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# Search and Competition in Expert Markets

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**Abstract.** We develop a model in which consumers sequentially search experts for recommendations and prices to treat a problem, and experts simultaneously compete in these two dimensions. Consumers have either zero or a positive search cost. In equilibrium, experts may “cheat” by recommending an unnecessary treatment with positive probabilities, prices follow distributions that depend on a consumer’s problem type and the treatment, and consumers search with Bayesian belief updating about their problem types. Remarkably, as search cost decreases, both expert cheating and prices can *increase* stochastically. However, if search cost is sufficiently small, competition forces all experts to behave honestly.

**Keywords:** search, experts, competition, credence good

**JEL Classification Number:** D8, L1

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## 1. INTRODUCTION

Consumers often need to search sellers to find product and price information. An extensive literature, going back to Stigler (1961), has devoted to the study of consumer search and the implications of search cost on competition. The literature typically assumes that the products are search goods, the value of which can be observed from inspection. However, in markets such as those for auto repair, medical service, IT service, and financial service, a seller typically possesses superior information about the service or product that a consumer needs, even after the consumer's purchase. A well-known problem in such markets, often referred to as credence-goods or expert markets, is that expert sellers may "cheat" by recommending unnecessary services to consumers. Consumers may then need to search for both honest recommendations and low prices, while expert sellers will also compete in these two dimensions. How do expert markets work under such multidimensional search and competition? How will search frictions shape the role of competition in disciplining expert behavior? These are important questions that are yet to be answered for the economics literatures on consumer search and on expert markets.

When firms sell homogeneous products with known values to consumers, Stahl (1989) provides a seminal analysis of oligopoly price competition when consumers conduct sequential search. As search frictions decrease, competition intensifies monotonically, and the classical Bertrand outcome (marginal cost pricing) and Diamond outcome (monopoly pricing) are obtained as the limiting cases of his model, respectively when search cost is zero and the fraction of consumers with zero search cost is zero.<sup>1</sup> In expert markets, if the nature of a consumer's problem were public information, competition could be analogous to that for a homogeneous product. However, because only the experts may learn a consumer's problem and the appropriate treatment, an expert's recommendation and price can reveal information about whether he is being honest, and a consumer may perform Bayesian belief updating about her problem type through sequential search. This can substantially complicate the strategic

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<sup>1</sup>For differentiated products, a seminal contribution is Wolinsky (1986), in which market power also increases with search friction but equilibrium price is above marginal cost even as search cost goes to zero, because product differentiation also softens price competition.

choices of consumers and experts, but—as it turns out—they can be fruitfully analyzed in a model built on Stahl (1989). As our analysis of this model will show, with the additional market interactions, search frictions have richer and novel effects on competition and expert behavior.

We consider an expert market in which each consumer has a problem that can be either major ( $M$ ) or minor ( $m$ ). A major treatment ( $T_M$ ) can fix both types of the problem, but a minor treatment ( $T_m$ ) can only fix a minor problem. When visiting an expert, a consumer’s problem is learned by the expert, who can then offer the consumer a recommended treatment at a certain price. At no time can a consumer observe her problem type, so the treatment by an expert is a credence good.<sup>2</sup> The consumer can either accept the offer or search other experts sequentially for additional offers. The expert is obligated to solve the consumer’s problem if his offer is accepted, and the treatment performed is verifiable by the consumer. However, there can be higher profits from  $T_M$  than from  $T_m$ , which provides an incentive for experts to recommend  $T_M$ —possibly with some probability  $\alpha$ —even for the minor problem. We extend Stahl (1989) to study search for—and competition in—recommendations and prices in this market. As in Stahl (1989), we assume that fraction  $\lambda$  of the consumers are shoppers who have zero search cost to visit any expert, whereas the rest of the consumers are searchers who must incur a search cost ( $s$ ) to visit an expert.

In a symmetric perfect Bayesian equilibrium of the model, a shopper will purchase from the expert who can solve her problem at the lowest price, whereas searchers will adopt an optimal reservation price for each recommended treatment. The tension between attracting the shoppers and exploiting the searchers implies that, as in Stahl (1989), experts will choose treatment prices with mixed strategies, and they will always recommend  $T_M$  for  $M$  but may cheat by recommending  $T_M$  also for  $m$ . Specifically, when  $s$  is above some threshold, initially experts will cheat with probability  $\alpha^* \in (0, 1)$ , which we term as the hybrid equilibrium; whereas when  $s$  is high enough, experts will always cheat (with  $\alpha^* = 1$ ), which we term as the

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<sup>2</sup>For example, the air conditioner in a consumer’s car is not cooling. The problem could be either a faulty compressor or inadequate refrigerant. Replacing the compressor will fix both types of the problem, but adding refrigerant can only fix the latter. An auto mechanic will know what the problem is but the consumer does not.

pooling equilibrium. However, when  $s$  is sufficiently small, the model has a unique separating equilibrium, where experts truthfully recommend  $T_m$  for  $m$  ( $\alpha^* = 0$ ). Despite the dynamic nature of consumers' Bayesian belief updating during their sequential search, we show that the beliefs are stationary given the experts' equilibrium strategy, which substantially facilitates the equilibrium analysis. The analysis of our model is made tractable also by the observation that because sellers will optimally choose not to price above the searchers' reservation prices, in equilibrium all searchers will purchase during their first visit when undertaking sequential searches.

The hybrid equilibrium, which our analysis focuses on, exhibits several interesting properties. First, there is an interval on which the two price distributions for treatment  $T_M$  associated with problems  $M$  and  $m$  have identical density, so that a consumer maintains the prior belief about her problem type when seeing a price in this interval; whereas prices for  $T_M$  in a lower interval will be chosen only for  $M$ . Hence, an honest expert is more likely to charge a lower price for the major treatment.<sup>3</sup> Second, an increase in experts' cheating probability ( $\alpha$ ) negatively impacts search benefit because consumers are less likely to encounter an honest seller from another search, but it also positively impacts search benefit because equilibrium prices and their dispersion are higher. A consumer's search benefit can thus be a decreasing, increasing, or non-monotonic function of  $\alpha$ . Third, there is a critical level of cheating  $\alpha^c$  such that as search frictions decrease (i.e.,  $\lambda$  rises or  $s$  falls), expert cheating decreases if  $\alpha^* > \alpha^c$  but increases if  $\alpha^* < \alpha^c$ .<sup>4</sup> Fourth, equilibrium prices stochastically increase in  $\alpha$ , and they depend on  $s$  only through  $\alpha$ . This implies that reductions in search cost, when it decreases (increases) cheating, also lowers (raises) prices and unambiguously benefits (harms) consumers.

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<sup>3</sup>When consumers lack information about product quality, a well-known result in the literature on pricing under asymmetric quality information is that a high price can serve as a signal for high quality (e.g., Bagwell & Riordan (1991)). Interestingly, in our setting a *lower* price can signal a high quality—honesty—of a recommendation

<sup>4</sup>This conclusion is valid for all equilibria of our model, including the pooling equilibrium with  $\alpha^* = 1$  and the separating equilibrium with  $\alpha^* = 0$ , if the “decreases” and “increases” are interpreted as “weakly increases” and “weakly increases”.

The result that equilibrium expert cheating can either rise or fall as search frictions decrease may seem surprising at first glance, but it has the following simple intuition. When cheating is sufficiently common in the market ( $\alpha^* > \alpha^c$ ), a marginal reduction in search friction leads to relatively more competition for dishonest experts, motivating experts to behave more honestly; but when cheating is sufficiently rare in the market ( $\alpha^* < \alpha^c$ ), a marginal reduction in search friction leads to relatively more competition for honest experts, motivating experts to behave less honestly.<sup>5</sup>

It is also interesting that in our model, despite experts' information advantage regarding the consumers' problem, the equilibrium outcome for each problem type reduces to that in Stahl (1989) when search cost is sufficiently small. In this case, where the separating equilibrium prevails, the expected profits for the two treatments are the same and experts always report consumers' problems truthfully.<sup>6</sup> Then, the equilibrium price distribution for each treatment has the same form as that in Stahl (1989). Therefore, although competition in expert markets with search cost generally works very differently from competition in other search markets, when search frictions are sufficiently small, competition can effectively discipline experts and the market operates as if consumers could observe their problem types. Furthermore, same as in Stahl (1989), the prices for the two treatments both approach their respective marginal costs when search cost approaches zero.

To the best of our knowledge, this is the first paper to study consumer search for both recommendations and prices in expert markets. It contributes to the search literature by providing a new framework to understand search and competition when only sellers can observe product features that match buyers' needs. As we mentioned earlier, in the extant literature, consumers either know the product value before price search for a homogeneous

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<sup>5</sup>The critical value  $\alpha^c$  depends on the extra cost for the major treatment ( $C$ ) and the number of sellers in the market ( $N$ ), both of which can affect the relationship between a consumer's search benefit and  $\alpha$ . Their influences can be so dominant that  $\alpha^c$  is either 0 or 1, so that a reduction in search friction always increases or decreases expert cheating, whereas in other situations  $\alpha^c \in (0, 1)$ .

<sup>6</sup>In the credence-goods literature, an important insight is that experts will not cheat if the price markups for the two treatments are equalized (Emons, 1997; Dulleck & Kerschbamer, 2006). Our result generalizes this insight to situations when prices follow mixed strategies and expected profits are equalized.

product (e.g., Stahl, 1989; Janssen et al., 2011), or they also search for a product’s value either under horizontal differentiation (e.g., Wolinsky, 1986; Anderson & Renault, 1999; Haan & Moraga-González, 2011; Rhodes, 2011) or (additionally) under vertical differentiation (e.g., Bar-Isaac et al., 2012; Chen & Zhang, 2018; Moraga-González & Sun, 2022). An exception is Chen et al. (2022b), in which consumers search for product matches without observing product quality before purchase, but in their model of experience goods, each firm produces only one product with a pre-determined quality, there is no role for product recommendation, and all firms set the same deterministic price in equilibrium. By contrast, in our model each seller may produce two products (either  $T_M$  or  $T_m$ ), and his choice of recommendations may interact with prices to influence consumer search and purchase. Our finding that changes in search cost can have non-monotonic effects on prices is broadly consistent with the results in the literature,<sup>7</sup> but the channel through which this happens in our mode is novel: lower search cost can increase false recommendations, which in turn leads to higher prices and lower consumer surplus.

Our paper is closely related to Janssen et al. (2011), in which firms have identical but stochastic production costs and the cost realization is unknown to consumers. In both papers, consumers update their beliefs in a Bayesian fashion when sequentially searching sellers, and in equilibrium all searchers purchase from their first-visited seller. One notable difference between the two papers is that production cost is unknown to consumers in Janssen et al. (2011), while treatment cost is verifiable in our setting. Also, in our model experts choose treatment (i.e. product) recommendations, whereas no such choice is made in Janssen et al. (2011); consequently, costs are exogenously determined in their model but depend on the experts’ recommendation choice in ours. Moreover, Janssen et al. (2011) focus on how production cost uncertainty matters for market outcomes and welfare, whereas we emphasize the effects of search frictions on experts’ cheating behavior. These differences make Janssen et al. (2011) especially suitable for retail markets such as gasoline, on which their analysis offers

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<sup>7</sup>While prices unambiguously increase in search cost in seminal papers such as Stahl (1989) and Wolinsky (1986), later contributions have shown that reductions in search frictions can sometimes increase price for homogeneous products (e.g., Chen & Zhang, 2011) or for differentiated products (e.g., Bar-Isaac et al., 2012; Zhou, 2014; Moraga-González et al., 2017; Choi et al., 2018; Chen et al., 2022b).

important insights, whereas our setting is more relevant for expert markets such as those for auto repair, medical/dental treatment, and IT service.

Our paper also contributes to the literature on credence goods and expert markets.<sup>8</sup> The literature has studied how various mechanisms may stop experts from cheating and improve efficiency, such as separating diagnosis from treatment (e.g., Wolinsky, 1993), liability (e.g., Fong, 2005; Dulleck et al., 2011; Bester & Dahm, 2018; Chen et al., 2022a), and reputation (e.g., Schneider, 2012; Fong et al., 2022). Several papers (Wolinsky, 1993, 1995; Pesendorfer & Wolinsky, 2003) have also examined the role of second opinions and expert competition, but in these studies prices are observable to all consumers without costly search, and expert behavior will become less favorable to consumers monotonically as consumer search cost increases. Our model allows experts to compete in—and consumers to search for—both recommendations and prices, and we demonstrate that the interactions between consumer search and competition in these two dimensions can substantially change how expert markets function. In particular, we show that search cost can affect expert behavior non-monotonically.

In the rest of the paper, we present our model in Section 2, which also contains results in the benchmark where consumers know their problem types so that for each treatment our model reduces to a version of Stahl (1989)’s model. Section 3 analyzes the hybrid equilibrium. Section 4 studies how changes in search frictions may affect the cheating probability and prices at the hybrid equilibrium. Section 5 characterizes the separating and pooling equilibria, and provides the conditions for their existence. Section 6 concludes.

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<sup>8</sup>The literature often considers products or services in expert markets as credence goods (e.g., Darby & Karni, 1973; Taylor, 1995; Emons, 1997, 2001; Fong, 2005; Alger & Salanie, 2006; Liu, 2011). See Dulleck & Kerschbamer (2006) for a review of the earlier literature, and Balafoutas & Kerschbamer (2020) for more recent contributions.



## 2. THE MODEL

The market contains a unit mass of consumers and  $N \geq 2$  experts. Each consumer has a problem that needs to be treated by an expert. A consumer's problem can be one of two types: major ( $i = M$ ) or minor ( $i = m$ ), each occurring with probability  $\theta$  or  $1 - \theta$ . The realization of the problem type,  $i$ , is independent across consumers. Any expert can solve the consumer's problem by a major treatment ( $T = T_M$ ) if  $i \in \{M, m\}$  or by a minor treatment ( $T = T_m$ ) if  $i = m$ .<sup>9</sup> We assume that each consumer is willing to pay at most  $V_i$  to have problem  $i \in \{M, m\}$  solved, with  $V_M \equiv V$  and  $V_m \equiv v$ . One natural interpretation of this assumption is that if problem  $i$  is not treated, the consumer will suffer a loss of  $-V_i$ , but we can also allow the possibility that  $V_i$  reflects consumers' (subjective) valuations of the treatment that solves problem  $i$ , knowing that  $i = M$  is much more costly to treat.

Consumers, who do not observe their problem types and the experts' prices, may sequentially search experts in random order for recommendations and prices. Following Stahl (1989), we assume that portion  $\lambda$  of consumers are *shoppers* who have zero cost to visit any expert,<sup>10</sup> whereas proportion  $1 - \lambda$  of consumers are *searchers* who incur a search cost  $s > 0$  to visit any expert except for a first visit. Whether a consumer is a shopper or searcher is her private information.

Since problem  $M$  can be solved only with  $T_M$ , any expert will always recommend  $T_M$  for  $M$ . However, an expert may recommend either  $T_M$  or  $T_m$  for  $m$ . A strategy of expert  $j$ ,  $j = 1, \dots, N$ , can thus be denoted as  $\gamma^j = (p^{ij}, q^j, \alpha^j)$ , where  $p^{Mj}$  is  $j$ 's price for  $T_M$  if  $i = M$ ,  $p^{mj}$  is  $j$ 's price for  $T_M$  if  $i = m$ ,  $q^j$  is  $j$ 's price for  $T_m$  if  $i = m$ , and  $\alpha^j$  is  $j$ 's probability to recommend  $T_M$  when  $i = m$ . The costs of the treatments are  $c_i$  for  $i \in \{M, m\}$ , where  $c_M = C > 0$  and  $c_m$  is normalized to 0. The type of treatment is verifiable, implying that an expert needs to incur cost  $C$  if he recommends a major treatment (even to fix the minor

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<sup>9</sup>This assumption of two problem types with two possible treatments is commonly made in the credence-goods literature. We also maintain this assumption for analytical tractability. Liu & Ma (2021), however, study a more general model in which a consumer's problem types are a continuum, though their analysis focuses on a monopoly expert.

<sup>10</sup>As argued in Stahl (1989), some consumers may enjoy finding the lowest price through search, which provides a justification for the existence of shoppers in the market.

problem).<sup>11</sup> We assume  $V - C \geq v$ , so that an expert can obtain a higher markup on the major treatment, which provides an incentive for the expert to cheat: recommend  $T_M$  when only  $T_m$  is needed to treat a consumer's problem.

Upon visiting the  $t^{\text{th}}$  expert in her random search, if the expert recommends  $T_M$ , a consumer holds the belief that her problem is  $M$  or  $m$  respectively with probabilities  $\mu_t$  and  $1 - \mu_t$ , for  $t = 1, \dots, N$ , where  $\mu_t$  may also depend on the expert's price for  $T_M$  and on offers from previously-visited experts (if  $t > 1$ ). Because an expert cannot solve a major problem with  $T_m$ , a consumer will hold belief  $\mu_t = 0$  once she has received recommendation  $T_m$  to solve her problem. A shopper's strategy is to search all experts and then decide which expert's offer to accept (if she accepts an offer at all). As in Stahl (1989) and Janssen et al. (2011), each searcher follows a reservation price strategy, which specifies a pair of reservation prices  $(r(\mu_t), r_m)$  for  $(T_M, T_m)$  in her  $t^{\text{th}}$  visit under belief  $\mu_t$ : she will accept recommendation  $T_M$  at price  $p \leq r(\mu_t)$ , and she will accept recommendation  $T_m$  at price  $q \leq r_m$ . Clearly,  $r(\mu_t) \leq V$  and  $r_m \leq v$ . As we shall argue later, a reservation price strategy will indeed be optimal for the searchers, given the optimal strategy of the experts.

The timing of the game is as follows: First, experts simultaneously choose their strategies. Next, shoppers search all experts, while searchers may sequentially search experts. When seeing a consumer and learning her problem, expert  $j$  offers his recommendation and price to the consumer according to  $\gamma^j$ . The consumer may (a) accept the offer, (b) search another expert, (c) possibly return to accept the offer from a previously-visited expert with no additional search cost, or (d) exit the market without receiving a treatment. The game ends if (a), (c), or (d) occurs for every consumer.

We will focus on symmetric perfect Bayesian equilibrium (PBE) where all experts choose the same strategy and so do all consumers. We can thus simplify notations by writing  $\gamma^j$  as  $\gamma$ . A PBE of our game is a profile of strategies by the experts and consumers, together with consumer beliefs, that satisfies:

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<sup>11</sup>The credence-goods literature has considered two alternative assumptions: the treatment is verifiable (e.g., Emons, 1997; Alger & Salanie, 2006; Chen et al., 2022a), or non-verifiable (e.g., Wolinsky, 1993; Taylor, 1995; Fong, 2005; Liu, 2011). We adopt the former, but will discuss in Section 5 how the results might change if treatment were not verifiable.

(i) For  $j = 1, \dots, N$ , given each consumer's strategy and all other experts' strategies,  $\gamma$  maximizes expert  $j$ 's expected profit and  $j$  has no incentive to change his strategy upon seeing any consumer.

(ii) Given  $\gamma$  for  $j = 1, \dots, N$ , each consumer chooses her strategy to maximize her expected surplus under her belief. Clearly, the optimal strategy for any shopper is to accept the offer from the expert with the lowest price, provided that this is better than no treatment (which—as it will become clear—must be true in equilibrium). Our equilibrium analysis will thus focus on the optimal strategy of the searchers, for whom the PBE imposes two requirements: given her belief and the experts' strategies, each searcher chooses her reservation price optimally, and it is indeed optimal for each searcher to follow a reservation price search strategy.

(iii) Consumers' beliefs are derived from the Bayes' rule along the equilibrium path. As in Janssen et al. (2011), we make the following assumption for out-of-equilibrium belief: if  $p^*$  is an equilibrium price, then when a consumer observes an out-of-equilibrium price  $p'$  in a small neighborhood of  $p^*$ , i.e.,  $p' \in (p - \epsilon, p + \epsilon)$ , her belief about the type of her problem associated with  $p'$  would be the same as that with  $p^*$ , i.e.,  $\mu_t(p') = \mu_t(p^*)$ .

Our model may have three types of equilibria, in which experts always recommend  $T_M$  if  $i = M$  but differ in their recommendation for  $i = m$ : (i) a hybrid equilibrium where experts recommend  $T_M$  for  $m$  with probability  $\alpha \in (0, 1)$ ; (ii) a separating equilibrium where experts always recommend  $T_m$  for  $m$  (i.e.,  $\alpha = 0$ ); and (iii) a pooling equilibrium where experts always recommend  $T_M$  for  $m$  (i.e.,  $\alpha = 1$ ). We will focus on the hybrid equilibrium, but will also provide results for the separating and pooling equilibria.

We conclude this section by considering a benchmark where each consumer can observe her  $i = \{M, m\}$ .

### **Benchmark: Problem Types are Observable to Consumers**

In this case, there is no possibility of expert cheating. For each  $i \in \{M, m\}$ , our model is then the same as that in Stahl (1989). Following Stahl (1989), there is a unique symmetric equilibrium where experts price according to price distribution  $F_i(p)$  for  $i = \{M, m\}$  and consumers search with reservation price  $r_i^o \leq V_i$  for  $T_i$ . The equilibrium can be derived as follows.

First, notice that, as in Stahl (1989), there can be no symmetric equilibrium where experts adopt a pure strategy. Suppose that, to the contrary, in equilibrium  $p = p^*$  for  $T_M$ . Then, if  $p^* > C$ , an expert can profitably deviate by lowering his price slightly to attract all the shoppers; while if  $p^* = C$ , an expert can profitably deviate by slightly raising the price which will be accepted by any searcher (who has a search cost  $s > 0$ ). Similarly a deterministic price  $q = q^*$  for  $T_m$  cannot be sustained in equilibrium. Next,  $F_i(p)$  must be atomless, as any price associated with a probability mass will also induce profitable deviations. Moreover, the upper bound of  $F_i(p)$  is  $r_i^o$  for  $i = M, m$ , the reservation price of searchers in their sequential search.

For  $i \in \{M, m\}$  and for any price  $p$  generated from  $F_i(p)$ , in equilibrium

$$(p - c_i) \left[ \frac{1 - \lambda}{N} + \lambda (1 - F_i(p))^{N-1} \right] = (r_i^o - c_i) \frac{1 - \lambda}{N},$$

where  $(1 - F_i(p))^{N-1}$  is the probability that an expert can sell to a shopper and  $r_i^o$  is the highest price in the support of  $F_i(p)$ . The equilibrium price distribution is

$$F_i(p) = 1 - \left[ \frac{(r_i^o - p)(1 - \lambda)}{(p - c_i)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{with} \quad p \in [b_i^o, r_i^o], \quad (1)$$

where  $b_M^o = \frac{r_M^o(1-\lambda)+C\lambda N}{\lambda N+1-\lambda}$  and  $b_m^o = \frac{r_m^o(1-\lambda)}{\lambda N+1-\lambda}$ .

Define  $r_M^o$  as the solution to

$$\int_{b_M^o}^{r_M^o} (r_M^o - p) dF_M(p) = s. \quad (2)$$

We can rearrange the term on the left-hand side in the above equation, which is the search benefit—a consumer's benefit from another search—as

$$\int_{b_M^o}^{r_M^o} (r_M^o - p) dF_M(p) = r_M^o + \int_{b_M^o}^{r_M^o} p d[1 - F_M(p)].$$

Define  $x = 1 - F_M(p)$ , and rewriting  $p$  as a function of  $x$  by (1), the search benefit under  $F_M(p)$  becomes

$$r_M^o + \int_{b_M^o}^{r_M^o} p d[1 - F_M(p)] = (r_M^o - C)(1 - \phi), \quad (3)$$

where

$$\phi \equiv \int_0^1 \frac{1 - \lambda}{\lambda N x^{N-1} + 1 - \lambda} dx < 1, \quad (4)$$

and  $\phi$  is a constant for given  $\lambda$  and  $N$ . Notice that  $\phi$  is lower when  $\lambda$  is higher or  $N$  is lower; and  $\phi \rightarrow 0$  when  $\lambda \rightarrow 1$  while  $\phi \rightarrow 1$  when  $\lambda \rightarrow 0$ .

For given  $V$  and  $v$ , there is an upper bound on search cost,  $\bar{s}$ , under which  $r_M^o = C + \frac{\bar{s}}{1-\phi}$  uniquely solves (2) and  $r_M^o = \theta V + (1-\theta)v < V$ . As it will become clear later, when  $s \leq \bar{s}$ , the consumer's reservation price is no higher than her expected willingness-to-pay for  $T_M$ , even in all equilibria of our model where problem types are not observable to consumers. Throughout the paper, we assume:

$$s \leq \bar{s} \equiv (1-\phi)[\theta V + (1-\theta)v - C]. \quad (5)$$

Furthermore, let  $r_m^o = \min\{v, \omega\}$ , where  $\omega$  solves

$$\int_{\frac{\omega(1-\lambda)}{\lambda N + 1 - \lambda}}^{\omega} (\omega - p) dF_m(p) = s. \quad (6)$$

Then  $r_m^o$  uniquely exists. The unique equilibrium  $F_i(p)$  is characterized by (1), (2) and (6). Notice that search benefit is strictly increasing in reservation prices  $r_M^o$  for  $T_M$  and  $r_m$  for  $T_m$ , which implies that it is optimal for searchers to adopt a reservation price strategy under both  $T_M$  and  $T_m$ .

We next return to the equilibrium analysis of our main model in which consumers do not observe their problem types.

### 3. HYBRID EQUILIBRIUM: PROBABILISTIC CHEATING

When only experts can privately learn a consumer's problem, they may cheat by recommending  $T_M$  even when  $i = m$ . This section analyzes the hybrid equilibrium where each expert cheats with probability  $\alpha \in (0, 1)$ .

As in the benchmark case, here there is also no equilibrium in which experts choose deterministic prices. To see this, consider first the price for  $T_m$ , and suppose  $q = q^*$  is the candidate equilibrium price for  $T_m$ . Then, any  $q^* > 0$  cannot be supported in equilibrium because an expert can profit from a deviation to a slightly lower price, while  $q^* = 0$  also cannot be supported in equilibrium because an expert can profitably deviate to a slightly higher price. Next, consider the price for  $T_M$ . Suppose that  $p = p^*$  is a candidate equilibrium price for  $T_M$  in equilibrium. Then, with all experts recommending  $T_M$  with probability 1 if

$i = M$  and with probability  $\alpha$  if  $i = m$ , each shopper—after seeing the recommendations from all experts—will form certain belief along the equilibrium path about the probability that her  $i = M$ . Clearly, some consumers must be willing to pay  $p^*$  for  $T_M$  in order for  $p^*$  to be an equilibrium price. Then, if  $p^* > C$ , an expert can deviate to a slightly lower price, for which consumers will still have the same belief as before under our assumption about off-equilibrium beliefs. It follows that the deviation is profitable to the expert by attracting all shoppers who would have purchased from other experts under  $p^*$ , and the deviation would not reduce the expert’s demand from searchers. On the other hand, if  $p^* = C$ , an expert can profitably deviate to a slightly higher price to sell to searchers for whom he happens to be the first expert they visit.<sup>12</sup>

In a potential symmetric mixed-strategy equilibrium, suppose that experts choose  $p$  according to distribution  $F(p)$  when recommending  $T_M$  for  $i = M$ , choose  $p$  according to distribution  $G(p)$  when recommending  $T_M$  for  $i = m$ , and choose  $q$  according to distribution  $H(q)$  when recommending  $T_m$  for  $i = m$ . From familiar arguments, the equilibrium price distributions are atomless.

Because a searcher can return to a previously-searched expert without cost, in equilibrium her reservation price for  $T_M$  or  $T_m$  under a certain belief must not increase in  $t$ . Let  $\mu \equiv \mu_1 \in [0, 1]$ , and let  $\{r(\mu), r_m\}$  be the searchers’ reservation prices for  $\{T_M, T_m\}$  in their first round of search. As we shall confirm later, in equilibrium consumers will indeed hold stationary belief  $\mu_t = \mu$  for all  $t$  and searchers will adopt a reservation-price search strategy. Then, the upper bound  $B$  of distributions  $F(p)$  and  $G(p)$  must be  $r \equiv r(\mu)$ , and the upper bound of  $H(q)$  must be  $r_m$ . To see this, suppose to the contrary that  $B \neq r$ . If  $B > r$ , then price  $p = B$  will not yield any sale to searchers during their first and possible future rounds of searches, and it will also not yield any sales to shoppers. By deviating to  $B = r$ , an expert will have a positive profit and hence the deviation is profitable. If  $B < r$ , since there is zero probability that  $B$  is the lowest price, an expert can profitably raise  $p = B$  to  $p = r$ , for which he will not lose any sales to shoppers but will have a higher profit from searchers who visit him and

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<sup>12</sup>For  $T_M$ , the experts may also choose a deterministic price  $p_1$  when  $i = M$  and  $p_2$  when  $i = m$ . From arguments similar to the above, there can be no deterministic equilibrium prices  $p_1^*$  or  $p_2^*$ .

who will pay  $r$  instead of  $B < r$ . The argument is similar for  $r_m$  when the recommended treatment is  $T_m$ . Therefore, in equilibrium all searchers will purchase at their first visit. As in Stahl (1989) and Janssen et al. (2011), our analysis is greatly facilitated by this feature of the model.

In subsection 3.1 below, we derive the equilibrium price distributions and cheating probability  $\alpha$ , given the consumers' strategies. In subsection 3.2, we then derive the optimal consumer strategy under the equilibrium expert strategy and fully characterize the hybrid equilibrium.

### 3.1 Price Distributions and Cheating Probability

We start by deriving the equilibrium price distributions, given that shoppers will purchase from the lowest-priced expert and searchers will search with reservation prices  $(r, r_m)$  for  $(T_M, T_m)$ . We consider in turn the cases where a consumer has a major problem ( $i = M$ ) and where she has a minor problem ( $i = m$ ).

First, suppose that  $i = M$ . Then, upon seeing the consumer, any expert will recommend  $T_M$  with a price  $p$  randomly drawn from  $F(p)$ . To determine  $F(p)$ , notice that an expert earns the same expected profit for any  $p \in [b_f, r]$  in the symmetric mixed strategy equilibrium:

$$(p - C) \left[ \frac{1 - \lambda}{N} + \lambda (1 - F(p))^{N-1} \right] = (r - C) \frac{1 - \lambda}{N},$$

where the expert can sell only to searchers if he sets  $p = r$ . Thus, the equilibrium price distribution is

$$F(p) = 1 - \left[ \frac{(r - p)(1 - \lambda)}{(p - C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{with } p \in [b_f, r], \quad (7)$$

where  $b_f = \frac{r(1-\lambda) + C\lambda N}{\lambda N + 1 - \lambda}$ . We have  $F(r) = 1$ ,  $F(b_f) = 0$ , and the probability density is

$$f(p) = \frac{1}{N-1} \left[ \frac{(r - p)(1 - \lambda)}{(p - C)\lambda N} \right]^{\frac{1}{N-1} - 1} \frac{r - C}{(p - C)^2} \left( \frac{1 - \lambda}{\lambda N} \right). \quad (8)$$

Notice that the price distribution has the same form as that for  $i = M$  when consumers can observe their problem types. However, the equilibrium  $r$  (to be derived) will differ from

$r_M^o$  in the benchmark case, because in optimally choosing  $r$  a consumer will now take into account the possibility that an expert may cheat by recommending  $T_M$  even when  $i = m$ .

Next, suppose that  $i = m$ . For such a consumer, an expert will recommend  $T_M$  with probability  $\alpha$  under a price  $p$  that is randomly drawn from  $G(p)$ . The expert earns equal profits from offering  $T_M$  to such consumers with any price  $p$  drawn from  $G(p)$  if

$$(p - C) \left[ \frac{1 - \lambda}{N} + \lambda \alpha^{N-1} (1 - G(p))^{N-1} \right] = (r - C) \frac{1 - \lambda}{N},$$

where  $\frac{1-\lambda}{N}$  of the  $m$ -type searchers will first visit the expert and will pay for  $T_M$  at  $p \leq r$ , while  $\alpha^{N-1} (1 - G(p))^{N-1}$  is the probability that the expert can sell to a shopper with  $i = m$  when other experts also cheat and price higher. Hence,  $G(p)$  is given by

$$G(p) = 1 - \frac{1}{\alpha} \left[ \frac{(r - p)(1 - \lambda)}{(p - C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{for } p \in [b_g, r], \quad (9)$$

where  $b_g = \frac{r(1-\lambda) + C\alpha^{N-1}\lambda N}{1-\lambda + \alpha^{N-1}\lambda N}$ . Moreover,  $G(r) = 1$ ,  $G(b_g) = 0$ , and the probability density is

$$g(p) = \frac{1}{\alpha} \frac{1}{N-1} \left[ \frac{(r - p)(1 - \lambda)}{(p - C)\lambda N} \right]^{\frac{1}{N-1}-1} \frac{r - C}{(p - C)^2} \left( \frac{1 - \lambda}{\lambda N} \right). \quad (10)$$

Next, with  $i = m$ , any expert will recommend  $T_m$  with probability  $1 - \alpha$  under a price  $q$  randomly drawn from  $H(q)$ . Suppose that all searchers' reservation price for  $T_m$  is  $r_m = \min\{v, \omega\}$ , where  $\omega$  is defined in (6). We construct the equilibrium under the assumption that  $v < \omega$  and will later show that  $v \geq \omega$  is not consistent with any hybrid equilibrium. An expert earns equal profits from recommending  $T_m$  to consumers with  $i = m$  under any price  $q$  drawn from  $H(q)$  if

$$q \left\{ \frac{1 - \lambda}{N} + \lambda [\alpha + (1 - \alpha) (1 - H(q))]^{N-1} \right\} = v \left( \frac{1 - \lambda}{N} + \lambda \alpha^{N-1} \right),$$

where we have further assumed that  $v < b_g$  so that prices under  $G(p)$  by experts who cheat are all higher than  $v$ .<sup>13</sup> Thus,

$$H(q) = \frac{1}{1 - \alpha} \left\{ 1 - \left[ \frac{v(1 - \lambda + \lambda N \alpha^{N-1}) - (1 - \lambda)q}{\lambda N q} \right]^{\frac{1}{N-1}} \right\} \quad \text{with } q \in [b_h, v], \quad (11)$$

<sup>13</sup>As we will see shortly in Lemma 1, this assumption is always satisfied in equilibrium.



where  $b_h = \frac{1-\lambda+\lambda N\alpha^{N-1}}{1-\lambda+\lambda N}v$ . Moreover,  $H(v) = 1$ ,  $H(b_h) = 0$ , and the probability density is

$$h(q) = \frac{1}{1-\alpha} \left[ \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{\lambda N q} - \frac{1-\lambda}{\lambda N} \right]^{\frac{1}{N-1}-1} \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{(N-1)\lambda N q^2}.$$

Finally, the experts must earn the same expected profit from recommending either  $T_M$  or  $T_m$  for  $i = m$ , and hence  $v \left( \frac{1-\lambda}{N} + \lambda\alpha^{N-1} \right) = (r - C) \frac{1-\lambda}{N}$ , which implies that  $r$  and  $\alpha$  are positively related in the following way:

$$r = \frac{1-\lambda+\lambda N\alpha^{N-1}}{1-\lambda}v + C, \quad (12)$$

We summarize the price distributions, their properties, and the cheating probability for given  $r$  at a hybrid equilibrium as follows:

**Lemma 1** *In a hybrid equilibrium, the price distributions  $F(p)$ ,  $G(p)$ , and  $H(q)$  are given by (7), (9), and (11), with the following properties: (i)  $b_f < b_g$  with  $F(b_g) = 1 - \alpha$ ; (ii)  $g(p) = \frac{1}{\alpha}f(p)$  for  $p \in [b_g, r]$ ; (iii)  $h(q) = \frac{1}{1-\alpha}f(q + C)$  for  $q \in [b_h, v]$ ; and (iv)  $b_g = v + C$  and  $b_f = b_h + C$ . Furthermore,  $\alpha$  is determined by (12) for given  $r$ .*

**Proof.** It suffices to prove properties (i)-(iv).

(i) Since  $b_g$  decreases in  $\alpha$  we have

$$b_g = \frac{r(1-\lambda) + C\alpha^{N-1}\lambda N}{1-\lambda + \alpha^{N-1}\lambda N} > \frac{r(1-\lambda) + C\lambda N}{1-\lambda + \lambda N} = b_f$$

for  $r > C$ . It is straightforward to also verify that  $F(b_g) = 1 - \alpha$  from (7).

(ii) From comparing (8) and (10), we immediately have  $g(p) = \frac{1}{\alpha}f(p)$  for  $p \in [b_g, r]$ .

(iii) From (11) and (12),

$$\begin{aligned} h(q) &= \frac{1}{1-\alpha} \left[ \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{\lambda N q} - \frac{1-\lambda}{\lambda N} \right]^{\frac{1}{N-1}-1} \frac{v(1-\lambda+\lambda N\alpha^{N-1})}{(N-1)\lambda N q^2} \\ &= \frac{1}{1-\alpha} \left[ \frac{(r-q-C)(1-\lambda)}{\lambda N q} \right]^{\frac{1}{N-1}-1} \frac{(r-C)(1-\lambda)}{(N-1)\lambda N q^2} \\ &= \frac{1}{1-\alpha} f(q + C). \end{aligned}$$

(iv) Substituting the  $r$  from (12) into

$$b_g = \frac{r(1-\lambda) + C\alpha^{N-1}\lambda N}{1-\lambda + \alpha^{N-1}\lambda N} \text{ and } b_f = \frac{r(1-\lambda) + C\lambda N}{\lambda N + 1 - \lambda},$$

we obtain  $b_g = v + C$  and  $b_f = b_h + C$ . ■

Part (i) in Lemma 1 implies that when a consumer receives a recommendation for  $T_M$  at a price  $p \in [b_f, b_g)$ , she can infer that the expert has made an honest recommendation:  $i = M$ , whereas when she receives a recommendation for  $T_M$  at a price  $p \in [b_g, r]$ , the true state can be either  $i = M$  or  $i = m$ .

Part (ii) in Lemma 1 implies that when  $T_M$  is being recommended at price  $p \in [b_g, r]$ , density  $g(p)$  is larger than  $f(p)$  with  $g(p) = \frac{1}{\alpha}f(p)$ . However, since an expert will recommend  $T_M$  when  $i = m$  only with probability  $\alpha$ , from the Bayes' rule a consumer's posterior belief when receiving recommendation  $T_M$  under a price  $p \in [b_g, r]$  is the same as her prior belief:<sup>14</sup>

$$\mu(p) = \frac{\theta f(p)}{\theta f(p) + (1 - \theta)\alpha g(p)} = \theta \quad \text{for } p \in [b_g, r]. \quad (13)$$

Part (iii) suggests that the price density function  $h(q)$  for  $T_m$  under  $i = m$  is a shift to the left by  $C$  from the density function  $f(p)$  for  $T_M$  under  $i = M$  on  $[b_f, b_g]$ . Part (iv) is based on the idea that when an expert recommends  $T_M$  with a price  $p \in [b_g, r]$ , a searcher's belief is  $\mu = \theta$  (as indicated in (ii) above), and hence her reservation price  $r$  for  $T_M$  is the same under both  $F(p)$  and  $G(p)$ . Therefore, both  $b_g$  and  $b_f$  are determined by the same  $r$  from (12) that makes the expert indifferent between recommending  $T_M$  and  $T_m$  for  $i = m$ .

Figure 1 below illustrates the relations between  $F(p)$ ,  $G(p)$ , and  $H(q)$ .

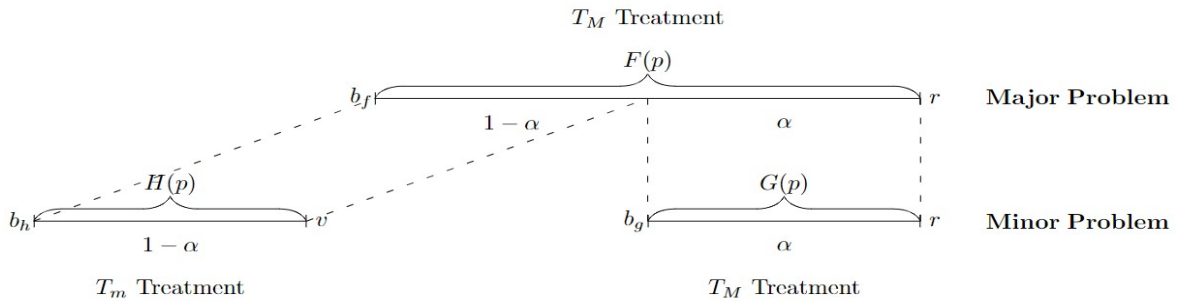


Figure 1: Equilibrium price distributions.

<sup>14</sup>Interestingly, under recommendation  $T_M$ , a lower price,  $p \in [b_f, b_g)$ , signals that the problem is indeed  $M$ , whereas a higher price,  $p \in [b_g, r]$ , does not provide useful information. This is because if an expert chooses to cheat—recommending  $T_M$  when  $i = m$ —he is unlikely to sell to shoppers and would thus rather charge a higher price to earn a higher profit when selling to searchers.

### 3.2 Optimal Consumer Search

We now characterize optimal consumer search given the experts' strategy described in the previous subsection. From the analysis of price distributions for our proposed equilibrium, a consumer's beliefs after receiving experts' offers of treatment and prices can be summarized in the following.

**Lemma 2** *Upon receiving the offer from the  $t^{\text{th}}$  expert that she visits, a consumer's belief  $\mu_t$  is consistent with the experts' equilibrium strategies when, for all  $t \geq 1$ : (i)  $\mu_t = 0$  if at least one of her visited experts recommends  $T_m$  with price  $q \leq v$ ; (ii)  $\mu_t = 1$  if at least one of her visited experts recommends  $T_M$  with price  $p \in [b_f, b_g]$ ; and (iii)  $\mu_t = \frac{\theta f(p_1) \dots f(p_t)}{\theta f(p_1) \dots f(p_t) + (1-\theta) \alpha^t g(p_1) \dots g(p_t)} = \theta$  if all her visited experts recommend  $T_M$  with prices  $p_1, \dots, p_t \in [b_g, r]$ .*

Since  $\mu_t$  is either 0, 1, or  $\theta$ , independent of  $t$ , we can simply denote a consumer's belief by  $\mu$ . Given  $\alpha$ , the price distributions, and belief  $\mu$  from Lemma 2, we can describe the optimal sequential search rule of a searcher as follows. (1) She will accept an offer that recommends  $T_m$  with price  $q \leq v$ . (2) She will accept an offer that recommends  $T_M$  with price  $p \leq r = r(\mu)$ , and  $r$  satisfies

$$\theta \int_{b_f}^r (r-p) dF(p) + (1-\theta) \alpha \int_{b_g}^r (r-p) dG(p) + (1-\theta)(1-\alpha) \int_{b_h}^v (r-q) dH(q) = s. \quad (14)$$

In the left-hand side of (14), which is the search benefit from visiting another expert, the first term is the expected benefit from finding a lower price when  $i = M$ , the second term is the expected benefit from finding a lower price when  $i = m$  but the expert recommends  $T_M$ , and the third term is the expected benefit from finding a lower price when  $i = m$  and the expert recommends  $T_m$ . Equation (14) says that at the optimal  $r$  the search benefit is equal to the search cost ( $s$ ).

So far, given experts' strategy, we have derived the searchers' (stationary) reservation prices, under the presumption that they follow a reservation price search strategy. We now argue that given experts' strategy, it is indeed optimal for searchers to adopt a reservation price strategy, which would be true if the search benefit is increasing in a sampled price under both  $T_m$  and  $T_M$ . Suppose first that a searcher is recommended  $T_m$ . Then, her belief is  $\mu = 0$ ,

and at the current offer  $\{T_m, q'\}$ , her benefit from another search is

$$(1 - \alpha) \int_{q \leq q'} (q' - q) dH(q),$$

which clearly increases with the sampled price  $q'$ . Next, suppose that a searcher is recommended  $T_M$ . At the current offer  $\{T_M, p'\}$ , the potential complication is that as the sampled price  $p'$  increases, a searcher's belief may change. In particular, if a lower  $p'$  were associated with a lower  $\mu$ , then the search benefit could be higher at a lower  $p'$ , because the lower  $\mu$  associated with  $p'$  would imply that with another search, it could be more likely for the searcher to encounter an honest expert that recommends  $T_m$  with a lower price. Fortunately, given the experts' strategy,  $\mu$  is weakly higher for lower  $p'$ , which ensures that search benefit increases in  $p'$ . Therefore, it is indeed optimal for searchers to adopt a reservation price search strategy under both  $T_m$  and  $T_M$ .<sup>15</sup>

Utilizing Lemma 1, we can simplify the left-hand side of equation (14) so that the equation is rewritten as

$$\tau(\alpha) \equiv \int_{b_f}^r (r - p) dF(p) + (1 - \theta)(1 - \alpha)C = s, \quad (15)$$

where  $r = r(\alpha)$  satisfies (12). The search benefit  $\tau(\alpha)$  in the above equation has an intuitive interpretation.<sup>16</sup> The first term, which we call the *price benefit*, is the benefit to a consumer from finding a lower price if  $i = M$ . The second term, which we call the *honesty benefit*, is the additional benefit from encountering an honest expert if  $i = m$ : the consumer then expects to pay a price that is lower by  $C$  than under  $T_M$ .

From (15),  $\int_{b_f}^r (r - p) dF(p) \leq s$ . It follows that  $r_M^o \geq r$ , where  $r_M^o$  satisfies (2).<sup>17</sup> Therefore, if a consumer receives a recommendation for  $T_M$  with a price  $p \in [b_f, b_g) < r$ , her updated

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<sup>15</sup>We may consider each shopper's expected value from having her problem solved, which depends on her belief about her problem type, as her reservation price: after searching all experts, she will purchase at the lowest price if it does not exceed her reservation price. For convenience, we sometimes also say that each shopper adopts a reservation price in search.

<sup>16</sup>Notice that  $\alpha$  is endogenously determined. As we illustrate later, there will be a range of intermediate values of  $s < \bar{s}$  under which the equation holds.

<sup>17</sup>Hence, if a consumer can observe her type, she will search with a higher reservation price for  $T_M$  than when she is recommended  $T_M$  but cannot observe whether her type is indeed  $i = M$ . In the latter case, there is a chance that her true type is  $i = m$  and she will receive a lower price if encountering an honest expert, which motivates her to lower the reservation price.

belief is  $\mu = 1$  and she will pay for  $T_M$  without searching further. Also, since  $v < \omega$  (as we have assumed), the consumer will also pay for the treatment without further searching when she is recommended  $T_m$  with a price  $q \in [b_h, v] < r$ , under which her updated belief is  $\mu = 0$ .

To solve the equilibrium, it remains to examine how  $\alpha^*$  is determined in (15), where  $r$  and the price distributions are all functions of  $\alpha$ . It is useful to note that an increase in  $\alpha$  has two opposing effects on the search benefit  $\tau(\alpha)$ :

$$\frac{d\tau(\alpha)}{d\alpha} = \underbrace{\frac{\partial\tau(\alpha)}{\partial\alpha}}_{\text{lower honesty benefit(-)}} + \underbrace{\frac{\partial\tau(\alpha)}{\partial r} \frac{\partial r}{\partial\alpha}}_{\text{higher price benefit(+)}}. \quad (16)$$

An increase in  $\alpha$  has a negative direct effect on the search benefit: As  $\alpha$  rises, experts are more likely to cheat, which reduces the honesty benefit of search, as can be seen from the first term of (16). On the other hand, an increase in  $\alpha$  has a positive indirect effect on search benefit: As  $\alpha$  rises, so does  $r = r(\alpha)$ , which in turn stochastically increases the equilibrium prices under  $T_M$  and hence also the price benefit of search, as can be seen from the second term of (16). We next show that, depending on parameter values,  $\tau(\alpha)$  can be a monotonically decreasing, monotonically increasing, or U-shaped function of  $\alpha$ . The result below refers to  $\hat{C}$  and  $\hat{\alpha}$  defined by

$$\hat{C} = \frac{(1-\phi)\lambda N(N-1)}{(1-\theta)(1-\lambda)}v, \quad \hat{\alpha} = \left( \frac{C(1-\theta)(1-\lambda)}{v\lambda N(N-1)(1-\phi)} \right)^{\frac{1}{N-2}}, \quad (17)$$

where  $\phi < 1$  is given by (4) and  $\hat{\alpha}$  is defined only if  $N > 2$ .

**Lemma 3** *The search benefit function in (15) can be written as*

$$\tau(\alpha) = (1-\phi) \frac{1-\lambda + \lambda N \alpha^{N-1}}{1-\lambda} v + (1-\theta)(1-\alpha)C. \quad (18)$$

*For all  $\alpha \in (0, 1)$ : when  $C \geq \hat{C}$ ,  $\tau(\alpha)$  monotonically decreases; when  $C < \hat{C}$ ,  $\tau(\alpha)$  monotonically increases if  $N = 2$ , but it first decreases and then increases—minimizing at  $\hat{\alpha} \in (0, 1)$ —if  $N > 2$ .*

**Proof.** By the argument leading to (3) and from (12),

$$\begin{aligned} \int_{b_f}^r (r-p) dF(p) &= (r-C)(1-\phi) \\ &= (1-\phi) \frac{v(1-\lambda + \lambda N \alpha^{N-1})}{1-\lambda}. \end{aligned}$$

Hence, the search benefit in (15) can be rewritten as (18), which is clearly positive. We then have

$$\tau'(\alpha) = -(1 - \theta)C + (1 - \phi) \frac{\lambda N(N - 1)\alpha^{N-2}}{1 - \lambda} v \quad (19)$$

and

$$\tau''(\alpha) = (1 - \phi) \frac{\lambda N(N - 1)(N - 2)\alpha^{N-3}}{1 - \lambda} v \geq 0,$$

where the weak inequality holds strictly if  $N > 2$ . Hence,  $\tau(\alpha)$  is a (weakly) convex function. When  $C \geq \hat{C}$ ,  $\tau'(\alpha) < 0$  for all  $\alpha \in (0, 1)$ . When  $C < \hat{C}$ ,  $\tau'(\alpha) > 0$  for all  $\alpha \in [0, 1]$  if  $N = 2$ ; but if  $N > 2$ ,  $\tau(\alpha)$  is minimized at

$$\hat{\alpha} = \left( \frac{C(1 - \theta)(1 - \lambda)}{(1 - \phi)\lambda N(N - 1)v} \right)^{\frac{1}{N-2}} < \left( \frac{\hat{C}(1 - \theta)(1 - \lambda)}{(1 - \phi)\lambda N(N - 1)v} \right)^{\frac{1}{N-2}} = 1.$$

Obviously  $\hat{\alpha} > 0$ . ■

To see the intuition about how  $\tau(\alpha)$  varies, we notice first that the honesty benefit is higher if  $C$  is larger (and it is independent of  $N$ ). The price benefit is independent of  $C$  but depends on the number of competing experts. Therefore, if  $C$  is sufficiently large ( $C \geq \hat{C}$  for given  $N$ ), then when  $\alpha$  increases, the reduction of the honesty benefit dominates, and hence  $\tau(\alpha)$  is decreasing.

Second, similar to Stahl (1989), the price benefit of search is high when prices are (stochastically) high. When  $\alpha$  is higher, so are  $r = r(\alpha)$  and prices. Hence, when  $C < \hat{C}$  and as  $\alpha$  increases, the price benefit of search dominates the reduction of honesty benefit either if  $N = 2$  or if  $N > 2$  and  $\alpha > \hat{\alpha}$ , so that  $\tau(\alpha)$  is increasing; but the price benefit is dominated if  $N > 2$  and  $\alpha < \hat{\alpha}$ , so that  $\tau(\alpha)$  is decreasing. Notice that when  $C < \hat{C}$ , the shape of  $\tau(\alpha)$  depends on  $N$ , because  $N$  affects the equilibrium price distribution and hence also the price benefit of search.

Figure 2 below illustrates how search benefit  $\tau(\alpha)$  varies with  $\alpha$ .

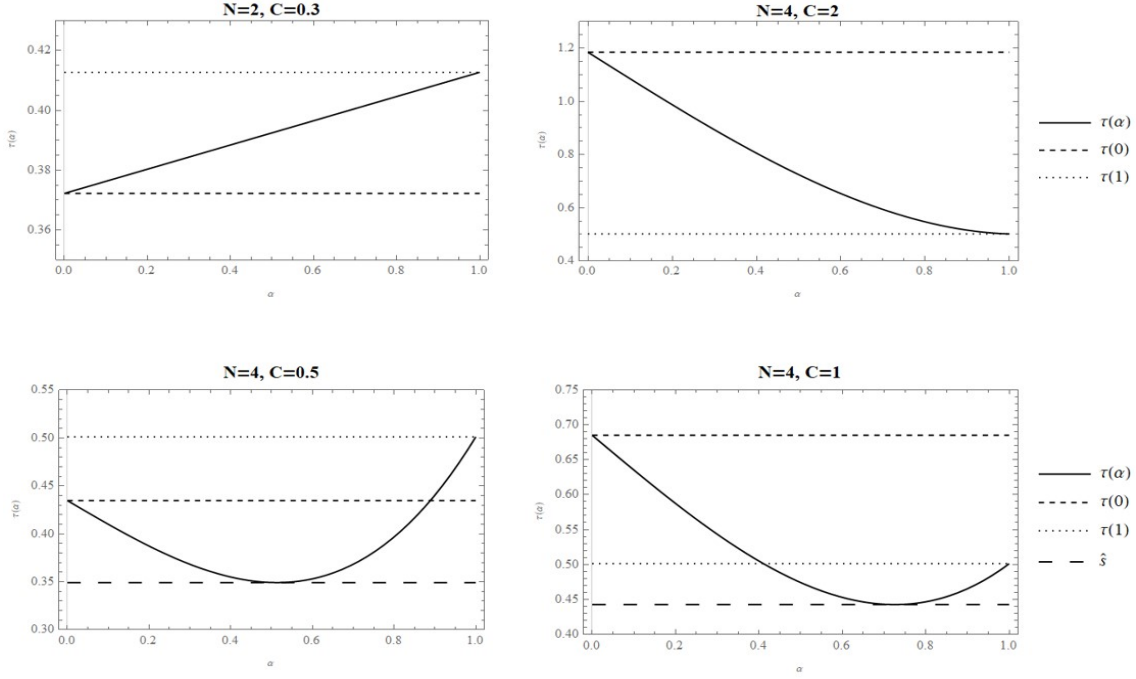


Figure 2: Search benefit  $\tau(\alpha)$  varies with  $\alpha$ .  
(Parameter values:  $\lambda = 0.3, \theta = 0.5, v = 0.8$ .)

We are now in a position to fully characterize the hybrid equilibrium. Define

$$\hat{s} \equiv \min_{\alpha \in [0,1]} \tau(\alpha). \quad (20)$$

By Lemma 3, when  $C \geq \hat{C}$ ,  $\tau(\alpha)$  decreases in  $\alpha$  and thus  $\hat{s} = \tau(1)$ ; when  $C < \hat{C}$  and  $N = 2$ ,  $\tau(\alpha)$  increases in  $\alpha$  and thus  $\hat{s} = \tau(0)$ ; but when  $C < \hat{C}$  and  $N > 2$ ,  $\tau(\alpha)$  is minimized at  $\hat{\alpha} \in (0, 1)$  and thus  $\hat{s} = \tau(\hat{\alpha})$ .

**Proposition 1** *Suppose that  $\hat{s} < s < \max\{\tau(0), \tau(1)\}$ . There exists a hybrid equilibrium, where  $\alpha^* \in (0, 1)$  and  $r^*$  satisfy (12) and (15). Moreover,  $\alpha^*$  is unique if  $C \geq \hat{C}$  or if  $C < \hat{C}$  and  $N = 2$ ; while  $\alpha^*$  may have either one or two values if  $C < \hat{C}$  and  $N > 2$ . Each expert's equilibrium strategy is: for  $i = M$ , recommend  $T_M$  and price from  $F(p)$ ; for  $i = m$ , recommend  $T_M$  and price from  $G(p)$  with probability  $\alpha^*$  but recommend  $T_m$  and price from  $H(q)$  with probability  $1 - \alpha^*$ . Searchers sequentially search experts with reservation prices  $r^*$  for  $T_M$  and  $v$  for  $T_m$ .*

**Proof.** The equilibrium strategies of the experts and consumers follow directly from the construction of the equilibrium. Thus, it suffices to show the existence and possible uniqueness of  $(\alpha^*, r^*)$ . We consider in turn three possible cases.

(i) When  $C \geq \hat{C}$ ,  $\tau(\alpha)$  decreases in  $\alpha$  for all  $\alpha \in (0, 1)$  by Lemma 3. For  $\tau(1) = \hat{s} < s < \tau(0)$ , there is a unique  $\alpha^* \in (0, 1)$  such that  $\tau(\alpha^*) = s$ , and the unique equilibrium  $r^*$  is then given by (12) with  $\alpha = \alpha^*$ .

(ii) When  $C < \hat{C}$ , by Lemma 3, if  $N = 2$ ,  $\tau(\alpha)$  monotonically increases, and hence for  $\tau(0) = \hat{s} < s < \tau(1)$ , there is a unique  $\alpha^*$  such that  $\tau(\alpha^*) = s$ , and the unique  $r^*$  is then given by (12) with  $\alpha = \alpha^*$ .

If  $N > 2$ ,  $\tau(\alpha)$  first decreases and then increases, reaching its minimum at  $\hat{\alpha} \in (0, 1)$ . Then

$$\hat{s} = \tau(\hat{\alpha}) = (1 - \theta)(1 - \hat{\alpha})C + (1 - \phi) \frac{(1 - \lambda + \lambda N \hat{\alpha}^{N-1})}{1 - \lambda} v > 0.$$

If  $\min\{\tau(0), \tau(1)\} < s < \max\{\tau(0), \tau(1)\}$ , there is a unique  $\alpha^*$  such that  $\tau(\alpha^*) = s$ ; whereas if  $\hat{s} < s < \min\{\tau(0), \tau(1)\}$ , there are two values of  $\alpha^*$ ,  $\alpha_1^* \in (0, \hat{\alpha})$  and  $\alpha_2^* \in (\hat{\alpha}, 1)$ , such that  $\tau(\alpha_1^*) = s$  and  $\tau(\alpha_2^*) = s$ . ■

The hybrid equilibrium has several notable features. First, each expert randomizes between recommending  $T_M$  and  $T_m$  for  $m$ , with the corresponding prices drawn from different distributions. While probabilistic cheating is a familiar equilibrium feature in the credence-goods literature, it is usually accompanied by random rejection of an expert's offer by consumers (as in the early contribution of Pitchik & Schotter (1987)). In our model, consumers adopt pure strategies, and an expert's indifference between honesty and dishonesty in equilibrium is due to the competition with other experts to balance the incentives to attract shoppers and to exploit searchers. Second, a price in the interval  $[b_f, b_g]$  indicates that the problem is  $M$  while a price in the interval  $[b_g, r^*]$  indicates the problem is  $M$  only with prior belief  $\theta$ . Hence, here a lower price for  $T_M$  can be a signal of high quality advice, namely that the expert is being truthful. Third, there is a gap between  $H(p)$  and  $G(p)$ :  $b_g = v + C$ , so that the prices for  $T_M$  when  $i = m$  are higher than the prices for  $T_m$  by at least  $C$ . This is because under our assumption of verifiable treatment, when recommending  $T_M$  for  $m$ , an expert needs



to incur  $C$  and will thus generally charge a price that is higher by more than  $C$  than the price when recommending  $T_m$ .

#### 4. EFFECTS OF SEARCH FRICTIONS

In this section, we study how search frictions affect equilibrium cheating probability and prices at the hybrid equilibrium. For this analysis, we assume  $\hat{s} < s < \max\{\tau(0), \tau(1)\}$ .

##### 4.1 Effects on the Cheating Probability

We are interested in whether under lower search frictions, in the sense that  $s$  is lower or  $\lambda$  is higher, competition by experts would reduce cheating in the market, with a lower  $\alpha^*$ .

**Proposition 2** *Suppose that  $\alpha^* \in (0, 1)$ . Then,  $\alpha^*$  increases in  $s$  and decreases in  $\lambda$  if  $\alpha^* > \alpha^c$ , but  $\alpha^*$  decreases in  $s$  and increases in  $\lambda$  if  $\alpha^* < \alpha^c$ , where*

$$\alpha^c = \begin{cases} \min\{\hat{\alpha}, N-2\} & \text{if } C < \hat{C} \\ 1 & \text{if } C \geq \hat{C} \end{cases}, \quad (21)$$

and  $0 < \alpha^c < 1$  if  $C < \hat{C}$  and  $N > 2$ .

**Proof.** From (15) and (18), the equilibrium expert cheating probability ( $\alpha^*$ ) and search cost ( $s$ ) satisfy consumers' optimal search rule:  $\tau(\alpha^*) - s = 0$ , or

$$\Psi(\alpha^*, s) \equiv (1 - \phi) \frac{v(1 - \lambda + \lambda N \alpha^{*N-1})}{1 - \lambda} + (1 - \theta)(1 - \alpha^*)C - s = 0.$$

We have  $\frac{\partial \Psi}{\partial s} = -1 < 0$ ,

$$\frac{\partial \Psi}{\partial \lambda} = -\frac{\partial \phi}{\partial \lambda} \frac{v(1 - \lambda + \lambda N \alpha^{*N-1})}{1 - \lambda} + (1 - \phi) v \frac{N \alpha^{*N-1}}{(1 - \lambda)^2} > 0$$

because  $\frac{\partial \phi}{\partial \lambda} < 0$ , and

$$\frac{\partial \Psi}{\partial \alpha^*} = (1 - \phi) \frac{v \lambda N (N - 1) \alpha^{*N-2}}{1 - \lambda} - (1 - \theta)C < 0 \quad \text{if } C \geq \hat{C} \equiv \frac{(1 - \phi) \lambda N (N - 1)}{(1 - \theta)(1 - \lambda)} v.$$

When  $C < \hat{C}$ ,  $\frac{\partial \Psi}{\partial \alpha^*} > 0$  if  $N = 2$ , while if  $N > 2$ , with  $\hat{\alpha}$  as defined in (17), we have

$$\frac{\partial \Psi}{\partial \alpha^*} < 0 \quad \text{if } \alpha^* < \hat{\alpha} \quad \text{and} \quad \frac{\partial \Psi}{\partial \alpha^*} > 0 \quad \text{if } \hat{\alpha} < \alpha^* < 1.$$

Therefore, when  $C < \hat{C}$  and either  $N = 2$  or  $\alpha^* > \hat{\alpha}$ , which is equivalent to  $\alpha^* > \alpha^c$ ,

$$\frac{\partial \alpha^*}{\partial s} = -\frac{\frac{\partial \Psi}{\partial s}}{\frac{\partial \Psi}{\partial \alpha^*}} > 0, \quad \frac{\partial \alpha^*}{\partial \lambda} = -\frac{\frac{\partial \Psi}{\partial \lambda}}{\frac{\partial \Psi}{\partial \alpha^*}} < 0;$$

whereas if  $C \geq \hat{C}$  or if  $C < \hat{C}$  but  $N > 2$  and  $\alpha^* < \hat{\alpha}$ , which is equivalent to  $\alpha^* < \alpha^c$ ,

$$\frac{\partial \alpha^*}{\partial s} < 0, \quad \frac{\partial \alpha^*}{\partial \lambda} > 0.$$

Moreover, since  $\hat{\alpha} \in (0, 1)$ ,  $\alpha^c \in (0, 1)$  if  $C < \hat{C}$  and  $N > 2$ . ■

At the hybrid equilibrium, if expert cheating in the market is pervasive enough ( $\alpha^* > \alpha^c$ ), increased competition due to lower search frictions can discipline experts, as one might expect. Surprisingly, if cheating is rare enough in the market ( $\alpha^* < \alpha^c$ ), lower search frictions actually increase expert cheating (i.e.,  $\alpha^*$  rises).

To see the intuition for these results, first notice that as  $\lambda$  increases, there are more shoppers in the market who will purchase from the lowest-priced seller, and offering  $T_m$  for  $i = m$  is more likely to have the lowest price if more experts are currently cheating by offering  $T_M$  for  $m$ . Hence, if the cheating probability in the market is currently above a critical level ( $\alpha^* > \alpha^c$ ), a higher  $\lambda$  makes it relatively more attractive for an expert to be honest, decreasing equilibrium cheating probability  $\alpha^*$ ; whereas if the cheating activity is currently below the critical level, competition for honest experts who offer  $T_m$  for  $m$  will be relatively more intense, and a higher  $\lambda$  increases the attractiveness of cheating, leading to a higher  $\alpha^*$ .

Next, suppose that at a hybrid equilibrium with some  $(\alpha^*, r^*)$  there is a marginal decrease in search cost  $s$ . This will have similar effects on experts' cheating as an increase in  $\lambda$ , but through somewhat different mechanisms. When  $\alpha^* > \alpha^c$ , cheating is sufficiently common in the market, and the lower  $s$  leads to relatively more competition for dishonesty experts, as reflected by a reduction in the searchers' reservation price for  $T_M$  but not for  $T_m$  (i.e.,  $r^*$  falls but  $r_m^* = v$  is unchanged). This motivates experts to be more honest, resulting in a decrease in  $\alpha^*$ . On the other hand, when  $\alpha^* < \alpha^c$ , cheating is sufficiently uncommon in the market, and the decrease in  $s$  leads to relatively more competition for honest experts who recommend  $T_m$  for  $m$ , as reflected by a *rise* in  $r^*$  while  $r_m^* = v$  is unchanged, causing  $r_m^*/r^*$  to fall. This motivates experts to cheat—recommending  $T_M$  instead of  $T_m$  for  $m$ —more, resulting in an increase in  $\alpha^*$ .

Notably, critical value  $\alpha^c$  depends on  $C$ , the additional cost for the major treatment, and it may also depend on  $N$ . Specifically, as Lemma 3 indicates, when  $C \geq \hat{C}$ , the loss in the honesty benefit of search from a higher  $\alpha$  dominates so that search benefit  $\tau(\alpha)$  monotonically decreases in  $\alpha$ . Then, when  $s$  falls,  $\alpha^*$  always rises to restore optimal search, and in this case  $\alpha^c = 1$ . On the other hand, when  $C < \hat{C}$ , if  $N = 2$  or if  $N > 2$  and  $\alpha > \hat{\alpha}$ , the gain in the price benefit of search from a higher  $\alpha$  dominates so that  $\tau(\alpha)$  increases; whereas if  $N > 2$  but  $\alpha < \hat{\alpha}$ , the honesty benefit again dominates so that  $\tau(\alpha)$  decreases. Hence, if  $C < \hat{C}$ ,  $\alpha^c = \min\{\hat{\alpha}, N - 2\} < 1$ .

The results in Proposition 2 suggest that the effects of search friction on expert behavior are rich and nuanced: they reflect complex interactions between experts' cheating probability, consumers' reservation price in sequential search, and experts' prices. Importantly, the optimal strategy of an expert or a consumer is determined by equilibrium considerations: each is optimal for the player given the strategies of all other market participants. Moreover, there is complementarity among experts' cheating incentives in the sense that when other experts increase (or decrease) their cheating probability, searchers have a higher (or lower) reservation price when being recommended the major treatment, which in turn increases (decreases) each expert's cheating incentive to recommend  $T_M$  for  $i = m$ .

Since  $\alpha^c$  becomes higher as  $N$  increases from 2 to 3 when  $C < \hat{C}$ , Proposition 2 indicates that reductions in search friction can adversely impact expert behavior when there are more sellers in the market. More generally, for given values of  $\lambda$  and  $s$ , it can be shown that an increase in the number of sellers can increase both expert cheating and prices. The intuition for this is somewhat similar to that in Stahl (1989) where sellers compete only in prices: as  $N$  rises, it becomes less likely to be the lowest-priced seller for an honest expert that offers  $T_m$  for  $m$ , which provides incentives for experts to increase cheating and raise prices. However, in our model experts also choose product offerings (i.e., recommendations), and the cheating incentive is influenced also by the magnitude of  $C$  (the cost of  $T_M$ ) and of  $\alpha^*$  (which depends on search friction). Therefore, unlike in Stahl (1989), where an increase in the number of sellers always stochastically increases prices, in our model the cheating probability and prices

can be either higher or lower as  $N$  increases, depending on parameter values.<sup>18</sup>

## 4.2 Effects on Equilibrium Prices

To examine the effects of search frictions on equilibrium prices, it is convenient to denote the equilibrium price distribution when  $i = m$  as

$$\Phi(p) = \begin{cases} (1 - \alpha) H(p) & \text{if } p \in [b_h, v] \\ 1 - \alpha & \text{if } p \in (v, b_g) \\ 1 - \alpha + \alpha G(p) & \text{if } p \in [b_g, r] \end{cases} .$$

Since  $H(v) = 1$ ,  $G(b_g) = 0$ , and both  $H(p)$  and  $G(p)$  increase in  $p$ ,  $\Phi(p)$  is continuous and weakly increases in  $p$ . Moreover,  $\Phi(r) = 1 - \alpha + \alpha G(r) = 1$ , and  $\Phi(b_h) = (1 - \alpha)H(b_h) = 0$ . Therefore,  $\Phi(p; \alpha)$  is a continuous c.d.f.

The following lemma is helpful for understanding the comparative statics on prices.

**Lemma 4** *Both  $F(p)$  and  $\Phi(p)$  decrease in  $\alpha$ .*

**Proof.** First, from (12),  $r$  increases in  $\alpha$ . From (7),  $\frac{\partial F(p)}{\partial r} < 0$  and thus

$$\frac{\partial F(p)}{\partial \alpha} = \frac{\partial F(p)}{\partial r} \frac{\partial r}{\partial \alpha} < 0.$$

Second,

$$1 - \alpha + \alpha G(p) = 1 - \left[ \frac{(r - p)(1 - \lambda)}{(p - C)\lambda N} \right]^{\frac{1}{N-1}} \quad \text{for } p \in [b_g, r],$$

which decreases in  $r$  and thus decreases in  $\alpha$ . From (11),

$$(1 - \alpha)H(p) = 1 - \left[ \frac{v(1 - \lambda + \lambda N \alpha^{N-1})}{p\lambda N} - \frac{1 - \lambda}{\lambda N} \right]^{\frac{1}{N-1}} \quad \text{for } p \in [b_h, v],$$

which decreases in  $\alpha$ . ■

Lemma 4 indicates that equilibrium prices are increasing in  $\alpha$  in the sense of first-order stochastic dominance (FSD). Notice that  $F(p)$  and  $\Phi(p)$  depend on  $s$  only through  $\alpha$  from (12) and (15). Thus, from Proposition 2, as  $s$  increases,  $\alpha^*$  and hence equilibrium prices are higher if  $\alpha^* > \alpha^c$  but lower if  $\alpha^* < \alpha^c$ . We thus have:

<sup>18</sup>The precise conditions for the effects of  $N$  are rather tedious. We thus choose not to include them in the paper, to avoid distractions from our focus on the effects of search friction.

**Proposition 3** *Suppose that prices are compared in the sense of FSD. Then, at a hybrid equilibrium, as search cost increases, both equilibrium prices and cheating probabilities are higher if  $\alpha^* > \alpha^c$  and both are lower if  $\alpha^* < \alpha^c$ .*

The effects of search cost on equilibrium prices and cheating probabilities are connected in interesting ways: they either both fall or both rise as search friction increases. Intriguingly, in an expert market, a reduction in search cost can hurt all consumers when the current level of expert cheating is relatively low, because in this case lower search cost will increase competition relatively more for honest experts and thus motivate experts to increase the frequency of recommending  $T_M$  for  $m$ , resulting in a higher  $\alpha^*$ . This in turn reduces consumers' search incentive, leading to higher equilibrium prices.

## 5. SEPARATING AND POOLING EQUILIBRIA

When search cost is sufficiently low or high, the model has equilibria that differ from the hybrid equilibrium.

### 5.1 Separating Equilibrium

When search cost is sufficiently low, there is a separating equilibrium where all experts recommend  $T_i$  for  $i \in \{M, m\}$ . At the equilibrium, experts will recommend  $T_M$  for  $i = M$  with prices drawn from  $F_M(p)$  given by (1), and searchers who are recommended  $T_M$  will search with reservation price  $r_M^o$  given by (2); whereas experts will recommend  $T_m$  for  $i = m$  with prices drawn from  $F_m(p)$  given by (1), and searchers who are recommended  $T_m$  will search with reservation price  $r_m^o = \min\{v, \omega\}$ .

To establish the equilibrium, it remains to show that no expert can benefit from choosing  $T = T_M$  for  $i = m$ . Suppose that a searcher with  $i = m$ , who is willing to pay  $p = r_M^o$  for  $T_M$ , visits such a deviating expert and mistakenly believes that her problem is  $i = M$ . Since other experts will still recommend  $T_m$  for  $i = m$  and price according to  $F_m(q)$ , resulting in stochastically lower prices than under  $F_M(p)$ , the deviating firm is less likely to sell to shoppers than the other experts. This implies that the most profitable deviation is for the

deviating expert to offer  $T = T_M$  for  $i = m$  with price  $p = r_M^o$ . The expert's profit under this deviation is

$$(r_M^o - C) \frac{1 - \lambda}{N},$$

whereas his profit when following the equilibrium strategy is

$$r_m^o \frac{1 - \lambda}{N}.$$

Hence, the separating equilibrium can be sustained if and only if

$$r_m^o \geq r_M^o - C.$$

**Proposition 4** *If  $s \leq v(1 - \phi)$ , there is a unique symmetric equilibrium in which experts are honest ( $\alpha^* = 0$ ). Equilibrium price distribution and optimal consumer search rules are the same as in the case where consumers can observe  $i \in (M, m)$ .*

**Proof.** When  $v \geq \omega$ ,  $r_m^o = \omega$ , which is determined by (6). Since

$$r_M^o = C + \frac{s}{1 - \phi} \quad \text{and} \quad \omega = \frac{s}{1 - \phi},$$

experts receive the same profit from recommending  $T_M$  or  $T_m$  when  $i = m$ . Hence recommending  $T_i$  for  $i \in \{M, m\}$  is optimal for experts.

Notice that  $\omega$  increases in  $s$ , and  $\omega = v$  when  $s = v(1 - \phi)$ . Thus, if  $s > v(1 - \phi)$ ,  $r_m^o = \min\{v, \omega\} = v < r_M^o - C$ . In this case, experts would deviate to recommending  $T_M$  for  $i = m$  with price  $r_M^o$ .

Therefore, if  $s \leq v(1 - \phi)$  and  $\omega$  solves (6), then there is a symmetric separating equilibrium in which  $\alpha^* = 0$ , and the equilibrium is unique because  $F_i(p)$  is unique. ■

Although the market in our model has search frictions, Proposition 4 indicates that if search cost is low enough, competition can effectively discipline experts so that they will all behave honestly.<sup>19</sup> Intuitively, as  $s$  becomes small, the price distribution  $H(q)$  shrinks so that its upper bound becomes  $r_m = \omega$ , under which the expected profit under  $T_m$  is the same as

<sup>19</sup>Notice that in our model a fraction of consumers have no search cost. If all consumers have a positive search cost, then the Diamond (1971) result holds: no matter how small the search cost is, experts will charge the monopoly price  $\theta V + (1 - \theta)v$ , and the separating equilibrium does not exist.

that under  $T_M$ . Experts will then have no incentive to offer  $T_M$  for  $m$ . Recall that  $\phi$  is lower when  $\lambda$  is higher or  $N$  is lower, and  $\phi \rightarrow 1$  as  $\lambda \rightarrow 0$ . Hence, the region of parameter values under which the separating equilibrium prevails is larger when  $\lambda$  is higher or  $N$  is lower, but the region vanishes as  $\lambda \rightarrow 0$ . This confirms that the presence of shoppers who can search without cost is essential for the existence of the separating equilibrium.

A key insight in the literature on credence goods is that experts will provide honest recommendations if there are equal markups for  $T_M$  and  $T_m$ . Our result extends this insight to situations under mixed-strategy pricing with consumer search: the experts will behave honestly when they expect to receive the same expected profit from  $T_M$  and  $T_m$  for problem  $m$ . Notice that from (18) and (20),  $\min\{\tau(0), \tau(1), \hat{s}\} > v(1 - \phi)$ .

Proposition 4 also extends Stahl (1989) to expert markets: When  $s$  is sufficiently small, experts will price and consumers will search in the same ways as in Stahl (1989), even though—unlike in Stahl—here consumers do not observe the value of the service they receive. Moreover, as  $s \rightarrow 0$ ,  $r_M^o = C + \frac{s}{1-\phi}$  and  $\omega = \frac{s}{1-\phi}$  approach  $C$  and 0, the respective marginal costs for  $T_M$  and  $T_m$ . Hence, same as in Stahl (1989), the Bertrand outcome is the limiting case of our model of expert markets when search cost tends to zero.

## 5.2 Pooling Equilibrium

When search cost is high enough, experts will always cheat, which yields a pooling equilibrium where experts always recommend  $T_M$  and follow the same pricing strategy for  $i \in \{M, m\}$ . Similar to the result in Lemma 1, experts would then price according to  $F(p)$  and  $G(p)$  respectively for  $i = M$  and  $i = m$ , which have the same upper bound. Setting  $\alpha = 1$  in  $G(p)$ , we have  $b_g = b_f$  and  $G(p) = F(p) = F_M(p)$ . The equilibrium upper bound for the common price distribution is then  $r_M^o$ , as given by (2).

At the proposed pooling equilibrium, if an expert deviates to  $T_m$  when  $i = m$ , it can save cost  $C$  and potentially capture all shoppers. The expert's optimal deviating price in this case is  $v$ , while he still prices according to  $F_M(p)$  if  $i = M$ . If  $s$  is high enough, such a deviation would not be profitable because the price reduction to  $v$  would be too large.

**Proposition 5** *If  $\tau(1) \leq s \leq \bar{s} = (1 - \phi)[\theta V + (1 - \theta)v - C]$ , there is a symmetric equilibrium in which experts always recommend  $T_M$  (i.e.,  $\alpha^* = 1$ ) and price according to  $F_M(p)$  for  $i \in \{M, m\}$ . All searchers will search with reservation price  $r_M^o \leq [\theta V + (1 - \theta)v]$ .*

**Proof.** At the proposed equilibrium, each expert's profit is

$$(r_M^o - C) \frac{1 - \lambda}{N}.$$

If an expert deviates to offering  $T_m$  for  $i = m$  with  $p = v$ , his profit is

$$v \left( \lambda + \frac{1 - \lambda}{N} \right).$$

Therefore, the equilibrium can be sustained if and only if

$$r_M^o = C + \frac{s}{1 - \phi} \geq C + \frac{1 - \lambda + \lambda N}{1 - \lambda} v,$$

or

$$s \geq \frac{(1 - \phi)(1 - \lambda + \lambda N)}{1 - \lambda} v = \tau(1).$$

In addition, to ensure the existence of the reservation price, we need  $r_M^o \leq [\theta V + (1 - \theta)v]$ , which is equivalent to

$$s \leq (1 - \phi)[\theta V + (1 - \theta)v - C] = \bar{s},$$

which holds under assumption (5). Notice that  $\tau(1) < \bar{s}$  when  $V$  is sufficiently large. ■

Intuitively, if  $s$  is high enough, search benefit is likely below  $s$ , which means that searchers have low incentives to search. Then, experts will charge high prices for treatment  $T_M$  (but a price only up to  $v$  for treatment  $T_m$ ). This motivates experts to always recommend  $T_M$  for  $m$  in equilibrium ( $\alpha^* = 1$ ), resulting in the pooling equilibrium. As  $s$  decreases, consumers search more intensively, which imposes downward pressure to the prices for both treatments; and when  $s$  is low enough, experts will cheat only with probabilities  $\alpha^* < 1$ , resulting in a hybrid equilibrium. Notice that the hybrid equilibrium and the pooling equilibrium coexist if  $\tau(1) < s < \tau(0)$ . When  $s$  is sufficiently low,  $v$  will not be a binding constraint for the price of  $T_m$ , and recommending  $T_i$  for  $i \in \{M, m\}$  is then optimal for experts because they need to incur  $C$  without getting a much higher price from  $T_M$  than from  $T_m$ , making cheating not



profitable. However, there may be a (small) region of  $s$  for which a symmetric equilibrium fails to exist. Notice that for given  $s$ ,  $\tau(1) \leq s$  holds if  $\lambda$  is small enough. Hence, a pooling equilibrium exists as  $\lambda$  approaches 0, and in this case the outcome in Stahl (1989) is also a limiting case of our model.

Notice that in the interior regions of  $s$  and  $\lambda$  in which the separating or pooling equilibrium exists, a marginal change in search frictions (either  $s$  or  $\lambda$ ) has no effect on  $\alpha^*$ , which is either 0 or 1. Hence, our results on the effects of search frictions on  $\alpha^*$ , based on the hybrid equilibrium, holds weakly at all equilibria of the model.

Finally, if we modify our model to assume alternatively that treatment is non-verifiable, then it can be shown that neither a separating nor a pooling equilibrium would exist,<sup>20</sup> but a hybrid equilibrium would. However, in this case the equilibrium price distributions for  $T_M$  and  $T_m$  under  $m$  will have no gap, because the experts would use treatment  $T_m$ , without incurring  $C$ , even when recommending  $T_M$  for  $m$ .<sup>21</sup> The waste of  $C$  to treat a minor problem is then avoided. Therefore, within our framework, if  $s$  is sufficiently small, in equilibrium experts will behave honestly when treatment is verifiable but not when it is unverifiable, whereas welfare is the same under the two alternative assumptions because in neither case experts would incur  $C$  for  $m$ . If  $s$  is higher, there is cheating in equilibrium under both assumptions. Not surprisingly, experts are more likely to cheat and earn higher expected profits—but welfare is weakly higher due to the avoidance of  $C$  for  $m$ —if treatment is not verifiable than when it is.

## 6. CONCLUSION

This paper has developed and analyzed a model of search and competition in expert markets. We extend Stahl (1989) to introduce sellers' private information about the appropriate treatment/service for consumers. The model shows that, due to search cost, for the

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<sup>20</sup>In this case, at a candidate separating equilibrium, an expert would always deviate to recommending  $T_M$  for  $m$ . At a candidate pooling equilibrium,  $T_M$  and  $T_m$  would have same prices. This would lead to different expected profits under  $M$  and  $m$  because only  $T_M$  costs  $C$ , which would in turn invalidate the equilibrium.

<sup>21</sup>The analysis is lengthy but largely parallels the analysis when treatment is verifiable and also the analysis in Janssen et al. (2011). We thus omit the detailed analysis under this alternative assumption.

same problem consumers may receive divergent recommendations and prices from different experts. Some experts may cheat by recommending an unnecessary treatment, and the dishonest experts also charge higher prices on average. Consumers search experts sequentially under Bayesian belief updating and with an optimal reservation price for each recommended treatment. The model further shows that search frictions can affect expert behavior non-monotonically: as they decrease, expert cheating can fall if it already occurs frequently enough in the market, but it can rise otherwise.

Despite the central importance of competition for economic efficiency, it is not surprising that competition may not work well when sellers possess superior product information relative to consumers. A novel insight of this paper, however, is that search cost can be a greater barrier to effective competition in expert markets. In fact, in our model if search cost is below some critical level, competition will drive all experts to make honest recommendations, and the equilibrium outcome coincides with that of Stahl (1989). Thus, a clear way to achieve efficiency gains from competition in expert markets is to make consumer search sufficiently convenient, even if it does not entirely eliminate search cost. However, in practice, because search cost may often be relatively high and its (marginal) reductions can—as we have shown—have non-monotonic effects, the role that competition plays in disciplining expert behavior is likely to be limited. This sentiment is echoed further by our finding that increases in the number of competing experts can result in more cheating.

Although not considered in our model, many products in expert markets may not be pure credence goods, in the sense that there is a (small) probability that a dishonest expert will be found to have been untruthful. Extending our model to include such a possibility will not change the analysis and results if consumers have no recourse *ex post* after detecting an expert's cheating, but it suggests that regulations can improve the performance of expert markets. For instance, regulators may be able to promote or set higher standards for professional conduct, inspect or gather information about the works performed by experts, and warn consumers about dishonest experts, especially when experts may interact with different consumers over time but each individual consumer lacks the knowledge about them. Regulations may also impose legal liability for unethical practices, as for instance in medical practices.

By showing the limits to effective competition in expert markets due to search frictions, our paper suggests the need for regulation in such markets, even when they may appear to be highly competitive with numerous providers.

## REFERENCE

- Alger, I., & Salanie, F. (2006). A theory of fraud and overtreatment in experts markets. *Journal of Economics & Management Strategy*, 15(4), 853–881.
- Anderson, S. P., & Renault, R. (1999). Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *RAND Journal of Economics*, 30(4), 719–735.
- Bagwell, K., & Riordan, M. H. (1991). High and declining prices signal product quality. *American Economic Review*, 81(1), 224–239.
- Balafoutas, L., & Kerschbamer, R. (2020). Credence goods in the literature: What the past fifteen years have taught us about fraud, incentives, and the role of institutions. *Journal of Behavioral and Experimental Finance*, 26, 100285.
- Bar-Isaac, H., Caruana, G., & Cuñat, V. (2012). Search, design, and market structure. *American Economic Review*, 102(2), 1140–60.
- Bester, H., & Dahm, M. (2018). Credence goods, costly diagnosis and subjective evaluation. *Economic Journal*, 128(611), 1367–1394.
- Chen, Y., Li, J., & Zhang, J. (2022a). Efficient liability in expert markets. *International Economic Review*, forthcoming.
- Chen, Y., Li, Z., & Zhang, T. (2022b). Experience goods and consumer search. *American Economic Journal: Microeconomics*, 14(3), 591–621.
- Chen, Y., & Zhang, T. (2011). Equilibrium price dispersion with heterogeneous searchers. *International Journal of Industrial Organization*, 29(6), 645–654.
- Chen, Y., & Zhang, T. (2018). Entry and welfare in search markets. *Economic Journal*, 128(608), 55–80.
- Choi, M., Dai, A. Y., & Kim, K. (2018). Consumer search and price competition. *Econometrica*, 86(4), 1257–1281.
- Darby, M. R., & Karni, E. (1973). Free competition and the optimal amount of fraud. *The Journal of Law and Economics*, 16(1), 67–88.

- Diamond, P. A. (1971). A model of price adjustment. *Journal of Economic Theory*, 3(2), 156–168.
- Dulleck, U., & Kerschbamer, R. (2006). On doctors, mechanics, and computer specialists: The economics of credence goods. *Journal of Economic Literature*, 44(1), 5–42.
- Dulleck, U., Kerschbamer, R., & Sutter, M. (2011). The economics of credence goods: An experiment on the role of liability, verifiability, reputation, and competition. *American Economic Review*, 101(2), 526–555.
- Emons, W. (1997). Credence goods and fraudulent experts. *RAND Journal of Economics*, 28(1), 107–119.
- Emons, W. (2001). Credence goods monopolists. *International Journal of Industrial Organization*, 19(3-4), 375–389.
- Fong, Y.-f. (2005). When do experts cheat and whom do they target? *RAND Journal of Economics*, 36(1), 113–130.
- Fong, Y.-f., Liu, T., & Meng, X. (2022). Trust building in credence goods markets. *American Economic Journal: Microeconomics*, 14(1), 490–528.
- Haan, M. A., & Moraga-González, J. L. (2011). Advertising for attention in a consumer search model. *Economic Journal*, 121(552), 552–579.
- Janssen, M., Pichler, P., & Weidenholzer, S. (2011). Oligopolistic markets with sequential search and production cost uncertainty. *RAND Journal of Economics*, 42(3), 444–470.
- Liu, T. (2011). Credence goods markets with conscientious and selfish experts. *International Economic Review*, 52(1), 227–244.
- Liu, T., & Ma, C.-t. A. (2021). Equilibrium information in credence good. *working paper*.
- Moraga-González, J. L., Sándor, Z., & Wildenbeest, M. R. (2017). Prices and heterogeneous search costs. *RAND Journal of Economics*, 48(1), 125–146.
- Moraga-González, J. L., & Sun, Y. (2022). Product quality and consumer search. *American Economic Journal: Microeconomics*, forthcoming.

- Pesendorfer, W., & Wolinsky, A. (2003). Second opinions and price competition: Inefficiency in the market for expert advice. *Review of Economic Studies*, 70(2), 417–437.
- Pitchik, C., & Schotter, A. (1987). Honesty in a model of strategic information transmission. *American Economic Review*, 77(5), 1032–1036.
- Rhodes, A. (2011). Can prominence matter even in an almost frictionless market? *Economic Journal*, 121(556), 297–308.
- Schneider, H. S. (2012). Agency problems and reputation in expert services: Evidence from auto repair. *Journal of Industrial Economics*, 60(3), 406–433.
- Stahl, D. O. (1989). Oligopolistic pricing with sequential consumer search. *American Economic Review*, 79(4), 700–712.
- Stigler, G. J. (1961). The economics of information. *Journal of Political Economy*, 69(3), 213–225.
- Taylor, C. R. (1995). The economics of breakdowns, checkups, and cures. *Journal of Political Economy*, 103(1), 53–74.
- Wolinsky, A. (1986). True monopolistic competition as a result of imperfect information. *Quarterly Journal of Economics*, 101(3), 493–511.
- Wolinsky, A. (1993). Competition in a market for informed experts' services. *RAND Journal of Economics*, 24(3), 380–398.
- Wolinsky, A. (1995). Competition in markets for credence goods. *Journal of Institutional and Theoretical Economics*, 151, 117–131.
- Zhou, J. (2014). Multiproduct search and the joint search effect. *American Economic Review*, 104(9), 2918–2939.