Data Envelopment Analysis To Measure Technical Efficiency In The Algerian Companies.

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Using Data Envelopment Analysis to measure technical efficiency in the Algerian companies

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1. Introduction

The agribusiness sector is a strategic sector. It occupies a privileged place in development policy adopted by the Algerian state in recent years. This policy aims to reposition the role of this sector in the economy of the country. Once this aim is reached, it will contribute to the improvement of GDP, employment creation and the revitalization of the agricultural sector. This aim seems delicate and difficult in today's global economy and in an environment characterized by fierce competition and an often difficult access to financial markets. Also, given the commitments of government with the European Union and its imminent accession to the WTO, Algeria has to pass these economic reforms to better integrate into the logic of the market, a goal which made possible, thanks to financial prosperity that saw the country following the rise in oil prices and the social climate that prevails between various economic operators, say, the privatization of agribusiness and particularly the grain industry remains the most likely strategic alternative for its growth and development. Such reform, if carried out, would in the medium term, substantially reduce the country's food dependence and achieve a degree of food self-sufficiency.

In the current globalized economy, characterized in particular by international economic and financial conditions and harsh logic of global markets expanding, Algeria, a developing country, has more than ever to confront the challenge of growth and development and to find effective and lasting solutions to its multiple economic and social crises, a difficult and sensitive goal, among others, which is to honor the commitments of the Algerian state to liberalize its economy particularly after its integration into the WTO and the Free Trade Area Euro-Mediterranean, crowned by the signature of the Association Agreement with the European Union.

A task made more difficult by a fragile economy, a ruthless international competition and an access, often difficult to credit and raw materials badly needed to operate the various sectors of national economic activities. Thus, the Algerian economic sector, despite the reforms, is still ineffective due to the inconsistency of different policy reforms and restructuring adopted by the state since independence by ending the so-called structural adjustment (PAS) applied at the beginning of the year 1994 (March 1994) which targeted the stabilization of major structural balances of the national economy leading though to a significant decrease of social demand itself and caused a dramatic drop in purchasing power. This reality does not provide favorable conditions for a real and lasting economic recovery and even a less effective integration of the national industrial components in the logic of market economy.

The development of agriculture and food sector is a major issue for Algeria in economic, political and social aspects. That was a conclusion of a report on the food chain in Algeria published on February 25th by the French Agency for international business development. Domestically, it currently employs 1.6 million people or 23% of the workforce; it is the second largest industry in the country after that of energy. Algerian households spend on average 45% of their expenditures on food. The distribution is mainly through supermarkets or grocery stores. The main items of agribusiness chains are cereal and milk, canned food, oil, mineral waters and sugar refining. Upstream of the food industry, in Algeria, there are more than a million farms covering more than 8.5 million hectares of arable land, exploited by the arboriculture (41%) of vegetable crops (26%) and crops (33%), mainly grain. Agriculture and agribusiness account for nearly 23% of the workforce. Agriculture contributes 10% to GDP in Algeria and the turnover achieved by the food industry accounts for 40% of the Algerian total income in non-hydrocarbon sectors.

2. The Concept of Technical Efficiency

Fundamentally, efficiency can be defined as the ratio of outputs to inputs. For many production scenarios, it is imperative to consider multiple inputs and outputs. Moreover, the computation of efficiency for the more realistic scenario of multiple inputs and outputs is difficult. This computation requires that weights be given to the different outputs and inputs. Given these weights, technical efficiency can be defined as

$$\text{Technical efficiency} = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}}.$$

Technical efficiency refers to the ability to:

1. Produce the maximum amount of outputs for a specific quantity of inputs (output increasing notion), and/or
2. Use the minimum amount of inputs to produce a specific quantity of outputs (input reducing notion).

Technical efficiency (TE) is an indicator of how close actual production is to the maximal production that could be produced taking into account the available fixed and variable factors of production. Technical efficiency also, however, may be an indicator of the minimum levels of inputs or factors of production necessary to produce a given level of output relative to the levels of inputs actually used to produce that same level of output.

Farrell considered firms that used two inputs $(X_1$ and $X_2)$ to produce a single output (Q), given constant returns to scale (CRS). Then, by constructing the unit isoquant for technically efficient firms, measures of technical efficiency and inefficiency could be developed (Figure 1).

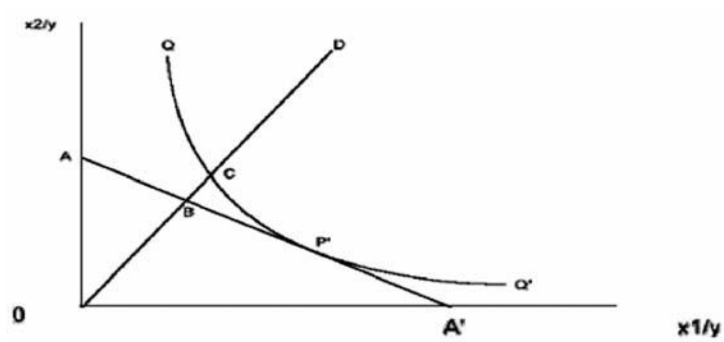

Figure 1. Technical and allocative efficiencies.

In Figure 1, any production along the unit isoquant, QQ', is technically efficient. If a producer uses input levels corresponding to point D to produce a unit of output along the isoquant, then production is inefficient and the level of inefficiency may be represented by the distance CD.

The distance CD represents the amount by which all inputs may be proportionally reduced without affecting output. The ratio $CD/0D$ is the percentage by which all inputs should be reduced to obtain technically efficient production. When the ratio $0C/0D$ is obtained, this is a measure of technical efficiency and equals 1.0 minus the level of inefficiency $(CD/0D)$. The input-oriented measure is restricted to values between 0.0 and 1.0; a value of 1.0 implies that production is technically efficient.

The input-oriented measure of technical efficiency for point C would equal 1.0. Further drawing upon the ideas of Farrell, allocative efficiency (AE), an economic based measure, may be developed. Given input prices for X_1 and X_2, the isocost line, AA', may be constructed. Allocative efficiency is determined by the ratio of $0B$ to $0C$. The distance BC is the reduction in costs if production occurred at the allocative efficiency point P'.

There is also a concept of overall or total economic efficiency [1]. An overall measure of economic efficiency may be defined by the ratio $0B/0D$ or by the product of technical and allocative efficiencies, $TE * AE$. All three efficiency measures are limited to values between 0.0 and 1.0.

The major difficulty of performance measurements is by defining the robust measures of inputs and outputs. In spite of this, it leads to better accountability improvements in the input mix and output quantities depending on the nature of the organization under consideration. There are three methods of measuring efficiency: index numbers (multi-factor productivity models, financial and operational ratios), econometric models (deterministic and stochastic frontier analysis (SFA) models), and linear programming (data envelopment analysis - DEA). Econometric models use average observations and linear programming models use best-practice observations.

The statistical method assumes an inexact relationship between inputs and outputs due to measurement errors and some other factors.

Output-oriented measures of technical and allocative efficiencies

The work of Farrell focused primarily on radial input-oriented measures of technical efficiency. Farrell did, however, recognize a symmetry between the input-based measure of TE and an output-based measure of TE.

The concept and literature on output-oriented measures of technical efficiency is substantially advanced [2-4]. In contrast to the input-oriented measure of TE which assesses TE relative to a radial input reduction given a constant output level, the radial output-oriented measure of TE provides a measure of the amount by which outputs may be proportionally expanded given inputs held constant. The output-oriented measure is illustrated in Figure 2 which depicts the production possibilities curve for a producer using one input X to produce two outputs Q_1 and Q_2.

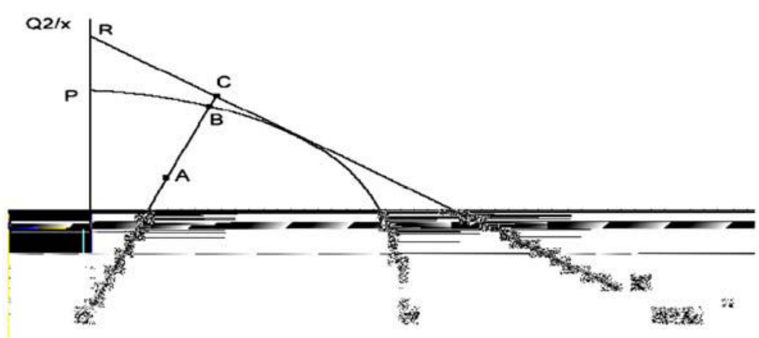

Figure 2. Technical efficiency: output orientation.

The curve PP' represents the production possibilities frontier. All points along the frontier are technically efficient (e.g., point B). All points on the interior of PP' represent technical inefficiency (e.g., point A). The distance defined by AB represents technical inefficiency; this is the amount by which outputs could be increased with no change in the level of x. The ratio $0A/0B$ is an output-oriented measure of technical efficiency. However, define technical efficiency in terms of $0B/0A$ which indicates the total efficient production level for each output [5]. Subtracting 1.0 from output-oriented measure indicates the proportional by which outputs may be expanded relative to their observed levels.

Not surprising, there is also an allocative measure of efficiency which corresponds to the mix of outputs that maximize revenue. The ratio $0B/0C$ is a measure of allocative efficiency which indicates the percent by which revenue may be increased without changing the input level. There is also an overall economic efficiency measure which equals the product of the output-oriented technical efficiency measure and the allocative efficiency measure; it equals the ratio $0A/0C$.

3. Method

There are several approaches to the measurement of the relative technical efficiency of firms in relation to an efficient frontier. These approaches can be placed into one of two broad categories of technique: programming (non-parametric) or statistical (parametric). Data envelopment analysis (DEA) is a linear programming approach, while stochastic frontier analysis (SFA) is a statistical technique.

4. DEA: Data Envelopment Analysis

Data envelopment analysis (DEA) is a relatively new “data oriented” approach for evaluating the performance of a set of peer entities called *decision making units* (*DMUs*) which convert multiple inputs into multiple outputs. The definition of a DMU is generic and flexible. Recent years have seen a great variety of applications of DEA for use in evaluating the performances of many different kinds of entities engaged in many different activities in many different contexts in many different countries.

This section is an introduction to data envelopment analysis (DEA) for people unfamiliar with the technique. For a more in-depth discussion of DEA, the interested reader is referred to the seminal work by Charnes et al. [7].

DEA is commonly used to evaluate the relative efficiency of a number of producers.

The procedure of finding the best virtual producer can be formulated as a linear program. Assume there are data on *k* inputs (denoted by the vector x_i) and *m* outputs (denoted by the vector y_i) on each of *N* firms or decision making units (DMUs). The $k * n$ input matrix, *X*, and the $m * n$ output matrix, *Y*, represent the data of all *N* DMUs. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier. For the simple example of an industry where one output is produced using two inputs, it can be visualized as a number of intersecting planes forming a tight cover over a scatter of points in two-dimensional space. To measure technical efficiency, one has to solve the following linear programming problem for each DMU_j, $j = 1, \ldots, N$ [7, 8]:

$$\max \theta = \frac{\sum_r u_r y_{rj_0}}{\sum_i v_i y_{ij_0}}$$

$$\text{subject to} \quad \frac{\sum_r u_r y_{rj_0}}{\sum_i v_i y_{ij_0}} \leq 1 \text{ for each unit } j$$

$$u_r, v_i \geq \varepsilon,$$

where μ_r is the weight of output *r*; y_{r_0} is the amount of output *r* produced by the DMU evaluated; *k* is the number of inputs; v_i is the weight of input *i* and X_{i_0} is the amount of input *i* used by the DMU.

The value of θ obtained will be the efficiency score for the *i*th DMU. It will satisfy $\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient DMU, according to Farrell [10].

In what follows, we present the methodology as described in Coelli [12].

The model is usually illustrated as follows. Assume that there are data on *K* inputs and *M* outputs on each *N* DMU. For the *i*th decision making unit (DMU), these are represented by the vectors X_i and Y_i, respectively. The $K \times N$ input matrix, X, and the $M \times N$ output matrix, Y, represent the data of all N firms. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all the observed points lie on or below the production frontier. The input-orientated DEA problem may be specified as:

$$
\begin{aligned}
&\min_{\theta, \lambda} \theta \\
&\text{subject to} \quad -y_i + Y\lambda \geq 0, \\
&\qquad\qquad\quad \theta \bullet x_i - X\lambda \geq 0, \\
&\qquad\qquad\quad \lambda \geq 0,
\end{aligned}
$$

where θ is a scalar and λ is an $N \times 1$ vector of constants. The value of θ obtained will be the efficiency score for the ith DMU. It reflects the amount by which the ith DMU can proportionally reduce inputs, without leaving the production possibility space. It will satisfy $\theta \leq 1$ with a value of 1 indicating a point on the frontier and, hence, a technically efficient DMU.

The constant returns to scale (CRS) assumption is only appropriate when all DMUs are operating at an optimal scale. However, the use of CRS models when all DMUs are not operating at optimal scale, will result in measures of TE (technical efficiency) which are confounded by scale efficiencies (SE). The use of a variable returns to scale (VRS) specification will permit the calculation of TE devoid of these SE effects.

The CRS linear programming problem can easily be modified to account for VRS by adding the convexity constraint $N1'\lambda = 1$ (where $N1$ is an $N \times 1$ vector of one) to equation (4). For a detailed exposition, see Coelli [12].

Basic DEA models

In DEA models, we evaluate *n productive* units, DMUs, where each DMU takes *m* different inputs to produce *s* different outputs. The essence of DEA models in measuring the efficiency of productive unit DMU_q lies in maximizing its efficiency rate.

However, subject to the condition that the efficiency rate of any other units in the population must not be greater than 1.

The models must include all characteristics considered, i.e., the weights of all inputs and outputs must be greater than zero. Such a model is defined as a linear divisive programming model:

DEA assumes DMUs must lie on or below the best practice frontier. Multiple inputs are aggregated into a composite input for each DMU. Efficiency measure is then the ratio of the composite output to the composite input. DEA can either be input-oriented or output-oriented. In the former, the method defines the frontier by seeking the maximum proportional reduction in inputs holding the outputs constant. The latter also seeks a maximum increase in outputs holding the inputs constant. The results from the two orientations are the same when a CRS technology [11] is invoked. In the case of a VRS technology [12], the technical efficiencies of the two measures are different.

The addition of the various percentages of the peers (or benchmark firms) gives an indication if the firm is in an increasing returns to scale (IRS) or decreasing returns to scale (DRS) relative to the best practice frontier (see Figure 3). DEA has the benefit of not requiring any production function to be specified but has these statistical errors already mentioned, in addition to the aggregating problem.

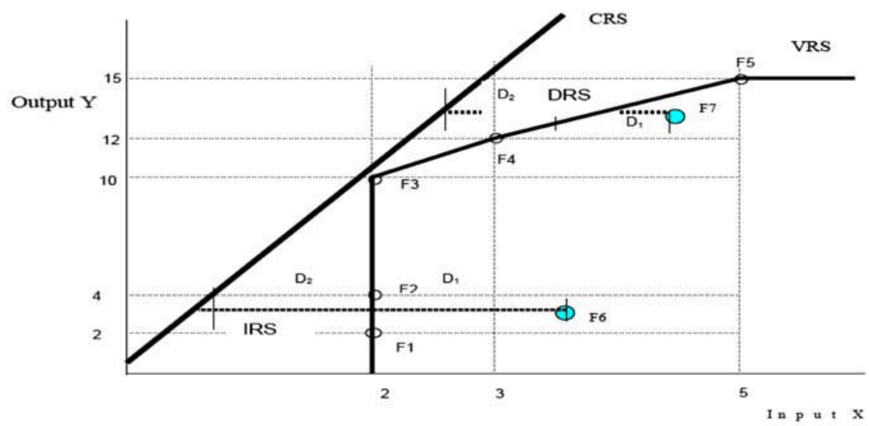

Figure 3. DEA frontier with CRS and VRS technologies: input orientation.

In Figure 3, F2 is the best practice DMU and the efficiencies of all other firms are measured with regards to this unit. When a VRS technology is used, units F1, F2, F3, F4 and F5 are all efficient and form the envelopment. Units F6 and F7 are inefficient with respect to both frontier technologies. The radial distances D_1 and D_2 measure the pure technical and scale inefficiencies of units F6 and F7. The two measures of inefficiencies give the total technical inefficiency of units F6 and F7.

Given the data, we measure the efficiency of each DMU once and hence need n optimizations, one for each DMU_j to be evaluated. Let the DMU_j to be evaluated on any trial be designated as DMU_o, where o ranges over $1, 2, \ldots, n$.

We solve the following fractional programming problem to obtain values for the input "weights" (v_i) $(i = 1, \ldots, m)$ and the output "weights" (μ_r) $(r = l, \ldots, s)$ as variables:

$$(FP_o) \qquad \max_{v, u} \theta = \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo}}$$

$$\text{subject to} \qquad \frac{u_1 y_{1j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + \cdots + v_m x_{mj}} \leq 1 \quad (j = 1, \ldots, n)$$

$$v_1, v_2, \ldots, v_m \geq 0$$

$$u_1, u_2, \ldots, u_s \geq 0.$$

The constraints mean that the ratio of "virtual output" vs. "virtual input" should not exceed 1 for every DMU. The objective is to obtain weights (v_i) and (u_r) that maximize the ratio of DMU_o, the DMU being evaluated. By virtue of the constraints, the optimal objective value θ is at most 1. Mathematically, the nonnegativity constraint is not sufficient for the fractional terms to have a positive value. We do not treat this assumption in explicit mathematical form at this time. Instead we put this in managerial terms by assuming that all outputs and inputs have some nonzero worth and this is to be reflected in the weights U_r and V_i being assigned some positive value.

From a fractional to a linear program

We now replace the above fractional program (FP_o)[1] by the following linear program (LP_o):

$$(LP_o) \quad \max_{\mu, \nu} \theta = \mu_1 y_{1o} + \cdots + \mu_s y_{so}$$

$$\text{subject to} \qquad \nu_1 x_{1o} + \cdots + \nu_m x_{mo} = 1$$

$$\mu_1 y_{1j} + \cdots + \mu_s y_{sj} \leq \nu_1 x_{1j} + \cdots + \nu_m x_{mj} \quad (j = 1, \ldots, n)$$

---

[1]Fractional program.

$$\nu_1, \nu_2, ..., \nu_m \geq 0$$

$$\mu_1, \mu_2, ..., \mu_s \geq 0.$$

Let an optimal solution of (LP_o) be $(u = u^*, v = v^*)$ and the optimal objective value θ^*. The solution $(u = u^*, v = v^*)$ is also optimal for (FP_o), since the above transformation is reversible under the assumptions above. (FP_o) and (LP_o) therefore have the same optimal objective value θ^*.

5. The CCR Model and Dual Problem

Based on the matrix (X, Y), the CCR model was formulated as an *LP* problem with row vector *v* for input multipliers and row vector *u* as output multipliers. These multipliers are treated as variables in the following *LP* problem (multiplier form):

$$(LP_o) \quad \max_{v, u} \quad uy_o$$

$$\text{subject to} \quad vx_o = 1$$

$$-vX + uY \leq 0$$

$$v \geq 0, \quad u \geq 0.$$

The dual problem of (LP_o) is expressed with a real variable θ and a nonnegative vector $\lambda = (\lambda_1, ..., \lambda_n)$ of variables as follows (envelopment form):

$$(DLP_o) \quad \min_{\theta, \lambda} \quad \theta$$

$$\text{subject to} \quad \theta x_o - X\lambda \geq 0$$

$$Y\lambda \geq y_o$$

$$\lambda \geq 0.$$

Correspondences between the primal (LP_o)[2] and the dual (DLP_o)[3] constraints and variables are displayed in the following table:

---

[2]Linear program.

[3]Dual linear program.

Constraint (LP_o)	Dual variable (DLP_o)	Constraint (DLP_o)	Primal variable (LP_o)
$vx_o = 1$	θ	$\theta x_o - X\lambda \geq 0$	$v \geq 0$
$-vX + uY \leq 0$	$\lambda \geq 0$	$Y\lambda \geq y_o$	$u \geq 0$

6. Strengths of DEA

As the earlier list of applications suggests, DEA can be a powerful tool when used wisely. A few of the characteristics that make it powerful are:

- DEA can handle multiple input and multiple output models.
- It does not require an assumption of a functional form relating inputs to outputs.
- DMUs are directly compared against a peer or combination of peers.
- Inputs and outputs can have very different units. For example, input 1 could be in units of lives saved and input 2 could be in units of dollars without requiring an a priori tradeoff between the two.

There are a lot of other items that we have not covered in this brief introduction to DEA such as returns to scale and input vs. output orientation.

7. Empirical Study

This section describes how to apply DEA to calculate the efficiency score of agro-alimentary companies.

We have chosen eleven companies from agro-alimentary sector in Sétif using two inputs and one output. The two inputs are fixed assets, and current ones, and number of employees. The output is the net income.

The table below shows the details of the eleven companies:

Table 1. Sétif companies' case

	A	B	C	D	E	F	G	H	I	J	K
Assets	28	31	25	19.5	24.2	65	44	38	24.6	52.1	29.3
Number of employees	45	30	27	40	19	32	21	28	31	25	48
Net income	15.2	3.7	7.7	12.1	9.3	12.4	8.2	10.3	11.9	14.7	16.1

Net income: 10^7 dinars, Assets (fixed current): 10^7 dinars.

We should take into consideration that this DEA frontier is the result of running five linear programming problems.

The linear program for A is:

$$< A > \max \theta = 15.2\mu$$

$$\begin{aligned}
\text{subject to} \quad & 28\nu_1 + 45\nu_2 = 1 \\
& 15.2\mu \le 28\nu_1 + 45\nu_2 \\
& 3.7\mu \le 31\nu_1 + 30\nu_2 \\
& 7.7\mu \le 25\nu_1 + 27\nu_2 \\
& 12.1\mu \le 19.5\nu_1 + 40\nu_2 \\
& 9.3\mu \le 24.2\nu_1 + 19\nu_2 \\
& 12.4\mu \le 65\nu_1 + 32\nu_2 \\
& 8.2\mu \le 44\nu_1 + 21\nu_2 \\
& 10.3\mu \le 38\nu_1 + 28\nu_2 \\
& 11.9\mu \le 24.6\nu_1 + 31\nu_2 \\
& 14.7\mu \le 52.1\nu_1 + 25\nu_2 \\
& 16.1\mu \le 29.3\nu_1 + 48\nu_2,
\end{aligned}$$

where all variables are constrained to be nonnegative.

The optimal solution is:

The CCR-efficiency of A is 0.998. By applying the optimal solution to the above constraints, the reference set of A is found to be D, E, F, ...

Now, let us observe the difference between the optimal weights ($\nu_1 = 0.0174$ and $\nu_2 = 0.0114$, $\theta^* = 0.998$). The ratio $\frac{\nu_1}{\nu_2} = 0.0174/0.0114 = 1.52$ suggests that it is an advantageous for B.

The optimal solution for DLP_A is: $\text{TE} = \theta^* = 0.998$.

Peer weights for DMU_4: $\lambda_4^* = 0.629$.

Peer weights for DMU_9: $\lambda_9^* = 0.638$.

Since $\lambda_4^* > 0$ and $\lambda_9^* > 0,$ the reference set of A is $E_A = \{\text{DMU}_4, \text{DMU}_9\}$. λ_4^*, λ_9^* show the proportion contributed by DMU_4 and DMU_9 to the point used to evaluate A. Hence, A is technically inefficient. No mix inefficiencies are achieved by reducing all inputs by 0.002 of their observed values.

In fact, based on this reference set and λ^*, we can express the inputs and outputs values needed to bring A into efficient status as:

$$0.998(\text{input of } A) = 0.629(\text{input of } \text{DMU}_4) + 0.638(\text{input of } \text{DMU}_9)$$

$$(\text{output of } A) = 0.629(\text{output of } \text{DMU}_4) + 0.638(\text{output of } \text{DMU}_9).$$

From the magnitude of coefficient on the right hand side, A has more similarity to 4 than 9.

A can be made efficient either by using these coefficients, $\lambda_4^* = 0.629$, $\lambda_9^* = 0.638$ or by reducing both of its inputs.

By reducing the input value radially in the ratio 0.998,

$$x_1 \leftarrow 0.998 * 28 = 27.955,$$

$$x_2 \leftarrow 0.998 * 45 = 44.928,$$

$$y \leftarrow Y = 15.2.$$

The optimal solution for the multiplier problem LP_A: $\nu_1^* = 0.002$, $\nu_2^* = 0.002$, $\mu^* = 0.998$ and we have $\mu^* * y = 0.998 * 1 = 0.998 = \theta^*$. In the optimal objective value of DLP_A, the optimal weighted inputs and outputs are

$$\nu_1 * x_1 = 0.002 * 28 = 0.056,$$

$$\nu_2 * x_2 = 0.002 * 45 = 0.09.$$

Table 2. Results

DMU	x_1	x_2	Y	CCR(θ)	Reference set	ν_1	ν_2	μ
A	28	45	15.2	0.998	D I	0.0174	0.0114	0.0656
B	31	30	3.7	0.282	E I	0.0168	0.0160	0.0763
C	25	27	7.7	0.691	I E	0.0197	0.0188	0.0897
D	19.5	40	12.1	1.000	D	0.0221	0.0142	0.0826
E	24.2	19	9.3	1.000	E	0.0397	0.00201	0.0707
F	65	32	12.4	0.666	E J	0.0150	0.00075	0.0267
G	44	21	8.2	0.664	J	0.0222	0.001123	0.0395
H	38	28	10.3	0.736	J E	0.0254	0.00128	0.0451
I	24.6	31	11.9	1.000	I	0.0382	0.00193	0.068
J	52.1	25	14.7	1.000	J	0.0187	0.00094	0.0333
K	29.3	48	13.1	0.814	I D	0.0315	0.00159	0.0561

In order to demonstrate the role of weights (v, u) for identifying the CCR efficiency of DMUs, we will show graphically the efficient frontier in the weight variables (multiplier) space. Our case has 2 inputs and 1 output, whose value is unitized to 1. For this simple example, we can illustrate the situations using a two-dimensional graph. The linear programming constraints for each DMU have the following inequalities in common with all variables being constrained to be nonnegative:

$$15.2\mu \leq 28\nu_1 + 45\nu_2, \tag{A}$$

$$3.7\mu \leq 31\nu_1 + 30\nu_2, \tag{B}$$

$$7.7\mu \leq 25\nu_1 + 27\nu_2, \tag{C}$$

$$12.1\mu \leq 19.5\nu_1 + 40\nu_2, \tag{D}$$

$$9.3\mu \leq 24.2\nu_1 + 19\nu_2, \tag{E}$$

$$12.4\mu \leq 65\nu_1 + 32\nu_2, \tag{F}$$

$$8.2\mu \leq 44\nu_1 + 21\nu_2, \tag{G}$$

$$10.3\mu \leq 38\nu_1 + 28\nu_2, \tag{H}$$

$$11.9\mu \leq 24.6\nu_1 + 31\nu_2, \tag{I}$$

$$14.7\mu \leq 52.1\nu_1 + 25\nu_2, \tag{J}$$

$$16.1\mu \leq 29.3\nu_1 + 48\nu_2. \tag{K}$$

Dividing these expressions by $u > 0$, we obtain the following inequalities:

$$15.2 \leq [28\nu_1/\mu] + [45\nu_2/\mu], \tag*{A}$$

$$3.7 \leq [31\nu_1/\mu] + [30\nu_2/\mu], \tag*{B}$$

$$7.7 \leq [25\nu_1/\mu] + [27\nu_2/\mu], \tag*{C}$$

$$12.1 \leq [19.5\nu_1/\mu] + [40\nu_2/\mu], \tag*{D}$$

$$9.3 \leq [24.2\nu_1/\mu] + [19\nu_2/\mu], \tag*{E}$$

$$12.4 \leq [65\nu_1/\mu] + [32\nu_2/\mu], \tag*{F}$$

$$8.2 \leq [44\nu_1/\mu] + [21\nu_2/\mu], \tag*{G}$$

$$10.3 \leq [38\nu_1/\mu] + [28\nu_2/\mu], \tag*{H}$$

$$11.9 \leq [24.6\nu_1/\mu] + [31\nu_2/\mu], \tag*{I}$$

$$14.7 \leq [52.1\nu_1/\mu] + [25\nu_2/\mu], \tag*{J}$$

$$16.1\mu \leq [29.3\nu_1/\mu] + [48\nu_2/\mu]. \tag*{K}$$

DMU	CCR-eff θ^*	Ref. set	Excess S_1^-	Excess S_2^-
A	0.998	D I	0	0
B	0.282	E I	0	0
C	0.691	I E	0	0
D	1.000	D	0	0
E	1.000	E	0	0
F	0.666	E J	0	0
G	0.664	J	0.175	0
H	0.736	J E	0	0
I	1.000	I	0	0
J	1.000	J	0	0
K	0.814	I D	0	0

Results from DEAP

The DEAP program (Data Envelopment Analysis Computer Program) was created by Tim Coelli from the "Centre for Efficiency and Productivity Analysis", the Econometrics Department of New England University of Armidale, Australia.

We execute DEAP.

Results from DEAP Version 2.1.

Instruction file = eg1-ins.txt

Data file = eg1-dta.txt

Input-orientated DEA

Scale assumption: CRS

Slacks calculated using multi-stage method

Efficiency summary:

Firm	TE
A	0.998
B	0.282
C	0.691
D	1.000
E	1.000
F	0.666
G	0.664
H	0.736
I	1.000
J	1.000
K	0.814

Conclusion

The most recent in efficiency is DEA, which measures the efficiency of decision making units by doing linear program for each in comparison to other units, and inefficient DMU should have deep change in inputs and/or outputs to improve their efficiencies.

Under the hypothesis of constant returns to scale (CRS), we counted the values of technical efficiency which means the firms' ability to maximize the outputs under the availability of some inputs.

In this paper, we have used a data envelopment analysis approach to estimate technical efficiency of Algerian agro-alimentary companies. Using assets and number of employees as inputs, net income as outputs, we have found the companies.

In this paper, we have not attempted to address the determinants of companies' efficiency rather than the characteristics of the companies themselves. The external environment in which the companies operate in Algeria is also an important factor affecting their performance. The technical efficiency of firms determines its ability to transform inputs into a maximum of outputs. Our results indicate that inefficiency is present in production. Efficiency scores are obtained using non-parametric models.

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