Average Inflation Targeting and Macroeconomic Stability

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Abstract

I study the dynamic consequences of average inflation targeting in a tractable monetary model with sticky prices. I demonstrate that in the case in which the central bank attaches a relatively high weight on the distant past, average inflation targeting not only ensures local determinacy of equilibrium but is also capable of eradicating the liquidity trap problem—differently from standard Taylor rules. Specifically, I show the existence of a saddle connection from the deflationary steady state to the target steady state, along which reflation occurs in equilibrium due to limited and gradual increases in expected nominal interest rates.

JEL Classification: E31; E52.

Keywords: Average Inflation Targeting; Local and Global Dynamics; Liquidity Traps.

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"[I]f inflation runs below 2 percent following economic downturns but never moves above 2 percent even when the economy is strong, then, over time, inflation will average less than 2 percent. Households and businesses will come to expect this result, meaning that inflation expectations would tend to move below our inflation goal and pull realized inflation down. To prevent this outcome and the adverse dynamics that could ensue, our new statement indicates that we will seek to achieve inflation that averages 2 percent over time. Therefore, following periods when inflation has been running below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time."

—Jerome H. Powell, Jackson Hole, August 27, 2020

1 Introduction

The alleged macroeconomic consequences of the adoption of a "flexible form of average inflation targeting" (Powell, 2020) by the Federal Reserve are at the center of the current monetary policy debate. Under the new policy framework, the central bank aims to compensate for undershooting the inflation target by tolerating overshooting paths—the so-called "make-up" strategy (e.g., Bernanke, 2019; Ascari et al., 2020). Using the discipline of a tractable general equilibrium model featured by sluggish price adjustment and the presence of both a demand- and a supply-side channel for the transmission of nominal interest-rate variations, in this paper I present an analytical investigation of local and global dynamics induced by this class of monetary regime. I find that in the case in which the central bank attaches a relatively high weight on the distant past, average inflation targeting not only guarantees local determinacy of equilibrium around the intended steady state but, in addition, is capable of eradicating the liquidity trap problem—unlike standard Taylor rules (Taylor, 1999, 2021; Woodford, 2003; Galí, 2015). Specifically, I show that when the policy parameter governing the exponential moving average of past inflation rates is sufficiently lower than the rate of time preference, there exists a saddle connection from the steady state exhibiting deflation or low inflation to the target steady state, along which reflation is pinned down in equilibrium by relatively limited and gradual increases in expected nominal interest rates.

The central reason why self-fulfilling deflationary paths leading to a liquidity trap are ruled out is as follows. Suppose that average inflation is below the target and that private agents expect further downward pressure on both inflation and output. Because of the Euler equation characterizing the households' optimal intertemporal decision between consumption and saving, the expected fall in output requires in equilibrium that the current real interest rate is below the rate of time preference. Since the nominal interest rate responds to the observed average inflation, the real interest rate is below the rate of time preference only if current inflation rises. From the Phillips curve, the increase in current inflation must be associated with an increase in current output. If the central bank places a sufficiently high weight on the distant past of below-target inflation rates, forward-looking agents expect a relatively low rise in the nominal interest rate, which prevents the expected real interest rate from overshooting the rate of time preference. This will continue to stimulate output and inflation towards the target steady state, thus invalidating the original expectations. Liquidity traps driven by arbitrary revisions in expectations cannot occur.

The present analysis is connected to four strands of literature. First, standard Taylor rules with no appropriate fiscal support (e.g., Mertens and Ravn, 2014; Schmidt, 2016; Bilbiie, 2021, 2022; Nakata and Schmidt, 2022) are found to bring about self-fulfilling liquidity traps, following the seminal studies by Benhabib et al. (2001a, 2002). This paper proves, by contrast, that the monetary strategy of targeting average inflation is potentially able to escape liquidity traps without an accompanying activist fiscal stance.

Second, average inflation targeting is found to increase welfare if compared to ordinary one-period inflation targeting (Nessen and Vestin, 2005; Budianto et al., 2020). This paper adds new insights to this line of research for it elucidates the complementary issue of local and global stability of the intended equilibrium.

Third, in the context of a sticky price model with exogenous labor supply and a production technology depending on real money balances, Benhabib et al. (2000, 2001b) find that sufficiently backward-looking interest rate rules sustain local determinacy under an active monetary stance, i.e., overreacting to average inflation. The present study provides novel insights along two important dimensions. On the one hand, I characterize the local stability properties of average inflation targeting in a more general sticky price environment allowing for endogenous labor supply and a production technology that employs both labor and real balances as factor inputs. In particular, I demonstrate that placing a relatively high weight on the historical path of inflation rates induces local determinacy around the target steady state under an active monetary stance and, at the same time, renders the liquidity-trap steady state unstable. On the other hand, while Benhabib et al. (2000, 2001b) concentrate on local dynamics, the central focus of the present investigation is to analyze the issue of global dynamics and explore the case for potential connections among different steady-state equilibria.

Forth, in the context of a New Keynesian model in which agents adopt adaptive learning rules, Honkapohja and McClung (2021) find that when policy details about are not publicly available, average inflation targeting is likely to cause local instability around the target steady state and, under plausible calibrations, typically fails to initiate an escape from a liquidity trap. They show, however, that targeting a discounted average of inflation can render the target steady state stable, while communicating the data window for the target enables the economy to escape from the zero-lower-bound regime, although the latter result is sensitive to the speed of learning. Conversely, the present study is intended to characterize analytically the complementary issue of macroeconomic dynamics induced by average inflation targeting under forward-looking optimizing agents with rational expectations. In this environment directly comparable with standard monetary theory, the paper's central contribution is to derive the conditions under which an exponential moving average policy rule stabilizes the economy both locally and globally.

The paper proceeds in three sections. In Section 2, I set up the model. In Section 3, I scrutinize equilibrium dynamics and establish the main results. In Section 4, I conclude with final remarks.

2 The Model

In this section, I set forth a continuous-time monetary model with monopolistic competition and sticky prices that reveals to be convenient in order to analyze transparently local and global equilibrium dynamics induced by average inflation targeting.

The economy consists of a continuum of household-firms indexed by j. In line with Rotemberg (1982), the unit j's lifetime utility function is of the form

$$U^j(0) = \int_0^\infty e^{-rt} \left[\frac{c^j(t)^{1-\sigma}}{1-\sigma} - \frac{h^j(t)^{1+\varphi}}{1+\varphi} - \frac{\gamma}{2} \left(\frac{\dot{P}^j(t)}{P^j(t)} - \pi^* \right)^2 \right] dt, \tag{1}$$

where σ, φ and γ are positive parameters, r denotes the rate of time preference, $c^j(t)$ consumption, $h^j(t)$ labor supply, $P^j(t)$ the price that the household-firm j sets for the differentiated good it sells, and π^* the steady state inflation rate that the monetary authority targets.

Money facilitates the production process, following Benhabib et al. (2001b). Hence, I shall additionally take into account the supply-side channel of monetary policy transmission. Specifically, the household-firm uses a technology for the production of good j given by

$$y^j(t) = h^j(t)^\alpha \, m^j(t)^{1-\alpha}, \tag{2}$$

where $0 < \alpha < 1$, and faces a demand function derived from Dixit-Stiglitz preferences, of the form

$$y^j(t) = \Psi\left(\frac{P^j(t)}{P(t)} \right) Y^d(t), \tag{3}$$

where $Y^d(t)$ denotes aggregate demand, $P^j(t)$ the product j's price, $P(t)$ the price level, and $\Psi(\cdot)$ obeys $\Psi' < 0$, $\Psi(1) = 1$ and $\Psi'(1) < -1$. The real value of financial wealth $a^j(t)$, consisting of interest-bearing government bonds and cash balances, evolves according to

$$\dot{a}^j(t) = (R(t) - \pi(t))\, a^j(t) + \frac{P^j(t)}{P(t)} h^j(t)^\alpha \, m^j(t)^{1-\alpha} - c^j(t) - R(t)\, m^j(t) - \tau(t), \tag{4}$$

where $R(t)$ denotes the nominal interest rate, $\pi(t) \equiv \dot{P}(t)/P(t)$ the inflation rate, and $\tau(t)$ real lump-sum taxes net of transfers. Ponzi's games are precluded, implying

$$\lim_{t\to\infty} e^{-\int_0^t (R(v)-\pi(v))dv} a^j(t) \geq 0. \tag{5}$$

Let $\lambda^j(t)$ be the co-state variable and $\mu^j(t)$ be the multiplier associated with the constraint that output is demand-determined. Optimality yields

$$c^j(t)^{-\sigma} = \lambda^j(t), \tag{6}$$

$$h^j(t)^{\varphi} = \left(\lambda^j(t)\frac{P^j(t)}{P(t)} - \mu^j(t)\right)\alpha h^j(t)^{\alpha-1} m^j(t)^{1-\alpha}, \tag{7}$$

$$\lambda^j(t) R(t) = \left(\lambda^j(t)\frac{P^j(t)}{P(t)} - \mu^j(t)\right)(1-\alpha) h^j(t)^{\alpha} m^j(t)^{-\alpha}, \tag{8}$$

$$\dot{\lambda}^j(t) = (r + \pi(t) - R(t))\lambda^j(t), \tag{9}$$

$$\begin{aligned}\dot{\pi}^j(t) &= r\left(\pi^j(t) - \pi^*\right) \\ &\quad -\frac{1}{\gamma}\left(\lambda^j(t)\frac{P^j(t)}{P(t)}h^j(t)^{\alpha} m^j(t)^{1-\alpha} + \mu^j(t)\frac{P^j(t)}{P(t)}\Psi'\left(\frac{P^j(t)}{P(t)}\right)Y^d(t)\right),\end{aligned} \tag{10}$$

$$\lim_{t\to\infty} e^{-\int_0^t (R(v)-\pi(v))dv} a^j(t) = 0, \tag{11}$$

where $\dot{\pi}^j(t) \equiv \dot{P}^j(t)/P^j(t)$.

Next, consider the public sector. On the grounds of my primary focus, I shall assume that the central bank adjusts the nominal interest rate according to nonlinear feedback rule of the form

$$R(t) = \Phi\left(\pi^A(t)\right), \tag{12}$$

where $\Phi(\cdot)$ is a continuous, positive, increasing and strictly convex function, and $\pi^A(t)$ is a weighted average of past inflation rates,

$$\pi^A(t) = \omega\int_{-\infty}^{t} e^{-\omega(t-v)}\pi(v)dv, \tag{13}$$

consistently with Budianto et al. (2020), where ω is a positive parameter. Furthermore, at the target level of steady-state inflation, I shall assume that monetary policy is "active" (Leeper, 1991), that is, $\Phi'(\pi^*) > 1$.

Budget deficits are financed by issuance of nominal bonds and money. Setting public consumption to zero, without loss of generality, the government's instantaneous budget constraint in real terms is thus given by

$$\dot{a}\left(t\right)=\left(R\left(t\right)-\pi\left(t\right)\right)a\left(t\right)-R\left(t\right)m\left(t\right)-\tau\left(t\right). \tag{14}$$

To concentrate on the dynamic implications of average inflation targeting, I restrict attention to the case in which fiscal policy is Ricardian, ensuring that the present discounted value of government liabilities converges to zero, so that equation (11) is verified, for all possible time paths of the remaining endogenous variables. To this end, the government is assumed to adjust net tax revenues inclusive of interest savings from the issuance of money according to a linear feedback rule of the form

$$\tau\left(t\right)+R\left(t\right)m\left(t\right)=\beta\left(t\right)a\left(t\right), \tag{15}$$

where $\beta\left(t\right)$ is an arbitrarily chosen positive function.

3 Local and Global Equilibrium Dynamics

Imposing the goods' market clearing condition along with equilibrium symmetry, equations (6)-(10), (12) and (13) yield a nonlinear dynamic system for output, inflation and average inflation given by

$$\dot{y}\left(t\right)=\frac{1}{\sigma}\left(\Phi\left(\pi^{A}\left(t\right)\right)-\pi(t)-r\right)y\left(t\right), \tag{16}$$

$$\begin{aligned}\dot{\pi}\left(t\right) &= r\left(\pi\left(t\right)-\pi^{*}\right)+\frac{\varepsilon-1}{\gamma y^{j}\left(t\right)^{\sigma-1}}\\ &\quad -\frac{\varepsilon}{\alpha\gamma}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} y^{j}\left(t\right)^{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\,\Phi\left(\pi^{A}\left(t\right)\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}},\end{aligned} \tag{17}$$

$$\dot{\pi}^{A}\left(t\right)=\omega\left(\pi(t)-\pi^{A}\left(t\right)\right), \tag{18}$$

where $\varepsilon\equiv-\Psi'\left(1\right)>1$.

The steady states of the economy are obtained setting $\dot{y}(t)=0$, $\dot{\pi}(t)=0$ and $\dot{\pi}^A(t)=0$ in (16)-(18), and thus solve

$$\Phi\left(\pi^A\right)=r+\pi, \tag{19}$$

$$r\left(\pi-\pi^*\right)=\frac{\varepsilon}{\alpha\gamma}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} y^{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\,\Phi\left(\pi^A\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}-\frac{\varepsilon-1}{\gamma y^{\sigma-1}}, \tag{20}$$

$$\pi=\pi^A. \tag{21}$$

Substituting (21) into the Fisher equation (19) yields $\Phi(\pi)=r+\pi$. Because the monetary policy reaction function $\Phi(\cdot)$ is positive—respecting the zero lower bound on nominal interest rates—increasing and strictly convex, the Fisher relation has two solutions, π^* and π^L. Since π^* is the target inflation rate at which monetary policy is active, $\Phi'(\pi^*)>1$, the alternative steady-state value π^L must obey $\pi^L<\pi^*$, is possibly negative, and necessarily features a "passive" monetary policy stance, $\Phi'\left(\pi^L\right)<1$. Using (20), the steady-state level of output associated to π^* is

$$y^*=\left[\frac{\alpha(\varepsilon-1)}{\varepsilon}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\Phi\left(\pi^*\right)^{-\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\right]^{\frac{1}{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}+(\sigma-1)}}, \tag{22}$$

while the steady-state level of output associated to π^L, y^L, is the solution to

$$\frac{\varepsilon-1}{\gamma y^{\sigma-1}}=\frac{\varepsilon}{\alpha\gamma}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} y^{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\,\Phi\left(\pi^L\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}+r\left(\pi^*-\pi^L\right). \tag{23}$$

I shall assume $1\leq\sigma\leq 1/(1-\alpha)$ in order to obtain uniqueness of y^L from the latter equation combined with a positively-slopped aggregate supply from equation (17).

Explore next local equilibrium dynamics. Linearizing the system (16)-(18) in the neighborhood of any steady-state point (y,π,π) gives

$$\begin{pmatrix}\dot{y}(t)\\ \dot{\pi}(t)\\ \dot{\pi}^A(t)\end{pmatrix}=J^{(y,\pi,\pi)}\begin{pmatrix}y(t)-y\\ \pi(t)-\pi\\ \pi^A(t)-\pi\end{pmatrix}, \tag{24}$$

where

$$J^{(y,\pi,\pi)} = \begin{pmatrix} 0 & -\frac{y}{\sigma} & \frac{y\Phi'(\pi)}{\sigma} \\ J_{21}^{(y,\pi,\pi)} & r & J_{23}^{(y,\pi,\pi)} \\ 0 & \omega & -\omega \end{pmatrix}, \tag{25}$$

with

$$\begin{aligned} J_{21}^{(y,\pi,\pi)} &= -\frac{(\varepsilon-1)(\sigma-1)}{\gamma} y^{-\sigma} \\ &\quad -\frac{\varepsilon\left[1-(1-\alpha)\sigma\right](1+\varphi)}{\alpha\gamma\left[(1-\alpha)(1+\varphi)+\alpha\right]}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} y^{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}-1}\Phi(\pi)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} \\ &< 0, \end{aligned}$$

$$\begin{aligned} J_{23}^{(y,\pi,\pi)} &= -\frac{\varepsilon(1-\alpha)(1+\varphi)}{\alpha\gamma\left[(1-\alpha)(1+\varphi)+\alpha\right]}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}} y^{\frac{[1-(1-\alpha)\sigma](1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}}\Phi(\pi)^{\frac{(1-\alpha)(1+\varphi)}{(1-\alpha)(1+\varphi)+\alpha}-1} \\ &< 0. \end{aligned}$$

The determinant and the trace of the Jacobian matrix $J^{(y,\pi,\pi)}$ are

$$\mathrm{Det}J^{(y,\pi,\pi)} = \frac{J_{21}^{(y,\pi,\pi)}\omega y}{\sigma}\left(\Phi'(\pi)-1\right), \tag{26}$$

$$\mathrm{Tr}J^{(y,\pi,\pi)} = r-\omega. \tag{27}$$

Observe that $\mathrm{Det}J^{(y^*,\pi^*,\pi^*)} < 0$, because $\Phi'(\pi^*) > 1$, and at the same time the number of variations of sign in the scheme (Gantmacher, 1960)

$$-1 \quad \mathrm{Tr}J^{(y^*,\pi^*,\pi^*)} \qquad -K^{(y^*,\pi^*,\pi^*)} + \frac{\mathrm{Det}J^{(y^*,\pi^*,\pi^*)}}{\mathrm{Tr}J^{(y^*,\pi^*,\pi^*)}} \quad \mathrm{Det}J^{(y^*,\pi^*,\pi^*)}, \tag{28}$$

where $K^{(y^*,\pi^*,\pi^*)}$ is the sum of the principal minors of $J^{(y^*,\pi^*,\pi^*)}$ given by $J_{21}^{(y^*,\pi^*,\pi^*)}y^*/\sigma - \omega \times \left(r + J_{23}^{(y^*,\pi^*,\pi^*)}\right)$, is higher than zero if either (Case I) $\mathrm{Tr}J^{(y^*,\pi^*,\pi^*)} > 0$ or (Case II) $-K^{(y^*,\pi^*,\pi^*)} + \mathrm{Det}J^{(y^*,\pi^*,\pi^*)}/\mathrm{Tr}J^{(y^*,\pi^*,\pi^*)} > 0$ when $\mathrm{Tr}J^{(y^*,\pi^*,\pi^*)} < 0$. Case I applies when the central bank attaches a relatively high weight on the distant past, such that the policy parameter governing the exponential moving average of past inflation rates is below the rate of time preference, $\omega < r$. If $-J_{23}^{(y^*,\pi^*,\pi^*)} > r$, setting

$$\Delta^{(y^*,\pi^*,\pi^*)} = \left[r\left(r + J_{23}^{(y^*,\pi^*,\pi^*)}\right) + \Phi'\left(\pi^*\right) J_{21}^{(y^*,\pi^*,\pi^*)} y^*/\sigma\right]^2 \\ - 4\left(r + J_{23}^{(y^*,\pi^*,\pi^*)}\right)\left(J_{21}^{(y^*,\pi^*,\pi^*)} r y^*/\sigma\right) > 0,$$

Case II also applies under a sufficiently history-dependent stance of monetary policy, such that

$$\omega < \frac{-\left[r\left(r + J_{23}^{(y^*,\pi^*,\pi^*)}\right) + \Phi'\left(\pi^*\right) J_{21}^{(y^*,\pi^*,\pi^*)} y^*/\sigma\right] + \sqrt{\Delta^{(y^*,\pi^*,\pi^*)}}}{-2\left(r + J_{23}^{(y^*,\pi^*,\pi^*)}\right)}. \tag{29}$$

In these circumstances, $J^{(y^*,\pi^*,\pi^*)}$ has necessarily two eigenvalues with positive real parts and one eigenvalue with a negative real part. Since $y(t)$ and $\pi(t)$ are jump variables with free initial conditions and $\pi^A(t)$ is a state variable,[1] according to the Blanchard-Kahn conditions this ensures that local determinacy of equilibrium applies in the neighborhood of the target steady state.[2] In particular, the unique trajectory of $\left(y(t), \pi(t), \pi^A(t)\right)$ converging asymptotically to (y^*, π^*, π^*) is given by the following saddle-path solution:

$$y(t) = y^* + \frac{(\zeta_1 - r) - \frac{\omega J_{23}^{(y^*,\pi^*,\pi^*)}}{\zeta_1 + \omega}}{J_{21}^{(y^*,\pi^*,\pi^*)}}\left(\pi(t) - \pi^*\right), \tag{30}$$

$$\pi(t) = \pi^* + \frac{\zeta_1 + \omega}{\omega}\left(\pi^A(t) - \pi^*\right), \tag{31}$$

$$\pi^A(t) = \pi^* + \left(\pi^A(0) - \pi^*\right) e^{\zeta_1 t}, \tag{32}$$

where ζ_1 is the negative root of $J^{(y^*,\pi^*,\pi^*)}$.

On the other hand, observe that $\mathrm{Det} J^{\left(y^L,\pi^L,\pi^L\right)} < 0$, because $\Phi'\left(\pi^L\right) < 1$. When $\mathrm{Tr} J^{\left(y^L,\pi^L,\pi^L\right)}$

[1] Average inflation must be counted as a state variable of the system (24) because its value cannot jump independently of the current inflation rate.

[2] It is worth observing that in the limiting case of price level targeting, whereby $\pi^* = 0$ and $\omega = 0$—so that the central bank aims at stabilizing the price level and the resulting averaging window is 'infinitely long' (see Budianto et al., 2020), Case I does apply, implying that local determinacy around the target steady state is guaranteed.

< 0, $J^{\left(y^L,\pi^L,\pi^L\right)}$ has necessarily one eigenvalue with a positive real part and two eigenvalues with negative real parts. In this case, there exists a continuum of trajectories of $\left(y\left(t\right),\pi\left(t\right),\pi^A\left(t\right)\right)$ converging asymptotically to $\left(y^L,\pi^L,\pi^L\right)$, so that equilibrium indeterminacy prevails. By contrast, and importantly for what will follow, when $\mathrm{Tr}J^{\left(y^L,\pi^L,\pi^L\right)} > 0$ and $-K^{\left(y^L,\pi^L,\pi^L\right)} + \mathrm{Det}J^{\left(y^L,\pi^L,\pi^L\right)}/\mathrm{Tr}J^{\left(y^L,\pi^L,\pi^L\right)} < 0$, $J^{\left(y^L,\pi^L,\pi^L\right)}$ has three eigenvalues with positive real parts. In this case, the unintended steady state $\left(y^L,\pi^L,\pi^L\right)$ becomes unstable. In particular, if $-J_{23}^{\left(y^L,\pi^L,\pi^L\right)} > r$ and $\Delta^{\left(y^L,\pi^L,\pi^L\right)} > 0$, instability applies under a monetary policy stance placing a relatively high weight on the distant past, such that ω is sufficiently lower than the rate of time preference:

$$\begin{aligned}
&\frac{-\left[r\left(r+J_{23}^{\left(y^L,\pi^L,\pi^L\right)}\right)+\Phi'\left(\pi^*\right)J_{21}^{\left(y^L,\pi^L,\pi^L\right)}y^*/\sigma\right]-\sqrt{\Delta^{\left(y^L,\pi^L,\pi^L\right)}}}{-2\left(r+J_{23}^{\left(y^L,\pi^L,\pi^L\right)}\right)} \\
< \ \ \omega < \ & \frac{-\left[r\left(r+J_{23}^{\left(y^L,\pi^L,\pi^L\right)}\right)+\Phi'\left(\pi^*\right)J_{21}^{\left(y^L,\pi^L,\pi^L\right)}y^*/\sigma\right]+\sqrt{\Delta^{\left(y^L,\pi^L,\pi^L\right)}}}{-2\left(r+J_{23}^{\left(y^L,\pi^L,\pi^L\right)}\right)}\left(< r\right). \qquad (33)
\end{aligned}$$

The two assumptions $-J_{23}^{\left(y^L,\pi^L,\pi^L\right)} > r$ and $\Delta^{\left(y^L,\pi^L,\pi^L\right)} > 0$ appear largely satisfied for empirically plausible model's parameterizations.[3]

The main consequence for global dynamics is that average inflation targeting is capable of removing the liquidity trap problem—that is, the typical existence under standard interest-rate feedback policies of a continuum of self-fulfilling decelerating inflation paths originating arbitrar-

[3]In particular, assuming that the time unit is a quarter, the baseline calibration sets $r = 0.04/4$, $\pi^* = 0.02/4$, $R\left(\pi^L\right) = 0.001/4$, $R'\left(\pi^L\right) = 0.1$, $1-\alpha = 0.1$, and, consistently with Benhabib et al. (2001a), $1/\sigma = 1/2$, $\varphi = 1$, $\varepsilon = 21$, implying a markup of prices over marginal costs of 5%, and $\gamma = -17.5\left(1-\varepsilon\right) = 350$. Under the baseline calibration, the assumptions $-J_{23}^{\left(y^L,\pi^L,\pi^L\right)} > r$ and $\Delta^{\left(y^L,\pi^L,\pi^L\right)} > 0$ are widely verified, and robustly continue to apply if: (a) r, the discount rate, is decreased from 0.04/4 to a value in the range $[0.03/4, 0.04/4)$; (b) $R'\left(\pi^L\right)$, the monetary policy feedback reaction to inflation at the liquidity-trap steady state, is decreased from 0.1 to a value in the range $[0.01, 0.1)$; (c) $1-\alpha$, the elasticity of output with respect to firms' real cash balances, is increased from 0.1 to a value in the range $(0.1, 0.3]$; (d) $1/\sigma$, the intertemporal elasticity of substitution, is decreased from 1/2 to a value in the range $[1/4, 1/2)$, consistently with Woodford (2003); (e) φ, the inverse of the Frisch elasticity, is decreased from 1 to a value in the range $[1/2, 1)$, consistently with Woodford (2003); (f) ε, the equilibrium price elasticity, is decreased from 21 to a value in the range $[6, 21)$, implying a markup of prices over marginal costs in the range $(5\%, 20\%]$, as in Woodford (2003) and Galí (2015); (g) γ, the parameter capturing the cost of deviating from the inflation target, is decreased from 350 to a value in the range $[35, 350)$.

ily close to π^* and converging to π^L. Consider indeed the case of values of ω satisfying condition (33), which implies that the steady state (y^*, π^*, π^*) is saddle-path stable and at the same time the steady state $\left(y^L, \pi^L, \pi^L\right)$ is unstable.[4] The resulting global behavior of equilibrium inflation is characterized in Figure 1. Panel (a) shows the case in which the roots associated to $J^{\left(y^L,\pi^L,\pi^L\right)}$

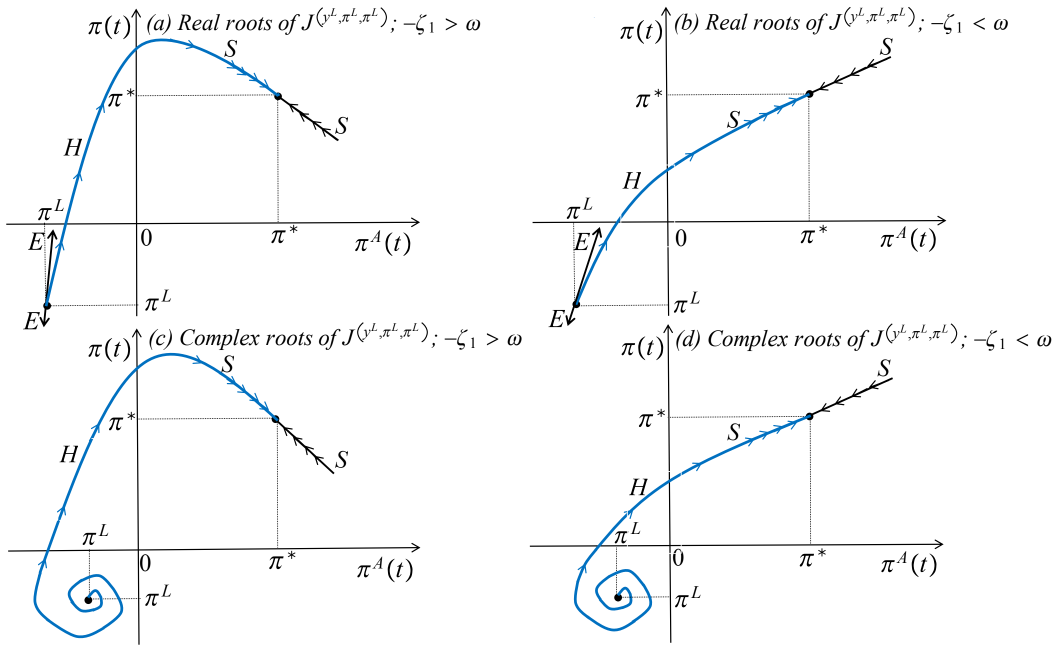

Figure 1: Global Equilibrium Dynamics

are real and $-\zeta_1 > \omega$. The stable arm of the saddle point passing through (y^*, π^*, π^*), SS, has a negative slope, given by $(\zeta_1 + \omega)/\omega < 0$. Since the steady state $\left(y^L, \pi^L, \pi^L\right)$ is an unstable node, there exists one trajectory—the heteroclinic orbit H—originating in the neighborhood of the steady state $\left(y^L, \pi^L, \pi^L\right)$, positively slopped around $\left(y^L, \pi^L, \pi^L\right)$ because tangent to the positively slopped eigenspace EE related to the nondominant eigenvalue of $J^{\left(y^L,\pi^L,\pi^L\right)}$, η_1—expressed by $\pi(t) = \pi^L + [(\eta_1 + \omega)/\omega]\left(\pi^A(t) - \pi^L\right)$—and converging asymptotically to the steady state (y^*, π^*, π^*), locally along the associated saddle path whose stable arm is given by equation (31). Along the heteroclinic orbit H, reflation takes place in equilibrium due to

[4] It is worth pointing out in the extreme case of price level targeting ($\omega = 0$), condition (33) is not verified, implying that instability cannot occur around the unintended steady state. Locally, multiple stable solutions exist. Globally, price level targeting does not eliminate as a result the existence of a continuum of self-fulfilling deflationary trajectories making the economy slide from the neighborhood of the target steady state to the unintended steady state—which acts as a trapping equilibrium.

relatively limited and gradual increases in expected nominal interest rates, which are linked to expected average inflation rates. By contrast, expectations of deflationary slumps are not sustained as equilibrium outcomes. Under such circumstances, the Euler equation requires reductions in the real interest rate, which under a sufficiently history-dependent monetary stance only occur if equilibrium inflation rises, thereby precluding the self-fulfillment of the original expectations. In particular, equilibrium inflation exhibits a nonmonotonic behavior, overshooting at some point the target level π^*. As shown in panel (b), overshoothing paths disappear if $-\zeta_1 < \omega$, for in this case the slope of the stable arm SS becomes positive. On the other hand, panels (c)-(d) show the case of complex roots associated to $J^{\left(y^L,\pi^L,\pi^L\right)}$, which imply that $\left(y^L,\pi^L,\pi^L\right)$ is an unstable spiral point. Even though here the inflation rate may fluctuate for relatively long periods of time in a region whereby monetary policy is necessarily passive due to the effective lower bound on the nominal interest rate, liquidity traps continue to be ruled out by the existence of the saddle connection H.

4 Conclusions

Does average inflation targeting possess stabilizing properties from a both a local- and a global-dynamics perspective? The analysis conducted here, based upon a tractable general equilibrium framework with sticky prices, suggests that the answer to this question is "yes, it does".

First, average inflation targeting induces local stability and uniqueness of equilibrium should the central bank be expected to place a sufficiently high weight on the historical path of inflation rates. Second, differently from conventional Taylor rules, average inflation targeting gives rise to a heteroclininc trajectory connecting the deflationary steady state with the intended steady state when the monetary authority is expected to set the policy parameter governing the exponential moving average of past inflation rates to a level sufficiently lower than the rate of time preference. Under such a monetary strategy, the economy turns to be insulated from the occurrence of expectations-driven liquidity traps. Intuitively, this is because expected demand-deficient deflationary slumps are not supported as equilibrium outcomes, for the associated falls in the real interest rate via the Euler equation in conjunction with a sufficiently inertial monetary rule require instead increases in equilibrium inflation.

Taken together, the foregoing results imply that the occurred Federal Reserve's switching from ordinary one-period inflation targeting to average inflation targeting is likely to strengthen the case for aggregate stability.

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