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Abstract: The demand and supply shocks in the U.S. and China are analyzed using the Blanchard and Quah (BQ) and ordinary least squares (OLS) methods. For the U.S. data, the aggregate supply (AS) curve has a positive slope, whereas the aggregate demand (AD) curve has a negative slope. However, the two methods yield inverse results when data from China are analyzed. In the BQ method, the AS curve slope is negative and AD curve slope is positive, indicating a "slope puzzle." In the OLS method, no "slope puzzle" is present.

Keywords: slope puzzle, BQ method, OLS.

JEL classification: E32.

1. Introduction

In this study, the Blanchard and Quah (BQ) and ordinary least squares (OLS) methods are used to estimate the aggregate supply (AS) and aggregate demand (AD) curves for the U.S. and China. According to Keynesian theory, the slope of the AS curve is always positive, whereas that of the AD curve is always negative. The case in which the empirical test results do not agree with the results predicted by the theory is referred to as a “slope puzzle.”

Blanchard and Quah (1989) proposed the BQ method, which is based on the assumption that AS shocks affect the long-run output, whereas AD shocks do not (long-run restriction). The OLS method also imposes this restriction; however, owing to the endogeneity problem, instrumental variables are needed in identifying AS shocks (Shapiro and Watson, 1988; Francis and Ramey, 2005; Fernald, 2007).

In previous studies, the results of these two methods were argued to be equivalent, i.e., the estimated AS and AD curves should be approximately the same. Francis and Ramey (2005) argued that "[u]sing lags one through p of Δx_t and Δn_t as instruments [the OLS method] yields estimates that are identical to those obtained using the matrix methods [the BQ method]." Fernald (2007) suggested that "these instruments [the OLS method] yield results identical to the Blanchard-Quah matrix methods." Ramey (2016) said "imposing the long-run restriction [the BQ method] is equivalent to identifying the error term in the following equation [the OLS method]." However, we find that these two methods are equivalent for the U.S. data but not for the Chinese data.

The analysis of macroeconomic data for different countries has led to different conclusions regarding the slope puzzle. In terms of the slope of the AS–AD curves for the U.S. and other countries in the Organization for Economic Co-operation and Development, research has revealed that the puzzle does not exist (Gamble, 1996; Spencer, 1996; Cover et al., 2005; Fernald, 2007; Cho, 2012). However, inconsistent conclusions were reached when estimating the AS–AD curves for China. Using the BQ method, Xu (2008), Gao (2010) and Zhu and Deng (2017) found the puzzle.

This study has two significant contributions: first, the BQ and OLS methods were used to identify the AS–AD curve in the U.S. and China and determine whether the two methods are equivalent in these two countries; second, the OLS method was used to estimate the AS–AD curve for China and evaluate whether a “slope puzzle” exists.

The remainder of this article is organized as follows. Section 2 introduces the BQ and OLS methods and the selection and processing of data. Section 3 presents the empirical results for the U.S. and China and a discussion on the possible reasons for inconsistencies between the two methods. Section 4 provides the study's conclusion.

2. Model

2.1 Empirical model and identification

Before the BQ and OLS methods are applied, ensuring that the variables are stationary is important. However, most macro variable series have unit roots, making estimates imprecise. Therefore, the variables need to be detrended before the BQ and OLS methods can be applied. Three main methods are used for data detrending: log difference, adding linear or nonlinear time trends, and filtering. Fernald (2007) found that different data treatments may make the results different, and low-frequency fluctuation needs to be considered. Additionally, Galí (1999), Francis and Ramey (2005), and Christiano et al. (2003) found that different methods for detrending variables will invariably lead to different results.

Based on previous research, in this study, the BQ and OLS methods with log difference and nonlinear time trends were used to identify supply and demand shocks. Model (I) and Model (II) utilized the BQ method, and Model (III) and Model (IV) employed the OLS method. Additionally, to capture the low-frequency price fluctuation, we used the quadratic time trend in Model (II) and Model (IV). Therefore, the main difference was that Model (I) and Model (III) used a quadratic time trend and log level of price, whereas Model (II) and Model (IV) used the log difference of the output and price.

2.1.1 BQ method

According to Blanchard and Quah (1989), Cover et al. (2006), and Gamber (1996), the log difference of the real gross domestic product (GDP) (Δy_t) and the log difference of price (Δp_t) can be decomposed in a bivariate vector structure autoregressive (SVAR) model to obtain supply and demand shocks. Therefore, we can identify Model (I) as follows:

$$\begin{bmatrix} \mathbf{\Delta y_t} \\ \mathbf{\Delta p_t} \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} \mathbf{C_{11}(L)} & \mathbf{C_{12}(L)} \\ \mathbf{C_{21}(L)} & \mathbf{C_{22}(L)} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t^s \\ \boldsymbol{\varepsilon}_t^d \end{bmatrix}, \tag{1}$$

where C(L) is a polynomial in the lag operator, and a and b are constants. According to Blanchard and Quah's (1989) assumption that demand shocks have no effect on the aggregate output in the long run, $\mathbf{C_{12}(1) = 0}$. We can identify demand and supply shocks by using the SVAR model.

Additionally, Zhang et al. (2019) reported that low-frequency fluctuations in the price level would lead to China's slope puzzle. Therefore, we use a quadratic time trend to capture the low-frequency of price fluctuation. The specific model is shown as follows by Model (II):

$$\begin{bmatrix}\boldsymbol{\Delta y_t}\\ \mathbf{p_t}\end{bmatrix}=\begin{bmatrix}a_1 & a_2\\ b_1 & b_2\end{bmatrix}\begin{bmatrix}t\\ t^2\end{bmatrix}+\begin{bmatrix}\mathbf{C_{11}(L)} & \mathbf{C_{12}(L)}\\ \mathbf{C_{21}(L)} & \mathbf{C_{22}(L)}\end{bmatrix}\begin{bmatrix}\boldsymbol{\varepsilon_t^s}\\ \boldsymbol{\varepsilon_t^d}\end{bmatrix}, \tag{2}$$

where $\mathbf{p_t}$ is the log level of the GDP deflator, and C(L) is a polynomial in the lag operator. Additionally, based on Blanchard and Quah's (1989) assumption, $\mathbf{C_{12}(1)} = \mathbf{0}.$ a_1 and b_1 are coefficients of linear time trends, and a_2 and b_2 are coefficients of quadratic time trends.

2.1.2 OLS method

Referring to Shapiro and Watson (1988), Francis and Ramey (2005), and Ramey (2016), we used the OLS method to identify supply and demand shocks. Specifically, we used the C (1) lower triangular property and assumed that supply and demand shocks were uncorrelated. We can derive the linear expressions of $\boldsymbol{\Delta y_t}$ from equation (1), and Model (III) can be written as follows:

$$\boldsymbol{\Delta y_t} = \mathrm{u}_1 + \sum_{\mathbf{j=1}}^{\mathbf{p}} \boldsymbol{\beta_{yy,j}\Delta y_{t-j}} + \sum_{\mathbf{j=0}}^{\mathbf{p-1}} \boldsymbol{\beta_{yp,j}\Delta^2 p_{t-j}} + \boldsymbol{\varepsilon_t^s} \tag{3}$$

and $$\boldsymbol{\Delta p_t} = \mathbf{u_2} + \sum_{\mathbf{j=1}}^{\mathbf{p}} \boldsymbol{\beta_{pp,j}}\Delta\, \mathrm{p}_{t-j} + \sum_{\mathbf{j=1}}^{\mathbf{p}} \boldsymbol{\beta_{py,j}\Delta y_{t-j}} + \boldsymbol{\beta_{ps}\varepsilon_t^s} + \boldsymbol{\varepsilon_t^d}, \tag{4}$$

where $\Delta \mathrm{y_t}$ is the log difference of the real GDP, and $\Delta \mathrm{p_t}$ is the log difference of the GDP deflator. According to Shapiro and Watson (1988), $\boldsymbol{\Delta^2 p_{t-j}}$ instead of $\boldsymbol{\Delta p_{t-j}}$ is inserted in equation (3). Given that a correlation may exist between $\boldsymbol{\Delta^2 p_{t-j}}$ and $\boldsymbol{\varepsilon_t^s}$ in equation (4), the coefficient estimates are biased and inconsistent when using the OLS method. Therefore, we used lags one through p of $\boldsymbol{\Delta y_t}$ and $\boldsymbol{\Delta p_t}$ as instrumental variables to estimate equation (3)[1].

Referring to Christiano et al.'s (2003) constant elasticity of variance (CEV) model, we used the log difference of the real GDP and the log level value of price, which adds the time linear and quadratic time trends. Model (IV) can be expressed as

$$\Delta \mathrm{y_t} = \mathrm{u_1} + \sum_{\mathrm{j=1}}^{\mathrm{n}} \beta_{\mathrm{yy,j}}\Delta \mathrm{y_{t-j}} + \sum_{\mathrm{j=0}}^{\mathrm{n-1}} \beta_{\mathrm{yp,j}}\Delta \mathrm{p_{t-j}} + \gamma_{11}\mathrm{t} + \gamma_{21}\mathrm{t}^2 + \varepsilon_\mathrm{t}^\mathrm{s} \tag{5}$$

and $$\mathrm{p_t} = \mathrm{u_2} + \sum_{\mathrm{j=1}}^{\mathrm{n}} \beta_{\mathrm{pp,j}}\, \mathrm{p_{t-j}} + \sum_{\mathrm{j=1}}^{\mathrm{n}} \beta_{\mathrm{py,j}}\Delta \mathrm{y_{t-j}} + \gamma_{21}\mathrm{t} + \gamma_{22}\mathrm{t}^2 + \beta_{\mathrm{ps}}\varepsilon_\mathrm{t}^\mathrm{s} + \varepsilon_\mathrm{t}^\mathrm{d}, \tag{6}$$

[1] All instrumental variables in this paper passed the test of weak instrumental variables.

where Δp_{t-j} is substituted into equation (5). We also used lags 1-n of Δy_t and p_t as instruments to estimate the equation. Next, we used the local projection framework proposed by Jordà (2005) to estimate AS–AD curves in Model (III) and Model (IV). As the error term may be serially correlated, the Newey–West method was used to estimate the covariance matrix consistently. Further details are provided in the Appendix.

2.2 Data selection and model setting

2.2.1 United States (U.S.)

The data used in this study were obtained from Ramey (2016)[2]. Referring to Cover et al. (2006), U.S. real GDP and GDP deflator were selected to identify supply and demand shocks. Due to the log first-order difference of the GDP deflator, we cannot reject the assumption of unit root at a 1% confidence level by the augmented Dickey-Fuller (ADF) test. Therefore, the log-difference values of real GDP (Δy_t) were used, and the log second-order differential values of GDP deflator ($\Delta \pi_t$). 1954Q1–2001Q4 were selected to identify demand and supply shocks[3].

We used $\Delta \pi_t$ and π_t in place of Δp_t and p_t respectively for both the BQ and OLS methods. Four lags were chosen considering the Akaike (AIC) and Schwartz (SC) information criteria and the literature (CEV, 2003; Francis and Ramey, 2005). Additionally, we used lags one through four of Δy_t and $\Delta \pi_t$ as instruments to estimate Model (III) and lags one through four of Δy_t and π_t as instruments to estimate Model (IV). Meanwhile, we used the Jordà local projection approach to calculate the impulse response functions of supply and demand shocks.

2.2.2 China

The data from China were obtained from the latest data published by Chang et al. (2016)[4]. Due to data limitations, the data selection interval of this study was 1992Q1–2019Q4, and the GDP (by value-added) and GDP deflator were used to measure output and price levels. We used the GDP deflator instead of the consumer price index (CPI) to estimate the price level for three main reasons: first, the GDP deflator

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[2] We use the data from the Handbook of Macroeconomics of technology shocks appendix. Source: https://econweb.ucsd.edu/~vramey/research.html#fluc

[3] The data interval is consistent with Cover et al. (2006)

[4] Latest version of data updated on June 30, 2021, source: https://www.atlantafed.org/cqer/research/chinaorg/cqer/research/China-macroeconomy.aspx?panel=1 They use TRAMO/SEATS (Signal extraction of ARIMA time series or time series regression with missing values) model to make seasonal adjustment of China's macroeconomic data. There are two main benefits. First, after seasonal adjustment, the data processing difference between China and the U.S. becomes smaller, reducing the impact of data problems on statistical results. The second is that removing seasonal factors reduces the noise in macro data, making it easier for economists to spot long-term economic patterns.

can be obtained from nominal and real GDP. China has been adjusting nominal and real quarterly GDP, which means that the GDP deflator are also revised; however, the CPI is revised less frequently. Second, the CPI can only reflect changes in the prices of some goods and does not reflect all price changes properly. Third, Nakamura et al. (2016) noted that the official CPI release in China would underestimate the real CPI volatility.

Considering previous literature (Zhang et al., 2019), AIC, and SC information criteria, we used six lags in the BQ methods for Model (I) and Model (II) and the OLS method for Model (III) and Model (IV). To properly estimate the OLS method, we used lags one through six of Δy_t and Δp_t as instruments to estimate Model (III) and lags one through six of Δy_t and p_t as instruments to estimate Model (IV). The impulse response function was estimated using the Jordà local projection framework by Model (III) and Model (IV).

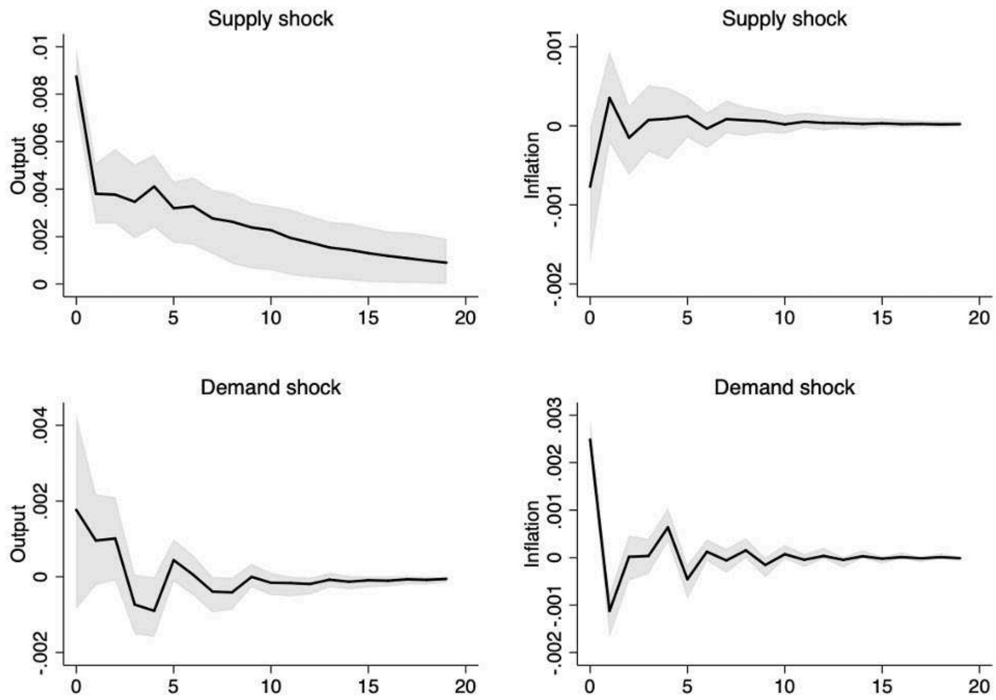
3. Empirical Analysis

3.1 BQ and OLS method in U.S.

Figures 1 and 2 show the results of BQ and OLS methods in the U.S., respectively. A one-unit positive supply shock causes output to increase and prices to decline and as supply shocks shift the supply curve, we can identify the demand curve. As the output increase and prices decrease, we obtain a downward-sloping demand curve. Additionally, a one-unit negative demand shock causes both output and prices to increase. The demand shock moves the demand curve so that we can identify the supply curve. The increase in output and prices means that the slope of the supply curve is positive. Therefore, we can identify the AS curve by demand shocks and the AD curve by supply shocks.

Based on the results in Figures 1 and 2, no slope puzzle exists in the U.S. (Cover et al., 2006; Shapiro and Waggoner, 1988; Galí, 1999; Spencer 1996; Gamber, 1996). Meanwhile, these two figures also indicate that the results of the BQ and OLS methods are the same and consistent with the literature (Francis and Ramey, 2005; Ramey, 2016).

(a) Model (I)



(b) Model (II)

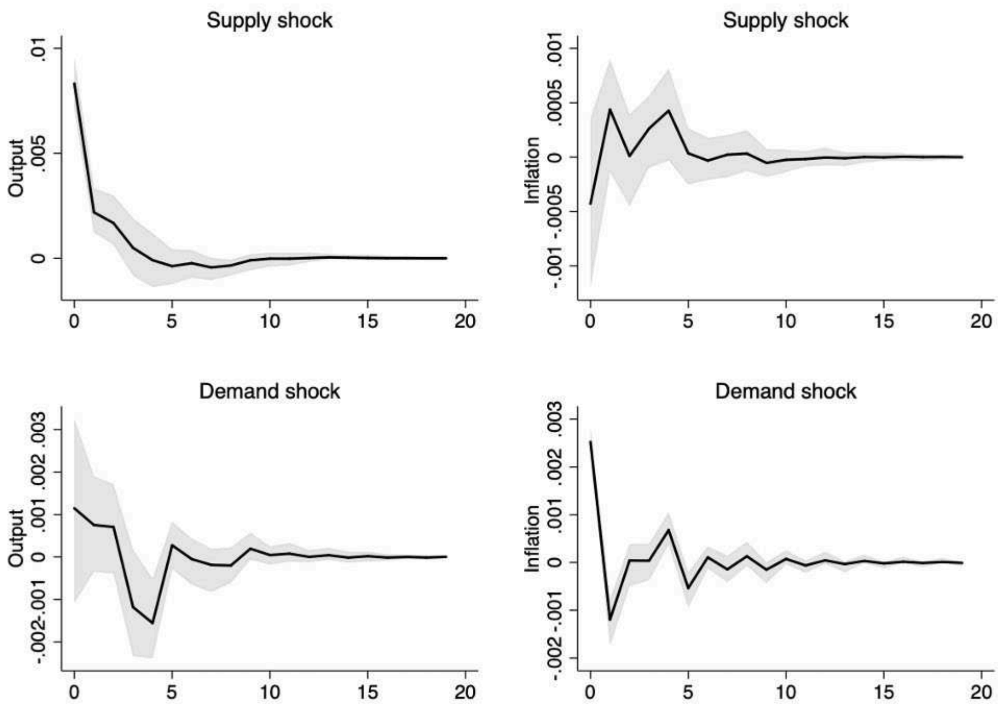
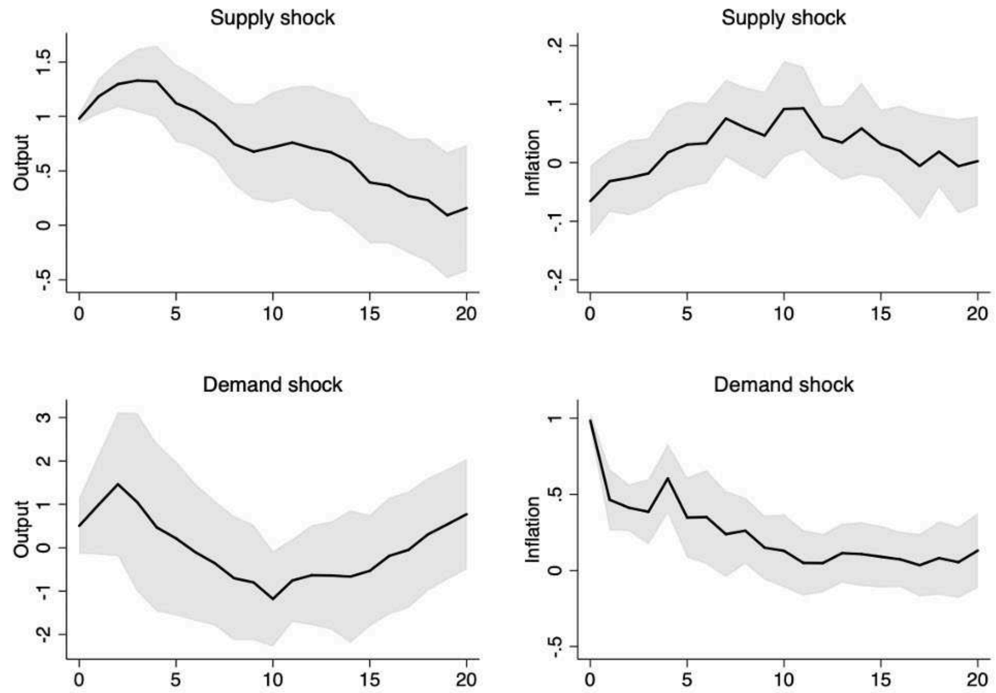


Figure 1. Impulse response function of the United States' supply and demand shocks using the BQ method. The data intervals are 1954Q1–2001Q4. The dotted line indicates the 95% confidence interval.

(c) Model (III)



(d) Model (IV)

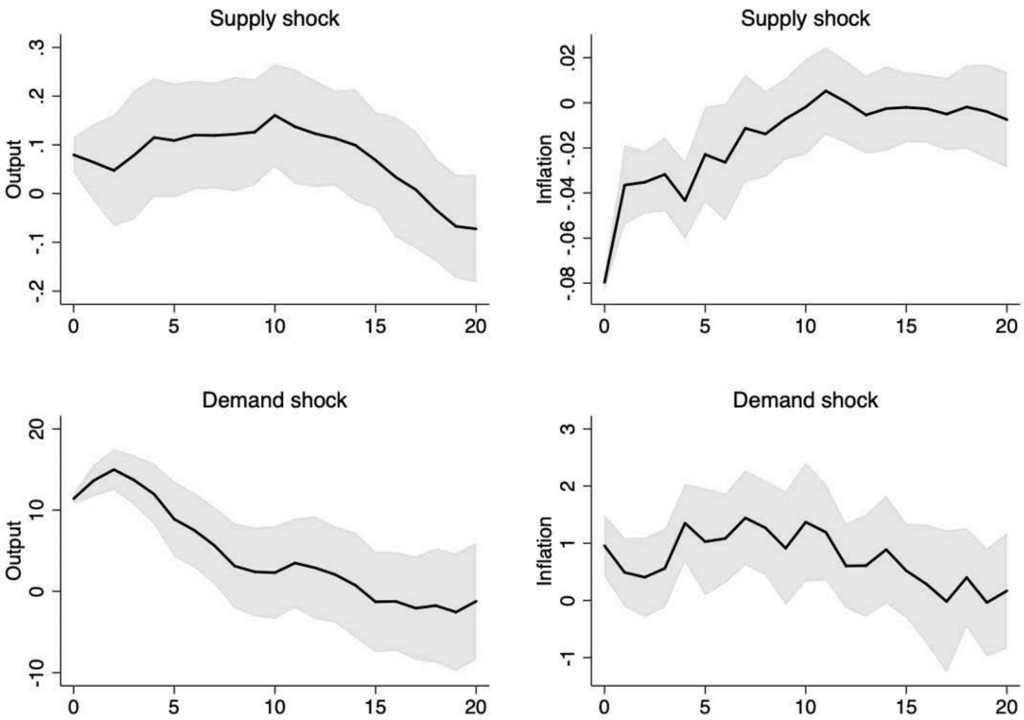


Figure 2. Impulse response function of the United States' supply and demand shocks using the OLS method. The data intervals are 1954Q1–2001Q4. The dotted line indicates the 95% confidence interval.

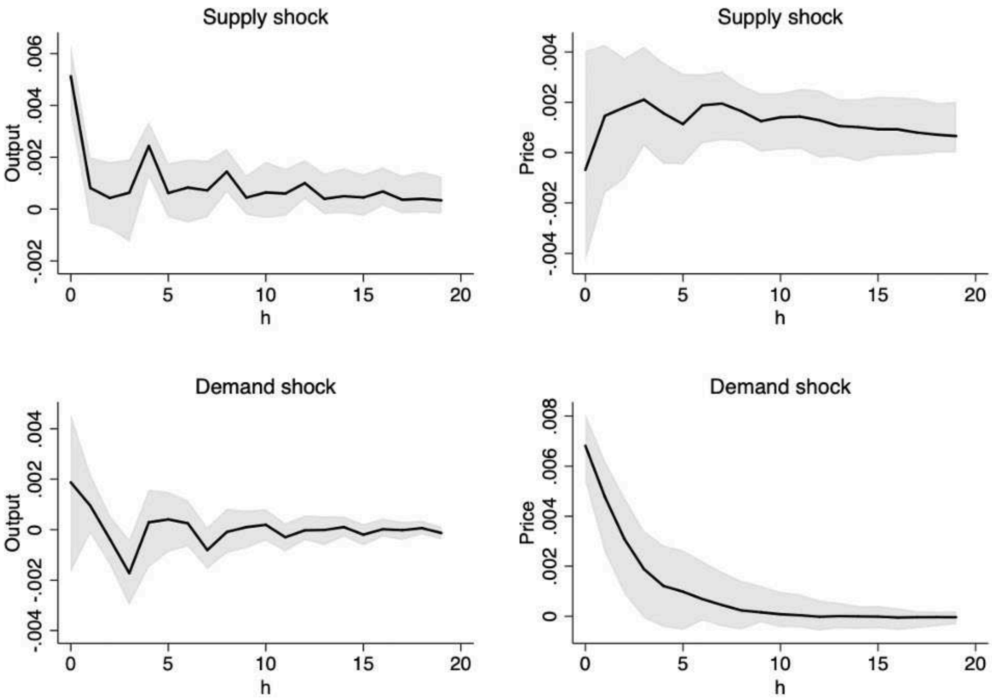
3.2 BQ and OLS method in China

The specific results are shown in Figures 3 and 4 for China’s demand and supply curves using the BQ and OLS methods.

As depicted in Figures 3, a one-unit positive supply shock causes both output and prices to increase and supply shocks shift the supply curve, meaning that we can identify the demand curve. Given that output and prices increase, we get an upward-sloping AD curve, which contradicts the Keynesian theory. Additionally, a one-unit negative demand shock causes both output and prices to increase. The demand shock moves the demand curve so that we can identify the supply curve and get an upward-sloping AS curve. If it is found that the slope of the AS curve is not positive, or the slope of the AD curve is not negative, then there is a "slope puzzle" in the economy. Therefore, the slope puzzle exists in China when we use the BQ method, which is consistent with the findings of Xu (2008).

As seen in Figure 4, a one-unit positive supply shock causes output to increase and prices to decrease, and we get a downward-sloping AD curve. Similarly, a one-unit negative demand shock causes both output and prices to increase and we get an upward-sloping AS curve. Therefore, we obtained an opposite result using the OLS method, with an upward-sloping AS curve and a downward-sloping AD curve, meaning that China has no slope puzzle. Furthermore, this indicates that the results of these two methods are not equivalent in terms of identifying demand and supply shocks in China, which contradicts the literature.

(a) Model (I)



(b) Model (II)

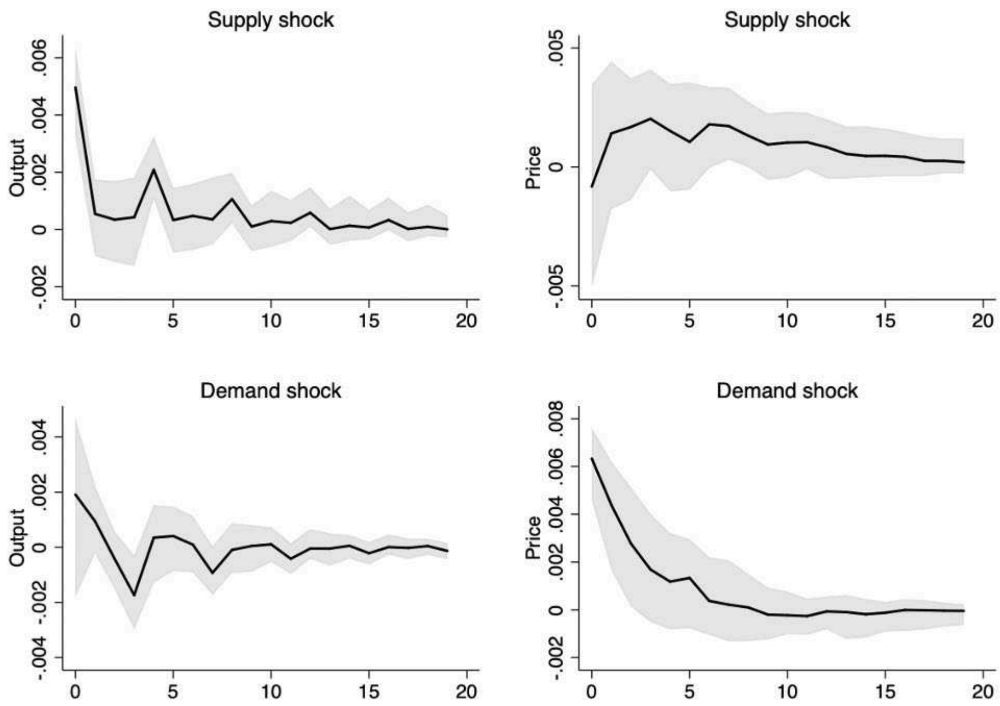
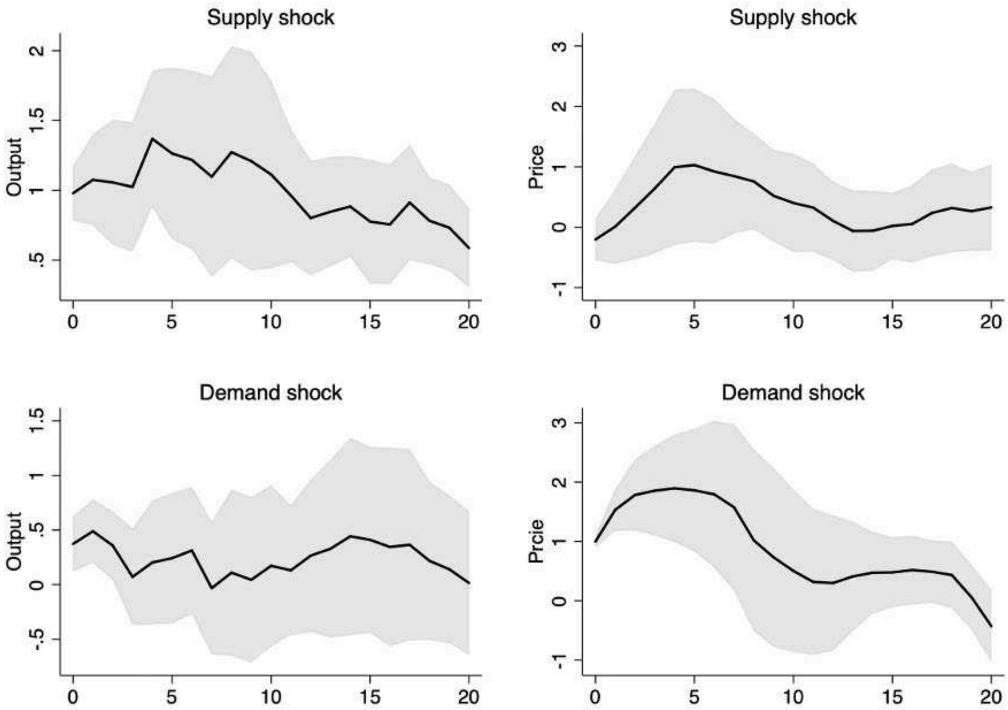


Figure 3. Impulse response function of China's supply and demand shocks using the BQ method. The data intervals are 1992Q1–2019Q4. The dotted line indicates the 95% confidence interval.

(c) Model (III)



(d) Model (IV)

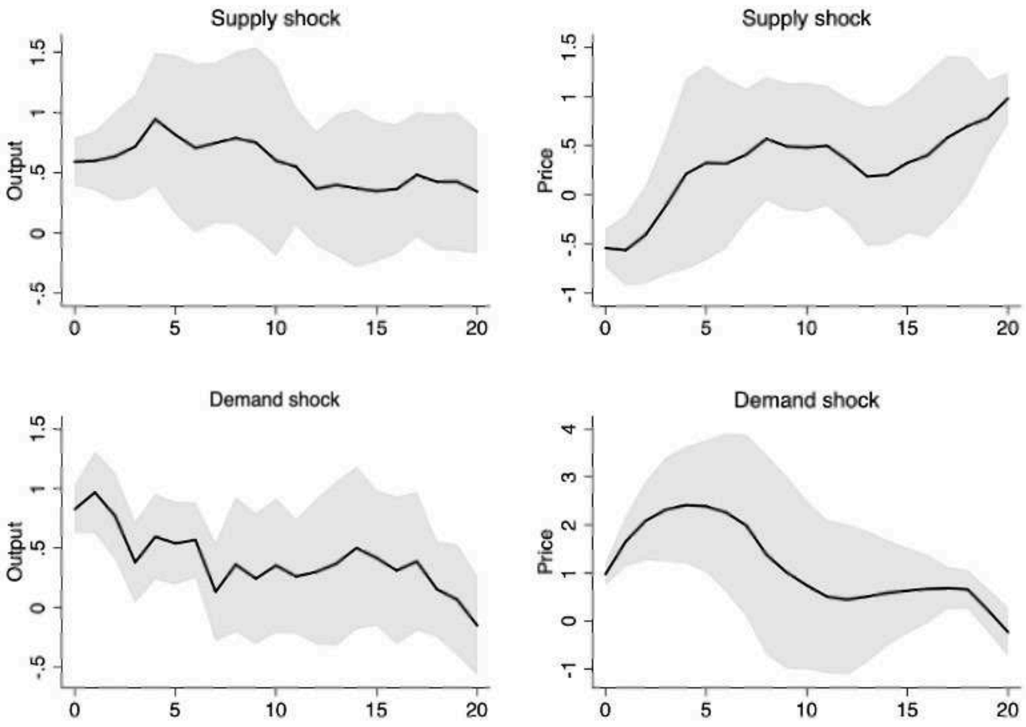


Figure 4. Impulse response function of China's supply and demand shocks using the OLS method. The data intervals are 1992Q1–2019Q4. The dotted line indicates the 95% confidence interval.

3.3 Explanation of inconsistent results of the two methods

Although literature has suggested that the BQ and OLS methods are equivalent, they do have several differences. The most pertinent difference is that the OLS method uses external instrument variables (IV) to identify demand and supply shocks.

For China's data, this method has several advantages over the BQ method. First, the OLS method uses Model (IV). Compared to the BQ method, Model (IV) uses information developed from "outside" the VAR (Ramey, 2016). Therefore, in the OLS method, we used more information than the BQ method to identify more accurate AS-AD curves.

Moreover, the identification of SVAR requires parameter restrictions that may be questioned (Mertens and Ravn, 2013). When supply shocks have a long-run effect on price ($C_{12}(1) = 0$), the BQ method results in inaccurate estimates. However, although we used the long-run restriction in Model (III) and Model (IV), we used $\Delta^2 p_{t-j}$ instead of Δp_{t-j} to identify supply shocks. Irrespective of whether the long-run multipliers for Δp_{t-j} (that is, $C_{12}(1)$) are zero, we can still obtain equation (3) and equation (5). Therefore, we can get the consistent and unbiased results when we use the OLS method; however, a biased estimate will be obtained when we use the BQ method if the long-run multipliers for Δp_{t-j} are not zero.

Finally, because of the endogeneity problem, the coefficient estimates were made unbiased and consistent by adding the instruments variables when using the OLS method. All these advantages make the OLS method more convincing and robust than the BQ method.

4. Conclusion

In this study, we identified AS–AD curves in the U.S. and China using the BQ and OLS methods. First, we found that these two methods are equivalent when identifying the AS–AD curves in the U.S.; this is consistent with the results reported in the literature. However, the results were different when we analyzed China's AS–AD curves, wherein the results of these two methods were not equivalent. Second, we used OLS to estimate AS-AD curves in China for the first time, and the results showed that there is no slope puzzle in China.

Therefore, both methods should be used carefully. Because of the use of a large amount of information and external Method (IV), the OLS method tends to be more robust and convincing than the BQ method when the two results are inconsistent. Therefore, the OLS method is more suitable for analyzing China's AS-AD curves.

The most pertinent finding in this study is that the results of the BQ and OLS methods are not equivalent. Additionally, the presence of a slope puzzle in China's macroeconomy is an essential academic issue. This study provides empirical evidence for future studies on the slope puzzle.

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Appendix

1. The BQ method

Following Blanchard and Quah (1989), we can identify AS-AD curves in a bivariate vector structure autoregressive model (SVAR):

$$\begin{bmatrix} \Delta y_t \\ \Delta p_t \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} C_{11}(L) \; C_{12}(L) \\ C_{21}(L) \; C_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix} \tag{1}$$

where ε_t^s denotes supply shock, and ε_t^d denotes demand shock. Δy_t is the log difference of real GDP and Δp_t. a and b are constants. Additionally, we assumed demand and supply shocks are uncorrelated. Therefore, $E\varepsilon_t^s\varepsilon_t^d = 0$ $\boldsymbol{\varepsilon_t}$ is the white noise, obeying a mean of 0 and a variance $\mathbf{I_2}$, that is, $var(\varepsilon_t^s) = var(\varepsilon_t^d) = 1$. $C(L)$ are lag polynomials. According to the BQ method, demand shocks do not affect the output in the long run; therefore, $C_{12}(1) = 0$.

However, we cannot directly identify demand and supply shocks using equation (1). Therefore, we used the following VAR model.

$$\begin{bmatrix} \Delta y_t \\ \Delta p_t \end{bmatrix} = \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \Delta p_{t-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{2}$$

where $A_{ij}(L)$ are lag polynomials, and c and d are constants. In the BQ method, (2) can be obtained by iterating (1); that is, (1) is equivalent to (2). Also, we can link $\boldsymbol{\varepsilon_t}$ with the reduced form VAR u_t:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \begin{bmatrix} c_{11}(0) & c_{12}(0) \\ c_{21}(0) & c_{22}(0) \end{bmatrix} \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix} \tag{3}$$

Therefore, if we determine the values of $c_{11}(0)$、$c_{12}(0)$、$c_{21}(0)$, and $c_{22}(0)$, we can identify the demand and supply shocks. If $E\varepsilon_t^s\varepsilon_t^d = 0$ and $var(\varepsilon_t^s) = var(\varepsilon_t^d) = 1$, we can construct the following three equations:

$$Var(e_{1t}) = c_{11}(0)^2 + c_{12}(0)^2 = 1 \tag{4}$$

$$Var(e_{2t}) = c_{21}(0)^2 + c_{22}(0)^2 = 1 \tag{5}$$

$$E(e_{1t}e_{2t}) = c_{11}(0)c_{21}(0) + c_{12}(0)c_{22}(0) = 0 \tag{6}$$

Using equation (3), we can obtain

$$[1 - A(L)L]Z_t = e_t \tag{7}$$

We define $X = [1 - A(L)L]$; therefore,

$$Z_t = X^{-1}e_t \tag{8}$$

The adjoint matrix of X is X^*, and we have the following results.

$$X^* = \begin{bmatrix} 1 - A_{22}(L)L & A_{12}(L)L \\ A_{21}(L)L & 1 - A_{11}(L)L \end{bmatrix} \tag{9}$$

where $A_{ij}(L) = \sum_{k=0}^{\infty} a_{ij}(k)L^k$. Equation (8) can be written as follows.

$$\begin{bmatrix} \Delta y_t \\ \Delta p_t \end{bmatrix} = \frac{1}{|X|} \begin{bmatrix} 1 - \sum a_{22}(k)L^{k+1} & \sum a_{12}(k)L^{k+1} \\ \sum a_{21}(k)L^{k+1} & 1 - \sum a_{11}(k)L^{k+1} \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \tag{10}$$

We can expand the first line of Equation (10) to get the following form

$$\Delta y_t = \frac{1}{|X|} \{[1 - \sum a_{22}(k)L^{k+1}]e_{1t} + \sum a_{12}(k)L^{k+1}e_{2t}\} \tag{11}$$

The following results can be obtained from equation (3)

$$e_{1t} = c_{11}(0)\varepsilon_t^s + c_{12}(0)\varepsilon_t^d \tag{12}$$

$$e_{2t} = c_{21}(0)\varepsilon_t^s + c_{22}(0)\varepsilon_t^d \tag{13}$$

Substitute Equation (12) and (13) into Equation (11).

$$[1 - \sum a_{22}(k)]c_{11}(0) + \sum a_{12}(k)c_{21}(0) = 0 \tag{14}$$

The values of $c_{11}(0)$、$c_{12}(0)$、$c_{21}(0)$, and $c_{22}(0)$ can be obtained from equations (4), (5), (6), and (14). We can identify demand and supply shocks in Model (II) using the same process.

2. The OLS method

Referring to Shapiro and Watson (1988), King et al. (1991), Francis et al. (2004), and Ramsey (2016), we have[5]:

$$X_t = C(L)\varepsilon_t \tag{15}$$

where $X_t = (\Delta y_t, \Delta p_t)^T$, $\varepsilon_t = (\varepsilon_t^s, \varepsilon_t^d)^T$, and Z_0 is constant. Assuming $C(\mathrm{L})$ has an inverse matrix, we can rewrite equation (15) as

$$D(L)X_t = \varepsilon_t \tag{16}$$

where $D(L) = C(L)^{-1}$. Because $C(1)$ is the lower triangular matrix, $D(1)$ is also the lower triangular matrix.

$$\begin{bmatrix} D_{11}(L) & D_{12}(L) \\ D_{21}(L) & D_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta p_t \end{bmatrix} = \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \end{bmatrix} \tag{17}$$

According to equation (17), Δy_t can be written as follows:

$$\Delta y_t = \beta_0 + \sum_{j=1}^{p} \beta_{yy,j}\Delta y_{t-j} + \sum_{j=0}^{p} \beta_{yp,j}\Delta p_{t-j} + \varepsilon_t^s \tag{18}$$

Therefore, because D (1) is the lower triangle matrix and the long-run multipliers

[5] We omit constant without loss of generality.

Δp_{t-j} are zero, the coefficients of its lags sum to zero. We define $\Delta^2 p_{t-j} = \Delta p_{t-j} - \Delta p_{t-(j-1)}$, and equation (18) is rewritten as:

$$\Delta y_t = b_1 + \sum_{j=1}^{p} \beta_{yy,j} \Delta y_{t-j} + \sum_{j=0}^{p-1} \gamma_{yp,j} \Delta^2 p_{t-j} + \varepsilon_t^s \qquad (19)$$

We can see that $\mathbf{\Delta^2 p_{t-j}}$ instead of $\mathbf{\Delta p_{t-j}}$ is inserted in equation (19). Given that there may be a correlation between $\mathbf{\Delta^2 p_{t-j}}$ and $\boldsymbol{\varepsilon_t^s}$, the coefficient estimates are biased and inconsistent when using the OLS method. Therefore, we used lags one through p of $\mathbf{\Delta y_t}$ and $\mathbf{\Delta p_t}$ as instrumental variables to estimate equation (19).

Similarly, we can write the following equation of Δp_t from equation (17)

$$\Delta p_t = b_2 + \sum_{j=1}^{p} \beta_{pp,j} \Delta p_{t-j} + \sum_{j=1}^{p} \beta_{py,j} \Delta y_{t-j} + \beta_{ps} \varepsilon_t^s + \varepsilon_t^d \qquad (20)$$

We can identify demand and supply shocks in Model (IV) using the same process and obtain the following equations:

$$\Delta y_t = u_1 + \sum_{j=1}^{n} \beta_{yy,j} \Delta y_{t-j} + \sum_{j=0}^{n-1} \beta_{yp,j} \Delta p_{t-j} + \gamma_{11} t + \gamma_{21} t^2 + \varepsilon_t^s \qquad (21)$$

$$p_t = u_2 + \sum_{j=1}^{n} \beta_{pp,j}\, p_{t-j} + \sum_{j=1}^{n} \beta_{py,j} \Delta y_{t-j} + \gamma_{21} t + \gamma_{22} t^2 + \beta_{ps} \varepsilon_t^s + \varepsilon_t^d \quad (22)$$

Similarly, we use lags one through p of $\mathbf{\Delta y_t}$ and $\mathbf{\Delta p_t}$ as instrumental variables to estimate equation (21).

Next, we used the local projection framework proposed by Jordà (2005) to estimate AS-AD curves in Models (III) and (IV). Finally, referring to Jordà and Òscar (2005) and Ramey (2016), we estimated the regression of the following equation.

$$z_{t+h} = \alpha_0 + \theta_h \mathrm{shock}_t + \delta_h(L)\gamma_{t-1} + \eta_1 t + \eta_2 t^2 + \varepsilon_{t+h} \qquad (23)$$

where z_{t+h} is our core variable for output and prices. The control variables are demand shocks, supply shocks for current values and lags of four. It was determined that γ_{t-1}, including the core variables, lagged by four periods. Considering that there may be a serial correlation in ε_{t+h}, we used the Newey–West method to estimate the covariance matrix consistently.