at his reach in the most important city of advanced capitalist nations was crucial. He constructed a theory of Capitalism endowed with two levels: the ‘exoteric’ level, based on the most visible phenomena of Capitalism, and the ‘esoteric’ level, built from a transformation of Ricardo’s theory of labor-value that was especially suited for his theory of the exploitation of wage labor in Capitalism. Both levels required the production of new conceptual categories and models. In the ‘esoteric’ level was the careful construction of his theory of *value* and surplus *value*. And in the ‘exoteric’ level were his distinctions between constant and variable capital (that were important to explain production prices), the precise distinction between fixed and circulating capital, the concepts of the production and circulation of commodities as a circular process, the duality between exchange values
(prices) and use values (quantities), the turnover of capital and related concepts, the
simple and extended reproduction of capital, etc. The theory of surplus value was a
sophisticated construction and Marx wrote the three volumes of *Theories of Surplus
Value* searching in the history of Political Economy for historical predecessors of that
theory. Marx and Engels considered the theory of surplus value a fundamental basis
for their political project. The communist society of the future had to eliminate surplus
value, source of all the incomes based on ownership and measure (through the rate of
surplus value) of the exploitation of wage labor in the capitalist mode of production.

As we have seen, the concept of surplus value was based on the notion of ‘unpaid
labor’, itself based on the notion that capitalist-entrepreneurs did not work, owing to
which the profits, interests, and ground rents into which the aggregate surplus value
was converted were illegitimate, at least from the perspective of the future surpassing
mode of production that Marx predicted and sought to bring about. We will not
repeat here the reasons for the invalidity of the theory of surplus value. We merely
want to point out that Marx had a consistent two-level theory of Capitalism, and that
its ‘exoteric’ portion was quite solid, even if we take into account that he was unable
to complete the part of his theory related to absolute ground rent, with the important
exception that it ignored entrepreneurial labor. It was a scientific theory, and like any
scientific theory it could be totally or partially refuted. Marx was a creature of his time.
In the last decades of his life Political Economy had a Neoclassical (or ‘Marginalist’)
revolution, the most advanced versions of which required the use of mathematical
methods (basically calculus) that could be usefully applied in several topics that could
not be adequately dealt with using the instruments Marx developed from his studies
of Classical Economics.

The second track of Marx’s life project, the political movement that he and Engels
inspired—and in certain phases of its development led—, started from the supposition
that Capitalism was reaching the end of its possibilities and the opinion that the
political, economic, and social interests of the working class, whose political power in
the most advanced capitalist countries was on the rise, hinged on the elimination of
their exploitation by the capitalist class. The Communist movement organized as a
political party was supposed to illuminate the working class in those ideas and become
its accepted representative. And once in power it was to replace Capitalism by a
superior mode of production, one not submitted to the laws of supply and demand; one
in which firms managed by their workers replaced private firms and central economic
planning replaced markets. There were at least two major flaws in this project. First,
the most advanced Capitalism of 1860-70 was very far from representing a fetter to
the further development of the productive forces. Second, replacing private firms and
markets by worker-managed firms and central planning was in the best of cases an
adventure full of difficulties and risks.

Historical reality has since then showed that it was in countries with scarcely de-
volved Capitalism where circumstances could combine so that a Communist Party
inspired by Marx and Engels could manage to take control over State power. This
made it even more difficult to realize the ‘historical task’ of doing away with private
firms and markets. And it triggered an extremely difficult competition with the de-
veloped capitalist world that was bound to fail in the long run. That failure materialized
in the breakup of the Soviet Union and the return to Capitalism by its former com-
ponents; and also in China’s return to Capitalism, where the Communist Party was able
to carry out that return without losing political control. One must recognize that it
was not possible in Marx’s time to fully understand the enormous intrinsic difficulties
to be encountered by trying to manage a whole complex economy from a single center
without the powerful mechanisms of the internal planning of private firms motivated by the expectation of profits, and without the use of markets of all types. Hence, Marx could not possibly understand how utopian it was to dispense with capital (and hence wage labor) and commodities (and hence markets) and still achieve a greater development of the productive forces.

It can be said that Capital was Marx’s attempt to give a scientific foundation to his political project of replacing capitalist society by communist society (as he understood it and not as it came about in historical reality a few decades later). But it is important to point out that the political project Marx and Engels forged was only one of the many that could have been conceived starting from Marx’s materialist conception of history (devoid of his millenarian conception of the future). As Engels wrote in 1890 in a letter to Bloch:

history is made in such a way that the ultimate result is invariably produced by the clash of many individual wills of which each in turn has been made what it is by a wide variety of living conditions; there are thus innumerable conflicting forces, an infinite number of parallelograms of forces, productive of one result—the historical event which itself may be seen as the product of a power operating unconsciously and involuntarily as a whole. For what each individual wants is obstructed by every other individual and the outcome is something that no one wanted (MECW 49, 35).

This brilliant paragraph itself implies that there can be no necessary correspondence between whatever materialist interpretation is (presently) given of (past and present) history and the political project any person or group (at present) proposes for the future. Any interpretation of concrete history is a hypothesis stated with limited empirical backing and from a biased point of view. And any political project is a gamble on the future course of events; a risky bet the success of which is dependent on the actions of millions of particular wills and the result of which it is impossible to predict.

The radicalism, utopianism, and millenarianism of Marx and its consequences

As we saw in Chapter 21, the millenarian aspects of Marx’s worldview were already firmly in place before he initiated his systematic economic research. Rubel (1973, 3) is right when he writes that “In the intellectual development of Marx the rejection of the State and Money and the affirmation that the proletariat was a liberating class came before his studies of political economy” if the latter refer to his deep and systematic studies once he was in London. But he is wrong when he adds that “they preceded also his discovery of the materialist conception of history, the ‘guiding line’ which directed his later historical researches.” It is more correct to say that he developed his ‘materialist conception of history’ during the same period in which he gave form to his political project, carefully differentiating it from other existing projects with which he shared some aspects. For both are present in the works he wrote in the 1843-1852 period. We have separated them (in Parts I and IV, respectively) because we deemed this to be the best way of avoiding an unnecessary tainting of the central part of this book—the exposition of Marx’s theory of Capitalism—with the millenarian and utopian aspects of his worldview of the future, even if in some occasions this was impossible to avoid. The naïvety and lack of realism of his millenarianism are in stark contrast to the sophistication and realism of his theory of the genesis and
functioning of Capitalism. It is not by chance that the weakest part of this theory (the ‘esoteric’ part and its overreach into the ‘exoteric’ part) is precisely the one Marx considered his most valuable contribution; for it was also the foundation of the utopian and millenarian aspects of his political project.

It may be pertinent to clarify that the ‘millenarianism’ we refer to is not Marx’s aim to transform society, not even his goal of making a political revolution in order to achieve this, but rather 1) the notion that a political party that manages to take over state power with a program of deep institutional transformations oriented towards a centrally planned economy devoid of private firms could avoid a political dictatorship that leads to the gestation of a new dominant class, and 2) the radicalism of Marx’s project for a new type of society. We respect to 1), we must observe that any human group will be necessarily heterogeneous in its ideas, unless it has a hierarchical structure in which the top has an undisputed supreme authority that all the rest obey. The latter was in the antipodes of Marx’s thought, so we may discard it as a feature of his project. The necessary heterogeneity implies that it is improbable that all the group members will coincide in the path to follow in every instance of their extremely difficult feat; there will not always be agreement on what the interests of the working class are, and on what the best way of ensuring that they prevail is. One can argue that for reaching a decision on each concrete issue a referendum would be necessary, or at least a well posed survey among those presumably represented. But it is difficult to believe that such a thing would be feasible in society after “it has just emerged after prolonged birth pangs from capitalist society” (Gotha, MECW 24, 87). It seems logical to expect the rise of dissent within the party initially in power in any concrete country. Even if there were initially democratic procedures within the party in decision-making, dissent would give rise to political struggles to hold on to or obtain power in the new government. During the short life of the Communist League at least one of these instances of dissent arose, and it led to a schism. The International Working Men’s Association also split in two in its 1872 Congress at The Hague, when the ‘anarchists’ left with Bakunin and the ‘statists’ stayed with Marx and Engels. And the political struggles, schisms, purges, persecutions, and mass assassinations that took place in the USSR especially after Lenin’s death are well known.

This matter is intimately linked with the degree of simultaneity that, in Marx’s view, a communist-led revolution would have in the most advanced capitalist countries. It can only be expected that capitalist States that were able to resist a communist revolution would turn against the new ‘Worker States’, leading to serious international conflicts and probably wars. In a war situation it is even more difficult than under peace that government decisions be based on the will of those presumably represented, and hence the very need to have an operational government would tend to make it less democratic. Therefore, dictatorial tendencies and wars would be forthcoming through the same dynamics that, during the French Revolution, the First Republic quickly led to the First Empire –Bonaparte’s personal dictatorship–, and the internal revolution led to ten years of international wars. These historical processes were well known to Marx and Engels, and they should have had a more concrete and realistic appraisal of the kind of difficulties that their schematic project would probably face. If dictatorial deviations did substantiate in their new revolutionary State, why would not a faction

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8That Marx had something along these lines in mind is confirmed by the criticism of the anarchist Bakunin, who knew Marx’s project from the inside since he was a member of the International Working Men’s Association (the First International) between 1868 and 1872. He wrote (in 1873): “So-called popular representatives and rulers of the state elected by the entire nation on the basis of universal suffrage –the last word of the Marxists, as well as of the democratic school– is a lie behind which the despotism of a ruling minority is concealed” (Bakunin 1990, 538).
take over that would eventually fundamentally represent themselves after purging those naive souls who –like Marx and Engels– insisted in defending what they considered to be the true class interests of workers? As in Capitalism, a new dominant class in formation would impose the idea that they represent the interests of the main portion of the population as they repress those who try to be loyal to the aims of the original project. This is precisely what happened in the USSR when Stalin gradually took control as Lenin was being left out of the daily political game as his health deteriorated starting 1921 (finally dying in January 1924).

With respect to the radicalism and utopianism of Marx’s project, one must recall that the project included not only the objective of eliminating capital, that is, the mode of production and circulation of commodities based on the private property of the means of production and wage labor, but much more than this: eliminating even commodities and money, and eliminating even the division of labor and social classes. Even if he relegated the elimination of the detailed division of labor in crafts and professions to the ‘second phase’ of communist society, that is, a possibly very remote future, he set the objective of eliminating the functional division into social classes, markets, commodities, and money already in the ‘first phase’. This implied a great lack of realism in not perceiving the enormous difficulties and perturbations that this could imply in practice, not only per se but also in the context of the international difficulties—and probably wars— that would ensue, as we have argued above. It is true that precisely ‘war economies’ tend to have a great deal of centralization and planning, especially in the part of the economy most linked to the war effort, and often include the rationing of consumption goods. But could it be realistic to expect that after a period of intense sacrifice in which the perils of war make people accept the inevitable bureaucratic inefficiencies that arise, they would continue accepting in resignation their continuation after peace has been reached? Marx did not appraise correctly the extent to which market mechanisms help in having a working economy that is increasingly complex. He did not apprehend the extent to which the increasing complexity and interlinking among individuals, among firms, and among individuals and firms, as well as the generation of ever increasing products and transactions, flows are facilitated if the products are commodities, that is, are purchased and sold in markets in which there is some degree of competition. He did not appreciate that without such help from markets the economic life of any country would be seriously injured. And this is independent of the tendency for increasing planning within firms (whether private or public) which he correctly highlighted but the importance of which he did not correctly appreciate. For various reasons, it is much more feasible and efficient for a government to intervene in markets in different ways and with varying intensities, than totally replacing markets with commanded flows of goods and services. On the other hand, capitalist commodity production implies not only the existence of markets and money but also the institution of private firms managed by capitalist entrepreneurs or executives that require the collaboration of wage workers. Replacing capitalist entrepreneurs or executives with State functionaries implies eliminating the organizing initiatives of thousands and even millions of persons endowed with the capacity to perceive where there is a deficit of production or a new commodity that can be introduced, i.e., where there are potential profits to be made. And Marx did not perceive the grave problems that a jump into the unknown he proposed would have on the economy of any capitalist country and the great sacrifices that would be required from workers to implement it, sacrifices that could only be maintained in time through the most despotic means.
‘Human nature’ and Bakunin’s critique of Marx’s ‘statism’

‘Anarchy’ was an important component of Marx and Engels’ political project. But they held that it should be reserved for the second phase of a successful revolution, i.e., for when there would no longer be social classes. In contrast, the ‘Anarchists’ (as they came to be known later) did not want a first phase in which there would still be a State (supposedly run by workers) but considered instead that the State should be immediately destroyed and be replaced by worker associations, at most politically organized in ‘communes’, as the one that was set up (in 1871) in Paris. That ‘anarchy’ was a component of Marx and Engel’s project is clear in the following sentences from a document directed against the Alliance (led by Bakunin) and signed by them and other members of the General Council of the International:

Anarchy, then, is the great war-horse of their master Bakunin... All socialists see anarchy as the following programme: once the aim of the proletarian movement, i.e., abolition of classes, is attained, the power of the State, which serves to keep the great majority of producers in bondage to a very small exploiter minority, disappears, and the functions of government become simple administrative functions. The Alliance reverses the whole process. It proclaims anarchy in proletarian ranks as the most infallible means of breaking the powerful concentration of social and political forces in the hands of the exploiters. Under this pretext, it asks the International, at a time when the old world is seeking a way of crushing it, to replace its organisation with anarchy (MECW 23, 121-2; italics added).

As Rubel (1973, 3) writes, Marx wanted “to lay down a rational basis for an anarchist utopia as the conscious aim of the revolutionary movement... the critique of the State led him to envisage the possibility of a society free from all political authority.” But if the anarchist (or ‘libertarian’) goal—the elimination of all authority—was utopian, then it could not possibly have a rational basis, for it was impossible to reach, not directly—as the ‘Anarchists’ wanted—and not even after an intermediate period of uncertain duration—as the ‘Marxists’ wanted. Regarding the first phase of Communism in Marx’s project, it was utopian to think that a political party could impose a process of social transformation taking exclusive command of the reins of State power without it becoming a political dictatorship of a faction (a proto-dominant class) over all of society, including the workers. Marx sharply denied what was evident to Bakunin: that whoever acceded to State power, even if they were of working class extraction or intended to represent working class interests, would themselves become an exploiting and dominant class. In Statism and Anarchy, The Struggle of the Two Parties in the International Working Men’s Association, Bakunin wrote:

If there is a state, then necessarily there is domination and consequently slavery. A state without slavery, open or camouflaged, is inconceivable—that is why we are enemies of the state. What does it mean, “the proletariat raised to a governing class?”... In the Marxists’ theory this dilemma is resolved in a simple fashion. By popular government they mean government of the people by a small number of representatives elected by the people... So, from whatever point of view we look at this question, it always comes down to the same dismal result: government of the vast majority of the people by a privileged minority. But this minority, the Marxists say, will consist of workers. Yes, perhaps of former workers, who, as soon as they
become rulers or representatives of the people will cease to be workers and will begin to look upon the whole workers' world from the heights of the state. They will no longer represent the people but themselves and their own pretensions to govern the people. Anyone who doubts this is not at all familiar with human nature (Bakunin 1990 [1873], 537-9).

Considering himself more knowledgeable on 'human nature' than his adversary, Bakunin wanted—as the anarchists who succeeded him—to simply eliminate the State, with no intermediate phase in which State power is used to eliminate social classes. Marx did not disagree on the final goal of eliminating the State as a mechanism of class oppression but was convinced that it was necessary that the working class control the State in order to be able to realize the (possibly extended) 'dictatorship of the proletariat' during which the great transformations he advocated were to be achieved. Concerning the second phase of Communism (the only phase for Bakunin and his epigones), it was utopian to believe that the elimination of the division of labor and social classes was compatible with an advanced society, taking into account that in history—as Marx himself highlighted many times—the greatest advances in the productive forces was realized under Capitalism, i.e., under the great (functional) division of labor between wage workers and capitalist entrepreneurs (along with the more detailed division of labor according to crafts, professions, branches of industry, etc.). Marx never accepted that a fundamental part of the division of labor in capitalist society is between entrepreneurial and non-entrepreneurial work, an idea that goes back at least to Cantillon, as we have seen in Chapter 19.

He had the utopian notion that eliminating capital, commodities, and money, human beings would come to behave in a radically different way, without egoism, envy, or the urge to obtain private gain from public administration. Let us recall that in On the Jewish Question he had quoted Rousseau's statement: "He who dares undertake to give institutions to a people ought to feel himself capable of changing, so to speak, human nature; of transforming every individual, who in himself is a perfect and solitary whole, into part of a greater whole." Let us also recall that in The Holy Family Marx had highlighted the close link between Communism and the ideas of materialism on "the original goodness and equal intellectual endowment of men, the omnipotence of experience, habit and education, and the influence of environment on man," from which was derived the conclusion that "If man is shaped by environment, his environment must be made human." Marx overvalued the degree to which "human nature" was molded by Capitalism, and thus undervalued how little that nature would change with the abolishment of Capitalism.

Marx did not properly see the extent to which the public property of all the means of production per se says nothing with respect to the control over such means, how it is exerted, who benefits from its control, a control which in any advanced—and hence complex—society requires a functional division of labor which in itself implies differentiation of power and some form of social class structure. Unwittingly, his general stance actually pointed to a kind of return to a simpler society, to the type of non-stratified human communities that existed in the remote past but is incompatible with any future society, unless there are survivors from a world war so devastating (and at the same time so little devastating) that there are residual human communities totally lacking in complexity. Although this is today a very real possibility (Ellsberg 2017), it is not what Marx contemplated. He acutely pointed to many real tendencies in the evolution of Capitalism. But he did not perceive the flexibility that Capitalism could develop in order to moderate several of its most defective facets (including the great fluctuations in the level of activity, the periodical mass unemployment, and the misery of structural
poverty), even though he did not hesitate to welcome the regulations introduced by
the most progressive sectors of the British government when they occurred. He did
not see how many of the problems of the Capitalism of his time, far from being solved
by his radical proposals, could actually intensify if they were used. He did not want
to accept that many of those problems could be ameliorated by means of adequate
reforms, more or less radical but conserving the functioning of markets (even if these
became strongly regulated), and conserving the functioning of private firms (even if
these were also constrained by legislation and regulation – without implying that this
was ever accomplished to a sufficient extent). Neither did Marx adequately perceive to
what extent the bellicose tendency of capitalist society is not actually due to Capital-
ism per se but to predatory tendencies of human society that have been present since
the dawn of history, but the destructive effects of which have become greater with the
advancement of the destructive force of war technology (that grew pari passu with the
advancement of the productive force of non-war technology).

The reformism of German Social Democracy

In the most advanced European countries the main working class parties that drew
inspiration from the ideas of Marx and Engels eventually had to leave behind the
most radical conceptions of their deceased predecessors as the evidence mounted that
there was no terminal crisis of Capitalism in sight in their countries and that neither
were their parties to gain sufficient power to take over the government and impose the
revolutionary agenda. Reformist socialists emerged that gained an important popular
support and a significant parliamentary representation, especially in Germany. An
outstanding case was that of Eduard Bernstein (1850-1932), who in 1872 had affiliated
to the Social Democratic Workers’ Party of Germany (founded in Eisenach in 1869
and led by Wilhelm Liebknecht and August Bebel). In the 1875 Congress held in
Gotha this party was unified with the General German Workers’ Association (that had
been led by the already deceased Ferdinand Lassalle (1825-1864)) to form the Socialist
Workers’ Party (SWP) of Germany. It was the party program that emerged from this
unifying Gotha Congress that was subjected by Marx to a strong critique. The SWP
obtained almost a half a million votes in the 1877 Reichstag elections, but due to
(or with the excuse of) two assassination attempts against the Kaiser Wilhelm I, the
following year Chancellor Bismarck prohibited all socialist organizations, assemblies,
and publications. However, Socialists were allowed to campaign and present themselves
as candidate on an individual level.

As other Socialist leaders, Bernstein had to go to exile in 1878, not returning to
Germany until 20 years later. First he established himself in Zurich, where he took
charge of the Party periodical and his reputation grew due to his many writings. Ten
years later, due to Bismarck’s pressure on the Swiss government, he (and other Socialist
Leaders) had to abandon Switzerland. He moved to London, where he became a close
friend of Engels. Meanwhile, the Socialist vote continued rising in Germany, reaching
763,000 in the 1887 elections (with 11 seats in the Reichstag). Bernstein generally
maintained an orthodox stance within the SWP, though his points of view gradually
changed. And after Engels’ death (in 1895) he began to propose profound changes to
the Party’s political stance (which after the ban was lifted in 1890 adopted the name
of Social Democratic Party of Germany – SDP). In the following years he published a
series of articles on the ‘problems of Socialism’ and in 1899 the book The Preconditions
of Socialism and the Tasks of Social Democracy (translation of the complete title of
the German edition). The latter sparked a great debate in the midst of the SDP. His
‘revisionism’ was attacked by several of the Party’s main leaders (Bebel, Kautsky, and Liebknecht). Rosa Luxemburg criticized his position in a series of articles which were collected in her 1900 book *Social Reform or Revolution*.

In his book Bernstein argued that the SDP had as theoretical basis for its activity the social doctrine developed by Marx and Engels that they called “scientific socialism.” This was only partially true. It was Engels who first used this term, and though Marx also did so on a few occasions, it was always in a political context. In his annotations (of 1874-75) on the margins of Bakunin’s *Statism and Anarchy* (of 1873), where the author complained about the use of the term ‘scientific socialism’, Marx wrote that he had only used it “in contrast to Utopian socialism which wishes to foist new illusions onto the people.” But the ‘doctrine’ that Marx and Engels did their utmost to propagate (and after Marx’s death came to be called ‘Marxism’, with various different interpretations) was also tainted with illusion. Its scientific camouflage endowed it with formidable convincing power, but with the benefit of hindsight it can be said that it was actually another variety of ‘Utopian socialism’, and possibly even much more Utopian than others if one considers the radicalism of its proposal.

Bernstein was a very intelligent man. But he did not have the necessary training to adequately address the philosophical aspects of Marx’s works, among them the use of the word ‘materialism’ (as opposed to ‘idealism’) and the sporadic use of metaphors of Hegelian root (such as ‘the negation of the negation’ and ‘dialectical’). He incorrectly identified materialism with the necessity of all events and, applying this concept to history, he identified the materialist conception of history with “the necessity of all historical events and developments”

From this he concluded that Marx initially had a deterministic conception, having identified as determining factors the “forces of production and relations of production” (Bernstein 1993 [1899], 13). He correctly stated, however, that Marx’s assertion (in the Preface to his *Contribution*) that “The bourgeois mode of production is the last antagonistic form of the social process of production” could only be a “hypothesis more or less grounded” (Ibid., 14). And he noted that the determinism he detected in Marx’s writings gradually became more relaxed with the passage of time, giving more room for the influence of ‘non-economic’ factors, especially in Engels’ later writings. Hence, “in addition to the development and influence of the forces of production and the relations of production” it was important “to take full account of the legal and moral concepts, the historical and religious traditions of every epoch, geographical and other natural influences, which include the nature of man himself and his intellectual dispositions”; and that this was “to be kept in mind most particularly where it is a matter not just of pure research into earlier historical epochs, but of projecting future developments, where the materialist conception of history is to be a guide to the future” (Ibid., 16). And he opined that, although “socialist criticism has quite rightly drawn attention to the great transformations which human nature has undergone in various countries in the course of time”, it was important to bear in mind that “even major changes in the ownership of property are unlikely to produce a rapid transformation of human nature, because economic and property relationships are only a part of the social environment which has a determining effect on human character” (Ibid., 17). The profound truths he was expressing certainly exceeded in importance his misuse of certain philosophical terms and his errors in the

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9 “To be a materialist means first of all to assert the necessity of all events... since the movement of matter determines the formation of ideas and the directions of the will, these too are necessitated, as are all human events” (Bernstein 1993 [1899], 12-3). “The application of materialism to the interpretation of history therefore means asserting, from the outset, the necessity of all historical events and developments” (Ibid, 13).
exegesis of Marx’s thought. Bernstein was also confused in his belief that Marx had been a Hegelian in his form of reasoning and that, although he had made a fundamental change (in relation to Hegel) in giving prevalence to the productive forces and the relations of production as conditioning factors in the development of ideas, he continued to think in Hegelian terms, according to “the laws of dialectics, as laid down by Hegel.” He wrongly interpreted that Marx’s conviction on the need for a proletarian revolution resulted “from a remnant of Hegelian contradiction dialectics” (Ibid., 32).

With challenging heterodoxy, Bernstein went on to state that there was a glaring contradiction at the heart of Marx’ praxis: “painstaking precision befitting the busy industry of genius in investigating the economic structure of society goes hand in hand with an almost incredible neglect of the most palpable facts; the very same theory that takes the determining influence of economics on power as its starting point concludes with a truly miraculous belief in the creative power of force” (Ibid., 35). Bernstein no longer believed in the feasibility of taking over State power by the SDP by means of a political revolution. And he was upset by the dogmatism that prevailed in the Party on the validity of all that Marx and Engels had written, and considered it a duty of the disciples to make all the necessary amendments so as to obtain an agreement between doctrine and reality.

Though sometimes using faulty arguments, Bernstein was correct in wanting to refute the traditional posture of the SDP on the ‘inevitable’ demise of Capitalism. He held that some of Marx and Engels’ forecasts on the tendencies of Capitalism could no longer be upheld. And he gathered data that showed that the centralization of capital was not leading to a reduction in the number of capitalists, that there were increasing numbers of stockholders and firms of all sizes. He stated that there was nothing to indicate an imminent or future collapse of Capitalism, and that the advancement in the repressive power of the State made the idea that the Party could take over power by force illusory. He also argued that the very considerable possibilities of introducing gradual reforms that benefited workers should not be rejected, even if in some measure this strengthened the power of the Imperial German government. Germany had had very strong growth and the working class could benefit by means of non violent and reformist political actions. The great growth in the electoral base of the Party had led to the resignation of Chancellor Bismarck and the new king seemed favorable to reformist policies. Because he no longer believed in the final goals that Marx had proclaimed and did believe in the possibility of introducing reforms that benefitted the working class, he stated in January 1898: “I frankly admit that I have extraordinarily little feeling for, or interest in, what is usually termed ‘the final goal of socialism’. This goal, whatever it may be, is nothing to me, the movement is everything” (Tudor 1993 [1899], xxviii).

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10It is curious that Bernstein’s arch enemy, Lenin, had a similarly erroneous interpretation of Marx’s ‘dialectical method’. In one of his last articles (“On the significance of militant materialism”) Lenin proposed to “arrange for the systematic study of Hegelian dialectics from a materialist standpoint, i.e., the dialectics which Marx applied practically in his Capital and in his historical and political works, and applied so successfully... Taking as our basis Marx’s method of applying materialistically conceived Hegelian dialectics, we can and should elaborate this dialectics from all aspects, print in the journal excerpts from Hegel’s principal works, interpret them materialistically and comment on them with the help of examples of the way Marx applied dialectics” (Lenin 1973, Vol. 33, 233-5). Marx’s ‘historical materialism’ and ‘dialectical method’ were thus amalgamated into a “Hegelian dialectics, materialistically interpreted” (Ibid.).
The experiences of real Communism in the 20th century and our future

All outstanding scientists are motivated by deep psychological factors that urge them to exert the greatest effort in research. As a scientist, Marx was able to generate scientific achievements that could help to better understand the functioning of the capitalist society of his time. This scientific endeavor was a part of the activities he decided to engage on in his youth, which were fundamentally motivated by the goal of finding lasting solutions to what he felt as intolerable exploitation of workers by the propertied classes all over the world and the miseries associated to prolonged worker unemployment. Finding solutions to these problems required understanding the economic functioning of Capitalism, and this was the main objective of his endeavors once established in London. But another fundamental part of his endeavors was seeking to implement solutions to these problems. And in his youth he reached the conclusion that this required leading workers through a party organization with clear objectives. He was also very successful in this task, for various socialist parties at least partially inspired by his ideas were able to gain an important influence on the European working classes. This was especially marked in Germany, where his ideas had the greatest impact.

With Engels, Marx generated a doctrine that not long after their deaths inspired two of the most important transforming experiences of the 20th century: the Communist revolutions of Russia and China, two countries that were in the periphery of world Capitalism but in very different situations. Whereas Russia was a great empire, China had been for a century the victim of European (including Russian), Japanese, and American imperialisms. The success of the Russian and Chinese Communists was only made possible by the devastating effects of the First and Second World Wars, respectively. And neither was able to consolidate power without first winning a bloody and prolonged civil war with widespread participation of foreign powers. In both cases the existing underdeveloped Capitalism was destroyed, the means of production were collectivized, and private firms were banned. In both cases very important achievements were eventually obtained, especially in bringing literacy and electricity to an overwhelmingly peasant population, but also in generating basic industries. And in both cases a dominant bureaucratic class was formed that exerted a strict control over the rest of society, a development that was very distant indeed from the increasingly libertarian society Marx had envisaged, one in which authority of man over man would fade as the last remnants of the bourgeoisie disappeared. Over the course of time, however, some of the most significant leaders of these societies became aware that they could not successfully compete with the more dynamic capitalist world if they maintained some of the basic restrictions that had been fundamental in shaping their societies. The ideological crises that ensued led in both cases, albeit in very different ways, to the explicit rehabilitation and encouragement of private firms and the formation of a new capitalist class.

Marx had given in 1859 a very synthetic exposition of what he viewed as the main motor of revolutionary change:

The mode of production of material life conditions the general process of social, political and intellectual life... At a certain stage of development, the material productive forces of society come into conflict with the existing relations of production or –this merely expresses the same thing in legal terms– with the property relations within the framework of which they have operated hitherto. From forms of development of the productive forces these relations turn into their fetters. Then begins an era of social
revolution (*Contribution*, MECW 29, 263).

The relations of production (characterized by many levels of bureaucratic authority over masses of workers in state-monopolized and centrally planned production and distribution) generated by the transformations inspired by the political project of Marx and Engels eventually proved to be fundamental impediments to successfully compete with the American-led capitalist world. They had become fetters for the further development of the productive forces that was necessary to keep apace of the capitalist world. Eras of social revolution opened up, first in China around 1979, when Deng Xiao Ping’s revolution (euphemized as ‘reform’) began to be implemented, and around 1989 in the Soviet Union, with the declarations of independence of the Eastern European countries (that had been an important component of the Soviet Empire since the end of World War II), the ensuing dissolution of the Soviet Union itself, and the decisions to reinstate Capitalism in its component countries. The experiences of the Soviet Union and China proved how much dynamism was lost with the prohibition of private firms, and how much clumsiness and oppression unchecked omnipotent bureaucratic dominant classes were capable of generating, even leading to massive famines and widespread political persecutions in the long early phases of Stalinism and Maoism, respectively.

The disappearance of the capitalist class and the centralization of the means of production in the hands of the State were not the best means to solve the very real and urgent problems faced by these countries with little capitalist development, although there were successful in solving some of the urgent problems that the previous dominant classes had been incapable of dealing with. It is likely that with less ideological rigidity (leading to the preservation of, or early return to, private firms and markets) the leaders of these countries could have achieved a faster and greater increase in living standards (even with all the investment in the military they incurred in) and could also have avoided some of the most socially costly phases they went through. Many of the most important reforms implemented in the capitalist countries consisted in introducing greater government interventions in the private economy, i.e., a direction that Marx had correctly detected in the most advanced capitalist countries of his time. But the most important steps in this direction were only introduced after the catastrophic effects of the economic depression of the 1930s. These effects mobilized socialist workers to such an extent that the scared conservative and reactionary forces turned their support to various forms of non-democratic government (Italian Fascism, Japanese Militarism, Spanish Francoism, German Nazism, etc.). The German and Japanese variants proved to be so aggressively expansionary that the catastrophic Second World War ensued, leading both to the Chinese Revolution and the long period of Pax Americana. In this period the armed competition between the greatest powers has been restrained by nuclear Mutually Assured Destruction (MAD) and hence takes place in the Third World through overt invasions and/or destabilizations that generate proxies susceptible to manipulation, resulting in millions of dead, maimed, and displaced.

Marx’s utopian and millenarian conception was used decades after his death as a guide for the design of the societies that emerged from the great communist revolutions. It was used in the Soviet Union to justify the mass expropriation of millions of peasants that had previously benefited from the expropriation of semi-feudal landlords, for which they fought to defend and extend the revolution that was threatened by Russian White forces backed by the imperialist powers. Once the revolutionary government succeeded in staving off these forces however the peasants’ lands and capital were seized because their potential growth could endanger the ‘Workers’ State’. Marx’s conception was
used to justify the consolidation of the omnipotent power of a state bureaucracy that became a dominant class (as Bakunin had predicted) of a new kind of industrial class society. This new dominant class was extremely authoritarian and wielded a far tighter political, economic, social, and ideological control over all of society than most of the capitalist variants. But its crucial weakness was in the economy, and this resulted first from the inherited anti-capitalist ideology but later from its need to keep in check, through the prohibition of private firms, the potential economic power of a capitalist class that could eventually succeed in putting limits to its power.

At the center of the communist conception was a paradoxical lack of understanding of the importance of the planning and dynamical adaptation to a changing economic and political environment exerted by entrepreneurs and executives in capitalist firms. Rational planning and flexible adaptive change tend to be more effective in private firms, with their mergers and acquisitions, their autonomous investment and divestment decisions, and their multiple forms of financing. There the risk of bankruptcy and the lure of profit are incentives that—at least in the long run—can hardly be replaced by state production and central planning. For in the exclusively state-run economy there tends to predominate a logic of political bureaucratic competition between cliques that is only held in check by an even tighter hierarchical control that weakens initiative and in the long run is no match for the flexibility attainable by Capitalism. The protection from competition in exclusively state-run economies tends to generate rigidity and defective adaptive behavior to an ever-changing environment. The general evolution of the events of the 20th century proved how flexible Capitalism can be and how many variants of political structures are possible on the basis of capitalist economic institutions. And that is why these events led to a politically motivated return to Capitalism by the very leaders of the principal countries in which political parties and projects inspired by Marx and Engels had successfully acceded to state power decades earlier.

Capitalist countries (whether of the bi-party plutocratic American kind or the Chinese single-party bureaucratic variant) have the advantage of being able to combine private planning with state planning in varying proportions that are sometimes the object of heated political debate and is settled daily by the power elite, the ‘deep State’ that is mostly hidden from the view of the majority. The desideratum of the 21st century is no longer Capitalism vs. Communism but what kind of Capitalism and what kind of popular participation in the political process (‘democracy’) is to exist. For it is clear that Capitalism can have widely diverging political institutions. It can, for example, have a political system that is rabidly nationalist, xenophobic, expansive, imperialist, fascist, and/or racist, or one in which the wildest ambitions of some powerful corporate and political minorities are held in check by a sufficiently enlightened majority; it can have a political system that maintains a compartmentalized educational system that ensures that the poor are effectively excluded from the quality education necessary for the best jobs or one which ensures not only that the natural talents of those born in less fortunate households can develop to the point of accessing positions of leadership in firms or government, but also that the great mass of the population is capable of discerning whether what the government is trying to impose on them is compatible with their interests or is instead another attempt to foiling them into accepting ventures that will probably reap great profits to a few and great losses (and possibly death) to themselves and their dearest. Capitalism can have institutions that impose ecological prudence or allow environmental depredation; institutions that allow a political-military elite to play with the survival of our species by engaging in crazy and illusory quests for global hegemony, or institutions that make political leaders peacefully coexist in a multi-polar world. Capitalism is compatible
with policies of solidarity with less favored individuals and social groups, or on the contrary, with policies that leave whole social groups in misery, oppressed, ignorant, and manipulable.

Such are the options of our time. One can only hope that the quest for radical but utopian solutions that characterized a whole era of human endeavor is in the past for good, an era that could well be associated to the name of Karl Marx. But the mysteries of complex human history are almost impenetrable when it comes to forming an image of the future. And the ups and downs of past history, that have swamped the surface of the planet with human blood, tend to repeat themselves in new and ever more threatening forms. If one of the most intelligent men that ever lived could have been so wrong in some of the most crucial aspects of feasible social organization it is of fundamental importance to beware of all political leadership prone to destabilize our problematic world for the worse. But for those of us who value human life and human happiness there is no option other than trying to find how to improve the human condition and its prospects of survival in this planet that is becoming all too small for us, and in which our human activity is generating ecological and social disequilibriums that are becoming catastrophic and will destroy us if we are unable to introduce profound changes in the behavior of those who have the power to make decisions on a global scale.
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\(^{12}\)This collection of articles published in *The New York Tribune* between October 1851 and December 1852 was for decades erroneously attributed to Marx, even by his daughter Eleanor who edited the book in 1896 and wrote for it an Introduction. Only after the Correspondence between Marx and Engels became available did it become known that the articles were written by Engels at Marx’s request and signed by Marx, who was the journal’s correspondent.
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