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5 July 2022

Online at <https://mpra.ub.uni-muenchen.de/113787/>  
MPRA Paper No. 113787, posted 20 Jul 2022 10:05 UTC

# The Problem of Combined Optimal Load Flow Control of Main Conveyor Line

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## Funding information:

Funding Agency  
Grant Number

## Acknowledgement:

## Abstract

The combined method of flow parameters control of conveyor transport system is researched in this article. A transport system distributed model with input accumulative bunker is developed for the optimal control synthesis. The transport system model is presented in dimensionless form. Expressions are obtained that determine the states of flow parameters along the transport route. The amount of transport delay was calculated for each technological position of the transport route at an arbitrary point in time. A system of characteristic equations is written down, the solution of which determines the trajectory of movement of a separate element of the transported material. Conditions are considered under which the material output flow does not depend on the initial filling of the conveyor section with the material. The algorithm of optimal control development of the material flow rate at the output from the accumulative bunker and the conveyor belt speed, which ensures the minimum deviation of the output cargo flow from a planned amount, is given. The optimal control algorithm takes into account the restrictions on the control modes of flow parameters and the volume of the accumulating bunker. It is shown that the developed control algorithm ensures the maximum filling of the transport system with material and forms a uniform distribution of material along the transportation route.

## Keywords

Conveyor optimal control, accumulative bunker, distributed transport system.

## Introduction

The use of conveyor transport systems is especially widespread in the mining industry, where the operation of transport conveyor systems is associated with a number of unique features and restrictions. The first of the features that should be noted is that the rock flow entering the transport conveyor system is non-stationary. The non-stationarity and stochasticity of the material flow are directly related to rock mining technology. The non-

uniform flow of material entering the transport system is the reason for the non-uniform distribution of material along the transport route. It results in the accumulation of material in some places of the transport route on the conveyor belt, but other places on the conveyor belt remain unfilled to the required level. In addition, two significant restrictions of the material along the transport route are imposed on the linear density: a) in order to avoid machinery breakdowns, the specific density of the material located on the conveyor belt should not exceed the maximum permissible value; b) the total mass of the transported material within one conveyor section should not exceed the maximum permitted value, which is limited by the power of electrical equipment that provides the conveyor belt motion. The presence of the features and restrictions mentioned above leads to the fact that if there are no effective control algorithms, the conveyor operates in modes that differ from the normative one for much of that time (Bebic and Ristic, 2018; He and Lodewijks, 2018; Wolstenholm, 1980; Hiltermann et al., 2011). In this case, the specific energy costs for the transportation of the rock can increase significantly, reaching multiple increases (Semenchenko et al., 2016). The load increase on conveyor systems as a result of using control systems of speed and amount of the material flow to the input of the conveyor section allows reducing the specific energy costs for transient and stationary operational modes (He et al., 2016a, 2016b; Zhang and Xia, 2011; DIN 22101, 2002; Antoniuk, 2010). The conventional level of filling the conveyor belt with the material is  $\varphi = 60-100\%$ , which makes it possible to reduce the specific energy consumption by up to 30% (Lauhoff, 2005; Semenchenko et al., 2016). Theoretical and experimental studies have shown that the control of the belt speed allows saving specific energy only if this leads to an increase in the level of filling of the conveyor belt material (Lauhoff, 2005). Non-uniform distribution of the rock on the conveyor belt along the transport route is of special importance for long conveyor systems (Yang, 2014). When designing the main conveyor, a popular solution is the division of the total transport route into sections (Marais and Pelzer, 2008).

A transport conveyor is a complex dynamic, stochastic and distributive system. This explains the presence of a large number of models of conveyor systems used to design algorithms for controlling the flow parameters of the conveyor. Special attention should be paid to the models based on the method of finite-element (He et al., 2018, 2016a, 2016b); finite difference method (Mathaba and Xia, 2015); Lagrangian equations (Yang, 2014); aggregative equations of state (Reutov, 2017); equations of neural net layers (Xinglei and Hongbin, 2015, Więcek et al., 2019); system dynamics equations (Wolstenholm, 1980) and regression equation (Andrejiova and Marasova, 2013). Despite the great variety and number of models, control algorithms designed with their help can be divided into two large groups that differ in the method they control the amount of the output cargo flow. The first method is to control the conveyor belt speed (Antoniuk, 2010). The second method involves the use of an accumulative bunker at the input to the conveyor, which determines the amount of material flow at the input of the transport system (Wolstenholm, 1980; Marais and Pelzer, 2008). Both control methods are based on the principle of the transport system accumulation of continuously incoming material. In the first case, the accumulation of material occurs on a conveyor belt. The amount of the output flow of material is controlled by the speed of the conveyor belt. In the second case, bunkers are used to accumulate incoming material. The amount of the incoming material to the entrance of the transport system is controlled by an alteration in the capacity of the bunker. In this paper, we consider the combined control system of the main conveyor line, equipped with an accumulative bunker and a belt speed control system (Reutov, 2017; Pihnastyi and Khodusov, 2019; Halepoto et al., 2016). This approach allows increasing the efficiency of the conveyor system control by taking advantage of each of the two methods of flow parameters control. The application of the combined system of control is very useful for long conveyor systems (Alspaugh, 2004; Kung, 2004; Siemens, 2018).

## Material and Methods

In this article, we use a distributive model of a long conveyor line to design the control system (Pihnastyi and Khodusov, 2017). The given model belongs to a new class of models of conveyor lines. This class of models allows you to accurately calculate the linear density  $[\chi]_0(t, S)$  of the material for a technological position  $S \in [0, S_d]$  of the transport route with the total length  $S_d$  at an arbitrary point in time  $t$ . In (Reutov, 2017; Pihnastyi and Khodusov, 2018a; Halepoto et al., 2016), a problem of the optimal control of conveyor belt motion speed  $a = a(t)$  is considered. Seeing that the flow of the material  $[\chi]_1(t, S)$  in any place of the transportation route  $S$  at the time  $t$  is the product of the belt speed  $a = a(t)$  and linear density of the material  $[\chi]_0(t, S)$

$$[\chi]_1(t, S) = a(t)[\chi]_0(t, S), \quad (1)$$

to ensure a required output flow of the material from the conveyor line,  $\sigma_{out}(t)$  the speed of the conveyor belt is to be defined with the formula

$$a(t) = \frac{[\chi]_1(t, S_d)}{[\chi]_0(t, S_d)} = \frac{\sigma_{out}(t)}{[\chi]_0(t, S_d)}. \quad (2)$$

The law mentioned above of belt speed control  $a = a(t)$  is the reason for the non-uniform distribution of linear density of the material along the transportation route. The linear density  $[\chi]_0(t, 0)$  of material entering the conveyor section depends on belt speed  $a(t)$  and material input flow  $\sigma_{in}(t) = [\chi]_1(t, 0)$ , which comes from the field of the mineral deposits

$$[\chi]_0(t, 0) = \frac{[\chi]_1(t, 0)}{a(t)} = \frac{\sigma_{in}(t)}{a(t)}. \quad (3)$$

It means that the non-uniformity of material distribution along the transportation route depends both on the amount of rock flow to the input of the conveyor  $\sigma_{in}(t)$  and on the required output material flow at the output of the conveyor line  $\sigma_{out}(t)$

$$[\chi]_0(t, 0) = \frac{\sigma_{in}(t)}{\sigma_{out}(t)} [\chi]_0(t, S_d). \quad (4)$$

As non-stationarity and stochasticity of the material flow  $\sigma_{in}(t)$  are directly linked to the technology of rock mining, then the usage of only belt speed control algorithms do not allow providing uniform distribution of the material along the technological route in the problems aimed to ensure a given flow of material  $\sigma_{out}(t)$  at the output of the transport system. The problem of optimal control of the amount of material flow intensity  $\lambda(t)$  that comes from the accumulative bunker to the input of the transport system is considered in (Pihnastyi and Khodusov, 2018a). It was presumed that the belt speed  $a(t)$  is not controlled and is constant; to ensure a required output material flow from the conveyor line  $\sigma_{out}(t)$ , it is necessary to provide a required value of the density of the material at the output of the transportation system

$$[\chi]_0(t, S_d) = [\chi]_1(t, S_d) / a = \sigma_{out}(t) / a. \quad (5)$$

When the belt speed  $a$  is constant, the linear density of the material at the output can be expressed in terms of linear density value at the input  $[\chi]_0(t, 0)$  (Pihnastyi and Khodusov, 2018b)

$$[\chi]_0(t, S_d) = [\chi]_0(t - S_d / a, 0) = \lambda(t - S_d / a) / a. \quad (6)$$

This ratio allows defining the algorithm of control of the material flow intensity amount, which comes from the bunker to the input of the transportation system if the belt speed is constant (Pihnastyi and Khodusov, 2019):

$$\lambda(t) = \sigma_{out}(t + S_d / a), \quad \Delta t_d = S_d / a, \quad a = const. \quad (7)$$

The delay  $\Delta t_d$  for the constant belt speed is proportional to the total length of the transportation route  $S_d$ . Non-uniform distribution of material flow along the route in case of unlimited bunker capacity is determined by the forecasting of output flow  $\sigma_{out}(t)$ . For long conveyor systems (Alspaugh, 2004; Kung, 2004; Siemens, 2018), the delay  $\Delta t_d$  can be up to several days. The required linear density at the input of the conveyor line (Pihnastyi and Khodusov, 2019) can be determined if the amount of the forecasted output flow is known

$$[\chi]_0(t, 0) = \lambda(t) / a = \sigma_{out}(t + S_d / a) / a. \quad (8)$$

For any arbitrary point of the transport route  $S$ , the linear density of the material at the time  $t$  is defined by the expression

$$[\chi]_0(t, S) = \sigma_{out}(t + (S_d - S)/a) / a \quad . \quad (9)$$

Uniform distribution along the transport route is achieved if  $\sigma_{out}(t) = const$ . The requirement to ensure certain unsteady output flow  $\sigma_{out}(t) \neq const$  from the conveyor line is the reason for the non-uniform distribution of the material along the transport route. Stating the purpose of the research, let us sum up the main advantages and disadvantages of the above-mentioned methods to control the amount of output material. The first method allows for controlling the amount of material flow at the output of the transport system without any delay. But under such regulation, the speed of the material is changed at any point of the transport route as a result of conveyor belt speed alteration. In many cases, this leads to a significant change in the power of the transport system as a whole. This problem is of special interest in the cases when the electricity tariff depends on the time of day and the conveyor belt is long. The second method of control needs forecasting a planned amount of the output flow  $\sigma_{out}(t)$  at the time  $(t + \Delta t_d)$ . The forecasted amount is used to develop a program to control input flow intensity

$$\lambda(t) = \sigma_{out}(t + \Delta t_d), \quad \Delta t_d = S_d / a = const \quad . \quad (10)$$

However, during the time interval  $0 \leq t \leq \Delta t_d$ , the output flow is not controlled. Thus, as a result of the application of both methods, an uneven distribution of material along the transport route occurs, which leads to the increased energy consumption of the conveyor system. Within this context, the purpose of our article is the development of a combined algorithm to control the conveyor transport system, which is based on the possibility to control the conveyor belt speed and material flow coming to the conveyor belt from the bunker. The combination of these two methods of control would allow ensuring uniform distribution of the material along the transportation route.

### Model of the conveyor line

A conveyor line is a kind of flow line. A feature of an industrial conveyor line is that the material moves at the same speed at different points of the conveyor line. The system of equations to model conveyor line with an accumulative bunker at the input takes the form:

$$\frac{\partial [\chi]_0(t, S)}{\partial t} + \frac{\partial [\chi]_1(t, S)}{\partial S} = \delta(S)\lambda(t), \quad [\chi]_1(t, S) = a(t)[\chi]_0(t, S), \quad (11)$$

$$dN_0(t)/dt = \sigma_{in}(t) - \lambda(t), \quad (12)$$

with the initial values for the linear density of the material along the transport route

$$[\chi]_0(0, S) = H(S)\Psi(S), \quad H(S) = \begin{cases} 0, & S < 0, \\ 1, & S \geq 0, \end{cases} \quad S \in [0; S_d], \quad (13)$$

and bulk volume  $N_0$  in the accumulative

$$N_0(0) = N_0, \quad 0 \leq N_0 \leq N_b, \quad 0 \leq \lambda(t) \leq \lambda_{max}. \quad (14)$$

Material density  $[\chi]_0(t, S)$  and flow  $[\chi]_1(t, S)$  are linked by the conveyor belt speed  $a = a(t)$  (1), (11). The right part of the equation (11), containing  $\delta(S)\lambda(t)$ , defines the position of the material flow with the intensity  $\lambda(t)$  ( $\delta(S)$  is a delta-function). At the initial time  $t = 0$ , the conveyor line is filled with the material whose linear density is  $[\chi]_0(0, S) = \Psi(S)$  (13). The system of equations (11)–(14) is closed regarding flow parameters  $[\chi]_0(t, S)$  and  $[\chi]_1(t, S)$ . The conditions (1) reflect the features of the conveyor line. The function  $N_0(t)$  sets the current amount of the material in the bunker with the maximal capacity  $N_b$ . The dependence of typical values of maximal bunker capacity  $N_b$  in relation to the intensity  $\lambda(t)$  is researched in (Wolstenholm, 1980). In this work, the recommendations how to choose the parameters of an accumulative bunker are given. We suppose that the material flow at the input of the accumulative bunker  $\sigma_{in}(t)$  is known, and the required flow  $\sigma(t) = \sigma_{out}(t)$  at

the output of the transportation system is defined by the schedule of rock removal. A schematic diagram of a conveyor line with the bunker at the input is given in Fig. 1 (Conveyorbeltguide, 2019). Rock flow  $\sigma_{in}(t)$  enters the bunker and fills it. The material comes to the input of the conveyor from the bunker with the regulated intensity  $\lambda(t)$ .

Conveyor line parameters will be described as dimensionless variables:

$$\tau = \frac{t}{T_d}, \quad \xi = \frac{S}{S_d}, \quad \theta_0(\tau, \xi) = \frac{[\chi]_0(t, S)}{[\chi]_{0\max}}, \quad \psi(\xi) = \frac{\Psi(S)}{[\chi]_{0\max}}, \quad (15)$$

$$n_0(\tau) = \frac{N_0(t)}{S_d[\chi]_{0\max}}, \quad g(\tau) = \frac{\sigma(t)T_d}{S_d[\chi]_{0\max}}, \quad \delta(\xi) = S_d\delta(S), \quad (16)$$

$$\gamma(\tau) = \frac{\lambda(t)T_d}{S_d[\chi]_{0\max}}, \quad g_{in}(\tau) = \frac{\sigma_{in}(t)T_d}{S_d[\chi]_{0\max}}, \quad g(\tau) = a(t)\frac{T_d}{S_d}. \quad (17)$$

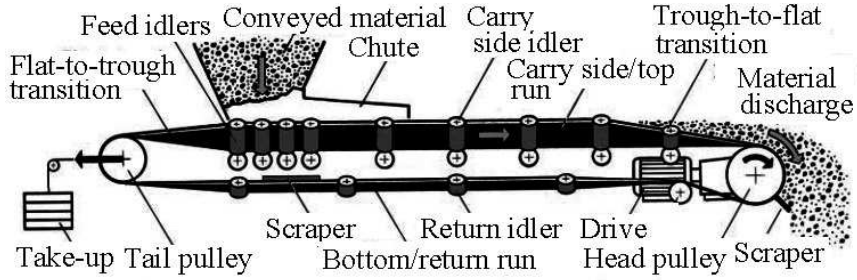


Figure 1. Schematic diagram of the main conveyor line (Conveyorbeltguide, 2019)

Let us assume that when the conveyor line stops at  $a(t)=0$ , the supply from the bunker is over, the intensity of material entry  $\lambda(t)\rightarrow 0$  and the value of loads per unit length on the conveyor line does not exceed the overload capacity:

$$\lim_{a(t)\rightarrow 0} (\lambda(t)/a(t)) = [\chi]_0(t, 0) \leq [\chi]_{0\max}. \quad (18)$$

Note that when  $n_0(\tau)=1.0$ , the accumulative bunker contains an amount of the material equal into  $N_0(t) = S_d[\chi]_{0\max}$ , which allows filling the conveyor line along the whole length with the maximum allowable linear density. Taking into account the dimensionless variables (15), flow parameters balance equation (11)–(14) is written in the dimensionless form:

$$\frac{\partial \theta_0(\tau, \xi)}{\partial \tau} + g(\tau) \frac{\partial \theta_0(\tau, \xi)}{\partial \xi} = \delta(\xi)\gamma(\tau), \quad \theta_0(0, \xi) = H(\xi)\psi(\xi), \quad (19)$$

$$dn_0(t)/dt = g_{in}(\tau) - \gamma(\tau), \quad n_0(0) = n_{00}, \quad 0 \leq n_0(\tau) \leq n_b, \quad 0 \leq \gamma(\tau) \leq \gamma_{\max}. \quad (20)$$

We supplement equation (7) with a system of characteristics

$$\frac{d\xi}{d\tau} = g(\tau), \quad \xi(\tau_0) = \xi_0, \quad \frac{d\theta_0}{d\tau} = \delta(\xi)\gamma(\tau), \quad \theta_0|_{\tau_0} = H(\xi_0)\psi(\xi_0). \quad (21)$$

The first characteristic equation has an explicit physical meaning. This equation defines a motion trajectory of the element of the material along the technological route. The solution of characteristic equations can be given in the form:

$$\xi = G(\tau) + \xi_0, \quad G(\tau) = \int_0^\tau g(\alpha) d\alpha, \quad \tau = G^{-1}(\xi - \xi_0), \quad (22)$$

$$\theta_0(\tau, \xi) = (H(\xi) - H(\xi - G(\tau))) \frac{\gamma(G^{-1}(G(\tau) - \xi))}{g(G^{-1}(G(\tau) - \xi))} + H(\xi - G(\tau))\psi(\xi - G(\tau)), \quad (23)$$

$$dn_0(t)/dt = \mathcal{G}_{in}(\tau) - \gamma(\tau), \quad n_0(0) = n_{00}, \quad 0 \leq n_0(\tau) \leq n_b, \quad 0 \leq \gamma(\tau) \leq \gamma_{\max}. \quad (24)$$

In the case when the speed  $g(\tau)$  of the conveyor belt is constant and equal to  $g_0$ , the linear density of the material along the transport route at any time can be determined if we know the intensity  $\gamma(\tau)$  of rock coming to the input

$$\theta_0(\tau, l) = \begin{cases} \gamma(\tau - l/g_0)/g_0, & \tau - l/g_0 \geq 0; \\ \psi(1 - g_0\tau), & \tau - l/g_0 < 0; \end{cases} \quad (25)$$

$$\theta_1(\tau, l) = \begin{cases} \gamma(\tau - l/g_0), & \tau - l/g_0 \geq 0; \\ \psi(1 - g_0\tau)g_0, & \tau - l/g_0 < 0. \end{cases} \quad (26)$$

For the case when the intensity of material flow  $\gamma(\tau)$  from the bunker to the input of the section is constant and equal  $\gamma_0$ , the values of the flow parameters at the output from the conveyor section take the form:

$$\theta_0(\tau, l) = \begin{cases} \frac{\gamma_0}{g(\tau - \Delta\tau_1(\tau))}, & G(\tau) - 1 \geq 0; \\ \psi(1 - G(\tau)), & G(\tau) - 1 < 0; \end{cases} \quad \Delta\tau_1(\tau) = \tau - G^{-1}(G(\tau) - 1); \quad (27)$$

$$\theta_1(\tau, l) = \begin{cases} \gamma_0 \frac{g(\tau)}{g(\tau - \Delta\tau_1(\tau))}, & G(\tau) - 1 \geq 0; \\ \psi(1 - G(\tau))g(\tau), & G(\tau) - 1 < 0. \end{cases} \quad (28)$$

where  $\Delta\tau_1(\tau)$  is the transport delay, the time interval between the moment when the material incoming at the section input and the moment when the material leaves the section.

In the case when the speed of the conveyor line and the intensity of the material flow are constants and, accordingly, equal to  $g_0$  and  $\gamma_0$ , we see a steady mode of the transport system operation:

$$\theta_0(\tau, l) = \begin{cases} \gamma_0/g_0, & \tau - l/g_0 \geq 0; \\ \psi(1 - g_0\tau), & \tau - l/g_0 < 0; \end{cases} \quad (29)$$

$$\theta_1(\tau, l) = \begin{cases} \gamma_0, & \tau - l/g_0 \geq 0; \\ \psi(1 - g_0\tau)g_0, & \tau - l/g_0 < 0. \end{cases} \quad (30)$$

For the steady mode of the operation, the values of flow parameters at the output of the conveyor line are constant

$$\theta_0(\tau, l) = \gamma_0/g_0, \quad \theta_1(\tau, l) = \gamma_0, \quad \tau - l/g_0 \geq 0, \quad (31)$$

and expressed in terms of input flow parameters without considering the delay  $\Delta\tau_1(\tau)$ . For the case of a constant belt speed  $g_0$ , the value of the transport delay  $\Delta\tau_1(\tau)$  is constant and equal to  $1/g_0$ .

### The optimal control problem

Let us consider the control problem for material flow  $\gamma(\tau) = u_\gamma(\tau)$ , which enters the input of the conveyor line from the bunker, and the speed of the conveyor belt  $g(\tau) = u_g(\tau)$ : to define the material output flow  $\theta_1(\tau, l)$  from the conveyor during the interval of time  $\tau = [0, \tau_k]$  with the controls  $u_\gamma(\tau)$  and  $u_g(\tau)$ , when

$$\frac{1}{2} \int_0^{\tau_k} (\theta_1(\tau, 1) - \mathcal{G}(\tau))^2 d\tau \rightarrow \min \quad (32)$$

with differential relations (19)

$$\frac{\partial \theta_0(\tau, \xi)}{\partial \tau} + u_g(\tau) \frac{\partial \theta_0(\tau, \xi)}{\partial \xi} = \delta(\xi) u_\gamma(\tau), \quad (33)$$

$$dn_0(t)/dt = \mathcal{G}_{in}(\tau) - u_\gamma(\tau), \quad (34)$$

restrictions to the linear density

$$0 \leq \theta_0(\tau, \xi) \leq 1, \quad (35)$$

restrictions to the amount of the material in the accumulative buffer (20)

$$0 \leq n_0(\tau) \leq n_b, \quad (36)$$

restrictions to control

$$u_{\gamma \min} \leq u_\gamma(\tau) \leq u_{\gamma \max}, \quad u_{g \min} \leq u_g(\tau) \leq u_{g \max} \quad (37)$$

and initial conditions (29)

$$\theta_0(0, \xi) = H(\xi) \psi(\xi), \quad n_0(0) = n_{00}. \quad (38)$$

Differential relation (33) determines the value of the output flow  $\theta_1(\tau, 1)$

$$\theta_1(\tau, 1) = (1 - H(1 - G(\tau))) \frac{\gamma(G^{-1}(G(\tau) - 1))}{g(G^{-1}(G(\tau) - 1))} g(\tau) + H(1 - G(\tau)) \psi(1 - G(\tau)) g(\tau). \quad (39)$$

The factor of quality in the form (32) is widely used when analysing conveyor flow lines (Zhang and Xia, 2011). Let us reformulate the factor of quality (32) with the due regard to the expression (29)

$$\int_0^{\tau_k} \left( \psi \left( 1 - \int_0^\tau u_g(\alpha) d\alpha \right) u_g(\tau) - \mathcal{G}(\tau) \right)^2 d\tau + \int_{\tau_r}^{\tau_k} \left( \frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) - \mathcal{G}(\tau) \right)^2 d\tau \rightarrow \min. \quad (40)$$

In the interval  $\tau \in [0, \tau_r[$  the output value  $\theta_1(\tau, 1)$  depends on the initial distribution of the linear density of the material  $\theta_0(0, \xi)$  (38). The value  $\tau = \tau_r$  is obtained by solving the equation (22). The material which enters the conveyor line input at the time  $\tau = 0$ , reaches the conveyor line output at the time  $\tau = \tau_r$ . During the interval  $\tau \in [0, \tau_r[$ , the control of the output material flow  $\theta_1(\tau, 1)$  can be performed only by controlling the speed of the conveyor belt  $g(\tau) = u_g(\tau)$ . Pontryagin function for the task under consideration takes the form:

$$H = -\frac{1}{2} \left( \psi \left( 1 - \int_0^\tau u_g(\alpha) d\alpha \right) u_g(\tau) - \mathcal{G}(\tau) \right)^2 + \psi_1 (\mathcal{G}_{in}(\tau) - u_\gamma(\tau)) \rightarrow \max, \quad \tau \in [0, \tau_r], \quad (41)$$

$$H = -\frac{1}{2} \left( \frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) - \mathcal{G}(\tau) \right)^2 + \psi_1 (\mathcal{G}_{in}(\tau) - u_\gamma(\tau)) \rightarrow \max, \quad \tau \in [\tau_r, \tau_k], \quad (42)$$

$$\Delta\tau_1 = \Delta\tau_1(\tau) = \tau - \tau_1, \quad \tau_1 = G^{-1}(G(\tau) - 1). \quad (43)$$



Taking into consideration the restrictions for phase variables, let us write down the Lagrangian function:

$$L = H + \mu_1 n_0 + \mu_2 (n_b - n_0), \quad \mu_1 \geq 0, \quad \mu_1 n_0 = 0, \quad \mu_2 \geq 0, \quad \mu_2 (n_b - n_0) = 0; \quad (44)$$

$$\frac{dn_0(t)}{dt} = g_{in}(\tau) - u_\gamma(\tau), \quad \frac{d\psi_1}{d\tau} = -\frac{\partial L}{\partial n_0} = -\mu_1 + \mu_2, \quad \psi_1(\tau_k) = 0. \quad (45)$$

The parameter  $\Delta\tau_1 = \Delta\tau_1(\tau)$  defines the time delay between two events: the output of the material at the time  $\tau$  and its arrival at the entrance of the conveyor line at the time  $\tau_1$ . Let us write down necessary conditions of extremum of Lagrangian function with respect to the controls  $u_g(\tau)$ ,  $u_\gamma(\tau)$  at the intervals  $\tau \in [0, \tau_{tr}[$  and  $\tau \in [\tau_{tr}, \tau_k]$ .

a) For the interval  $\tau \in [0, \tau_{tr}[$  the condition for extremum of Lagrangian function with respect to the control  $u_g(\tau)$  takes the form:

$$\frac{\partial L}{\partial u_g} = -\left(\psi(1-G(\tau)) u_g(\tau) - \mathcal{G}(\tau)\right) \left(\psi(1-G(\tau)) + \frac{d\psi(1-G(\tau))}{du_g(\tau)} u_g(\tau)\right) = 0, \quad (46)$$

$$G(\tau) = \int_0^\tau u_g(\alpha) d\alpha. \quad (47)$$

The solution of the latest equation is attained when

$$\psi(1-G(\tau)) u_g(\tau) = \mathcal{G}(\tau) \quad \text{or} \quad \psi(1-G(\tau))^{-1} = C_g u_g(\tau), \quad (48)$$

where  $C_g$  is the integration constant. As  $\psi(1-G(\tau)) \geq 0$ ,  $u_g(\tau) \geq 0$ , there is a solution

$$\psi(1-G(\tau)) u_g(\tau) = \mathcal{G}(\tau), \quad u_{g \min} \leq u_g(\tau) \leq u_{g \max}, \quad (49)$$

$$u_g(\tau) = u_{g \min}, \quad u_{g \min} > \frac{\mathcal{G}(\tau)}{\psi(1-G(\tau))}, \quad u_g(\tau) = u_{g \max}, \quad u_{g \max} < \frac{\mathcal{G}(\tau)}{\psi(1-G(\tau))}, \quad (50)$$

when the optimal control  $u_g(\tau)$  of the conveyor line is attained. Since the Lagrangian function is linear on control, the maximum value of the function will be attained at the ends of control:

$$\begin{aligned} u_\gamma(\tau) &= u_{\gamma \min}, & \text{when} & & \psi_1 > 0, & (51) \\ u_{\gamma \min} \leq u_\gamma(\tau) \leq u_{\gamma \max}, & \text{when} & \psi_1 = 0, & & u_\gamma(\tau) = u_{\gamma \max}, & \text{when} & \psi_1 < 0. \end{aligned}$$

b) For the interval  $\tau \in [\tau_{tr}, \tau_k]$ , the condition for the extremum of Lagrangian function with respect to the control  $u_g(\tau)$ ,  $u_\gamma(\tau)$  take the form:

$$\frac{\partial L}{\partial u_g} = -\left(\frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) - \mathcal{G}(\tau)\right) \frac{\partial}{\partial u_g} \left(\frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau)\right) = 0. \quad (52)$$

Consequently,

$$\frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) = \mathcal{G}(\tau) \quad \text{or} \quad \frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) = C_{2g}, \quad (53)$$

where  $C_{2g}$  is the integration constant. The last equation corresponds to the case when the factor of quality (40) over the interval  $\tau \in [\tau_{tr}, \tau_k]$  does not depend on the control  $u_g(\tau)$ . Taking into consideration that

$$\frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} = \theta_0(\tau - \Delta\tau_1, 0) = \theta_0(\tau, 1) \quad (54)$$

it follows that the output flow of the material is constant in value

$$\theta_0(\tau, 1)u_g(\tau) = \theta_1(\tau, 1) = C_{2g}, \quad (55)$$

and the factor of quality over the interval  $\tau \in [\tau_{tr}, \tau_k]$  is expressed as

$$\frac{1}{2} \int_{\tau_{tr}}^{\tau_k} (C_{2g} - \mathcal{G}(\tau))^2 d\tau \rightarrow \min. \quad (56)$$

In such a way, the optimal control  $u_g(\tau)$  in the interval  $\tau \in [\tau_{tr}, \tau_k]$  takes the form

$$u_{g \min} \leq \mathcal{G}(\tau) \frac{u_g(\tau - \Delta\tau_1)}{u_\gamma(\tau - \Delta\tau_1)} \leq u_{g \max}, \quad u_g(\tau) = \mathcal{G}(\tau) \frac{u_g(\tau - \Delta\tau_1)}{u_\gamma(\tau - \Delta\tau_1)}, \quad (57)$$

$$u_{g \min} \geq \mathcal{G}(\tau) \frac{u_g(\tau - \Delta\tau_1)}{u_\gamma(\tau - \Delta\tau_1)}, \quad u_g(\tau) = u_{g \min}, \quad (58)$$

$$u_{g \max} \leq \mathcal{G}(\tau) \frac{u_g(\tau - \Delta\tau_1)}{u_\gamma(\tau - \Delta\tau_1)}, \quad u_g(\tau) = u_{g \max}. \quad (59)$$

From the conditions for extremum with respect to the control  $u_\gamma(\tau)$  extremum

$$\frac{\partial L}{\partial u_\gamma} = - \left( \frac{u_\gamma(\tau - \Delta\tau_1)}{u_g(\tau - \Delta\tau_1)} u_g(\tau) - \mathcal{G}(\tau) \right) \frac{u_g(\tau)}{u_g(\tau - \Delta\tau_1)} - \psi_1(\tau) = 0, \quad (60)$$

follows for the cases when the restriction  $u_g(\tau + \Delta\tau_1) = u_{g \min}$  is attained or  $u_g(\tau + \Delta\tau_1) = u_{g \max}$ , then optimal control  $u_\gamma(\tau)$  is expressed by the equation

$$u_\gamma(\tau) = \mathcal{G}(\tau + \Delta\tau_1) \frac{u_g(\tau)}{u_g(\tau + \Delta\tau_1)} - \left( \frac{u_g(\tau)}{u_g(\tau + \Delta\tau_1)} \right)^2 \psi_1(\tau + \Delta\tau_1). \quad (61)$$

In the case of  $u_{g \min} < u_g(\tau) < u_{g \max}$ , the Lagrangian function is linear in control  $u_\gamma(\tau)$ , and the maximum value of the Lagrangian function will be attained at the end of the control  $u_\gamma(\tau)$ .

## Results and Discussion

For the synthesis of optimal equations, let us define the formula for the functions  $\mathcal{G}(\tau)$ ,  $\mathcal{G}_m(\tau)$  and  $\psi(\xi)$ . Let us assume that the material comes into the bunker with a constant intensity  $\mathcal{G}_m(\tau) = 1$ , the initial linear density of the material distribution is  $\psi(\xi) = 1$ , and the given output flow from the transport system is  $\mathcal{G}(\tau) = 1 + \sin(\pi\tau)$ . Let us also assume that there are some restrictions to the size of the accumulative bunker  $0 \leq n_0(\tau) \leq n_b = (2/\pi + 0.2)$  (Pihnastyi and Khodusov, 2018a), to the input flow control

$u_{\gamma \min} = 0 \leq u_{\gamma}(\tau) \leq u_{\gamma \max} = 2$  and to the control of conveyor belt speed  $u_{g \min} = 0 \leq u_g(\tau) \leq u_{g \max} = 2$ . The decision to choose a test formula for the function  $\mathcal{G}(\tau)$ , which defines an estimated flow at the output of the production line, is based on the possibility of the comparative analyses of the research results in (Pihnastyi and Khodusov, 2018a; He et al., 2016b). Intensity value  $\mathcal{G}_{in}(\tau) = 1$  corresponds to the case of the transport system operation when in a special time  $T_d$  the transport system along all its length  $S_d$  is filled with the material, the linear density value of which is  $[\chi]_{0\max}$ . The assumption  $\psi(\xi) = 1$  corresponds to the case of transport system filling where the material is with the maximum allowed amount of the linear density  $[\chi]_{0\max}$  at the initial time. On the interval  $\tau \in [0, \tau_{tr}[$  when  $\mathcal{G}_{in}(\tau) = 1$ ,  $\psi(\xi) = 1$ ,  $\mathcal{G}(\tau) = 1 + \sin(\pi\tau)$  the Lagrangian function for the control takes the form

$$L = -\frac{1}{2}(u_g(\tau) - \mathcal{G}(\tau))^2 + \psi_1(1 - u_{\gamma}(\tau)) + \mu_1 n_0 + \mu_2(n_b - n_0) \rightarrow \max. \quad (62)$$

When  $u_{g \min} < u_g(\tau) < u_{g \max}$ , the Lagrangian function is linear relating to the control  $u_{\gamma}(\tau)$  and reaches its maximum value when controls are  $u_{\gamma}(\tau)$  and  $u_g(\tau)$ , which meet the conditions:

$$\begin{cases} \psi_1 < 0, & u_{\gamma}(\tau) = \min(u_g(\tau), u_{\gamma \max}), \\ \psi_1 = 0, & u_{\gamma \min} \leq u_{\gamma}(\tau) \leq u_{\gamma \max}, & u_g(\tau) = \mathcal{G}(\tau). \\ \psi_1 > 0, & u_{\gamma}(\tau) = \max(u_g(\tau), u_{\gamma \min}), \end{cases} \quad (63)$$

With reference to (63), it follows that when  $u_{g \min} < u_g(\tau) < u_{g \max}$ ,  $u_{\gamma \min} < u_{\gamma}(\tau) < u_{\gamma \max}$ ,  $0 < n_0(\tau) < n_b$  the optimal control  $u_{\gamma}(\tau)$  provides the loading of the material on the conveyor line with the maximum density

$$\theta_0(\tau, 0) = u_{\gamma}(\tau) / u_g(\tau) = 1. \quad (64)$$

Let us consider the synthesis of the optimal control when at the initial time  $\tau = 0$ , none of the constraints  $n_0(\tau)$  for the bunker is accomplished. When this is the case, regardless of the value of the conjugate variable  $\psi_1(0)$  at the initial time, the control is

$$u_{\gamma}(\tau) = u_g(\tau) = \mathcal{G}(\tau). \quad (65)$$

When there are no restrictions, the conjugate equation takes the form

$$d\psi_1/d\tau = 0, \quad \psi_1(\tau_k) = 0. \quad (66)$$

Let us consider the case when a phase constraint is attained. Let  $\psi_1(0) < 0$  at the initial time. As on the initial interval  $\mathcal{G}(\tau) > \mathcal{G}_{in}(\tau)$ , then taking into account the constraint (17), the amount of the material in the  $n_0(\tau)$  will decrease. Unless the phase constraint  $n_0(\tau) = 0$  is attained, the value of the conjugate variable is not changed but remains constant  $\psi_1(\tau) = \text{const} < 0$ . When the lower phase constraint  $n_0(\tau) = 0$  is attained, the optimal control is

$$u_{\gamma}(\tau) = 1, \quad u_g(\tau) = \mathcal{G}(\tau), \quad \tau < \tau_{tr}, \quad u_{\gamma}(\tau) = 1, \quad (67)$$

$$u_g(\tau) = \mathcal{G}(\tau) \frac{u_g(\tau - \Delta\tau_1)}{u_{\gamma}(\tau - \Delta\tau_1)}, \quad \tau \geq \tau_{tr}. \quad (68)$$

In such a case, the value of the density of the material at the input is jump-like  $\theta_0(\tau, 0) = u_g^{-1}(\tau) < 1$ . The value of the conjugate function  $\psi_1(\tau)$  decreases by virtue of the equation (51)  $d\psi_1 = -\mu_1 d\tau$  and remains

negative. This mode of behaviour is kept until the moment when the belt speed becomes less than one  $u_g(\tau) < u_\gamma(\tau) = 1$ . Taking into consideration the density constraints  $u_\gamma(\tau) \leq u_g(\tau)$  (63), the phase coordinate leaves its lower constraint  $n_0(\tau) = 0$ . There is the process of filling the bunker with the material. The optimal control is

$$u_\gamma(\tau) = u_g(\tau) = \mathcal{G}(\tau), \quad \tau < \tau_{II}, \quad (69)$$

When the ceiling constraint is attained, the control (67) is established again. The value of the conjugate function  $\psi_1(\tau)$  will increase  $d\psi_1 = \mu_2 d\tau$  by virtue of (51). The phase coordinate will be at the upper limit. The above-mentioned function, defining the linear density of the material at the output, will be kept unless the value of the belt motion speed becomes more than one  $u_g(\tau) > u_\gamma(\tau) = 1$ . Thus, regardless of the sign of the conjugate  $\psi_1(\tau)$ , we have the control (69) unless the phase constraint is attained once again. By reaching the lower phase limit, control is again established (67). The phase coordinate will be kept at a limit until the tape speed satisfies the inequality  $u_g(\tau) < u_\gamma(\tau) = 1$ . If  $\psi_1(0) = 0$ , the low limit phase  $n_0(\tau) = 0$  is attained, then the conjugate variable will have a negative value. The subsequent behaviour of the phase trajectory will be similar to the case when the inequality holds at the phase constraint  $\psi_1 < 0$ . If at the initial value of the conjugate variable  $\psi_1 < 0$  phase constraints are not attained, then the value of the conjugate value remains constant along with all control interval  $\tau \in [0, \tau_k]$ . The control  $u_\gamma(\tau)$  is defined by the constraints for control  $u_\gamma(\tau) \leq u_g(\tau)$  (35) and  $u_{\gamma \min} \leq u_\gamma(\tau) \leq u_{\gamma \max}$  (37). If phase constraints are unattainable in the case of the optimal control of the belt speed  $u_g(\tau)$ , the condition of the material in the bunker  $n_0(\tau)$  and the value of the material linear density at the input of the conveyor  $\theta_0(\tau, 0)$  is shown in Fig. 2. Oscillation periods of the phase coordinate  $n_0(\tau)$  and controls  $u_\gamma(\tau) = u_g(\tau)$  are equal in value. The extreme of the function  $n_0(\tau)$  corresponds to values of material supply intensity at the input of the conveyor belt  $u_\gamma(\tau) = 1$  at times  $\tau = 0, 1, 2, \dots$ . There is a domain of initial values of the material in the bunker where phase constraints cannot be attained. For the functions  $\mathcal{G}(\tau)$ ,  $\mathcal{G}_{in}(\tau)$ ,  $\psi(\xi)$  this domain is given with the inequality  $(2/\pi - 0.2) \leq n_0(0) \leq (2/\pi + 0.2)$ . When the bunker is filled to 80%, the maximum value provides the best economy at the minimal risk of industrial losses (Marais and Pelzer, 2008). A simulated model of the transport system with the bunkers is given in (Marais and Pelzer, 2008), where the oscillations of bunker loading level depending on the time-of-day electricity tariff are researched. In (Wolstenholm, 1980), the evaluation is accomplished using the dynamic system model, which allows choosing the capacity of the bunker when the overloading of the bunker with the material is not admitted. It should also be noted that the lack of effects associated with the transport delay of the material  $\Delta\tau_1$  leads to significant additional control costs due to the presence of control errors. Hence, the results of the research given in this and many other works pictorially prove the urgency of the problem of how to design the control system for the accumulative bunker load level and the conveyor belt speed. Fig. 2 shows the transport delay value  $\Delta\tau_1(\tau)$  for various times  $\tau$  of the transport system operation with a control algorithm that ensures that the phase coordinate does not reach phase restrictions. The value of the transport delay  $\Delta\tau_1 = \Delta\tau_1(\tau)$  is defined according to the equation

$$1 = G(\tau) - G(\tau - \Delta\tau_1) = \int_{\tau - \Delta\tau_1}^{\tau} u_g(\alpha) d\alpha. \quad (70)$$

Considering the control  $u_g(\tau) = \mathcal{G}(\tau)$  (Fig. 2) analysed algorithm for the case when the phase constrains are not attained, the value of the transport delay oscillates periodically with the oscillation period  $T = 2$  (Fig. 3). The transport delay  $\Delta\tau_1(\tau)$  sets the interval of time during which the material moves along the transportation route and leaves the conveyor line at a time  $\tau$ .

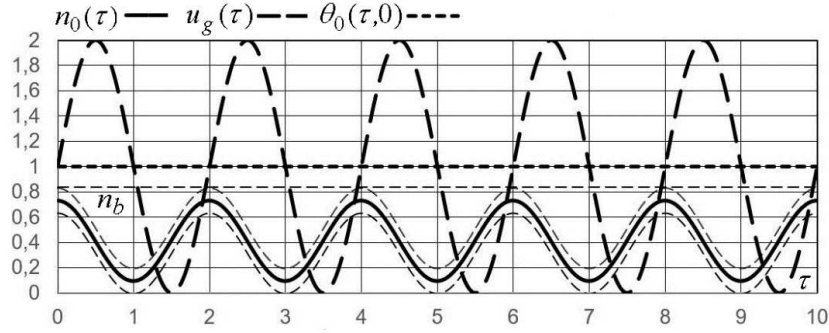


Figure 2. Condition of the material in bunker  $n_0(\tau)$  and value of material linear density at the input of the conveyor  $\theta_0(\tau,0)$  when the optimal control of the belt speed is  $u_g(\tau)$

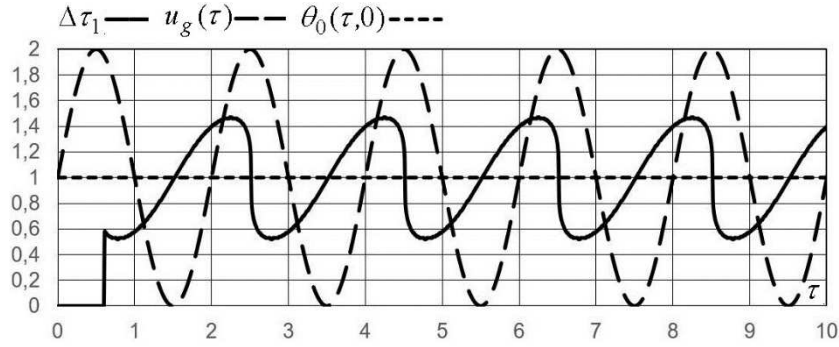


Figure 3. Transport delay  $\Delta\tau_1(\tau)$  of the conveyor line for the control of the belt speed is  $u_g(\tau)$

The density of the material at the output of the conveyor line  $\theta_0(\tau,1)$  at a time  $\tau$  is defined by the value of the flow intensity from the accumulative bunker  $u_\gamma(\tau_1)$  and the conveyor belt motion speed  $u_g(\tau_1)$  at a time  $\tau_1$

$$\theta_0(\tau,1) = \theta_0(\tau_1,0) = u_\gamma(\tau_1) / u_g(\tau_1). \quad (71)$$

The interrelation between time points  $\tau_1$  and  $\tau$  is given in fig.4.

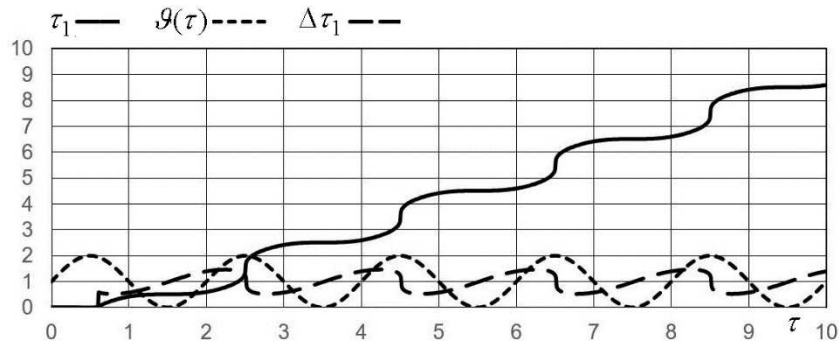


Fig.4. Dependence between the time moment of material loading at the input of the conveyor line  $\tau_1(\tau)$  and transport delay  $\Delta\tau_1(\tau)$  at the output flow  $\mathcal{G}(\tau)$

It should be noted that in the case of combined optimal controls  $u_\gamma(\tau_1)$   $u_g(\tau)$  and the presence of phase constraints, the linear density of the material  $\theta_0(\tau,0)$  remains constant unless the constraints for the phase variables  $n_0(\tau)$  and controls  $u_\gamma(\tau_1)$  are attained. The value of the linear density at times when the phase coordinate is between the constraints corresponds to the maximum allowed values.

## Conclusions

The combined method of optimal control of the flow parameters of a conveyor transport system is considered. A distributed model of a conveyor line with an accumulative input bunker has been developed to synthesise optimal controls. For the given criterion of the performance of the output flow control from the conveyor line, the task of optimal control of the transport system with constraints on phase coordinates and control is set. Using the Pontryagin maximum principle, algorithms for optimal control of the material flow intensity at the output from the accumulative bunker  $u_\gamma(\tau)$  and the conveyor belt speed  $u_g(\tau)$  for the criterion of the performance considered has been researched for the first time in this paper. The results presented in work allow us to draw the following important conclusions: a) the combined method of controlling the output flow of the conveyor line allows providing minimum deviation of the amount of the material output flow from the required value, ensuring uniform distribution of material along the transport route. The exception is the sections when the phase coordinate was at the phase constraints; b) it is possible to create conditions that allow achieving constraints for the phase coordinate  $n_0(\tau)$  and controls  $u_\gamma(\tau)$ ,  $u_g(\tau)$ . These conditions allow a wealth of variations for the synthesis of the algorithms of the control  $u_\gamma(\tau)$ . However, each of these controls results in an increase of specific energy costs for the transportation of the material; c) combined control, aimed at eliminating the non-uniform distribution of rock along the transport route and increasing its linear density to the maximum value, allows reducing the specific energy consumption to move unit mass material along the transport system; d) at the initial period  $\tau \in [0, \tau_{tr}]$  the control of the output flow value is accomplished by changing the belt speed.

Prospects for further research are a) synthesis of optimal controls for a conveyor system with an input and output of accumulative bunker; b) research of the influence of the initial distribution of material on the formation of non-uniform distribution of material along the transport route.

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