How Do Firing Costs Affect Innovation and Growth when Workers’ Ability is Unknown? – Employment Protection as a Burden on a Firm’s Screening Process

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August 2008

Online at https://mpra.ub.uni-muenchen.de/11410/
MPRA Paper No. 11410, posted 8. November 2008 15:50 UTC
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Abstract
This paper analyzes the implication of employment protection legislation on a firm's screening process. We present a model in which human-capital-intensive firms (high-tech) with imperfect information about their workers' type attempt during a trial period to identify those incompetent workers who they will subsequently dismiss. Employment protection measures, however, place a burden on this screening process and thereby motivate innovators to embark on medium-tech projects which are more flexible in their human capital requirements. Employment protection legislation thereby distorts the pattern of specialization in favor of medium-tech firms rather than high-tech firms and consequently slows down the process of economic growth. The results of the paper are consistent with documented data on Europe versus US productivity growth and specialization patterns as well as with employment protection legislation in those economies.
1. Introduction

In recent decades there has been a dramatic rise in US productivity and output growth accompanied by a remarkable expansion of US human-capital-intensive industries. These trends, however, were paralleled by different paths in most European countries of lower growth rates and a tendency to specialize in less human-capital-intensive technologies. Several recent theories suggest that these differences can be related to a host of labor market regulations (such as, minimum wage laws, unemployment subsidies and firing costs) that are rarely applied in the US but are extensively used in Europe. For example, Hopenhayn and Rogerson (1993) argue that a tax on job destruction, such as a firing cost in various European countries, may slow down the reallocation of resources from declining industries to growing industries thereby hampering economic growth by reducing productivity. Another explanation by Saint-Paul (1997) and (2002) is that, due to innovation risks, employment protection legislation might distort the pattern of specialization in favor of mature goods rather than primary innovation, which negatively affects productivity and growth. Other more recent works emphasize the effect of labor market regulation on delays and barriers to technology adoption (see Gust and Marquez (2004) and Alesina and Zeire (2006)).

In this paper we offer another explanation to the differences between European and US productivity and specialization patterns which is based on the burden that firing costs impose on a firm’s screening process. We show that high-tech firms with imperfect information about their workers' ability attempt during a trial period to identify those incompetent workers who they will subsequently dismiss. Firing costs stemming from employment protection legislation, however, place a burden on this

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screening process, thereby motivating innovators to embark on projects which are more flexible in their labor requirements (such as medium-tech projects). Employment protection legislation therefore distorts the pattern of specialization in favor of medium-tech firms rather than high-tech firms and consequently slows down the process of economic growth.

The paper presents a model in which a final good is produced by many intermediate goods that can be upgraded in a quality-ladder fashion (see Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992)). These intermediate goods, however, are not identical, since they differ in their productivity rates per quality rank and their labor requirements. Specifically:

1) High-tech intermediate goods are much more human-capital-intensive than medium-tech goods, and suffer from lower substitutability between skilled and unskilled workers.

2) Per quality rank, high-tech intermediate goods are much more productive than medium-tech goods and therefore generate higher economic growth.

An important assumption of the paper is that the workers' type is unknown, and is revealed only after a certain period of employment (which we henceforth term the "trial period"). Following this trial period, both medium-tech and high-tech firms have the opportunity to dismiss any incompetent workers. However, due to differences in labor requirements, only high-tech firms have the incentive to dismiss incompetent workers, whereas medium-tech firms can continue to keep them with no significant loss of profits. Thus, firing costs affect the profit function of high-tech firms significantly more than medium-tech firms and therefore affect the decisions of innovators of whether to embark on a high or medium-tech project.
The paper has three central results. First, employment protection legislation (and various firing costs that stem from them) biases the pattern of specialization from human-capital-intensive products toward less human-capital-intensive products. Second, firing costs can negatively affect economic growth. In closed economies this negative effect is unambiguous, while in open economies the magnitude of this negative effect depends on firms' adjustment costs. Third, employment protection might trap the economy into adopting inferior technologies which can affect the trajectory of innovation and growth over a long period of time. A major consequence of this latter result is that measures taken belatedly to reduce firing costs might prove ineffective.

The paper also relates to two other important issues. The first concerns the impact of firing costs on worker flows in the face of adverse selection. This issue was recently analyzed by Kugler and Saint-Paul (2004) who showed that firms who face high firing costs tend to hire workers from a pool of already employed workers rather than from a pool of unemployed workers who are most likely to be lemons. In our paper, imperfect information on workers' skills and firing costs also play a significant role, however not by affecting worker flows, but rather by affecting project selection and thereby specialization and growth.

The second issue our model addresses is why income and growth vary across countries. This problem has been addressed in a literature that focuses mainly on income and growth differences between developed and underdeveloped economies, that are either caused by impediments to technology adoption and/or differences in institutions and human capital accumulation. Unlike this literature, our paper

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compares the pattern of growth between two developed economies that differ in growth patterns due to differences in regulation and specialization.

The rest of the paper is organized as follows. Section 2 sets up the basic model; Section 3 relates employment protection legislation to growth in closed as well as in open economies; Section 4 extends the basic model; and Section 5 concludes. The mathematical proofs appear in an appendix.

2. The Model

Consider a closed economy whose activities extend over an infinite discrete time. The economy consists of three types of goods: a final good $Y$ that is used for consumption only, and two types of continuum intermediate goods $x_i$ and $z_i$ which we denote by “medium” and “high,” respectively. The quality of both the “medium” and “high” intermediate goods can be potentially improved over time in a quality-ladder fashion (see Grossman and Helpman (1991a, 1991b) and Aghion and Howitt (1992)), however they differ in their improvement rate (i.e., quality rank intervals). Formally, the final good is produced by intermediate goods $x_i$ and $z_i$ in a constant return to scale technology which is given by:

$$Y = \left[ \int_0^\infty \left( \sum_{j=0}^{\infty} \left( (\lambda_1(i))^j \cdot x_{i,j} \right) \right)^\sigma di + \int_0^\infty \left( \sum_{j=0}^{\infty} \left( (\lambda_2(i))^j \cdot z_{i,j} \right) \right)^\sigma di \right]^{1/\sigma}$$

(1)

where $0<\sigma<1$ (i.e., the elasticity of substitution between factors of production $1/(1-\sigma)$ is higher than one); $x_{i,j}$ and $z_{i,j}$ are the quantities of intermediate goods of types $x$ and $z$ of quality $j$ in product line $i$; and $\lambda_1(i)$ and $\lambda_2(i)$ are parameters that reflect the improvement rate of intermediate goods $x_i$ and $z_i$, respectively.

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3The open economy analysis is left for the next section.
To keep the analysis simple and to highlight the effects of interest, several assumptions are used:

(1) High-tech products are much more productive per quality rank than the medium-tech products, which implies that for any two product lines $i_x$ and $i_z$ of intermediate goods of types $x$ and $z$, the inequality $1 < \lambda_x(i_x) < \lambda_z(i_z)$ must hold.

(2) All products of type $x$ have identical quality rank intervals (such that $\lambda_1$ is constant across all intermediate goods of type $x$).

(3) Intermediate goods of type $z$ are positioned by a decreasing order of their quality rank intervals such that $\lambda_z(i_1) \geq \lambda_z(i_2)$ for all $i_1 < i_2$. We assume that $\lambda_z(i)$ is a twice-differentiable monotonically non-increasing and weakly convex function of $i$ (i.e., $\lambda_z'(i) \leq 0$ and $\lambda_z''(i) \geq 0$).

(4) To reach an equilibrium in which innovators always employ a limit pricing strategy we also assume that $\lambda_1 < \lambda_z(i) < \sigma^{(\sigma-1)}$.

The quality rank intervals of high-tech and medium-tech products are shown in Figure (1).

[Insert Figure 1 Here]

2.1 Individuals

At each period of time, a generation $L$ of individuals is born. All individuals live for one period only and have identical concave preferences denoted by $u = u(c)$. There are two types of individuals in the economy: a portion $(1-\mu)$ of less-competent individuals and a portion $\mu$ of most-competent individuals $(0 < \mu < 1)$. Less-competent and most-competent individuals differ in productivity when employed in the production process of intermediate goods.
Within the population of competent individuals, there exists a measure $\nu > 0$ of individuals who we henceforth refer to as "innovators." It is assumed that innovators are the only individuals in the economy who have the skill to upgrade existing intermediate goods. This skill is manifested in the ability of innovators to establish firms through which they upgrade existing products and subsequently manufacture them.\(^4\)

Ex-ante, members of the economy know the probabilities of being either incompetent or competent (i.e., $\mu$ and $(1-\mu)$, respectively), however, they do not know their own type nor the types of others.\(^5\) During the production process, however, employers can reveal their workers' type if they employ them for at least a $0 < \beta < 1$ unit of time. We therefore refer to the sub-period $[0, \beta)$ as a "trial-period."\(^6\)

2.2 Intermediate Goods Production

All intermediate goods are produced by a linear production function in which labor is the only primary factor. Intermediate goods differ, however, in their labor requirements. We assume that one unit of intermediate good of type $x$ (regardless of whether $x$ is of an old vintage or a state-of-the-art product) is produced by either one unit of competent workers or $1/\theta$ units of less-competent workers (where $\theta > 1$).

\(^4\) To simplify the analysis, we ignore the fact that when innovators establish innovative firms, their skill is revealed. We justify this by assuming that the number of innovators $\nu$ is relatively very small compared to the total number of skilled individuals in the economy.

\(^5\) In section 4 we show that the results of the model hold even when we relax this assumption and assume instead asymmetric information (i.e., that all individuals know their own type but do not know the type of others).

\(^6\) In this model we assumed an "ex-post screening process" whereby workers are employed until detected and then are dismissed. An alternative modeling would be to assume an "ex-ante screening process" in which employers test workers before they are employed. Obviously, the "ex-ante screening process" is less accurate than the "ex-post screening process", and become more reliable when firms spend more resources on screening. Thus, both type of screening (ex-ante and ex-post) become more costly when firing costs rise and therefore create a similar (negative) effect on high-tech firms' profit function.
Intermediate goods of type $z$, however, differ in their production technology according to whether they are of an old or new vintage. Specifically, one unit of an old vintage of $z_{i,j}$ can be produced by either one unit of less-competent or $(1/\theta)<1$ units of competent workers, whereas a state-of-the-art product $z_{i,j}$ can be produced by competent workers only. Formally, the production functions of intermediate goods $x$ and $z$ are given by:

$$
\begin{align*}
    x_{i,j} &= l_u + \theta \cdot l_s \\
    z_{i,j} &= \begin{cases} 
        l_u + \theta \cdot l_s & \text{if } z_{i,j} \text{ is of an old vintage} \\
        \theta \cdot l_s & \text{if } z_{i,j} \text{ is a state-of-the-art product}
    \end{cases}
\end{align*}
$$

where $l_u$ and $l_s$ are labor inputs of less-competent and most-competent workers, respectively. The differences in labor requirements between high tech and medium tech products are a core assumption in our model. It implies that the appropriability of workers in the high-tech firms is much more significant than in the medium-tech firms, since, competent and less-competent workers are highly substitutable in the medium-tech sector, whereas in the high-tech sector, less-competent workers are totally unproductive. The production function (2) implies that medium-tech producers have no incentive to dismiss workers, while high-tech producers would like to dismiss incompetent workers as soon as they are revealed (i.e., subsequent to the trial period).  

For the sake of concreteness, we add two additional assumptions. The first is that the production technology of old-vintage products can be instantly and costlessly adopted, while the production technology of the state-of-the-art products can be

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7 The specific production functions in equation (2) are chosen for simplicity only, and the results carry through with other production functions in which less-competent and most-competent workers exhibit significantly higher substitutability in medium-tech than high-tech products.
adopted only after one period of time. Second, we assume that all innovators can ensure the success of the innovation process by carrying-out some R&D activities that cost $G$ units of the final good $Y$. $G$ is assumed to be very small and identical for all innovators in all sectors.

The first assumption implies that innovators who upgrade the quality of intermediate goods became monopolistic producers for one period only, and then are replaced. The second assumption allows us to focus on the effect of incomplete information in the allocation of resources by disentangling it from other effects (such as differences in R&D risks as in Saint-Paul (1997) and (2002)).

2.4 Equilibrium

We start our analysis by describing how wages are determined. During the trial period, all workers (competent as well as incompetent) are paid in equilibrium a salary $w_u$ which is exactly the marginal productivity of incompetent workers. When the workers type is revealed, however, the salary of competent workers (who can now be identified) becomes $\theta w_u$, while the salary of incompetent workers remains $w_u$. We also assume that competent workers have a sufficient bargaining power to demand a full compensation for the loss of their income $[\beta(\theta-1)w_u]$ caused by the inability of their employers to identify them during the trial period.\(^8\) We assume that workers who were dismissed from their high-tech jobs do not bear frictional searching costs since they immediately find an alternative job in the medium tech sector.\(^9\)

Let the final good $Y$ serve as a numeraire. Profit maximization by firms, which produce the final good, leads to the following first order condition:

\(^8\)This assumption does not affect the results of the paper, but it does significantly simplify the innovators' profit functions.

\(^9\)In section 4 we relax this assumption and show that the results of the paper still hold even when workers who were dismissed bear a certain level of frictional searching costs.
\[ p(x_{i,j}) = \frac{\partial Y}{\partial x_{i,j}} = \left( \frac{Y}{x_{i,j}} \right)^{1-\sigma} (\lambda_i)^{\sigma-j} \]

\[ p(z_{i,j}) = \frac{\partial Y}{\partial z_{i,j}} = \left( \frac{Y}{z_{i,j}} \right)^{1-\sigma} (\lambda_2(i))^{\sigma-j} \]

At each period of time \( t \), final good producers can purchase intermediate goods from both competitive as well as monopolist producers. In the case where intermediate good \( x_{i,j} \) and \( z_{i,j} \) are purchased from competitive firms, their competitive prices must be equal to their constant marginal costs (i.e., \( p(x_{i,j}) = p(z_{i,j}) = w \)). If, however, the intermediate goods \( x_{i(k)i} \) and \( z_{i(k)i} \) are purchased from monopolistic innovators (who just tapped the state-of-the-art products), then final good firms will be willing to pay a premium for these products as long as their prices are not higher than \( \lambda_1 p(x) \) and \( \lambda_2(i) p(z_i) \), respectively. Innovators, who own the monopolistic firms, would clearly not charge prices below \( \lambda_1 p(x) \) and \( \lambda_2 p(z_i) \), respectively. Thus, the monopolistic prices are given by:

\[ p(x) = \lambda_1 w \]
\[ p(z_i) = \lambda_2(i) w \]

where \( w \) is the real wage of unskilled workers. By substituting equation (4) into equation (3), we get the final good producers demand functions for products \( x \) and \( z \):

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*10 The assumption that \( 1 < \lambda_1 < \lambda_2(i) < \sigma^{(\sigma-1)} \) implies that firms who produce the state-of-the-art products (leaders) always employ a limit-pricing strategy. Under an alternative condition that \( w \sigma^{(\sigma-1)} < \lambda_1 < \lambda_2 \), innovators would set a price which is not lower than \( w \sigma^{(\sigma-1)} \). The assumption that \( \lambda_1 < \lambda_2 < \sigma^{(\sigma-1)} \), however, ensures that monopolistic prices can never be reached and instead the innovators can set a price that is sufficiently below the monopoly price so as to make it just slightly unprofitable for potential producers of the last version of the product.*
To characterize the repercussion of mandatory firing costs on the pattern of specialization, we now describe how innovators rank intermediate goods in terms of profits. Note that due to labor requirements, only the innovators who operate in the high-tech sector (z products) might be motivated to dismiss workers. Thus, mandatory firing costs, if they exist, appear only in the high-tech firms' profit function.

Suppose that at the beginning of some period $t$, an innovator considers whether to operate in the high-tech or the medium-tech sector. If the innovator decides to operate in the medium-tech sector then his profit function is given by:

$$ \pi(x) = wx(\lambda - 1) - G $$

(6)

If the innovator decides to operate in the high-tech sector (i.e., to produce a state-of-the-art product of type $z$), then he cannot detect, at least not during the trial period $[0, \beta)$, whether the workers he hires are productive or not. Thus, innovators who improve and produce intermediate goods of type $z$ hire workers only of whom a portion $\mu$ is productive. Innovators therefore face two alternative paths. They can either dismiss the unproductive workers when detected (after a $[0, \beta$) sub-period of time) and bear the mandatory firing costs, or can continue hiring them throughout the production process while bearing the costs of paying them salaries. The profit functions under each alternative path are given by:

Innovators may use another screening process in which they test workers before they are employed. It is easy to see that this "ex-ante screening process" is equivalent to the "ex-post screening process" assumed above. The higher are firing costs $F$, the more reliable is the required screening process and the higher are the firms' outlays. Thus, both type of screening (ex-ante and ex-post) become more...
\[ \Pi_z(F) = z \cdot w \left[ (\lambda_z(i) - 1) - \beta(1 + F) \frac{(1-\mu)}{\mu\theta} \right] - G \]  
\[ \Pi_{\text{do not dismiss}} = z \cdot w \left[ (\lambda_z(i) - 1) - \frac{(1-\mu)}{\mu\theta} \right] - G \]  

where \( F \) is a mandatory rate of firing costs expressed by the portion of wage income that employers must compensate dismissed employees. The following lemma relates the mandatory firing costs to operating profits in the high-tech sector.

**Lemma 1:**

(i) Whenever the mandatory firing cost \( F \) is lower than \( \beta \frac{(1-\mu)}{\mu\theta} \), innovators who upgrade and produce a state-of-the-art product of type \( z \) would rather dismiss unproductive workers than continue hiring them. If, however, \( F > \beta \frac{(1-\mu)}{\mu\theta} \), innovators who upgrade and produce a state-of-the-art product of type \( z \) would rather continue to hire unproductive workers than dismiss them.

(ii) For any product \( i \) of type \( z \), such that \( \lambda_z(i) > \frac{1+\mu(\theta-1)}{\mu\theta} \), operating profits are positive for all possible mandatory firing costs \( F \).

(iii) For any product \( i \) of type \( z \), such that \( \frac{\beta + \mu(\theta-\beta)}{\mu\theta} < \lambda_z(i) < \frac{1+\mu(\theta-1)}{\mu\theta} \), there exists a threshold value \( \tilde{f}(i) = \frac{(\mu\theta)\lambda_z(i) - (\beta + \mu(\theta-\beta))}{\beta(1-\mu)} \) such that any mandatory firing cost \( F > \tilde{f}(i) \) necessarily leads to negative operating profits.

**Proof:** Follows immediately from equation (7). □

Innovators' profit functions in each sector as given in equations (6) and (7) allow us also to establish the conditions under which innovators rank a certain \( z_r \)-
project either higher or lower than $x$-projects. Obviously, these ranking conditions also depend on the mandatory firing cost $F$. The following Lemma states these conditions:

**Lemma 2:** Suppose that $1 < \lambda_i < \lambda_z(i) < 1 + \frac{1}{\mu \beta}$, and that at period $t-1$, the quality rank of all products of type $x$ is $(j_x-1)$. Then, for any product $i$ of type $z$ with quality rank $(j_z-1)$, there exists a threshold value:

$$\tilde{f}(i) = \min \left\{ \frac{(\lambda_z(i) - 1)}{\beta}, \frac{(\lambda_z(i) - 1)}{\lambda_z(i) - 1} \left( \frac{1}{\beta}, \frac{\beta^{1/2}}{\mu \beta} \right)^{1/2} \left( \frac{1}{\beta}, \frac{\beta^{1/2}}{\mu \beta} \right)^{1/2} \right\}$$

such that whenever the mandatory firing cost $F$ is higher than $\tilde{f}(i)$, innovators rank the $x$ projects higher than project $z_i$ (i.e., $\Pi_{z_i}(F) < \pi(x)$), and, vice-versa, whenever the mandatory firing cost $F$ is lower than $\tilde{f}(i)$, innovators rank the $z_i$ project higher than the $x$ projects (i.e., $\pi(x) < \Pi_{z_i}(F)$).

**Proof:** By substituting equation (5) into the profit functions that are given in equations (6) and (7) we get that: (i) if $1 < \lambda_i < \lambda_z(i) < 1 + \frac{1}{\mu \beta}$ then whenever $F \geq \frac{(1-\beta)}{\mu \beta}$, $x$ projects are always more lucrative than $z$ projects, and (ii) if $F < \frac{(1-\beta)}{\mu \beta}$ and $F < \tilde{f}(i)$ then project $z_i$ is more lucrative than projects of type $x$.\[\square\]

Note that since innovators' profits depend on the quality rank of the product they are updating and producing, the threshold value $\tilde{f}(i)$ must increase with the incipient quality rank of products $z$ $(j_z)$, and must decrease with $j_x$.

**Lemma 3:** $\tilde{f}(i)$ and $\tilde{f}(i)$ are non-increasing functions of $i$. 

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Proof: It easy to see that since $\mathcal{L}_2'(i) \leq 0$, the functions $\tilde{f}(i)$ and $\hat{f}(i)$ must satisfy

$$\frac{\partial \tilde{f}}{\partial(i)} \leq 0 \quad \text{and} \quad \frac{\partial \hat{f}}{\partial(i)} \leq 0. \Box$$

We now examine the effect of mandatory firing costs on the specialization pattern in the economy. Let us denote by $X_t \subseteq [0,1]$ and $Z_t \subseteq [0,1]$ the sets of intermediate goods of type $x$ and $z$, respectively, that innovators upgrade and produce at period $t$. Since intermediate goods of type $z$ are ordered by decreasing rates of improvement (per quality rank), and since innovators always prefer to be engaged in the most rewarding projects, then the set $Z_t$ must be either an empty or a closed interval of the form $Z_t = [0, \hat{z}_t]$.\footnote{The set $Z_t$, if nonempty, always contains a continuum of intermediate goods such that if $i \in Z_t$, then every intermediate good $j$ of type $z$ with lower index $j < i$ (which is by definition more or at least equally rewarding than $i$) must belong to $Z_t$ as well.} Note that the set $X_t$ need not be path-connected, however, since all projects in $X_t$ are equally rewarding, we can assume, with no loss of generality, that the set $X_t$, if nonempty, is always a closed interval of the form $[0,a]$.

**Definition 1:** Let $A$ and $B$ be two economies with an identical number of innovators $\nu$. Economy $A$ is said to be more specialized in high-tech projects than economy $B$ (while economy $B$ is said to be more specialized in medium-tech projects than $A$) if $X_t(A) \subset X_t(B)$ and $Z_t(A) \supset Z_t(B)$.

We now show that due to Lemmas (1),(2) and (3), high mandatory firing costs bias the pattern of specialization toward medium-tech products. Specifically, we show that whenever the incipient quality rank of products $z$ are at least equal to the incipient quality rank of products $x$ (i.e., $j_z \leq j_x$), then $Z_t(F)$ is a non-increasing monotonic set.
function of the firing cost $F$ while $X_i(F)$ is non-decreasing monotonic set function of $F$\textsuperscript{13}. We start our analysis with the following definition and notations.

**Definition 2:** For a given firing cost $F$:

1. Let $\tilde{i}(F)$ denote the highest index $i$ of intermediate goods of type $z$ that yields non-negative operating profits. Formally, $\tilde{i}(F) = \tilde{f}^{-1}(F)$.

2. Let $\bar{i}(F)$ denote the lowest index $i$ of intermediate goods of type $z$ that is ranked either higher or equal to the most profitable projects of type $x$. Formally: $\bar{i}(F) = \tilde{f}^{-1}(F)$\textsuperscript{14}.

3. Let $F^*$ and $F^{**}$ denote the lowest mandatory firing cost such that $\tilde{i}(F^*) = 0$, and $\bar{i}(F^{**}) = 0$, respectively\textsuperscript{15}. The threshold conditions expressed by the functions $\tilde{i}(F)$ and $\bar{i}(F)$ as well as the points $F^*$ and $F^{**}$ are shown in Figure 2.

![Insert Figure 2 Here](image)

**Lemma 4:** If the incipient quality rank of high and medium tech sectors are identical (i.e., $j_x = j_z$), then, the set functions $Z_i(F)$ and $X_i(F)$ are uniquely determined by the number of innovators $\nu$ and the functions $\tilde{i}(F)$ and $\bar{i}(F)$. Namely, if the number of innovators is lower than one (i.e., $0 \leq \nu \leq 1$), then

$$Z_i(F) = \begin{cases} [0, \nu] & \text{if } \tilde{i}(F) > \nu \\ [0, \bar{i}(F)] & \text{if } \bar{i}(F) \leq \nu \text{ and } F < F^{**} \\ \phi & \text{if } F \geq F^{**} \end{cases}$$

and

\textsuperscript{13} Thus, if $F_1 < F_2$ then $X_i(F_1) \subseteq X_i(F_2)$ while $Z_i(F_2) \subseteq Z_i(F_1)$.

\textsuperscript{14} The function $\bar{i}(F) = \tilde{f}^{-1}(F)$ can be considered as the marginal index of products $z$ that makes innovators indifferent between project $x$ and project $z_i$.

\textsuperscript{15} Note that since all projects of type $x$ gain positive operating profits, then whenever $j_i \leq j_i$, the inequalities $\tilde{i}(F) < \bar{i}(F)$ and thereby $F^* < F^{**}$ must hold.
\[
X_1(F) = \begin{cases} 
\phi & \text{if } \bar{i}(F) > \nu \\
[0, \nu - \bar{i}(F)] & \text{if } \bar{i}(F) \leq \nu \text{ and } F < F^{**} \\
[0, \nu] & \text{if } F \geq F^{**}
\end{cases}
\]

(This case is shown in Figure 3-A below.)

If the number of innovators is higher than one but lower than two (i.e., \(1 \leq \nu < 2\)) then

\[
Z_1(F) = \begin{cases} 
[0, \bar{i}(F)] & \text{if } \bar{i}(F) > \nu - 1 \\
[0, \bar{i}(F)] & \text{if } \bar{i}(F) \leq \nu - 1 \\
\phi & \text{if } F > F^{**}
\end{cases}
\]

and

\[
X_1(F) = \begin{cases} 
[0, (\nu - \bar{i}(F))] & \text{if } F \leq F^{**} \\
[0, 1] & \text{if } F > F^{**}
\end{cases}
\]

(This case is shown in Figure 3-B below.)

If the number of innovators is higher than two (i.e., \(2 \leq \nu\)) then

\[
Z_1(F) = \begin{cases} 
[0, (1 - \bar{i}(F))] & \text{if } F \leq F^* \\
\phi & \text{if } F > F^*
\end{cases}
\]

and

\[
X_1(F) = [0, 1]
\]

(This case is shown in Figure 3-C below.)

An immediate consequence of Lemma 4 is the following fundamental result.

**Proposition 1:** Whenever \( j_i = j_\nu \), high mandatory firing costs bias the pattern of specialization toward medium-tech products. Namely, the inclusion relations

\( X_1(F_1) \subseteq X_1(F_2) \) and \( Z_1(F_1) \supseteq Z_1(F_2) \) hold for any two mandatory firing costs \( F_1 \) and \( F_2 \) such that \( F_1 < F_2 \).

[Insert Figures 3-A, 3-B, and 3-C here]
costs might, under certain conditions, create technological traps, in that, if a policy of firing-cost-reduction is adopted too late, it will be ineffective in shifting innovators from medium to high-tech products. To demonstrate this phenomenon let us assume, for example, that due to high firing costs, a certain economy has specialized in medium-tech products for a significant period of time, whereby the quality rank of products that innovators persistently developed and produced grew, while the quality rank of high-tech projects that were previously abandoned stagnated. Under such conditions, a policy that strives to reduce firing costs might not be effective since profits (even with zero firing costs) in abandoned high-tech products become significantly lower than already developed and produced products. The following proposition state the condition under which this "technological trap" occurs.

**Proposition 2:** Suppose that the number of innovators in a certain economy is not higher than the number of intermediate goods (i.e., 0<v<2), and suppose that after some period $t>0$, the mandatory firing cost $F$ is reduced to zero. If $t>0$ is sufficiently high, then the reduction of $F$ will not shift specialization toward high-tech products.

**Proof:** See the Appendix.

To complete the equilibrium analysis, we briefly describe real wage and aggregate output determination (the calculations are presented in appendix A-2). Suppose that $\hat{m}$ innovators operate in the medium-tech sector while $\hat{h}$ innovators operate in the high-tech sector. Suppose also that in a certain period $t$, the quality ranks of the state-of-the-art products in the medium and high tech sectors are $j_x$ and $j_z$.,
respectively. By substituting equation (5) and the parameters \( \hat{m}, \hat{h}, j, j_i \) into equation (1) we get the equilibrium wage rates:

\[
w_u = \left[ \hat{m} \left( \frac{\lambda_0}{\lambda_1} \right)^{\frac{\hat{h}}{j}} \right]^{\frac{1}{\sigma}} + \frac{1}{\lambda_1} \int_{0}^{\frac{\hat{h}}{j}} \left( \frac{\lambda_2(i)}{\lambda_1(i)} \right)^{\frac{\hat{h}}{j}} \, di
\]

\[
w_s = \theta \cdot w_u
\]

Equation (8) can also be obtained by substituting equation (5) into the following labor-market clearing condition:

\[
\begin{align*}
L &= \hat{m} \cdot \frac{1}{[1 - \mu + \mu \theta]} \cdot \left( \frac{Y(i)}{\lambda_1(i)} \right)^{\frac{\hat{h}}{j}} + \frac{1}{\theta} \int_{0}^{\frac{\hat{h}}{j}} \left( \frac{Y(i)}{\lambda_2(i)} \right)^{\frac{\hat{h}}{j}} \, di \quad \text{during } [0, \beta) \\
L &= \hat{m} \cdot \frac{1}{[1 - \mu + \theta \alpha]} \cdot \left( \frac{Y(i)}{\lambda_1(i)} \right)^{\frac{\hat{h}}{j}} + \frac{1}{\theta} \int_{0}^{\frac{\hat{h}}{j}} \left( \frac{Y(i)}{\lambda_2(i)} \right)^{\frac{\hat{h}}{j}} \, di \quad \text{during } [\beta, 1]
\end{align*}
\]

where \( \alpha = \mu - \frac{1}{\theta L} \cdot \int_{0}^{\frac{\hat{h}}{j}} \left( \frac{Y(i)}{\lambda_2(i)} \right)^{\frac{\hat{h}}{j}} \, di \) (For more details see Appendix A-2.)

From equations (1), (5) and (9) we can derive the final good output, which is given by:

16 Note that the CRS property of the production technology as well as the linearity property of intermediate goods’ production technology ensures that the equilibrium wage rate \( w_u \) does not depend on labor supply nor output. Thus, equilibrium wage rates \( w_u \) and \( w_s \) are not affected by the movement of incompetent workers from the high to the medium-tech sector at time \( \beta \).

17 Note that if \( \hat{m} > 0 \) and \( \hat{h} > 0 \), then aggregate production of the final good \( Y \) must rise after the trial period \([0, \beta)\) since unskilled workers, who were totally unproductive in the high tech sector during \([0, \beta),\) are dismissed and then start working in the medium tech sector where they become productive.
3. Comparative Analysis with Macroeconomic Applications

To demonstrate how employment protection legislation affects economic performance we compare between two economies, one with high mandatory firing cost and the other with low mandatory firing costs. We start by assuming that both economies are closed to trade. In the next subsection we relax this assumption and discuss the applications of mandatory firing costs to open economies.

3.1 The Closed Economies Case

Assumptions:

(A-1) There exist two closed economies A and B with identical population size $L$ and identical number of innovators $\nu$.

(A-2) Both economies are launched at period $t=0$, such that quality ranks in both economies at period $t=0$ is $j=j_x=j_z=0$.

(A-3) The mandatory firing cost in economies A and B denoted by $F_A$ and $F_B$ satisfies the following condition: $0 < F_A < F_B < \frac{\mu}{\rho}$.

Assumptions (1)-(3) as well as Lemma 4 and Proposition 1 guarantee that economy A is more (less) specialized in high-tech (medium-tech) production than
economy $B$, and that this pattern of specialization persists through all periods of time. The next Proposition and Lemma prove that this pattern of specialization affects the level as well as the growth rates of output and wages in each economy.

**Lemma 5:** Suppose that in a certain closed economy $0 < \hat{m} \leq 1$ innovators operate in the medium-tech sector while $0 < \hat{h} < 1$ innovators operate in the high-tech sector. Any reallocation of innovators from the medium-tech to the high-tech sector will necessarily lead to an increase in the level and growth rates of wage and output.

**Proof:** See the Appendix.

**Proposition 3:** Output and wage rates in economy $A$ are higher and grow faster than in economy $B$.

**Proof:** This proposition follows immediately from Lemma 5.

### 3.2 Open Economies

We now relax the assumption that economies $A$ and $B$ are closed, and assume instead that both economies $A$ and $B$ are open to trade. Since final good producers from both economies can now import new intermediate goods from abroad, they can potentially expand their product variety. Interestingly, if final good producers do not face installation (or adjustment) costs then Proposition (3) need not hold, and economies $A$ and $B$ might grow at similar rates. If, on the other hand, final good producers face adjustment costs when installing new brands of products, then economy $A$ might grow faster than economy $B$, at least for several periods.

To demonstrate how mandatory firing costs might affect growth patterns in open economies, let us assume that final good producers face high adjustment costs when purchasing new intermediate goods from abroad, but face relatively low adjustment
costs when purchasing these new intermediate goods from domestic sources. The rationale of this assumption is that final good producers who purchase new intermediate goods from domestic sources procure access to local expertise and proficiency which reduces their adjustment costs.

Let us denote by $M_t$ and $H_t$ the number of intermediate goods of types $x$ and $z$ that are already employed in the final good production process at the beginning of period $t$. Let us also denote by $\Delta M_t = M_{t+1} - M_t$ and $\Delta H_t = H_{t+1} - H_t$ the number of new intermediate goods of types $x$ and $z$ that final good producers intend to purchase and to install at period $t$. We assume that the higher the quantities and the quality ranks of the new installed intermediate goods are, the higher are the adjustment costs that final good producers must bear.

Formally, the adjustment costs function is given by:

$$C(\Delta M_t, \Delta H_t) = a \left[ n_x \frac{\Delta M_t^2}{2} \xi_x (\cdot) + n_z \frac{\Delta H_t^2}{2} \xi_z (\cdot) \right]$$

(11)

where $a = \left[ (1 - \sigma) / \sigma \right]$ is a constant parameter,

$n_x$, $n_z$ are parameters that reflect the effect of imports on adjustment costs (specifically, $n=1$ when intermediate goods are purchased from abroad and $n=\varepsilon$ ((where $\varepsilon > 0$ is very small)) when intermediate goods are purchased from domestic producers),

$\xi_x (\cdot)$ and $\xi_z (\cdot)$ are:

$$\xi_x (j_x, j_z, M_t, H_t) \overset{\text{def}}{=} \left( \frac{\lambda_x}{\lambda_x^{(j_x)}(j_z)} \right)^{\frac{1}{\sigma}} \times \left[ \left( \lambda_x^{(j_x+1)}(j_z) \right)^{\frac{1}{\sigma}} \left( \lambda_x(j_z) \right)^{\frac{1}{\sigma}} \right]^{\gamma} \left( \lambda_x^{(j_x)}(j_z) \right)^{\frac{1}{\sigma}} \frac{d\lambda_x}{\lambda_x}$$

$$\xi_z (j_x, j_z, M_t, H_t) \overset{\text{def}}{=} \left( \frac{\lambda_z}{\lambda_z(i)} \right)^{\frac{1}{\sigma}} \times \left[ \left( \lambda_z(j_x+1) \right)^{\frac{1}{\sigma}} \left( \lambda_z(j_x) \right)^{\frac{1}{\sigma}} \right]^{\gamma} \left( \lambda_z(i) \right)^{\frac{1}{\sigma}} \frac{d\lambda_z}{\lambda_z}$$
The profit function of a final good firm in both economies A and B is therefore:

\[ \Pi = Y - \int_0^{M_{t+1}} p_{z,j}(k)z_jdj - \int_0^{H_{t+1}} p_{x,j}(k)x_jdj - C(\Delta M_t, \Delta H_t) \]  

(12)

Profit maximization by firms leads to the following first order condition:

\[
\begin{bmatrix}
Y\left(\frac{(\lambda_2(i))^\sigma}{(\lambda_2(i)w)^\sigma}\right)[((1-\sigma)/\sigma)] = \frac{\partial}{\partial H_{t+1}} C(\Delta M_t, \Delta H_t) \\
Y\left(\frac{(\lambda_1(i))^\sigma}{(\lambda_1w)^\sigma}\right)[((1-\sigma)/\sigma)] = \frac{\partial}{\partial M_{t+1}} C(\Delta M_t, \Delta H_t)
\end{bmatrix}
\]

(13)

By substituting equations (8), (10) and (11) into equation (12) we get that the rate at which final good producers import medium and high tech products from abroad cannot be higher than 

\[
(\Delta M) = \frac{1}{(\lambda_1)^\sigma} \quad \text{and} \quad (\Delta H) = \frac{1}{(\lambda_2(H_{t+1}))^\sigma}, \text{ respectively.}
\]

Thus:

\[
(\Delta M) < \frac{1}{(\lambda_1)^\sigma} \quad \text{and} \quad (\Delta H) < \frac{1}{(\lambda_2(H_{t+1}))^\sigma}
\]

(14)

We now sketch a plausible trade-growth dynamics for economies A and B, which in our view might, at least to some extent, explain the US and the European trade-growth patterns from the mid 1980s until early 2000.

Let us assume that previous to period \( t= T \) all intermediate goods that exist in both economies are of type \( x \) only. At period \( t= T \), however, new technologies which allow innovators to develop intermediate goods of type \( z \), emerge. The emergence of these new types of intermediate goods is analogous to the ICT and other high tech products that appeared in the early 1980's mostly in the US (see OECD (2003) for a

\[ \text{Derivation of the profit function with respect to } M_{t+1} \text{ and } H_{t+1} \text{ yields:} \]

\[
y^{1-\sigma} \cdot (1/\sigma) \left(\left[(\lambda_2(i))^{\sigma} \cdot (\lambda_2(i)w)^{\sigma}\right] - p_{z,j}(k)z_j - \frac{\partial}{\partial H_{t+1}} C(\Delta M_t, \Delta H_t) \right)
\]

\[
y^{1-\sigma} \cdot (1/\sigma) \left(\left[(\lambda_1(i))^{\sigma} \cdot (\lambda_1w)^{\sigma}\right] - p_{x,j}(k)x_j = \frac{\partial}{\partial M_{t+1}} C(\Delta M_t, \Delta H_t) \right)
\]

Substituting equation (5) into these equations yield equation (13) above.
However, these technologies are not being developed everywhere. Suppose that firing costs in economy $A$ are lower than in economy $B$, and that the parameters $F_A$, $F_B$ and $T$ (the period at which the new technologies emerge) are such that the conditions in proposition (1) hold. Innovators in economy $A$, who face low firing costs, become biased toward high-tech intermediate goods projects (and therefore embark on $z$-projects), while innovators in economy $B$ who face relatively high firing costs continue to be engaged in medium-tech projects only. This change in the pattern of project selection affects growth and trade. Final good producers in economy $B$ import high-tech products (adopt technology) from producers in Economy $A$, while final good producers in economy $A$ import medium-tech products. However, since final good producers in economy $A$ have already used $x$ products (before period $T$) they do not face any adjustment costs when importing $x$ products from economy $B$, while final good producers in economy $B$ are required to adjust to the new $z$-products they import from economy $A$. Since this adjustment is costly, importation of $z$-products from economy $A$ is gradual rather than immediate. Economy $A$ thereby grows at a higher rate than economy $B$, while $B$ economy's growth rates converge to that of $A$.$^{19}$

4. Asymmetric Information

In the basic model we made two assumptions that to some readers may seem rather restrictive. The first assumption is that all individuals are symmetrically uninformed about their type (i.e., they do not know their own type or that of others), and the second assumption is that workers who were dismissed from their job, immediately find an alternative job and therefore do not bear frictional searching

---

$^{19}$ This pattern of growth trivially follows from proposition 5.
costs. In this section we show that the results of the paper carry through even when these two assumptions are relaxed. To adjust the basic model to this new setting, several additional assumptions are used. Specifically, we assume that:

- All individuals have a constant relative risk aversion utility function (i.e.,
  \[ u(c) = \left( \frac{1}{1-\gamma} \right)^{c(1-\gamma)} \text{ where } \gamma > 0. \]

- The production function of intermediate goods is given by:
  \[
  x_{i,j} = l_u + \theta_1 \cdot l_s \\
  z_{i,j} = \begin{cases} 
  l_u + \theta_1 \cdot l_s & \text{if } z_{i,j} \text{ is of an old vintage} \\
  \theta_2 \cdot l_s & \text{if } z_{i,j} \text{ is a state-of-the-art product}
  \end{cases} \quad (2')
  \]
  where \( l_u \) and \( l_s \) are labor inputs of less competent and competent workers, respectively, and \( 1 < \theta_1 < \theta_2 \) (i.e., competent workers are more productive in the high-tech firms than in the medium-tech firms).\(^{20}\)

- All individuals who are dismissed from their job lose a portion \( 0 < d < 1 \) of their salaries (i.e., an employee who was dismissed lose \( d \cdot w \)).\(^{21}\)

- Subsequent to the trial period \([0, \beta]\), employers can correctly identify the type of their workers with probability \((1-\varepsilon)\) and misidentify their workers' type with probability \( \varepsilon > 0 \) (\( \varepsilon \) is assumed to be relatively small and is common knowledge among all individuals).

We also assume that \( \varepsilon > 0 \) is, on the one hand small enough to ensure that all competent individuals would be willing to risk working in the high tech firms but, on

\(^{20}\) The modification we made to the intermediate goods' production function (equation (2')) in conjunction with the subsequent assumptions ensures that all workers (competent as well as less-competent workers) prefer to work in high-tech firms rather than medium-tech firms.

\(^{21}\) This is a simple way of modeling frictional costs of workers who search for new jobs. The parameter \( d \) can be regarded as the average portion of time that workers need to spend to find a new job (and therefore \( d \cdot w \) is their income loss).
the other hand is sufficiently high to ensure that competent individuals would also be willing risk working in the high-tech sector. Specifically:

\[
\frac{u(1) - u((1 - d))}{[u(1) - u((1 - d)) + u(\theta_2) - u(\theta_1)]} < \varepsilon < \frac{u(\theta_2) - u(\theta_1)}{[u(1) - u((1 - d)) + u(\theta_2) - u(\theta_1)]}
\]

We now adjust the innovators' profit functions \(6\) and \(7\) that have been formulized to the new assumptions of asymmetric information and frictional costs. The profit that innovators can gain in the \(x\) sector is:

\[
\pi_x = x|\pi|_{x} - G \quad (6')
\]

where \(c = \frac{[(1 - \varepsilon)\mu\rho + \varepsilon(1 - \mu)\rho_{i} + \varepsilon(1 - \mu)\rho_{i} + \varepsilon\rho]}{[\theta_1 + \mu(1 - \rho)]}\), and the profit that innovators can gain in the \(z\) sector is:

\[
\pi_z = \begin{cases} 
    w \cdot z \left[ \lambda_z(i) \cdot c - b \right] - \beta(1 + F) \frac{(1 - \mu)(1 - \varepsilon) + \varepsilon \mu}{\theta_2 \mu} & \text{dismiss} \\
    w \cdot z \left[ \lambda_z(i) \cdot c - b \right] - \frac{(1 - \mu)(1 - \varepsilon) + \varepsilon \mu}{\theta_2 \mu} & \text{do not dismiss}
\end{cases} \quad (7')
\]

where \(b = 1 + \frac{\varepsilon(1 - \mu)}{\mu(1 - \varepsilon)}\).

Lemmas 1 and 2 in the basic model can be easily adjusted to the new profit functions as follows.

**Lemma 1':**

(i) Whenever the mandatory firing cost \(F\) is lower than \(\frac{(1 - \beta)}{\beta}\), innovators who upgrade and produce a state-of-the-art product of type \(z\) would rather dismiss unproductive workers than continue to hire them. If, however, \(F > \frac{(1 - \beta)}{\beta}\), innovators who upgrade

22In order to ensure that less competent workers will always prefer working in the high-tech firms the parameters \(d, \varepsilon\) must satisfy: \((1 - \varepsilon) \cdot u(w) + \varepsilon \cdot u(w(1 - d)) < (1 - \varepsilon) \cdot u(w(1 - d)) + \varepsilon \cdot u(w(1 - d))\).

And to ensure that competent workers will always prefer working in the high-tech firms the parameters \(d, \varepsilon\) must satisfy: \((1 - \varepsilon) \cdot u(\theta_1 w) + \varepsilon \cdot u(w) < (1 - \varepsilon) \cdot u(\theta_2 w) + \varepsilon \cdot u(w(1 - d))\).
and produce a state-of-the-art product of type $z$ would rather continue to hire unproductive workers than to dismiss them.

(ii) For any product $i$ of type $z$, such that \( \lambda_z(i) > \frac{b \theta_{t, \mu} + (1 - \mu) X_{1x} + \gamma_{\mu}}{\epsilon_{\mu} \lambda_z} \), operating profits are positive for all possible mandatory firing costs $F$.

(iii) For any product $i$ of type $z$, such that \( \lambda_z(i) < \frac{b \theta_{t, \mu} + (1 - \mu) X_{1x} + \gamma_{\mu}}{\epsilon_{\mu} \lambda_z} \), there exists a threshold value \( \tilde{f}(i) = \frac{1}{\beta^{1 - \mu (1 - x) + \gamma_{\mu}} \lambda_z} - 1 \) such that any mandatory firing cost $F > \tilde{f}(i)$ necessarily leads to negative operating profits.

Lemma 2': Suppose that $1 < \lambda_t < \lambda_z(i) < \frac{b \theta_{t, \mu} + (1 - \mu) X_{1x} + \gamma_{\mu}}{\epsilon_{\mu} \lambda_z}$, and that at period $t-1$, the quality rank of all products of type $x$ is $(j_x - 1)$. Then, for any product $i$ of type $z$ with quality rank $(j_z - 1)$, there exists a threshold value:

\[
\tilde{f}(i) = \min\left\{ \frac{\theta_{t, \mu} (\lambda_z(i))^{1 - \frac{\epsilon_{\mu} \lambda_z}{\epsilon_{\mu} \lambda_z}} \left[ \frac{(1 - \mu) X_{1x} + \gamma_{\mu}}{\epsilon_{\mu} \lambda_z} \right]^{1 - \frac{\epsilon_{\mu} \lambda_z}{\epsilon_{\mu} \lambda_z}} - \left( \frac{\epsilon_{\mu} \lambda_z}{\epsilon_{\mu} \lambda_z} \right) \right\}
\]

such that whenever the mandatory firing cost $F$ is higher than $\tilde{f}(i)$, innovators rank the $x$ projects higher than project $z_i$ (i.e., $\Pi_{z_i}(F) < \Pi(x)$), and vice versa whenever the mandatory firing cost $F$ is lower than $\tilde{f}(i)$, innovators rank the $z_i$ project higher than the $x$ projects (i.e. $\Pi(x) < \Pi_{z_i}(F)$).

Lemmas 1' and 2' imply that the qualitative properties of the threshold conditions in the basic model are preserved under the new assumptions of the model. Since they guarantee that lemma 3 and lemma 4 in the previous section still hold, then the main result of the paper that employment protection legislation distorts the pattern of
specialization in favor of medium-tech firms rather than high-tech firms (and thereby slows down the process of economic growth) carries through.

5. Summary

This paper focuses on the burden that high firing costs place on the screening process of human-capital-intensive firms (high-tech). It is shown that when firing costs are high and workers productivity is ex ante unknown, innovators will embark on relatively less human-capital intensive projects, since the screening process become expensive. Firing costs distort the pattern of specialization toward medium-tech industries rather than high-tech industries and thereby affect output and labor productivity growth negatively. This negative effect becomes significant when adjustment costs of new products are high. These results are consistent with the US productivity revival in the 1990's as well as the evolving US-EU productivity gap.

References


OECD (2003), "ICT and Economic Growth- evidence from OECD countries industries and firms".

Appendix

A-1 Mathematical Proofs

Proof of Proposition 2: Let $\tilde{Z}$ denote the set of all products $i$ of type $z$ that innovators in economy $B$ did not produce due to high firing cost (i.e., $\tilde{Z} = [0,1] \setminus Z(F_B)$). Note that $j_{z_i} = 0$ for all products in $\tilde{Z}$.

Suppose now that at some period $T > 0$ where
\[
T > \frac{1}{\beta \sigma \log(\lambda_i)} \log \left( \frac{\left( \lambda_z(0) - 1 \right) - \left( \beta \frac{1 - \mu}{\mu} \right) \lambda_f / \lambda_z(0) \right)^{\frac{1}{\rho}}}{(\lambda_i - 1)} \right),
\]
Firing cost $F_B$ in economy $B$ is reduced to zero.

Since the quality rank of all other intermediate goods that are produced is equal to $j = T$ (and thereby $j_i = T$ to all products is $X_i(F_B)$) then the threshold value
\[
\bar{f}(i) = \min \left\{ \frac{\left( \lambda_z(0) - 1 \right) - \left( \beta \frac{1 - \mu}{\mu} \right) \lambda_f / \lambda_z(0) \right)^{\frac{1}{\rho}}}{(\lambda_i - 1)} \right),
\]
become less than zero for all products in $\tilde{Z}$.

Thus, even if firing cost $F_B$ has declined to zero, innovators will not find it optimal to shift their activities from $x$ projects to $z$ projects since the $z$-projects in $\tilde{Z}$ are less profitable than the $x$-projects that where already developed and produced. □

Proof of Lemma 5: First note that whenever $0 < \hat{m}, \hat{h}$ the quality rank of all intermediate goods that are employed in the economy in period $t$ is $j = t$.

Let $w_j = g(\hat{m}, \hat{h}, j)$ and $Y_i = f(\hat{m}, \hat{h}, j)$ denote real wage and aggregate output, respectively, as given in equations (8) and (10). It is easy to verify that:

(i) $\frac{d}{dt} g(\hat{m} - t, \hat{h} + t, j) > 0$,

(ii) $\frac{d}{dt} f(\hat{m} - t, \hat{h} + t, j) > 0$,
(iii) \( \frac{d}{dt} \left[ \frac{g(\hat{m} - t, \hat{h} + t, j + 1)}{g(\hat{m} - t, \hat{h} + t, j)} \right] > 0 \),

(iv) \( \frac{d}{dt} \left[ \frac{f(\hat{m} - t, \hat{h} + t, j + 1)}{f(\hat{m} - t, \hat{h} + t, j)} \right] > 0 \).

A-2 Labor Market Equilibrium and Final Good's Output Determination

Market clearing conditions in the labor market are:

\[
\begin{align*}
\bar{L} &= \hat{m} \cdot x^* + \frac{\hat{h}}{\mu} \cdot z^* \quad \text{during the } [0, \beta) \text{ testing period} \\
\bar{L} &= \hat{m} \cdot x^* + \hat{h} \cdot z^* \quad \text{during } [\beta, 1]
\end{align*}
\]

Substituting equation (5) into equation (*) yields:

\[
\begin{align*}
\bar{L} &= \hat{m} \cdot \left( \frac{Y(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_1^{\frac{1}{\sigma}}]} \right) + \frac{\hat{h}}{\mu} \cdot \left( \frac{Y(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_2^{\frac{1}{\sigma}}]} \right) \quad \text{during the } [0, \beta) \text{ testing period} \\
\bar{L} &= \hat{m} \cdot \left( \frac{Y(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_1^{\frac{1}{\sigma}}]} \right) + \frac{\hat{h}}{\mu} \cdot \left( \frac{Y(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_2^{\frac{1}{\sigma}}]} \right) \quad \text{during } [\beta, 1]
\end{align*}
\]

and therefore

\[
\begin{align*}
w &= \left( \frac{Y}{\bar{L}} \right)^{1-\sigma} \left\{ \hat{m} \cdot \left( \frac{(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_1^{\frac{1}{\sigma}}]} \right) + \frac{\hat{h}}{\mu} \cdot \left( \frac{(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_2^{\frac{1}{\sigma}}]} \right) \right\}^{1-\sigma} \quad \text{during the } [0, \beta) \text{ testing period} \\
w &= \left( \frac{Y}{\bar{L}} \right)^{1-\sigma} \left\{ \hat{m} \cdot \left( \frac{(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_1^{\frac{1}{\sigma}}]} \right) + \frac{\hat{h}}{\mu} \cdot \left( \frac{(\lambda_j)^{\frac{j}{\sigma}}}{[\lambda_2^{\frac{1}{\sigma}}]} \right) \right\}^{1-\sigma} \quad \text{during } [\beta, 1]
\end{align*}
\]

By substituting the last equation into equation (5) and then into equation (1) we get that at the trial period [0,\beta)
and at sub-period $[\beta, 1]$

\[
Y = \left[ \hat{\lambda} \left( \frac{\lambda_1}{\lambda_i^{\hat{\sigma}}} \right)^\sigma + \hat{\mu} \left( \frac{\lambda_2}{\lambda_i^{\hat{\sigma}}} \right)^\sigma \right] \cdot \overline{L}
\]

Substituting this final good output equation into the wage equation (***) yields the wage rate equilibrium:

\[
w = \left[ \hat{m} \left( \frac{\lambda_1}{\lambda_i^{\hat{\sigma}}} \right)^\sigma + \hat{h} \left( \frac{\lambda_2}{\lambda_i^{\hat{\sigma}}} \right)^\sigma \right]^{\frac{\beta}{\hat{\sigma}}}.\]
Figures

Figure 1.

Quality Rank’s Interval of Intermediate Goods

Figure 2
Figure 3-A

\[ X_t \]

\[ v \]

\[ Z_t \]

\[ v \]

\[ F \quad F^{**} \quad F^* \]
Figure 3-B