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Long-run relationship between inflation and growth in a New Keynesian framework*

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Abstract

This paper examines the steady-state growth effect of inflation in an endogenous growth model in which the Calvo-type nominal rigidity with endogenous contract duration and monetary friction via wage-payment-in-advance constraint are assumed. On the balanced-growth path in this model, the marginal growth effect of inflation is weakly negative or even positive at low inflation rates because the effect on average markup offsets the negative marginal growth effect through the monetary friction but the growth effect of inflation is negative and convex at higher inflation rates because the frequency of price adjustment approaches that of the flexible-price economy and the growth effect through the nominal rigidity is dominated by the growth effect through the monetary friction. With a plausible calibration of the structural parameters, this model generates a relationship between inflation and growth that is consistent with empirical evidence, especially in industrial countries.

Keywords: Inflation and growth; Sticky prices; Endogenous contract duration

JEL classification: E31; O42

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1 Introduction

Recent empirical studies have found that the relationship between inflation and growth is non-linear. The stylized facts is as follows. First, there is a threshold inflation rate above which the marginal effect of inflation on growth is negative and below which the one is nonsignificant or even positive. Second, above the threshold inflation rate, the relationship between inflation and growth is convex in the sence that the negative marginal effect is weaker as inflation is high.

On the other hand, most theoretical studies fail to generate this non-linear relationship. For example, in flexible-price monetary endogenous growth models with cash-in-advance constraint, the marginal growth effect of inflation is always negative as surveyed in Gillman and Kejak (2005). In monetary endogenous growth models with the prototypical Calvo-type nominal rigidity as in Funk and Kromen (2006) and Kuwahara and Sudo (2007), there is the threshold inflation rate but above it the relationship is concave.

In this paper we show that, with a plausible calibration of the structural parameters, a monetary endogenous growth model with the Calvo-type staggerred price setting with endogenous contract duration as in Levin and Yun (2007) can generate the non-linear relationship consistent with the empirical evidence for industrial countries, in the wide range of inflation. In this model, there is a threshold inflation rate below which the marginal effect of inflation on growth is weakly negative or even positive because, at low inflation rates, steady-state inflation affects average markup through the nominal rigidity, which offsets the negative marginal growth effect through the monetary friction. As inflation is high, nominal rigidity becomes weaker and the situation approaches that of flexible-price economy hence the marginal effect becomes negative and the inflation-growth relationship is convex. In our numerical result, the threshold inflation rate is about 0.1%, which is consistent with empirical evidence in Khan and Senhadji (2001) that the one is below 1% for five-year averaged data in industrial countries.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 shows the mechanisms and the numerical results of the growth effect in the flexible-price economy, in the prototypical Calvo-type sticky-price economy, and in the Calvo economy with endogenous contract duration. Section 4 provides concluding remarks.

2 The Model

The model considered in this paper is a simple two-capital endogneous growth model in which monetary friction via wage-payment-in-advance constraint of firms is introduced. Time is discrete. There are three types of agents in this economy. The representative household, the monopolistically competitive firms, and the monetary authority. For simplicity, fiscal policy is ignored.

The representative household maximizes the following discounted sum of utility[1]:

$$\sum_{t=0}^{\infty} \beta^t \{\log C_t + \psi \log[(1-n_t)H_t]\}, \tag{1}$$

where C denotes aggregate consumption, $n \in (0,1)$ denotes hour worked, H denotes human capital stock, and $\psi > 0$ and $\beta \in (0,1)$ is exogenous parameters. The intertemporal budget constraint is as follows:

$$\begin{aligned} \frac{B_t}{P_t} + C_t + K_{t+1} - (1-\delta_k)K_t + H_{t+1} - (1-\delta_H)H_t \\ = \frac{i_{t-1}B_{t-1}}{P_t} + w_t n_t H_t + r_t^K K_t + \Phi_t, \end{aligned} \tag{2}$$

where B denotes the quantity of nominal financial asset which earns the gross nominal interest rate i, K denotes physical capital stock, π denotes gross rate of inflation, w denotes real wage rate, r^K denotes real gross rate of return on physical capital, Φ denotes real dividend income from firms they own, δ_K is an exogenous parameter representing depreciation rate of physical capital, and δ_H is an exogenous parameter representing depreciation rate of human capital.

Each individual firm j ($\in [0,1]$) monopolistically supplies the variety j, using a Cobb-Douglas production technology:

$$Y_t(j) = AK_t(j)^\alpha Z_t(j)^{1-\alpha}, \tag{3}$$

where A and α denote exogenous parameters representing aggregate productivity and capital share respectively, and where $K(j)$ and $Z(j)$ denote the demand for physical capital and for effective labor respectively, each of which must satisfy the resource constraints, $\int_0^1 K_t(j)di = K_t$ and $\int_0^1 Z_t(j)di = n_t H_t$. It is assumed that workers must be paid their wage-bill by cash in advance of production. Hence firm j borrows its nominal wage payment, $P_t w_t Z_t(j)$, from a financial intermediary at the beginning of period t. Repayment occurs at the end of period t at the gross nominal interest rate i_t. Consequently, the total real production cost of firm j is $r_t^K K_t(j) + i_t w_t Z_t(j)$. From the first-order conditions of cost minimization with respect to $K_t(j)$ and $Z_t(j)$, it is holds that $r_t^K = \frac{\alpha A\left(\frac{K_t}{n_t H_t}\right)^{\alpha-1}}{\mu_t}$ and $w_t = \frac{(1-\alpha)A\left(\frac{K_t}{n_t H_t}\right)^{\alpha}}{i_t \mu_t}$, where μ denotes average markup, which is defined as reciprocal of the real marginal cost (the Lagrange multiplier with respect to (3)).

[1]To keep the model tractable, we assume log utility and quality time of leisure. Our final result is robust even if the instantaneous utility function is assumed to be CRRA form, $\frac{C_t^{1-\sigma}[(1-n_t)H_t]^{\psi(1-\sigma)}}{1-\sigma}$ or to depend on raw time of leisure, $\frac{C_t^{1-\sigma}(1-n_t)^{\psi(1-\sigma)}}{1-\sigma}$, though its mechanism become more complicated.

The aggregate demand index Y is assembled using the Dixit-Stiglitz aggregator, $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}$, where $\theta > 1$ denotes the parameter representing the elasticity of substitution. Hence firm j faces a downward-sloping demand function:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t, \tag{4}$$

where $P(j)$ denotes the price of variety j and the aggregate price level P is defined as $P_t = \left(\int_0^1 P_t(j)^{1-\theta} di\right)^{\frac{1}{1-\theta}}$. Each firm maximizes its profit by optimally setting its price subject to (4). Its details will be described later.

At the beginning of the period t, financial intermediaries have nominal money balances $P_{t-1}M_{t-1}$ and receive a monetary transfer $P_tM_t - P_{t-1}M_{t-1}$ from the monetary authority, where M denotes real money balances, and lend all their money to firms for their wage payments $\int_0^1 P_t w_t Z_t(j) dj$. Hence loan market clearing condition is $M_t = w_t n_t H_t$.

The aggregate demand consists of the aggregate consumption, the aggregate physical capital investment, the aggregate human capital investment, and the aggregate menu cost,[2] hence,

$$Y_t = C_t + K_{t+1} - (1-\delta_K)K_t + H_{t+1} - (1-\delta_H)H_t + (1-\xi)\Omega_t. \tag{5}$$

The monetary authority sets inflation rate $\{\pi_t\}$.[3]

3 Growth Effect of Inflation

Given μ and i, the steady-state growth rate of output γ is determined by:

$$\gamma = \beta r, \qquad \text{(Euler equation)} \tag{6}$$

$$r = \frac{\alpha A \left(\frac{K}{nH}\right)^{\alpha-1}}{\mu} + 1 - \delta_K, \qquad \text{(No-arbitrage condition)} \tag{7}$$

$$r = \frac{(1-\alpha) A \left(\frac{K}{nH}\right)^{\alpha}}{i\mu} + 1 - \delta_H. \qquad \text{(No-arbitrage condition)} \tag{8}$$

Equation (6) implies that households' saving behavior determines growth rate of output, depending only on real rate of interest, r. Equations (7) and (8) imply that arbitrage

[2]The final term of RHS in (5) denotes the aggregate menu cost. The datail is described later.

[3]This assumption is equivalent to assuming that the monetary authority sets nominal interest rate $\{i_t\}$ or money growth rate $\{\eta_t\} \equiv \{\frac{P_tM_t}{P_{t-1}M_{t-1}}\}$. Stability of the equilibrium depends on the monetary policy rule, but we ignore the detail of the rule because in this paper we focus on the steady state.

between physical capital, human capital, and financial assets determines physical-capital-to-effective-labor ratio and real rate of interest, for given μ and i.[4] Hence inflation has growth effect if inflation affects real rate of interest through the change of nominal rate of interest and/or average markup. In the following subsections, we consider the flexible-price economy in order to see the growth effect through the change of nominal rate of interest, and the sticky-price economy in order to see the effect through the change of average markup.

3.1 Flexible-price Economy

Let us consider the flexible-price economy, in which inflation affects real rate of interest through only the change of nominal interest rate because steady-state average markup is constant, $\mu = \frac{\theta}{\theta - 1}$. The reason why nominal rate of interest affects real rate of interest is that there exists a monetary friction by wage-payment-in-advance assumption. By this reason, we refer to this growth effect of inflation as *monetary-friction effect*. The relationship between inflation rate, real and nominal rate of interest is described by the Fisher equation:

$$i = r\pi. \qquad \text{(Fisher equation)} \tag{9}$$

Substituting (9) into (8), it is holds that:

$$r = \frac{1}{2}\left(1 - \delta_H + \sqrt{(1-\delta_H)^2 + \frac{4}{\pi\mu}(1-\alpha)A\left(\frac{K}{nH}\right)^{\alpha}}\right). \tag{10}$$

Given π, equations (7) and (10) determine real rate of interest. Figure 1 illustrates the determination of real interest rate. When π rises, (10) shifts downward and r falls. Therefore, the marginal monetary-friction effect of inflation on real interest rate and growth rate is necesserily negative as in standard monetary endogenous growth models.[5]

3.2 Sticky-price economy with exogenous contract duration

In this subsection we consider the sticky-price economy with exogenous contract duration (the prototypical Calvo model), in which each firm can reset its price with probability $1 - \xi$ and in which ξ is constant. In this economy, inflation has an effect on real interest

[4] Note that real rate of interest depends only on $\frac{K}{nH}$ because we assume quality time for utility from leisure. If it is assumed that utility from leisure depends only on raw time, $1 - n_t$, then the determination of real interest rate becomes more complicated. However, our main results is numerically robust.

[5] This monetary friction works similarly as cash-in-advance constraint in standard monetary endogenous growth models. In our model, cash-in-advance constraint of households does not affects growth because cash-in-advance constraint affects only hour worked, n. If we assume that utility from leisure depends only on raw time, cash-in-advance constraint has the growth effect as in standard monetary endogenous growth models.

rate and growth since the existence of nominal rigidity causes the effect of inflation on average markup, in addition to the monetary-friction effect. We refer to the effect on average markup as *sticky-price effect*. As illustrated in figure 2, a rise of μ makes r fall because (7) and (10) shift downward. Therefore, a rise (fall) of average markup gives rise to a fall (rise) of growth rate of output.

In this economy, for given π, the economywide average markup μ is determined by the optimal pricing behavior of firms and the price level equation as follows:[6]

$$\tilde{\mu} = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi\pi^{\theta-1}}{1 - \beta\xi\pi^{\theta}}, \qquad \text{(Optimal pricing behavior)} \qquad (11)$$

$$\mu^{1-\theta} = \xi \left(\frac{\mu}{\pi}\right)^{1-\theta} + (1-\xi)\tilde{\mu}^{1-\theta}, \qquad \text{(Price level equation)} \qquad (12)$$

where $\tilde{\mu} \equiv \frac{\tilde{P}}{P}\mu$ denotes the optimal markup set by firms which can reset their prices. From these equations, average markup is described as:

$$\mu = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi\pi^{\theta-1}}{1 - \beta\xi\pi^{\theta}} \left(\frac{1 - \xi\pi^{\theta-1}}{1 - \xi}\right)^{\frac{1}{\theta-1}}, \qquad (13)$$

hence average markup depends only on inflation rate.[7] As shown in Panel C of Figure 3, The relationship between inflation and average markup is U-shaped.[8] The intuition of the U-shaped average markup is as follows. Equation (12) implies that the economywide average markup μ depends on the average markup on firms which *cannot* reset their nominal prices μ/π and the markup of firms which *can* reset their nominal prices $\tilde{\mu}$. On the one hand, μ/π is decreasing in π for given μ. It is because the average relative price on firms which cannot reset their nominal prices falls at inflation rate but real marginal cost is constant on balanced-growth path. On the other hand, from (11), we see that $\tilde{\mu}$ is increasing in π. The reason is that as inflation rate is high, firms which can reset their nominal prices set their markup to be higher because they are concerned by the possibility that their markup would keep declining in the future when they cannot reset their prices. By these two opposing effects, inflation has an U-shaped impact on economywide average markup.

Since a rise of average markup brings a fall of real interest rate and growth, the marginal sticky-price effect on growth is positive at low inflation rates and negative at high inflation rates. Figure 3 shows the numerical result of this relationship for various values of ξ.[9] We can see that the U-shaped average markup becomes more flat as ξ

[6] Derivation of (11) and (12) is in appendix A.

[7] Our assumption of log utility simplifies the analysis. If instantaneous utility has a more general form, $\frac{C_t^{1-\sigma}[(1-n_t)H_t]^{\psi(1-\sigma)}}{1-\sigma}$, then average markup depends not only on inflation but growth rate of output hence the mechanism becomes more complicated. However, even in the case, our results is robust.

[8] Under the assumption of log utility, we can prove this U-shaped relationship analytically.

[9] The values of the structural parameters is in appendix B. The Matlab programs for our numerical analysis are on the author's website. (http://sites.google.com/site/hirokiarato/)

decreases and that sticky-price effect disappears when $\xi = 0$. It is because the decrease of ξ means that nominal rigidity become weaker and the situation approaches that of the flexible-price economy. When ξ is sufficiently high, there is a threshold inflation rate below which the marginal growth effect is positive because the sticky-price effect offsets the monetary-friction effect. However, this relationship is concave in the whole range of inflation, which is inconsistent with empirical evidence at high inflation rates.[10]

3.3 Sticky-price economy with endogeneous contract duration

Finally, we consider the Calvo model with endogenous contract duration as in Levin and Yun (2007). In this model, each firm is allowed to choose not only its price but also its average contract duration (or the probability of changing its price). For simplicity, we assume that the economy is on a balanced-growth path.[11] In each period, firm j can reset the nominal price of their variety with probability $1 - \xi(j)$. Moreover, firms must pay fixed menu cost $\Omega_t \equiv \omega Y_t$ when they can change their prices. Given these assumptions, each firm maximizes the expected present-value of its current and future profits subject to the demand function (4), choosing its price and the probability of changing its price. Following Levin and Yun (2007), we restrict our analysis to a symmetric Nash equilibrium in which all firms choose the same probability of changing prices hence $\xi(j) = \xi$ for all j. In this economy, ξ and μ are determined according to (11), (12), and the optimal condition with respect to contract duration of firms which is described as:

$$\frac{\tilde{\mu}^{1-\theta}(\pi^{\theta-1} - 1)}{(1 - \beta\xi\pi^{\theta-1})^2} = \frac{\tilde{\mu}^{-\theta}(\pi^{\theta} - 1)}{(1 - \beta\xi\pi^{\theta})^2} - \omega\mu^{1-\theta}, \qquad \text{(Optimal contract duration)} \tag{14}$$

when the internal solution exists.[12] By allowing firms to choose the frequency of changing prices, firms chnage their price more frequently as inflation deviates from zero, as shown in Panel D of figure 4. The reason of this relationship is the existence of fixed menu cost. If inflation is near zero, the opportunity cost of unchanging their prices is small because the deviation of price-unchanging firms' markup from optimal one is small. Hence firms choose low frequency of changing price, concerned by fixed menu cost. As inflation deviates from zero, the opportunity cost becomes larger hence firms choose higher frequency even if they must pay the menu cost more frequently. If inflation is extremely high, all firms change their prices in every period hence the situation is same as the flexible-price economy.

Varying the frequency of changing prices makes the sticky-price effect more complex. In addition to the U-shaped markup effect in the previous subsection, there is the effect

[10] Moreover, this model can analyze the growth effect only at moderate inflation. This model has an equilibrium only if $\beta\xi\pi^{\theta} < 1$ because $\lim_{\pi \to (\frac{1}{\beta\xi})^{\frac{1}{\theta}}} \tilde{\mu} = \infty$.

[11] For the firm's behavior in stochastic economy, see the working paper version of Levin and Yun (2007).

[12] Derivation of (14) is in appendix A.

that this U-shaped relationship become flatter as inflation deviates from zero. Figure 4 and 5 indicate the numerical result, which is consistent with empirical evidence. First, there is a threshold inflation rate about 0.1% in year at which the marginal growth effect changes from positive to negative. Readers may have a question that at severe deflation (below minus 0.1% in year) the marginal effect is negative. We think that the reason of the positive or nonsignificant marginal effect of growth in empirical studies is rare observations of severe deflation. Since most of the observations below the threshold inflation rate are distributed around zero inflation, the regression analysis would show an upward-sloping or nonsignificant relationship between inflation and growth. Second, above the threshold inflation rate, the relationship between inflation and growth is decreasing and convex because sticky-price effect is weaker as inflation is high and the situation approaches the flexible-economy in which only monetary-friction effect affects growth. As the result, this economy can genarate the plausible inflation-growth relationship in wider range of inflation than the sticky-price economy with exogenous contract duration. Third, our model can be calibrated more accurately than endogenous growth models with imperfect information in credit market in Bose (2002) and Hung (2007), which also show the existence of threshold inflation rate. With our calibration of the structural parameters, the threshold inflation rate is 0.1%. In the empirical study in Khan and Senhadji (2001), the threshold inflation rate is below 1% in industrial countries and 11% in developed countries for five-year averaged data. With this empirical evidence, we conclude that our model can genarate the plausible threshold inflation rate in industrial countries.

4 Concluding Remarks

In this paper we show that the monetary endogenous growth model with the Calvo-type nominal rigidity with endogenous contract duration can generate the plausible relationship between inflation and growth, especially in industrial countries. However, there are some open questions in our analysis. First, our model suggests the existence of a lower alternative threshold inflation rate below which the marginal growth effect become negative. This hypothesis is potentially testble. If we had more observations of deflation, we could test the existence of the alternative threshold inflation rate by dividing the low-inflation observations into two subsamples. Second, our model can not replicate the plausible threshold inflation rate in developing countries, that is shown 11% for five-year averaged data in Khan and Senhadji (2001). This result suggests that the analysis for developing countries needs some alternative assumptions. For example, imperfect information in credit market as in Bose (2002) and Hung (2007). However, The measurement of the degree of imperfect information is difficult. In order to analyze the growth effect of inflation in developing countries quantitatively, we must obtain more empirical evidence about market structure and about imperfect information in developing countries.

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Appendix

A. Derivation of (11), (12), and (14)

In the prototypical Calvo model, firms which can reset their price at time t, choose their steady-state relative price $\tilde{p} \equiv \tilde{P}_t/P_t$, which is constant, to maximize the discounted sum of its expected profit until they reset their prices,

$$\begin{aligned}\Phi_t(\tilde{p},\xi) &= \sum_{s=0}^{\infty}\left(\frac{\xi}{r}\right)^s\left[\left(\frac{\tilde{p}}{\pi^s}\right)^{1-\theta}Y_{t+s} - \left(\frac{\tilde{p}}{\pi^s}\right)^{-\theta}\frac{Y_{t+s}}{\mu}\right] \\ &= Y_t\sum_{s=0}^{\infty}\left(\frac{\xi\gamma}{r}\right)^s\left[\left(\frac{\tilde{p}}{\pi^s}\right)^{1-\theta} - \left(\frac{\tilde{p}}{\pi^s}\right)^{-\theta}\frac{1}{\mu}\right] \end{aligned} \tag{15}$$

on a balanced-growth path. The first-order condition is described as:

$$\sum_{s=0}^{\infty}\left(\frac{\xi\gamma}{r}\right)^s\left[\pi^{s\theta} - \frac{\theta-1}{\theta}\left(\mu\tilde{p}\right)\pi^{s(\theta-1)}\right] = 0, \tag{16}$$

Using the Fisher equation $\gamma = \beta r$ and the definition of optimal markup $\tilde{\mu} = \mu \tilde{p}$,

$$\sum_{s=0}^{\infty} (\beta\xi)^s \left(\pi^{s\theta} - \frac{\theta - 1}{\theta} \tilde{\mu} \pi^{s(\theta-1)} \right) = 0. \tag{17}$$

After some calculation, we obtain (11).

See the aggregate price level equation,

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{18}$$

When the probability that each firm can reset their prices is $1 - \xi$, (18) can be rewritten as:

$$P_t^{1-\theta} = \xi P_{t-1}^{1-\theta} + (1 - \xi) \tilde{P}_t^{1-\theta}. \tag{19}$$

Dividing $(\frac{P_t}{\mu})^{1-\theta}$ into this equation, we can see that (12) holds on balanced growth path.

Next consider the Calvo model with endogenous contract duration. Assume that firm j can change its price at time t. If it sets their relative price to be $\tilde{p}(j)$, then the discounted sum of expected profit until it resets its price is $\Phi_t(\tilde{p}(j), \xi(j)) - \omega Y_t$. Therefore, its total discounted sum of profit $V_t(\tilde{p}(j), \xi(j))$ is:

$$\begin{aligned} V_t(\tilde{p}(j), \xi(j)) = (\Phi_t(\tilde{p}(j), \xi(j)) - \omega Y_t) + \frac{1 - \xi(j)}{r} (\Phi_{t+1}(\tilde{p}(j), \xi(j)) - \omega Y_{t+1}) \\ + \frac{1 - \xi(j)}{r^2} (\Phi_{t+2}(\tilde{p}(j), \xi(j)) - \omega Y_{t+2}) + \cdots \end{aligned} \tag{20}$$

Hence, $V_t(\tilde{p}(j), \xi(j))$ can be rewritten as:

$$\begin{aligned} V_t(\tilde{p}(j), \xi(j)) &= Y_t \phi(\tilde{p}(j), \xi(j)) \left[1 + (1 - \xi(j)) \frac{\gamma}{r} + (1 - \xi(j)) \left(\frac{\gamma}{r} \right)^2 + \cdots \right] \\ &= Y_t \phi(\tilde{p}(j), \xi(j)) \left[1 + (1 - \xi(j))\beta + (1 - \xi(j))\beta^2 + \cdots \right] \\ &= Y_t \phi(\tilde{p}(j), \xi(j)) \frac{1 - \beta\xi(j)}{1 - \beta}, \end{aligned} \tag{21}$$

where,

$$\begin{aligned} \phi(\tilde{p}(j), \xi(j)) &\equiv \frac{\Phi_t(\tilde{p}(j), \xi(j)) - \omega Y_t}{Y_t} \\ &= \left\{ \sum_{s=0}^{\infty} \left(\frac{\xi(j)\gamma}{r} \right)^s \left[\left(\frac{\tilde{p}(j)}{\pi^s} \right)^{1-\theta} - \left(\frac{\tilde{p}(j)}{\pi^s} \right)^{-\theta} \frac{1}{\mu} \right] \right\} - \omega, \end{aligned} \tag{22}$$

which is constant on balanced-growth path.

The profit maximization problem of firm j has two steps. First, given $\xi(j)$, firm j chooses its optimal relative price $\tilde{p}^*(\xi(j))$. Hence,

$$\begin{aligned} \tilde{p}^*(\xi(j)) &= \arg\max_{\tilde{p}(j)} V_t(\tilde{p}(j), \xi(j)) \\ &= \arg\max_{\tilde{p}(j)} \phi(\tilde{p}(j), \xi(j)). \end{aligned} \tag{23}$$

We can solve this problem as in the prototypical Calvo model and obtain:

$$\tilde{\mu}(\xi(j)) = \frac{\theta}{\theta - 1} \frac{1 - \beta\xi(j)\pi^{\theta-1}}{1 - \beta\xi(j)\pi^{\theta}}. \tag{24}$$

Second, given ξ, firm j chooses its optimal frequency of changing price $1 - \xi^*(j)$. Hence,

$$\xi^*(j) = \arg\max_{\xi(j)} \eta_t(\xi(j)), \tag{25}$$

where,

$$\eta_t(\xi(j)) \equiv V_t(\tilde{p}^*(\xi(j)), \xi(j)). \tag{26}$$

By envelop theorem, the first-order condition is

$$\frac{d\eta_t}{d\xi(j)} = \frac{\partial V_t}{\partial \xi(j)} = 0. \tag{27}$$

By (21), the first-order condition can be written as:

$$\frac{\partial}{\partial \xi(j)} \left[\phi(\tilde{p}^*, \xi(j)) \frac{1 - \beta\xi(j)}{1 - \beta} \right] = 0. \tag{28}$$

Some calculations arrange it as:

$$\frac{\tilde{\mu}^{1-\theta}(\pi^{\theta-1} - 1)}{(1 - \beta\xi^*(j)\pi^{\theta-1})^2} = \frac{\tilde{\mu}^{-\theta}(\pi^{\theta} - 1)}{(1 - \beta\xi^*(j)\pi^{\theta})^2} - \omega\mu^{1-\theta}. \tag{29}$$

In a symmetric Nash equilibrium, $\xi^*(j) = \xi$, for all j. Therefore, we obtain (11) and (14) from (24) and (29), respectively.

B. Calibration

In order to calibrate our model, in addtion to the equilibrium conditions which have already been derived, that is, the Euler equation (6), the no-arbitrage conditions (7) and (8), the Fisher equation (9), the optimal pricing equation (11), the price level equation

(12), and the optimal contract duration equation (14), the other conditions are needed. First, the optimal labor supply equation,

$$\frac{\psi}{1-n}\frac{C}{H} = \frac{(1-\alpha)A(\frac{K}{nH})^{\alpha}}{i\mu}. \tag{30}$$

Second, the aggregate good market clearing condition,

$$\frac{A(\frac{K}{H})^{\alpha}n^{1-\alpha}}{s} = \frac{\frac{C}{H} + (\gamma - 1 + \delta_K)\frac{K}{H} + (\gamma - 1 + \delta_H)}{1-(1-\xi)\omega}, \tag{31}$$

where $s \equiv \int_0^1 (\frac{P_t(j)}{P_t})^{-\theta} dj \geq 1$ denotes the degree of relative price dispersion, which is described as:

$$s = (1-\xi)^{\frac{1}{1-\theta}} \frac{(1-\xi\pi^{\theta-1})^{\frac{\theta}{\theta-1}}}{1-\xi\pi^{\theta}}. \tag{32}$$

Time unit is assumed to be quarter. α, δ_H, δ_K, ω, θ are set to be the values used in growth and business cycle literature. A, β, and ψ are set such that the annual real interest rate is 3% and that the Frisch elasticity of labor supply is unity (hence $n = 0.5$) at the steady state with $\pi = 1.042^{1/4}$ and $\gamma = 1.0045$. The values of the structural parameters are shown in Table 1.

Table 1: Structural parameters

A	α	β	δ_H	δ_K	ω	ψ	θ
0.0445	0.36	$1.0045/1.03^{1/4}$	0.005	0.025	0.029	807.4	4.33

Figure 1: Monetary-friction effect (when π increases)

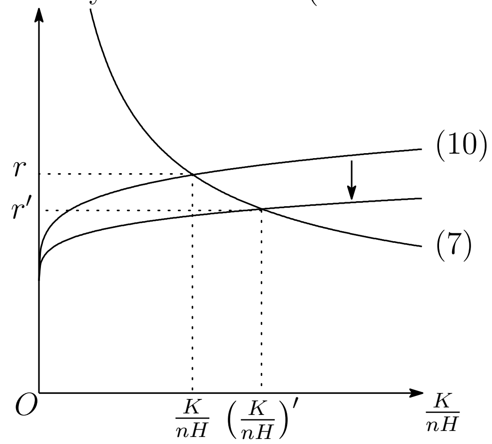


Figure 2: Sticky-price effect (when μ increases)

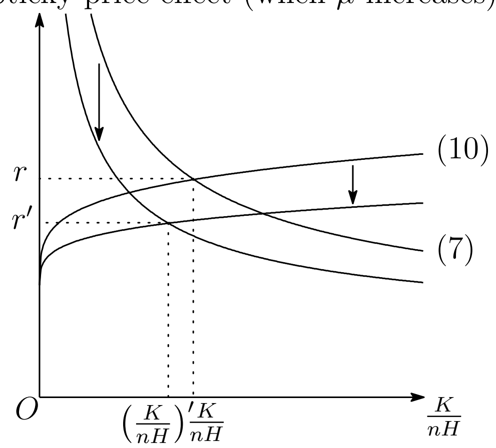
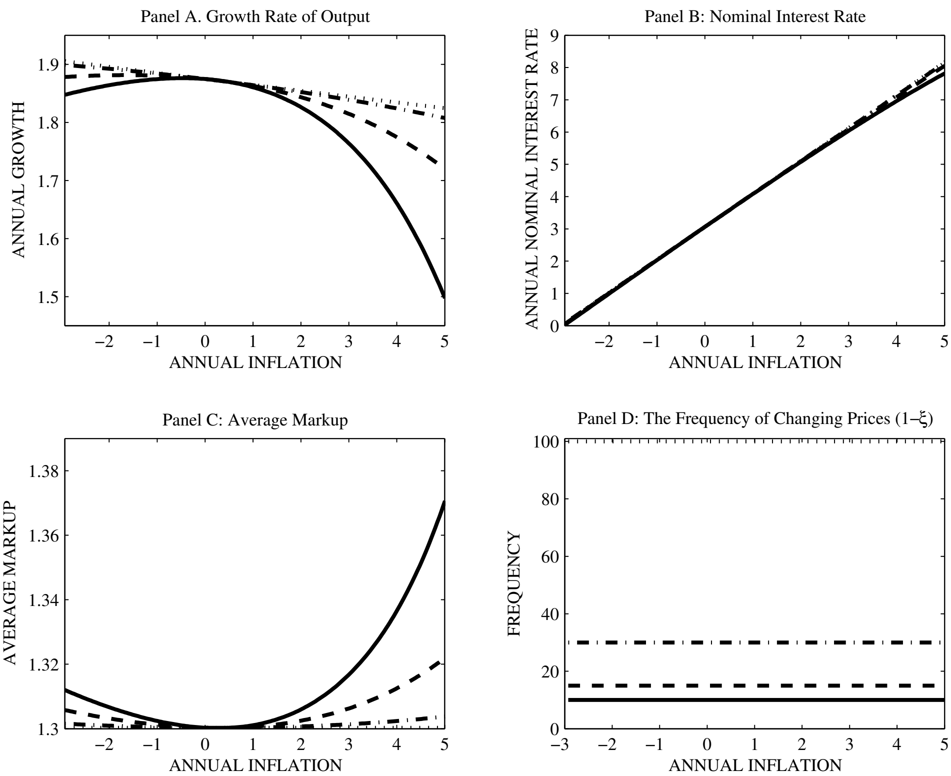


Figure 3: Effects of inflation in the exogenous contract duration model



Note: Solid line when $\xi = 0.9$, broken line when $\xi = 0.85$, dash–dotted line when $\xi = 0.7$, dotted line when $\xi = 0$ (flexible-price economy).

Figure 4: Effects of inflation in the endogenous contract duration model

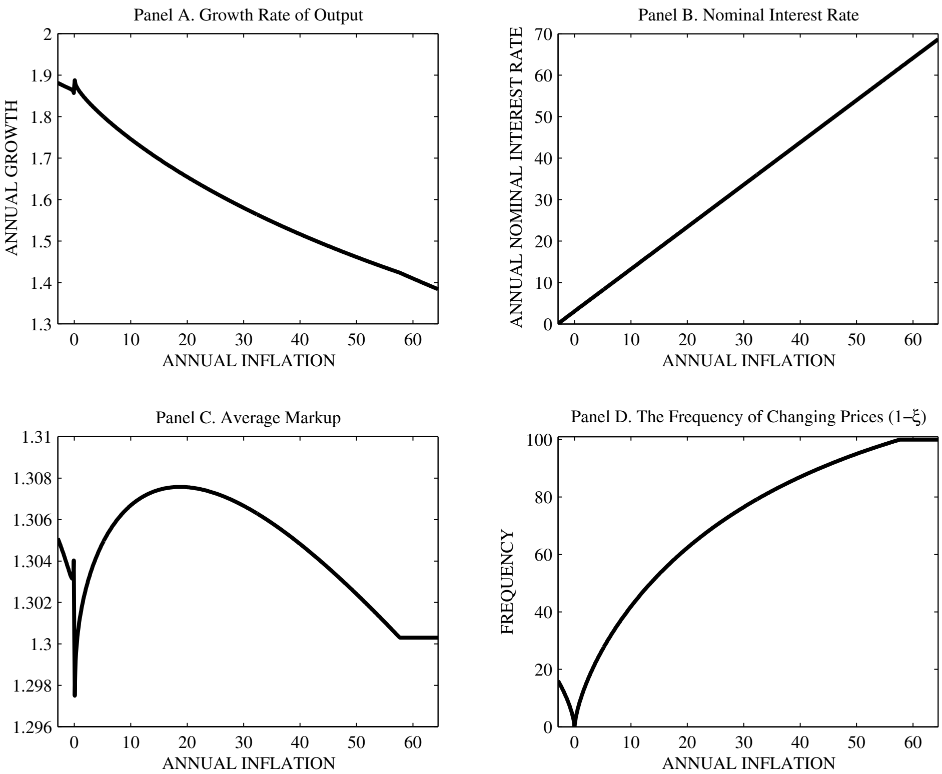


Figure 5: Effects of inflation in the endogenous contract duration model (around zero inflation)