Economic Growth Analysis When Balanced Growth Paths May Be Time Varying

Mountford, Andrew

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Andrew Mountford

Royal Holloway, University of London

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Abstract

The determinants of an economy's balanced growth path for income per capita may vary over time. In this paper we apply unobserved components analysis to an otherwise standard panel model of economic growth dynamics so that an economy's long run relative income per capita can change at any point of time. We apply this model to data for the world economy from 1970-2019 and for US States from 1929-2021. In both datasets an economy's initial relative income per capita is a good predictor of its long run relative income per capita. While we find evidence for (σ) convergence in relative income in US States in the years 1929-1970, there is little convergence in subsequent periods. Overall these results provide support for the 'Poor Stay Poor' hypothesis of Canova and Marcet (1995).

Keywords: Bayesian Econometrics, Economic Growth, State Space Models, Macroeconomics

*Address for correspondence: A.Mountford, Department of Economics, Royal Holloway, University of London, Egham, TW20 OEX, U.K. Tel: +44 1784 443906 email: A.Mountford@rhul.ac.uk;

Introduction

Economic growth theory describes how an economy's income per capita on its balanced growth path may be determined by, *inter alia*, its saving propensity, human capital accumulation, population growth, institutional quality and economic policy. All of these factors may be changing over time and influencing an economy's growth dynamics. Even without the anchor of economic growth theory it is intuitive that an economy's long run path is determined by factors that are likely to change over time. In this paper we therefore add unobserved components analysis to an otherwise standard empirical model of economic growth dynamics so that an economy's balanced growth path can change at any point in time. We estimate the model using two publicly available datasets; the Penn World Tables for GDP per capita in the world economy from 1970-2019, and US Bureau of Economic Analysis (BEA) data for per capita personal income for the 48 contiguous US States from 1929-2019. We find that for US States there was significant convergence in States' long run balanced growth paths during the years 1929-1970 but that the last 50 years have seen little further convergence. The world economy over the last 50 years is more varied but shows a similar pattern to the US States over this same period. The analysis demonstrates that convergence in one sub-period does not imply further subsequent convergence and that much of the variation in relative GDP per capita over time can be ascribed to temporary movements away from a stable balanced growth path with little mobility in relative rankings. These results provide support for 'The Poor Stay Poor' hypothesis of Canova and Marcet (1995).

This paper combines the analysis of two important literatures; the empirical economic growth literature and the Bayesian macroeconomic time series literature. In the empirical growth literature a key question is whether one should think of the world as made up of economies that are slowly converging to the same balanced growth path or whether one should think that economies are converging to their own individual balanced growth paths. Examples of the former view include Barro and Sala-i-Martin (1991) on the convergence of US States, and recently Patel, Sandefur and Subramanian (2021) on convergence in the world economy. The latter view emphasizes country fixed effects in panel data estimations and include seminal contributions by Canova and Marcet (1995), Caselli, Esquivel and Lefort (1996) and Shioji (2004). Shioji (2004) argued that US States' income per capita data was more consistent with a slow convergence of States to the same balanced growth path. This was because panel models produced parameter estimates that implied a relatively fast rate of convergence to the long run balanced growth path and this seemed inconsistent with the large distance of many States' initial conditions from their long run balanced growth paths. This paper addresses this issue by allowing the long run balanced growth path to change over time so that, for example, an economy could initially be close to its initial balanced growth path but far away from its ultimate balanced growth path.

The empirical growth literature has also analysed the effects of changes in the economic environment on economic growth using panel methods. In this literature an economy's fixed effect may change if important features of the economy change. Notable examples are Acemoglu *et al* (2019), Cerra and Saxena (2008) and Wacziarg and Welch (2008) who showed how changes to democracy, financial and political stability and trade openness can affect long run growth. These papers relate to the 'The Poor Stay Poor' hypothesis as they imply that economies that remain undemocratic, unstable and closed to trade will converge to a lower long run growth path. The contribution of this paper is to estimate a reduced form model of income per capita dynamics where an economy's long run balanced growth path can change at any point in time and is not tied to a specific observable policy or institutional change. The model is applied to two important datasets from the literature. The US States dataset is of contiguous, free trading, democratic economies operating under free interstate capital mobility. The conditions for convergence in this dataset therefore are intuitively as good as could reasonably be expected. The Penn World Tables is an authorativate dataset used throughout the world and is therefore of intrinsic interest.

This paper is also an application of Bayesian macroeconomic time series analysis to the issue of economic growth. Bayesian macroeconomic time series analysis has applied unobserved components models to, *inter alia*, decompose series such as GDP and inflation into a trend and cyclical component.[1] A contribution of this paper is to apply this analysis to the empirical growth literature, so that an economy's change in relative income per capita can be decomposed into a long run growth path component which can change through time and a transitory component around this path. We estimate two variations of this model; (i) A model where each individual economy's intercept terms follow an independent local level model but with the same convergence coefficients across economies. (ii) A hierarchical model where the convergence parameters as well as the intercept terms can vary across economies but where the convergence terms are drawn from a common population distribution. The estimation process follows the Gibbs sampling procedures set out in Chan *et al* (2019).

The paper is organised as follows. Section 1 describes the estimated models and their relationship to the empirical growth literature. Sections 2 and 3 describe the results from applying the models to data on US States and the world economy respectively. Section 4 concludes. The Gibbs sampling algorithms used in the estimation and some further results are presented in the Appendix.

1 Growth Dynamics with a Time Varying Balanced Growth Path

There is a large literature analyzing economic growth dynamics using dynamic panels models, see for example Chen *er al* (2019). These models have the form

$$Y_{i,t} = \alpha_i + \gamma_t + \sum_{l}^{L} \beta_l Y_{t-l} + \delta D_{i,t} + e_{i,t}$$

where $e_{i,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$ and α_i are individual fixed effects, γ_t are time effects, $Y_{i,t}$ is the log of GDP per capita in country i at time t and Y_{t-l} is its l'th lag. The $D_{i,t}$ is an indicator variable

[1] This literature is very large indeed but relevant contributions to this paper include the seminal state space analysis of Dejong and Shephard (1995) and Chan, Koop and Potter's (2013) analysis of trend inflation.

for a significant change. Notable examples of this estimated model include Acemoglu *et al* (2019) where $D_{i,t}$ was an indicator for democracy, Wacziarg and Welch (2008) where $D_{i,t}$ was an indicator for trade openness and Cerra and Saxena (2008) where $D_{i,t}$ was an indicator for financial instability.

The model can also be estimated in terms of relative income per capita, where the time effects cancel out, so that the estimated equation becomes,

$$Y^*_{i,t} = \alpha_i + \sum_{l}^{L} \beta_l Y^*_{t-l} + \delta D_{i,t} + e_{i,t} \tag{1}$$

where $Y^*_{i,t}$ is the deviation of the log of the income per capita in economy i at time t, from the average across all economies at time t or a comparator economy such as the 'frontier' economy, and where $D_{i,t}$ is similarly redefined. The relative income per capita form of the model was used by Canova and Marcet (1995) and Shioji (2004) and we use this form in our estimation below.

The literature also focusses on the long-run dynamic effect of changes in policies or institutions. An economy i's long run relative income per capita in this model, denoted, $Y_i^{*,LR}$ is given by

$$Y_i^{*,LR} = \frac{\alpha_i + \delta}{1 - \sum_l^L \beta_l} \tag{2}$$

1.1 An Unobserved Components Model of Time Varying Balanced Growth Paths

The contribution of this paper is to estimate income per capita dynamics where the country specific intercept term, now denoted $\alpha_{i,t}$, is free to move persistently at any point time, rather than being tied to particular policies or institutional variables.

We estimate two variations of model. Our baseline model has the intercept term for each economy following an independent local level model. Following the literaure the β coefficients are estimated using the aggregate of all economies as in equation (1). Our second model estimates a hierarchical model where an individual economy's β coefficients can vary but are drawn from a common population distribution. For both cases we estimate the models with looser and tighter priors to illustrate the implications of prior choice. Tighter priors imply a smoother long run balanced growth. One's choice of prior will reflect the degree of variation one thinks is consistent with the concept of a balanced growth path. We describe the models in more detail below.

Model 1: The Baseline Model

The baseline model varies equation (1) by treating each economy's intercept term as a local levels model, so that

$$\begin{aligned} Y^*_{i,t} &= \alpha_{i,t} + \sum_{l}^{L} \beta_l Y^*_{t-l} + e_{i,t} \; \forall i \\ \alpha_{i,t} &= \alpha_{i,t-1} + \nu_{i,t} \; \forall i \end{aligned} \tag{3}$$

where $e_{i,t} \overset{iid}{\sim} \mathcal{N}(0,\sigma^2)$ and $\nu_{i,t} \overset{iid}{\sim} \mathcal{N}(0,\omega_i^2)$. Thus α_i follows a random walk. One could choose other specifications for α_i, such as local linear trends, see for example Chan *et al* (2019). However as Figures 3 and 6 show, the random walk specification is able to fit the trends in the US States dataset before 1970 and the growth paths of economies that have strong or volatile trends, such as North Dakota in the US States datatset and China in the Penn World Tables Dataset, as Figures 4 and 7 highlight respectively.

We estimate this model using Bayesian methods set out in Chan et al (2019). The prior for the β parameters, $\underline{\beta} \equiv [\beta_1 \ldots \beta_l]'$ is is distributed $\mathcal{N}(\beta_0, S_\beta)$, where $\mathcal{N}$ denotes the Normal distribution. The prior for the initial value α, denoted α_0 in each economy is also normal with distribution $\mathcal{N}(a_0, b_0)$. The prior for σ^2 is distributed so that $\sigma^{-2} \sim \mathcal{G}(\nu_\sigma, S_\sigma)$ where $\mathcal{G}$ denotes the gamma distribution.[2]. We set $\nu_\sigma = 2.5$ and $S_\sigma = \frac{1}{4} \times 10^{-4}$. The prior values of β_0 is taken from a pooled OLS regression of all economies with a common intercept term and we set $S_\beta = 10^8 I_p$. The parameters for the priors of α_0 and σ^2 are chosen to be diffuse: $a_0 = 0$,$b_0 = 1000$.

The prior for ω_i^2 is very important as it impacts on the variation in the path of $\alpha_{i,t}$ and so on the variation in the balanced growth path. We assume the priors for ω_i^2 is distributed so that $\omega_i^{-2} \sim \mathcal{G}(\nu_\omega, S_\omega^{-1})$ Intuitively, if ω_i^2 is restricted to be small (e.g. if $S_\omega = 0.01$) then balanced growth path will not be able to explain much of the variation in $Y^*_{i,t}$ over time and the model will resemble the panel model of equation 1 with fixed effects but without the $D_{i,t}$ indicator variable. Conversely one also does not want to set S_ω so high that the balanced growth path is very variable, as this would conflict somehwhat with the notion of a long run balanced growth path, which intuitively should be slow moving. For illustration we choose two specifications for these parameters; a loose prior case and a tight prior case. In our loose prior case we set $\nu_\omega = 3$ and $S_\omega = 16$ this lets the unobserved component explain alot of the movement in $Y^*_{i,t}$ but still produces a smooth balanced growth path. In our tight prior case we set $\nu_\omega = 3$ and $S_\omega = 4$. This results in much less variation in the balanced growth path through time.

Given the $\underline{\beta}$ coefficients, the model described by equation (3) is a local levels model for each economy, i, where the dependent variable is $\widehat{Y_{i,t}}$ where $\widehat{Y_{i,t}} \equiv Y_{i,t} - \sum_l^L \beta_l Y_{i,t-l}$. Similarly given vectors $\underline{\alpha_i} \equiv [\alpha_1, \ldots, \alpha_T]$ for each economy i the model is a simple linear regression model of the form $\tilde{Y}_t = \sum_l^L \beta_l Y_{t-l}$ where $\tilde{Y}_t$ is the stacked vector $Y_{i,t} - \alpha_{i,t}$. Thus given $\underline{\alpha_i}$ for each economy i the model can be estimated using standard methods and the model overall can be estimated with a Gibbs sampler as detailed Chan *et al* (2019) and described in the Appendix.

[2]We are the following form for Gamma Distribution. A random variable y $y \sim \mathcal{G}(\nu_\omega, S_\omega)$ has a gamma distribution with the following form

$$f_\gamma(y \mid \nu, S) = \begin{cases} \frac{S^{-\nu}}{\Gamma(\nu)} y^{\nu-1} e^{-\frac{y}{S}} & \nu, S, > 0 \qquad y > 0, \\ 0 & \text{otherwise} \end{cases}$$

This implies that y has a mean of νS and y^{-1} has a mean of $\frac{1}{S(\nu-1)}$.

Model 2: Hierarchical Model

The baseline model assumes that the $\underline{\beta}$ coefficients are the same for all economies. We relax this assumption by estimating a hierarchical model where these coefficients, now denoted $\underline{\beta_i}$, are drawn from a population distribution so that each individual economy's coefficients can vary from each other to the extent allowed by the variance of the population distribution. This model is thus described as follows,

$$\begin{aligned} Y^*_{i,t} &= \alpha_{i,t} + \sum_{l}^{L} \beta_{l,i} Y^*_{t-l} + e_{i,t} \ \forall i \\ \alpha_{i,t} &= \alpha_{i,t-1} + \nu_{i,t} \ \forall i \end{aligned} \tag{4}$$

where now $\underline{\beta_i} \sim \mathcal{N}(\underline{\overline{\beta_l}}, \overline{\Sigma}_{\overline{\beta}^2}) \quad \forall i$, and where $\underline{\overline{\beta_l}}$ and $\overline{\Sigma}_{\overline{\beta}^2}$ are the mean and variance of the population distribution respectively. The prior for the mean of the population distribution is distributed normally, i.e. $\underline{\overline{\beta_l}} \sim \mathcal{N}(\psi, C)$, where ψ is taken from a pooled OLS regression of all economies with a common intercept term and where we set $C = I_p \times 10^{-1}$. The prior for the variance $\overline{\Sigma}_{\overline{\beta}^2}$ is distributed with a Inverse Wishart density function so that $(\overline{\Sigma}_{\overline{\beta}^2})^{-1} \sim \mathcal{W}([\rho R]^{-1}, \rho))$, where where $\mathcal{W}$ denotes the Wishart distribution. We set $R = I_p \times 10^2$ $\rho = 10$. This allows for considerable variation across economies.

The other priors are set as in the baseline model so that the prior for the initial value α, denoted α_0 in each economy is Normal with distribution $\mathcal{N}(a_0, b_0)$ and the priors for σ^2 and ω_i^2 are distributed so that $\sigma^{-2} \sim \mathcal{G}(\nu_\sigma, S_\sigma)$ and $\omega_i^{-2} \sim \mathcal{G}(\nu_\omega, S_\omega^{-1})$ with the same parameter values as in the baseline model. That is, we estimate the model with both loose and tight priors for ω_i^{-2}, where $\nu_\omega = 3$ and $S_\omega = 16$ in the loose priors case and $S_\omega = 4$ in the tight priors case.

This model can also be estimated using Gibbs sampling. Given the $\underline{\beta_i}$ coefficients, the model described by equation (4) is a local levels model for each economy, i, where the dependent variable is $\widehat{Y_{i,t}}$ where $\widehat{Y_{i,t}} \equiv Y_{i,t} - \sum_l^L \beta_{l,i} Y_{i,t-l}$. Similarly given vectors $\underline{\alpha_i} \equiv [\alpha_1, \ldots, \alpha_T]$ for each economy i then each economy's model is a simple linear regression model with dependent variable $\hat{Y_{i,t}} = Y_{i,t} - \alpha_{i,t}$ and so given the population parameters, the $\underline{\beta_i}$ can be estimated using standard methods for each economy i. Given the estimated parameters for each individual economy the population parameters $\underline{\beta_i}$, $\overline{\Sigma}_{\overline{\beta}^2}$ and the inverse variance $(\frac{1}{\sigma^2})$ can be shown to be distributed, Normal, Inverse Wishart, and Gamma respectively, as described in Chan *et al* (2019) and noted in the Appendix.

2 Application to US States Growth Dynamics 1929-2019

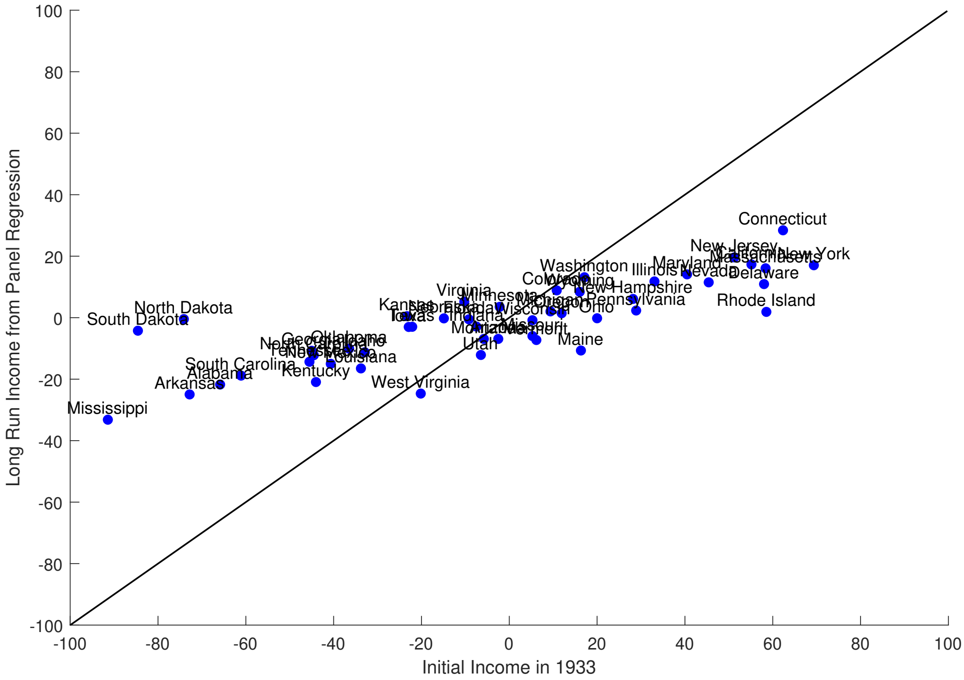
We estimate the models described in section 1.1 using data for per capita personal income for the 48 contiguous US States for the period 1929-2021 which is publically available from the BEA. This is the same dataset used by Shioji (2004) for the period 1929-2001. We estimate the model using per capita personal income relative to the mean as in (1) by subtracting the average level per capita income across States in each period.

2.1 Initial Conditions and Long Run Growth

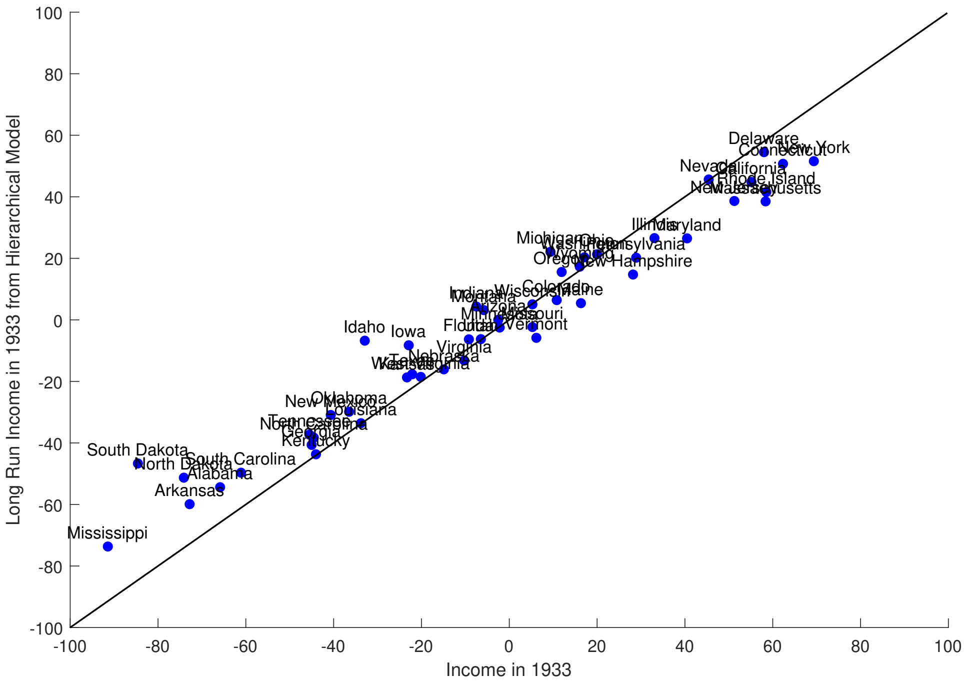
Shioji (2004) found a very significant relationship between initial level of relative income per capita and long run relative income per capita, where the latter was derived from the (time invariant) fixed effect from a panel regression using data from 1929-2001. However the relationship was not close to the 45° line. We have plotted this relationship using our updated dataset in Figure 1a and found virtually the same pattern. Shioji (2004) argued that these initial conditions were too far away from the long run income level to be consistent with the estimated rate of convergence. However if States' long run balanced growth paths are allowed to vary through time then a State's initial conditions may have been close to its initial balanced growth path but far away from its present day balanced growth path. This idea is illustrated in Figure 1b which plots the initial level of relative income per capita, in 1933, against the initial value of long run balanced growth derived from the mean of the initial level of $\alpha_{i,t}$ from the hierarchical model of Section 1.1 with loose priors. This relationship fits the 45° line much more closely. In the Appendix Figure B1 we plot this relationship for the remaining models described in section 1.1. This shows that all the baseline unobserved components models are closer to the 45° line than the panel regression model but that the relationship is closest for the hierarchical version of the model, with loose priors, which is intuitive as this allows the balanced growth paths greater scope to change through time.

The change in the balanced growth path over time is illustrated in Figure 2, which plots the same relationships as in Figure 1 but for the sample period 1970-2021. Figure 2a plots the relationship from the panel regression and Figure 2b plots the relationship from the hierarchical unobserved components model. Note that the scaling of the y axis in Figure 2 is much reduced indicating convergence in the distribution from 1933-1974, as is presented in Figure 3 below. Both Figures 2a and 2b now fit the 45° line. The difference between the two graphs stems from the different β coefficients used to calculate the balanced growth path but also from the distance between a State's balanced growth in 1974 and in the rest of the sample, which the hierarchical model is able to track. The similarity of the two Figures implies that there is not a great deal of movement in States' balanced growth paths for most economies over this period. Thus although the unobserved component models allow States' balanced growth paths to change over time, over the last half century most of the estimated paths do not vary very much. The evolution of the balanced growth paths and the relative stability over the sub sample 1974-2021 is discussed in more detail in section 2.2 below.

Figure 1: Contrasting the Association Between US States' Long Run Relative Income per Capita and Initial Conditions with a Panel Regression and the Hierarchical Unobserved Components Model Using Data from 1929-2021



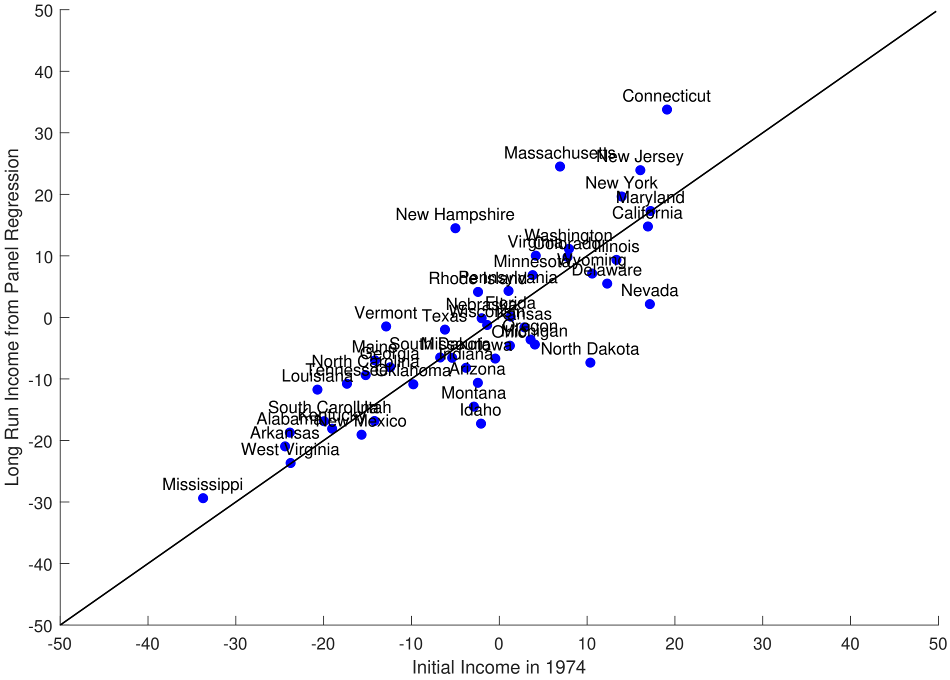
(a) Panel Regression



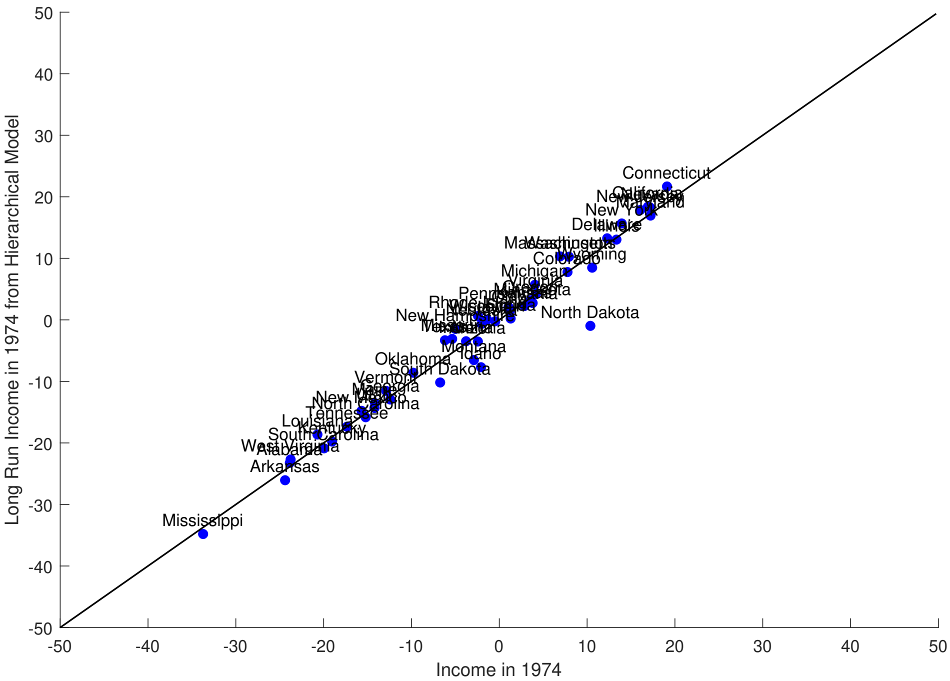
(b) Hierarchical Model with Loose Priors

States' initial Income per capita in the dataset in 1933 against the estimated long run balanced growth path in 1933. Panel a) show the results from a panel regression model with 4 lags using data from 1929-2021. Panel b) shows the results for the mean posterior value of the balanced growth path from the hierarchical model with loose priors of Section 1.1 using the same data and the same number of lags.

Figure 2: Contrasting the Association Between US States' Long Run Relative Income per Capita and Initial Conditions with a Panel Regression and the Hierarchical Unobserved Components Model Using Data from 1970-2021



(a) Panel Regression)



(b) Hierarchical Model with Loose Priors

States' initial Income per capita in the dataset 1974 against the estimated long run balanced growth path in 1974. Panel a) show the results from a panel regression model with 4 lags using data from 1970-2021. Panel b) shows the results for the mean posterior value of the balanced growth path from the hierarchical model of Section 1.1 wity loose priors using the same data and the same number of lags.

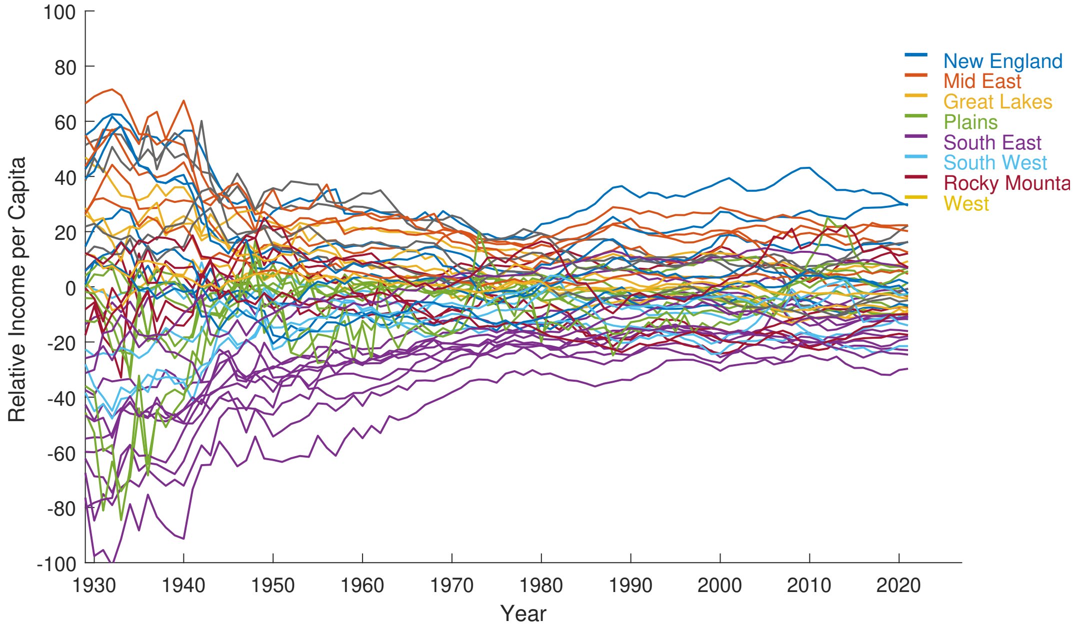
2.2 The Evolution of Long Run Income

Figure 3 plots the evolution of US States relative income per capita. Figure 3a plots the raw data and Figure 3b plots the evolution of the balanced growth path as estimated by the hierarchical unobserved components model with loose priors. The hierarchical growth paths are noticeably smoother. The balanced growth paths of the baseline unobserved component models are also smoother, with the model with tight priors being smoothest which illustrates the effect of the prior ω_i^{-2} discussed above. These are shown in Figure B3 in the Appendix. Figure 3 shows that from 1933 to around the mid-1970's there was a substantial - (σ) - convergence in the distribution of income per capita with the variance around the mean dropping noticeably. However since the mid-1970's the variance of the distribution has remained approximately constant. Figures 3b and B3 show that after the mid 1970's, the balanced growth path of most States has been largely stable and slow moving. This is consistent with the concept of a balanced growth path. although there are exceptions to this, notably North Dakota, which we discuss below.

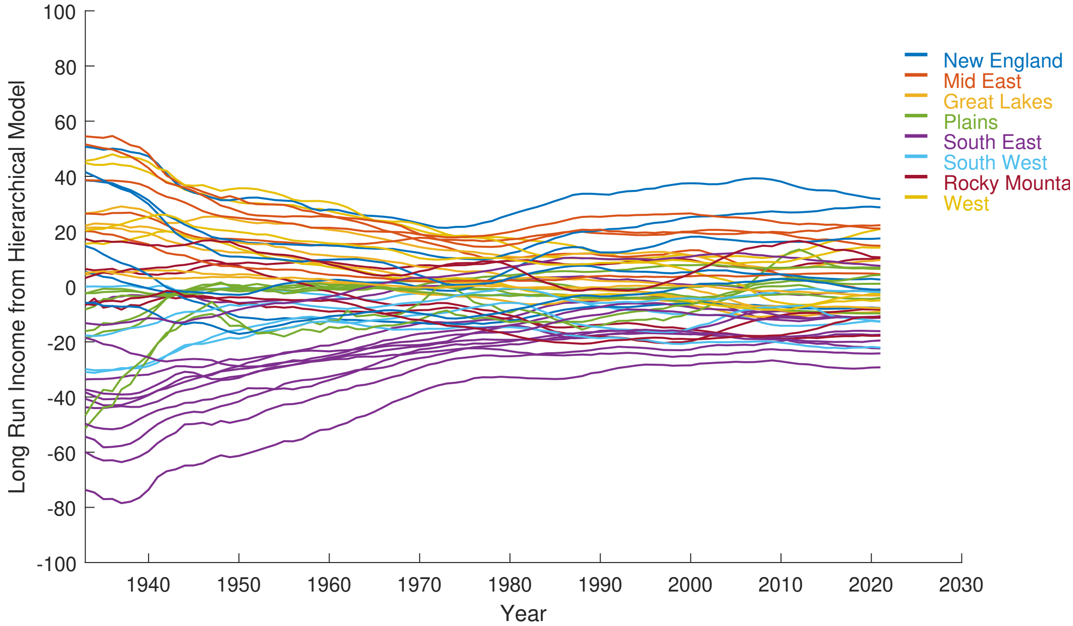
2.2.1 Income Mobility

Figure 4 highlights the evolution of the balanced growth paths for California, Georgia, New York North Dakota and Texas. All except North Dakota are smooth and slow moving. The paths do sometimes cross with their neighbours' in the distribution but tend to stay in the same part of the distribution. This is consistent with the raw data on income mobility presented in Table 1. This shows that over the 93 years of the sample no state has moved from the top quartile to the bottom quartile and only one State has moved from the bottom quartile to the top quartile. This state is North Dakota which was ranked 41st in 1929 and 12th in 2021 and is a state with a small population and whose income per capita is highly influenced by the price of oil. Its unusual degree of volatility is evident in Figure 4. One may think the level of volatility for this state is inconsistent with the notion of a balanced growth path. If so one would prefer the models with tighter priors which provides a smoother estimate of the balanced growth path, see Figure B3b in the Appendix. Table 1 also shows that two States that were in the first quartile in 1929 dropped to the third quartile in 2021. These were Nevada which was ranked 9th in 1929 and 26th in 2021 and Michigan that was ranked 10th in 1929 and 32nd in 2021. Thus change does happen but as Figures 3 and 4 show, this occurs against a backdrop of stability.

Figure 3: The US States Dataset and the Evolution of the Balanced Growth Path



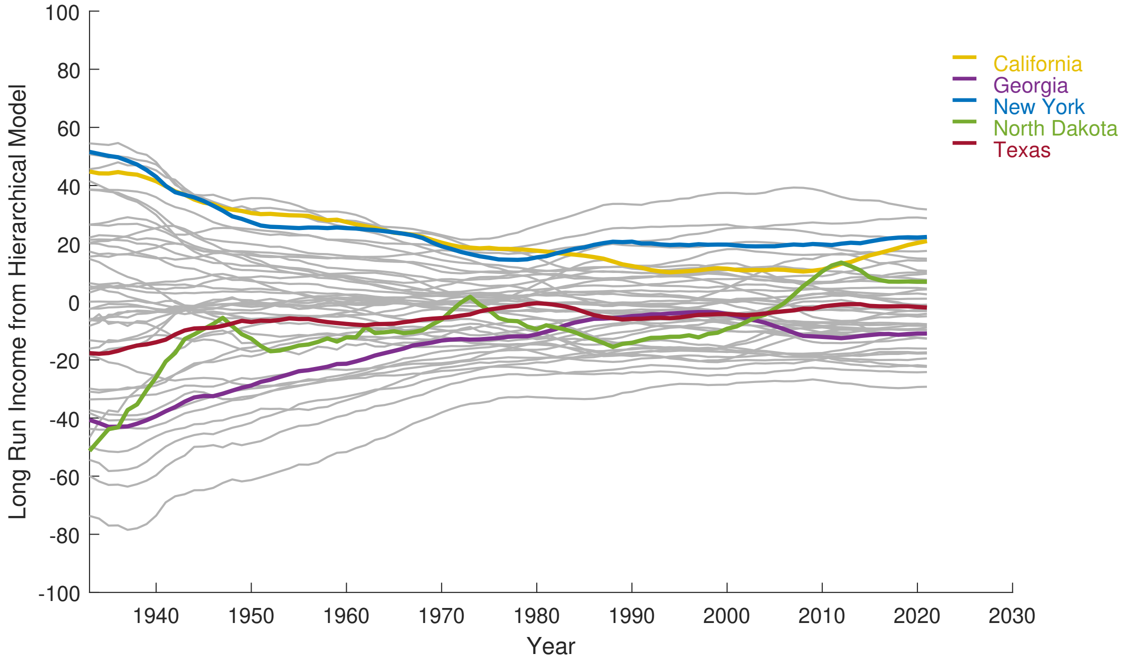
(a) States Income per capita 1929-2021



(b) States' Long run Growth path 1933-2021 from the Hierarchical model with Loose priors

The evolution of the income per capita and balanced growth path for the 48 contiguous US States from 1933-2021. Panel a) plots the raw data and Panel b) plots the evolution of the balanced growth income per capita using the mean posterior value from the hierarchical model with loose priors of Section 1.1 estimated on the same dataset using 4 lags.

Figure 4: Selected States' Balanced Growth Paths for Income per Capita 1933-2021



The evolution of the balanced growth path of selected States using the mean posterior value of the balanced growth path from the hierarchical model with loose priors of Section 1.1 estimated with with 4 lags using data from 1929-2021.

Table 1: Income Mobility of US States by Quartile 1929-2021

	Quartile in 2021			
Quartile in 1929	**First**	**Second**	**Third**	**Fourth**
First	7	3	2	0
Second	4	2	5	1
Third	0	6	3	3
Fourth	1	1	2	8

The interquartile mobility in income per capita in US States from 1929-2021

3 Application to World Growth Dynamics 1970-2019

In this section we carry out the same analysis as in Section 2 but for the world economy using the Penn World Tables dataset. Following, for example, Patel *et al* (2021), we exclude oil producing economies and small economies from the dataset which leaves a dataset of 123 countries.[3] As we discuss below the results of the analysis for the world economy share many of the characteristics of that for US States above, most notably that an economy's initial level of income per capita is a strong predictor of its long run balanced growth path and that there is little interquartile mobility.

3.1 Balanced Growth Paths and Income Mobility

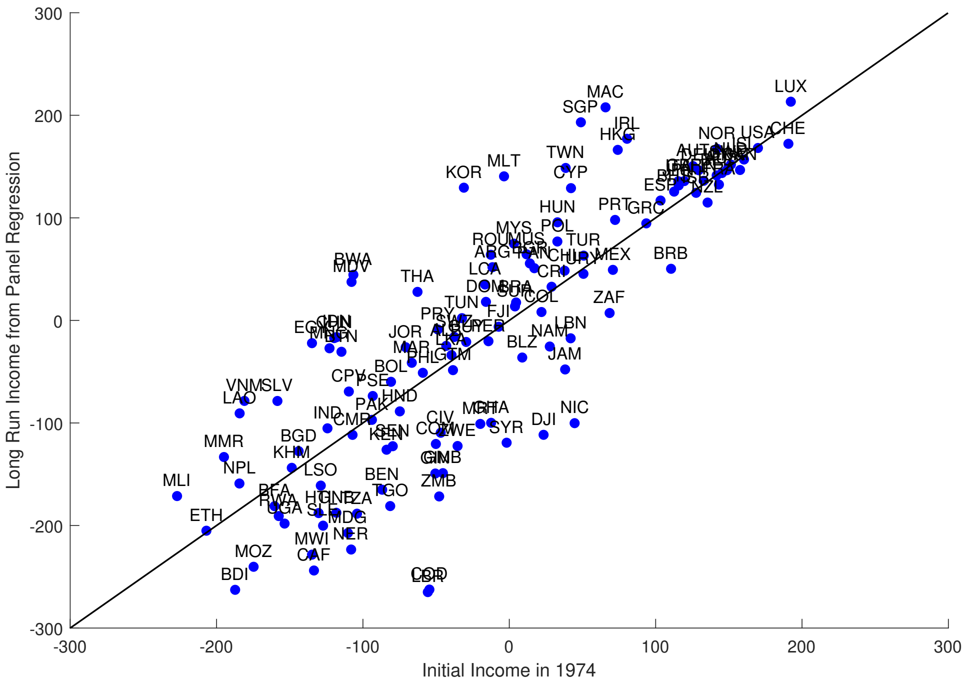
The relationship between an economy's initial conditions and its estimated balanced growth path is shown in Figure 5. The first panel Figure 5a plots the estimated balanced growth path from a panel regression against the initial level of relative income per capita. This relationship is close to the 45° line but with a noticeable degree of noise. Figure 5b shows this relationship is much tighter when the balanced growth path is derived from the hierarchical unobserved components model with loose priors. Figure B2 in the Appendix plots this relationship for the remaining models described in Section 1.1. This shows that, as with the US States dataset, all these models are closer to the 45° line than the panel regression model but that the relationship is closest for the hierarchical version of the model with loose priors.

Figure 6 plots the raw data and the estimated balanced growth paths for this dataset for the hierarchical model with loose priors. The estimated growth paths are smoother than the raw data but less noticeable so than for the US States. The estimated balanced growth paths from the baseline unobserved components models with loose and tight priors are also smoother for most economies. These are shown in Figure B4 in the Appendix. All models, however, have very volatile growth paths for the Lebanon and Liberia which are associated with civil war episodes in these countries. This volatility is smallest in the hierarchical model with loose priors shown in Figure 6b.

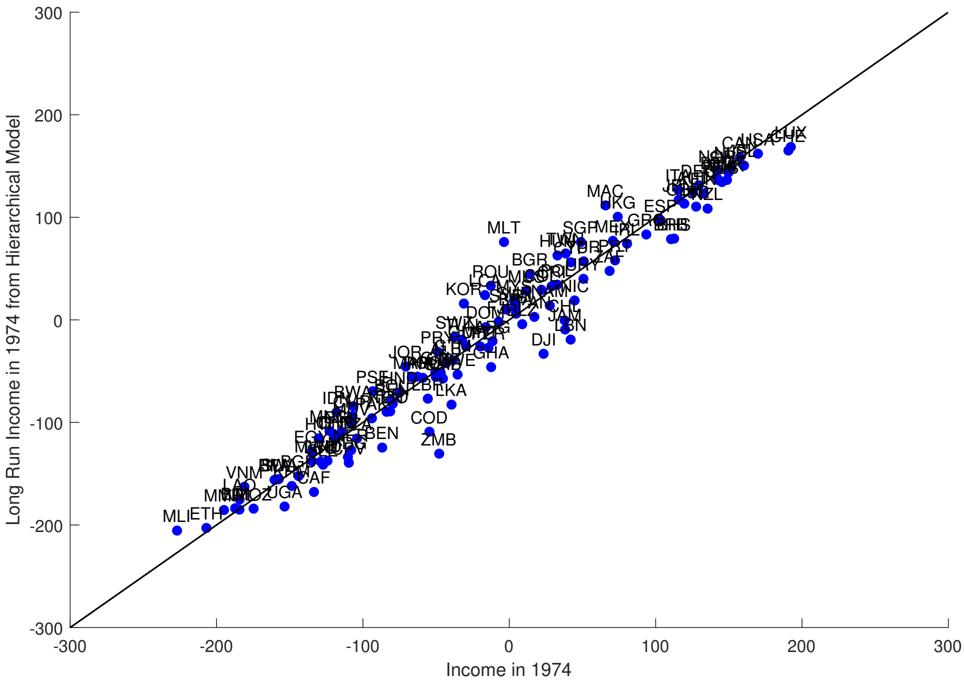
Figure 7 highlights the evolution of the balanced growth paths for Brazil, China, India, South Africa and the USA. All of these have smooth growth paths although those of China and India has a marked upward trend. China rose from a ranking of 101st in 1970 to 63 in 2019 and India from 103rd in 1970 to 83rd in 2019. However, as with US States dataset, this change is occuring in a broadly stable setting. This stability is again consistent with the raw data on income mobility presented in Table 2. This shows that over the 50 years of the sample no country has moved from the top quartile to the bottom half of the distribution and no country moved from the bottom quartile to the top quartile. One country moved from the third quartile in 1970 to the first quartile in 2019. This country is the Republic of Korea which was ranked 64th in 1970 and 27th in 2019.

[3]The oil producing countries are the same as in Patel *et al* (2021), and small countries are those with a population less than 100,000 in 1970.

Figure 5: Contrasting the Association Between Long Run GDP per Capita and Initial Conditions with a Panel Regression and the Hierarchical Unobserved Components Model in the Penn World Tables Dataset 1970-2019



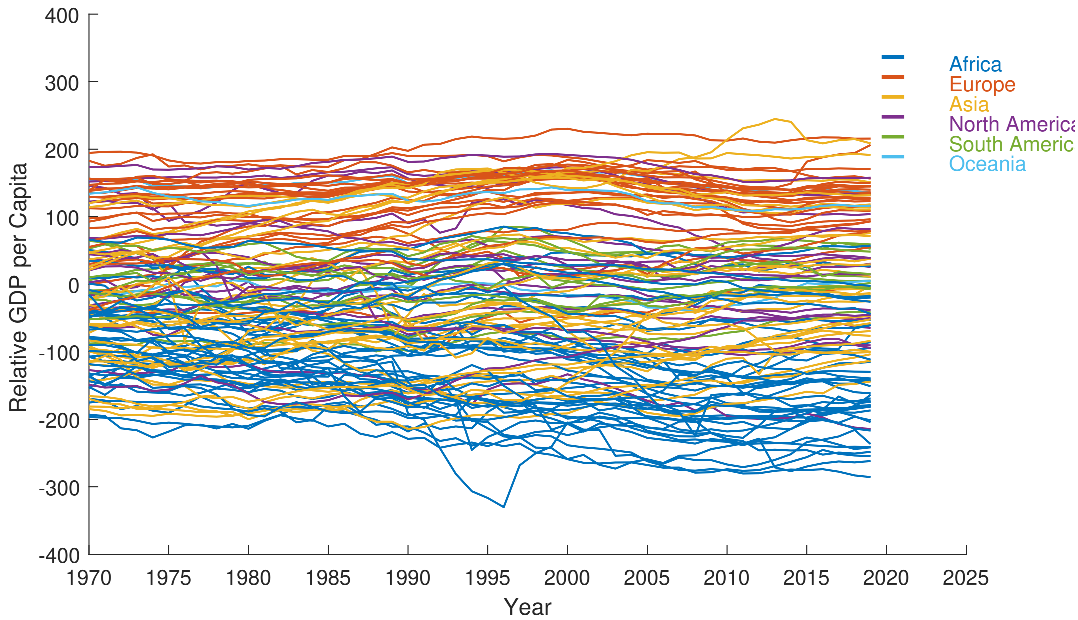
(a) Panel Regression



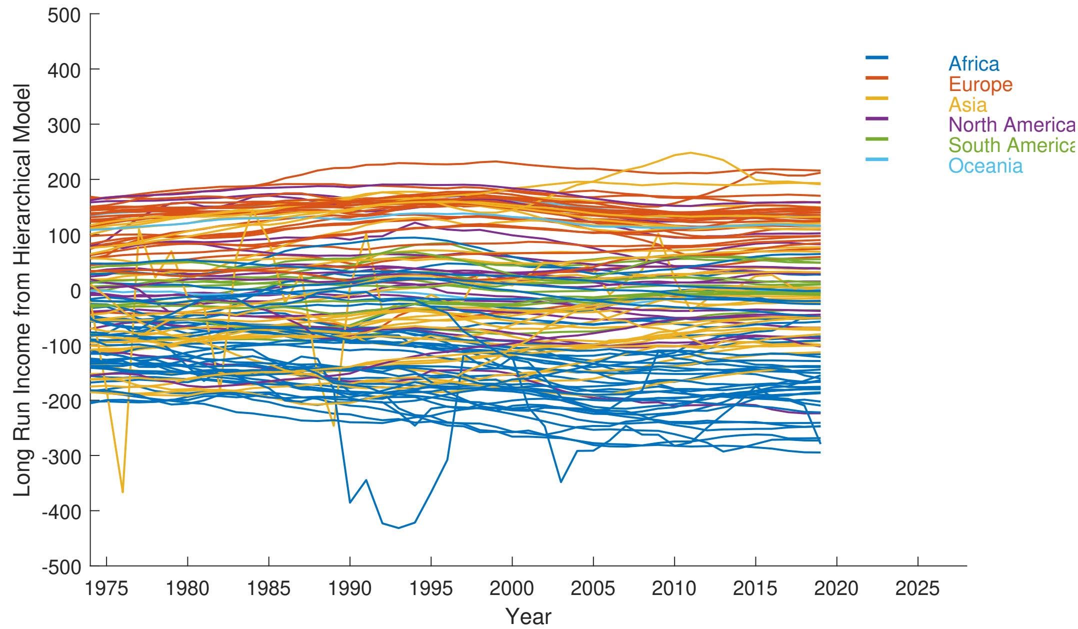
(b) Hierarchical Model with Loose Priors

Initial GDP per capita in 1974 in the PWT dataset against the estimated long run balanced growth path in 1974. Panel a) show the results for panel regression Model with 4 lags using data from 1970-2019. Panel b) shows the results for the mean posterior value of the balanced growth path from the hierarchical model with loose priors of Section 1.1 using the same data and the same number of lags. Labels in both panels are three letter country codes.

Figure 6: GDP per capita and the Evolution of the Balanced Growth Path in the World Economy 1970-2019



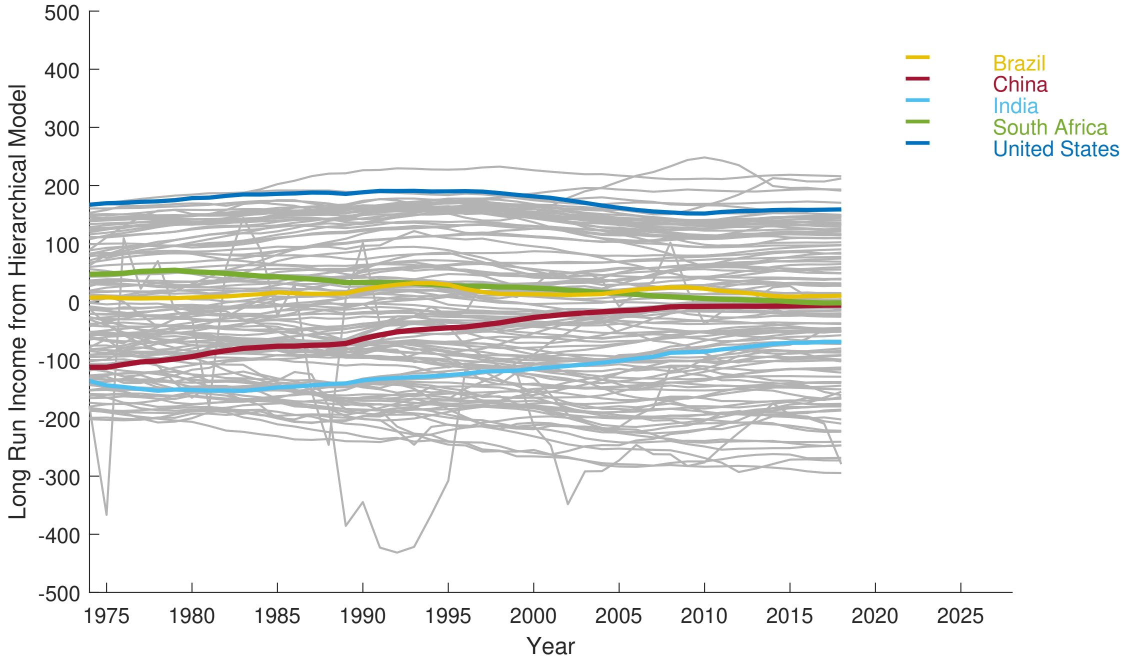
(a) GDP per capita in the World Economy 1974 -2019



(b) Countries' Long run Growth path 1974-2019 in the Hierarchical model with Loose Priors

The evolution of GDP per capita and estimated balanced growth paths in the world economy from 1974-2019. Panel a) plots the raw data and Panel b) plots the evolution of the balanced growth GDP per capita using the mean posterior value from the hierarchical model with loose priors estimated on the same dataset using the same number of lags.

Figure 7: Selected Countries Balanced Growth Paths for GDP per capita 1974-2019



The evolution of the balanced growth path of selected countries using the mean posterior value of the balanced growth path from the hierarchical model with loose priors of Section 1.1 estimated with 4 lags using data from 1970-2019.

Table 2: Income Mobility in World Economy by Quartile 1970-2019

	Quartile in 2019			
Quartile in 1970	**First**	**Second**	**Third**	**Fourth**
First	25	5	0	0
Second	4	19	8	0
Third	1	5	12	13
Fourth	0	2	11	18

The interquartile mobility in GDP per Capita in The World Economy from 1970-2019

4 Conclusion

In this paper we have added unobserved components analysis to an otherwise standard empirical model of economic growth dynamics so that an economy's balanced growth path can change at any point in time. We applied this model to data on income per capita from US States and the world economy. We found that an economy's initial level of income per capita is a good predictor of its long run balanced growth path and that although the empirical model allows growth paths to change through time, for most economies they have been quite stable over the last 50 years. This is consistent with the low interquartile mobility observed in the data. We did find convergence in US States' long run balanced growth paths during the years 1929-1970, but little evidence of further subsequent convergence. 'The Poor' stay relatively poor.

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Appendix A

Estimation Algorithms

Draws from the posterior for all the models are made iteratively via a Gibbs sampling algorithm which draws in sequence from the conditional posterior distributions described below. We follow Chan *et al* (2019) closely here, including the notation, and further detail and derivations can also be found at this reference.

Model 1: The Baseline Model

The baseline model of equation (3) with T time periods and N individual economies has the set of estimated parameters, Γ, where $\Gamma = [\{\underline{\alpha_i}\}_{i=1}^N, \{\omega_i^2\}_{i=1}^N, \{\alpha_{0,i}\}_{i=1}^N, \underline{\beta}, \sigma^2]$ where $\underline{\alpha_i} \equiv [\alpha_1, \ldots, \alpha_T]'$ and $\{\psi_i\}_{i=1}^N$ denotes the set of the ψ_i coefficients for all economies, $i = 1 : N$.

Denoting all parameters other than ψ by $\Gamma_{-\psi}$, then the conditional posterior distribution for parameter ψ given all the other parameters and the data, Y can be written $p(\psi \mid \Gamma_{-\phi}, Y)$. Using this notation, the Gibbs sampler algorithm can be described as follows:-

Choose starting values for $\underline{\beta}$, ω^2, σ^2 and $\alpha_{0,i}$, and also the number of draws, n^{draw}. Then cycle through draws from the condition posterior distributions described in (i)-(v) below, n^{draw} times, saving the draws after discarding an initial number, n^{burnin}. We choose $n^{\text{draw}} = 10^5$ and $n^{\text{burnin}} = 5 \times 10^4$ in all our estimations.

(i) Draw from $p(\underline{\alpha_i} \mid \Gamma_{-\underline{\alpha_i}}, \underline{Y_i})$, for each economy i, separately. This has distribution $\mathcal{N}(\underline{\hat{\alpha_i}}, V_\alpha)$ where

$$\underline{\hat{\alpha_i}} \;=\; V_\alpha\left(\frac{\alpha_{0,i}}{\omega_i^2} C'C 1_T + \frac{1}{\sigma^2}\widehat{Y_i}\right) \quad V_\alpha = \left[\frac{1}{\omega^2}C'C + \frac{1}{\sigma^2}I_T\right]^{-1}$$

where where $\widehat{Y_{i,t}} \equiv Y_{i,t} - \sum_l^L \beta_l Y_{i,t-l}$ and where $\widehat{Y_i} = [\widehat{Y_{i,1}}, \widehat{Y_{i,2}}, \ldots, \widehat{Y_{i,N}}]'$, 1_T is a $T \times 1$ vector of ones and C is a $T \times T$ matrix with 1 on the diagonal -1 below the diagonal and zeros elsewhere.

(ii) Draw from $p(\omega_i^2 \mid \Gamma_{-\omega_i^2}, \underline{Y_i})$, for each economy i, separately. $p(\frac{1}{\omega_i^2} \mid \Gamma_{-\omega_i^2}, \underline{Y_i})$ has a Gamma distribution with parameters $\nu_\omega + \frac{T}{2}$ and $\frac{1}{S_\omega} + \frac{1}{2}[(\underline{\alpha_i} - \alpha_0 1_T)'C'C(\underline{\alpha_i} - \alpha_0 1_T)]$.

(iii) For each economy i, draw from $p(\alpha_{0,i} \mid \Gamma_{-\alpha_{0,i}}, Y_i)$. $p(\alpha_{0,i} \mid \Gamma_{-\alpha_{0,i}}, Y_i) \sim \mathcal{N}(\hat{\alpha_{0,i}}, V_{\alpha i0})$ where

$$\hat{\alpha_{0,i}} \;=\; V_{\alpha i0}\left(\frac{a_0}{b_0} + \frac{\underline{\underline{\alpha_i(1)}}}{\omega_i^2}\right) \quad V_{\alpha i0} = \left[\frac{1}{b_0} + \frac{1}{\omega_i^2}\right]^{-1}$$

(iv) Given the parameters $\underline{\alpha}$ from all economies and defining $\underline{Y_i^\star} = \underline{Y_i} - \underline{\alpha_i}$ and stacking this variable across economies to create $Y^\star = [\underline{Y_1^{\star\prime}}, \underline{Y_2^{\star\prime}}, \ldots, \underline{Y_2^{\star\prime}}]'$ then the system becomes a linear regression model $Y^\star = X\underline{\beta} + u$ where X is the stacked system of lagged values of Y. i.e. $X = [Y_{-1}, Y_{-2}, \ldots, Y_{-p}]$, where Y_{-j} is the stacked system of $Y_{i,t}$'s lagged by j time periods. This can be estimated in the standard way. Given the prior for the β parameters, the conditional posterior distribution for $\underline{\beta}$, is Normally distributed, $p(\underline{\beta} \mid \Gamma_{-\underline{\beta}}, Y) \sim \mathcal{N}(\hat{\beta}, V_\beta)$ where

$$\hat{\beta} \;=\; V_\beta\left(S_\beta^{-1}\beta_0 + \frac{1}{\sigma^2}X'X\beta^{OLS}\right) \quad V_\beta = \left[S_\beta^{-1} + \frac{1}{\sigma^2}X'X\right]^{-1}$$

(v) Draw from $p(\frac{1}{\sigma^2} \mid \Gamma_{-\sigma^2}, Y)$ using the stacked system. $\frac{1}{\sigma^2}$ will have a Gamma distribution with parameters $\frac{\nu_\sigma + T\times N}{2}$ and $\frac{2}{\frac{\nu_\sigma}{S_\sigma}+[Y^\star - X\underline{\beta})'(Y^\star - X\underline{\beta})]}$.

For the initial values of $\alpha_{0,i}$ we choose the intercept term from an economy, i level OLS regression of $Y_{i,t} = \alpha_i + \sum_l^L \beta_l Y_{t-l}$ and for the initial values of $\underline{\beta}$ we use the β_l values from an OLS regression using the stacked system.

Model 2: The Hierarchical Model

The hierarchical model extends the baseline model by allowing the $\underline{\beta_i}$ coefficients to vary across economies by being randomly drawn from a common higher level distribution. This can be estimated in the same way as in the baseline model except for having a separate draw for each individual economy's $\underline{\beta_i}$ before there is a draw for $\overline{\beta}$ and $\overline{\Sigma}_{\overline{\beta}}$.

Using the same notation as in the Baseline model above the Gibbs sampler algorithm can be described as follows. Firstly choose starting values for $\underline{\beta}$,$\underline{\beta_i}$, ω^2, σ^2 and $\alpha_{0,i}$, and also the number of draws, n^{draw}. Then cycle through draws from condition posterior distributions (i)-(viii) below, n^{draw} times, saving the draws after discarding an initial number, n^{burnin}.

(i) Draw from $p(\underline{\alpha_i} \mid \Gamma_{-\underline{\alpha_i}}, \underline{Y_i})$, for each economy i. This is the same as in the Baseline Model

(ii) Draw from $p(\omega_i^2 \mid \Gamma_{-\omega_i^2}, \underline{Y_i})$, for each economy i. This is the same as in the Baseline Model

(iii) For each economy i, draw from $p(\alpha_{0,i} \mid \Gamma_{-\alpha_{0,i}}, Y_i)$. This is the same as in the Baseline Model

(iv) For each economy i, draw from $p(\underline{\beta_i} \mid \Gamma_{-\underline{\beta_i}}, Y)$ Given $\underline{\alpha_i}$ each economy is a linear regression model with dependent variable $\hat{Y_{i,t}} = Y_{i,t} - \alpha_{i,t}$. Given the population parameters $\underline{\overline{\beta_l}}, \overline{\Sigma}_{\overline{\beta}^2}$, $\underline{\beta_i}$ this can be estimated using standard methods so that $p(\underline{\beta_i} \mid \Gamma_{-\underline{\beta}}, Y) \sim \mathcal{N}(\hat{\beta}_i, V_{\beta,i})$

$$\hat{\beta}_i \quad = \quad V_{\beta,i}((\overline{\Sigma}_{\overline{\beta}})^{-1}\underline{\overline{\beta_l}} + \frac{1}{\sigma^2}X_i'X_i\beta_i^{OLS}) \quad V_{\beta,i} = [(\overline{\Sigma}_{\overline{\beta}})^{-1} + + \frac{1}{\sigma^2}X_i'X_i]^{-1}$$

where where X_i is the matrix of lagged values of Y_i. i.e. $X_i = [Y_{i,-1}, Y_{i,-2}, \ldots, Y_{i,-p}]$

(v) Collect all the error vectors and the $\underline{\beta_i}$ vectors and calculate the sum of squared residuals and the mean of the $\underline{\beta_i}$ vectors denoted SSE and β^{mean} respectively.

(vi) Draw from $p(\underline{\overline{\beta_l}} \mid \Gamma_{-\underline{\overline{\beta_l}}}, Y)$ which following Chan *et al* (2019) is distributed $\mathcal{N}(\hat{\beta}, V_\beta)$ where

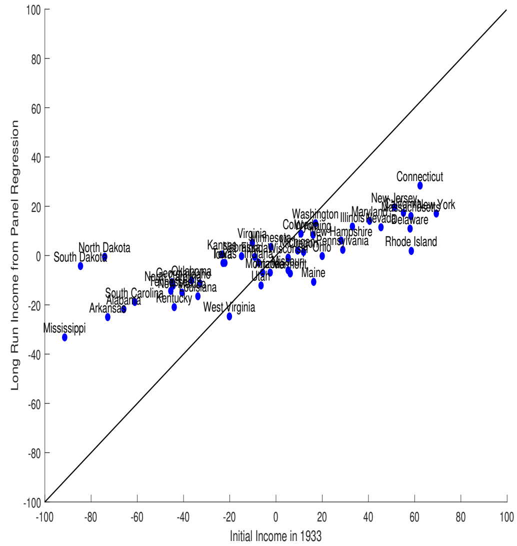
$$\hat{\beta} \quad = \quad V_\beta(N \times (\overline{\Sigma}_{\overline{\beta}})^{-1}\beta^{\text{mean}} + C^{-1}\psi) \quad V_\beta = [N \times (\overline{\Sigma}_{\overline{\beta}})^{-1} + C^{-1}]^{-1}$$

(vii) Draw from $p(\frac{1}{\sigma^2} \mid \Gamma_{-\sigma^2}, Y)$. As in the baseline model $\frac{1}{\sigma^2}$ will have a Gamma distribution will have a Gamma distribution with parameters $\frac{\nu_\sigma + T\times N}{2}$ and $\frac{2}{\nu_\sigma S_\sigma + SSE}$.

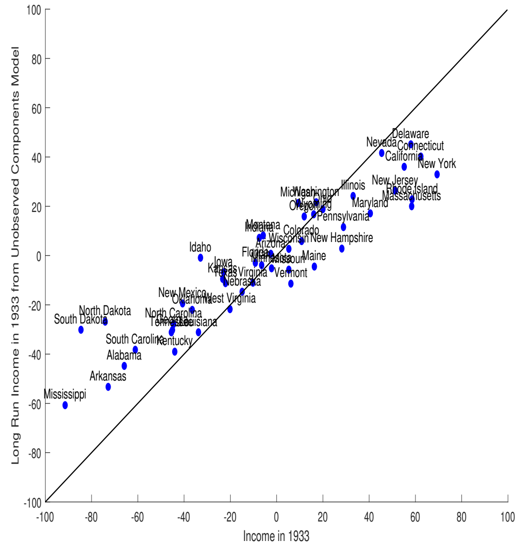
(viii) Draw from $p(\overline{\Sigma}_{\overline{\beta}^2} \mid \Gamma_{-\overline{\Sigma}_{\overline{\beta}^2}}, Y)$. $(\overline{\Sigma}_{\overline{\beta}^2})^{-1}$ will be distributed $\mathcal{W}([\rho R + M]^{-1}, \rho + N))$. where $M = \Sigma_{i=1}^N(\underline{\beta_i} - \underline{\overline{\beta_l}})'(\underline{\beta_i} - \underline{\overline{\beta_l}})$

Appendix B

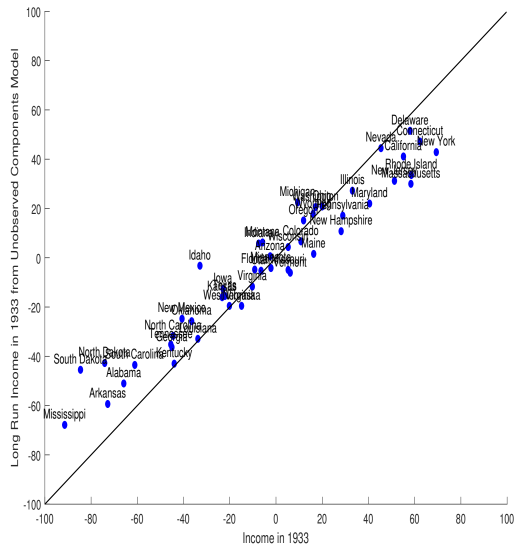
Figure B1: Comparison Between the Models: Initial Income and Long Run Growth in US States



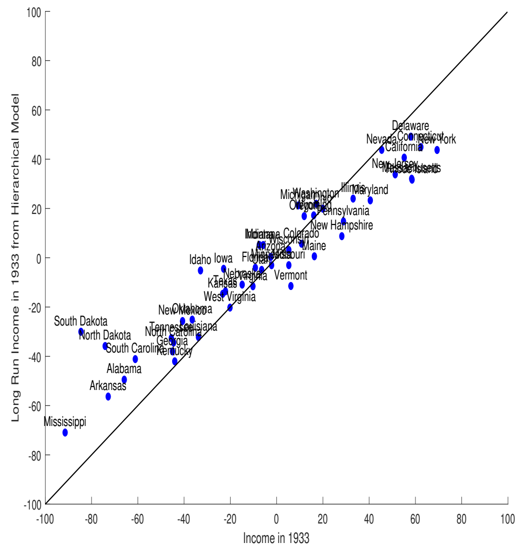
(a) Panel Regression



(b) Unobserved Components Model Tight Priors



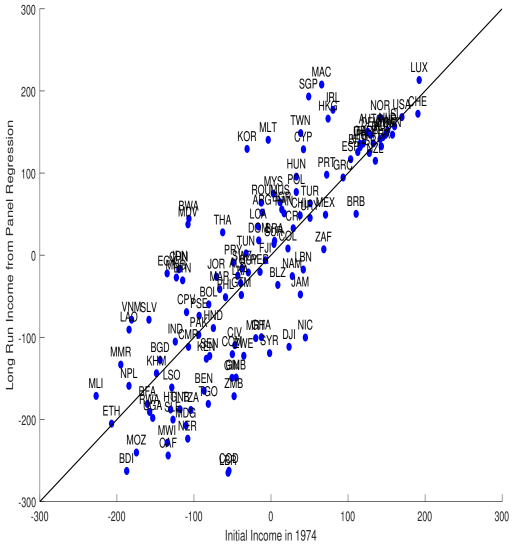
(c) Unobserved Components Model Loose Priors



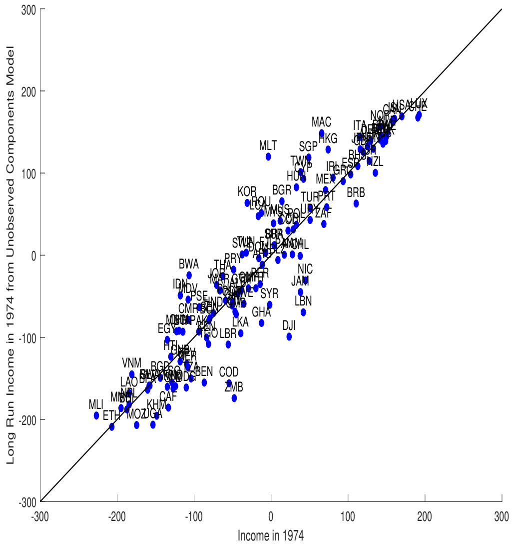
(d) Hierarachical Model Tight Priors

States' initial relative income per capita in 1933 against the estimated long run balanced growth path level of relative income per capita in 1933. Panel a) show the results for a panel regression Model with 4 lags using data from 1929-2021. Panels b) c) and d) shows the results for the Unobserved Components model, with ltight and loose priors and the Hierarchical model with tight priors described in section 1.1, respectively using the same data and the same number of lags.

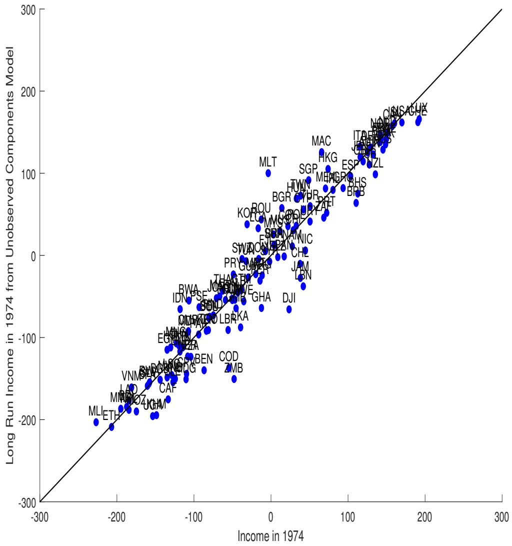
Figure B2: Comparison Between the Models: Initial Income and Long Run Growth in World Economy



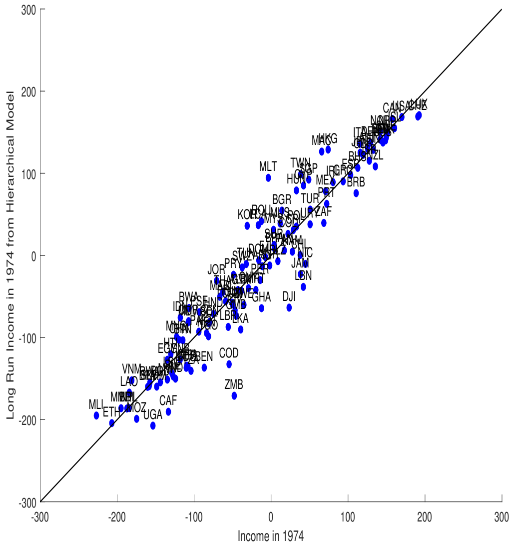
(a) Panel Regression



(b) Unobserved Components Model Tight Priors



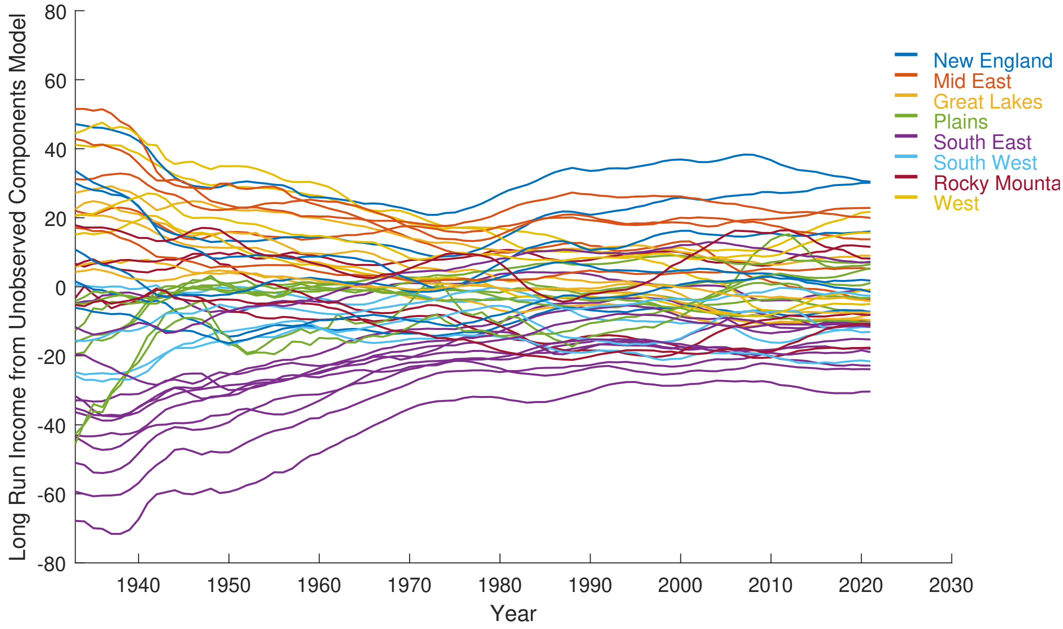
(c) Unobserved Components Model Loose Priors



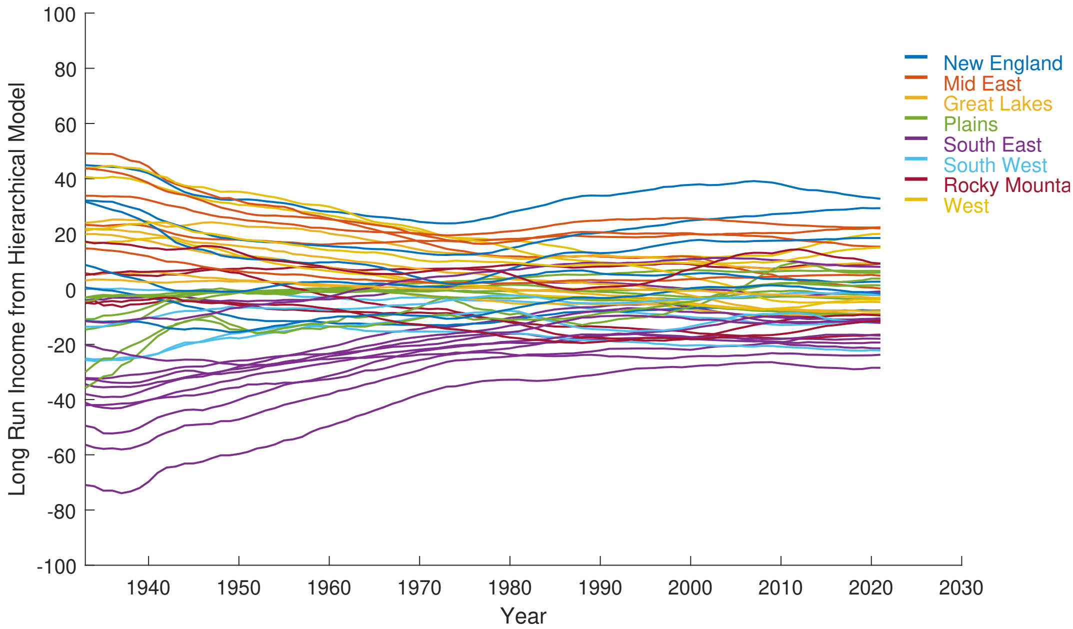
(d) Hierarachical Model Tight Priors

Economies' initial relative GDP per capita in 1974 against the estimated long run balanced growth path level of relative income per capita in 1974. Panel a) show the results for a panel regression with 4 lags using data from 1970-2019. Panels b) c) and d) shows the results for the unobserved components model with tight and loose priors and the hierarchical model with tight priordescribed in section 1.1, respectively using the same data and the same number of lags.

Figure B3: Evolution of the Balanced Growth Path in the Regional Dataset in the Baseline Unobserved Componets Models



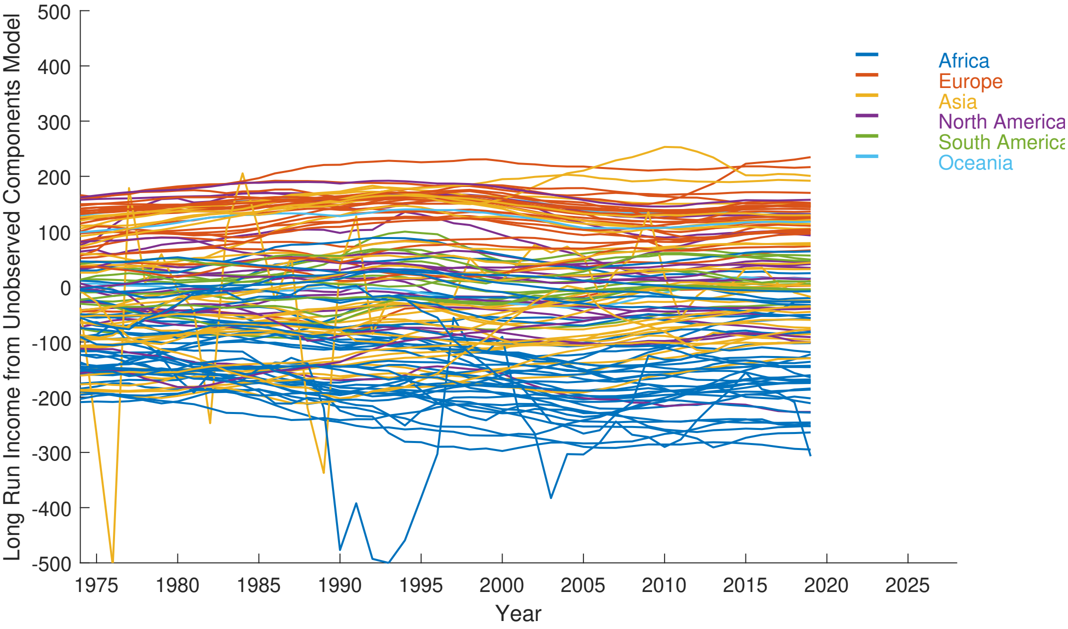
(a) States' Long run Growth paths 1933-2021 from the Baseline Model With Loose Priors



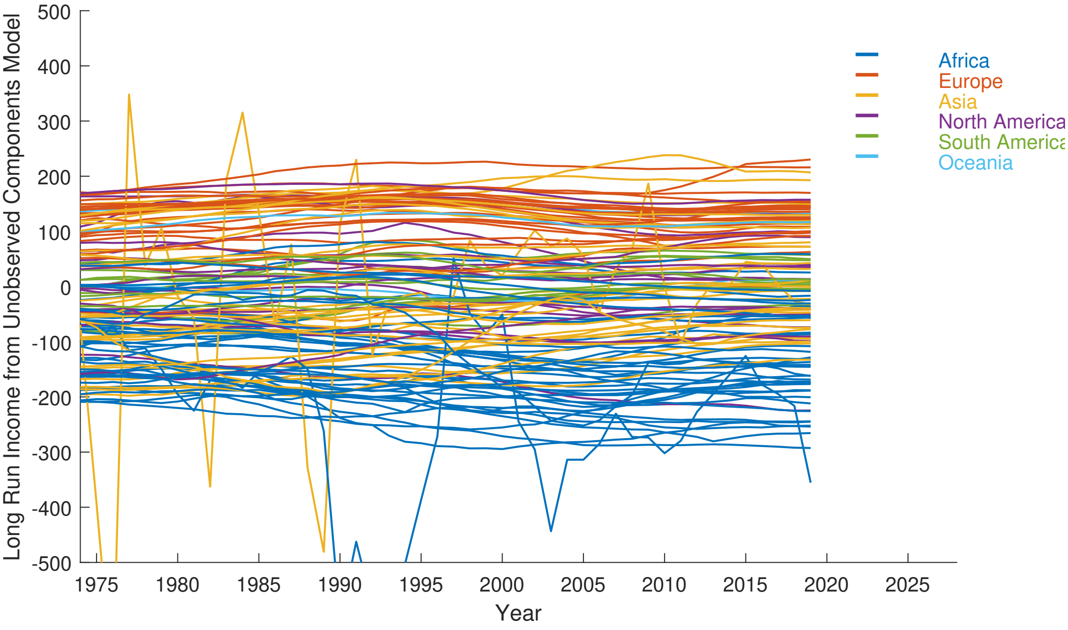
(b) States' Long run Growth paths 1933-2021 from the Baseline Model With Tight Priors

The evolution of the balanced growth path for the 48 contiguous US States from 1933-2021. Panel a) plots the results for the baseline model unobserved components model with loose priors and Panel b) plots the results from the same model with tight priors. Both panels plot the mean posterior value of the long run growth path from the model using 4 lags.

Figure B4: Evolution of the Balanced Growth Path in the World Economy in the Baseline Unobserved Componets Models



(a) Long run Growth in the World Economy 1974-2019 from the Baseline Model With Loose Priors



(b) Long run Growth in the World Economy 1974-2019 from the Baseline Model With Tight Priors

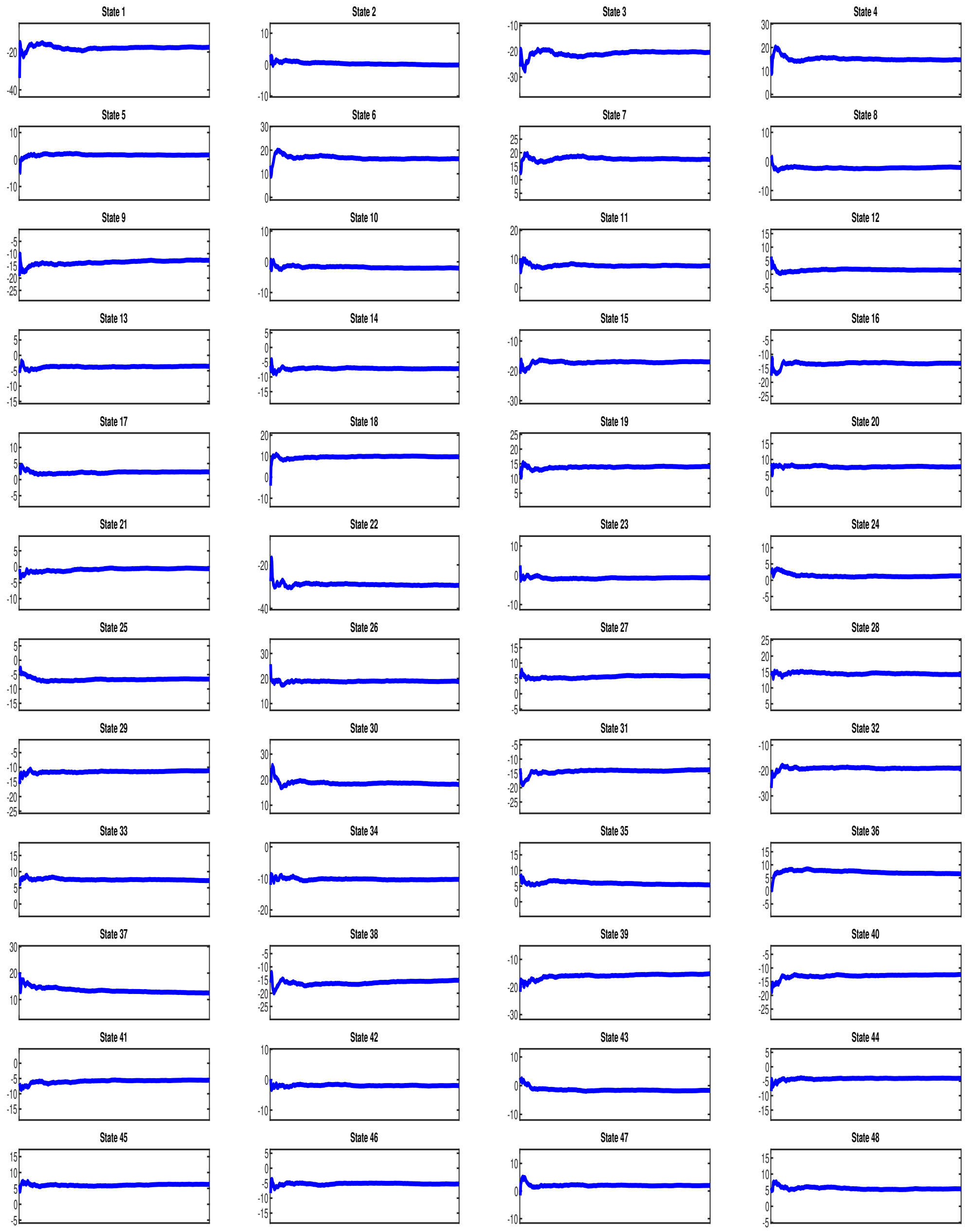
The evolution of the balanced growth path for the world economy from 1974-2019.Panel a) plots the results for the baseline model unobserved components model with loose priors. The highly volatile Asian economy is Lebanon which had a civil war 1975-1990. The highly volatile African economy Liberia which had a civil war 1989-1997. Panel b) plots the results from the same model with tight priors. Both panels plot the mean posterior value of the long run growth path from the model using 4 lags.

Appendix C - (Not Intended for Publication)

Convergence Figures for Gibbs Sampler

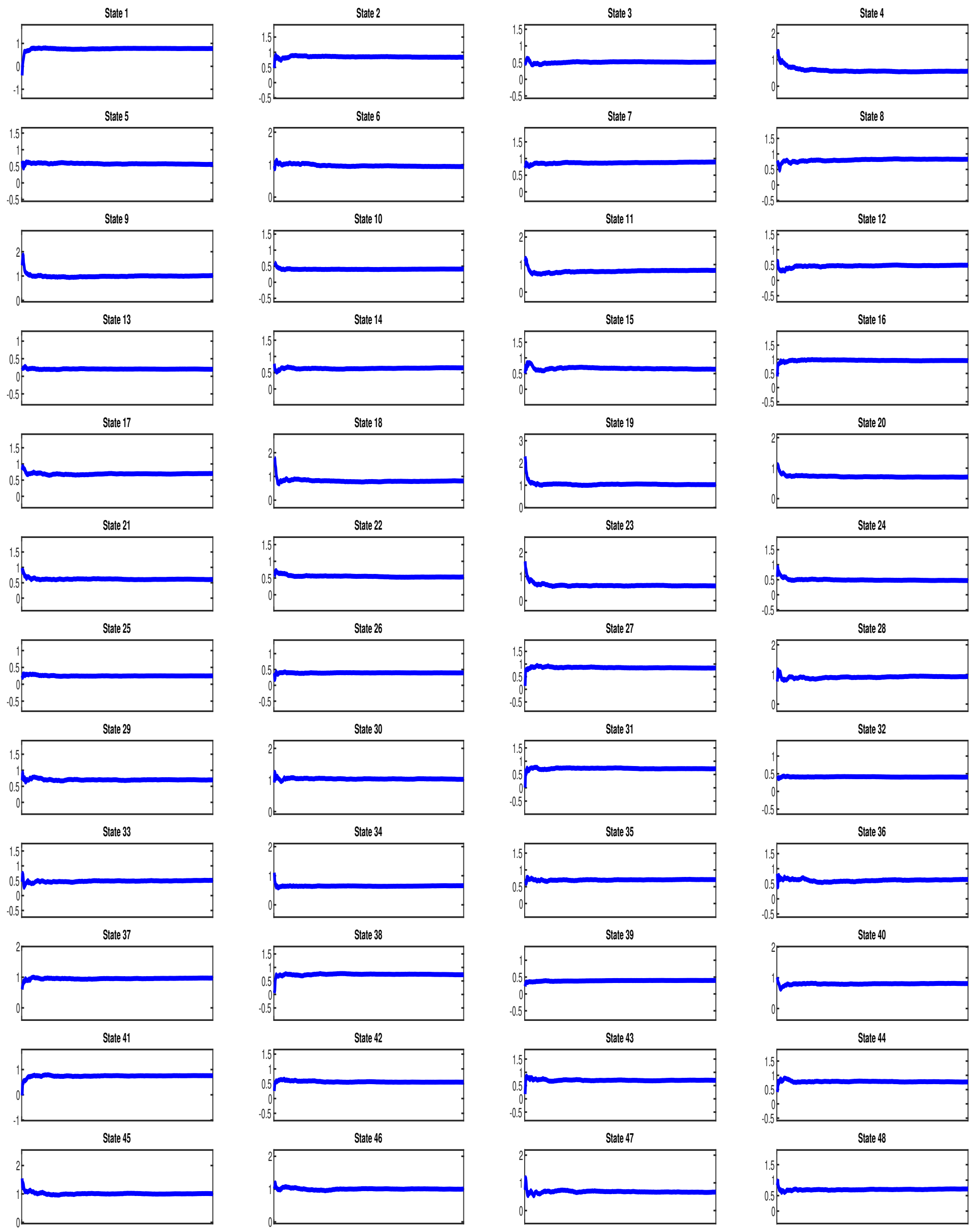
We illustrate the convergence properties of the Gibbs Sampler by plotting the recursive means of two of the coefficients for each economy in the hierarchical model with loose priors using the US States dataset.

Figure C1: Recursive Mean for α_0 for Each State from Gibbs Sampler in Hierarchical Model



The recursive mean of the α_0 coefficient for each state in the estimated hierarchical model using US States dataset.

Figure C2: Recursive Mean for β_1 for Each State from Gibbs Sampler in Hierarchical Model



The recursive mean of the β_1 coefficient for each state in the estimated hierarchical model using US States dataset.