Growth with Automation Capital and Declining Population

Sasaki, Hiroaki

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Hiroaki Sasaki*

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Abstract

This study investigates how the long-run growth rate of per capita output is determined when automation capital is introduced in final goods production and when the population is declining. The results indicate that even though the population is declining, per capita output can continue to grow at a positive rate depending on condition.

Keywords: growth; automation technology; declining population

JEL Classification: J11; O33, O41

1 Introduction

Using a simpler economic growth model, we investigate whether per capita output attains sustainable growth and under what conditions it is achievable when the population declines and automation technology advances. Recently, numerous empirical studies have predicted that almost half of the jobs in developed economies have been replaced by artificial intelligence (AI) and robots (Frey and Osborne, 2013, 2017). In addition, population decline is becoming a serious problem.[1] Japan first experienced population decline in 2005 and the population has continued to decline since then 2010. In addition, according to the United Nations World Population Prospects 2019, many countries are predicted to experience population decline.

*Graduate School of Economics, Kyoto University. Yoshida-honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail: sasaki.hiroaki.7x@kyoto-u.ac.jp

[1]For growth models that consider declining population, see Christiaans (2011), Jones (2022), Sasaki (2019), and Sasaki and Hoshida (2017).

If automation technology substitutes human labor and labor force continues to decline with population decline, it is considered to complement human labor. Advances in automation technology may be desirable for a population declining economy rather than for a population increasing economy. Therefore, investigating how advances in automation technology affect economic growth under a population declining economy is of significance.

Numerous studies analyze how advances in automation technology affect economic growth. We use the automation capital approach following Steigum (2011).[2] In addition to labor and capital (traditional capital), this approach introduces a third input factor, that is, automation capital, and assumes that automation capital and labor are substitutes. As automation capital accumulates, fewer labor inputs are required in final goods production. In contrast, traditional and automation capital inputs are relatively higher, and as long as these two kinds of capitals accumulate, an economy can attain sustainable growth. Prettner (2019) further extends this automation capital approach. He assumes that automation capital is a perfect substitute for labor, and demonstrates that the long-run growth rate of per capita output is positive and the labor share of income will be zero in the long run. Critically examining Prettner (2019), Heer and Irmen (2019) reveals that with the no-arbitrage condition between traditional and automation capital, the production function becomes AK type, and per capita output can attain sustainable growth depending on some conditions.

We consider negative population growth in a Steigum–Prettner–Heer–Irmen's growth model with automation capital. This study derives the following results. When the population growth rate is negative and its absolute value is small, the long-run growth rate of per capita output is positive if the saving rate of households is high, whereas it is zero if the saving rate is lower. When the population growth rate is negative and its absolute value is large, the long-run growth rate of per capita output is positive irrespective of the saving rate. Further, we provide a numerical example for the Japanese economy, and demonstrate that under the present population decline, per capita output cannot grow in the long run, but can if the saving rate of households increases.

The remainder of this paper is organized as follows. Section 2 explains how the introduction of automation capital affects the growth rate of per capita output when the population growth rate is positive. Section 3 investigates the case where the population growth rate is negative. Section 4 presents a numerical example based on

[2]For growth models with automation technology, see Acemoglu and Restrepo (2018, 2020), Aghion et al. (2019), Antony and Klarl (2020), DeCanio (2016), Eden and Gaggl (2018), Gasteiger and Prettner (2022), Irmen (2021), Pillai (2022), Prettner and Strulik (2020), and Stähler (2021).

the Japanese economy. Finally, concluding remarks are provided in section 5.

2 Automation capital and growth

Based on Prettner (2019) and Irmen and Heer (2019), we present a growth model with automation capital. First, we consider positive population growth, followed by negative population growth.

The production function of the final goods is specified as follows:

$$Y = F(K, L, P) = K^{\alpha}(L + P)^{1-\alpha}, \tag{1}$$

where Y denotes output, K, traditional capital, L, labor, and P, automation capital. Households own K and P as stockholdings. The model is a one-good model; the final good is used for consumption and investment in traditional and automation capital.

Let w, R^k, and R^p denote the wage rate, rental price of capital, and rental price of automation capital, respectively. As per the firms' profit maximization perspective, their factor prices are equal to their marginal productivities.

$$w = (1-\alpha)\frac{Y}{L+P} = (1-\alpha)\left(\frac{K}{L+P}\right)^{\alpha}, \tag{2}$$

$$R^k = \alpha\frac{Y}{K} = \alpha\left(\frac{K}{L+P}\right)^{-(1-\alpha)}, \tag{3}$$

$$R^p = (1-\alpha)\frac{Y}{L+P} = (1-\alpha)\left(\frac{K}{L+P}\right)^{\alpha}. \tag{4}$$

From equations (2) and (4), w and R^p increase in K but decrease in P, whereas from equation (3), R^k decreases in K but increases in P.

Heer and Irmen (2019) impose a no-arbitrage condition between two assets, K and P, such that $R^k = R^p$. From this, we obtain[3]

$$P = \left(\frac{1-\alpha}{\alpha}\right)K - L \Longrightarrow P = \max\left\{0, \left(\frac{1-\alpha}{\alpha}\right)K - L\right\}, \tag{5}$$

When $K > [\alpha/(1-\alpha)]L \equiv \bar{K}$, P begins to accumulate. Therefore, when $0 < K < \bar{K}$, $P = 0$, and when $\bar{K} < K$, $P > 0$.

[3]This kind of no-arbitrage condition between the two kinds of assets is also used in a one-sector human capital growth model presented by Barro and Sala-i-Martin (2003), in which the rate of return from human capital is equal to that from physical capital.

Using the no-arbitrage condition, the production function can be rewritten as

$$Y = \begin{cases} K^\alpha L^{1-\alpha} & \text{if } 0 < K < \bar{K}, \\ \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} K & \text{if } \bar{K} \le K. \end{cases} \Longrightarrow y = \begin{cases} k^\alpha & \text{if } 0 < k < \bar{k}, \\ \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} k & \text{if } \bar{k} \le k. \end{cases} \tag{6}$$

Therefore, we obtain the usual Cobb–Douglas production function with $P = 0$ when K is small, and the AK type production function when K is large. It should be noted that although P and L do not appear in the production function, they are input through the no-arbitrage condition $P = (\frac{1-\alpha}{\alpha})K - L$.

At equilibrium, by substituting $K/(L+P) = \alpha/(1-\alpha)$ into equations (2)–(4), we have

$$w = R^k = R^p = \alpha^\alpha (1-\alpha)^{1-\alpha} \equiv R. \tag{7}$$

Using the above information, the equilibrium labor share of income leads to

$$\sigma_L \equiv \frac{wL}{Y} = \frac{\alpha}{k}. \tag{8}$$

Therefore, it is decreasing in k. If we know the dynamics of k, then we also know those of σ_L.

The asset holdings of households are given by $A = K+P$, where A denotes the asset holdings. Note that our model is a one-good model, and hence, K and P have the same price as the price of Y, which is normalized to unity. Let s denote the saving rate of households. Now, the saving of households S is given by $S = s(wL+R^kK+R^pP) = sY$. The dynamics of A are given by $\dot{A} = S - \delta A = s(wL + R^kK + R^pP) - \delta A$, where $\delta \in [0, 1]$ is the capital depreciation rate. For simplicity, we assume a common capital depreciation rate for traditional and automation capital.

Let $a = A/L$ be per capita assets. The differential equation for a is given by $\dot{a} = sy - (\delta + n)a$. From the no-arbitrage condition, we have $\dot{a} = \dot{k}/\alpha$. Therefore, we finally obtain the differential equation for k as follows:

$$\dot{k} = Bk + \alpha(\delta + n), \quad k > \frac{\alpha}{1-\alpha} \equiv \bar{k}, \; B \equiv sR - (\delta + n), \; R \equiv \alpha^\alpha (1-\alpha)^{1-\alpha}. \tag{9}$$

By investigating the dynamics of k, we know those of other variables too. Note that for automation capital to be accumulated, the constraint $k > \bar{k}$ is necessary.

We define k^* such that $\dot{k} = 0$. In the dynamic equation of k given by (9), we need to consider three cases according to whether the coefficient of k is positive or negative and which is larger, k^* or $\bar{k}$. As a result, the following three cases arise according to the size of the saving rate of households.

Case 1 $\frac{\delta+n}{R} < s$, i.e., the saving rate is high.

Case 2 $\frac{\alpha(\delta+n)}{R} < s < \frac{\delta+n}{R}$, i.e., the saving rate is intermediate.

Case 3 $s < \frac{\alpha(\delta+n)}{R}$, i.e., the saving rate is low.

Figures 1–3 illustrate the dynamics of k in these three cases.

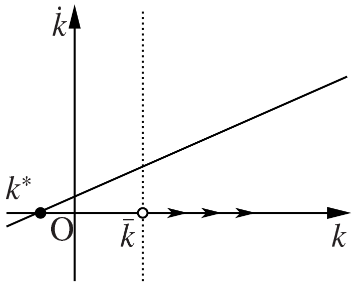

Figure 1: Dynamics of k in Case 1

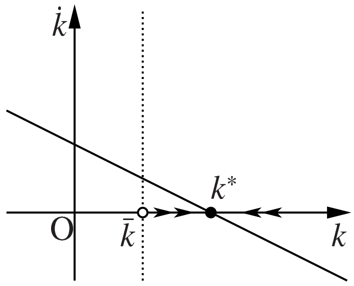

Figure 2: Dynamics of k in Case 2

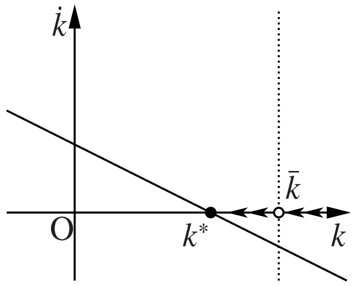

Figure 3: Dynamics of k in Case 3

In Case 1, k continues to increase persistently. The growth rate of y is given by $\dot{y}/y = \dot{k}/k = B + [\alpha(\delta+n)/k]$. Since we have $\lim_{k\to+\infty} \dot{k}/k = B$, the long-run growth rate of per capita output is $\dot{y}/y = \dot{k}/k = B > 0$. In this case, the labor share of income converges to zero.

In Case 2, k converges to k^*. The long-run growth rate of per capita output is zero, and the labor share of income converges to a positive constant value.

In Case 3, k becomes less than $\bar{k}$, and converges to k^*. In this case, households do not own automation capital, that is, $P = 0$. Firms produce the final goods by using only labor and traditional capital as the standard Solow growth model. The long-run growth rate of per capita output is zero, and the labor share of income converges to a positive constant value.

3 Growth with declining population

Next, we consider the case in which population growth is negative, i.e., $n < 0$.

Even if $n < 0$, Cases 1–3 also hold true as long as $\delta + n > 0$. Therefore, when the population growth rate is negative and its absolute value is small, there are the three cases: the long-run growth rate of per capita output is positive when the saving rate of households is high while it is zero when the saving rate is intermediate or low.

However, when $n < 0$ and $\delta + n < 0$, i.e., the population growth rate is negative and its absolute value is large, a different case arises, which is referred to as Case 4. The dynamics of k in this case are illustrated in Figure 4. As the figure shows, per capita capital k continues to increase. In this case, the long-run growth rate of per capita output becomes $g_y^* = B > 0$, and the labor share of income converges to zero.

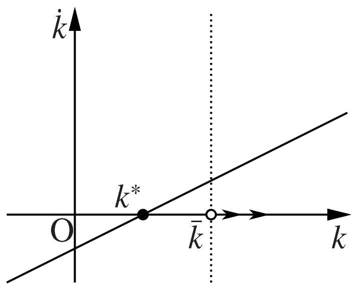

Figure 4: Dynamics of k in Case 4

Summarizing the above results, we obtain the following proposition.

Proposition 1. *When the population growth rate is negative and its absolute value is relatively small, we obtain the three cases as in a population increasing economy. The long-run growth rate of per capita output is positive when the saving rate is high whereas it is zero when the saving rate is relatively low. Alternatively, when the absolute value of the rate of population decline is large, the long-run growth rate of per capita output is positive irrespective the size of the saving rate.*

4 Numerical example

This section presents numerical examples for the Japanese economy as Japan has experienced steady population decline since 2010. For this purpose, we need to set the parameter values based on actual data and existing studies.

For the population growth rate, we use the long-run economic statistics of the Annual Report on the Japanese Economy and Public Finance 2021. The annual average rate of population decline in 2010–2020 is 0.19%, and hence, we have $n = -0.0019$. For capital share, we use $\alpha = 0.3$, which is commonly used for numerical simulations in macroeconomics. For the depreciation rate, we use $\delta = 0.07$, which is reasonable for the Japanese economy. For the saving rate of households, we employ Unayama and Yoneda's (2018) empirical analysis. They calculated adjusted saving rates and according to their results, the saving rate of households in Japan averages out around 10%. Therefore, we use $s = 0.1$.

These parameter values corresponds to Case 2 of our model, from which the Japanese economy converges to its steady state in the long run. Then, we have

$$g_y^* = 0, \quad \sigma_L^* = -\frac{B}{\delta + n} = 0.20. \tag{10}$$

Accordingly, without exogenous technological progress, the growth rate of per capita output in Japan will be zero in the long run.

If the saving rate increased to $s = 0.15$, the Japanese economy would correspond to Case 1, from which we obtain

$$g_y^* = B = 0.013, \quad \sigma_L^* = 0. \tag{11}$$

Therefore, under the present population decline, per capita output in Japan can attain sustainable growth if the saving rate is raised, that is, $s > (\delta + n)/R = 0.125$.

5 Conclusion

This study presents a growth model with automation capital, and investigates whether per capita output can attain sustainable growth when the population is declining. The results indicate that it can grow at a positive constant rate depending on the conditions.

Our model is based on the Solow growth model in which the saving rate of households is constant. In this case, in the long run, the growth rate of per capita capital and that of per capita output are the same, and hence, that of per capita consumption is also equal to that of per capita capital. However, in the Ramsey type growth model, per capita consumption follows the Euler equation, and its growth rate is not necessarily equal to that of per capita capital. Therefore, we may obtain different results compared to the Solow type model. Analysis using the Ramsey model is left for future research.

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