Forecasting using Fuzzy Time Series

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"The human brain works as a binary computer and can only analyze the exact information-based zeros and ones (or black and white). Our heart is more like a chemical computer that uses fuzzy logic to analyze information that can't be easily defined in zeros and ones." **Naveen Jain**

Preface

This chapter is a very short introduction to Fuzzy Time Series (FTS) models. The aim is to present an overview of the concepts of fuzzy Logic, fuzzy set theory and fuzzy time series framework. Accordingly, the chapter has a full application dimension of the FTS models as a main vocation. The R program was used to fit and forecast the principal FTS models, where real datasets of road traffic accident in Algeria have been used. This chapter is organized as follows; first section presents the concept of fuzzy logic, second section is devoted to Fuzzy Time Series, where we defining fuzzy set, and universe of discourse. Third section summarizes the main models of fuzzy time series, precisely; we presented the (Song & Chissom, 1993) model, the (Chen, 1996) model, the Heuristic (Huarng, 2001) model, the (Abbasov & Mamedova, 2003) model, the (Chen & Hsu, 2004) model and the (Singh, 2008) model. Fourth section is a case application of these models on number of accidents in Algeria; the "AnalyzeTS" package of the R program was used to demonstrate the steps of estimation and forecasting.

1. Fuzzy Logic

Fuzzy logic (FL) is a multi-valued logic where the truth values of variables - instead of being true or false - are reels between 0 and 1. In this sense, it extends classical Boolean logic with partial truth values. The FL method imitates the way of decision making in a human which consider all the possibilities between digital values True and False. The fuzzy logic is considered as a support of decision making. As depicted by (Asli et al, 2017) "Both degrees of truth and probabilities range between 0 and 1 and hence may seem similar at first, but fuzzy logic uses degrees of truth as a mathematical model of vagueness, while probability is a mathematical model of ignorance".

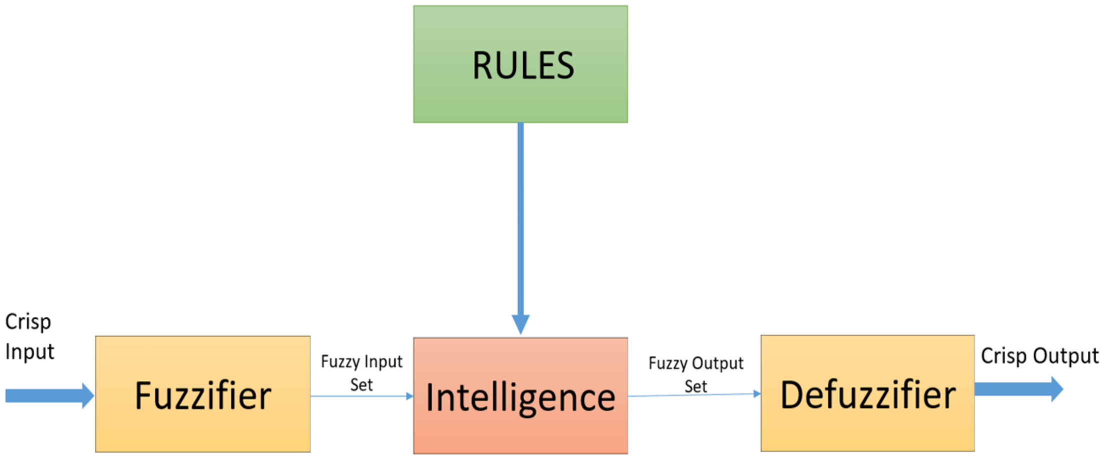

Fig.1: Fuzzy Logic Structure.

The diagram of a fuzzy system is shown in Figure 1. The system has as input a precise value (x_t), the latter is fuzzified (transformed into degree of membership in the input fuzzy set, see Definition (1) below; then it is transmitted to the fuzzy inference engine. Using the fuzzy IF-THEN rules stored in the rule base, the inference engine produces a fuzzy value that will be defuzzified giving the result to be usable. Partitioning the crisp dataset (Figure 1), the identification of fuzzy logical relationships and the defuzzification play a very important role on the forecasting performance of the model, see (Bose & Kalyani, 2019).

In this context, the most well-known system used in fuzzy logic and is the Mamdani system, see (Mamdani, 1974). Briefly, the system uses the following steps: *first,* fuzzify all input values into fuzzy membership functions (Fuzzify in Fig.1), *second,* execute all applicable rules in the rulebase to compute the fuzzy output functions, *third,* De-fuzzify the fuzzy output functions to get "crisp" output values.

2. Fuzzy Time Series

In this section, we are trying to present fundamental concepts of fuzzy time series. For this purpose, the content is mainly based on the works conducted by (Chen, 1996; Chen & Hsu, 2004; Huarng, 2001). The main difference between the fuzzy time series and classical time series is that the values of the former are fuzzy sets while the values of the latter are real numbers.

2.1. Fuzzy sets and Universe of Discourse

We put Ω the universe of discourse that contains n elements as $\Omega = \{u_1, u_2, \dots, u_n\}$, and we define a fuzzy set $\mathcal{M}$ of U as:

$$\mathcal{M} = \left\{\frac{u_{\mathcal{M}}(u_1)}{u_1}, \frac{u_{\mathcal{M}}(u_2)}{u_2}, \dots, \frac{u_{\mathcal{M}}(u_n)}{u_n}\right\}$$

With: $u_{\mathcal{M}}(u_i)$ is the membership function of Ω, taking values in the interval $[0, 1]$, where $1 \leq i \leq n$. In relation with the universe of discourse Ω (or the reference set), each element u_i has three possible situations,

1) u_i is called ***not included*** in the fuzzy set if (no member);
2) u_i ***fully included*** if (full member);
3) u_i ***partially included*** if (fuzzy member)

In mathematics, fuzzy sets (or *uncertain sets*) are sets whose elements have degrees of membership. Fuzzy sets were introduced independently by Lotfi A. Zadeh in 1965 as an extension of the classical notion of set. Fuzzy relations, which are now used throughout fuzzy mathematics and have applications in several areas are special cases of L-relations when L is the unit interval $[0, 1]$.

Fig2. **Lotfi A. Zadeh** (4 February 1921 – 6 September 2017*), was mathematician, computer scientist, electrical engineer, artificial intelligence researcher, and professor of computer science at the University of California, Berkeley*

2.2. Fuzzy Time Series

Mathematically, we have subset, $\mathcal{H}_t$, $(t = 1, 2, \dots)$ of real numbers be the universe of discourse by which we define a fuzzy sets $m_i(t)$are defined. If $\mathcal{M}\ (t)$is a collection of : $m_1(t), m_2(t), \dots$, then, $\mathcal{M}(t)$ is called a Fuzzy time series (FTS) defined on $\mathcal{H}_t$. *As cited by (Chen, 1996), f*uzzy time series is a new concept in statistical modeling process which can be used to deal with forecasting problems in which historical data are linguistic values.

In literature according to time scale, we have two kinds of fuzzy time series (**FTS**) models: time variant and time invariant (Song & Chissom,1993). Accordingly, $\mathcal{M}(t)$ *is called* a time-invariant fuzzy time series If *:* $\forall\, t, i$: $R(t, t - i)$ *is independent of* t, *where* $R(t, t - i)$ *is the fuzzy relationship between* $F(t - i)$ *and* $F(t)$, *if not* $\mathcal{M}$(t) is a time-variant fuzzy time series model, for this category, we found a few number of studies, see (Liu et al, 2010 ; Jiang et al, 2017)

Another decomposition of FTS models is when varying the values of $\boldsymbol{i}$ in $R(t, t - i)$; (*i.e.*): $\mathcal{M}(t)$ *depends on :* $\mathcal{M}(t-1), \mathcal{M}(t-2), \dots, \mathcal{M}(t-i)$,

$$\begin{cases} if\ i = 1, & we\ have\ a\ first\ order\ FTS\ model \\ if\ i > 1, & we\ have\ a\ high\ order\ FTS\ model \end{cases}$$

In this chapter, especially for the fuzzy time series part, we are focused mainly only on applications. For theoretical advanced, the references cited here are sufficient to give the reader a full overview of Fuzzy logic theory in general and FTS in particular.

3. Main models of FTS

The aim through this section is to present a short description of the well-know models of fuzzy time series. Specifically, the models proposed by (Song & Chissom, 1993), the (Chen, 1996) model, the Heuristic (Huarng, 2001) model, the (Abbasov & Mamedova, 2003) model, the (Chen & Hsu, 2004) model and the (Singh, 2008) model

3.1. The (Song & Chissom, 1993) model

The method presented by (Song & Chissom, 1993) uses the following model for forecasting university enrollments:

$$A_i = A_{i-1} \circ R$$

Where A_{i-1} is the observation of period $i - 1$ represented by a fuzzy set, and A_i is the forecasted observation of period i represented by a fuzzy set, and " $\circ$ " is the max-min composition operator, and R is a fuzzy relation indicating fuzzy relationships between fuzzy time series. As limitation of this method, (Chen, 1996) indicated that the forecasted method presented by (Song & Chissom, 1993) requires a large amount of computations to derive the fuzzy relation R, and the max-min composition operations of this method will take a large amount of computation time when the fuzzy relation R of is very big.

3.2. The (Chen, 1996) model

The new method proposed by (Chen, 1996) was essentially a modification of the (Song & Chissom, 1993) method. In his article (Chen, 1996) indicated that the proposed methods is more efficient than the one presented in (Song & Chissom, 1993); according to him, the optimality of his method is due to the fact that it uses simplified arithmetic operations rather than the complicated max-min composition operations presented in the equation cited in the (Song & Chissom, 1993) model.

The dataset used by (Chen, 1996) to illustrate the forecasting steps of the new method was the historical enrollments of the University of Alabama. Firstly, we define D_{min} and D_{max} be the minimum value and the maximum value of known historical data. Based on D_{min} and

D_{max} we define the universe of discourse U as define $[D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers. Hereafter, we cite the steps of this algortithm,

(1) *Step 1:* Partition the universe of discourse $U = [D_{min} - D_1, D_{max} + D_2]$ into even lengthy and equal length intervals $u_1, u_2, \dots, u_m$.

(2) *Step 2*: Define fuzzy sets $A_1, A_2, \dots, A_k$ on the universe of discourse U as follows:

$$\begin{aligned} A_1 &= a_{11}/\mu_1 + a_{12}/\mu_2 + \dots + a_{1m}/\mu_m \,, \\ A_2 &= a_{21}/\mu_1 + a_{22}/\mu_2 + \dots + a_{2m}/\mu_m \,, \\ &\vdots \\ A_k &= a_{k1}/\mu_1 + a_{k2}/\mu_2 + \dots + a_{km}/\mu_m \,, \end{aligned} \quad \text{Where } a_{ij} \in [0,1], 1 \le i \le k \text{ , and } 1 \le j \le m$$

(3) *Step 3:* Divide the derived fuzzy logical relationships into groups based on the current states of the enrollments of fuzzy logical relationships.

(4) *Step 4:* Calculate the forecasted outputs, where the calculations are carried out by respecting some principles cited by (Chen, 1996).

3.3. The Heuristic (Huarng, 2001) model

In fact, the Heuristic model (or approach) proposed by (Huarng, 2001) was not a new model, but it was just an improvement in terms of estimating optimal length of intervals in defining the fuzzy sets. For this purpose, (Huarng, 2001) suggested two approaches to determine the length of the intervals; the first approach was called "*Algorithm for distribution-based length*", the second approach was called "*Algorithm for average-based length*". The empirical results showed that the proposed approaches were more effective an accurate (in term of forecasting) than the other unreason methods as that followed by (Song & Chissom, 1993) and many other researchers after.

3.4. The (Abbasov & Mamedova, 2003) model

This method was developed by to forecast the pattern of population dynamic, as showed in the original article, the steps to be followed in forecasting are summarize hereafter,

1. Definition of universal set U containing the interval between the least and greatest variations in total population.
2. Division of the universal set U into equal-length intervals containing variation values corresponding to different population growth rates.

3. The qualitative description of variation values of total population as a linguistic variable, that's to say, determining the respective values of linguistic variable or the set of fuzzy sets F(t).
4. Fuzzifying the input data or the conversion of numerical values into fuzzy values. This operation enables us to reflect the corresponding numerical/qualitative values of qualitative representations of population growth rates in the value of membership function.
5. Selection of parameter W>1, corresponding to the time period prior to the concerned year, calculation of fuzzy relationships matrix Pw(τ) and forecasting of population growth in the next year.
6. Defuzzifying the obtained results or conversion of fuzzy values into qualitative values.

3.5. The (Chen & Hsu, 2004) model

First, the proposed method defines the universe of discourse and partitions the universe of discourse into some even and equal length intervals. Then, it gets the statistical distributions of the historical enrollment data in each interval and re-divided each interval. Then, it defines linguistic values represented by fuzzy sets based on the re-divided intervals and fuzzify the historical enrollments to get fuzzified enrollments. Then, it establishes fuzzy logical relationships based on the fuzzified enrollments. Finally, it uses a set of rules to determine whether the trend of the forecasting goes up or down and to forecast the enrollments. Assume that we want to forecast the enrollment of year n, then the "difference of differences" of the enrollments between years n-1 and n-2 and between years n-2 and n-3 = (the enrollment of year n-1 - the enrollment of year n-2) - (the enrollment of year n-2 - the enrollment of year n-3)

3.6. The (Singh, 2008) model

This model was proposed by (Singh, 2008) to cope up with the situation of high uncertainty having large fluctuations in the consecutive values of historical time series. Briefly, the steps to fit and forecast time series with this method are described as follows,

(1) *Define the Universe of discourse,* U based on the range of available historical time series data, by rule: $U = [Dmin - D1, Dmax + D2]$ where $D1$ and $D2$ are two proper positive numbers.

(2) *Partition the Universe of discourse* U into equal length of intervals: $u_1, u_2, \ldots, u_m$. The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets) $A_1, A_2, \ldots, A_m$ to be considered.

(3) *Construct the fuzzy sets* A_i in accordance with the intervals in Step 2 and apply the triangular membership rule to each interval in each fuzzy set so constructed.

(4) *Fuzzify the historical data* and establish the fuzzy logical relationships by the rule: If A_i is the fuzzy production of year n and A_j is the fuzzify production of year $n+1$, then the fuzzy logical relation is denoted as $A_i \rightarrow A_j$. Here A_i is called current state and A_j is next state.

(5) *Rules for forecasting.*

3.7. Accuracy comparison criterion

As in classical (no-fuzzy time series), the performance and forecast accuracy of the fuzzy time series models is measured and evaluated in terms of the well-known accuracy statistics: ME – Mean Error, RMSE – Root Mean Squared Error, MAE – Mean Absolute Error, MPE – Mean Percentage Error, MAPE – Mean Absolute Percentage Error. However, the default criterion that we used through this chapter was the Root Mean Square Errors (RMSE). The formula of RMSE is defined as:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{n}} \quad ; \quad U = \frac{\sqrt{\sum_{t=1}^{n-1}\left(\frac{\hat{y}_{i+1} - y_i}{y_i}\right)^2}}{\sqrt{\sum_{t=1}^{n-1}\left(\frac{y_{i+1} - y_i}{y_i}\right)^2}}$$

With: $\hat{y}_i$: are predicted values, and y_i: observed values. Sometimes, for Fuzzy Time Series models, we can deal with the Theil's U statistics; this statistic takes equals 1 under the naïve forecasting method. If U is less than one, this indicate greater forecasting accuracy of the proposed model than the naïve forecasts, and U can be greater than one, which indicate the opposite of the second case.

4. Application Estimation and Prediction for Fuzzy models With R program

In this section, we aim to show in as simple way the steps to fit and forecast dynamic of variables using the FTS models with the R programs. For this purpose, a time series of number of road traffic accidents in Algeria over the period (2016 to 2021) was used. In the R program, the first step is to load the necessary packages to fit any model you want to work with. In our case, main packages that work with time series framework are gathered by (Hyndman & Killick, 2022) in one link, and the user can simply execute this command to install all the core packages.

```
ctv::install.views("TimeSeries", coreOnly = TRUE)
```

And the command in R program "`fuzzy.ts1`" is to estimate the (Chen, 1996) model, the Heuristic (Huarng, 2001) model, the (Abbasov & Mamedova, 2003) model, the (Chen & Hsu, 2004) model and the (Singh, 2008),

```
fuzzy.ts1(ts, n = 5, D1 = 0, D2 = 0, type = c("Chen", "Singh", "Heuristic", "Chen-Hsu"), bin = NULL, trace = FALSE, plot = FALSE, grid = FALSE)
```

We can rewrite the explanation of each element in this command as it was indicated in the tutorial of the "AnalyzeTS" package developed by (Tran et al, 2016)

ts	*The name of the univariate time series*
n	*Number of fuzzy set.*
D1 , D2	*Two proper positive numbers (see the definitions of the models above)*
type	*Type of the model*
trace	*It is recommended to let trace=TRUE to print all of calculation results out to screen*

4.1. Application of FTS models to forecast the number of Road Traffic Accident in Algeria

In this application, we want to model and forecast the trajectories of the number of roads traffic accident in Algeria. The dataset is provided by the DNSR (Délégation Nationale de la Surété Routière). The dataset was used previously by (Chellai, 2022) to fit hybrid models, and we suggest readers to read this study for more detail and information about the data structure and forecasting results. Hereafter, we put the figure showed the trajectories of number of accidents over the period (2015.2020)

Fig3. Evolution of number of accidents over the period (2015-2020) in Algeria

Over the study period we can see a decreasing in number of accidents, there is a seasonality component in the time series where the months of summer (especially June and July) recorded the higher number of accidents. As indicated above, we want to go forward to fit the Fuzzy Time Series on this dataset, and in the end we will do a comparison amongst the FTS models.

Step 1: Determination of Universe of Discourse

We define the universe of discourse Ω as an interval containing all observations. $\Omega = [Min(y_t) - d_1; Max(y_t) + d_2]$. For the "number of accidents" dataset, the smallest number ever recorded was 1398 accidents, so $Min(y_t) = 1398$ and the maximum monthly number of accidents over the study period was 4146 accidents, so $Max\ (y_t) = 4146$. As cited above, the values of $d_1$andd_2 are arbitrarily selected, .we put, $d_1 = 18$ and $d_2 = 14$. Thus, the universe of discourse of the dataset would be defined as: $\Omega = [1380\ , 4160]$.

Step 2: Definition of Fuzzy Sets

The challenge in working with FTS is how to determine the number of fuzzy sets of the universe of discourse. The universe of discourse: $\Omega = [1380\ , 4160]$ is divided into equal sub-intervals according to the well known formula in statistics, $k = 1 + 3.32 * \log(T)$; With, k: number of sub-intervals desired and T : number of observations, $T = 72$ observations, so, $k = 1 + 3.32 * log(72) \approx 15$.

Table 1: Fuzzy Sets

id	set	dow	up	mid	num
1	A1	1380	1565.333	1472.667	3
2	A2	1565.333	1750.667	1658	7
3	A3	1750.667	1936	1843.333	9
4	A4	1936	2121.333	2028.667	10
5	A5	2121.333	2306.667	2214	9
6	A6	2306.667	2492	2399.333	7
7	A7	2492	2677.333	2584.667	6
8	A8	2677.333	2862.667	2770	4
9	A9	2862.667	3048	2955.333	3
10	A10	3048	3233.333	3140.667	4
11	A11	3233.333	3418.667	3326	3
12	A12	3418.667	3604	3511.333	2
13	A13	3604	3789.333	3696.667	2
14	A14	3789.333	3974.667	3882	2
15	A15	3974.667	4160	4067.333	1

Source: Author calculation based on R outputs

To estimate the (Chen, 1996), (Singh, 2008), (Huarng, 2001) and (Chen & Hsu, 2004) models, the command below is used in R. as showed we select in “type” the appropriate model.

```
fuzzy.ts1(accident, n = 15, D1 = 18, D2 = 14, type = c("Chen", "Singh",
"Heuristic", "Chen-Hsu"), bin = NULL, trace = TRUE, plot = TRUE, grid =
TRUE)
```

According to these sub-intervals and the function of membership- see Definition (1) - , we define the fuzzy sets $\mathcal{M}_i, i: 1,2, \ldots, 15$, as:

Note1: in R package, to fit the (Chen & Hsu, 2004) model, we have some specific commands as below, first, we execute the command

`A= fuzzy.ts1(accident,n=15,type="Chen-Hsu", plot=1) #` here you can change the name of time series and the number of fuzzy set.

Then, we execute the second command as:

`B= ChenHsu.bin(a$table1,n.subset=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)) #` here, we need the output of the first command

Finally, we execute the final command to obtain the estimation of the model as

```
Chen-Hsu= fuzzy.ts1(accident,type="Chen-Hsu",bin=b,plot=1,trace=1)
```

Step3. Fuzzification of original time series (number of accidents)

This step consist in transforming the original time series to fuzzy series, this was indicated in definition (1), u_i are the sub-intervals, while numbers in nominator of fuzzy sets m_i denote membership degrees of u_i to m_i, taking values in the interval $[0,1]$. The results for this application is summarize in appendix for each observation (2015-Feb to 2020-Dec)

Step4: Definition of fuzzy logic relationships

In this step, fuzzy logical relationships are defined between the fuzzyfied data, and then, fuzzy relationship groups are formed.

Table 2: Fuzzy relationships of the 15 fuzzy sets

"A1->A3,A4"	"A2->A1,A2,A3,A4,A5"	"A3->A2,A3,A4,A6"
"A4->A2,A3,A4,A6"	"A5->A4,A5,A6,A7,A8"	"A6->A4,A5,A6,A7"
"A7->A5,A6,A7,A10"	"A8->A6,A7,A10,A11"	"A9->A7,A8"
"A10->A8,A9,A12,A13"	"A11->A9,A10,A11"	"A12->A10,A14"
"A13->A11,A14"	"A14->A13,A15"	"A15->A12"

Source: Author calculates using R

Before to summarize the estimates of each model, the implementation of the (Abbasov & Mamedova, 2010) has some specific feature and commands in R, we put two of these commands as notes, the first one is about the principal command to fit and estimate the models,

Note2: When we fit the (Abbasov & Mamedova, 2010) model in R package, there is as specific command different from those in "fuzzy.ts1"

```
fuzzy.ts2(accident, n = 15, w = 7, D1 = 18, D2 = 14, C = 0.001, forecast = 5, r =
    12, trace = TRUE, plot = TRUE, grid = TRUE, type = "Abbasov-Mamedova")
```

The second command in this model allows estimating (using simulation) the best or the optimal C value. Specifically, this estimation is based on some criterion of accuracy (of forecasting), such as: to "ME","MAE" , "MPE", "MAPE", "MSE" (as default), or "RMSE".

Note3: working with the (Abbasov & Mamedova, 2004) model

In R program, the command below is used

```
GDOC(accident, n = 15, w = 7, D1 = 18, D2 = 14, error = 1e-06, k = 500, r = 13,
    CEF = "MSE", type = "Abbasov-Mamedova", show.complete = TRUE)
```

However, the formula proposed by (Abbasov & Mamedova, 2004) was,

$$\mu_{A_i}(\mu_i) = \frac{1}{1 + \left[C \,.\, (U - \mu_m^i)\right]^2}$$

According to the authors of this method," C is chosen in such a way that it ensures the conversion of definite quantitative values into fuzzy values or their belonging to the interval"

Step5: Forecasting and Select best Fuzzy Time series Models

Figure4 illustrates the actual and forecasted values of number of accidents using four fuzzy time series models. In the same point, Table 3 summarizes the accuracy measures of forecasting of these models. Based on accuracy measures, it's seems that the Singh model outperforms the other models.

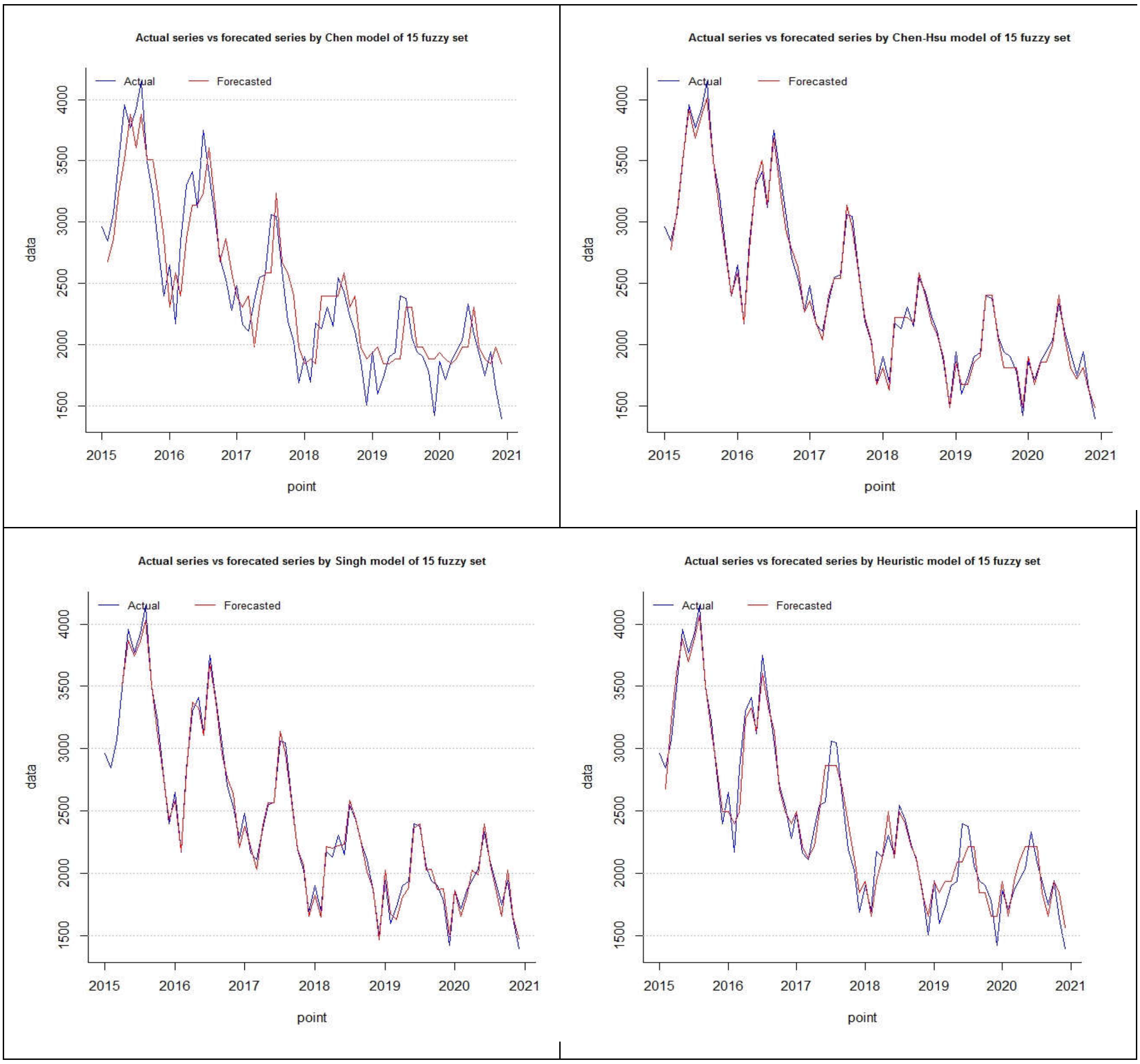

Fig4. Actual values and forecasted values of the time series by the four FTS models

The Sing model recorded the lowest values of the accuracy measures (ME, MAE…), the second optimal model that fit better the dataset was the (Chen & Hsu, 2004) model, except the Mean Errors (ME) and Mean Percentage Errors (MPE) indicators, we see that these two models have almost the level of accuracy.

Table 3: Select optimal fuzzy time series models

Models	ME	MAE	MPE	MAPE	MSE	RMSE	U
Chen,(1996)	-33.61	222.31	-3.043	9.69	71584.26	267.55	0.879
Heuristic Huarng, (2001)	-8.037	109.36	-0.98	4.95	18231.37	135.02	0.444
Abbasov – Manedova(2010)	-28.82	261.50	-2.013	11.18	94860.76	307.99	1.003
Chen & Hsu, (2004)	21.49	54.80	0.843	2.38	4423.37	66.51	0.218
Singh,(2008)	9.02	54.29	0.246	2.42	3974.37	63.04	0.205

Source: Own calculates using R program

Conclusions

This chapter is a practical guide that can help you to understand the idea of the Fuzzy Time Series. Furthermore, it is a pure practical guide to fit the FTS models with R program. We hope it would be beneficial for students, researchers and even professionals in this area of research. As a limitation of this approach is that require higher subjectivity, particularly during the fuzzifcation process: number of fuzzy sets, lengths of fuzzy sets.

Appendix

Table the change fuzzy of original time series

Obs	Estimated Membership Function for each observations (2015-Feb, 2020-Dec)														
[2] "A[2015 Feb]=	{(0.800021333191/u1)	(0.856151182120/u2)	(0.907199512652/u3)	(0.949854591457/u4)	(0.980857496814/u5)	(0.997542585982/u6)	(0.998367813942/u7)	(0.983254922850/u8)	(0.953607332784/u9)	(0.911998469836/u10)	(0.861652631307/u11)	(0.805897934303/u12)	(0.747737703998/u13)	(0.689604374728/u14)	(0.633284040872/u15)}"
[3] "A[2015 Mar]=	{(0.579997868327/u1)	(0.633324732994/u2)	(0.689646915147/u3)	(0.747781004114/u4)	(0.805940432185/u5)	(0.861692296208/u6)	(0.912032916313/u7)	(0.953634072963/u8)	(0.983271740991/u9)	(0.998373183067/u10)	(0.997535996310/u11)	(0.980839572496/u12)	(0.949826948126/u13)	(0.907164413507/u14)	(0.856111120126/u15)}"
[4] "A[2015 Apr]=	{(0.482822153166/u1)	(0.528307785099/u2)	(0.577787526693/u3)	(0.630967352263/u4)	(0.687181277163/u5)	(0.745269698078/u6)	(0.803473455744/u7)	(0.859386900494/u8)	(0.910027144097/u9)	(0.952072298601/u10)	(0.982283061386/u11)	(0.998047232982/u12)	(0.997903709221/u13)	(0.981866101525/u14)	(0.951419624903/u15)}"
[5] "A[2015 May]=	{(0.456122976361/u1)	(0.499050902498/u2)	(0.546018425300/u3)	(0.596907854556/u4)	(0.651302488719/u5)	(0.708369144771/u6)	(0.766738404784/u7)	(0.824414372098/u8)	(0.878763408895/u9)	(0.926640467733/u10)	(0.964696274205/u11)	(0.989855772173/u12)	(0.999875321102/u13)	(0.993813303485/u14)	(0.972242728291/u15)}"
[6] "A[2015 Jun]=	{(0.842133534417/u1)	(0.894790201270/u2)	(0.939916147469/u3)	(0.974190775686/u4)	(0.994742497425/u5)	(0.999698489824/u6)	(0.988589774807/u7)	(0.962458118033/u8)	(0.923621500734/u9)	(0.875191607765/u10)	(0.820517337986/u11)	(0.762716202193/u12)	(0.704380103539/u13)	(0.647460229447/u14)	(0.593285620121/u15)}"
[7] "A[2015 Jul]=	{(0.624776121455/u1)	(0.680694412583/u2)	(0.738646980520/u3)	(0.796946491371/u4)	(0.853259728191/u5)	(0.904660702475/u6)	(0.947847963278/u7)	(0.979546817562/u8)	(0.997045765582/u9)	(0.998731875736/u10)	(0.984445282163/u11)	(0.955515722417/u12)	(0.914466285088/u13)	(0.864500888495/u14)	(0.808954412750/u15)}"
[8] "A[2015 Aug]=	{(0.584592662467/u1)	(0.638219761031/u2)	(0.694759002479/u3)	(0.752977090072/u4)	(0.811030475127/u5)	(0.866430295945/u6)	(0.916131305064/u7)	(0.956794737628/u8)	(0.985231458165/u9)	(0.998953495208/u10)	(0.996681757391/u11)	(0.978631591167/u12)	(0.946463576966/u13)	(0.902919527992/u14)	(0.851283979471/u15)}"
[9] "A[2015 Sep]=	{(0.999269732569/u1)	(0.986473076718/u2)	(0.958848129349/u3)	(0.918824050308/u4)	(0.869564241967/u5)	(0.814412538523/u6)	(0.756440759603/u7)	(0.698174709493/u8)	(0.641496103942/u9)	(0.587671958308/u10)	(0.537453151082/u11)	(0.491195568042/u12)	(0.448975163679/u13)	(0.410683620628/u14)	(0.376101299668/u15)}"
[10] "A[2015 Oct]=	{(0.895302276436/u1)	(0.940332507780/u2)	(0.974478384052/u3)	(0.994875936809/u4)	(0.999665222113/u5)	(0.988392970781/u6)	(0.962115927424/u7)	(0.923163061291/u8)	(0.874651336887/u9)	(0.819929400051/u10)	(0.762110493995/u11)	(0.703780192417/u12)	(0.646882961208/u13)	(0.592741801949/u14)	(0.542152422863/u15)}"
[11] "A[2015 Nov]=	{(0.952640311477/u1)	(0.982644136379/u2)	(0.998168665494/u3)	(0.997773978449/u4)	(0.981497505882/u5)	(0.950845471772/u6)	(0.908460007474/u7)	(0.857591561299/u8)	(0.801556566565/u9)	(0.743321478889/u10)	(0.685270720549/u11)	(0.629142251173/u12)	(0.576077341839/u13)	(0.526727632264/u14)	(0.481376783556/u15)}"
[12] "A[2015 Dec]=	{(0.962287193969/u1)	(0.988491575793/u2)	(0.999682073331/u3)	(0.994809428165/u4)	(0.974334765387/u5)	(0.940124474005/u6)	(0.895046340074/u7)	(0.842420062233/u8)	(0.785499107647/u9)	(0.727103721340/u10)	(0.669439355050/u11)	(0.614069943490/u12)	(0.561988862348/u13)	(0.513733533228/u14)	(0.469506021827/u15)}"
[13] "A[2016 Jan]=	{(0.567514085909/u1)	(0.619988781798/u2)	(0.675667445417/u3)	(0.733499387869/u4)	(0.791852603226/u5)	(0.848450649124/u6)	(0.900413713989/u7)	(0.944459857504/u8)	(0.977291726539/u9)	(0.996125469267/u10)	(0.999233298282/u11)	(0.986320729852/u12)	(0.958593832316/u13)	(0.918489216399/u14)	(0.869173609751/u15)}"
[14] "A[2016 Feb]=	{(0.980243645679/u1)	(0.997313626043/u2)	(0.998545424427/u3)	(0.983822235941/u4)	(0.954512797625/u5)	(0.913166953387/u6)	(0.862999575819/u7)	(0.807342130805/u8)	(0.749209927538/u9)	(0.691051317953/u10)	(0.634668504294/u11)	(0.581258531918/u12)	(0.531516813596/u13)	(0.485758717786/u14)	(0.444032697638/u15)}"
[15] "A[2016 Mar]=	{(0.374647194580/u1)	(0.409070967067/u2)	(0.447192519770/u3)	(0.489235086155/u4)	(0.535313310235/u5)	(0.585361284604/u6)	(0.639037883194/u7)	(0.695612372787/u8)	(0.753843061143/u9)	(0.811876870237/u10)	(0.867215673736/u11)	(0.916807493475/u12)	(0.957312148299/u13)	(0.985546749947/u14)	(0.999037858829/u15)}"
[16] "A[2016 Apr]=	{(0.460691101527/u1)	(0.504066401957/u2)	(0.551479703532/u3)	(0.602785427658/u4)	(0.657526656015/u5)	(0.714816379783/u6)	(0.773219120025/u7)	(0.830666720106/u8)	(0.884459473044/u9)	(0.931411015939/u10)	(0.968176343716/u11)	(0.991745187990/u12)	(0.999998638890/u13)	(0.992158760958/u14)	(0.968964955757/u15)}"
[17] "A[2016 May]=	{(0.650516453215/u1)	(0.707553639588/u2)	(0.765916885805/u3)	(0.823619436814/u4)	(0.878036135910/u5)	(0.926027445019/u6)	(0.964243980773/u7)	(0.989602983652/u8)	(0.999845436115/u9)	(0.994009171883/u10)	(0.972645994625/u11)	(0.937696047824/u12)	(0.892070187388/u13)	(0.839097647479/u14)	(0.782014418415/u15)}"
[18] "A[2016 Jun]=	{(0.903951314925/u1)	(0.947284731936/u2)	(0.979175520611/u3)	(0.996899720513/u4)	(0.998824892502/u5)	(0.984768518577/u6)	(0.956039712313/u7)	(0.915147312406/u8)	(0.865289292952/u9)	(0.809802190509/u10)	(0.751721391244/u11)	(0.693522290437/u12)	(0.637034634918/u13)	(0.583479588813/u14)	(0.533571614242/u15)}"
[19] "A[2016 Jul]=	{(0.391018951423/u1)	(0.427216710345/u2)	(0.467231200495/u3)	(0.511240215713/u4)	(0.559280584855/u5)	(0.611165155804/u6)	(0.666377641900/u7)	(0.723952418227/u8)	(0.782358797326/u9)	(0.839426449343/u10)	(0.892365314120/u11)	(0.937937617266/u12)	(0.972814982561/u13)	(0.994090717199/u14)	(0.999831893819/u15)}"
[20] "A[2016 Aug]=	{(0.932537384768/u1)	(0.968987356178/u2)	(0.992170424893/u3)	(0.999998478891/u4)	(0.991733219321/u5)	(0.968153680834/u6)	(0.931379624105/u7)	(0.884421772783/u8)	(0.830625180683/u9)	(0.773175948409/u10)	(0.714773347980/u11)	(0.657485053963/u12)	(0.602746101191/u13)	(0.551443134679/u14)	(0.504032799752/u15)}"
[21] "A[2016 Sep]=	{(0.933004549855/u1)	(0.969322484585/u2)	(0.992344366429/u3)	(0.999995012247/u4)	(0.991552675209/u5)	(0.967812865760/u6)	(0.930908076034/u7)	(0.883855821019/u8)	(0.830001852684/u9)	(0.772528315589/u10)	(0.714127948119/u11)	(0.656861194082/u12)	(0.602156432580/u13)	(0.550894859562/u14)	(0.503529033303/u15)}"
[22] "A[2016 Oct]=	{(0.925865800848/u1)	(0.964124530913/u2)	(0.989535959585/u3)	(0.999837038782/u4)	(0.994060202015/u5)	(0.972751668287/u6)	(0.937847072778/u7)	(0.892254671789/u8)	(0.839303165413/u9)	(0.782229660488/u10)	(0.723822965960/u11)	(0.666251967259/u12)	(0.611045990393/u13)	(0.559169527233/u14)	(0.511138003494/u15)}"
[23] "A[2016 Nov]=	{(0.832243740563/u1)	(0.885889299220/u2)	(0.932599746918/u3)	(0.969032135085/u4)	(0.992193727333/u5)	(0.999998132225/u6)	(0.991709256621/u7)	(0.968108333258/u8)	(0.931316823657/u9)	(0.884346361048/u10)	(0.830542095905/u11)	(0.773089603698/u12)	(0.714687286294/u13)	(0.657401854126/u14)	(0.602667453962/u15)}"
[24] "A[2016 Dec]=	{(0.881317754846/u1)	(0.928786579992/u2)	(0.966270684466/u3)	(0.990723079338/u4)	(0.999955112015/u5)	(0.993097968018/u6)	(0.970799212900/u7)	(0.935075597355/u8)	(0.888881471447/u9)	(0.835554142631/u10)	(0.778309637398/u11)	(0.719898425787/u12)	(0.662445551345/u13)	(0.607439231733/u14)	(0.555809869105/u15)}"
[25] "A[2017 Jan]=	{(0.596205461864/u1)	(0.650557810060/u2)	(0.707596554154/u3)	(0.765960126691/u4)	(0.823661291565/u5)	(0.878074445104/u6)	(0.926059757757/u7)	(0.964267849501/u8)	(0.989616363440/u9)	(0.999847088940/u10)	(0.993998940138/u11)	(0.972624837421/u12)	(0.937665825224/u13)	(0.892033278443/u14)	(0.839056537196/u15)}"
[26] "A[2017 Feb]=	{(0.913885873491/u1)	(0.955068232414/u2)	(0.984167990258/u3)	(0.998650037085/u4)	(0.997167157793/u5)	(0.979859957347/u6)	(0.948324722683/u7)	(0.905262266926/u8)	(0.853943700207/u9)	(0.797673006325/u10)	(0.739382637235/u11)	(0.681413892245/u12)	(0.625462049230/u13)	(0.572631683684/u14)	(0.523545845872/u15)}"
[27] "A[2017 Mar]=	{(0.756094422855/u1)	(0.814074759953/u2)	(0.869251761846/u3)	(0.918556223656/u4)	(0.958644746283/u5)	(0.986351264644/u6)	(0.999240655930/u7)	(0.996108949416/u8)	(0.977252887385/u9)	(0.944402162218/u10)	(0.900341804862/u11)	(0.848369513306/u12)	(0.791766873745/u13)	(0.733412908144/u14)	(0.675583102024/u15)}"
[28] "A[2017 Apr]=	{(0.570331535641/u1)	(0.623003115370/u2)	(0.678833768864/u3)	(0.736743233201/u4)	(0.795064690001/u5)	(0.851485876109/u6)	(0.903097689193/u7)	(0.946605531726/u8)	(0.978725839871/u9)	(0.996719860280/u10)	(0.998931852159/u11)	(0.985152127558/u12)	(0.956664963891/u13)	(0.915961947679/u14)	(0.866233756850/u15)}"
[29] "A[2017 May]=	{(0.606806671389/u1)	(0.661777450450/u2)	(0.719208855874/u3)	(0.777619859216/u4)	(0.834893129379/u5)	(0.888285012120/u6)	(0.934583351303/u7)	(0.970449568040/u8)	(0.992921474940/u9)	(0.999968266562/u10)	(0.990924626715/u11)	(0.966641908425/u12)	(0.929295458520/u13)	(0.881925355231/u14)	(0.827879263975/u15)}"

[30] "A[2017 Jun]=	{(0.701724576486/u1)	(0.760033423533/u2)	(0.817911145359/u3)	(0.872793957147/u4)	(0.921583484280/u5)	(0.960932333962/u6)	(0.987705904978/u7)	(0.999537963576/u8)	(0.995320611840/u9)	(0.975453174979/u10)	(0.941751102736/u11)	(0.897051777436/u12)	(0.844667341727/u13)	(0.787862375034/u14)	(0.729479521605/u15)}"
[31] "A[2017 Jul]=	{(0.448915617167/u1)	(0.491130089894/u2)	(0.537381696352/u3)	(0.587594819988/u4)	(0.641414070696/u5)	(0.698089245421/u6)	(0.756354176185/u7)	(0.814328102971/u8)	(0.869486141197/u9)	(0.918757124023/u10)	(0.958797324522/u11)	(0.986442672776/u12)	(0.999262516505/u13)	(0.996059181493/u14)	(0.977136184754/u15)}"
[32] "A[2017 Aug]=	{(0.732072638580/u1)	(0.790437595817/u2)	(0.847110626835/u3)	(0.899225003829/u4)	(0.943504738360/u5)	(0.976646938206/u6)	(0.995848452751/u7)	(0.999350172550/u8)	(0.986820368164/u9)	(0.959430419675/u10)	(0.919592242867/u11)	(0.870461473960/u12)	(0.815383108319/u13)	(0.757436429439/u14)	(0.699157803288/u15)}"
[33] "A[2017 Sep]=	{(0.968964955757/u1)	(0.992158760958/u2)	(0.999998638890/u3)	(0.991745187990/u4)	(0.968176343716/u5)	(0.931411015939/u6)	(0.884459473044/u7)	(0.830666720106/u8)	(0.773219120025/u9)	(0.714816379783/u10)	(0.657526656015/u11)	(0.602785427658/u12)	(0.551479703532/u13)	(0.504066401957/u14)	(0.460691101527/u15)}"
[34] "A[2017 Oct]=	{(0.958542891089/u1)	(0.986290162364/u2)	(0.999225905242/u3)	(0.996141954385/u4)	(0.977330534819/u5)	(0.944517528120/u6)	(0.900485605773/u7)	(0.848531774863/u8)	(0.791938328974/u9)	(0.733585868855/u10)	(0.675751793636/u11)	(0.620069034917/u12)	(0.567589065642/u13)	(0.518893856738/u14)	(0.474217100511/u15)}"
[35] "A[2017 Nov]=	{(0.827254026560/u1)	(0.881355757127/u2)	(0.928818426085/u3)	(0.966293939858/u4)	(0.990735739084/u5)	(0.999956000824/u6)	(0.993087001111/u7)	(0.970777415542/u8)	(0.935044875073/u9)	(0.888844221942/u10)	(0.835512845178/u11)	(0.778266530792/u12)	(0.719855323477/u13)	(0.662403784773/u14)	(0.607399682673/u15)}"
[36] "A[2017 Dec]=	{(0.926350330661/u1)	(0.964482349158/u2)	(0.989736405977/u3)	(0.999861564722/u4)	(0.993906468769/u5)	(0.972434085746/u6)	(0.937393557985/u7)	(0.891700917270/u8)	(0.838686444482/u9)	(0.781583882186/u10)	(0.723175770857/u11)	(0.665623772763/u12)	(0.610450412527/u13)	(0.558614525594/u14)	(0.510627241959/u15)}"
[37] "A[2018 Jan]=	{(0.590376810367/u1)	(0.644371245446/u2)	(0.701168179297/u3)	(0.759470797676/u4)	(0.817363888825/u5)	(0.872289591855/u6)	(0.921153621394/u7)	(0.960609026122/u8)	(0.987516551998/u9)	(0.999499982364/u10)	(0.995437620707/u11)	(0.975714161214/u12)	(0.942132914070/u13)	(0.897523938780/u14)	(0.845197463253/u15)}"
[38] "A[2018 Feb]=	{(0.854305488124/u1)	(0.905580213347/u2)	(0.948576383151/u3)	(0.980024819433/u4)	(0.997230401976/u5)	(0.998605679067/u6)	(0.984020244865/u7)	(0.954830547416/u8)	(0.913578026931/u9)	(0.863474144798/u10)	(0.807851482709/u11)	(0.749729541940/u12)	(0.691562281263/u13)	(0.635157595080/u14)	(0.581717502712/u15)}"
[39] "A[2018 Mar]=	{(0.449809721266/u1)	(0.492113194636/u2)	(0.538454433854/u3)	(0.588752731192/u4)	(0.642645234722/u5)	(0.699371581452/u6)	(0.757652869163/u7)	(0.815593994047/u8)	(0.870656297786/u9)	(0.919758884881/u10)	(0.959556525215/u11)	(0.986895292643/u12)	(0.999367039781/u13)	(0.995805656616/u14)	(0.976548511917/u15)}"
[40] "A[2018 Apr]=	{(0.750249157119/u1)	(0.808360641305/u2)	(0.863948280600/u3)	(0.913988401460/u4)	(0.955147341076/u5)	(0.984217093593/u6)	(0.998664664394/u7)	(0.997145919093/u8)	(0.979804862041/u9)	(0.948240721666/u10)	(0.905156203303/u11)	(0.853823055677/u12)	(0.797544820301/u13)	(0.739252811014/u14)	(0.681286901389/u15)}"
[41] "A[2018 May]=	{(0.614548375353/u1)	(0.669943310268/u2)	(0.727621971119/u3)	(0.786014936489/u4)	(0.842910986686/u5)	(0.895484963916/u6)	(0.940480923844/u7)	(0.974580741456/u8)	(0.994923184462/u9)	(0.999652919395/u10)	(0.988322290198/u11)	(0.961993384715/u12)	(0.922999083487/u13)	(0.874458221757/u14)	(0.819719342651/u15)}"
[42] "A[2018 Jun]=	{(0.819719342651/u1)	(0.874458221757/u2)	(0.922999083487/u3)	(0.961993384715/u4)	(0.988322290198/u5)	(0.999652919395/u6)	(0.994923184462/u7)	(0.974580741456/u8)	(0.940480923844/u9)	(0.895484963916/u10)	(0.842910986686/u11)	(0.786014936489/u12)	(0.727621971119/u13)	(0.669943310268/u14)	(0.614548375353/u15)}"
[43] "A[2018 Jul]=	{(0.492572660765/u1)	(0.538955716493/u2)	(0.589293702196/u3)	(0.643220264127/u4)	(0.699970277162/u5)	(0.758258879255/u6)	(0.816184267288/u7)	(0.871201371445/u8)	(0.920224802862/u9)	(0.959908707818/u10)	(0.987103989796/u11)	(0.999413090220/u12)	(0.995684674960/u13)	(0.976271897034/u14)	(0.942951792676/u15)}"
[44] "A[2018 Aug]=	{(0.797459357574/u1)	(0.853742612533/u2)	(0.905085471620/u3)	(0.948184689297/u4)	(0.979768092615/u5)	(0.997131716272/u6)	(0.998674371844/u7)	(0.984249788816/u8)	(0.955200046914/u9)	(0.914056729088/u10)	(0.864027260958/u11)	(0.808445482140/u12)	(0.750335759620/u13)	(0.692158585907/u14)	(0.635728500759/u15)}"
[45] "A[2018 Sep]=	{(0.851283979471/u1)	(0.902919527992/u2)	(0.946463576966/u3)	(0.978631591167/u4)	(0.996681757391/u5)	(0.998953495208/u6)	(0.985231458165/u7)	(0.956794737628/u8)	(0.916131305064/u9)	(0.866430295945/u10)	(0.811030475127/u11)	(0.752977090072/u12)	(0.694759002479/u13)	(0.638219761031/u14)	(0.584592662467/u15)}"
[46] "A[2018 Oct]=	{(0.796176906407/u1)	(0.852534680663/u2)	(0.904022334374/u3)	(0.947341168540/u4)	(0.979212791057/u5)	(0.996914482109/u6)	(0.998815749667/u7)	(0.984736340537/u8)	(0.955987433689/u9)	(0.915079298011/u10)	(0.865210507576/u11)	(0.809717437683/u12)	(0.751634790009/u13)	(0.693437029721/u14)	(0.636952952505/u15)}"
[47] "A[2018 Nov]=	{(0.878457343713/u1)	(0.926382589654/u2)	(0.964506146998/u3)	(0.989749702296/u4)	(0.999863128736/u5)	(0.993896151331/u6)	(0.972412853707/u7)	(0.937363276778/u8)	(0.891663968202/u9)	(0.838645311962/u10)	(0.781540824773/u11)	(0.723132628480/u12)	(0.665581903745/u13)	(0.610410722118/u14)	(0.558577542467/u15)}"
[48] "A[2018 Dec]=	{(0.934398380402/u1)	(0.970317963784/u2)	(0.992854727895/u3)	(0.999972612972/u4)	(0.990999651970/u5)	(0.966780642620/u6)	(0.929485924912/u7)	(0.882152964883/u8)	(0.828129239604/u9)	(0.770584784832/u10)	(0.712192636086/u11)	(0.654991549520/u12)	(0.600390003339/u13)	(0.549252936090/u14)	(0.502020733577/u15)}"
[49] "A[2019 Jan]=	{(0.471384508851/u1)	(0.515791681869/u2)	(0.564223270106/u3)	(0.616464716786/u4)	(0.671960961970/u5)	(0.729695563608/u6)	(0.788077094769/u7)	(0.844871282803/u8)	(0.897233462267/u9)	(0.941898074339/u10)	(0.975553706967/u11)	(0.995365788129/u12)	(0.999523532684/u13)	(0.987633241724/u14)	(0.960808123157/u15)}"
[50] "A[2019 Feb]=	{(0.929231925665/u1)	(0.966595606101/u2)	(0.990899550998/u3)	(0.999966746661/u4)	(0.992943655839/u5)	(0.970493377083/u6)	(0.934644962425/u7)	(0.888359623966/u8)	(0.834975785595/u9)	(0.777706089886/u10)	(0.719295044246/u11)	(0.661860943870/u12)	(0.606885715210/u13)	(0.555294564025/u14)	(0.507573160463/u15)}"
[51] "A[2019 Mar]=	{(0.635728500759/u1)	(0.692158585907/u2)	(0.750335759620/u3)	(0.808445482140/u4)	(0.864027260958/u5)	(0.914056729088/u6)	(0.955200046914/u7)	(0.984249788816/u8)	(0.998674371844/u9)	(0.997131716272/u10)	(0.979768092615/u11)	(0.948184689297/u12)	(0.905085471620/u13)	(0.853742612533/u14)	(0.797459357574/u15)}"
[52] "A[2019 Apr]=	{(0.618144926867/u1)	(0.673728777046/u2)	(0.731510686405/u3)	(0.789879897962/u4)	(0.846581995881/u5)	(0.898755438520/u6)	(0.943126645427/u7)	(0.976390630457/u8)	(0.995736732160/u9)	(0.999393566873/u10)	(0.987014745181/u11)	(0.959757937275/u12)	(0.920025246515/u13)	(0.870967846732/u14)	(0.815931330479/u15)}"
[53] "A[2019 May]=	{(0.699799203737/u1)	(0.758085736782/u2)	(0.816015649004/u3)	(0.871045701368/u4)	(0.920091785780/u5)	(0.959808221615/u6)	(0.987044526233/u7)	(0.999400110083/u8)	(0.995719414407/u9)	(0.976351083334/u10)	(0.943068385594/u11)	(0.898683133542/u12)	(0.846500631054/u13)	(0.789794084965/u14)	(0.731424237459/u15)}"
[54] "A[2019 Jun]=	{(0.460691101527/u1)	(0.504066401957/u2)	(0.551479703532/u3)	(0.602785427658/u4)	(0.657526656015/u5)	(0.714816379783/u6)	(0.773219120025/u7)	(0.830666720106/u8)	(0.884459473044/u9)	(0.931411015939/u10)	(0.968176343716/u11)	(0.991745187990/u12)	(0.999998638890/u13)	(0.992158760958/u14)	(0.968964955757/u15)}"
[55] "A[2019 Jul]=	{(0.732072638580/u1)	(0.790437595817/u2)	(0.847110626835/u3)	(0.899225003829/u4)	(0.943504738360/u5)	(0.976646938206/u6)	(0.995848452751/u7)	(0.999350172550/u8)	(0.986820368164/u9)	(0.959430419675/u10)	(0.919592242867/u11)	(0.870461473960/u12)	(0.815383108319/u13)	(0.757436429439/u14)	(0.699157803288/u15)}"
[56] "A[2019 Aug]=	{(0.917952431503/u1)	(0.958185540000/u2)	(0.986075274959/u3)	(0.999173163113/u4)	(0.996256377373/u5)	(0.977601327641/u6)	(0.944920530813/u7)	(0.900988361708/u8)	(0.849099371947/u9)	(0.792538303999/u10)	(0.734191270584/u11)	(0.676342365945/u12)	(0.620631005542/u13)	(0.568114157722/u14)	(0.519378028583/u15)}"
[57] "A[2019 Sep]=	{(0.797459357574/u1)	(0.853742612533/u2)	(0.905085471620/u3)	(0.948184689297/u4)	(0.979768092615/u5)	(0.997131716272/u6)	(0.998674371844/u7)	(0.984249788816/u8)	(0.955200046914/u9)	(0.914056729088/u10)	(0.864027260958/u11)	(0.808445482140/u12)	(0.750335759620/u13)	(0.692158585907/u14)	(0.635728500759/u15)}"
[58] "A[2019 Oct]=	{(0.744403794970/u1)	(0.802621805865/u2)	(0.858589682971/u3)	(0.909331809457/u4)	(0.951528663625/u5)	(0.981935913077/u6)	(0.997927981065/u7)	(0.998023663640/u8)	(0.982213886505/u9)	(0.951963779690/u10)	(0.909888227601/u11)	(0.859227548571/u12)	(0.803303164762/u13)	(0.745096514251/u14)	(0.687011362643/u15)}"
[59] "A[2019 Nov]=	{(0.802579210642/u1)	(0.858549792123/u2)	(0.909296993447/u3)	(0.951501413673/u4)	(0.981918472114/u5)	(0.997921926271/u6)	(0.998029569152/u7)	(0.982231192175/u8)	(0.951990919182/u9)	(0.909922963788/u10)	(0.859267390856/u11)	(0.803345739345/u12)	(0.745139810065/u13)	(0.687053839689/u14)	(0.630845572346/u15)}"
[60] "A[2019 Dec]=	{(0.934860420628/u1)	(0.970646476081/u2)	(0.993021020682/u3)	(0.999961147065/u4)	(0.990811521104/u5)	(0.966433321377/u6)	(0.929009387411/u7)	(0.881583694580/u8)	(0.827504172508/u9)	(0.769936741338/u10)	(0.711547835770/u11)	(0.654368984793/u12)	(0.599802055325/u13)	(0.548706596766/u14)	(0.501518966933/u15)}"
[61] "A[2020 Jan]=	{(0.469037637086/u1)	(0.513220242960/u2)	(0.561431446826/u3)	(0.613472272769/u4)	(0.668809671073/u5)	(0.726455996424/u6)	(0.784854164098/u7)	(0.841805934689/u8)	(0.894497223927/u9)	(0.939677702095/u10)	(0.974025762143/u11)	(0.994665488520/u12)	(0.999716719159/u13)	(0.988701504796/u14)	(0.962653039946/u15)}"

[62] "A[2020 Feb]=	{(0.817 826969 750/u1)	(0.8727 163989 98/u2)	(0.9215 174084 49/u3)	(0.9608 826704 08/u4)	(0.9876 768644 07/u5)	(0.9995 322178 11/u6)	(0.9953 387082 82/u7)	(0.9754 934106 63/u8)
[63] "A[2020 Mar]=	{(0.630 845572 346/u1)	(0.6870 538396 89/u2)	(0.7451 398100 65/u3)	(0.8033 457393 45/u4)	(0.8592 673908 56/u5)	(0.9099 229637 88/u6)	(0.9519 909191 82/u7)	(0.9822 311921 75/u8)
[64] "A[2020 Apr]=	{(0.667 383465 675/u1)	(0.7249 881919 36/u2)	(0.7833 916615 57/u3)	(0.8404 119920 33/u4)	(0.8932 491436 48/u5)	(0.9386 600655 30/u6)	(0.9733 190652 40/u7)	(0.9943 320586 82/u8)
[65] "A[2020 May]=	{(0.666 754737 065/u1)	(0.7243 408011 59/u2)	(0.7827 461697 02/u3)	(0.8397 961800 11/u4)	(0.8926 970232 99/u5)	(0.9382 089331 47/u6)	(0.9730 045204 09/u7)	(0.9941 817995 63/u8)
[66] "A[2020 Jun]=	{(0.541 612444 917/u1)	(0.5921 595669 17/u2)	(0.6462 647932 47/u3)	(0.7031 376075 86/u4)	(0.7614 614665 96/u5)	(0.8192 991030 72/u6)	(0.8740 717378 78/u7)	(0.9226 707350 29/u8)
[67] "A[2020 Jul]=	{(0.875 577106 612/u1)	(0.9239 483257 50/u2)	(0.9627 017003 79/u3)	(0.9887 293543 43/u4)	(0.9997 211877 57/u5)	(0.9946 461502 16/u6)	(0.9739 844331 68/u7)	(0.9396 180311 76/u8)
[68] "A[2020 Aug]=	{(0.819 719342 651/u1)	(0.8744 582217 57/u2)	(0.9229 990834 87/u3)	(0.9619 933847 15/u4)	(0.9883 222901 98/u5)	(0.9996 529193 95/u6)	(0.9949 231844 62/u7)	(0.9745 807414 56/u8)
[69] "A[2020 Sep]=	{(0.840 288870 362/u1)	(0.8931 387925 80/u2)	(0.9385 699452 95/u3)	(0.9732 562915 62/u4)	(0.9943 021614 07/u5)	(0.9997 936414 81/u6)	(0.9892 113434 41/u7)	(0.9635 487113 61/u8)
[70] "A[2020 Oct]=	{(0.600 899907 300/u1)	(0.6555 313694 34/u2)	(0.7127 515872 03/u3)	(0.7711 463435 44/u4)	(0.8286 706178 39/u5)	(0.8826 456671 50/u6)	(0.9298 979207 96/u7)	(0.9670 803433 42/u8)
[71] "A[2020 Nov]=	{(0.908 704405 899/u1)	(0.9510 371730 85/u2)	(0.9816 207600 35/u3)	(0.9978 176519 48/u4)	(0.9981 286186 34/u5)	(0.9825 241690 21/u6)	(0.9524 512936 80/u7)	(0.9105 127575 35/u8)
[72] "A[2020 Dec]=	{(0.874 419588 574/u1)	(0.9229 662722 00/u2)	(0.9619 688552 42/u3)	(0.9883 081292 48/u4)	(0.9996 504322 40/u5)	(0.9949 326081 34/u6)	(0.9746 011901 87/u7)	(0.9405 105890 91/u8)

[62] "A[2020 Feb]=	(0.9418 099095 26/u9)	(0.8971 244640 00/u10)	(0.8447 489253 71/u11)	(0.7879 482654 42/u12)	(0.7295 659372 47/u13)	(0.6718 347735 78/u14)	(0.61634 4823304 /u15)}"
[63] "A[2020 Mar]=	(0.9980 295691 52/u9)	(0.9979 219262 71/u10)	(0.9819 184721 14/u11)	(0.9515 014136 73/u12)	(0.9092 969934 47/u13)	(0.8585 497921 23/u14)	(0.80257 9210642 /u15)}"
[64] "A[2020 Apr]=	(0.9997 878572 36/u9)	(0.9891 704291 53/u10)	(0.9634 764480 92/u11)	(0.9249 906110 11/u12)	(0.8768 083960 05/u13)	(0.8222 791537 24/u14)	(0.76453 3007298 /u15)}"
[65] "A[2020 May]=	(0.9998 159794 25/u9)	(0.9893 742513 57/u10)	(0.9638 371296 83/u11)	(0.9254 773084 17/u12)	(0.8773 843437 44/u13)	(0.8229 076353 41/u14)	(0.76518 1740725 /u15)}"
[66] "A[2020 Jun]=	(0.9617 477762 83/u9)	(0.9881 803083 55/u10)	(0.9996 276486 97/u11)	(0.9950 170331 93/u12)	(0.9747 848873 14/u13)	(0.9407 773065 24/u14)	(0.89585 0027602 /u15)}"
[67] "A[2020 Jul]=	(0.8944 239384 56/u9)	(0.8417 240116 01/u10)	(0.7847 681585 71/u11)	(0.7263 696405 56/u12)	(0.6687 257351 24/u13)	(0.6133 926143 76/u14)	(0.56135 7160673 /u15)}"
[68] "A[2020 Aug]=	(0.9404 809238 44/u9)	(0.8954 849639 16/u10)	(0.8429 109866 86/u11)	(0.7860 149364 89/u12)	(0.7276 219711 19/u13)	(0.6699 433102 68/u14)	(0.61454 8375353 /u15)}"
[69] "A[2020 Sep]=	(0.9250 880464 62/u9)	(0.8769 236480 92/u10)	(0.8224 048814 53/u11)	(0.7646 627595 21/u12)	(0.7063 094485 70/u13)	(0.6493 177576 11/u14)	(0.59503 6207559 /u15)}"
[70] "A[2020 Oct]=	(0.9911 611672 12/u9)	(0.9999 809325 85/u10)	(0.9927 090582 48/u11)	(0.9700 319106 76/u12)	(0.9339 969035 40/u13)	(0.8875 754271 58/u14)	(0.83410 7476978 /u15)}"
[71] "A[2020 Nov]=	(0.8599 442697 44/u9)	(0.8040 693190 62/u10)	(0.7458 758528 87/u11)	(0.6877 761106 33/u12)	(0.6315 358588 95/u13)	(0.5783 204292 02/u14)	(0.52880 0297058 /u15)}"
[72] "A[2020 Dec]=	(0.8955 214889 73/u9)	(0.8429 518818 31/u10)	(0.7860 579171 08/u11)	(0.7276 651613 57/u12)	(0.6699 853148 52/u13)	(0.6145 882565 05/u14)	(0.56247 2340071 /u15)}"

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