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Licensing in a Stackelberg industry, product differentiation, and welfare

Manel Antelo* **and Lluís Bru****

Abstract

In a differentiated Stackelberg duopoly, we explore the licensing behaviour of an inside patent holder owning a cost-reducing innovation and that may play as a leader or follower in setting the output level in the marketplace. We find that, regardless of whether the licensor is the leader or the follower, the licensing contract always involves royalties: per-unit or ad-valorem (depending on the degree of product differentiation and the size of the innovation) when the licensor is the leading firm, and per-unit royalties (alone or combined with a fixed payment) when it is the follower. We also show that, as compared to the pre-licensing context, licensing by a market follower is never welfare reducing, and licensing by a market leader is only welfare reducing when the products are very close substitutes.

Keywords: Stackelberg industry, licensing, differentiated products, per-unit and ad-valorem royalties, welfare

JEL Classification: L13, L24

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1. Introduction

Firms that spend on R&D and patent their innovations manage intellectual property as a business asset not very different from a product on a shelf. A common way to profit from an innovation is to either use it to produce a product and/or license it to other firms to use in the production of their products, and which may or may not play as direct competitors in the product market.[1] Licensing to direct competitors has several advantages for the innovating firm: royalty revenues are earned, competitors are discouraged from conducting their own in-house R&D to develop a leapfrogging innovation that could establish an industry standard, and market position is protected and even improved (Moreira et al., 2020). This could explain why patents are predominantly out-licensed to firms not affiliated with the licensors and that most licensing arrangements are between competitors in the same industry (Radauer and Dudenbostel, 2013; Jiang and Shi, 2018). Prominent examples of licensing between direct competitors include Microsoft, which has licensed mobile operating system features to Samsung and HTC (Hoffman, 2014), Apple, which obtained an eight-year licence from Microsoft for Applesoft BASIC, a dialect of Microsoft BASIC adapted to Apple II personal computer services, and General Motors, which has licensed the OnStar service, a satellite-based mapping service, to other car makers including Toyota and Honda (Montinaro et al., 2020).

A key issue in the licensing literature is the design of the optimal contract from the licensor's perspective, given that the revenue accrued by any (inside) licensor will depend on the structure of the deal (Sen and Tauman, 2009). Of the aspects that the licensor has to consider, especially important is the staus of the licensor and of the licensee in the marketplace. Both firms are sometimes in the same market position, and, accordingly, either Cournot or Bertrand models fit the licensing game well (see, e.g., Mukherjee and Balasubramanian, 2001; Faulí-Oller and Sandonís, 2002; Sen and Tauman, 2007; San Martín and Saracho, 2010, 2012). The literature shows, for different levels of generality, that when the licensor and licensee are Cournot competitors in the marketplace and information is complete, the licensor prefers ad-valorem to per-unit royalties because they lead to a more collusive industry. As a result, licensing, as compared to non-licensing, is detrimental for consumers and society as a whole. Similar results are derived when the

[1] https://www.economist.com/special-report/2005/10/22/the-arms-race

analysis is extended to a differentiated duopoly, where the optimal royalty modality depends on whether products are substitutes or complements and on the degree of product differentiation.

However, in industries consisting of a well-established firm with sound assets, we expect leaders and followers in setting output level (or market price) to emerge; therefore, a market interaction leadership model can preferably be used to explore the impact of a licensing process[2] (Fjell and Heywood, 2002). Analysis of the licensing game in a Stackelberg framework is particularly relevant for industries in which either the leader firm or the follower in the marketplace may be the (main) innovator. This may be the case, e.g., of former state monopolies with a first-mover advantage, which, however, face increased competition once the market is fully opened up, e.g., telecommunications, electricity, post, etc. Procter & Gamble and Ford are two firms that play as leaders in their product markets and that frequently license their patented innovations to direct competitors (Jiang and Shi, 2018), while Qualcomm is a dominant firm and also a large licensor in the industry of baseband processor chips that manage device wireless connections.

However, the innovation owner, who is, hence, the potential licensor, does not necessarily have to be the market leader (the most established firm), but could also be a follower firm that entered the industry later. For example, Advanced Micro Devices Inc (AMD) is a licensor that acts as follower in the market of desktop and laptop microprocessors, where Intel plays as the leader. AMD, up until about 2016, controlled around 25% of the central processing unit market, in comparison with Intel, with more than 70%; in that year, however, AMD licensed the x86 processor and system-on-chip technology to a company called THATIC, creating a new rival for Intel.[3] Note that AMD's technological advantage and the fact that it has become a licensor does not mean that it has overtaken Intel in terms of market share.

In this paper, we investigate the licensing behaviour of a firm that may play as a Stackelberg leader or Stackelberg follower in setting the output level in the marketplace

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[2] For example, when a firm supplies interconnection services to a network of customers, it needs to be able to anticipate customer demand to invest in network capacity and, once capacity is installed, customers determine how much demand there is for whatever proportion of the service. The provider then behaves in the market as a classic Stackelberg leader, while the rival follows, taking the constraints imposed by the leader as given (Hoesel, 2008).

[3] https://www.infoworld.com/article/3060454/a-new-amd-licensing-deal-could-create-more-x86-rivals-for-intel.html

(Filippini, 2005; Kabiraj, 2005; Yi and Yanagawa, 2011), in a context in which both licensor and licensee produce differentiated goods and information is complete.[4] This setting has practical significance, because a number of products exists in the real world whose markets can be approximated as a leadership structure (Kabiraj, 2002; Cumbul, 2021). Moreover, the degree of product differentiation assumes great importance in many industrial markets. In our framework, we show that differentiation, coupled with the size of the innovation and the licensor's market position, play a crucial role in framing the licensing agreement and its welfare impact.

Our results contribute to the existing literature in three ways. First, we find that royalties are always present in licensing deals, regardless of whether the licensor acts as leader or follower in the marketplace. However, while royalties are per unit when the licensor is the market follower, this does not always hold when the licensor is the market leader: royalties may be per unit or ad valorem, depending on the innovation size and the degree of product differentiation. In particular, if the licensor is the market follower, the way for the licence to increase industry profits is to use per-unit royalties (combined or not with a fixed payment) rather than ad-valorem royalties. A licence consisting of an ad-valorem royalty would greatly increase the leader/licensee's output (and would reduce the follower/licensor's output), and the industry would consequently be more competitive. In contrast, a per-unit royalty allows the licensor to "control" the leader/licensee's output level, which is either maintained (when the per-unit royalty is equal to the innovation size, and therefore, leads the licensee to have the same marginal cost as without licensing), or is slightly increased (when the per-unit royalty is placed below the innovation size, and therefore, leads the licensee to have a lower marginal cost than without licensing). The use of one or the other royalty depends on the innovation size and the degree of product differentiation; thus, if the innovation is small (large), the contract is a pure "high" per-unit royalty (a "low" unit royalty combined with a fixed fee) that leads leader/licensee efficiency to remain unchanged (to improve). Hence, the existence of non-identical products allows (as in the case of homogeneous products) not only the presence of pure per-unit royalty contracts, but also the emergence of two-part tariff (2PT) contracts in which the licensee gains productive efficiency.

[4] Zhang et al. (2016) study the effects of product differentiation and technology spillover on the optimal licensing strategy for a stochastic R&D firm in a differentiated Stackelberg industry. However, they do not consider ad-valorem royalties.

Second, when the licensor is the market leader, we show that licensing may not only feature per-unit royalties, but also ad-valorem royalties. In fact, when products are close substitutes, or are distant substitutes but the innovation size is sufficiently large, then the licence consists of a (pure) ad-valorem royalty. By contrast, when products are very differentiated, a per-unit royalty (combined or not with a fixed-fee payment as in a 2PT contract) is preferred. The intuition behind this result is as follows. If products are close substitutes, the main objective of the leader/licensor is to restrict competition, since the licensing contract redistributes any extra profit arising from increased collusion between the follower/licensee and the leader/licensor. To achieve a more collusive outcome, therefore, the transfer of the innovation must induce a reduction in the licensee's own production, which is better achieved with ad-valorem royalties. If products are highly differentiated, however, rivalry between firms is already weak, and alleviating market competition thus becomes a secondary concern for the leader/licensor; rather, their main objective is now to improve industry efficiency, which is better achieved by means of a licensing contract involving per-unit royalties.

Therefore, we can make a straightforward theoretical prediction: in a leadership industry in which firms set quantities, we should implement technology transfer arrangements featuring per-unit royalties when the licensor is the follower; and when the licensor is the leader, we should implement contracts based not only on per-unit royalties, but also on ad-valorem royalties.

Third, the licensor's market status also plays a role in the welfare impact of licensing. As compared with a no-licensing scenario, licensing of a cost-reducing innovation from market follower to market leader never harms consumers or society as a whole, and sometimes even benefits both. However, this does not always hold when it is the market leader who licenses to the market follower, as, when products are identical or close substitutes, licensing harms both consumers and society as a whole. This is because, when licensing is by means of ad-valorem royalties, it improves follower/licensee efficiency and leads to a redistribution of production levels that outweighs the beneficial effects of increased efficiency to the point that welfare is reduced.

Summing up, in a scenario of product differentiation, as compared to a homogenous good scenario, the anti-competitive impact of licensing is reduced to the point that licensing becomes pro-competitive in almost the entire region of admissible parameters. Beyond this, while licensing plays an important role in disseminating an innovation within an industry, to obtain clear-cut results about the social impact, it is necessary to analyse the structure of the industry and the licensing modality that is implemented.

The rest of the paper is structured as follows. Section 2 outlines the model, Section 3 describes how we determine the optimal licensing scheme, Section 4 analyses the welfare impact of licensing, and Section 5 concludes.

2. The model

Consider an industry consisting of a firm that can make output commitment (market leader) and a rival that chooses its quantity after observing the quantity chosen by the leader (market follower). Each firm i, $i = 1, 2$, produces a differentiated product and faces the residual inverse demand function $p_i(q_i, q_j) = 1 - q_i - bq_j$,[5] where q_i and q_j denote, respectively, the quantity produced by firms i and j, $i, j = 1, 2; i \neq j$, and where parameter b, $0 \leq b \leq 1$, measures the degree of product differentiation, ranging from entirely different products to perfect substitutes. Currently, each firm produces with a technology that displays linear and constant marginal cost $c > 0$ and with no fixed costs. However, it is assumed that a firm, whether the leader or follower, has developed a new technology that reduces the marginal cost from c to 0 and that can be licensed to the direct competitor.[6] In order to ensure that the innovation is non-drastic, i.e., that both firms remain active in the market (irrespective of the degree of product differentiation and even if licensing does not occur), we assume that, for an innovation size c in $0 < c < 1$, the degree of product differentiation, b, satisfies $0 < b < \min\{b_{max}(c), 1\}$, where $b_{max}(c) \equiv \overline{b}(c) = \frac{\sqrt{1+4(1-c)^2}-1}{1-c}$ when the innovation owner is the leader firm and $b_{max}(c) \equiv \tilde{b}(c) = 2(1-c)$ when the owner is the follower firm.

The innovating firm – either the leader or the follower in setting the output level in the marketplace – licenses their innovation to their rival by means of a 2PT contract, involving an upfront fee plus a non-negative royalty, which may be either a rate per unit produced by the licensee (per-unit royalty) or a percentage of licensee's sales (ad-valorem

[5] This demand comes from a utility function, $u(q_1, q_2) = q_1 + q_2 - bq_1q_2 - (q_1^2 + q_2^2)/2$.
[6] Of course, zero marginal costs must not be interpreted literally; what matters is that the wedge from the intercept of demand to marginal costs of production increases from $1 - c$ to 1.

royalty).[7] The marginal cost of selling the licence is zero and the licensor has all the bargaining power, while the licensee only obtains their outside option if the licensor's offer is rejected.

The analysis of the licensing deal follows a four-stage non-cooperative game with the following timing. In the first stage, the licensor – either the leader or the follower in setting the output level in the marketplace – offers a 2PT licensing contract to the competitor. In the second stage, the licensee – either the follower or the leader in setting the output level – decides whether to accept or reject the offer. In the third stage, if the licensing deal is accepted, the market leader – either the licensor or the licensee – commits to a capacity level (or to an output production level). Finally, in the fourth stage, the market follower – either the licensor or the licensee – chooses their own output level aware of the leader's output. We look for the subgame perfect Nash equilibrium of this licensing and production game.

3. The game when the licensor plays as the market leader

In this section, we proceed to resolution of the model when the licensor is the market leader. We first evaluate Stackelberg interaction without licensing (the benchmark), and then consider 2PT licensing involving per-unit and ad-valorem royalties.[8] Finally, we study which modality the licensor offers as a function of product differentiation and innovation size.

3.1. Benchmark: no licence

If the market leader does not transfer the cost-reducing technology, the quantity the follower chooses in the fourth stage of the game is:[9]

[7] It is common to see 2PT licensing contracts in the literature and in the real world. For example, Fan et al. (2018b) show that, under incomplete information, an inside licensor generally increases their payoff by using a per-unit 2PT rather than a pure per-unit royalty contract. Licensing by means of a pure fixed-fee payment is obviated, because such an arrangement is never optimal for the licensor, irrespective of whether it plays as market leader or market follower.

[8] https://www.economist.com/business/2022/06/22/why-everyone-wants-arm

[9] Throughout the analysis, superscripts n, u and v denote no licensing, and per-unit royalty and ad-valorem royalty licensing, respectively, whereas subscripts L and F denote, respectively, the leader and follower firm in setting the quantity to be produced.

$$q_F = \text{argmax}_q\ \pi_F^n = (1 - c - bq_L - q)q \qquad (1)$$

which leads to $q_F(q_L) = (1 - c - bq_L)/2$. Thus, the leader commits to produce in the third stage:

$$q_L = \text{argmax}_q\ \pi_L^n = \left(1 - q - \frac{b(1-c-bq)}{2}\right)q \qquad (2)$$

yielding $(q_L^n, q_F^n) = \left(\frac{2-(1-c)b}{2(2-b^2)}, \frac{4-2b-b^2-(4-b^2)c}{4(2-b^2)}\right)$ as the Nash equilibrium of the production subgame. Finally, the follower/licensee's profit if licensing does not occur amounts to $\pi_F^n = \left(\frac{4-2b-b^2-(4-b^2)c}{4(2-b^2)}\right)^2$, which represents the outside option when evaluating whether or not to accept the leader/licensor's offer.

3.2. The royalty involved in the contract is per unit

Assume now that the market leading firm licenses their superior technology by means of a 2PT contract, (r, f), where r is a non-negative uniform royalty rate per unit sold, such that $r \leq c$,[10] and f is a non-negative upfront fee. If the follower accepts the licensor's offer, then, in the fourth stage of the game, it produces the output level:

$$q_F = \text{argmax}_q\ \pi_F^u = (1 - r - bq_L - q)q \qquad (3)$$

which affords $q_F(q_L) = (1 - r - bq_L)/2$. Thus, the licensor's optimal quantity in the third stage of the game is defined as:

$$q_L = \text{argmax}_q \pi_L^u = \left\{\left(1 - q - \frac{b(1-r-bq)}{2}\right)q + \frac{r(1-r-bq)}{2}\right\} \qquad (4)$$

which yields $q_L = \frac{2-b}{2(2-b^2)}$, and as result, $q_F(r) = \frac{4-2b-b^2-2(2-b^2)r}{4(2-b^2)}$. Market profits obtained by the licensor and licensee are then $\pi_L^u(r) = \frac{(2-b)(2-b-2br)}{8(2-b^2)}$ and $\pi_F^u(r) = \frac{[4-2b-b^2-2(2-b^2)r]^2}{16(2-b^2)^2}$, respectively. In the second stage of the game, the licensee accepts the licensor's offer if and only if their profits are at least as high as $\pi_F^n = \left(\frac{4-2b-b^2-(4-b^2)c}{4(2-b^2)}\right)^2$,

[10] The follower/licensee could accept a per-unit royalty rate $r > c$ if this led to a sufficiently high price through more collusive behaviour by the leader/licensor. Of course, firms could not offer an efficiency rationale for such a royalty rate (the follower would produce under higher marginal costs of production), and a competition authority would dictate that the royalty rate may not exceed the cost reduction induced by the technology transfer as a scheme geared to raising the equilibrium price.

the profit obtained with no licence. Finally, the licensor chooses, in the first stage, the feasible 2PT contract (r, f) that maximizes the sum of licensing income and the profit obtained from production, i.e.:

$$\max_{(r,f)} \pi_L^u = \pi_L^u(r) + f, \text{s.t:} \, \pi_F^u(r) - f \geq \pi_F^n \text{ and } 0 \leq r \leq c \tag{5}$$

The fact that $0 \leq r \leq c$ implies that the licensee's profit, net of fee payment, strictly increases,[11] $\pi_F^u(r) > \pi_F^n$, and consequently, the licence can involve a positive fixed fee, $f > 0$. In particular, from the licensee's acceptance of the constraint as stated in Eq. (5), the licensor can charge as the fixed payment:

$$f = \pi_F^u(r) - \pi_F^n = \left(\frac{4-2b-b^2-2(2-b^2)r}{4(2-b^2)}\right)^2 - \left(\frac{4-2b-b^2-(4-b^2)c}{4(2-b^2)}\right)^2, \tag{6}$$

which leads the licensor to become the residual claimant of industry profits (equivalent to industry revenues, given our normalization of zero marginal production costs when the new technology is employed). The licensor then chooses the per-unit royalty that maximizes industry profits (or industry revenues), i.e.:

$$R^u(r, b) = (1 - q_L - bq_F)q_L + (1 - q_F - bq_L)q_F = \frac{(2-b)(2-b+2br)}{8(2-b^2)} + \left(\frac{4-2b-b^2-2(2-b^2)r}{4(2-b^2)}\right)^2, \tag{7}$$

which takes into account how the per-unit royalty affects production q_L and q_F in the third and fourth stages, respectively. Hence, the problem stated in Eq. (4) can be rewritten as:

$$\max_{r} \pi_L^u(r, b, c) = R^u(r, b) - \pi_F^n, \, \text{s.t:} \, r \leq c \tag{8}$$

and, if solved, allows us to establish the following result.

Lemma 1. *When royalties are per unit, the optimal contract for a market-leader licensor is a 2PT contract,* $(r^*, f^*) = \begin{cases} \left(\frac{b(2-b)}{2(2-b^2)}, \left(\frac{1-b}{2-b^2}\right)^2 - \pi_F^n\right), & if \; 0 \leq b \leq min\{b^u(c), \overline{b}(c)\} \\ (c, \pi_F^u(c) - \pi_F^n), & otherwise \end{cases}$

[11] The licensor reduces its production level if $r > 0$, because it internalizes revenues $rq_F(q_L)$ and therefore, other things being equal, the final price increases, and as a result, the licensee's profits strictly increase under the licence. Since, moreover, their marginal costs of production do not increase under the licence (the unit royalty r cannot exceed c), then the licensee's profits must increase.

where $b^u(c) \equiv \frac{1-\sqrt{1-4c+8c^2}}{1-2c}$.

Thus, under a per-unit royalty, the optimal contract for a market-leader licensor is always a 2PT contract. Since a per-unit royalty allows the licensor to control the licensee's behaviour as well as their own behaviour (both in production terms), the licensor sets the per-unit royalty that allows maximization of industry profits under the restriction that firms choose quantities in sequence.

However, industry profits are still below the highest possible industry profits, which would be achieved if both firms produced $q^* = \frac{1}{2(1+b)}$ and which would amount to $R(q^*, q^*) = 2(1-(1+b)q^*)q^* = \frac{1}{2(1+b)}$, since $q_F(r^*) = \max\left\{\frac{4-2b-b^2-2(2-b^2)c}{4(2-b^2)}, \frac{1-b}{2-b^2}\right\} < q^* < q_L(r^*) = \frac{2-b}{2(2-b^2)}$. In other words, the optimal per-unit royalty cannot completely resolve the Stackelberg strategic distortion that leads the licensor to produce too much and the licensee too little. More precisely, if c, the innovation size is sufficiently large, $c > .4245$, the licensor sets, as the variable part of the tariff, a per-unit royalty below the innovation size, regardless of the degree of product differentiation.

If, on the other hand, the innovation size is small, $c < .4245$, the size of the royalty charged depends on the degree of product differentiation. If the goods are sufficiently differentiated and market competition is thus sufficiently weak, the licensor charges a per-unit royalty below the innovation size, $r(b) = \frac{b(2-b)}{2(2-b^2)} < c$, and the licensee's production efficiency increases with the license. Intuitively in this case, there is little competition between firms (the market is approaching a monopoly), and, because licensing results in little threat to the licensor, there is no need to worry about competition from a more efficient licensee.

However, if the innovation size is sufficiently small, $c < .4245$, but the degree of product differentiation is sufficiently small, then $r = c$, and the licensee faces the same marginal cost as with the old technology. In this case, the licensing contract again includes a fixed payment that also allows the licensor to be the residual claimant of industry profits, and $r = c$ allows it to soft market competition.

The content of Lemma 1 is graphically illustrated in Figure 1.

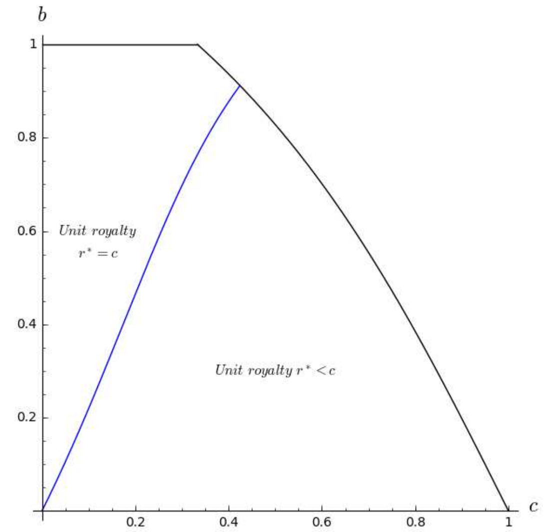

Figure 1. The royalty chosen when the licensor is the market leading firm and royalties are per unit.

Finally, taking into account both the licensing revenue and internal profit, as the licensor's payoff, we obtain:

$$\pi_L^u = \begin{cases} \frac{1}{2(1+b)} - \frac{(1-b)b^2}{4(1+b)(2-b^2)^2} - \pi_F^n, & \text{if } \ 0 \le b \le \min\left\{b^u(c), \overline{b}(c)\right\} \\ \frac{1}{2(1+b)} - \frac{(18-4b-3b^2+b^3)b^2-4b(1+b)(2-b)(2-b^2)c+4(1+b)(2-b^2)^2c^2}{4(1+b)(2-b^2)^2} - \pi_F^n, & \text{otherwise} \end{cases} \tag{9}$$

3.3. The royalty is ad valorem

Assume now that the leader/licensor issues the licence by means of a 2PT contract (d, f), where d, $0 < d < 1$, is a royalty rate in the form of a percentage of the licensee's revenue and f is a fixed-fee payment. If the licensee accepts the contract, in the fourth stage of the game it chooses the output level defined as:

$$q_F = \text{argmax}_q \, \pi_F^v = (1-d)(1-q-bq_L)q \tag{10}$$

i.e., $q_F(q_L) = (1-bq_L)/2$, and the licensor, in the third stage of the game, chooses the quantity:

$$q_L = \text{argmax}_q \, \pi_L^v = \big(1-q-bq_F(q)\big)q + d\big(1-q-bq_F(q)\big)q_F(q) \tag{11}$$

which yields $q_L = \frac{2-(1+d)b}{2(2-b^2)-b^2 d}$. The licensee then reacts by producing $q_F = \frac{4-2b-b^2}{2[2(2-b^2)-b^2 d]}$, and as a result, their equilibrium profit amounts to:

$$\pi_F^v(d) = (1-d)\left(\frac{4-2b-b^2}{2(2(2-b^2)-b^2 d)}\right)^2 \tag{12}$$

Finally, in the first stage of the game, the licensor chooses the feasible 2PT contract (d, f) that maximizes the reward for innovation transfer, i.e., the sum of licensing income and their own market profit. Formally, it solves the problem:

$$\max_{(d,f)} \pi_L^v = \pi_L^v(d) + f, \text{ s.t: } \pi_F^v(d) - f \geq \pi_F^n \tag{13}$$

and, as in the case of per-unit royalty, the fixed fee makes the licensor the residual claimant of industry profits. It then chooses the ad-valorem royalty that maximizes industry profits, taking into account that the licensee's net profits must be at least those of their outside option:

$$R^v(d,b) = (1 - q_L - bq_F)q_L + (1 - q_F - bq_L)q_F$$

$$= \frac{(2-b-b^2 d)[(2-b)(2-b^2)+2b(1-b)d]}{[2(2-b^2)-b^2 d]^2} + \left(\frac{4-2b-b^2}{2(2(2-b^2)-b^2 d)}\right)^2 \tag{14}$$

Thus, the licensor's problem as stated in Eq. (13) can be rewritten as:

$$\max_{d} \pi_L^v(d,b,c) = R^v(d,b) - \pi_F^n, \text{ s.t: } (1-d)\left(\frac{4-2b-b^2}{2[2(2-b^2)-b^2 d]}\right)^2 \geq \pi_F^n \tag{15}$$

And, provided that industry revenues increase in d:

$$\frac{\partial R^v(d,b)}{\partial d} = (1-d)\frac{4b^2(1-b)(4-b^2)+b^6}{2[2(2-b^2)-b^2 d]^3} > 0,$$

and that the licensee's profits decrease in d, it follows that $f^* = 0$, and the optimal licensing contract degenerates into the largest ad-valorem royalty rate d^* that the licensee accepts, i.e., that which satisfies $\pi_F^v(d^*, b) - \pi_F^n = 0$. This is recorded in Lemma 2.

Lemma 2. *When royalties are ad valorem, the optimal contract for a market leading licensor consists of* $f^* = 0$ *and* $d^* =$

$\frac{1}{8b^4}\left\{\sqrt{[H(b,c)-16b^2(2-b^2)]^2+16b^4[H(b,c)-16(2-b^2)^2]}-H(b,c)+16b^2(2-b^2)\right\}$, *where* $H(b,c) \equiv \frac{(4-2b-b^2)^2}{\pi_F^n} = \left(\frac{4(2-b^2)(4-2b-b^2)}{(4-2b-b^2)-(4-b^2)c}\right)^2$

In this case, the 2PT contract degenerates into a pure ad-valorem royalty, because setting the maximum possible ad-valorem royalty allows the licensor to commit to the least aggressive behaviour possible in the product market. This is due to the fact that $\frac{\partial q_L}{\partial d} < 0$ and, despite that, the licensee reacts by increasing their production, $\frac{\partial q_F}{\partial d} > 0$, total industry production decreases as compared to the pre-licensing scenario and, thus, the market becomes more collusive. In other words, with pure ad-valorem royalty, the first-mover advantage of the (market-leader) licensor is reduced as compared to a no-licensing scenario.

3.4. Which royalty does a market leading licensor prefer?

By comparing the licensing profits under each royalty modality, we can determine the optimal licensing contract from the point of view of a licensor that commits to a production level in the product marketplace. Such a contract depends on the innovation size, c, and the degree of product differentiation, b, as stated in Proposition 1.

Proposition 1. *The optimal contract for a licensor that commits to the output level in the product market depends on the innovation size,* c*, and the degree of product differentiation,* b*, as follows:*

- **(i)** *Given* c*, if* $0 \leq b \leq b^{uv}(c)$*, a per-unit royalty* r^* *combined with a fixed-fee payment as stated in Lemma 1, where* $b^{uv}(c)$ *is the solution to the equation* $R^v(d^*,b) - R^u(r^*,b) = 0$.
- **(ii)** *In any other case, a pure ad-valorem royalty as stated in Lemma 2.*

Proof. From the comparison of the licensor's payoff as derived in Lemmas 1 and 2.

The optimal licensing contract for a market leading innovator crucially depends on the innovation size and the degree of product differentiation. When the innovation size, c, is small, $c < 0.3$, the licensor prefers to use a per-unit royalty below the innovation size (combined with a fixed-fee payment) if, given c, the products of licensor and licensee are distant substitutes, but a per-unit royalty equal to the innovation size (combined with a fixed-fee payment) if, given c, the goods are less differentiated. In both cases, collusion via product differentiation is already sufficiently high and the licensor uses per-unit rather ad-valorem royalties in the licensing arrangement. However, if, given c, the goods are close substitutes, the best contract is a pure ad-valorem royalty that leads the licensee and licensor to increase and reduce output levels, respectively, and that allows the licensor to strengthen industry collusion.

On the other hand, when the innovation size is large, $c > 0.3$, the licensor's choice is between a 2PT contract featuring a per-unit royalty below the innovation size and a pure ad-valorem royalty. The first option is optimal when, given c, product differentiation is sufficiently high, and the second option is optimal when, given c, product differentiation is sufficiently low, with greater collusion in the industry achieved by an ad-valorem royalty contract.

The content of Proposition 1 is graphically illustrated in Figure 2.

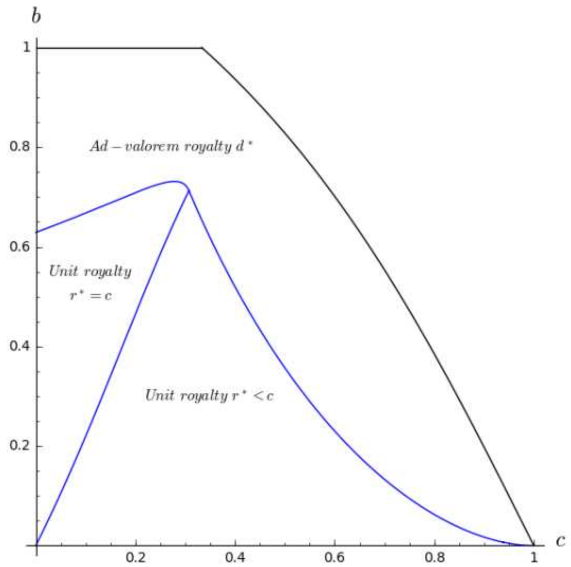

Figure 2. The optimal licensing contract for a market leading licensor.

4. The game when the licensor plays as the follower in the product market

In this section, we determine the optimal licence for a licensor that plays as a follower in the product market. We first consider that licensing does not occur, then assume that licensing takes place and royalties can be per unit or ad valorem. We finally study which licensing arrangement is preferable for the licensor.

4.1 No licence

If we consider a status quo in which the follower does not license their superior technology, then chosen, in the fourth stage of the game, is the quantity defined as:

$$q_F = \text{argmax}_q \, \pi_F^n \equiv (1 - bq_L - q)q \tag{16}$$

which leads to $q_F(q_L) = \frac{1-bq_L}{2}$. The leader's optimal production in the third stage is then:

$$q_L = \text{argmax}_q \, \pi_L^n \equiv \left(1 - c - q - b\frac{1-bq}{2}\right)q \tag{17}$$

which affords $(q_L^n, q_F^n) = \left(\frac{2(1-c)-b}{2(2-b^2)}, \frac{4-2(1-c)b-b^2}{4(2-b^2)}\right)$ as the subgame perfect Nash equilibrium of the game when licensing does not hold.[12] As result, the leader/licensee's profit amounts to $\pi_L^n = \frac{(2(1-c)-b)^2}{8(2-b^2)}$, which represents their outside option when evaluating whether or not to accept the licensing contract offered by the follower.

4.2. Licensing

If the licence consists of a per-unit royalty r, $0 < r \le c$, combined with a non-negative fixed-fee payment f, then, in the fourth stage of the game the follower/licensor chooses to produce the quantity:

$$q_F = \arg\max_q \pi_F^u \equiv (1 - bq_L - q)q + rq_L \tag{18}$$

[12] The fact that $b \le \min\{2(1-c), 1\}$ ensures that the leader/licensee never exits the market even when the follower/licensor does not license their innovation and, as result, the leader's marginal cost is c.

Solving Eq. (18) affords $q_F(q_L) = \frac{1-bq_L}{2}$, the same best reaction as under no licensing; thus, licensing merely affects market outcome if $r < c$, and as a result, the leader/licensee sees how their effective marginal cost decreases with the licence. Thus, the licensee chooses in the third stage to produce:

$$q_L = \arg\max_q \pi_L^u \equiv \big(1 - r - q - bq_F(q)\big)q \qquad (19)$$

which yields $q_L = \frac{2(1-r)-b}{2(2-b^2)}$, and consequently, $q_F = \frac{4-2(1-r)b-b^2}{4(2-b^2)}$. Hence, the contract that the licensor offers is that which solves the problem:

$$\max_{(r,f)} \pi_F^u = (1 - q_F - bq_L)q_F + rq_L + f, \text{ s.t: } (1 - r - q_L - bq_F)q_L - f \geq \pi_L^n \text{ and } r \leq c \qquad (20)$$

which, once saturated the licensee's participation constraint, $f = (1 - r - q_L - bq_F)q_L - \pi_L^n = \frac{(2(1-r)-b)^2}{8(2-b^2)} - \frac{(2(1-c)-b)^2}{8(2-b^2)}$, becomes:

$$\max_r \pi_F^u(r) = (1 - q_F - bq_L)q_F + (1 - q_L - bq_F)q_L, \text{ s.t: } r \leq c \qquad (21)$$

The licensor chooses the per-unit royalty to maximize industry profits, subject to two restrictions. The first restriction is that the per-unit royalty cannot exceed the leader/licensee's marginal cost with the old technology; the second restriction is behavioural, in the sense that the follower/licensor cannot directly modify the market behaviour of the leader/licensee, but can only indirectly affect it through the per-unit royalty r.

The licensor's profits evolve in the royalty rate as follows:

$$\frac{\partial \pi_F^u(r)}{\partial r} = -bq_F \frac{\partial q_L}{\partial r} - \left(bq_L \frac{\partial q_F}{\partial r} - r\frac{\partial q_L}{\partial r}\right) \qquad (22)$$

where the first term in Eq. (22) is positive, $-bq_F \frac{\partial q_L}{\partial r} > 0$, indicating the collusive effect of an increase in the per-unit royalty, r: the leader/licensee reduces their production level, and therefore, the follower/licensor's market profits increase. The second term in Eq. (22) has two negative effects resulting from increasing the per-unit royalty: first, there is a pro-competitive effect in that the follower/licensor increases their production, which consequently reduces the leader/licensee's market profits, $bq_L \frac{\partial q_F}{\partial r} < 0$; and second, the increase in the per-unit royalty increases the licensee's costs, $r\frac{\partial q_L}{\partial r} < 0$, and hence it does not correctly internalize the real costs of production. As a consequence of the trade-off between these two effects, the optimal royalty rate is never equal to zero, as Proposition 2 attests.

Proposition 2. *An innovation owned by a follower in the marketplace is never licensed through an ad-valorem royalty, but through a per-unit royalty (alone or combined with a fixed fee) as* $(r^*, f^*) = \begin{cases} (c, \quad 0), & if\ 0 \leq c \leq \frac{b(4-2b-b^2)}{2(4-3b^2)}\ and\ b^f(c) \leq b \leq 1 \\ \left(\frac{b(4-2b-b^2)}{2(4-3b^2)}, \frac{16(1-b)^2(2-b^2)^2-(2-b-2c)^2}{8(2-b^2)^2(4-3b^2)^2}\right), & otherwise \end{cases}$

where $b^f(c)$ *solves the equation* $c - \frac{b(4-2b-b^2)}{2(4-3b^2)} = 0.$

Proof. See the Appendix.

As occurs when the licensor is the leading firm in the product market, the optimal licensing agreement for a licensor acting as a follower in the marketplace also depends on c, the innovation size, and *b*, the degree of product differentiation. If the innovation size is such that $c < 0.5$, the licence consists of a per-unit royalty equal to the innovation size (and with no fixed payment) when, given *c*, the products are little differentiated to a sufficient degree. The explanation is that the follower/licensor does not want a more efficient leader/licensee, because their market share would be very reduced, and the decreased internal profit would not be compensated for by the increased royalty revenues. Thus, the licensor prefers to increase their production at the expense of the licensee.

With an equilibrium per-unit royalty at $r = c$, market competition is not reduced (production levels remain unchanged), and the licensee obtains the same profit as with the old technology, leading it to be indifferent between accepting or rejecting the licence; in contrast, the licensor's payoff amounts to the profit under a no-licensing scenario (the licensor's profit remains unchanged) plus a reaped royalty revenue, i.e., $\pi_L^u = \left(\frac{4-2(1-c)b-b^2}{4(2-b^2)}\right)^2 + \frac{(2(1-c)-b)c}{2(2-b^2)}$.

However, when, given c, the degree of product differentiation is sufficiently high, the licensor sets a per-unit royalty below the innovation size (in combination with a fixed-fee payment). In this case, the licensor seeks to have a more efficient licensee and so extract their increased profits by means of the fixed payment.

In contrast, when the innovation size is sufficiently large, $c > 0.5$, the licence consists of a 2PT contract featuring a per-unit royalty below the innovation size, combined with a fixed payment. In this case, the licensor allows the licensee to become more efficient, because this would increase royalty revenues and would not overly reduce licensor market share due to product differentiation. Moreover, the licensor reaps the increase in profits through the fixed fee.

The content of Lemma 3 is graphically illustrated in Figure 3.

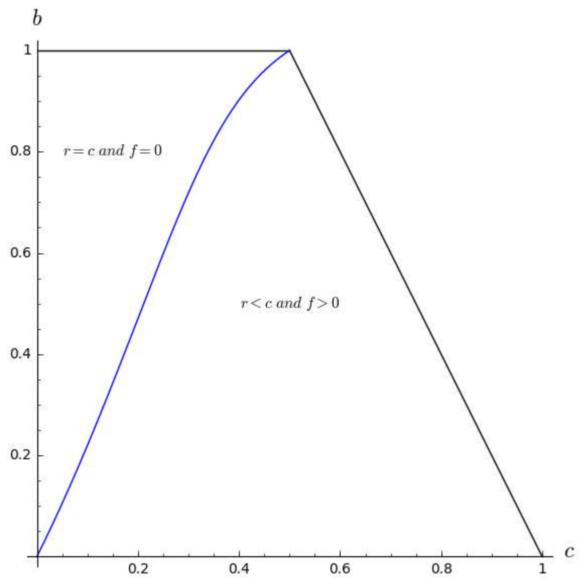

Figure 3. The optimal licensing contract for a market following licensor.

Finally, the licensor's payoff from the contract defined in Proposition 2 amounts to:

$$\pi_F^u = \begin{cases} \left(\frac{4-2(1-c)b-b^2}{4(2-b^2)}\right)^2 + \frac{(2(1-c)-b)c}{2(2-b^2)}, & \text{if } 0 \leq c \leq \frac{b(4-2b-b^2)}{2(4-3b^2)} \text{ and } b^f(c) \leq b \leq 1 \\ \frac{(4-2b-b^2)^2+4(2-b)(4-3b^2)c-4(4-3b^2)c^2}{8(8-10b^2+3b^4)}, & \text{otherwise} \end{cases} \quad (23)$$

5. Licensing and welfare

In this section, we evaluate the effects of innovation licensing on both consumers and society as a whole compared to a context in which the innovation is not licensed. To this end, we examine the impact of the licensor's contract on consumer surplus, defined as $CS = \frac{q_L^2+2bq_Lq_F+q_F^2}{2}$, and aggregate welfare, W, measured as the non-weighted sum of consumer surplus and industry profits, i.e., $W = CS + \pi_L + \pi_F$. As indicated by Farrell and Katz (2006), numerous studies have been performed on whether consumer surplus or social welfare should be employed as a welfare standard for regulation guidelines. As pointed out by Blair and Sokol (2013), the European Union, the United States, and Japan adopt consumer surplus as the criterion, while Canada, Australia, and New Zealand adopt social welfare as the criterion (see Takashima and Ouchida, 2019). It is then important to compare the results with each standard and clarify the difference between them, because findings derived from each criterion are needed for policy recommendations.

5.1. Consumer surplus

When the licensor plays as the leader in the product market, if we compare consumer surplus if licensing does not occur:

$$CS_L^n = \frac{16-12b^2+4b^3(1-c)+\left(16-20b^2+5b^4\right)(1-c)^2}{32(2-b^2)^2} \tag{24}$$

with consumer surplus when licensing takes place by means of a (pure) ad-valorem royalty:

$$CS_L^v = \frac{32-32b^2+4b^3+5b^4-4b\left(4+2b-2b^2-b^3\right)d^*+4b^2(d^*)^2}{8[2(2-b^2)-b^2d^*]^2} \tag{25}$$

where d^* is stated in Lemma 2, then we can conclude that licensing based on an ad-valorem royalty hurts consumers when the products are identical or close substitutes, but otherwise benefits consumers. In other words, licensing by means of ad-valorem royalty hurts consumers when the goods are identical (Antelo and Bru, 2022) no longer holds when the products are sufficiently differentiated, when consumer appreciation for variety outweighs the damage caused by the collusive effect of the licence.

In contrast, when licensing occurs by means of a contract involving a per-unit royalty (alone or combined with a fixed-fee payment), consumer surplus amounts to:

$$CS_L^u = \begin{cases} \frac{4(5-8c+4c^2)-4b-(19-32c+16c^2)b^2+4b^3+(3-8c+4c^2)b^4}{8(2-b^2)^2}, \text{if } r^* = c \\ \frac{8-4b-7b^2+4b^3}{8(2-b^2)^2}, \quad \text{if } r^* < c \end{cases} \tag{26}$$

and, if compared to the Stackelberg equilibrium when licensing does not hold, leads us to conclude that consumers are affected as follows after licensing:

$$CS_L^u - CS_L^n = \begin{cases} \frac{48(1-c)^2-16b-44(1-c)^2b^2+4(3+c)b^3+(7-22c+11c^2)b^4}{32(2-b^2)^2}, \text{if } r^* = c \\ \frac{-b(16-4b-12b^2+5b^3)+(16-20b^2+5b^4)(2-c)c+4b^3c}{32(2-b^2)^2}, \quad \text{if } r^* < c \end{cases} \tag{27}$$

According to Eq. (27), licensing leads consumers to be better off when industry efficiency is improved, due to $r^* < c$, and, given c, the products are sufficiently differentiated. If $r^* < c$ and b is not sufficiently high, $CS_L^u < CS_L^n$, in which case consumers are worse off after licensing, despite the innovation improving industry efficiency. Finally, licensing is prejudicial for consumers when the royalty featured in the contract is $r^* = c$, in which case the improvement in industry efficiency disappears.

On the other hand, when the licensor plays as the market follower and the licence is based on a pure per-unit royalty $r^* = c$, both firms produce the same quantities as in a no-licensing scenario, and consumers are therefore left with the same surplus. Likewise, when the per-unit royalty charged is below the innovation size, $r^* < c$, even if the follower/licensor reduces production as a result of the licence, that reduction is more than compensated for by an increase in the leader/licensee's production, and, as a consequence, the consumer surplus increases, from:

$$CS_F^n = \frac{32-32b^2+4b^3+5b^4-4(8-6b^2+b^3)c+4(4-3b^2)c^2}{32(2-b^2)^2} \tag{28}$$

when the innovation is not licensed to:

$$CS_F^u = \frac{-b(4-3b)(48-36b^2+3b^4)+12(32-48b^2+4b^3+18b^4-3b^5)c-12(4-3b^2)^2c^2}{96(2-b^2)^2(8-10b^2+3b^4)} \tag{29}$$

when the innovation is licensed.

Consideration of Eq. (27) when the licensor is the market leader, and comparison of Eqs. (28) and (29) when it is the market follower, allow us to state the following result regarding the impact of licensing on consumers.

Proposition 3. *As compared to a no-licensing scenario, the following hold when licensing occurs:*

(i) *Licensing by a market leading firm benefits consumers:*

(i.1) *When the licence features a per-unit royalty* $r^* < c$, *if* $0 \leq b \leq min\{b_{cs}^{u}(c), b^{uv}(c)\}$, *where* $b_{cs}^{u}(c)$ *is the solution to the equation* $CS_L^u - CS_L^n = 0$ *and* $b^{uv}(c)$ *is the cut-off value defined in Proposition 1.*

(i.2) *When the licence consists of a pure ad-valorem royalty, if* $c^{nv} \leq c \leq 1$ *and* $b^{uv}(c) < b < min\{b^{nv}(c), \overline{b}(c)\}$, *where* $c^{nv} \approx 0.3176$ *and* $b^{nv}(c)$ *is the solution to the equation* $CS_L^n - CS_L^v = 0$.

(ii) *Licensing by a market leading firm is detrimental for consumers in any other circumstances.*

(iii) *Licensing by a market following firm benefits consumers (is innocuous for consumers) when the contract features a per-unit royalty* $r^* < c$ *(features a per-unit royalty* $r^* = c$*).*

In our analytical framework, licensing a cost-reducing innovation causes three effects that impact on consumer and society as a whole: an improvement in industry efficiency (efficiency effect), which, in turn, will affect the quantities produced in the industry, and a strategic effect, sought by the licensor to modify the industry profits (collusive effect).[13]

When licensing is by a market leading firm and the per-unit royalty amounts to $r^* < c$, consumers benefit if, given c, the products are sufficiently differentiated. Three different effects lead to that $r^* < c$ implying production redistribution from the leader/licensor to

[13] When $b = 0$, we have two local monopolies and licensing always benefit consumers, since, whatever the contract is used, the collusive effect disappears. On the other hand, when $b = 1$ or is close to 1, the collusive effect dominates and licensing leads consumers to be worse off.

the follower/licensee. First, the licensor internalizes industry profits through royalty revenues and reduces production, thus reflecting a collusive effect. Second, because the licensor reduces production, the licensee is induced to increase production, thus reflecting a competitive effect. Finally, another reason – lower marginal production costs – induces the follower/licensee to increase production, thus reflecting an efficiency effect; this efficiency improvement effect of the industry must be present (and is not always sufficient) for the increase in licensee production to be large enough to outweigh the reduction in licensor production. When, on the other hand, licensing features a per-unit royalty $r^* = c$, it has a negative impact on consumers, because of the collusive effect that it induces.

Finally, when licensing occurs through ad-valorem royalty, despite the collusive effect, consumers may benefit when product differentiation is sufficiently high. In particular, licensing hurts consumers when product differentiation is sufficiently low, in which case the collusive effect predominates over the preference for variety and the efficiency improvement effect. The opposite holds when the products are sufficiently differentiated, and consumers are worse off after licensing.

Thus, our findings suggest that, in general, for licensing to lead to an improvement in consumer surplus, some degree of product differentiation is necessary. Meanwhile, there is always a cut-off value of parameter b for which licensing a cost-reducing innovation benefits (harms) consumers if, given c, the degree of product differentiation is above (below) that critical cut-off value.

Figure 4 illustrates the content of parts (i) and (ii) of Proposition 2. In that figure, the region in which the contract per-unit royalty is $r^* < c$ is separated from the region in which it is $r^* = c$ by a broken line.

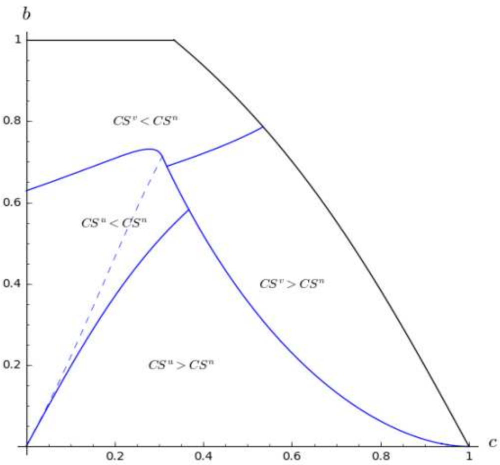

Figure 4. Licensing and consumer surplus when the licensor is the market leader.

This result underlines the importance of the policymaker, in the interest of regulating innovation diffusion, to be fully informed as to whether firms involved in licensing deals define the same market (their products are close substitutes) or different markets (their products are distant substitutes). In our case, this information is summarized in the value of parameter b and, as we have seen, its value (along with the innovation size) are crucial to deciding the type of licence, and thus, the effect on consumers.

5.2 Aggregate welfare

If the licensor is the market leader and both consumers and firms are taken into account, then aggregate welfare in the Stackelberg equilibrium amounts to, in a no-licensing scenario:

$$W_L^n = \frac{96-64b-48b^2+28b^3+3b^4-2(48-32b-28b^2+14b^3+3b^4)c+\left(48-28b^2+3b^3\right)c^2}{32(2-b^2)^2} \quad (30)$$

and when licensing is by means of a 2PT contract involving per-unit royalties:

$$W_L^u = \begin{cases} \dfrac{24-20b-9b^2+8b^3}{8(2-b^2)^2}, if\ r^* < c \\ \dfrac{96-64b-48b^2+28b^3+3b^4-4\left(8-4b-6b^2+2b^3+b^4\right)c-4(2-b^2)^2c^2}{32(2-b^2)^2}, if\ r^* = c \end{cases} \quad (31)$$

Considering the difference in total welfare when licensing features a per-unit royalty and when licensing does not occur, it follows from Eqs. (31) and (30) that:

$$W_L^u - W_L^n = \begin{cases} \frac{48(2-c)c-16(1+4c)b+4(3-14c+7c^2)b^2+(4+28c-3c^2)b^3-3(1-2c)b^4}{32(2-b^2)^2}, & \text{if } r^* < c \\ \frac{c(64(1-c)-48b-4(8-11c)b^2+(20-3c)b^3+2(1-2c)b^4)}{32(2-b^2)^2}, & \text{if } r^* = c \end{cases} \tag{32}$$

and Eq. (32) is strictly positive. On the other hand, when licensing consists of a pure ad-valorem royalty, total welfare amounts to:

$$W_L^v = \frac{(96-64b-48b^2+28b^3+3b^4)-8b(2+3b-3b^2)d^*-4b^2(1-2b)(d^*)^2}{8(2(2-b^2)-b^2d^*)^2} \tag{33}$$

where d^* is given in Lemma 2. From Eqs. (30), (32), and (33), the following result can be stated.

Proposition 4. *As compared to a no-licensing scenario, the following hold after licensing:*

(i) *When the licensor is the market leader, welfare is increased:*

(i.1) If it uses a per-unit royalty.

(i.2) If it uses an ad-valorem royalty whenever $0 \leq b \leq b_w^v$*, where* b_w^v *is the solution in* $[0, 1]$ *to the equation* $W_L^v - W_L^n = 0$.

(ii) *When the licensor is the market leader, welfare is reduced if it uses an ad-valorem royalty whenever* $b_w^v < b \leq 1$.

(iii) *When the licensor is the market follower, welfare is increased (remains unchanged) whenever it uses a per-unit royalty* $r < c$ *(per-unit royalty* $r = c$*).*

Proposition 4 shows that the diffusion of a cost-reducing technology reduces social welfare only in a very small region of parameters. In fact, licensing is welfare-enhancing when it benefits consumers, provided that industry profits unequivocally increase with the licence, and even when it harms consumers, but the increase in industry profits outweighs the reduction in consumer surplus. The diffusion of the innovation is welfare-reducing, only when licensing greatly reduces consumer surplus − which occurs if the licensor, as market leader, implements the licensing arrangement by means of an ad-valorem royalty, and the products of the licensor and licensee are very close substitutes − and that reduction is not compensated for by increased industry profits.

Figure 5 illustrates the result stated in parts (i) and (ii) of Proposition 4. The broken line separates the region in which the leader/licensor uses a per-unit royalty, $r^* < c$ or $r^* = c$, from the region in which the royalty used is ad valorem.

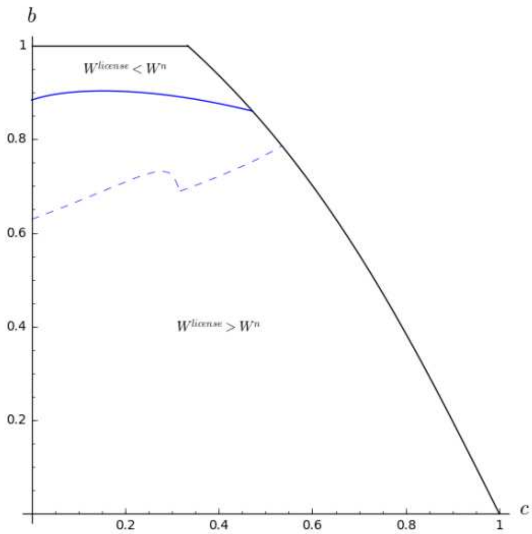

Figure 5. Licensing and social welfare when the licensor is the market leading firm.

Our result suggests that minimal differentiation between the products of the licensor and licensee for licensing a cost-reducing innovation is sufficient not to reduce social welfare. This conclusion highlights the crucial role that product differentiation plays in an issue as important for the dynamics of society as the diffusion of technologies that reduce the production costs of firms.

6. Conclusions

We showed that an inside licensor with a cost-reducing innovation chooses the licensing contract that makes the industry maximally collusive. In pursuit of that goal, both the licensor's position in the product market, the relationship between the industry products, and the size of the cost-reduction achieved by the innovation all play a key role in the optimal licensing contract and its impact on both consumers and society as a whole.

We examine this issue in a Stackelberg duopoly model in which the licensor plays as the market leader or the market follower. If the licensor is the market leader, the licence consists of a pure ad-valorem royalty contract when products are close substitutes or, if distant substitutes, when the innovation size is sufficiently large. In both cases, ad-valorem royalty leads to more relaxed market competition than would be the case if a per-unit royalty were used. However, this advantage of an ad-valorem over a per-unit royalty is mitigated when products are sufficiently differentiated, but the innovation size is not very large. In this case, the licensor prefers a 2PT contract that involves a per-unit royalty (combined with a positive fixed fee), which is equal to (below) the cost reduction achieved by the innovation if that cost reduction is small (moderate).

In contrast, when the licensor is the market follower, we show that a per-unit royalty is preferred over an ad-valorem royalty. In particular, the contract may consist of a pure royalty-based contract (with the royalty equal to the cost reduction achieved by the innovation, when, given the low product differentiation, the innovation size is small), or a 2PT contract (with the royalty below the innovation size when that size is large).

Our results suggest that either per-unit or ad-valorem royalties should be observed in licensing deals. Specifically, the wide use of both ad-valorem and per-unit royalties indicated by the empirical literature (Bousquet et al., 1998) can be theoretically rationalized by the licensor's different position in the product market, the innovation size, and the degree of product differentiation.

We further show that licensing by a market leading licensor may benefit consumers, irrespective of whether the licence features a per-unit or an ad-valorem royalty. In the case of a per-unit royalty, sufficient product differentiation coupled with a sufficiently large innovation size is necessary to increase production efficiency, thanks to the fact that licensing involving a per-unit royalty below the innovation size is translated into a larger consumer surplus. When licensing is by means of a pure ad-valorem royalty, consumers benefit from licensing if the innovation size is sufficiently large, whereas the contrary holds if it is sufficiently small. On the other hand, if we consider aggregate welfare, licensing by a market-leader licensor is almost always welfare-enhancing; this is not only true when consumers benefit from licensing, but also even when consumers are harmed, provided, given the innovation size, the products of the licensor and licensee are minimally differentiated.

On the other hand, when the patent holder is the market follower, licensing is by means of per-unit royalties, combined or not with a fixed payment, depending on product differentiation and the innovation size. Depending on the degree of product differentiation, if the innovation size is sufficiently small, then the licence consists of either a pure per-unit royalty at the level of the cost reduction induced by the innovation, or if the innovation size is sufficiently large, a per-unit royalty lower than the cost reduction induced by the innovation combined with a fixed payment. Finally, licensing by a market follower benefits both consumers and social welfare (when it is in the form of a 2PT contract involving a royalty rate below the cost reduction due to the innovation), or it leads to the same level of both consumer surplus and social welfare as before licensing (when the licence consists of a royalty rate equal to the cost reduction).

The model presented here can potentially be extended. For comparison of the results with those of this paper, an extension that would be worth exploring could consist of determining optimal licensing in a differentiated-goods Stackelberg industry when the licensor and licensee compete by setting prices rather than quantities. Licensing contracts in this framework may drastically differ from those that emerge under quantity competition, because of the differing strategic values for market position in setting price rather than quantity. This would substantially modify the impact of the licensor's market position on the optimal licensing contract and the impact on welfare. A second worthwhile extension would be to perform the analysis in a setup consisting of several licensees in the industry (one leader and several followers if the leader is the patent holder, or several leaders and one follower if the latter is the patent holder). In this case, the licensor would have to decide not only the type of innovation-transfer licensing contract, but also the optimal number of licences to grant – undoubtedly an important dimension of every licensing arrangement.

Acknowlegment

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References

Antelo, M. and L. Bru (2022), Per-unit versus ad-valorem royalty licensing in a Stackelberg market, *Journal of Industrial and Business Economics* 49, 95-109.

Blair, R.D. and D.D. Sokol (2013). Welfare standards in US and EU antitrust enforcement, *Fordham Law Review* 81, 2497–2541.

Bousquet, A., H. Cremer, M. Ivaldi and M. Wolkowicz (1986), Risk sharing in licensing, *International Journal of Industrial Organization* 16, 535-554.

Fan, C., B.H. Jun and E.G. Wolfstetter (2018a), Per unit vs. ad valorem royalty licensing, *Economics Letters* 170, 71-75.

Fan, C., B.H. Jun and E.G. Wolfstetter (2018b), Optimal licensing under incomplete information: the case of the inside patent holder, *Economic Theory* 66, 979-1005.

Farrell, J., and M.L. Katz (2006). The economics of welfare standards in antitrust, *Competition Policy International* 2, 3-28.

Filippini, L. (2005), Licensing contract in a Stackelberg model, *Manchester School* 73, 582-598.

Hoesel, S.V. (2008), An overview of Stackelberg pricing in networks, *European Journal of Operational Research,* 189, 1393-1402.

Hoffman, C. (2014), Why Microsoft Makes $5 to $15 From Every Android Device Sold (https://www.howtogeek.com/183766/why-microsoft-makes-5-to-15-from-every-android-device-sold/)

Hsu, J., L. Liu, X.H. Wang and C. Zeng (2019), Ad Valorem Versus Per-unit Royalty Licensing in a Cournot Duopoly Model, Manchester School 87, 890-901.

Jiang, B. and H. Shi (2018), Inter-competitor licensing and product innovation, *Journal of Marketing Research* 55, 738-751.

Kabiraj, T. (2005), Technology transfer in a Stackelberg structure: Licensing contracts and welfare, *Manchester School* 73, 1-28.

Li, Y. and T. Yanagawa (2011), Patent licensing of Stackelberg manufacturer in a differentiated product market, *International Journal of Economic Theory* 7, 7-20.

Lyarskaya, N. (2009), Design of technology licensing agreements: New empirical evidences (http://citeseerx.ist.psu.edu/viewdoc/download;jsessionid=694337FDA654B3DBB4025 15D955C1C72?doi=10.1.1.485.1768&rep=rep1&type=pdf)

Ludlow, T. (2014), Trends in Technology IP Licensing, *IP Law Journal*, December, 1-11.

Montinaro, M., R. Pal and M. Scimitore (2020), Per unit and ad valorem royalties in a patent licensing game. Working Paper, Research in Agricultural & Applied Economics.

Moreira, S., T.M. Klueter and S. Tasselli (2020), When Licensing New Tech Is Better Than Building It In-House, Harvard Business Review (https://hbr.org/2020/06/when-licensing-new-tech-is-better-than-building-it-in-house).

Mukherjee, A. and N. Balasubramanian (2001), Technology transfer in a horizontally differentiated product market. *Research in Economics* 55, 257-274.

Radauer, A. and T. Dudenbostel (2013), PATLICE Survey- Survey on patent licensing activities by patenting firms, European Comission (https://www.technopolis-group.com/wp-content/uploads/2020/02/A-Survey-on-Patent-Licensing-Activities-by-Patenting-Firms.pdf)

San Martín, M. and A.I. Saracho (2010), Royalty licensing, *Economics Letters* 107, 284-287.

San Martín, M. and A.I. Saracho (2015), Optimal two-part tariff licensing mechanisms, *The Manchester School* 83, 288-306.

Takashima, N. and Y. Ouchida (2020). Quality-improving R&D and merger policy in a differentiated duopoly: Cournot and Bertrand equilibria, *Managerial and Decision Economics* 41, 1338-1348.

Trombini, G. and A. Comacchio (2012), Corporate markets for ideas: when does technology licensing combine with R&D partnerships? Working Paper 8, Department of Management, Università Ca'Foscari Venezia.

Zhang H., P. Qing, X. Hong and X. Wang (2016), Optimal licensing of uncertain patents in a differentiated Stackelberg duopolistic competition market, *International Review of Economics & Finance* 45, 215-229.

Zuniga M.P. and D. Guellec (2009), Who licenses out patents and why? Lessons from a business survey. STI Working Paper 2009/5. Statistical Analysis of Science, Technology and Industry.

Appendix

Proof of Proposition 2. Here we prove that, for a licensor playing as a follower in the product market, ad-valorem royalty is never superior to per-unit royalty. If the licensor uses a 2PT contract, (d, f), where d is a non-negative ad-valorem royalty such that $0 \leq d \leq 1$, and f is a non-negative upfront fee, then they choose to produce, in the fourth stage of the game:

$$q_F = \arg\max_q \pi_F^v \equiv (1 - bq_L - q)q + d(1 - q_L - bq)q_L \tag{A1}$$

which yields $q_F(q_L) = \frac{1-b(1+d)q_L}{2}$. Thus, the licensor internalizes the effect of a higher market price on royalty revenue by reducing their production. In turn, the licensee chooses to produce, in the third stage of the game, the quantity:

$$q_L = \arg\max_q = \pi_L^v = (1 - d)\big(1 - q - bq_F(q_L)\big)q \tag{A2}$$

which affords $q_L = \frac{2-b}{2(2-(1+d)b^2)}$, a larger amount than without the royalty, because the licensee knows that the licensor will reduce their production when compared to a Stackelberg game without a license, from $\frac{1-q_L}{2}$ to $\frac{1-b(1+d)q_L}{2}$. Thus, $q_F(q_L) = \frac{4-2b-b^2-b(2+b)d}{4(2-(1+d)b^2)}$, which is positive only if $d < \frac{4-2b-b^2}{b(2+b)}$, and, as a result, the licensee's profit amounts to:

$$\pi_L^v(d) = \frac{(1-d)(2-b)^2}{8(2-b^2(1+d))} \tag{A3}$$

The licensor can then charge a fixed fee equal to the increased profit of the licensee, i.e., $f = \frac{(1-d)(2-b)^2}{8(2-b^2(1+d))} - \frac{(2-b-2c)^2}{8(2-b^2)}$, and their payoff, i.e., royalty revenues plus the fixed payment and their own market profit, amounts to:

$$\pi_F^v(d,b) = \left(\frac{(1-d)(2-b)^2}{8(2-b^2(1+d))} - \frac{(2-b-2c)^2}{8(2-b^2)}\right) + \frac{d(2-b)^2}{8(2-b^2(1+d))} + \frac{(4-2b-b^2-b(2+b)d)(4-2b-b^2+b(2-3b)d)}{16(2-b^2(1+d))^2} \tag{A4}$$

This allows us to state the following result.

Lemma A1. *There is a cut-off value for the innovation size,* $\tilde{c}(b) = 1 - \frac{b-\sqrt{b(2-b)(2-b^2)}}{2}$, *such that, with ad-valorem royalties, the optimal licensing deal for a market following licensor is:*

$$(d^*, f^*) = \begin{cases} \left(\frac{2(2-b^2)\left((2-b)c-c^2\right)}{(1-b)(4-3b^2+b^3)+2b^2(2-b)c-2b^2c^2}, 0\right), & if\ 0 \le c \le \tilde{c}(b) \\ \left(\frac{2-b-b^2}{2+b+b^2}, \pi_L^v(d^*) - \pi_L^n\right), & if\ \tilde{c}(b) \le c \le \bar{c}(b) \end{cases}$$

The content of Lemma A1 is graphically illustrated in Figure A1.

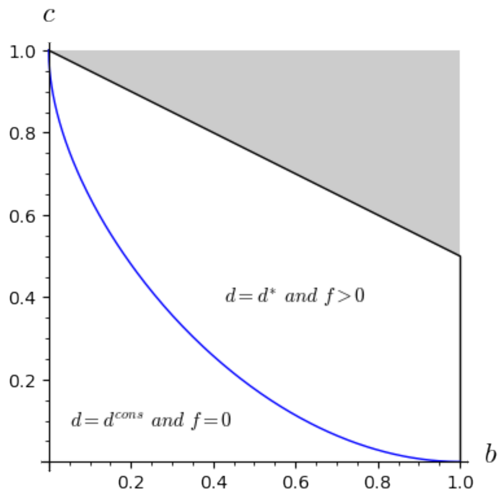

Figure A1. The licensing contract of a market following licensor if royalties are ad valorem.

Finally, when comparing the licensor's payoffs in Proposition 2 and Lemma A1, we can state that the follower always prefers a contract involving a per-unit to an ad-valorem royalty. ■