A Kaleckian growth model of secular stagnation with induced innovation

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12 July 2022

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Abstract The present paper works out a demand-led growth model of a labour-constrained economy with an endogenous direction of technical change. It draws on the Kaleckian-Steindlian tradition to examine the short-run relation between income distribution, capacity utilization, and capital accumulation; on Goodwin-type growth cycle models to investigate the dynamic interaction between labour market and distributive conflict; on the induced innovation literature to link labour productivity growth to income distribution. The model defines a two-dimensional system of differential equations in the wage share and the employment rate at full capacity to investigate the properties of the long-run equilibrium. In a Kaleckian fashion, an endogenous rate of capacity utilization allows effective demand and income distribution to affect the long-run equilibrium. We find that: i) an exogenous increase in workers' bargaining power raises the long-run labour share, capital accumulation, labour productivity growth, and real wage growth, regardless of the short-run demand and growth regime of the economy; ii) a positive institutional shock to the labour share may cause the long-run employment rate to fall even in a wage-led demand regime; conversely, iii) positive technology shocks reduce the long-run rate of growth of the economy in a wage-led growth regime; thus, strengthening labour market regulation emerges as an unambiguously better strategy to raise the long-run labour share, capital accumulation, and labour productivity growth.

Keywords Functional income distribution, Effective demand, Growth regimes, Endogenous technical change

JEL classification D33, E12, E24, E25, O40

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1. INTRODUCTION

The weak recovery that followed the global crisis of 2008 has given new energy to the old issue of "secular stagnation", that is, a long-lasting period of low or even negative growth in mature capitalist economies, originally put forward by Hansen (1939) in the aftermath of the Great Depression. After having been, for a long time, a topic reserved to those economists closer to the deep message of the Keynesian revolution, it has received renewed attention in the mainstream economic agenda with Summers' speech at the IMF in 2013.

The current popular explanation for secular stagnation rests on the pre-Keynesian notion of the Wicksellian "natural interest rate", i.e. the real interest rate clearing the market for loanable funds and associated with full employment (Summers, 2014a; 2014b; 2015). In this view, modern economies are confronted with a substantial increase in saving supply and reduced investment demand, which have led to a significant decline in the equilibrium interest rate for full-employment saving and investment. Low or even negative equilibrium interest rates have made it more difficult for central banks to achieve full employment, due to the zero-lower-bound constraint for the short-term nominal interest rates.

From a classical-Keynesian standpoint, the conventional debate on secular stagnation is largely unsatisfactory, since it ignores the effective demand dynamics and the linkage between income distribution and the long-run performance of the economy. Instead, the relation between income distribution, capital accumulation, and labour productivity growth has always been at the very core of the theories of demand-led growth and the classical-Marxian approach to the theory of induced technical change.

The literature on demand-led growth points to the chronic lack of effective demand as an intrinsic characteristic of a capitalist economy and hence as the main determinant of secular stagnation. Based on the seminal works of Michal Kalecki (1971) and Josef Steindl (1976), the Kaleckian-Steindlian approach to demand-led growth claims that income redistribution from wages to profits may imply a lower level of economic activity and slower capital accumulation. Since capitalists have a higher propensity to save than workers, a decrease in the labour share of income is supposed to dampen consumption demand and – via Keynesian accelerator – investment demand.[1] The Kaleckian-Steindlian approach relies upon the assumption of an endogenous rate of capacity utilization, which allows the contractionary effect of a decline in the labour share to have long-run effects.

The classical-Marxian approach to induced innovation points to the increase in unit labour costs as the main driver of labour productivity growth. A decline in the labour share of income (that is, the counterpart of unit labour costs at a macro level) is supposed to lessen the firms' incentive to invest in labour-saving innovations.[2] Recent contributions in this tradition (e.g. Petach and Tavani, 2020; Barrales-Ruiz, *et al.*, 2021; Luzuriaga and Tavani, 2021) claim that in mature economies, in which labour supply poses constraints to growth, a decline in the labour share leads to slower capital accumulation via a reduced pressure to innovate rather than via the Kaleckian-Steindlian "underconsumptionist" channel. The key argument is that, in mature economies, capital accumulation is anchored to the natural growth rate, and hence the poor macroeconomic performance of these countries over the last decades must be explained by factors affecting labour productivity growth.

The present paper adds to this debate by presenting a Kaleckian model of a labour-constrained

[1] See Hein (2016) for a comparison between the theoretical foundations of the conventional view and the alternative Kaleckian-Steindlian paradigm on secular stagnation. For a critique of the zero-lower-bound economic theory and the potentially detrimental effects of its policy recommendations, see Palley (2019).

[2] See Hein (2017) or Dutt (2018) for a survey of the heterodox models of income distribution and growth. A survey of endogenous technical change theories falling within non-mainstream traditions is provided by Tavani and Zamparelli (2018).

economy with induced innovation. Three modifications are made to the standard model with a Bhaduri-Marglin investment function: (i) I introduce a labour market, which allows income distribution and the employment rate to interact with each other along Goodwinian lines, while emphasizing the role of labour supply constraints to economic growth; (ii) based on the arguments by Kalecki (1979), Steindl (1979), and Skott (1989; 2010), I modify the standard Bhaduri-Marglin investment function by assuming that the employment rate affects negatively firms' investment decisions; and (iii) labour productivity growth is assumed to be endogenous to the labour share, in accordance with the classical-Marxian view of wage-led labour-saving innovations.

We find that an increase in workers' bargaining power unambiguously raises the long-run labour share, capital accumulation, and labour productivity growth, irrespective of the short-run demand and growth regime of the economy. Thus, conditional on institutional shocks, the long-run rate of growth of the economy is increasing in the labour share of income. Differently from conventional Kaleckian findings, the long-run effects of distribution on capital accumulation do not rest on the assumption of an endogenous rate of capacity utilization, as a decline in the labour share impacts the long-run rate of growth via the induced innovation channel rather than underconsumption – a result in line with a classical account of secular stagnation.[3] Conversely, the long-run effects of exogenous technology shocks on capital accumulation and labour productivity growth rely entirely upon Kaleckian mechanisms since they are dependent on endogenous capacity utilization and the short-run demand and growth regime of the economy. Conditional on technology shocks, the long-run rate of growth of the economy is increasing (decreasing) in the labour share if the economy exhibits a wage-led (profit-led) demand and growth regime in the short run. Thus, since exogenous productivity shocks reduce the long-run labour share, strengthening labour market regulation emerges as an unambiguously better strategy to improve income distribution and raise long-run capital accumulation and labour productivity growth if the short-run growth regime is wage-led. However, positive institutional shocks to the labour share may cause the employment rate to fall even in a wage-led demand regime.

The remainder of this paper is organized as follows. Section 2 provides an extensive discussion of the related literature and the main contributions of this paper. Section 3 proposes a simple theoretical model and derives the basic equations for the analysis. Section 4 discusses the characteristics of the short-run equilibrium. Section 5 details the properties of the dynamical system and the long-run equilibrium. Section 6 derives the necessary and sufficient conditions for the local stability of the long-run equilibrium. Section 7 details the main results of comparative statics analysis. Section 8 then concludes.

2. RELATED LITERATURE

The present paper is related to different streams of literature. First, the model economy in the short run is formalized according to the Kaleckian-Steindlian tradition. Lavoie (2014) lists the distinctive features of this class of models: (i) an investment function, independent of savings, including the rate of capacity utilization, which captures the Keynesian accelerator principle; (ii) a fixed mark-up on unit variable costs; (iii) class-based saving behaviour, with workers having a lower propensity to save than capitalists; (iv) an endogenous rate of capacity utilization that may diverge

[3] A similar argument is put forward by Rada, *et al.* (2021) in a neo-Goodwinian framework.

from the normal level even in the long run. In fact, only (iv) is essential to this tradition,[4] since the assumption of an endogenous rate of capacity utilization is crucial for this class of models to allow functional income distribution to affect steady-state capital accumulation.[5]

The earlier formulations of the Kaleckian growth models, developed by Rowthorn (1981), Dutt (1984), Taylor (1985), and Amadeo (1986), reflect the "underconsumptionist" view that income redistribution from profits to wages has long-run positive effects on capital accumulation via increased consumption demand. The more flexible Bhaduri-Marglin model, that has progressively become a benchmark model in the Kaleckian-Steindlian literature, claims that, if the dual role of wages as a cost of production to the firm and as the main source of demand in the economy is considered, economic growth may either be "wage-led" or "profit-led", depending on the differential in propensities to save and the relative responsiveness of investment to the profit share and the rate of capacity utilization (Bhaduri and Marglin, 1990; Marglin and Bhaduri, 1990).

However, traditional Kaleckian growth models fail to consider the labour market satisfactorily, since they do not define an equilibrium rate of employment and rely on the implicit assumption that firms face an infinitely elastic labour supply at a constant real wage rate. Thus, the canonical framework does not allow considering the positive impact of the employment rate on the real wage rate and its feedback effect on capital accumulation and labour productivity growth. Some exceptions are the works of Dutt (1992), Lima (2004), and Sasaki (2013), which incorporate a theory of inflation based on conflicting income claims of workers and firms and make the workers' target share in income depend on the employment rate, so that capacity utilization, employment rate, capital accumulation, and income distribution are all endogenously determined.[6]

This is a severe limitation since mature economies, although demand-led, are in fact labour-constrained economies, that is, they face relevant labour supply constraints to economic growth. As Flaschel and Skott (2006) and Skott (1989, 2010) claim, assuming that capital accumulation is not constrained by the growth of the labour force may be reasonable only for less developed countries, having a hidden reserve army in agriculture, among women, and from immigration. In OECD countries, from at least the 1960s, "[t]he hidden reserve army gradually became depleted (…) and immigration was hampered by growing political resistance. As a result, the economy became mature in Kaldor's (1966) sense of the term: its growth rate became constrained by the growth in the labour force" (Flaschel and Skott, 2006, pp. 327-328). In his well-known list of six "stylized facts" of economic growth in industrialized economies, Kaldor (1957) posited that, over long periods, factor shares of national income are roughly constant, and hence the real wage rate grows at the same rate as labour productivity. Taylor, *et al.* (2018) also include a constant employment rate in the long run.[7]

In the present paper, capital accumulation is assumed to be constrained by the growth rate of

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[4] The assumptions of class-based saving behaviour and demand-driven investment are indeed common to all demand-led growth models. Furthermore, Kaleckian growth models have been investigated under different pricing procedures (see, for instance, Lavoie (2019), who assumes that the profit share depends on target-return pricing at a firm level). In this paper, I assume that the profit share is determined along classical-Marxian lines as a residual after workers are paid their share in national income, without any specific assumption on firms' pricing procedures.

[5] The potential divergence of the rate of capacity utilization from its normal rate is controversial and has been questioned by a number of authors. For a discussion, see Lavoie (1995), Nikiforos (2016), Girardi and Pariboni (2019), and Trezzini and Pignalosa (2021).

[6] Cassetti (2003) incorporates conflicting claims inflation into a Kaleckian model, but makes the workers' target share depend on the *growth* of the employment rate, rather than on the *level* of the employment rate, thus he can endogenize income distribution through the labour market channel even without stabilizing the employment rate. Stockhammer (2004) finds that in a wage-led growth regime the long-run equilibrium rate of employment is unstable. Yet, labour productivity growth is exogenous, hence he does not consider the stabilizing effect of induced technical change.

[7] The downward trend of the labour share and the upward trend of the employment rate over the last decades need not imply a rejection of the steady-state growth assumption, since they can be interpreted as a transition towards a new steady-state growth path.

effective labour supply so that the employment rate is constant in the long run. For the long-run analysis, I define a two-dimensional system of differential equations in the wage share and the long-run component of the employment rate, describing the motion of the two variables around their long-run equilibrium values. The assumption that the economy is labour-constrained allows dealing with labour scarcity, which is a distinctive feature of a mature economy, while keeping that growth is demand-driven.

The introduction of a labour market relates this paper to Goodwin's (1967) seminal work on the growth cycle, which formalizes Marx's (1976) view of economic fluctuations, based on the interaction between profit-constrained capital accumulation and an employment-driven labour share. The model predicts a counterclockwise cycle in the employment rate-labour share space, as a result of profit-led employment and profit squeeze in income distribution. In the upturn of the business cycle, that is, a period of faster capital accumulation, and hence faster growth of labour demand, the labour share rises; the erosion of profitability leads to the downward phase of the business cycle, in which capital accumulation and the growth of labour demand slow down; as a result, the labour share falls, restoring profitability and inducing a new phase of expansion.

The original model has been progressively extended in different directions. Since Goodwin (1967) assumes the validity of Say's law, several contributions have been proposed to incorporate the disequilibrium in the goods market and a counterclockwise cycle in the utilization-labour share plane of the kind observed in the US and other OECD countries (Zipperer and Skott, 2011). Barbosa-Filho and Taylor (2006) develop a "structuralist Goodwin model", with a system of differential equations in the wage share and the rate of capacity utilization. Sasaki (2013) examines the motion of the rate of capacity utilization jointly with the wage share and the employment rate in a model with endogenous technical change. Von Arnim and Barrales (2015) examine Harrodian and Kaleckian narratives on demand-driven distributive cycles with fluctuations in employment, labour share, and capacity utilization.

Following You (1994) and Lima (2004), in the present paper, I distinguish between a short-run component and a long-run component of the employment rate. The short-run component follows the cyclical fluctuations of the rate of capacity utilization, which is supposed to adjust instantaneously to bring the goods market back to equilibrium. The long-run component, i.e. the employment rate at full capacity utilization, is treated as a state variable that adjusts slowly in the long run, along with the wage share.[8] Thus, differently from the literature originated from the Goodwin model of growth cycle, this paper does not adopt a dynamic specification for the rate of capacity utilization, since it assumes that the goods market clears much faster than income distribution and the long-run component of the employment rate. Naturally, both the cyclical and the long run components of the employment rate exert downward pressure on profitability.

In addition, following Flaschel and Skott (2006) and Skott (1989; 2010), I modify the standard Bhaduri-Marglin investment function letting the employment rate affect negatively firms' investment decisions. The inclusion of the employment rate as an argument of the investment function can be motivated on both macroeconomic and microeconomic grounds. In Flaschel and Skott (2006), this assumption represents an attempt to formalize the argument put forward by Kalecki (1971) and Steindl (1979) that persistently high employment, by strengthening the economic and social position of workers and trade unions, undermines the capitalists' state of confidence. Even though the original argument referred to a political opposition of capitalists to full employment, it seems indeed reasonable to expect that the capitalists' general decline of confidence associated with high employment could impact negatively firms' investment plans. At a micro level, the employment rate affects firms' investment decisions through its effects on the costs of recruitment. Due to the not

[8] The Goodwin original model considers implicitly only the long-run component since it assumes full capacity utilization.

perfectly competitive nature of the labour markets, higher employment rates make it harder for firms to hire and retain workers with the desired skills (Skott, 1989; 2010). Thus, in a mature economy, the costs of changing output are associated positively with labour market tightness, as measured by the employment rate. Furthermore, a fall in the size of the reserve army, "may lead to increased worker militancy, and increased monitoring and additional managerial input may also be needed in order to maintain discipline and prevent shirking" (Skott, 2010, p. 120). The impact of labour market tightness on the costs of recruitment, shirking, worker militancy, and power relations in capitalist economies motivates the inclusion of the employment rate as an additional argument in the investment function, in that these effects are independent of the impact of labour market tightness on the wage-profit divide.

The assumption of a constant rate of employment in the long run implies that the actual rate of growth is equal to the natural rate of growth, thus relating the present paper to the literature concerned with the reconciliation of aggregate demand and aggregate supply in economic growth theory (Dutt, 2006; 2010; Storm and Naastepad, 2012a; 2012b). In this paper, the natural rate of growth is made endogenous to the labour share according to the theory of distribution-induced technical change.

The core idea of the theory of distribution-induced technical change is that the direction of technical change is endogenously determined by the relative size of the labour and capital shares in total costs. An increase in the labour share of income is then supposed to encourage firms to adopt innovations that allow them to save on unit labour costs. Within the classical-Marxian tradition, induced technical change is thought of as an instrument in the hands of capitalists in the class conflict to regenerate the reserve army of labour in the face of rising workers' bargaining power (Tavani and Zamparelli, 2018; Foley, *et al.*, 2019). Within the neoclassical tradition, it has been conceived as being driven by relative factor endowments (Brugger and Gehrke, 2017). This interpretation dates back to Hicks's (1932) claim that an increase in relative input prices stimulates innovations that replaces the factor of production that has become relatively more scarce. This argument has been formalized by Kennedy (1964) and Samuelson (1965) by means of a decreasing and concave "innovation possibility frontier", whose shape and position are exogenously given by technical factors, and representing all feasible combinations of labour- and capital-saving innovations. The microeconomic choice of the optimal direction of technical change makes labour productivity growth an increasing function of the labour share at a macro level.

The induced innovation hypothesis has been integrated into Goodwinian and classical-Marxian models, as a firm's maximization problem *à la* Kennedy is consistent with the Okishio (1961) rule for viable innovations in the classical analysis of the choice of techniques (Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2005; Tavani, 2012, 2013; Zamparelli, 2015). However, some contributions in classical-Marxian and post-Keynesian literature simply postulate a positive dependence of labour productivity growth on the labour share, with no microeconomic foundations (e.g. Lima, 2004; Dutt, 2013).

More recent works in classical-Marxian and neo-Goodwinian traditions have shown that the theory of induced technical change can be deployed to build a classical narrative on secular stagnation for labour-constrained economies, in which the slowdown in capital accumulation is linked to the decline in the labour share via the balanced growth condition rather than the Kaleckian-Steindlian underconsumptionist channel (Petach and Tavani, 2020; Barrales-Ruiz, *et al.*, 2021; Luzuriaga and Tavani, 2021, Rada, *et al.*, 2021). The central argument is that, provided that technical change is induced by income distribution, the natural rate of growth is endogenous and positively related to the labour share. Thus, in labour-constrained economies, in which the actual rate of growth is anchored to the natural rate of growth, the steady-state rate of capital accumulation is wage-led, even though capital accumulation is profit-led at business cycle frequencies. Thus, an institutional shock against the labour share, despite its initial positive impact on capital accumulation, unambiguously lowers the actual rate of growth of the economy in the long run.

This paper shows that, even in a Kaleckian model of a labour-constrained economy, labour market deregulation is detrimental to the long-run actual rate of growth, irrespective of the short-run demand and growth regime of the economy and the assumption of an endogenous rate of capacity utilization, as the relevant channel is induced innovation. Thus, conditional on institutional shocks, long-run capital accumulation is increasing in the labour share even if the economy exhibits profit-led activity in the short run, as predicted by more recent works in the classical tradition. However, the short-run demand and growth regime still appears to be crucial for assessing the long-run effects of technology shocks.

3. THE STRUCTURE OF THE MODEL

Consider a closed economy with no government, in which only one good is produced with two homogeneous inputs, labour and a non-depreciating capital. The homogeneous good is used for both consumption and investment. There are two social classes: capitalists, who own the economy's capital stock and receive profits, and workers, that inelastically supply one unit of labour in each period and receive wages. The relation between labour and capital inputs and the homogenous output is represented by a fixed-coefficients or Leontief production function:

$$Y = \min\{a_L L, a_K u K\} \tag{1}$$

where Y denotes actual output in real terms; L, labour employed in production; K, capital; $u = Y/Y_p$, the rate of capacity utilization, with Y_p being full-capacity output; $a_L = Y/L$, labour productivity; and $a_K = Y_p/K$, the ratio of full-capacity output to capital stock. The assumption of a fixed-coefficients production function implies that demands for labour and capital are inelastic to input prices, and one or both inputs may not be fully employed. Since the economy is demand-constrained, we assume that the economy operates at less than full capacity and full employment of labour.[9] Following the Kalecki-Steindl tradition, we consider the rate of capacity utilization as an accommodating variable, that adjusts to bring saving into equilibrium with investment, without any tendency to converge to a unique normal rate.

Denoting the workers' real wage rate by w and the profit rate on capital stock by $r = \pi u a_K$, where π is the profit share, in each period t national income in real terms is given by:

$$Y = wL + rK \tag{2}$$

National income accrues to the two social classes in the economy, workers and capitalists, which only receive wages and profits, respectively. Workers devote all their income to consumption, whereas capitalists have propensity to save $s \in (0,1)$. These behavioural assumptions are in line with the classical and Keynesian traditions. They follow Kaldor (1955-56; 1966) argument that the functional nature of profits implies that a major part of profits is retained for investment purposes and, to the extent that profits and wages are unequally distributed across individuals, they are also consistent with Keynes's (1936) absolute income hypothesis that the propensity to save of high income individuals exceeds the propensity to save of low income earners.

The profit share is defined along classical-Marxian lines as a residual after the workers are paid their share in national income:

[9] For the reasons for which firms voluntarily choose to hold excess capacity, see Hein (2014).

$$\pi = 1 - \omega \tag{3}$$

where the wage share is given by $\omega = w/a_L$.

Since total savings are given by $S = srK$, the saving function, i.e. the ratio of savings to capital stock, is:

$$g^s \equiv \frac{S}{K} = s\pi u a_K \tag{4}$$

Let us assume an investment function independent of savings, in which the ratio of investment to capital stock is increasing in the profit share, the rate of capacity utilization, and an exogenous variable γ, and decreasing in the employment rate. Denoting the employment rate by $e = L/N$, with N being labour supply, we have:

$$g^i \equiv \frac{I}{K} = g^i(u, \pi, e, \gamma) \qquad g_u^{i'} > 0, \;\; g_\pi^{i'} > 0, \;\; g_e^{i'} < 0, \;\; g_\gamma^{i'} > 0 \tag{5}$$

We define equation (5) as a "Bhaduri-Marglin-Skott investment function". The rate of capacity utilization captures the Keynesian accelerator principle, namely the positive response of investment to a positive variation in actual output. The economic rationale for the inclusion of the profit share is that it may be considered as an index for firms' profit margins. Thus, based on the argument of Bhaduri and Marglin (1990) and Marglin and Bhaduri (1990), the rate of capacity utilization and the profit share capture the demand-side and the cost-side of the expected profit rate, respectively, provided that the actual profit rate can be seen as an indicator for expected profitability. Moreover, since firms operate in incomplete financial markets and realized profits provide internal funds for firms' investment plans, higher profit share and capacity utilization, as components of the realized profit rate, make it easier to have access to external funding. The employment rate has a negative effect on firms' investment decisions because a fall in the reserve army of labour strengthens the economic and political position of workers, thus making capitalists' state of confidence decline (Kalecki, 1971; Steindl, 1979), and increases firms' costs of recruitment and monitoring to prevent workers' militancy and shirking (Skott, 2010). The exogenous parameter γ represents autonomous investment, and can be interpreted as the "animal spirits" of capitalists or the expected trend of future sales (Lavoie, 2014).

At each point in time, labour supply is $N = N_0 e^{nt}$, where N_0 denotes the initial value of labour supply and $n > 0$ denotes the exogenous growth rate of N.

Since balanced growth requires the ratio of full-capacity output to capital stock to be constant, and the equilibrium condition in the goods market requires the rate of capacity utilization to be also constant, in the long run actual output and capital stock grow at the same rate $g = \hat{K} = \hat{Y}$.[10] For the sake of simplicity and without loss of generality, we assume $a_K = 1$ in the reminder of the paper. Then, we define $a_L \equiv a$ to save notation.

The fixed-coefficients nature of the production function implies that the rate of capacity utilization and the employment rate cannot be taken as independent of each other, since an increase in output in the short run will necessarily be associated with an increase in the employment rate. From equation (1), the employment rate is related to the state of the goods market by:

$$e = uk \tag{6}$$

where $k = K/aN$. Equation (6) shows that the employment rate comprises two components: a short-

[10] For any variable x, $\dot{x} = dx/dt$ and $\hat{x} = \dot{x}/x$.

run component, following the cyclical fluctuations of output as measured by variations in the rate of capacity utilization, and a long-run component k, namely the current capital stock in effective labour supply units. The latter is the employment rate at full capacity utilization and is determined by long-run changes in the growth of labour productivity, capital accumulation, and the growth of labour supply.

Substituting from equation (6) into equation (5), the investment function becomes:

$$g^i \equiv \frac{I}{K} = G(u, \pi, k, \gamma) \qquad G'_u > 0, \;\; G'_\pi > 0, \;\; G'_k < 0, \;\; G'_\gamma > 0 \tag{7}$$

A rise in output will have both a positive effect on firms' investment decision via the Keynesian accelerator principle and a negative effect due to the resulting fall in the size of the reserve army. The sign $G'_u > 0$ means that we assume that the first effect dominates, consistently with the observation that investment responds positively to an increase in capacity utilization.

As in Goodwin (1967) original model, the growth rate of the real wage is assumed to be an increasing function of the employment rate and an exogenous variable, that we call α.

$$\frac{\dot{w}}{w} = h(e, \alpha) \qquad h'_e > 0, \;\; h'_\alpha > 0 \tag{8}$$

Equation (8) formalizes the source of the Marxian profit-squeeze mechanism and is consistent with the real Phillips curve in mainstream economics. An increase in the employment rate and labour market tightness raises workers' relative bargaining strength, leading to faster growth of the real wage rate.[11] However, in contrast to the original model, the employment rate is also determined by the cyclical fluctuations of the aggregate demand (equation (6)). We interpret the exogenous variable α in a broad sense as a parameter that captures all institutional factors favouring workers' relative bargaining power.

As stated above, Harrod-neutral technical change is the only one consistent with balanced growth, thus we assume that only labour productivity growth is affected by technological innovations. The growth rate of labour productivity depends on the prevailing income distribution, being positively related to the wage share in national income, and an exogenous variable τ:

$$\frac{\dot{a}}{a} = f(\omega, \tau) \qquad f'_\omega > 0, \;\; f'_\tau > 0 \tag{9}$$

Equation (9) is consistent with a classical-Marxian approach to induced innovation, in which labour-saving innovations are regarded as a weapon of capitalists for restoring profitability in the face of rising unit labour costs. It can also be seen as the solution of a firm's maximization problem *à la* Kennedy, which results in a growth rate of labour productivity being positively related to the labour share at a macro level (Kennedy, 1964; Samuelson, 1965).[12] The variable τ represents all exogenous factors affecting labour productivity growth.

[11] For the sake of simplicity, we expressed all variables in real terms, thus avoiding to frame the distributive conflict in the context of a conflicting income claims theory of inflation. However, the growth rate of the real wage as determined by equation (8) can be seen as the outcome of the bargaining process between workers and firms after their conflicting targets have been reconciled by the price inflation rate, provided that the workers' target wage share is made to depend on the employment rate.

[12] In our version, however, differently from models with microfounded induced technical change, the output-capital ratio is assumed to be constant even out of the steady state.

4. THE SHORT-RUN EQUILIBRIUM

The short run is defined as a time period in which the capital stock K, the labour supply N, the real wage rate w, and the labour productivity a are all taken as given. The rate of capacity utilization is the adjusting variable in the goods market, thus it will increase (decrease) when demand exceeds (falls short of) supply in the goods market. Any disequilibrium in the goods market will be self-correcting (and hence the short-run equilibrium will be stable) if the investment/capital ratio is less responsive than the saving/capital ratio to changes in the rate of capacity utilization (the so-called Keynesian stability condition):

$$G'_u < s(1-\omega) \tag{10}$$

In what follows, we assume that the Keynesian stability condition holds, and hence inequality (10) is always satisfied.[13] The goods market is in equilibrium when $g^s = g^i$.

Total differentiation of $g^s = g^i$, with (3), (4), and (7), with respect to the wage share ω in the equilibrium point yields:

$$\frac{du^*}{d\omega} = \frac{su - G'_\pi}{s(1-\omega) - G'_u} \tag{11}$$

where "$*$" stands for short-run equilibrium.

Using equation (11) and totally differentiating equations (6) and (7) we obtain:

$$\frac{de^*}{d\omega} = \frac{(su - G'_\pi)k}{s(1-\omega) - G'_u} \tag{12}$$

$$\frac{dg^*}{d\omega} = \frac{s[G'_u u - G'_\pi(1-\omega)]}{s(1-\omega) - G'_u} \tag{13}$$

Given the Keynesian stability condition (equation (10)), $du^*/d\omega > 0$ (and $de^*/d\omega > 0$) if $su > G'_\pi$, that is, when the propensity to save s is high and the partial effect G'_π is weak, whereas $dg^*/d\omega > 0$ if $G'_u u > G'_\pi(1-\omega)$, namely with a strong partial effect G'_u and a weak partial effect G'_π. Since we have $G'_\pi < s(1-\omega)G'_\pi/G'_u$ from the Keynesian stability condition, $dg^*/d\omega > 0$ implies $du^*/d\omega > 0$ (and $de^*/d\omega > 0$), whereas $dg^*/d\omega < 0$ is compatible with either $du^*/d\omega > 0$ (and $de^*/d\omega > 0$) or $du^*/d\omega < 0$ (and $de^*/d\omega < 0$). Thus, three different configurations are possible, depending on the relative sizes of the elasticities of the investment rate to the profit share G'_π and the rate of capacity utilization G'_u, and the capitalists' propensity to save s:[14]

i) a "pure" profit-led growth regime (i.e. profit-led demand and profit-led growth), in which $dg^*/d\omega < 0$, $du^*/d\omega < 0$, and $de^*/d\omega < 0$;
ii) an intermediate case (i.e. wage-led demand and profit-led growth), in which $dg^*/d\omega < 0$, $du^*/d\omega > 0$, and $de^*/d\omega > 0$;
iii) a "pure" wage-led growth regime (i.e. wage-led demand and wage-led growth), in which $dg^*/d\omega > 0$, $du^*/d\omega > 0$, and $de^*/d\omega > 0$.

[13] This is a standard assumption in the relevant literature. However, some have criticized this hypothesis on both theoretical and empirical grounds. See, for instance, Skott (2017).

[14] The three cases were originally called "exhilarationism", "conflicting stagnationism", and "cooperative stagnationism" respectively by Bhaduri and Marglin (1990).

These results are in line with the relevant literature originated from Bhaduri and Marglin (1990) and Marglin and Bhaduri (1990). A redistribution from profits to wages has an expansionary effect on consumption demand and a direct contractionary effect on investment demand. The overall effect on the rates of capacity utilization and employment will be positive (negative) if the effect on consumption (investment) dominates the effect on investment (consumption). Moreover, a higher wage share causes the cost-side profitability of investment to fall, but has an ambiguous effect on the demand-side profitability; thus, it results in an overall decline in the rate of capital accumulation only if the new equilibrium rate of capacity utilization is lower or it is impossible for the rate of capacity utilization to rise enough to offset the negative effect of the fall in cost-side profitability.[15]

In a pure wage-led (profit-led) growth regime, in which the partial effect of the profit share on investment G'_π is weak (strong) relative to the capitalists' propensity to save s and the responsiveness of investment to capacity utilization G'_u, a redistribution from profits to wages results in an increase (decrease) in capacity utilization, employment rate and capital accumulation. In the intermediate case, capitalists' propensity to save s is high relative to the response of the investment rate to the profit share G'_π, but the response of the investment rate to capacity utilization G'_u is too weak to prevent firms' investment from decreasing; thus, a redistribution from profits to wages results in higher capacity utilization and employment but slower capital accumulation.

Both a fall in the propensity to save out of profits and an increase in autonomous investment lead to higher capacity utilization and employment and faster capital accumulation.

Totally differentiating $g^s = g^i$, with (3), (4), and (7), with respect to the propensity to save s in the equilibrium yields:

$$\frac{du^*}{ds} = -\frac{(1-\omega)u}{s(1-\omega) - G'_u} < 0 \tag{14}$$

If we totally differentiate equations (6) and (7), using equation (14), we obtain:

$$\frac{de^*}{ds} = -\frac{(1-\omega)uk}{s(1-\omega) - G'_u} < 0 \tag{15}$$

$$\frac{dg^*}{ds} = -\frac{(1-\omega)G'_u u}{s(1-\omega) - G'_u} < 0 \tag{16}$$

Equations (14), (15), and (16) show that in the short-run equilibrium the paradox of thrift holds. A fall in propensity to save out of profits expands consumption demand and hence causes the rate of capacity utilization and employment to increase. The rise in output stimulates investment and hence capital accumulation via the Keynesian accelerator. No counterbalancing effects lowering investment demand are exerted now, thus a fall in the propensity to save out of profits unambiguously leads to higher capacity utilization, employment, and capital accumulation.

If we totally differentiate $g^s = g^i$, with (3), (4), and (7), with respect to the autonomous investment parameter γ (the "animal spirits"), we have:

$$\frac{du^*}{d\gamma} = \frac{G'_\gamma}{s(1-\omega) - G'_u} > 0 \tag{17}$$

Using equation (17), total differentiation of equations (6) and (7) yields:

[15] Naturally, this result is strictly dependent on the assumptions of an investment function independent of savings and the rate of capacity utilization as an accommodating variable in the goods market. If $u = \bar{u}$, from equation (4) the equilibrium rate of capital accumulation would be $g^* = s(1-\omega)\bar{u}$, being unambiguously negatively related to the wage share, as in classical-Marxian growth models.

$$\frac{de^*}{d\gamma} = \frac{G'_\gamma k}{s(1-\omega) - G'_u} > 0 \tag{18}$$

$$\frac{dg^*}{d\gamma} = \frac{s(1-\omega)G'_\gamma}{s(1-\omega) - G'_u} > 0 \tag{19}$$

An increase in autonomous investment or "animal spirits" has a direct positive effect on capital accumulation. The increase in autonomous investment, as a component of the aggregate demand, affects positively the rate of capacity utilization, leading to a further increase in capital accumulation. As a result, the short-run equilibrium rates of capacity utilization, employment, and capital accumulation will rise.

Finally, we may assess the effect of an increase in the long-run component of the employment rate on the short-run equilibrium values of capacity utilization, employment rate, and capital accumulation. Total differentiation of $g^s = g^i$, with (3), (4), and (7), with respect to k in the equilibrium point yields:

$$\frac{du^*}{dk} = \frac{G'_k}{s(1-\omega) - G'_u} < 0 \tag{20}$$

Using equation (20) and totally differentiating equations (6) and (7) we have:

$$\frac{de^*}{dk} = \frac{G'_k k + [s(1-\omega) - G'_u]u}{s(1-\omega) - G'_u} \tag{21}$$

$$\frac{dg^*}{dk} = \frac{s(1-\omega)G'_k}{s(1-\omega) - G'_u} < 0 \tag{22}$$

An increase in the long-run component of the employment rate reduces the rate of capital accumulation both directly, since a fall in the reserve army is detrimental for the economic and political position of capitalists, and indirectly, through its negative effect on the rate of capacity utilization. Conversely, an increase in k has an ambiguous effect on the employment rate, since the positive direct effect may be offset by the decline in the rate of capacity utilization. In what follows, we assume $[s(1-\omega) - G'_u]u > -G'_k k$, meaning that the first effect dominates and $de^*/dk > 0$. Even though this assumption may be questionable for the short run, in which the level of the employment rate is mainly determined by the cyclical fluctuations of output, it seems reasonable for a long-run analysis, in which the employment rate is fully endogenized and its long-run component will presumably play a major role in directly determining the employment rate.

5. THE LONG-RUN EQUILIBRIUM

In the long run we assume that the economy has already attained the short-run equilibrium values of capital accumulation $g^*(\omega, k, s, \gamma)$, capacity utilization $u^*(\omega, k, s, \gamma)$, and employment $e^*(\omega, k, s, \gamma)$. Thus, the long-run dynamics is the movement over time of the short-run equilibrium due to variations in capital stock K, labour supply N, real wage rate w, and labour productivity a. We examine the dynamic behaviour of these variables by defining a two-dimensional system of differential equations in the wage share ω and the ratio of capital stock to effective labour supply k. From the definitions of these variables, we have:

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a} \tag{23}$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{a}}{a} - n \tag{24}$$

Substituting from the short-run equilibrium rate of employment into equation (8), and then from the resulting expression, along with equation (9), into equation (23), we obtain the equation of motion for the wage share. Substituting from the short-run equilibrium rate of capital accumulation and equation (9) into (24), we obtain the equation of motion for the capital stock in effective labour supply units.

$$\frac{\dot{\omega}}{\omega} = h[e(\omega, k, s, \gamma), \alpha] - f(\omega, \tau) \tag{25}$$

$$\frac{\dot{k}}{k} = g(\omega, k, s, \gamma) - f(\omega, \tau) - n \tag{26}$$

where "$*$" has been omitted to save notation.[16] It is immediate to check that $\dot{\omega}/\omega = \dot{k}/k = 0$ implies $\dot{u}/u = \dot{e}/e = 0$, that is, if the income shares and the ratio of the capital stock to effective labour supply are constant, the rate of capacity utilization and the employment rate will also be constant over time, thus making our long-run analysis consistent with the short-run analysis.[17] Remind that, in a long-run analysis, the short-run demand and growth regime are reflected in the signs of e'_ω and g'_ω. Thus, the short-run demand regime is profit-led (wage-led) if $e'_\omega < (>)\ 0$; the short-run growth regime is profit-led (wage-led) if $g'_\omega < (>)\ 0$. Moreover, our assumption about the short-run effect of k on the employment rate translates into $e'_k > 0$.

Equations (25) and (26) imply that the equilibrium size of the long-run component of the reserve army of labour and the equilibrium wage share will stabilize at the level that makes the rate of capital accumulation equal to the rate at which the reserve army is replenished (i.e. labour supply growth plus labour productivity growth) and that makes real wages grow at the same rate as labour productivity. Thus, the economy is labour-constrained: in the steady-state equilibrium, economy will grow at the same rate as the full-employment growth rate.[18] However, the dynamic interaction between the two variables is quite different from the Goodwin model (with or without induced technical change). In the latter, the steady-state value of the wage share is fully determined by the dynamic equation of the employment rate at full capacity, and the dynamic equation of the wage share determines the size of the reserve army of labour that makes the workers' bargaining power compatible with it. Conversely, in the present model, the size of the reserve army affects the dynamic behaviour of both variables, and hence both the wage share and the long-run component of the employment rate will adjust to make the employment rate at full capacity stable.[19]

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[16] Total effects of exogenous variables in the short-run analysis are now converted into partial effects. The effect of any variable x on the short-run equilibrium value of a generic y, i.e. dy^*/dx, is now denoted by y'_x.

[17] See Appendix A.

[18] Naturally, this does not imply that the economy will achieve the full-employment *level* of output. Steady-state growth only requires that the employment rate would be constant. In our model, the equilibrium rate of employment is jointly determined by the cyclical fluctuations of the aggregate demand and by a slowly-adjusting ratio of capital stock to effective labour supply.

[19] A further difference is that the employment rate is also affected by the aggregate demand, whereas the original Goodwin model assumes implicitly that Say's law holds. It is also worth remembering that the present model departs from growth cycle models with cyclical fluctuations of aggregate demand (e.g. Barbosa-Filho and Taylor, 2006; Sasaki,

In the long-run equilibrium, we have $\dot{\omega} = \dot{k} = 0$. Therefore, the steady-state values of the wage share $\omega^*(\alpha, \tau, n, s, \gamma)$ and the employment rate at full capacity $k^*(\alpha, \tau, n, s, \gamma)$ solve the following two equations:

$$h[e(\omega, k, s, \gamma), \alpha] - f(\omega, \tau) = 0 \tag{27}$$

$$g(\omega, k, s, \gamma) - f(\omega, \tau) - n = 0 \tag{28}$$

Equation (27) gives the conditions on the labour share and the ratio of capital to effective labour supply that keep income distribution constant. Equation (28) gives the corresponding conditions for the equilibrium in the labour market. Let us call the two isoclines Ω and X, respectively. Figure 1 depicts four alternative configurations of the Ω and X isoclines.[20] For the sake of simplicity, both curves are assumed to be linear.

In the (ω, e) plane, the Ω isocline is upward (downward) sloping if $h'_e e'_\omega < (>) f'_\omega$,[21] which implies that it unambiguously slopes upward in a short-run profit-led demand and growth regime. An increase in the long-run component of employment exerts upward pressure on the wage share, as a tighter labour market allows workers to claim for higher real wages. In a "pure" profit-led regime, an increase in the labour share unambiguously has negative feedback on itself, as any positive deviation would be self-corrected via both a decrease in employment and real wages and an increase in labour productivity. Thus, a constant income distribution requires ω and k to go in the same direction. In intermediate and "pure" wage-led regimes, the slope of the Ω isocline may be either positive or negative, depending on the relative sizes of the induced innovation effect (f'_ω), the wage-led demand (e'_ω), and the reserve-army effect (h'_e). With strongly wage-led demand and a strong reserve-army effect relative to the induced innovation effect, the Ω isocline turns downward sloping.

The X isocline is upward (downward) sloping if $g'_\omega > (<) f'_\omega$, which implies that it always has a negative slope in a short-run profit-led growth regime. An increase in the ratio of capital to effective labour supply now unambiguously puts downward pressure on the employment rate, as the Bhaduri-Marglin-Skott investment function postulates an adverse effect of labour market tightness on firms' investment plans. Conversely, an increase in the labour share gives rise to two effects on the employment rate: on the one hand, it induces a higher rate of labour-saving innovations, which puts downward pressure on employment; on the other hand, it has a negative (positive) effect on capital accumulation if the short-run growth regime is profit-led (wage-led), thus exerting downward (upward) pressure on employment. Thus, in intermediate and pure profit-led regimes, an increase in k needs to be counteracted by a decrease in ω to keep the labour market in equilibrium, whereas the slope of the X isocline turns positive only in a strongly wage-led growth regime with a relatively weak induced innovation effect ($g'_\omega > f'_\omega$).

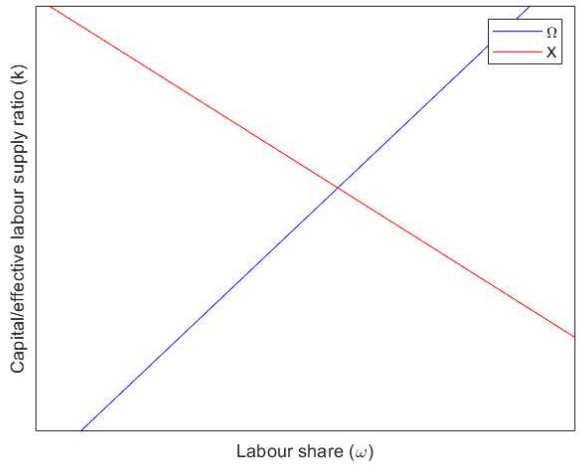
In what follows, we assume that there exists a unique economically meaningful pair of long-run equilibrium solution (ω^{**}, k^{**}).

2013; Rada, *et al.*, 2021), in that the rate of capacity utilization is supposed to instantaneously adjust to clear the saving-investment market, while the full adjustment of the wage share and the employment rate takes place more gradually.

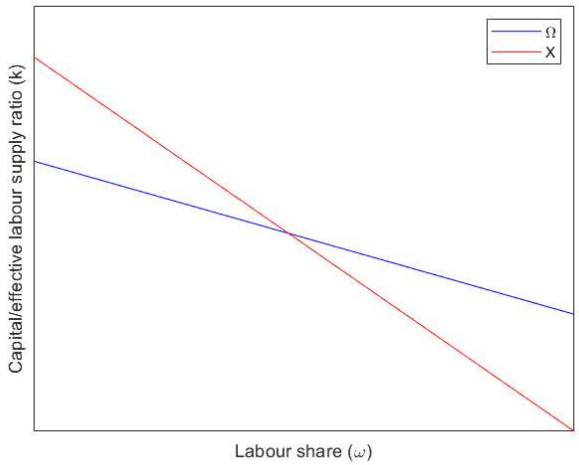
[20] The next section shows that only in the first three configurations the long-run equilibrium is locally stable.

[21] For the computation of the slopes of the Ω and X isoclines, see Appendix B.

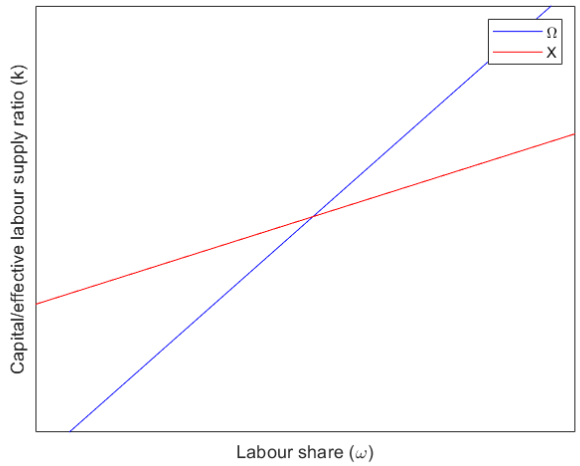
Fig. 1. *Ω and X isoclines*



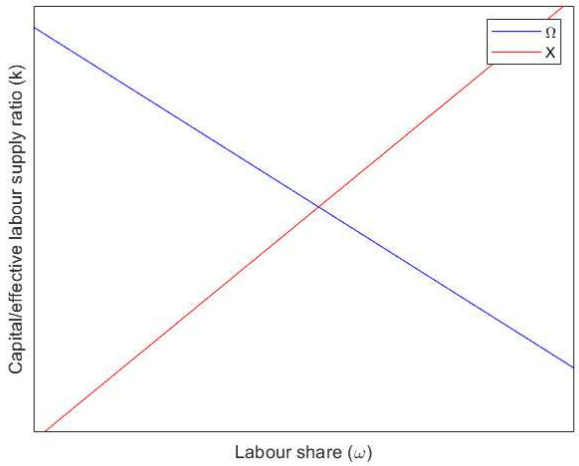
a) Case I: $g'_\omega < f'_\omega$, $h'_e e'_\omega < f'_\omega$



b) Case II: $g'_\omega < f'_\omega$, $h'_e e'_\omega > f'_\omega$



c) Case III: $g'_\omega > f'_\omega$, $h'_e e'_\omega < f'_\omega$



d) Case IV: $g'_\omega > f'_\omega$, $h'_e e'_\omega > f'_\omega$

6. LOCAL STABILITY OF THE LONG-RUN EQUILIBRIUM

We investigate the local stability of the long-run equilibrium linearizing the system of differential equations (25) and (26) around the steady-state equilibrium values of the wage share and the capital/effective labour supply ratio:

$$\begin{bmatrix} \dot{\omega} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} \omega - \omega^{**} \\ k - k^{**} \end{bmatrix} \tag{29}$$

where the elements of the Jacobian matrix $\boldsymbol{J}$ evaluated at the steady-state values $\omega^{**}(\alpha, \tau, n, s, \gamma)$ and $k^{**}(\alpha, \tau, n, s, \gamma)$ are given by:

$$J_{11} \equiv \left.\frac{\partial \dot{\omega}}{\partial \omega}\right|_{\omega=\omega^{**},k=k^{**}} = (h'_e e'_\omega - f'_\omega)\omega^{**} \tag{30}$$

$$J_{12} \equiv \left.\frac{\partial \dot{\omega}}{\partial k}\right|_{\omega=\omega^{**},k=k^{**}} = h'_e e'_k \omega^{**} > 0 \tag{31}$$

$$J_{21} \equiv \left.\frac{\partial \dot{k}}{\partial \omega}\right|_{\omega=\omega^{**},k=k^{**}} = (g'_\omega - f'_\omega)k^{**} \tag{32}$$

$$J_{22} \equiv \left.\frac{\partial \dot{k}}{\partial k}\right|_{\omega=\omega^{**},k=k^{**}} = g'_k k^{**} < 0 \tag{33}$$

Only partial derivatives (31) and (33) are unambiguously signed, whereas the signs of (30) and (32) are crucially dependent on the direct impact of income distribution on capacity utilization and capital accumulation, and on the strength of the reserve-army effect. Equation (31) shows that an increase in the ratio of capital stock to effective labour supply, by increasing the employment rate, will raise the growth rate of the wage share. Equation (33) shows that an increase in k affects negatively its growth rate, since the resulting increase in the employment rate has a negative effect on capital accumulation via Bhaduri-Marglin-Skott investment function. Equation (32) shows that the effect of an increase in the wage share on the growth rate of the long-run component of the employment rate is mediated by its impact on labour productivity growth and capital accumulation. The growth of the capital/effective labour supply ratio is negatively affected in a profit-led growth regime and in a weakly wage-led growth regime with a relatively strong induced innovation effect. Equation (30) shows that the effect of an increase in the wage share on its growth rate is negative in a short-run profit-led demand regime and in a weakly wage-led demand regime with a weak reserve-army effect and a relatively strong induced innovation effect.

The characteristic equation of the Jacobian matrix $\boldsymbol{J}$ in (29) is given by:

$$\lambda^2 + a_1\lambda + a_2 = 0 \tag{34}$$

in which λ denotes a characteristic root and $a_1 = -\mathrm{Tr}(\boldsymbol{J})$ and $a_2 = \mathrm{Det}(\boldsymbol{J})$.

A necessary and sufficient condition for the local stability of the dynamic system is that all characteristic roots are negative or have a negative real part, which occurs when $a_1 > 0$ and $a_2 > 0$ or, equivalently:

$$\mathrm{Tr}(\boldsymbol{J}) < 0, \;\; \mathrm{Det}(\boldsymbol{J}) > 0 \tag{35}$$

where:

$$\mathrm{Tr}(\boldsymbol{J}) = (h'_e e'_\omega - f'_\omega)\omega^{**} + g'_k k^{**} \tag{36}$$

$$\mathrm{Det}(\boldsymbol{J}) = [(h'_e e'_\omega - f'_\omega)g'_k - (g'_\omega - f'_\omega)h'_e e'_k]\omega^{**}k^{**} \tag{37}$$

From equation (36), we have that a sufficient condition for $\mathrm{Tr}(\boldsymbol{J}) < 0$ is $h'_e e'_\omega < f'_\omega$. Equation (37) implies that a necessary condition for $\mathrm{Det}(\boldsymbol{J}) > 0$ is $f'_\omega > \min\{h'_e e'_\omega, g'_\omega\}$, whereas a sufficient condition is $f'_\omega > \max\{h'_e e'_\omega, g'_\omega\}$. Thus, if the economy exhibits both profit-led demand and profit-led growth in the short run, the conditions for the local stability of the long-run equilibrium are always satisfied. If the economy has a short-run intermediate regime, with wage-led demand and profit-led growth, $h'_e e'_\omega < f'_\omega$ is a sufficient, albeit not necessary, condition for the long-run equilibrium to be locally stable. If the economy exhibits both wage-led demand and wage-led growth in the short run,

we need to evaluate the more general necessary and sufficient conditions stated above.

The intuition is straightforward. An increase in the capital/effective labour supply ratio is self-stabilizing via Bhaduri-Marglin-Skott investment function (equation (33)) but has a destabilization effect on the rate of growth of the labour share via reserve-army effect (equation (31)). An increase in the labour share has a stabilization effect via distribution-induced technical change and profit-led demand and growth. By contrast, wage-led demand and wage-led growth act as destabilization forces, as any deviation of ω from its steady-state value will be exacerbated by the positive response of capacity utilization, capital accumulation, and real wages to the labour share (equations (30) and (32)). Thus, a necessary condition for the local stability of the long-run equilibrium is that the induced innovation effect would be strong enough to prevent income distribution from causing an explosive growth of the labour share *or* the capital/effective labour supply ratio (i.e. J_{11} *or* J_{21} must be negatively signed). If the induced innovation effect offset the destabilizing effect of wage-led demand and growth so as to allow the labour share to have a negative effect on *both* its own rate of growth and the rate of growth of the capital/effective labour supply ratio, the long-run equilibrium will be always locally stable (i.e. a negative sign for *both* J_{11} and J_{21} is a sufficient condition for the local stability).

The conditions for the local stability of the long-run equilibrium can be phrased in terms of conditions on the slopes of the Ω and X isoclines. Equation (36) implies that an upward-sloping Ω isocline is a sufficient condition for stability. Equation (37) implies that the equilibrium will be stable only if the slope of the Ω isocline is greater than the slope of the X isocline:[22]

$$\left.\frac{dk}{d\omega}\right|_{\Omega} > \left.\frac{dk}{d\omega}\right|_{X} \tag{38}$$

where:

$$\left.\frac{dk}{d\omega}\right|_{\Omega} = -\frac{h'_e e'_\omega - f'_\omega}{h'_e e'_k} \tag{39}$$

$$\left.\frac{dk}{d\omega}\right|_{X} = -\frac{g'_\omega - f'_\omega}{g'_k} \tag{40}$$

Figure 1 depicts four alternative configurations depending on the slopes of the Ω and X isoclines. In panel a), we have $g'_\omega < f'_\omega$ and $h'_e e'_\omega < f'_\omega$, so the Ω isocline slopes upward while the X isocline slopes downward. In this scenario, the long-run equilibrium is always locally stable, as the conditions $\mathrm{Tr}(\boldsymbol{J}) < 0$ and $\mathrm{Det}(\boldsymbol{J}) > 0$ are both satisfied. This case is consistent with all possible short-run demand and growth regimes, as no restrictions are imposed on the signs of e'_ω and g'_ω. In panel b), we have $g'_\omega < f'_\omega < h'_e e'_\omega$. Both curves slope downward but the Ω isocline is flatter than the X isocline, which implies that the long-run equilibrium is locally stable provided that $\mathrm{Tr}(\boldsymbol{J}) < 0$. This scenario rules out a "pure" profit-led regime, as capital accumulation may be either profit- or wage-led but capacity utilization is unambiguously wage-led. In panel c), in which $h'_e e'_\omega < f'_\omega < g'_\omega$ both curves slope upward but the X isocline is steeper. Thus, both necessary and sufficient conditions for local stability are satisfied. This case is consistent only with a strongly wage-led growth regime, in which both capacity utilization and capital accumulation respond positively to the labour share in the short-run, and the induced innovation effect is relatively weak as compared to the partial effect of distribution on capital accumulation. In panel d), we have $g'_\omega > f'_\omega$ and $h'_e e'_\omega > f'_\omega$, so the Ω isocline slopes downward while the X isocline slopes upward. In this scenario, the long-run equilibrium will be unambiguously unstable since the induced innovation effect is too weak to prevent wage-led

[22] See Appendix B.

demand and growth from causing an explosive growth of ω and k. Suppose, for instance, that a temporary shock brings the capital/effective labour supply above its steady-state value. The increase in k raises the labour share via reserve-army effect. If the Ω curve is downward sloping, as in panel d), the expansionary effect of an increase in the labour share causes the labour share to rise even further, which in turn speeds up capital accumulation and causes the long-run component of employment to increase more, and so on in an explosive way. If the Ω curve were upward sloping, as in panel c), the positive feedback of the wage share on itself would turn into a negative one. Thus, any deviation of the labour share from its steady-state value would be self-correcting rather than self-reinforcing and would bring the economy back to its long-run equilibrium point.

It is worth emphasizing the role of the induced innovation effect and a Bhaduri-Marglin-Skott investment function in the stability of an economy with a short-run wage-led growth regime. Suppose that $f'_\omega = g'_k = 0$. If the economy exhibits a wage-led growth regime in the short run, and labour productivity does not react to the labour share, an increase in the labour share will always have a destabilizing effect on both its own rate of growth and the rate of growth of k. The presence of the induced innovation effect (i.e. $f'_\omega > 0$) opens up the possibility of a stable wage-led growth regime, provided that $f'_\omega > \max\{h'_e e'_\omega, g'_\omega\}$. If we postulate that the employment rate has a dampening effect on firms' investment plans (i.e. $g'_k < 0$), as in a Bhaduri-Marglin-Skott investment function, the necessary condition for a stable wage-led growth regime becomes even weaker (i.e. $f'_\omega > \min\{h'_e e'_\omega, g'_\omega\}$). In terms of conditions on the slopes of the Ω and X isoclines, if $f'_\omega = g'_k = 0$, only the scenario depicted in panel a), with a vertical straight line for X and a short-run profit-led demand and growth regime, would be locally stable. The induced innovation effect makes it possible for the long-run equilibrium in panel a) with a vertical X to be locally stable even if the economy exhibits wage-led demand and growth in the short run. Postulating a negative effect of the employment rate on capital accumulation allows for a stable long-run equilibrium in the cases depicted in panels b) and c). However, it has to be noted that assuming a Bhaduri-Marglin investment function is not sufficient to make an economy with a short-run wage-led growth regime stable. Indeed, even if $g'_k < 0$, a "pure" wage-led growth regime without induced innovation would never satisfy the conditions to be locally stable. The Bhaduri-Marglin-Skott investment function may only increase the chance for a stable wage-led growth regime with distribution-induced technical change.

7. COMPARATIVE STATICS ANALYSIS

This section addresses the long-run effects of institutional and technological shocks on income distribution, employment, and the macroeconomic outcomes of the economy. The parameters of major concern are the institutional variable α, denoting all institutional and political factors which strengthen the workers' bargaining power, and the exogenous variable τ, representing all exogenous factors affecting labour productivity growth. However, we can also evaluate the effects of changes in the growth rate of labour supply n, the propensity to save out of profits s, and the exogenous component of investment γ. We then investigate the effects of changes in the exogenous variables on the steady-state values of the wage share, the capital/effective labour supply ratio, the employment rate, the rate of capital accumulation, and the growth rates of real wages and labour productivity both analytically and graphically. The main results of comparative statics analysis are summarized in Table 1.

Let us define $\Gamma \equiv (h'_e e'_\omega - f'_\omega) g'_k - (g'_\omega - f'_\omega) h'_e e'_k$. Since the implementation of a comparative statics analysis requires the stability of the equilibrium, in what follows we assume $\Gamma > 0$.

Furthermore, for the sake of simplicity, we limit ourselves to the discussion of the case of an upward sloping Ω isocline. This corresponds to the scenarios depicted in panels a) and c) in Figure 1, in which both necessary and sufficient conditions for local stability are satisfied.[23]

Proposition 1 The long-run labour share, capital accumulation, labour productivity growth, and real wage growth are increasing in α; the equilibrium capital/effective labour supply ratio is a positive function of α if and only if the economy exhibits a short-run wage-led growth with $g'_\omega > f'_\omega$; the long-run employment rate is decreasing (increasing) in α if the economy exhibits a short-run profit-led demand and growth regime (wage-led demand and growth regime with $g'_\omega > f'_\omega$).

Proof Total differentiation of equations (27) and (28) with respect to α yields:

$$\frac{d\omega^{**}}{d\alpha} = -\frac{g'_k h'_z}{\Gamma} > 0 \tag{41}$$

$$\frac{dk^{**}}{d\alpha} = \frac{(g'_\omega - f'_\omega)h'_z}{\Gamma} \tag{42}$$

Given that $\widehat{w}^{**} = \hat{a}^{**}$, if we totally differentiate equations (9) and $e(\omega, k, s, \gamma)$, using equations (28), (41), and (42), we obtain:

$$\frac{de^{**}}{d\alpha} = \frac{[(g'_\omega - f'_\omega)e'_k - g'_k e'_\omega]h'_z}{\Gamma} \tag{43}$$

$$\frac{dg^{**}}{d\alpha} = \frac{d\widehat{w}^{**}}{d\alpha} = \frac{d\hat{a}^{**}}{d\alpha} = -\frac{g'_k f'_\omega h'_z}{\Gamma} > 0 \tag{44}$$

An increase in workers' bargaining strength, as measured by the institutional variable α, leads to a downward shift in the Ω isocline, while leaving the X isocline unaffected (Figure 2).[24]

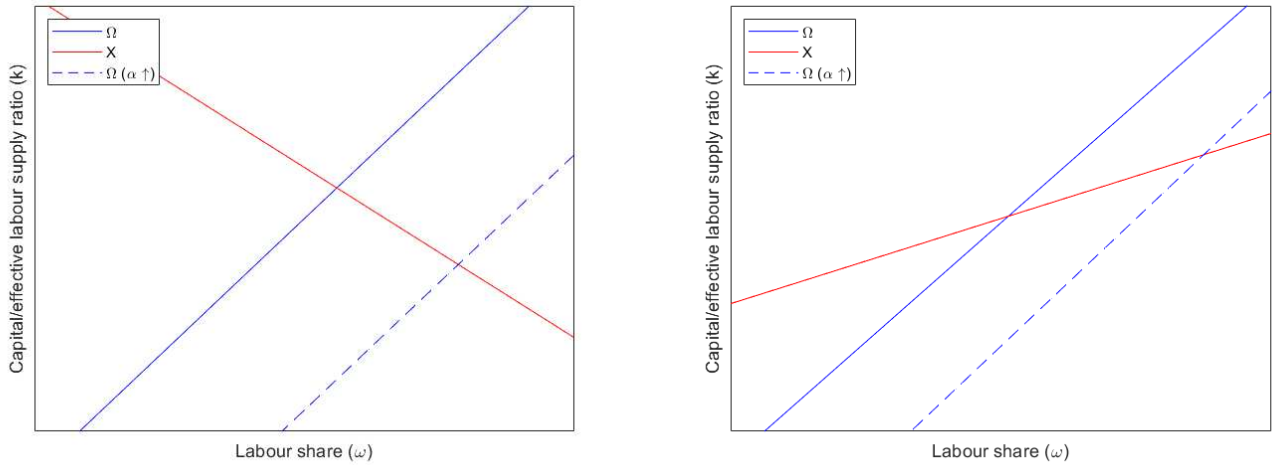
Positive institutional shocks allow workers to claim for higher real wages, for a given capital/effective labour supply ratio, thus increasing the income share they are able to attain in the labour market. The increase in the labour share puts pressure on the employment rate at full capacity via both the goods market and induced technical change. Indeed, rising labour costs induce the adoption of labour-saving innovations and also affect capital accumulation according to the nature of the short-run demand and growth regime of the economy. In the case of a downward sloping X isocline, corresponding to profit-led or weakly wage-led growth regimes, increases in the income share of workers exert downward pressure on the capital/effective labour supply ratio, thus k must decrease to restore a constant employment rate at full capacity (Figure 2a). In the case of a strongly wage-led growth regime, in which the X isocline slopes upward, a constant employment rate at full capacity requires k to increase (Figure 2b). In both cases, the stronger the partial effect of k on capital accumulation $|g'_k|$, the flatter the X isocline, and the more institutional shocks will affect long-run income distribution.

The increase in the long-run wage share is associated with an increase in steady-state labour productivity growth and also with an increase in steady-state capital accumulation, which in a labour-constrained economy is anchored to labour productivity growth via the dynamic equation of employment (equation (28)). Thus, irrespective of the short-run demand and growth regime, redistribution from capital to labour fosters the long-run rate of growth of the economy via the induced

[23] The coefficient a_2 of the Jacobian matrix is positive if and only if $\Gamma > 0$. An upward sloping Ω isocline is a sufficient condition for $a_1 > 0$.

[24] For a formal proof, see Appendix B.

Fig. 2. *The effect of an institutional shock*



a) $g'_\omega < f'_\omega,\ h'_e e'_\omega < f'_\omega$ *b)* $g'_\omega > f'_\omega,\ h'_e e'_\omega < f'_\omega$

innovation effect, that is, capital accumulation is wage-led in the long run. Conversely, the long-run effect of institutional shocks on employment crucially depends on the short-run demand and growth regime of the economy. If the economy exhibits profit-led demand and growth in the short run, an increase in workers' bargaining power will unambiguously lower the long-run employment rate. Indeed, in the scenario depicted in panel a) with profit-led demand, the capital/effective labour supply decreases and the increase in the labour share is associated with a negative partial effect on capacity utilization, which unambiguously lead to a reduction in the employment rate. If the economy exhibits a strongly wage-led demand and growth regime in the short run, as in panel b), positive institutional shocks will raise the long-run employment rate, as the capital/effective labour supply rises and the increase in the labour share is associated with a positive partial effect on utilization. In intermediate and weakly wage-led growth regimes, the effect on long-run employment depends on the relative strength of the partial effect of distribution on capacity utilization and the partial effect of the capital/effective labour supply ratio on employment. In the case depicted in panel a), the long-run employment rate will then rise if and only if $e'_\omega d\omega^{**}/d\alpha > e'_k |dk^{**}/d\alpha|$.

Our results are close to the ones identified by the more recent classical literature on secular stagnation in labour-constrained economies, which links the slowdown in capital accumulation to labour market deregulation (e.g. Petach and Tavani, 2020; Barrales-Ruiz, *et al.*, 2021; Luzuriaga and Tavani, 2021). The key message of this literature is that, provided that technical change is distribution-induced, the long-run rate of growth of an economy turns out to be wage-led even if capital accumulation is assumed to be profit-led at business cycle frequencies. Indeed, in a labour-constrained economy, in which the actual rate of growth is equal to the natural rate of growth, long-run capital accumulation is linked positively to the labour share via the balanced growth condition rather than via the Kaleckian-Steindlian mechanism of a boosting effect of the labour share on consumption demand.

We find that, in a neo-Kaleckian model of a labour-constrained economy with induced innovation, labour market institutions affect growth in the long run via the natural rate of growth rather than via the "traditional" underconsumptionist channel. Indeed, conditional on institutional shocks, the short-run demand and growth regime of the economy does not matter for addressing the long-run effects of income distribution on capital accumulation and labour productivity growth. Thus,

labour market deregulation will depress both the labour share and the long-run rate of growth even when a higher labour share is associated with slower capital accumulation in the short run, as is the case in economies with profit-led activity.

These results rely upon the presence of an adverse effect of labour market tightness on firms' investment plans, as made explicit by the Bhaduri-Marglin-Skott investment function.[25] Consider a simple version of a Kaleckian model of a labour-constrained economy with induced innovation and a standard Bhaduri-Marglin investment function:

$$\frac{\dot{\omega}}{\omega} = h[e(\omega, k), \alpha] - f(\omega) \tag{45}$$

$$\frac{\dot{k}}{k} = g(\omega) - f(\omega) - n \tag{46}$$

where $e(\omega, k) = u(\omega)k$, short-run demand is profit-led (wage-led) if $e'_\omega < (>)\ 0$, short-run growth is profit-led (wage-led) if $g'_\omega < (>)\ 0$, and s and γ have been omitted for the sake of simplicity. In this model, the steady-state level of the wage share is determined by the dynamic equation of the employment rate at full capacity, whereas the dynamic equation of the wage share determines the corresponding equilibrium employment rate at full capacity. Thus, a positive institutional shock only lowers the long-run employment rate, while leaving long-run income distribution, capacity utilization, capital accumulation, and labour productivity growth unaffected. It is indeed immediate to check that $d\omega^*/d\alpha = du^*/d\alpha = dg^*/d\alpha = d\hat{a}^*/d\alpha = 0$, whereas $dk^*/d\alpha = -h'_\alpha/h'_e e'_k < 0$ and $de^*/d\alpha = -h'_\alpha/h'_e < 0$. Conversely, postulating a Bhaduri-Marglin-Skott investment function restores a channel through which labour market institutions may have long-run effects, as different combinations of ω and k are consistent with a constant employment rate at full capacity.

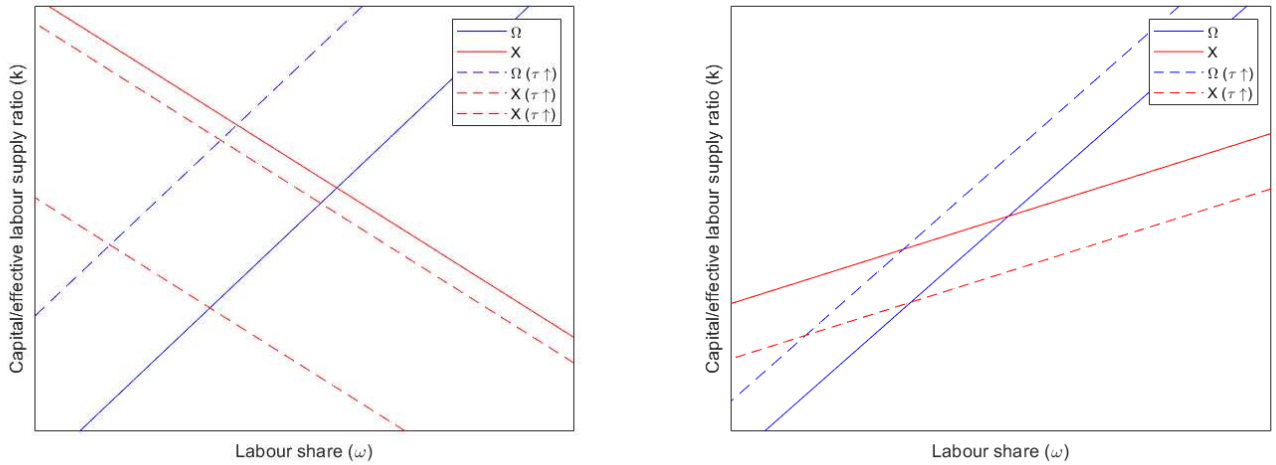
The key conclusion of the model about the effects of labour market institutions on long-run income distribution, capital accumulation, and labour productivity growth is independent of the endogeneity of the rate of capacity utilization. Indeed, it would hold even if we assume that the rate of capacity utilization adjusts to a predetermined and exogenously given normal level, that in our model would be equivalent to assume $g'_\omega < 0$ and $e'_\omega = 0$ – a result which is still different from standard Kaleckian findings, where the possibility of wage-led growth rests on an endogenous rate of capacity utilization.

The impact of labour market institutions on the long-run employment rate is instead dependent on the short-run demand and growth regime of the economy. In a pure profit-led growth regime, the overall impact of α on the equilibrium employment rate will be unambiguously negative, since a positive institutional shock is associated with a fall in both the short-run and the long-run components of the employment rate. However, employment needs not necessarily increase in intermediate and wage-led growth regimes. A positive institutional shock to the labour share causes the employment rate to fall even in intermediate and wage-led growth regimes, if labour productivity growth is more sensitive than capital accumulation to the labour share.

Proposition 2 The long-run labour share is decreasing in τ; the long-run employment rate, capital accumulation, labour productivity growth, and real wage growth are increasing (decreasing) in τ if the economy exhibits a short-run profit-led (wage-led) demand and growth regime.

[25] In a similar vein, Rada, *et al.* (2021) find a positive effect of labour market institutions on the natural rate of growth in a neo-Goodwinian framework.

Fig. 3. *The effect of a technology shock*



a) $g'_\omega < f'_\omega$, $h'_e e'_\omega < f'_\omega$ *b)* $g'_\omega > f'_\omega$, $h'_e e'_\omega < f'_\omega$

Proof If we totally differentiate equations (27) and (28) with respect to τ, we have:

$$\frac{d\omega^{**}}{d\tau} = -\frac{(h'_e e'_k - g'_k) f'_\tau}{\Gamma} < 0 \tag{47}$$

$$\frac{dk^{**}}{d\tau} = \frac{(h'_e e'_\omega - g'_\omega) f'_\tau}{\Gamma} \tag{48}$$

Since we know that $\hat{w}^{**} = \hat{a}^{**}$, total differentiation of equations (9) and $e(\omega, k, s, \gamma)$, after using equations (28), (47), and (48), yields:

$$\frac{de^{**}}{d\tau} = \frac{(g'_k e'_\omega - g'_\omega e'_k) f'_\tau}{\Gamma} \tag{49}$$

$$\frac{dg^{**}}{d\tau} = \frac{d\hat{w}^{**}}{d\tau} = \frac{d\hat{a}^{**}}{d\tau} = \frac{(g'_k e'_\omega - g'_\omega e'_k) h'_e f'_\tau}{\Gamma} \tag{50}$$

An exogenous increase in labour productivity, as measured by the variable τ, leads to an upward shift in the Ω isocline and a downward shift in the X isocline (Figure 3).[26] Indeed, technology shocks exert downward pressure on both the labour share and the capital/effective labour supply ratio, as labour productivity growth rises. Therefore, for a given labour share, the employment rate (then k) must rise to restore a stable income distribution. Conversely, for a given labour share, a constant capital/effective labour supply ratio requires k to rise.

As shown in panels a) and b), irrespective of the short-run demand and growth regime, the labour share unambiguously declines following a technology shock. In the scenario depicted in panel a), the effect on the employment rate at full capacity depends on the relative responsiveness of the Ω and X isoclines to an increase in τ. In a strongly wage-led growth regime, depicted in panel b), the capital/effective labour supply ratio unambiguously declines. However, irrespective of the slope of Ω and X and their response to technology shocks, equation (49) shows that in a pure profit-led regime

[26] For a formal proof, see Appendix B.

an exogenous increase in labour productivity growth unambiguously raises the long-run employment rate, whereas in a pure wage-led regime a technology shock unambiguously lowers long-run employment; in an intermediate regime, the long-run employment rate will rise if and only if $e'_\omega|d\omega^{**}/d\tau| < e'_k dk^{**}/d\tau$. As real wage growth is determined by the employment rate (equation (8)), labour productivity growth is anchored to real wage growth via the dynamic equation of the labour share (equation (27)), and in a labour-constrained economy capital accumulation is tied up with labour productivity growth via the dynamic equation of the employment rate at full capacity (equation (28)), following a technology shock the steady-state values of real wages, capital accumulation, and labour productivity growth move together with the long-run employment rate. Thus, following a productivity shock, real wage growth, capital accumulation, and labour productivity growth unambiguously rise in a short-run profit-led demand and growth regime but fall in a short-run wage-led demand and growth regime.

Our results show that, differently from institutional shocks, the effect of technology shocks on the long-run rate of growth of the economy is crucially dependent on the short-run association between income distribution and capital accumulation. Conditional on technology shocks, if the economy exhibits profit-led demand and growth in the short run, a decrease in the labour share is associated with faster capital accumulation and labour productivity growth in the long run, whereas if the short-run demand and growth regime of the economy is wage-led, a decrease in the labour share is associated with slower capital accumulation and labour productivity growth. Therefore, if the economy has a pure profit-led regime in the short run, both institutional and technology shocks are conducive to faster capital accumulation and labour productivity growth but lead to opposite distributional outcomes; if the economy exhibits a pure wage-led regime in the short run, improving the workers' bargaining position represents an unambiguously better strategy to raise the long-run labour share, capital accumulation, and labour productivity growth, since exogenous productivity shocks would lead to worse outcomes in terms of both income distribution and long-run rate of growth.

Proposition 3 The long-run labour share, employment rate, labour productivity growth, and real wage growth are decreasing in n; the long-run capital accumulation is a positive function of n if the economy exhibits a short-run profit-led demand and growth regime.

Proof Totally differentiating equations (27) and (28) with respect to n, we have:

$$\frac{d\omega^{**}}{dn} = -\frac{h'_e e'_k}{\Gamma} < 0 \tag{51}$$

$$\frac{dk^{**}}{dn} = \frac{h'_e e'_\omega - f'_\omega}{\Gamma} \tag{52}$$

Using equations (28), (51), and (52), total differentiation of equations (9) and $e(\omega, k, s, \gamma)$, yields:

$$\frac{de^{**}}{dn} = -\frac{e'_k f'_\omega}{\Gamma} < 0 \tag{53}$$

$$\frac{d\hat{w}^{**}}{dn} = \frac{d\hat{a}^{**}}{dn} = -\frac{h'_e e'_k f'_\omega}{\Gamma} < 0 \tag{54}$$

$$\frac{dg^{**}}{dn} = \frac{(h'_e e'_\omega - f'_\omega) g'_k - g'_\omega h'_e e'_k}{\Gamma} \tag{55}$$

For what concerns the effects on long-run income distribution, employment, real wages, and

labour productivity growth, our comparative statics analysis results with respect to labour supply growth are in line with those of a standard classical growth model. An increase in labour supply growth lowers the steady-state labour share, which in turn induces less labour-saving innovations and then slower real wage growth in the long run. The steady-state employment rate falls to the level associated with a lower rate of growth of real wages. If the economy exhibits a pure profit-led regime, as is the case in a standard classical growth model, the lower labour share induces faster capital accumulation.

Proposition 4 The long-run labour share, employment rate, capital accumulation, labour productivity growth, and real wage growth are decreasing in s.

Proof Total differentiation of equations (27) and (28) with respect to s yields:

$$\frac{d\omega^{**}}{ds} = -\frac{(g'_k e'_s - e'_k g'_s)h'_e}{\Gamma} < 0 \tag{56}$$

$$\frac{dk^{**}}{ds} = \frac{(g'_\omega - f'_\omega)h'_e e'_s - (h'_e e'_\omega - f'_\omega)g'_s}{\Gamma} \tag{57}$$

Since we know that $\hat{w}^{**} = \hat{a}^{**}$, total differentiation of equations (9) and $e(\omega, k, s, \gamma)$, after using equations (28), (56), and (57), yields:

$$\frac{de^{**}}{ds} = -\frac{(g'_k e'_s - e'_k g'_s)f'_\omega}{\Gamma} < 0 \tag{58}$$

$$\frac{dg^{**}}{ds} = \frac{d\hat{w}^{**}}{ds} = \frac{d\hat{a}^{**}}{ds} = -\frac{(g'_k e'_s - e'_k g'_s)h'_e f'_\omega}{\Gamma} < 0 \tag{59}$$

Comparative statics analysis with respect to s shows that the paradox of thrift holds even in the long run. An increase in the propensity to save out of profits reduces the steady-state values of employment rate, capital accumulation, labour productivity growth, and real wage growth, irrespective of the short-run demand and growth regime of the economy. Slower labour productivity growth is associated with a lower labour share in the long run. The appearance of the paradox of thrift in the long run relies on a genuine Kaleckian mechanism, that is, a contractionary effect of an increase in the propensity to save which is allowed to have long-run effects by means of an endogenous rate of capacity utilization. If we assume that the rate of capacity utilization adjusts to a predetermined and exogenously given normal level (i.e. $g'_s > 0$ and $e'_s = 0$), an increase in the propensity to save out of profits would have an expansionary effect on the long-run rate of growth and employment, and the labour share would increase accordingly, as in classical growth models.

Proposition 5 The long-run labour share, employment rate, capital accumulation, labour productivity growth, and real wage growth are increasing in γ.

Proof Totally differentiating equations (27) and (28) with respect to γ, we have:

$$\frac{d\omega^{**}}{d\gamma} = \frac{(e'_k g'_\gamma - g'_k e'_\gamma)h'_e}{\Gamma} > 0 \tag{60}$$

$$\frac{dk^{**}}{d\gamma} = \frac{(g'_\omega - f'_\omega)h'_e e'_\gamma - (h'_e e'_\omega - f'_\omega)g'_\gamma}{\Gamma} \tag{61}$$

Tab. 1. *Results of comparative statics analysis*

	α			
	Short-run profit-led demand and growth	Short-run wage-led demand and profit-led growth	Short-run wage-led demand and growth with $g'_\omega < f'_\omega$	Short-run wage-led demand and growth with $g'_\omega > f'_\omega$
ω^{**}	$+$	$+$	$+$	$+$
e^{**}	$-$	$+/-$	$+/-$	$+$
g^{**}	$+$	$+$	$+$	$+$
$\hat{a}^{**}$	$+$	$+$	$+$	$+$
$\hat{w}^{**}$	$+$	$+$	$+$	$+$
k^{**}	$-$	$-$	$-$	$+$
u^{**}	$+/-$	$+$	$+$	$+/-$
	τ			
	Short-run profit-led demand and growth	Short-run wage-led demand and profit-led growth	Short-run wage-led demand and growth with $g'_\omega < f'_\omega$	Short-run wage-led demand and growth with $g'_\omega > f'_\omega$
ω^{**}	$-$	$-$	$-$	$-$
e^{**}	$+$	$+/-$	$-$	$-$
g^{**}	$+$	$+/-$	$-$	$-$
$\hat{a}^{**}$	$+$	$+/-$	$-$	$-$
$\hat{w}^{**}$	$+$	$+/-$	$-$	$-$
k^{**}	$+/-$	$+/-$	$+/-$	$-$
u^{**}	$+/-$	$+/-$	$+/-$	$+/-$

Using equations (28), (60), and (61), total differentiation of equations (9) and $e(\omega, k, s, \gamma)$, yields:

$$\frac{de^{**}}{d\gamma} = \frac{(e'_k g'_\gamma - g'_k e'_\gamma) f'_\omega}{\Gamma} > 0 \tag{62}$$

$$\frac{dg^{**}}{d\gamma} = \frac{d\hat{w}^{**}}{d\gamma} = \frac{d\hat{a}^{**}}{d\gamma} = \frac{(e'_k g'_\gamma - g'_k e'_\gamma) h'_e f'_\omega}{\Gamma} > 0 \tag{63}$$

An increase in autonomous investment has an expansionary effect on the long-run rate of growth of the economy. Acceleration in capital accumulation requires faster labour productivity growth in order to keep the employment rate at full capacity constant in the long run, which in turn requires a higher labour share. Real wage growth and the employment rate rise accordingly.

8. CONCLUDING REMARKS

This paper presented a labour-constrained model of demand-led growth. More specifically, we have drawn on the Kaleckian-Steindlian tradition to examine the short-run relation between income distribution, capacity utilization, and capital accumulation; on growth cycle models *à la* Goodwin to formalize the dynamic interaction between employment rate at full capacity and distributive shares of national income; on the classical-Marxian approach to induced technical change literature to link labour productivity growth to the prevailing income distribution. The goods market is assumed to clear instantaneously through changes in the rate of capacity utilization, whereas variations in the wage share and the long-run component of the employment rate are supposed to take place at a lower speed.

We considered the adverse effect of persistently high employment on firms' investment plans, based on the argument that a high employment rate raises the adjustment costs related to monitoring and is associated with a decline in capitalists' "state of confidence", as a tighter labour market increases the chance for worker militancy and is detrimental to the economic, political and social position of capitalists *vis-à-vis* workers. The modification of the standard Bhaduri-Marglin investment function has been motivated on the ground that this effect is independent of the impact of labour market tightness on the wage-profit divide. We have shown that the negative impact of employment on capital accumulation acts as a stabilizing factor for a wage-led growth regime with induced technical change, since it allows firms, by reducing investment, to regenerate the reserve army of labour, thus counteracting the potentially explosive growth of the wage share and the employment rate.

As in canonical Kaleckian models, we left the rate of capacity utilization diverge from the normal level even in the long run. However, we have shown that the long-run effects of institutional shocks to the labour share are independent of the endogeneity of the rate of capacity utilization and the short-run demand and growth regime of the economy. Institutional factors strengthening the relative bargaining power of workers are found to have an unambiguously positive effect on wage share, capital accumulation, labour productivity growth, and real wages growth in the long run, as income distribution impacts the long-run rate of growth via the induced innovation channel rather than via underconsumption. Thus, improving the social protection system and labour market regulation, centralizing the industrial relation system and the collective bargaining structure, and reducing market concentration, along with any institutional change altering the balance of power in favour of workers, would lead to better results in terms of both income distribution and long-run performance of the economy. Provided that wages and profits are unequally distributed across individuals, a pro-labour distributive strategy reduces personal income inequality while stimulating economic growth and labour productivity.

Yet, a pro-labour distributive strategy may be detrimental for employment in the long run. Improving the bargaining power of workers results in a lower employment rate if the economy exhibits a short-run profit-led demand and growth regime. Furthermore, conditional on institutional shocks, the employment rate may be decreasing in the labour share even in a wage-led demand regime, if labour productivity growth is highly responsive to income distribution. Therefore, a pro-labour distributive policy does not emerge as an effective tool to counteract long-run unemployment, despite the positive effect on capital accumulation and labour productivity growth.

Differently from institutional shocks, the impact of positive exogenous shocks on labour productivity growth on the long-run rate of growth relies entirely upon Kaleckian mechanisms, since they are dependent on the assumption of an endogenous rate of capacity utilization and the short-run demand and growth regime of the economy. Technology shocks reduce the long-run labour share, thus long-run capital accumulation and labour productivity growth are decreasing in the labour share,

conditional on technology shocks, if the economy exhibits a wage-led growth regime in the short run. Accordingly, if capital accumulation comoves positively with the labour share in the short run, improving the bargaining strength of workers emerges as an ambiguously better strategy than stimulating labour productivity growth exogenously.

However, in the present model we have only examined the role of institutional factors affecting the labour share, while other drivers of labour productivity growth are left unexplained. Thus, more work is needed to provide a more complete view of the interaction between income distribution, capital accumulation, and labour productivity growth. This analysis is left for future research.

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APPENDIX A

Taking logarithms of $\dot{u} = g^i - g^s$ with (4) and (7) and differentiating with respect to time, we find:

$$\frac{\dot{\pi}}{\pi} + \frac{\dot{u}}{u} = \frac{G'_u \dot{u} + G'_\pi \dot{\pi} + G'_k \dot{k}}{G(u, \pi, k, \gamma)} \tag{A1}$$

After rearranging the terms:

$$\frac{\dot{u}}{u} = \left[\frac{G'_\pi \pi - G(u, \pi, k, \gamma)}{G(u, \pi, k, \gamma) - G'_u u}\right]\frac{\dot{\pi}}{\pi} + \left[\frac{G'_k k}{G(u, \pi, k, \gamma) - G'_u u}\right]\frac{\dot{k}}{k} \tag{A2}$$

Thus, $\dot{\pi}/\pi = \dot{k}/k = 0$ implies $\dot{u}/u = 0$. Since $e = uk$, it immediately follows that $\dot{e}/e = \dot{u}/u + \dot{k}/k = 0$.

APPENDIX B

Differentiating equations (27) and (28) with respect to ω, we find:

$$h'_e\left(e'_k \left.\frac{dk}{d\omega}\right|_\Omega + e'_\omega\right) - f'_\omega = 0 \tag{B1}$$

$$g'_\omega + g'_k \left.\frac{dk}{d\omega}\right|_X - f'_\omega = 0 \tag{B2}$$

After rearranging the terms, we have:

$$\left.\frac{dk}{d\omega}\right|_\Omega = -\frac{h'_e e'_\omega - f'_\omega}{h'_e e'_k} \tag{B3}$$

$$\left.\frac{dk}{d\omega}\right|_X = -\frac{g'_\omega - f'_\omega}{g'_k} \tag{B4}$$

Accordingly, $dk/d\omega|_\Omega > 0$ if and only if $h'_e e'_\omega < f'_\omega$, whereas $dk/d\omega|_X > 0$ if and only if $g'_\omega > f'_\omega$. Differentiating equation (27) with respect to α, we find:

$$\left.\frac{\partial k}{\partial \alpha}\right|_\Omega = -\frac{h'_\alpha}{h'_e e'_k} < 0 \tag{B5}$$

Differentiating equations (27) and (28) with respect to τ, we find:

$$\left.\frac{\partial k}{\partial \tau}\right|_\Omega = \frac{f'_\tau}{h'_e e'_k} > 0 \tag{B6}$$

$$\left.\frac{\partial k}{\partial \tau}\right|_X = \frac{f'_\tau}{g'_k} < 0 \tag{B7}$$

APPENDIX C

Since we have that $e'_\omega = u'_\omega k$, $e'_k = u'_k k + u$, $e'_s = u'_s k$, and $e'_\gamma = u'_\gamma k$, total differentiation of $u(\omega, k, s, \gamma)$ evaluated at the equilibrium point with respect to α, τ, n, s, and γ, after using equations (41), (42), (47), (48), (51), (52), (56), (57), (60), and (61), yields:

$$\frac{du^{**}}{d\alpha} = \frac{[(g'_\omega - f'_\omega)u'_k - g'_k u'_\omega]h'_z}{\Gamma} \tag{C1}$$

$$\frac{du^{**}}{d\tau} = \frac{[(g'_k - h'_e u)u'_\omega - g'_\omega u'_k]f'_\tau}{\Gamma} \tag{C2}$$

$$\frac{du^{**}}{dn} = -\frac{h'_e u u'_\omega + u'_k f'_\omega}{\Gamma} \tag{C3}$$

$$\frac{du^{*}}{ds} = -\frac{(g'_k u'_s - u'_k g'_s)f'_\omega - [u'_\omega g'_s - (g'_\omega - f'_\omega)u'_s]h'_e u}{\Gamma} \tag{C4}$$

$$\frac{du^{*}}{d\gamma} = \frac{(u'_k g'_\gamma - g'_k u'_\gamma)f'_\omega + \left[u'_\omega g'_\gamma - (g'_\omega - f'_\omega)u'_\gamma\right]h'_e u}{\Gamma} \tag{C5}$$