# Disposition Effect and its outcome on endogenous price fluctuations

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**Abstract**

We develop a financial market model where a group of traders is affected by Disposition Effect, namely they are reluctant to realize losses. In particular, we present a set of stylized facts of financial markets (fat tails, volatility clustering, etc...) that can also be caused by the DE when the trading behaviour of agents are consistent with the findings of Ben-David and Hirshleifer (2012). In order to do that, we show that the version of the model where a class of agents is endowed with a high degree of Disposition Effect, permits to obtain simulated time series whose features are closer to those of real financial market with respect to the version of the model where traders are not affected by it. This happens both for the deterministic version and the stochastic one.

**Keywords**: Disposition Effect, Behavioural finance, Heterogeneous agents, Financial Markets.

**JEL classification**: D84, G12, C62.

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# 1 Introduction

Over the last years behavioral finance has focused to the design and the collection of systematic biases exhibited by financial actors to understand how these phenomena may impact on trading performance (for an exhaustive review, see Shleifer, 2000; Barberis and Thaler, 2003; Campbell, 2006). Several trading patterns, such as the January effect (Ritter, 1988) or the weekend effect (Obsorne, 1962; Lakonishok and Maberly, 1990), to cite some, are worthy of attention. However, one of the most common is the tendency to prefer selling a winning title with respect to a losing one. This well established behaviour of traders is known as *Disposition Effect* (DE henceforth) from the pioneering work of Shefrin and Statman (1985).

The cause of this regularity has been often identified in the prospect theory preferences of investors (Camerer 2000, Henderson 2012, Li and Yang 2013 and many others): as demonstrated by Kahneman and Tversky (1979), people are more risk averse in gain domain and more risk seeking in loss domain. However, some criticism and alternative explanations are recently emerging. Kaustia (2010) uses data from the Finnish Central Securities Depository to show that the propensity to sell an asset is basically constant on losses and increasing or constant on gains. Meng and Weng (2018) put into question the existence of DE if the initial wealth is used as reference point. Barberis and Huang (2009) compare preferences defined over annual gains and losses to preferences defined over realized gains and losses, finding that the realized gain/loss model better predicts the DE. Hens and Vlcek (2011) show that investors who sell winning stocks and hold losing assets would not have invested in stocks in the first place. That is, the standard prospect theory argument is sound ex-post, assuming that the investment occurred, but not ex-ante, requiring also that the investment has to be made in the first place. Lehenkari (2012) using data from the finnish stock market shows that the explanation of the DE referring the escalation of commitment seems to be more consistent with the data than the explanation based on prospect theory preferences.

Focusing on the consequences of this trading regularity, Grinblatt and Han (2005) showed how DE can be both at the origin of the persisting spread between a stock's fundamental value and its market price, and the tendency for rising asset prices to rise furtherly, known as *Momentum Effect*. Other theoretical models have been developed considering investors biased with DE, such as Barberis and Xiong (2011), Hens and Vleck (2011), Ingersoll and Jin (2013) and Polach and Kukacka (2019). Furthermore, Rau (2014) analyses gender differences in DE and loss aversion in an experimental framework, findind that female investors sell a higher amount of stocks, showing a significant degree of DE. This bias has been repeatedly analysed in the behaviorual literature also using an experimental approach (for instance, Weber and Camerer 1998; Da Costa Jr et al., 2013; Frydman and Rangel, 2014; Talpsepp et al., 2014) and in an empirical framework (Odean 1998; Weber and Camerer 1998; Dhar and Zhu 2006 and Jin and Scherbina, 2011 that deepened the relationship between DE and taxes, and Firth 2015, among the others).

Since this article tries to identify some other stylized facts of financial markets that can be better explained by considering investors affected by DE[1], our work properly belongs to this literature. We develop a simple model of asset market where a group of traders presents a trading behaviour consistent with the findings of Ben-David and Hirshleifer (2012) which include the DE. We further demonstrate how this bias may facilitate the occurrence of price fluctuations and boom-bust dynamics, even starting from a stable scenario. Our work also contributes to the literature on heterogeneous, boundedly rational, interacting traders, surveyed in Hommes (2013).

Moreover, given the empirical evidence highlighting the presence of a set of stylized empirical facts emerging from the statistical analysis of price variations in various types of assets in financial markets (Cont, 2001), we demonstrate that our agent-based model is capable of reproducing some of these regularities, such as absence of autocorrelation of returns, heavy tails, inferred volatility and volatility clustering. In particular, along the line of Pruna et al. (2020), we test how the presence of different degrees of DE in our simulated time series allows to simulate financial time series whose feature are closer to observed properties of real-financial markets data (for a complete review of the literature on agent-based modeling of financial markets, see Chen et al. 2012). By including a stochastic version of our model, we show that if the asset is hit by purely Gaussian uncorrelated shocks, fat-tail distributed time series arise via the endogenous transmission mechanisms embodied in the system.

The remainder of the paper is organised as follows. In the next Section, we present our a simple discrete-time agent-based model. Section 3 introduces some analytical and numerical results. Section 4, by focusing on the stochastic version of the model, discuss the statistical properties of our simulated time series and the empirical validation of the model. We conclude with some final considerations.

## 2 The base model

We consider an asset market where a market maker adjusts the stock log-price $P_t$ following the rule:

$$P_{t+1} = P_t + \alpha D_t, \tag{1}$$

where $\alpha \geq 0$ is the reactivity of the market maker to the total excess demand $D_t$, obtained by summing up the excess demands of all the traders.

We consider a group of fundamentalists and a group of chartists operating and interacting in this market.

Fundamentalists buy the asset when its current price is lower than the (exogenously given) log-fundamental value ($F$) and sell it when the price is higher

[1] Frazzini (2006) and Statman et al. (2006) are two of the most relevant contributions in this sense. The former finds a connection between DE and post-earning announcement drifts, while the latters explore the consequences of DE for trading volumes.

than it. We formalize their excess demand as folows:

$$D^F = f(F - P_t)^3, \tag{2}$$

where $f > 0$ is their speed of adjustment. The cubic function captures an intuition that dates back at least from Day and Huang (1990), who argue that fundamentalists trade increasingly aggressively as the market's mispricing increases for two main reasons: a first one concerns the increasing convinction of fundamentalists that a fundamental price correction is about to set as the misalignment increases, while the second one is related with the increasing gain potential of fundamental analysis.

Chartists behave at the opposite. They buy the asset when it is overvalued ($P_t > F$) and they sell it when it is undervalued ($P_t < F$), betting on the persistence, at least in the short run, of the current scenario. Typically their excess demand ($D^C$) is the following:

$$D^C = c_t(P_t - F), \tag{3}$$

where $c_t$ is a time-varying variable measuring the reactivity of chartists. We introduce here the findings of Ben-David and Hirshleifer (2012)[2], who empirically discovered that the function giving the probability of buying or selling an asset for any amount of profit has a V-shape. In particular it has an asymmetric V-shape. If we consider as profit the difference between the current asset price and the price at which the asset has been bought, then the higher is this difference in absolute value the larger is the probability of buying/selling the asset. Considering selling decisions, the right branch of the function (gain domain) is steeper than the left one (loss domain). This asymmetry is clearly related with the DE. The opposite is true when buying decisions are considered (Figure 1).

In our work we use the reactivity $c$ as a proxy of the probability of selling/buying the asset. So, instead of considering it an exogenous parameter as it usually happens in the literature, we consider it a time-varying variable that at each time step changes value in a way that is consistent with the findings of Ben-David and Hirshleifer (2012). Following Grinblatt and Han (2005) we define the traders' profit at the aggregate level as the difference between the current asset price and a weighted average of the past prices ($\widetilde{P}$), forming the reference price. Summarizing, we model the reactivity of chartists at time time $t$ ($c_t$) as follows:

$$c_t = c(P_t, \widetilde{P}_t) = \begin{cases} \widehat{c} + s_g(P_t - \widetilde{P}_t) & \text{if} \quad P_t < F \ \cup \ P_t \geq \widetilde{P}_t \\ \widehat{c} - s_l(P_t - \widetilde{P}_t) & \text{if} \quad P_t < F \ \cup \ P_t < \widetilde{P}_t \\ \widehat{c} + b_g(P_t - \widetilde{P}_t) & \text{if} \quad P_t \geq F \ \cup \ P_t \geq \widetilde{P}_t \\ \widehat{c} - b_l(P_t - \widetilde{P}_t) & \text{if} \quad P_t \geq F \ \cup \ P_t < \widetilde{P}_t \end{cases}, \tag{4}$$

[2] In our work we relate the disposition effect only to the behavior of chartists, who typically are assumed to be the kind of traders more inclined to be victim of cognitive biases and to use heuristics and rules of thumbs (see for instance Kaizoji et al., 2015).

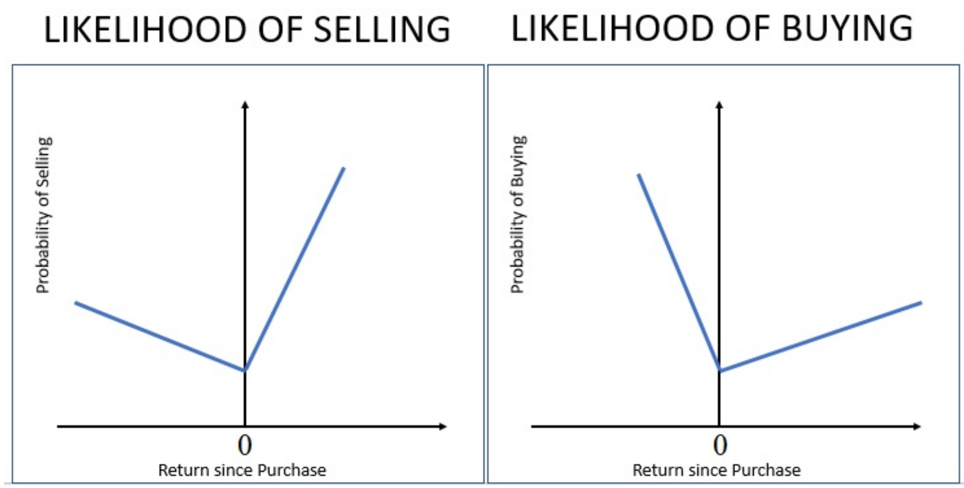


Figure 1: V-shaped probability of either selling or buying an asset with respect to the comparison between current and reference price. On the orizzontal axis there are profits measured as the return since purchase, or the difference between current and purchase price, used as reference price.

with $s_g, s_l, b_g$ and $b_l$ positive parameters regulating the slope of the branches of the V function. To be consistent with the empirical evidence, we suppose $s_g \geq s_l$ and $b_l \geq b_g$. Parameter $\widehat{c}$ and f can be interpreted as measures of the relevance in the market of the two groups of traders. This relevance can be due by the relative number of each group and/or by the relative aggressiveness. In Section 3 we will separate these two components. Moreover, we have that:

$$\widetilde{P}_{t+1} = \lambda \widetilde{P}_t + (1 - \lambda) P_t, \tag{5}$$

with $\lambda \in [0, 1]$ regulating how gradual is the fading importance of past prices.

By inserting the endogenous reactivity (4) in the chartists' trading rule (3) and by using it together with the fundamentalists' trading rule (2) in the market maker equation (1), we finally obtain the two-dimensional piecewise-defined nonlinear map regulating the dynamics of asset price and reference price:

$$T : \begin{cases} P_{t+1} = P_t + \alpha \left[ f(F - P_t)^3 + c_t(P_t - F) \right] \\ \widetilde{P}_{t+1} = \lambda \widetilde{P}_t + (1 - \lambda) P_t \end{cases} \tag{6}$$

## 2.1 Analytical results

Let us first consider the equilibria of the model. We denote with an asterisk ($^*$) the equilibrium values of the model's variables. A first result is the following:

**Proposition 1** *The equilibria of the dynamical system (6) are such that $P^* = \widetilde{P}^*$, that is at the equilibria the asset price is equal to the reference price.*

**Proof.** By using the equilibrium conditions $P_{t+1} = P_t = P^*$ and $\widetilde{P}_{t+1} = \widetilde{P}_t = \widetilde{P}^*$ on the dynamic equation of the reference price we immediately get $P^* = \widetilde{P}^*$. ■

An immediate consequence is that the parameter $\lambda$ plays no role in the equilibrium values.

Concerning the number of the equilibria we have the next result:

**Proposition 2** *The dynamical system (6) admits up to three equilibria. The first one is the fundamental one ($E_0$) where the asset price is equal to its fundamental value ($P_0^* = F$), while the other two are non-fundamental equilibria ($E_{1,2}$). In the latters, the asset price is different from the fundamental value ($P_{1,2}^* = F \pm \sqrt{\frac{\widehat{c}}{f}}$).*

**Proof.** Using the equilibrium conditions $P_{t+1} = P_t = P^*$ and $\widetilde{P}_{t+1} = \widetilde{P}_t = \widetilde{P}^*$ on the first equation of the dynamical system (6), we first get that at the equilibria the reactivity of chartists ($c^*$) must be equal to $\widehat{c}$. In fact, as we know from Proposition 1, at the equilibrium the asset price is equal to the reference price, so all the four possible dynamic equations of the reactivity (4) reduce to $c_t = \widehat{c}$. Then, we get that at the equilibrium:

$$f(F - P^*)^3 + \widehat{c}(P^* - F) = 0$$

from which we obtain the three equilibrium values of the asset price (and of the reference price):

$$\begin{aligned} &E_0 \to P_0^* = \widetilde{P}_0^* = F \\ &E_{1,2} \to P_{1,2}^* = \widetilde{P}_{1,2}^* = F \pm \sqrt{\tfrac{\widehat{c}}{f}} \end{aligned}$$

If we consider strictly positive values of $\widehat{c}$ and $f$ then the three equilibria always exist. ■

The next results concern the local stability of the equilibria.

**Proposition 3** *The fundamental equilibrium $E_0$ is unstable*

**Proof.** The local stability of the equilibria must be studied by using the Jacobian matrix of (6) calculated at the equilibria. Considering the fundamental equilibrium, we have:

$$J(E_0) : \begin{bmatrix} 1 + \alpha\widehat{c} & 0 \\ 1 - \lambda & \lambda \end{bmatrix}, \tag{7}$$

where in the main diagonal there are the two eigenvalues ($\xi_1 = 1 + \alpha\widehat{c}$ and $\xi_2 = \lambda$). Given the positivity of the parameters $\alpha$ and $\widehat{c}$ we have that $\xi_1 > 1$ so the equilibrium is unstable. ■

It is important to stress that the parameters related to the Disposition Effect do not play any role, neither in the equilibrium values nor in the stability of the fundamental equilibrium. Disposition Effect, in fact, can be relevant when transactions are classified as gains or losses, so outside the equilibrium.

We know that besides the fundamental equilibrium, other two (non - fundamental) equilibria exist. The analytical study of their local stability is more complicated because they are located on the line separating two regions of the phase space characterized by different dynamic equations. Let us consider, for instance, the equilibrium $E_1$, where the asset price and the reference price are higher than the fundamental value. In any neighborhood of $E_1$ we can find points of the phase plane where the asset price is larger that the reference price, so the reactivity of chartists is $\widehat{c} + b_g(P_t - \widetilde{P}_t)$, but also points when the opposite occurs, so the reactivity is $\widehat{c} - b_l(P_t - \widetilde{P}_t)$, because the equilibrium is on the boundary. The same happens for the other non-fundamental equilibrium. For this reason we prefer to rely on numerical simulations to investigate their stability.

## 2.2 Numerical results

In order to say something about the local stability of the non-fundamental equilibrium, we have computed some numerical simulations. In all the simulations we keep fixed these parameters' values: $\alpha = 1$, $f = 0.45$, $\lambda = 0.9$, $F = 1$[3], and to reduce the number of parameters dealing with Disposition Effect we also assume that $s_l = b_g = s_g/2 = b_l/2$. This assumption is not arbitrary but coherent with the measures about the gain-loss asymmetry of the DE, according to which investors are about twice as likely to realize a gain rather than a loss (see Barber et al. 2007, Einiö et al. 2008, Seru et al. 2010). These simulations allow us to understand how large the exogenous component of chartists' reactivity ($\widehat{c}$) must be to cause the loss of stability of the non-fundamental equilibria. In other words, the larger is the share of investors affected by DE, the more probable is that dynamics are chaotic. The role played by the parameters regulating how much they are affected by DE is similar. In fact, panel a in Fig. 2 shows that by increasing the relvance of chartists the non-fundamental equilibria become unstable and dynamics turn to periodic and eventually chaotic for higher values of the reactivity parameter. Panel b instead explores the role of Disposition Effect demonstrating that by increasing the values of the four parameters related to this bias the presence of chaotic motion becomes more probable. As a result, the deeper is the DE the more unstable and unpredictable is the asset price dynamics and the larger are price fluctuations. DE makes more extreme the actions of traders, especially when price differs from the reference one. Transactions of larger volumes of the asset cause the emergence of fluctuations and bubbles and crashes phenomena even when the market would be stable without DE (panel c of Fig. 2).

This result seems to be robust to changes in the combinations of parameters kept fixed.

[3]The parameter values (with the exception of the scale parameters $\alpha$ and $F$) have been determined by following the Westerhoff's trial and error approach (Westerhoff, 2008) in ranges of values coherent with some existing empirical results (Gilli and Winker 2003, Alfarano et al. 2005, Boswijk et al. 2007, Winker et al. 2007, Bekaert and Wu 2000).

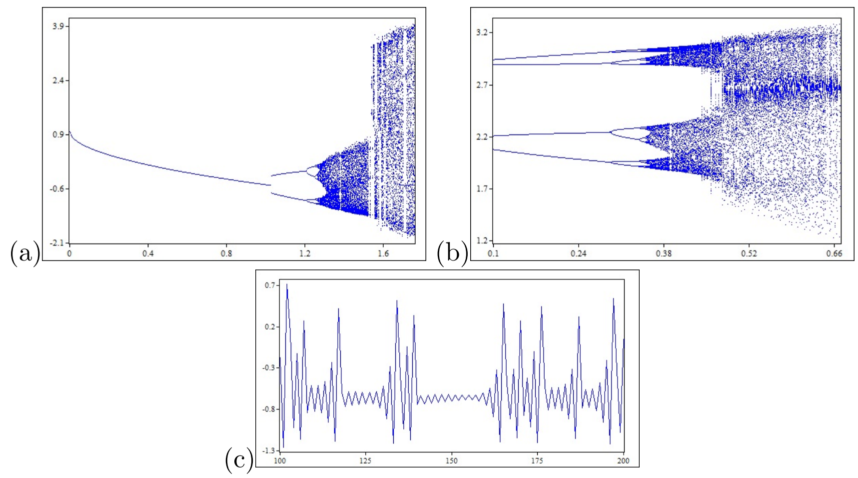

Figure 2: In panel (a) bifurcation diagram when the exogenous component $\widehat{c}$ of the chartists' reactivity varies. In panel (b) bifurcation diagram with respect to the DE effect parameter $s_g$ with $\widehat{c} = 1.23$. In panel (c) a typical chaotic timeplot obtained with $\widehat{c} = 1.23$ and $s_g = 0.63$.

In the next section we propose an evolutive version of our model, where we consider that looking at the results of the two trading strategies, some agents may decide to change their behavior moving from fundamentalist to chartist and *viceversa*.

# 3 Enriching the model: An evolutive version

The switching of traders from one behavioral strategy to another one is a typical source of instability of the market (see Brock & Hommes, 1997, 1998). Moreover, it is a quite realistic assumption because it is hard to justify that agents who constantly gather lower profits from their strategy with respect to the alternative one, keep using it forever.

We have decided to enrich our model with an evolutive version, so we can look at the combined effect of DE and strategy switching.

Let us consider $N$ the fixed amount of traders. To simplify we normalize $N$ to unity, so $N = 1$. The share of fundamentalists and chartists at each time period $t$ is denoted by $n_t^f$ and $n_t^c = 1 - n_t^f$, respectively.[4] It is clear that we only need to specify how the share of fundamentalists evolves as time passes, to have also the complementary share of chartists.

We introduce a measure of the attractiveness of the fundamentalists' trading strategy ($a_t$) that take into consideration the success of this strategy. In

[4] Now $\widehat{c}$ and $f$ only represent the aggressiveness of the two groups of traders, and by multiplying their shares you get what we previously called relevance

particular, we consider the fundamental strategy successful in a certain period $t$ if the asset price increases when its current value is below the fundamental value, and decreases when its current value is below it. Putting it into a formula we have that if the product $(P_t - P_{t-1})(F - P_{t-1})$ is positive, then $a_{t+1} > a_t$, otherwise $a_{t+1} < a_t$. So, the attractiveness of fundamentalists'trading strategy dynamically evolves according to:

$$a_{t+1} = (1-\tau)a_t + \tau(P_t - P_{t-1})(F - P_{t-1}) \tag{8}$$

where $\tau \in [0,1]$ regulates the importance of the last price movements or, in other words, how myopic are investors.

On this basis, the market share of fundamentalists at time $t+1$ can be determined by the formula:

$$n^f_{t+1} = \frac{1}{1+e^{-\beta a_t}} \tag{9}$$

where $\beta \geq 0$ is the so-called intensity of choice. Note that in the limit case $\beta = 0$ the two shares remain fixed and $n^f = n^c = 0.5$. So, the larger is the intensity of choice parameter, the more important is the switching mechanism.

The dynamical system including all the dynamic equations we have introduced is the following:

$$\widetilde{T} : \begin{cases} P_{t+1} = P_t + \alpha \left[ n^f_t \cdot f(F - P_t)^3 + (1 - n^f_t)c_t(P_t - F) \right] \\ \widetilde{P}_{t+1} = \lambda \widetilde{P}_t + (1-\lambda)P_t \\ a_{t+1} = (1-\tau)a_t + \tau(P_t - Z_t)(F - Z_t) \\ n^f_{t+1} = \frac{1}{1+e^{-\beta a_t}} \\ Z_{t+1} = P_t \end{cases} \tag{10}$$

where the auxiliary lagged variable $Z$ permits to have a system of difference equations of the first order.

In the next we illustrate the results of this augmented version of the model.

## 3.1 Analytical results

The dynamical system (10) is five-dimensional and there are several nonlinearities, so it is harder to obtain analytical results. Nevertheless it is possible to state the following Proposition about the equilibria of the evolutive model:

**Proposition 4** *The dynamical system (10) admits the same three equilibria of the system (6): The first fundamental one ($E_0$) where the asset price is equal to its fundamental value ($P^*_0 = F$), and two are non-fundamental equilibria ($E_{1,2}$), where the asset price is different from the fundamental value ($P^*_{1,2} = F \pm \sqrt{\frac{c}{f}}$). At the equilibria, there are 50% of fundamentalists and 50% of chartists.*

**Proof.** Using the equilibrium conditions $P_{t+1} = P_t = P^*$, $\widetilde{P}_{t+1} = \widetilde{P}_t = \widetilde{P}^*$, $n^f_{t+1} = n^f_t = n^{f*}$, $a_{t+1} = a_t = a^*$ and $Z_{t+1} = Z_t = Z^*$on the second and fifth equation of the dynamical system we can easily get that $P^* = \widetilde{P}^* = Z^*$, so also in this case at the equilibria asset and reference price are coincident. Then, as a consequence also in this case the reactivity of chartists at the equilibria is equal to its exogenous component $c^* = \widehat{c}$. The third equation provides the equilibrium value of the attractiveness of the fundamentalists' trading strategy, which is $a^* = 0$, which, applied to the equation regulating the share of fundamentalists permits to obtain the equilibrium share of fundamentalists, that is $n^{f*} = 0.5$. Now, moving to the asset price equation and by applying the results we have just obtained, we find that the equilibria are also in this case those values of the asset price solving the equation:

$$f(F - P^*)^3 + \widehat{c}(P^* - F) = 0$$

from which we obtain the three equilibrium values of the asset price (and of the reference price):

$$\begin{array}{l} E_0 \to P_0^* = \widetilde{P}_0^* = F \\ E_{1,2} \to P_{1,2}^* = \widetilde{P}_{1,2}^* = F \pm \sqrt{\frac{\widehat{c}}{f}} \end{array}$$

■

It is difficult to study analytically the stability of the equilibria, but numerical simulations can give us insights about the role of the switching mechanism.

### 3.2 Numerical results

In order to check if the intensity of switching parameter $\beta$ has a destabilizing effect, we numerically obtain the bifurcation diagrams for different parameters' configurations (see Fig. 3).

Panels on the left show the loss of stability of the equilibrium as a consequence of the increasing of the intensity of switching, while panels on the right show the destabilizing role of DE when switching is admitted.

We used three different configurations of parameters to test the robustness of these results.

Now we are interested in exploring if with the introduction of stochastic elements, DE is able to explain some more stylized facts of financial markets.

## 4 The stochastic version of the model

The bifurcation diagrams in Figs. 2 and 3 illustrate how our deterministic model is already able to mimic some qualitative features of financial markets such as bubbles and crashes and some kind of excess volatility, at least in the chaotic region of the parameters' space. This is a first contribution of the DE to let the model better replicate the dynamic evolution of financial markets. In order to deeper investigate the role played by DE in asset price dynamics, we have to perform a more complete analysis by adding some stochastic components. The

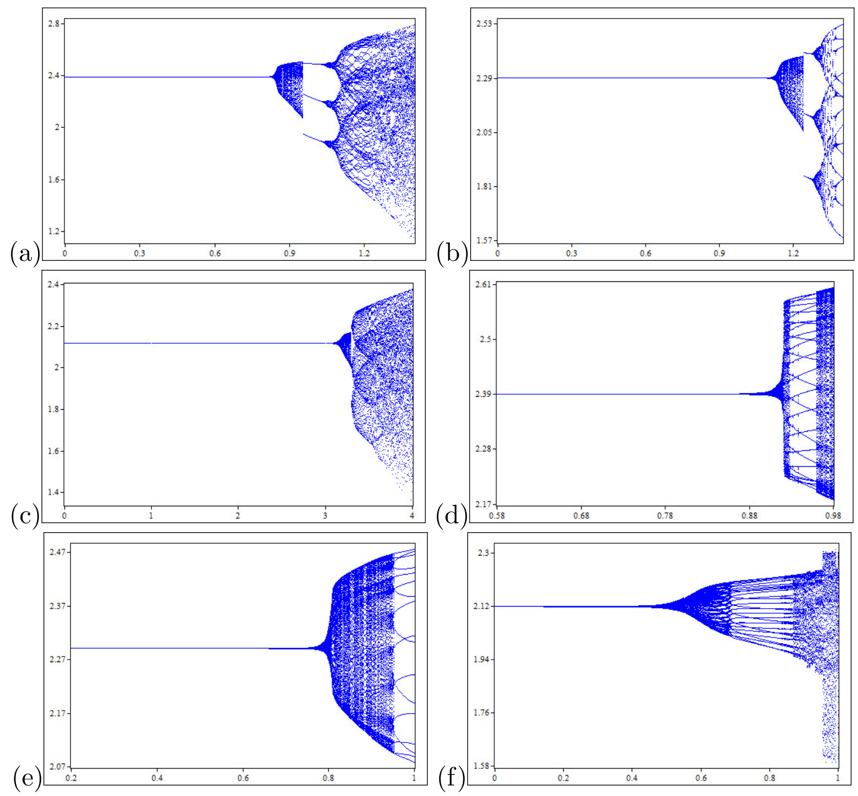

Figure 3: In panels (a), (c) and (e) we have bifurcation diagrams when the intensity of choice parameter $\beta$ varies. Panel (a) is obtained by keeping fixed $\alpha = 1$, $f = 0.45$, $\widehat{c} = 0.87$, $F = 1$, $\lambda = 0.9$, $s_g = 0.2$ and $\tau = 0.9$. For panel (c) we fixed $\alpha = 1$, $f = 0.6$, $\widehat{c} = 1$, $F = 1$, $\lambda = 0.7$, $s_g = 0.2$ and $\tau = 0.7$. For panel (e) we used $\alpha = 1$, $f = 0.8$, $\widehat{c} = 1$, $F = 1$, $\lambda = 0.5$, $s_g = 0.2$ and $\tau = 0.4$. In panels (b), (d) and (f) we have bifurcation diagrams with respect to the DE effect parameter $s_g$ For panel (b) we used the same parameters' values of panel (a), with $\beta = 0.7$. Panel (d) is obtained with the same parameters' values of panel (c) and $\beta = 1$. Panel (f) is obtained with the same parameters' values of panel (e) and $\beta = 3$.

aim of this last step of our study is to demonstrate that DE may permit to obtain simulated returns better reproducing some important quantitative features that are observed in financial markets. To do so, we need a more sophisticated investigation on the features of the time series and of its descriptive power. First of all, it is not realistic to assume a constant, exogenously given, fundamental value. In what follows, the fundamental value is supposed to follow a random walk capturing unexpected or unpredictable events that may hit a financial asset. The fundamental value now follows a geometric Brownian motion, with its log-value becoming:

$$F_{t+1} = F_t + \xi_{F,t}, \qquad \text{with } \xi_{F,t} \sim N(\mu_F, \sigma_F^2),$$

where $\xi_{F,t}$ is independent and identically distributed.

Moreover, it is reasonable to assume that also the amount of chartists (measured by the proxy $\widehat{c}$) varies with time. We assume:

$$\widehat{c}_{t+1} = \widehat{c}_t + \xi_{\widehat{c},t}, \qquad \text{with } \xi_{\widehat{c},t} \sim N(\mu_{\widehat{c}}, \sigma_{\widehat{c}}^2)$$

where $\xi_{\widehat{c},t}$ is independent and identically distributed.

To test the model we rely on the same parameters' values fixed in Fig. 3 (c), that is: $\alpha = 1$, $f = 0.6$, $\lambda = 0.9$, $\tau = 0.7$. We have set as initial values $P_0 = 0.97$ and $\tilde{P}_0 = 1.02$. We note that the initial values of the dynamic variables have a negligible effect on the results of the stochastic model. We have thus calibrated the variance of the noise with a trial and error calibration approach, that consists in finding a value that does not undermine the stability of the model. We have chosen as initial values, average and variances:

$$\begin{array}{llll} F \longrightarrow & F_0 = 1; & \mu_F = 0; & \sigma_F^2 = 0.1 \\ \widehat{c} \longrightarrow & \widehat{c}_0 = 1.3; & \mu_{\widehat{c}} = 0; & \sigma_{\widehat{c}}^2 = 0.05 \end{array}$$

Then we have considered nine different scenarios, which differ for three different values of $s_g$ (and consequently the other parameters graduating DE) and $\beta$:

1. $s_g = 0$ and $\beta = 0$ (no DE no switching scenario)
2. $s_g = 0$ and $\beta = 0.2$ (no DE slow switching scenario)
3. $s_g = 0$ and $\beta = 0.4$ (no DE fast switching scenario)
4. $s_g = 0.2$ and $\beta = 0$ (weak DE no switching scenario)
5. $s_g = 0.2$ and $\beta = 0.2$ (weak DE slow switching scenario)
6. $s_g = 0.2$ and $\beta = 0.4$ (weak DE fast switching scenario)
7. $s_g = 0.35$ and $\beta = 0$ (strong DE no switching scenario)
8. $s_g = 0.35$ and $\beta = 0.2$ (strong DE slow switching scenario)

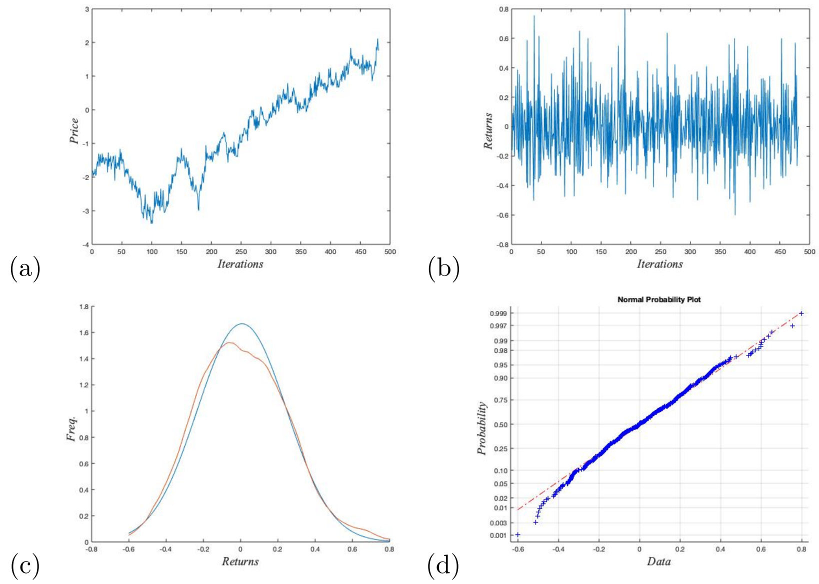


Figure 4: Scenario 1 ($s_g = 0$ and $\beta = 0$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

9. $s_g = 0.35$ and $\beta = 0.4$ (strong DE fast switching scenario)

For every scenario we performed 1000 runs of Monte Carlo simulations, each one of size 500 iterations of the dynamical system (10) with stochastic fundamental value. Monte Carlo method works by selecting a random sequence of $F$ and $\widehat{c}$ for each simulation, building sampling based on those values. This process has been repeated a thousand times to obtain, as output, the distribution of our simulated time series.

Several stylized facts, i.e. qualitative features common to a wide set of assets, are frequently observed in real financial markets. On this topic, Mantegna and Stanley (2000), Cont (2001) and Lux and Ausloos (2002) provide an exhaustive guide.

The volatile nature of returns is discussed thoroughly in Shiller (2015) and here is measured by using the variance of simulated time series of returns.

We know that in this case extreme events are more likely to happen. Formally, an excess kurtosis implies a peakiness bigger than normal and a slow asymptotic decay of the probability density function. This non-normal decay

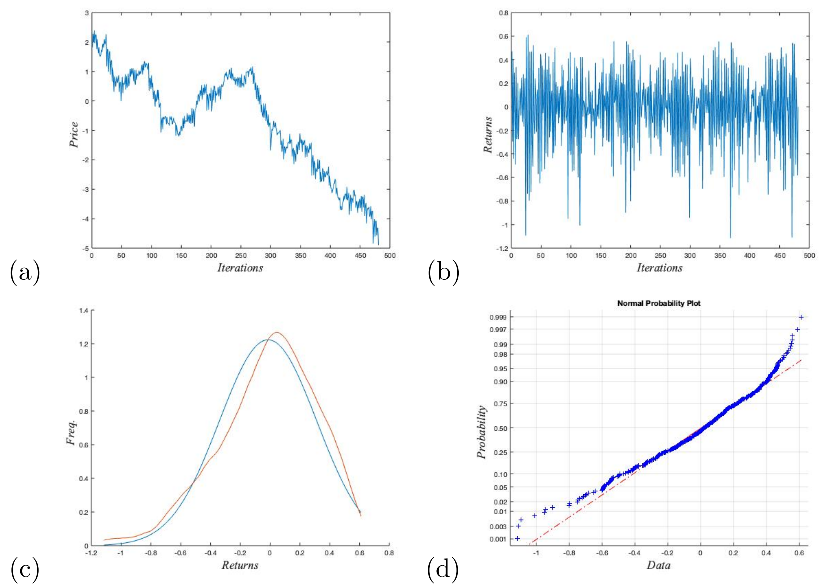


Figure 5: Scenario 2 ($s_g = 0$ and $\beta = 0.2$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

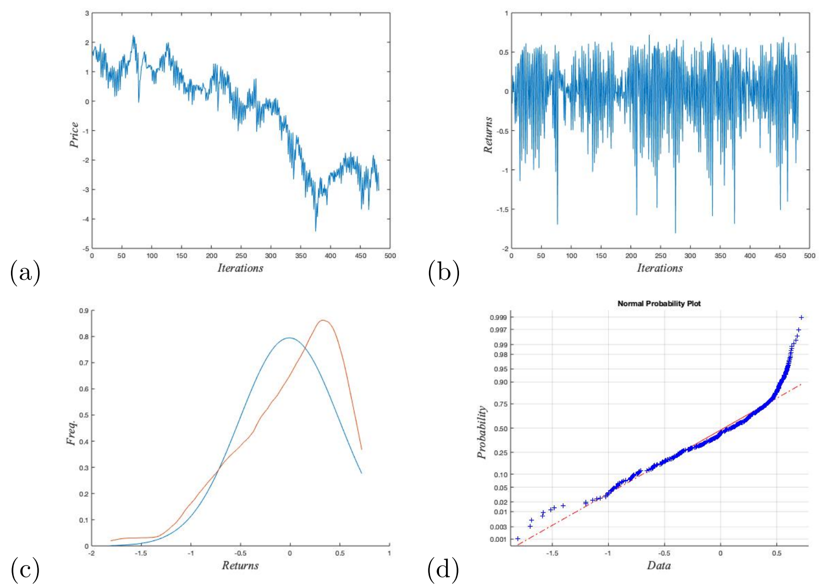


Figure 6: Scenario 3 ($s_g = 0$ and $\beta = 0.35$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

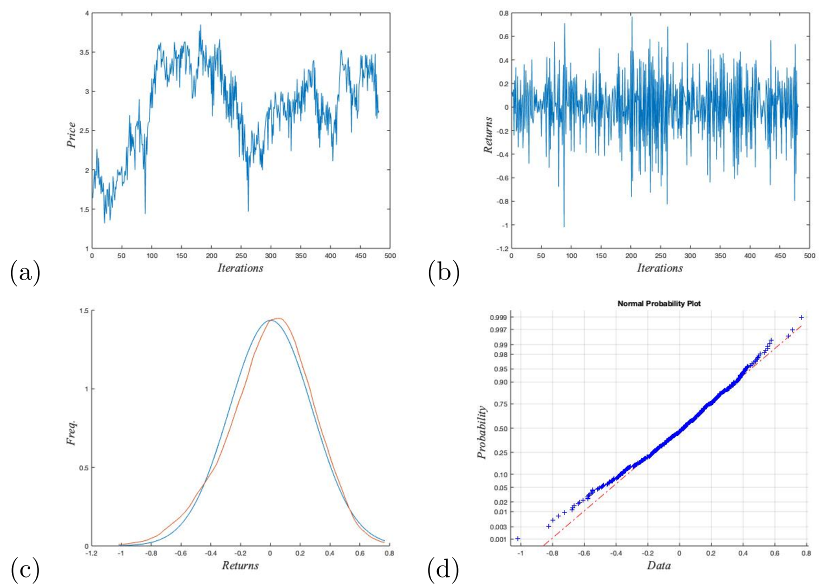


Figure 7: Scenario 4 ($s_g = 0.2$ and $\beta = 0$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

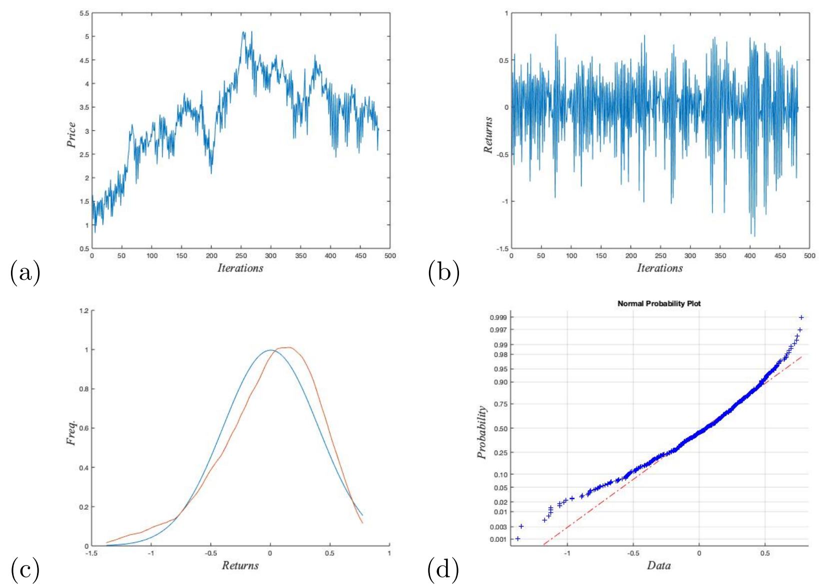


Figure 8: Scenario 5 ($s_g = 0.2$ and $\beta = 0.2$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

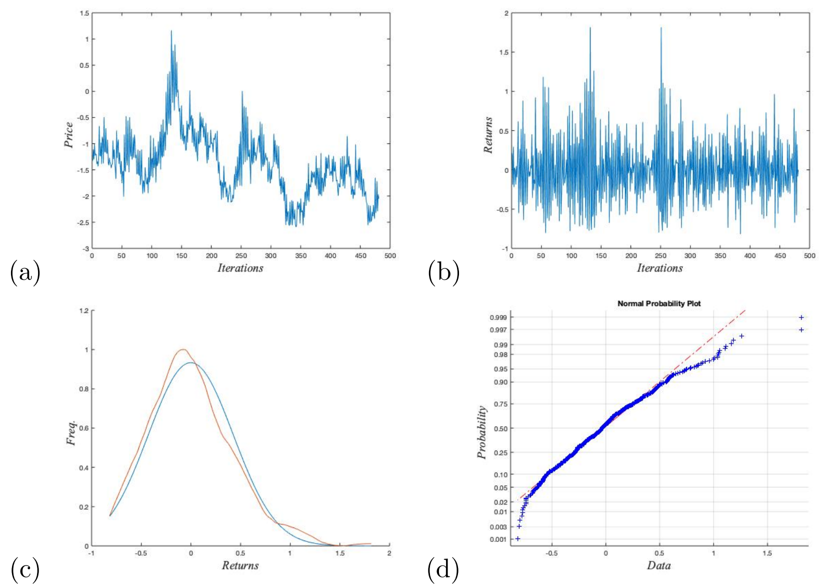


Figure 9: Scenario 6 ($s_g = 0.2$ and $\beta = 0.35$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

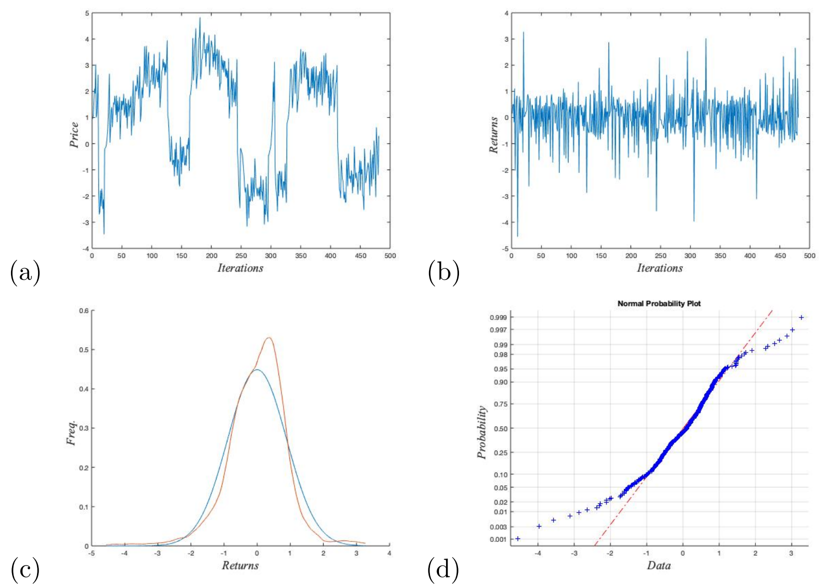


Figure 10: Scenario 7 ($s_g = 0.35$ and $\beta = 0$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

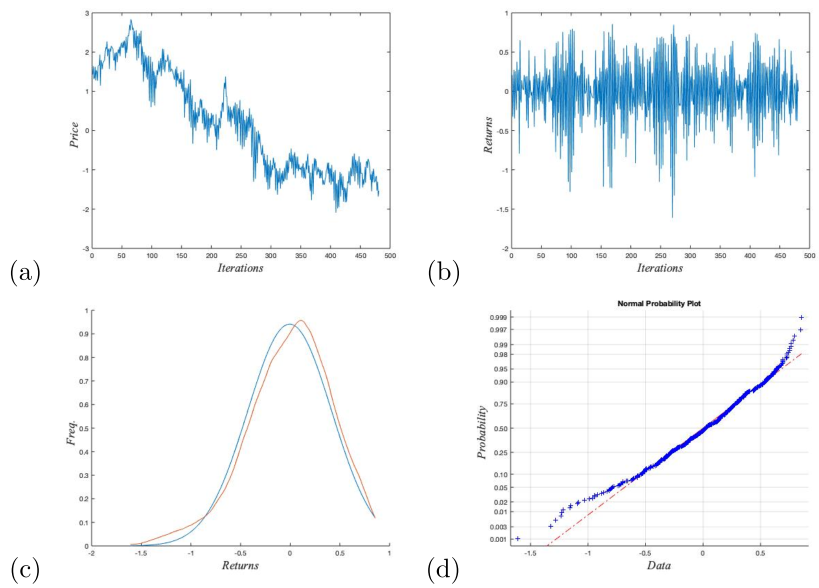


Figure 11: Scenario 8 ($s_g = 0.35$ and $\beta = 0.2$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

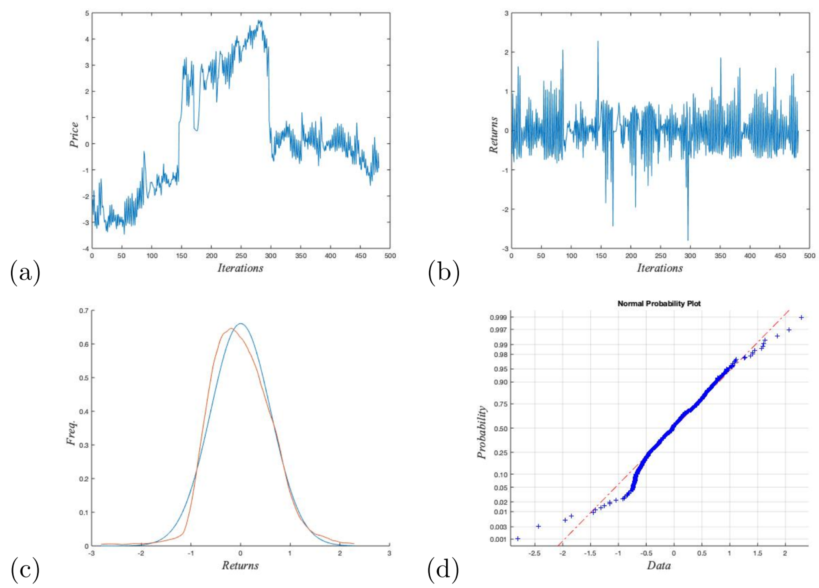


Figure 12: Scenario 9 ($s_g = 0.35$ and $\beta = 0.35$). Panels (a) and (b) show the pattern of the simulated price and returns. They are single, representative, simulations obtained with a sequence of random realizations of the fundamental value. Panel (c) illustrates the theoretical kernel estimator (in blue) compared with the simulated distribution (in red). In panel (d) figures the relative probability plot. Average measures and statistics of 1000 Monte Carlo simulations (length 500 iterations each).

| $S_g$, $\beta$ | Kurtosis | Volatility | Jarque-Bera (p-value) | Shapiro-Wilk (p-value) |
|---|---|---|---|---|
| $S_g = 0$, $\beta = 0$ | 2.94 | 0.0472 | 0.129 | 0.0823 |
| $S_g = 0$, $\beta = 0.2$ | 3.23 | 0.0629 | 0 | 0 |
| $S_g = 0$, $\beta = 0.3$ | 3.32 | 0.094 | 0 | 0 |
| $S_g = 0.2$, $\beta = 0$ | 3.22 | 0.0534 | 0.005 | 0.03 |
| $S_g = 0.2$, $\beta = 0.2$ | 3.63 | 0.0758 | 0 | 0 |
| $S_g = 0.2$, $\beta = 0.3$ | 3.94 | 0.081 | 0 | 0 |
| $S_g = 0.35$, $\beta = 0$ | 6.4 | 0.16 | 0 | 0 |
| $S_g = 0.35$, $\beta = 0.2$ | 3.4 | 0.08 | 0 | 0 |
| $S_g = 0.35$, $\beta = 0.3$ | 4.75 | 0.11 | 0 | 0 |
| $S\&P500$ | 17.47 | 0.94 | 0 | 0 |

Table 1: Average values of kurtosis, volatility and normality tests for different degrees of $s_g$ and $\beta$.

is the so-called heavy tail (LeBaron and Samanta 2005). However, the precise form of the tails is difficult to determine. We measure the presence of heavy tails by referring to the value of the kurtosis of the distribution, defined as:

$$\kappa = \frac{\left\langle (r(t,T) - \left\langle r(t,T) \right\rangle)^4 \right\rangle}{\sigma(T)^4} - 3$$

with $\sigma(T)^2$ indicating the variance of log-returns and a positive value of $\kappa$ suggesting the presence of fat tails.

However, the presence of a high level of kurtosis is not sufficient *per se* to identify the distribution of returns: additional tests to explore their normality are needed.

The normality of the distribution of returns is tested by performing the Jarque-Bera test and the Shapiro-Wilk test (as suggested in the complete guide of Yap and Sim (2011), by plotting the Q-Q plot and by comparing the theoretical kernel estimator versus the simulated distribution.

Table 1 illustrates different values of kurtosis, volatility for different degrees of $s_g$ and $\beta$ and the p-value of the two different normality tests. We computed the Jarque-Bera test as a moment test and the Shapiro-Wilk test as a test based on regression. To have a term of comparison we have also added the values obtained by using the *Standard and Poor 500* (S&P 500) index time series. The analyses are computed on 2590 observations, that is the closing market price from May 6, 2011 to April 30, 2021[5].

The results clearly show that scenarios with high DE are characterized by higher kurtosis, even if not so high as those of the S&P 500 index. Moreover,

[5]Data have been retrieved from Fred website. Not seasonally adjusted.

| Scenario | Negative returns (mean) | Positive returns (mean) |
|---|---|---|
| Scenario 1 | -0.193 | 0.207 |
| Scenario 2 | -0.291 | 0.232 |
| Scenario 3 | -0.459 | 0.355 |
| Scenario 4 | -0.236 | 0.27 |
| Scenario 5 | -0.35 | 0.293 |
| Scenario 6 | -0.307 | 0.364 |
| Scenario 7 | -0.718 | 0.559 |
| Scenario 8 | -0.371 | 0.311 |
| Scenario 9 | -0.431 | 0.493 |

Table 2: Gain/loss asymmetry for our generated time-series of returns

it seems that scenario 7 (strong DE but with no switching), is the one more promising. Variability is not so high as the one of the S&P 500, but is higher than what we get from the other scenarios and the null hypothesis of normality of the distribution of returns are rejected by both the Jarque-Bera and the Shapiro-Wilk test.

Fig. 4-12 display the simulated time-series of price and returns for all the nine scenarios together with the pattern observed in financial markets previously discussed.

Panels (a) and (b) of these figures show a single sequence of the evolution of price and returns for different combinations of $s_g$ and $\beta$. In addition, panels (c) show the distribution of our simulated time series (in red) compared with the theoretical kernel normal one (in blue). Finally, panels (d) present the Probability plot (QQ-Plot). It helps to understand how far are the processes to normality.

These analyses confirm that Scenario 7 is the one to prefer, where deviations from the normality distribution of returns are consistent enough. One last proof of that is provided in Table 2, where the average values of negative and positive returns are compared. We can notice that Scenario 7 is the one where the difference is more pronounced and coherent with the gain/loss asymmetry (Cont, 2001).

## 4.1 Scenario 7, with high DE and no switching

We have decided to better investigate Scenario 7, where DE is high ($s_g = 0.35$) and there is no switching between trading strategies ($\beta = 0$).

We want to see if this Scenario permits to replicate also other stylized facts of financial markets. In particular, by following the list of Cont (2011), we want to see if a typical simulation run is able to replicate:

1. **Absence of autocorrelations**: (linear) autocorrelations of asset returns are often insignificant, excluding the possibility of nonlinear dependencies

of returns;

2. **Slow decay of autocorrelation in absolute returns**: the autocorrelation function of absolute returns decays slowly as a function of the time lag. This is sometimes interpreted as a sign of long-range dependence and volatility clustering (Mandelbrot, 1963);

3. **Power-law consistent distribution of returns' tails**: the heavy tails of the distribution of returns seem consistent with a power-law distribution.

Figure 13 shows the autocorrelation functions obtained in a typical simulation run of 4000 iterations, up to 100 lags. As we can see, the autocorrelations become quite soon insignificant, while in the case of absolute returns they remain significant for the whole interval, and they are also slowly decreasing. This is consistent with the presence of volatility clustering. In order to confirm that in Figure 14, panel (a), we show the returns corresponding to the 4000 iterations. There is a clear alternation of periods characterized by high and low volatility. We explain this feature with the corresponding alternation of periods where traders affected with DE are more relevant (high variability) and periods when they are less relevant (low variability). In panel (b) we plot the corresponding values of $\widehat{c}_t$[6] and it seems evident that when the value is high there is also high variability, and when it is low, variability is also low.

Figure 15 represents a log-log plot of extreme returns[7] and also in this case we can state that at least a part of the returns seem to be placed in a line, as it should be if their distribution follows a power-law.

Concluding, we want to underline two things. First, our results suggest that the DE of a share of traders may contribute a better comprehension of the dynamics of financial markets. Secondly, our aim is not so ambitious to make us expect that our model perfectly mimics real financial time series and all the stylized facts of financial markets. Traders are affected by several biases and we only consider one of them, trying to understand how relevant it could be. In order to explain understand the dynamics of asset prices we should consider all the behavioral features of traders and combine them. This is out of the scope of this paper, which is limited to DE.

[6]We have limited the range of variability of $\widehat{c}_t$ between a maximum (1.5) and a minimum (0.8) in order to avoid explosions of prices and returns and a too strong stability of the equilibrium. When the value becomes larger than the maximum it will take the maximum value, when it reaches a lower than the minimum value, it takes the minimum value.

[7]We have normalized returns and used the positive tails. with the negative tail we obtain a similar graph.

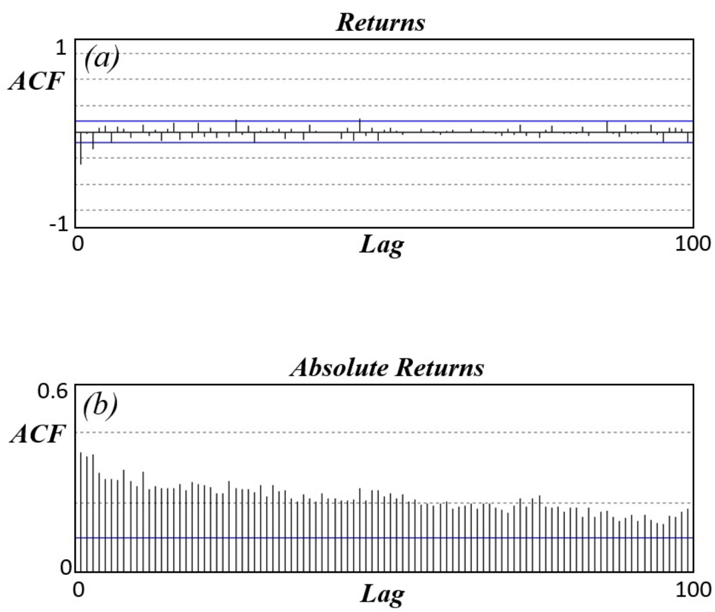


Figure 13: Autocorrelation function o returns (panel a) and absolute returns (panel b) obtained by fixing $\alpha = 1$, $f = 0.45$, $\beta = 0$, $\lambda = 0.9$ and $s_g = 0.35$.R

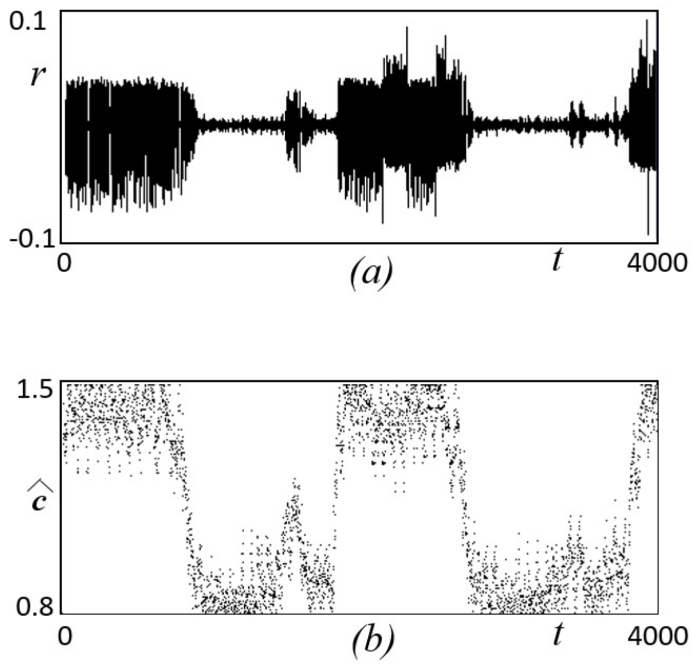


Figure 14: A simulation run. Returns are plotted in panel a, the values of $\widehat{c}_t$ are in panel b.

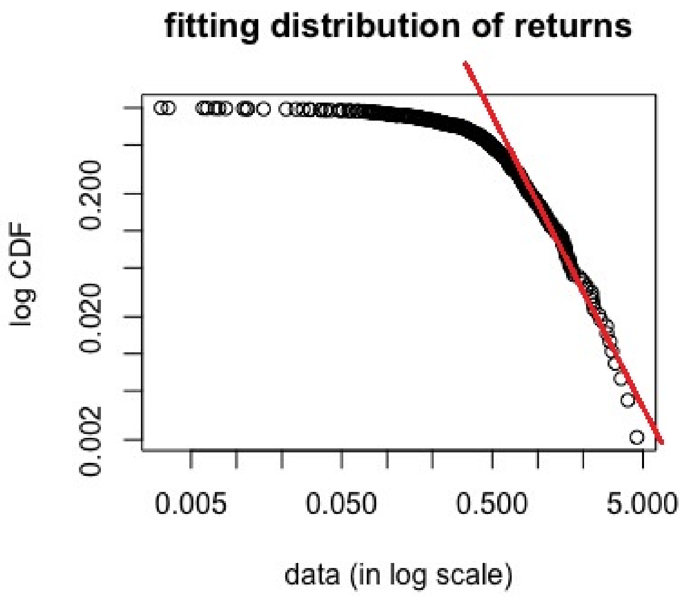


Figure 15: Log-log plot of the positive tail of normalized returns for the simulation run with Scenario 7.

# 5 Conclusions

In this paper we investigate the consequences of a trading irregularity known as *Disposition Effect* (DE). We develop a simple financial market model where heterogeneous agents coexist and where a group of traders behave according to the empirical findings of Ben-David and Hirshleifer (2012). As a consequence of this behaviour, the existence of the DE emerges.

We find that when DE is particularly accentuated the stock market is more likely to become unstable and bubbles and crashes may appear. So this psychological feature of investors, which leads to a strong reaction to price changes, at the aggregate level may be one of the causes of the main features of the dynamics of stock markets. The panic of selling causes sudden and more frequent transactions, making the market more unstable, more volatile and less predictable.

The version of our model that more closely replicates important characteristics of financial time series, such as the presence of heavy tails, skewness, high volatility and gain/loss asymmetry, is the one with traders affected by DE are relevant and highly affected by this bias. We want to stress that the role played by the behavioral parameter has been anticipated by analyzing the deterministic skeleton of the model. The stochastic version of the model allows to better mimic the quantitative stylized facts of financial markets, but qualitatively they can already be detected from the deterministic version. We think that this kind of studies (deterministic and stochastic) can be a good way to explore the consequences of some behavioral features of investors on the asset price dynamics. We will continue in our future works to analyze this class of models.

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