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# **Harroddian Instability: A Marxian Perspective**

Chatzarakis Nikolaos\* and Persefoni Tsaliki \*\*

## **Abstract**

The analysis of a sustained long-run equilibrium path of economic growth goes back to Marx's discussion of the schemes of reproduction and capital accumulation. In this paper, we indicate that the Harroddian 'knife edge' instability is a feature of the inner nature of the capitalist mode of production that is explained by the evolution of the rate of surplus-value. A fundamental category in Marx's analysis, which is not restricted to income distribution, but also has further repercussions, which we grapple with in our analysis. In particular, we argue that the unbalanced economic growth path is the macroeconomic manifestation of the consequent antithesis between productive forces and productive relations formed during the process of capital accumulation and confined by the evolution of the rate of surplus-value.

**Keywords:** economic growth, capital accumulation, rate of surplus-value, knife-edge instability

**JEL Classification:** E11, E12, E32, P16

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## 1. Introduction

The theory of economic growth has been haunted from its very beginning by the ‘ghost’ of instability. Harrod (1939) and Domar (1946) revealed that a free-market economy goes along with serious setbacks, that is, ‘disinvestment’ and ‘knife-edge’ instability while a balanced growth path is far from being attainable. The approach set out by Solow (1956) and Swan (1956) attempted to bypass these problems by relying on strict analytical assumptions whose inconsistencies had been soon pointed out; as a consequence, a large part of the extant literature was devoted to expand and improve the proposed growth model in an effort to exemplify the nature of the instability. In addition, the post-Keynesians attempted to link economic cycles with economic growth; over the years, however, they could not manage to present a consistent theory of cyclical growth puzzled, at the same time, by the extent to which such a theory can be brought together in a single model.

Indeed, the short-run multiplier analysis and the mystified concept of ‘animal spirits’ in the Harrod and Domar analysis were proved inappropriate to grapple successfully with such issues while the purified and polished neoclassical proposals were proved to be inadequate to tackle with reality. Nevertheless, years before Harrod and Domar and from a different theoretical perspective, the tradition founded on the classical political economy and especially on Marx’s writings argued that a balanced growth path is an idea at odds with the turbulent nature of capital accumulation while instability manifests a vital feature of the long-run dynamics of capitalism. In the present work, we argue that a theory of economic growth should be founded and be intrinsically linked to the inner nature of capitalism while the roots of the instability could be traced in the evolution of the rate of surplus-value and in the unbalanced growth of constant and variable capital. We also argue that Marx’s analysis of reproduction and economic crisis probes to deeper understanding of the turbulent evolution of capital accumulation. Essentially, we show that the surfacing of instability is just the macroeconomic manifestation of what Marx identified as the consequent contradiction between productive forces and productive relations featuring the capitalist mode of production which is detained by the evolution of the rate of surplus-value.

The paper is structured as follows: Section 2 briefly presents the main concerns regarding the theory of economic growth set out by Harrod and Domar and its amendments. Section 3 reviews the basics of Marx’s theory of capital reproduction and shows that persistent instability is in the

very core of the capitalist system. Section 4 brings forth the essential features of Marx's analysis of capital accumulation according to which the evolution of the capitalist system is presented as a continuous battle between the expanding productive forces and the restricting productive relations of the system. Finally, Section 5 concludes the paper.

## 2. The Instability Issues in the Current Theory of Economic Growth

From its initiation in the last quarter of the 19<sup>th</sup> century, economic theory, as known today, was preoccupied with a static picture of an economy at equilibrium. Pigou's (1933) attempt to set a macroeconomic theory on neoclassical premises was overshadowed by Keynes (1936), who offered the first solid analysis of an economy as a whole, at least, in the short-run. In turn, Harrod (1939 and 1948) and Domar (1946 and 1947) set up the framework for extending the Keynesian theory of income multiplier to the long-run horizon. Harrod's interests lay with the analysis of the business cycle with constant interest rate and 'warranted' capital-output ratio. Domar's interests lay with the possibility of a long-run equilibrium with full employment of capital and labour and constant capital-output ratio.<sup>1</sup>

Harrod and Domar presented a 'warranted' growth rate,  $g_w$ , identified with the rate of capital accumulation in the form

$$g_w = \frac{s^*}{\sigma_w}$$

where  $s^*$  is the propensity to save at equilibrium and  $\sigma_w$  is the 'warranted' capital-output ratio. Their main conclusion was that the 'warranted' growth rate cannot be secured within the analytical frame of a free-market economy. Domar argued that stepping away from the above defined balanced growth path, inflationary pressures come forth which cause disinvestment leading to further divergence; hence, to a non-stable economy. Harrod went a step further and discussed two main problems:

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<sup>1</sup> It is worth noting that both considered 'fixed proportions' only at equilibrium while their results remain valid without resorting to any production function.

1. First Problem: In equilibrium and given that the conditions of full employment of capital and labour are met, the  $g_w$  should be equal to actual growth rate of capital stock,  $g_a$  and to natural rate of growth of the labour force,  $g_n$ . That is

$$g_w = g_a = g_n$$

$$\frac{s^*}{\sigma_w} - \delta = \frac{s}{\sigma(u)} - \delta = n + \tau$$

where  $s$  is the actual propensity to save ( $\neq s^*$ ),  $\sigma(u)$  is the actual capital-output ratio depending on the rate of capacity utilization  $u$ ,  $n$  is the rate of change of the population and by extent the rate of change of labour force,  $\delta$  is the rate of capital depreciation and  $\tau$  is the rate of a labour-saving technological change.<sup>2</sup> The first problem emerges because in a free market economy, the different parameters and variables involved in the above relation have no *à priori* reason to be such so that the warranted, actual and natural rate of growth to coincide.

2. Second Problem: In his attempt to delve deeper in the reasons of the surfacing instability, Harrod stated that capitalists decide on their future investments (hence, decide on the value of  $s$ ) based on their current returns (hence, on the current value of  $\sigma(u)$ ). Having accumulated a large capital stock (hence, a high value of  $\sigma(u)$ ), their returns drop; thus, they tend to decrease their savings and *vice versa*. The combination of all the above leads to a constant decrease (or increase) of the actual growth rate with respect to the ‘warranted’ one. Consequently, the ‘animal spirits’ of the entrepreneurs deem the ‘warranted’ growth rate fundamentally unattainable at any possible deviation. This problem came to be known in the literature as the ‘knife-edge’ instability.

A Third Problem, concerning the uniqueness of the equilibrium, was also raised in the relevant literature. Unlike the common belief, Harrod did not explicitly consider a unique balanced growth path; in fact, he allowed for many possible equilibria, for any possible pairs of  $s^*$  and  $\sigma_w$  values. Kregel (1972: ch. 8) showed that as capitalists, guided from their ‘animal spirits’, attempt to retain their profits lead the economy to evolve chaotically among the so defined multitude of unstable equilibria.

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<sup>2</sup> In Harrod’s analysis, the depreciation and technological change are omitted. However, the introduction of both generalizes their arguments.

A non-stabilized economy, due to free market functioning, was not an appealing result for the neoclassical tradition. Soon, Solow (1956) and Swan (1956) proposed a different vision of economic growth founded on neoclassical hypotheses, such as the Walras and Say's Laws, perfect competition, homogeneous production function, perfect substitutability between factors of production, *etc.* whose implementation promised and pledged a balanced economic growth. Nevertheless, Solow and his successors just simply circumvented Harrod's First Problem as in their analysis presuppose that the markets are always at equilibrium and do not evolve out of proportions. They wiped out Harrod's Second Problem as well, as the attainment of equilibrium relies only on a set of mathematical prerequisites (Inada conditions); also the consent of Say's Law and Phelps' (1961) 'golden rule' leaves no room for instability 'worries'.<sup>3</sup> According to Solow, the assumption of 'fixed proportions' is the basic source of instability in the Harrod-Domar growth model; however, from the outset it is clear that this assumption rules the Harrod-Domar proposal only at equilibrium and applies to Solow's analysis as well (Hagemann 2009); in addition, as argued by Darity (2009) and others, altering one of the exacting assumptions of the Solow-Swan model, the 'knife-edge' instability issues return into the picture. Farther, Chatzarakis and Tsaliki (2021) showed that, even without altering any of the strict assumptions, the same problems re-emerge in Solow's model by simply introducing the dynamics of the rate of profit next to those of the capital-labour ratio; after all, the rate of profit and the capital-labour ratio are two facets and phases of the same dynamic process of capital accumulation and must be examined simultaneously.<sup>4</sup>

The (post-)Keynesian tradition also tempted to respond to the 'problems' raised by Harrod and Domar, taking into account different aspects of the growth process.<sup>5</sup> Robinson (1956) was the first to recognize the two horizons involved in the analysis of growth; the short-run horizon, where the acting economic forces are supposed to converge to some temporary state of equilibrium and the long-run horizon, where the consequences of technological change and the 'animal spirits' idea apply. Her initiative was that the long-run horizon cannot be formed as a sequence of pseudo-static

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<sup>3</sup> For details on the way Solow side-stepped the problems raised by Harrod see Chatzarakis and Tsaliki (2021).

<sup>4</sup> The number of theoretical and empirical concerns referring to the Solow-Swan growth model, such as the 'Solow residual', the 'productivity' and the 'convergence paradox', *etc.* discussed in the literature, indicate, to a great extent, the 'weak' theoretical ground upon which the neoclassical theory of growth is built on.

<sup>5</sup> It is important to note at the outset that many post-Keynesians followed Harrod's original idea to construct not a theory of growth, but a theory of economic dynamics.

‘short-run’ states of equilibrium. Consequently, the shift from short- to long-run analysis, as attempted by Solow, is impossible and one should emphasize on the factors of growth, such as technological change and investment, whose trends guide economic growth in the long-run, rather than on the demand and supply of the factors of production whose trends define the short-run behavior of the system.

Kaldor (1955) and Pasinetti (1962), sharing a similar viewpoint with Robison, argued that the distribution of total income between wages and profits determines the propensity to save; hence, it determines investment and eventually economic growth in the long-run. Both concluded that it is the portion of profits invested that determines growth in the long-run. Kaldor (1957), by introducing ‘stylized facts’ of the long-run evolution of a capitalist economy, linked the rate of growth to the rate of change in labour productivity. At a later point, Kaldor (1988) proposed that effective demand bears a long-run character related to investment; therefore, it can be viewed as a stabilizing force in economic dynamics. This idea had been already discussed in a quasi-Keynesian framework by Kalecki (1954), who attempted to construct a model introducing both the long-run tendency of economic growth and the short-run (cyclical) fluctuations around it. The works of both initiated a series of models which emphasized either on income distribution and its role in a demand-led growth (Bhaduri and Marglin 1990; Lavoie 1995) or on the autonomous component of effective demand (Park 2000; Skott 2019) or, influenced by the Sraffian tradition, on a super-multiplier where business investment is induced by total income while autonomous expenditures determine growth (Freitas and Serrano 2015).

Early enough, Samuelson (1939), followed by Duesenberry (1949) and Hicks (1950) *inter alia*, turned their attention to the short-run implications of economic growth, expecting to unveil what Harrod originally hoped to: the cyclical fluctuations. Their models were mostly based on the interplay of the multiplier with the accelerator principle, deeming possible the generation of periodic solutions around a uniform trend; however, distinguishing the trend from the cycle was proved to be erroneous as both are aspects of the same process. Despite all these efforts over the years, little has been done to offer a theoretically solid cyclical model of economic growth and although models linking growth to cyclical pattern are still offered (*e.g.*, Sasaki 2013), little

progress has been added up in countering the neoclassical orthodoxy of the theory of economic growth.

### **3. The Schemes of Expanded Reproduction as a Proper Analytical Framework to Study Growth**

The neoclassical models of economic growth concealed the actual problem of stability while the post-Keynesian models did not manage to offer a viable alternative; within these strands of economic analysis, the idea to construct a unified model of cycle and growth continues to be shattered as Pasinetti (1960) has pointed out over fifty years ago. Nevertheless, many of the current troubling issues concerning the evolution of a market economy have been already introduced and explored by the old classical economists (Smith and Ricardo) and were formed into a solid analytical system by Marx in his schemes of expanded reproduction (SER). From the start, Marx paid attention to the long-run tendencies of the capitalist system which are shaped by the Laws of Capital Accumulation (*Capital I*) and Falling Tendency of the Rate of Profit (*Capital III*).

The Law of Capital Accumulation states that economic growth is the process of replacing living (variable capital,  $V$ ) by dead (constant capital,  $C$ ) labour<sup>6</sup> leading to a rising organic composition of capital (OCC), which is defined as

$$\gamma = \frac{C}{C + V}$$

The displaced labour forms the industrial reserve army of labour (IRAL), an essential feature of capitalist development which reflects and, at the same time, relates capital accumulation with the rate of profit as it affects the value of the variable capital and surplus produced. The imposed, by competition and profit motive, continuous technological change raises the OCC faster than the rate of surplus-value (RSV); consequently, the rate of profit (ROP) defined as

$$\pi = \frac{S}{C + V} = e(1 - \gamma)$$

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<sup>6</sup> Usually,  $C$  and  $V$  are interpreted as the circulating constant and variable capital, respectively. However, in what follows,  $C$  is considered to be the stock of fixed and circulating constant capital while  $V$  is the stock of variable capital (both multiplied by the respective turnover times).



where  $S$  is the surplus-value and  $e = S/V$  is the RSV, eventually displays a long-run falling tendency (second Law). This indirect relation between OCC and ROP has been empirically verified for many different economies and time spans (see Shaikh 1992 and 2016; Tsoulfidis and Tsaliki 2019; Li 2020, *inter alia*).

Marx's formation of economic growth is presented in the schemes of reproduction (*Capital II*) where the economy is divided in two departments; Department I produces means of production (elements of constant capital) and Department II produces means of consumption (elements of variable capital). In the analysis of SER, a balanced growth is attained when

$$V_I + S_I = C_{II} + \Delta C_I + \Delta C_{II} \quad (1)$$

where  $\Delta$  stands for the (discrete) change over time and subscripts I and II denote the two departments. As shown in equation (1), the balance path of economic growth requires that the net output produced in Department I ( $V_I + S_I$ ) to be equal to new and old requirements of constant capital in Department II ( $C_{II} + \Delta C_{II}$ ) and to new requirements of constant capital in Department I ( $\Delta C_I$ ). From the beginning, Luxemburg (1913) pointed out that there is no mechanism to form the specific investment behavior for the attainment of the balanced growth path described in equation (1); thus, the 'knife-edge' and 'disinvestment' problems set out by Harrod and Domar several years later, simply anticipated her views. It has also been argued that the violation of equation (1) caused by the disproportional growth of the two departments sooner or later, lead to overproduction (Tugan-Baranovskii 1901) or under-consumption (Luxemburg 1913) while Bukharin (1924) presented the first as the possibility and the second as the cause for the disruption of the accumulation process.

The equation (1) together with the adoption of 'fixed proportions' (translated into constant OCC and RSV) led to the conclusion that Marx abandoned major features of his own theory of accumulation according to which capital accumulation displays a turbulent and cyclical nature forming a sequence of phases of expansion and contraction. Apparently, Marx's SER should be viewed as a general theoretical framework that provides a balanced growth path under specific conditions, which cannot generally hold; nevertheless, they provide the analytical groundwork to develop a theory for the causes of the instability and the conditions for the establishment of a

balanced growth path. Grossman (1929) was the first to point out that in Marx's analysis profitability is beyond doubt the driving motive that shapes capitalists' attitudes towards investment and accumulation; as such, it should be regarded as the key variable in exploring the dynamics of capital accumulation. Following Grossman's idea, a series of studies attributed the periodicity of the long-run economic cycles to the evolution and falling tendency of the rate of profit. Shaikh (1978 and 2016) and Tsoulfidis (2006) described the built-in mechanism of Marx's theory of capital accumulation, starting the analysis from the sphere of production and placing the emphasis on capitalists' pursuit of profit. They showed that the motive of profit maximization provides an answer for the transition from one phase of capital accumulation to another and allows for a prioritization of the different possible causes of crises, deeming the decline of profitability as the main driving force.

Going one step further, in this paper we attempt to offer an explanation of the emerging instability issues by rejecting the indeterminate psychological factor of 'animal spirits' and introducing the evolution of the RSV as the key parameter in shaping the cyclical behaviour of capital accumulation. In so doing, we bring into the discussion a vital Marxian category whose roots are deeply embedded in the interplay between productive forces and productive relations that shape the nature of the capitalist system. We proceed the analysis by acknowledging that the constant capital is the output of Department I and the variable capital is the output of Department II; hence, the growth rates of the two departments can be approximated by the growth rates of constant and variable capital, respectively. The change in constant capital,  $\Delta C$ , equals the portion of surplus-value (profits) invested in capital stock,  $s_c$ , minus the depreciation,  $\delta$ , that is

$$\Delta C = s_c S - \delta C$$

In turn, the rate of growth of constant capital,  $g_c$ , can be defined as

$$g_c = \frac{\Delta C}{C} = s_c \frac{S}{C} - \delta = s_c \frac{S/(C+V)}{C/(C+V)} - \delta = s_c \frac{\pi}{\gamma} - \delta \quad (2)$$

In addition, the change in variable capital,  $\Delta V$ , is equal to the portion of surplus-value (profits) spent (or saved) to hire (or fire) labourers,  $s_v$ , plus the enhancing labour productivity technological change,  $\tau$ , that is

$$\Delta V = s_v S + \tau V$$

Since  $S/V = e$ , the rate of growth of variable capital,  $g_v$ , can be defined as

$$g_V = \frac{\Delta V}{V} = s_V \frac{S}{V} + \tau = s_V e + \tau \quad (3)$$

The assumption of fixed proportions in SER implies that the constant and variable capital must grow at the same rate; hence the following relation holds

$$g_V = g_C \quad (4)$$

which expresses the simultaneous equilibrium growth rates of the two departments or the ‘warranted’ growth path in a Harrod-Domar sense.

The assumption of fixed proportions holds only at equilibrium; as it is dropped, the balanced growth path of equation (4) should be perceived as a trajectory (long-run tendency) in agreement to Marx’s theory of capital accumulation (rising OCC and falling ROP) characterized by asymptotic stability (the solutions of the system do not diverge from it) and structural instability (the solutions of the system do not converge on it either). The long-wave-wise oscillations of the system around this trajectory are due to the structural instability already discussed in Marx’s theory of capital accumulation and ‘naively’ described, many years later, by Harrod as the second problem caused by the ‘animal spirits’. As the  $g_C$  and  $g_V$  differ over time, the evolution of ROP and OCC, the two phases of the same process of capital accumulation, are attracted to a common trajectory defined by the following two equations:

$$\begin{aligned} \text{rate of change of OCC:} \quad & \frac{\Delta \gamma}{\gamma} = (1 - \gamma)(g_C - g_V) \\ \text{rate of change ROP:} \quad & \frac{\Delta \pi}{\pi} = g_e - \frac{\gamma}{1 - \gamma}(g_C - g_V) \end{aligned}$$

This equilibrium growth path implies that the growth rates of constant and variable capital differ depending on the dynamics of RSV,  $g_e$ .

$$g_C - g_V = g_e \quad (5)$$

In other words, the  $g_e$  captures the difference between the growth rates of Department I and Department II in Marx's analysis, the deviation from the balanced growth rate in Harrod-Domar analysis, and the extent of disproportions of the two departments in Tugan-Baranovskii analysis.<sup>7</sup>

#### 4. 'Animal Spirits' and the Rate of Surplus-Value

To explore the 'knife-edge' instability of growth within Harrod's analysis, one must resort and dive into the mystifying idea of capitalists' 'animal spirits', that is, to their fears, hopes, passions, *etc.* that guide their investing behavior. In Marx's analysis, the instability is explained by the evolution of the RSV, a key variable that reflects the interface between capital and labour not only as factors entering into the production process but also as social classes which confront each other and form the state of affairs upon which the capitalist economy and society are based. Although the interconnection between capital and labour embraces all spheres of reproduction, the production sphere retains the prime area in which the intense struggle between capitalists (owners of the means of production) and workers (owners of the labour power) is manifested and the exploitative nature of the system is revealed.

In the sphere of production, the Law of Capital Accumulation is manifested by a rising OCC; the dead labour (constant capital) by replacing the living (variable capital) takes the control over the production process, increases the dexterity of labourers, intensifies their efforts, degrades their skills and transforms production into a process of labour consumption by capital. In the sphere of circulation, the already in place subordination of labour to capital takes the form of a 'fair

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<sup>7</sup> A simple way to prove equation (5) is to reorder the equation for the evolution of the ROP as

$$\frac{\Delta\pi}{\pi} + \frac{\gamma}{1-\gamma}(g_C - g_V) - (g_C - g_V) = g_e - (g_C - g_V)$$

After simple manipulations we arrive at

$$g_e - (g_C - g_V) = \frac{\Delta\pi}{\pi} - \frac{1-2\gamma}{1-\gamma}(g_C - g_V)$$

By proving that the right-hand side term of the above relation is equal to zero, equation (5) becomes self-evident. In fact, over the long-run, the right-hand side term can be zero as it describes the interplay of ROP and OCC during a complete cycle according to Marx's analysis of capital accumulation. Hence,

$$\frac{\Delta\pi}{\pi} = \frac{1-2\gamma}{1-\gamma}(g_C - g_V)$$

which shows that the ROP initially rises together with OCC (when  $g_C > g_V$  and  $0 < \gamma < 1/2$ ; a phase of expansion) and then drops as the OCC rises (when given that  $g_C > g_V$  and  $1/2 < \gamma < 1$ ; a phase of contraction). For details, see Chatzarakis and Tsaliki (2021).

exchange' between capitalists and workers and it is concealed by the legal and moral status which becomes the social and political *status quo* upon which the society as a whole functions. Labour is transformed into a mere commodity (a factor of production) always in abundance due to the ever-present IRAL. This *status quo* becomes more apparent in the sphere of distribution, where the antithesis between capital and labour turns into a struggle over the distribution of income among the various social classes participating in the reproduction process. The objective of this struggle is the satisfaction of the ever-increasing need for higher profits (as realized surplus-value), a prerequisite of the self-expanding inner nature of capital.

As pointed out by Marx, the RSV encapsulates all the factors involved in the accumulation of capital manifesting, at the same time, the fundamental contradictions of the **capitalist mode of production**. Marx noted "At a certain stage of development, the material productive forces of society come into conflict with the existing relations of production or – this merely expresses the same thing in legal terms – with the property relations within the framework of which they have operated hitherto. From forms of development of the productive forces these relations turn into their fetters" (*Preface to Contribution: 4*). The productive forces, when they are aligned with productive relations, initiate an expansion of economic activity while, when they are at conflict, the economy enters into a phase of contraction. The sequence of expansions and contractions in economic activity is the 'normal' state of affairs in the process of reproduction in the long-run while the evolution of the RSV shows the possibility of its interruption and defines the size of the disproportion/divergence from the balanced path. In fact, the RSV brings out of the shadows the role that the falling tendency in the RP eventually plays in destabilizing the growth path as it reaches all layers of the system: when it is expressed as a profit-to-wage ratio, it becomes a key distributive variable reflecting the level of class struggle in the sphere of exchange; when it is expressed as a surplus-over-living labour ratio, it reflects the intensity of labourers' exploitation in the sphere of production; finally, when it is expressed as a surplus-to-necessary output ratio, it reveals the level of technological change induced in the labour process. Essentially, the RSV is a variable that stands for and reveals the antithesis between the productive forces and the productive (social) relations, penetrating all spheres of the capitalist mode of production. More importantly, it is a feature that openly describes the process of capitalist production and reproduction, without obscuring it under the uncertain, complex and short-run market mechanisms.

#### 4.1. The effects of $g_C$ and $g_V$ on $g_e$

Exploring farther the equation (5) and the effects that the evolution of constant and variable capital exert on the evolution of the RSV, we start the analysis from constant capital whose rise implies, with a given technology, more labour in use and output produced. The introduction of more constant capital, however, enables “the capitalist to replace the value of a day’s labour-power by a smaller portion of the value of a day’s product” (*Capital I*: p. 277); hence, the output increases by more due to rise in labour productivity “...as the use of machinery spreads... the rapidity and intensity of labour increase as a natural consequence” (*Capital I*: p. 279). *Ceteris paribus*, as constant capital increases but variable capital remains constant, the working day becomes more intense and each commodity produced embodies less living labour. Furthermore, “...machinery becomes in the hands of capital the objective means, systematically employed for squeezing out more labour in a given time” (*Capital I*: p. 280). As a result, with an increase of constant capital more surplus product is produced from a fixed labour force leading to a rising RSV.

The impact of constant capital on the RSV is straightforward; in contrast, the evolution of variable capital exerts multiple and different effects. As the variable capital increases, more labourers are employed, more labour hours are spent in the production process and, therefore, output and consequently surplus product are expected to rise. So long as a specific production technique continues to be in place, the effect on the RSV is neutral as both, the surplus product and variable capital, rise proportionately. The extracted surplus-value from a constant labour force may increase through a common practice of extending the working day beyond its legal, moral and cultural limitations; hence, the RSV rises as the surplus-value produced increases while the number of labourers and the value of the labour power remain unchanged.

The effects of the changes in variable capital presented above, though important, are generally considered minor regarding their impact on the evolution of the RSV. The reason is that the analysis of the evolution of capital accumulation requires a long-run perspective and more effective channels to reduce the value of the variable capital and to raise the surplus-value produced. For instance, the value of variable capital can decrease (through the wage-fund paid) as a direct side effect caused by the mechanization of the production process as labourers are being

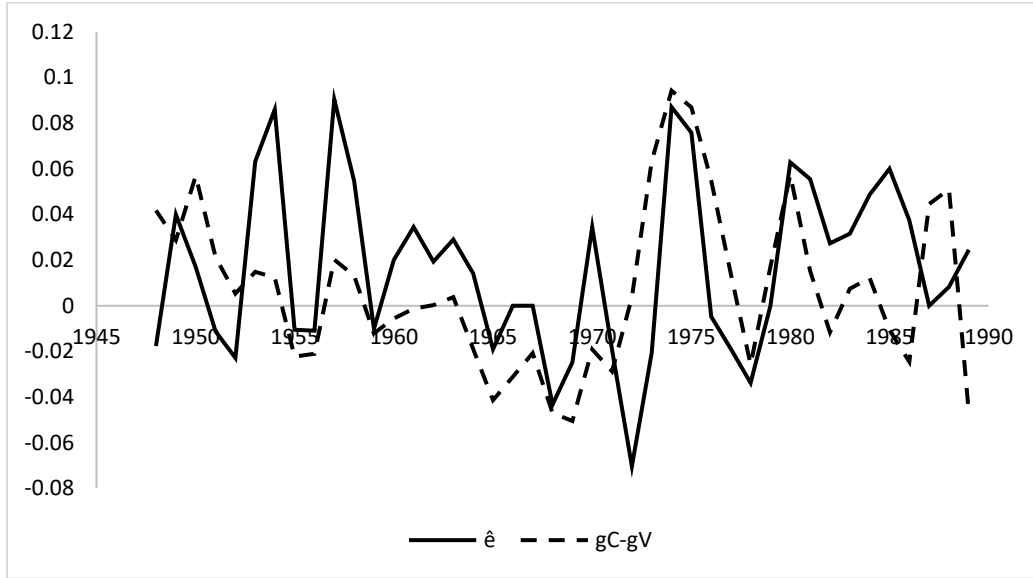
laid off. Hence, “this accelerated relative diminution of the variable constituent that goes along with the accelerated increase of the total capital, and moves more rapidly than this increase, takes the inverse form, at the other pole, of an apparently absolute increase of the labouring population, an increase always moving more rapidly than that of the variable capital or the means of employment” (*Capital I*: p. 443). The pressure from an increasing IRAL keeps the wage share constant, increases the discipline of the labour force and leads to higher level of labour intensity. In addition, the value of variable capital can indirectly decrease through the decreasing labour content of the commodities produced caused by technological advancements; hence, the value of the variable capital is decreased while, at the same time, the surplus-value produced and the RSV increase.

In general, “with a given rate of surplus-value, and a given value of labour-power, ..., the masses of surplus-value produced vary directly as the amounts of the variable capitals advanced” (*Capital I*: p. 214). On further consideration, with a given working day and hourly wage, the mass of surplus-value produced depends on labour productivity whose rise comes mainly through the investment in constant capital; hence, “an increase in surplus-value is accompanied by an increase in constant capital, and the growing exploitation of labour by greater outlays of the means of production through which labour is exploited, *i.e.*, by a greater investment of capital” (*Capital III*: p. 50). The fierce competition forces capitalists to continuously invest in constant capital and to adopt new methods of production in order to reduce the per unit cost of the output produced and, eventually, to increase the profit yields; the latter make up the necessary funds needed to finance investment projects while its source is the surplus-value produced by the unpaid labour employed in the production process (Tsaliki 2006).

#### **4.2 The evolution of $g_C$ , $g_V$ and $g_e$**

Equation (5) presents a growth path that is formed by the dynamic interaction of Marx’s key variables of capital accumulation, OCC and RP, while the interplay of the evolution of constant and variable capital with that of surplus-value captures the turbulent nature of capitalism which is manifested by the succession of phases of expansion and contraction in economic activity. In general, the periods of expansion are characterized by a declining RSV and growing constant capital while the variable one first increases and then follows a declining trend; in contrast, the

periods of recession are characterized by a rising RSV and contraction of investments in constant and variable capital. Figure 1 depicts the behaviour of  $(g_C - g_V)$  and  $g_e$  derived from actual data for the US economy for the period 1949-1989 (Shaikh and Tonak 1997).



**Figure 1:** The evolution of  $(g_C - g_V)$  and  $g_e$ , USA, 1949-1989

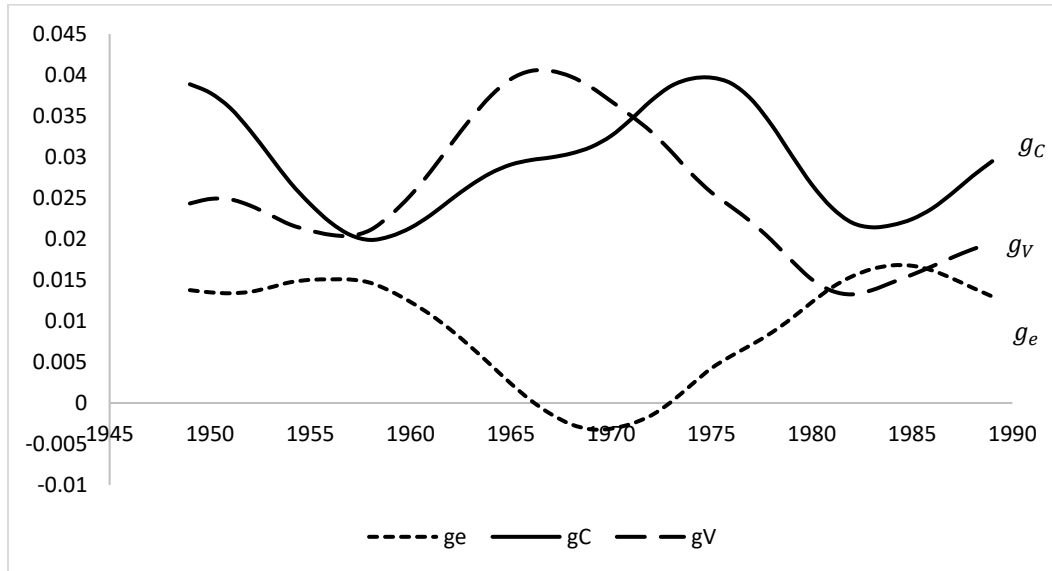
The two time series,  $(g_C - g_V)$  and  $g_e$ , do not fully coincide; however, they display common behavior as the mean of their differences is 0.0091 and their variance is 0.0017, while they are proved stationary by means of the Phillips-Perron and Augmented Dickey-Fuller unit root tests (Table 1). Consequently, as these differences are negligible, the parameters  $(g_C - g_V)$  and  $g_e$  can be considered as if they converge in the long run.

**Table 1:** Unit Root Tests for  $(g_C - g_V) - g_e$

	t-Statistic	Probability value
<b><i>Phillips-Perron Unit Root Test</i></b>		
No constant or trend	-5.3435	0.0006***
Constant, no trend	-5.3435	0.0001***
Constant and trend	-5.2608	0***
<b><i>Augmented Dickey-Fuller Unit Root Test</i></b>		
No constant or trend	-5.3435	0.0006***
Constant, no trend	-5.3435	0.0001***
Constant and trend	-5.2608	0***



The long-run cyclical behaviour of the three variables,  $g_C$ ,  $g_V$  and  $g_e$ , are presented in Figure 2. We filter out the short-run deviations by means of a Hondrick-Prescott filter.



**Figure 2:** The long-run cyclical behaviour of  $g_C$ ,  $g_V$  and  $g_e$ , USA, 1949-1989

The cyclical behaviour of  $g_C$ ,  $g_V$  and  $g_e$  presented in Figure 2 allow us to attempt a description of an economic cycle by tracing their dynamics by means of actual data and by utilizing Marx's theory of capital accumulation. During the expansion phase, the constant and variable capital increase, technological changes are introduced securing higher labour productivity, high RSV and relatively high mass of profits. The increasing  $g_V$ , however, eventually causes the growth of RSV to decline which in combination with a rising OCC eventually leads to a falling ROP. As the economy approaches the over-accumulation point, the  $g_C$  reaches its peak and the mass of profits stagnates (Tsoulfidis and Tsaliki 2019). The variable capital which has been already in a diminishing trend due to introduction of labour-saving techniques is further reduced, either by laying off workers or by keeping wages constant. At this moment, the RSV stagnates. After  $g_C$  reaches its peak, the economy sinks deeper into stagnation, capital accumulation is low and the IRAL rises. The cost of production (especially that associated with wages) is reduced, the discipline of the workers raises and more surplus-value is squeezed out of them; thus, the RSV starts its rising trend once again. However, the surplus-value produced in this phase either is not realized or is hoarded. The necessary condition for the new phase of accumulation to be in place

is the capital stock to devalue so that the ROP to start rising. A massive devaluation of capital stock takes place due to induced technological changes; the latter are introduced only if the variable capital is increased and sufficiently exploited. During the era of expansion, the IRAL contracts and variable capital increases as capitalists are in need to set in motion their newly invested capital and prepare the production process for its revolutionization. The change of the RSV reaches its peak, the ROP is at a maximum and a new phase of accumulation kicks off.<sup>8</sup>

Looking in Figure 2, it becomes apparent that capital accumulation only slows down and rarely or never stops; this observation implies that the OCC is always rising: in other words,  $g_c > 0$  always. The secular component of the ROP, which is related to the rising OCC caused by the relentless mechanization of the production process, gives rise to its long-run falling tendency. As Shaikh (2016) and Tsoulfidis and Tsaliki (2019) argued, the above constitutes the general condition for crises, in the sense that capitalism suffers inherently and regularly from the possibility of a breakdown. The actuality of the crisis, however, is related to its cyclical component, which is associated with the evolution of  $g_e$ . The passage of  $g_e$  from an upswing to a downswing pathway and *vice versa* signals on the one hand the moment of over-accumulation and the emergence of a crisis and on the other hand the appearance of recovery and a new phase of expansion. The coexistence of the general condition and the cause of crises in equation (5) allows for the construction of a structural theory of crisis within the premises of Marx's analysis of capital accumulation, in which the cyclical evolution of  $g_e$  is the counterpart to the long-run falling tendency of ROP.

## 5. Conclusions

The 'knife-edge' instability introduced by the Harod-Domar model remains an open issue in modern theory of economic growth. The neoclassical strand of economics, by altering the original assumptions of the model, removed the conditions for instability and, by replacing them by neoclassical assumptions paved the path for the attainment of a stable equilibrium; however, their

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<sup>8</sup> In Chatzarakis *et al.* (2022), there is a full description of the interaction of all the afore mentioned variables (investment, employment, technological change, devaluation, profitability) incorporated in five 'stylized facts' that shape the cyclical process of growth.

analysis excludes the cyclical nature of capitalist development while Harrod's 'ghost' still hangs around. The (post-)Keynesian school of thought attempted to unveil an inherent mechanism that would stabilize the economy in the long-run; however, they have not, as of now, concluded to a solid counter-paradigm to the neoclassical 'orthodoxy'. Some in this tradition emphasized on the formation of cyclical fluctuations around a given trend, utilizing the multiplier and accelerator principle; but as Kalecki and Pasinetti argued, such models are unable to provide the trend and the cycles at once. Until today, post-Keynesians failed to fully develop Robinson's ideas on the long-run tendencies of accumulation and Kaldor's 'stylized facts' and to present a growth model. In addition, the issue of 'animal spirits' is yet unresolved, despite the consensus that profitability drives these spirits in the long run.

A careful study of Marx's SER reveals that he had already constructed a balanced growth path long before the Harrod-Domar 'warranted' growth path. In addition, his Laws of capital accumulation and falling ROP together with the ideas by Tugan-Baranovskii's on disproportions and by Luxemburg's on the investment behaviour of capitalists had already posed the question on the stability of the accumulation process long before Harrod and Domar resort to 'animal spirits' as a plausible but insufficient explanation. What is more interesting though is that Marx's analysis provides a more fundamental reasoning behind this instability, rather than swaying around a simple, yet indeterminate psychological factor. In fact, the answer to 'knife-edge' problem is located in the fundamental contradictions of the capitalist mode of production as these are put in a nutshell by the evolution of the RSV. Our analysis reveals that the growth rates of constant and variable capital (also standing for the growth rates of the two Departments) do not coincide as the conditions for balanced growth would require, but differ by a specific factor – the dynamics of the RSV.

The RSV is a key variable in Marx's analysis of the capitalist mode of production which runs through the spheres of production, circulation and distribution and reflects the fundamental contradictions of the system; namely, the contradictions between productive forces (technological change, increase in labour productivity, *etc.*) and productive relations (property rights, income distribution, intensity of exploitation, class struggle, *etc.*). Hence, in Marx's analysis of capital accumulation, the Harrodian instability is crystalized not as a stochastic factor attributed to 'animal

spirits', but it is fully determined by the inherent nature of capitalism, which is the pursuit of maximum profit in any way and at any cost.

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