What Are the Impacts of Credit Crunch on the Bank-Enterprise System? An Analysis Through Dynamic Modeling and an Italian Dataset

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What are the impacts of credit crunch on the bank-enterprise system? An analysis through dynamic modeling and an Italian dataset

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Abstract

There is an intrinsic and mutualistic dependence between the bio-economic performance of banks and that of enterprises. This supposition is supported by correlations identified in a comprehensive analysis of the Italian banking sector, which reveal particularly strong relations between financial intermediaries and smaller enterprises.

Concentrating on developments within the bank-enterprise system (and by extension, in households), we discuss the positive effects, including on macroeconomics, generated when the banking sector supplies funding to productive infrastructure to understand how the industry remains healthy and efficient. The negative effects produced by the disappearance of such a cycle are also considered. This paper thus presents a mathematical argument through dynamic modelling to evaluate the structural trends in bank and company populations that result from more and less expansive credit strategies assumed by banks.

Empirical observations of this data also reflect the critical stress factor of the (micro)enterprise population that allows it to generate positive economic variations as financial leverage decreases. The ensuing assessment of stable and unstable points of equilibrium as well as bifurcations and their irreversibility (hysteresis) reveals that banks have stagnating profits and increasing numbers of non-performing loans.

Finally, we investigate the possibility of an optimal minimum level of credit leverage and how to improve the stabilizing measures that are conferred to the system itself, especially given the uncertainty caused by the COVID-19 pandemic.

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JEL CODES: C61, C62, E32, E44, G21

1. Introduction

Banks, especially commercial banks – that is, institutions that cannot, due to regulatory restrictions or limitations, engage in business other than financial intermediation – are acutely dependent on the performance of their customers. It is well-established that banks implement lending strategies based on the criteria of commerce and profit: loans are represented by risk portfolios, the overall weighting of which is derived from the aggregate of segmented units, which are themselves subject to precise individual evaluation. Studying the systematization of these two criteria and its effects, this paper determines whether an ideal level, or quantitative *optimum*, of total bank loans disbursed to companies can be defined. With the reasoning that each bank has an average portfolio of clients, all n client positions comprising this portfolio [5] then have the same PD – Probability of Default, LGD – Loss Given Default and EAD – Exposure at Default. The TEL_n – total expected losses, represented as costs in the bank's income statement, are therefore[3]:

$$TEL_n = n * PD * EAD * LGD \qquad (1)$$

Unexpected losses are then added. Representing a capital constraint for banks, the extent of these losses determines the level of caution that banks will increasingly adopt in the disbursement of loans.

Applying the Vasicek model and taking a time $t = 1$ year:

$$TL_n = \sum_{i=1}^{n} U_i LGD_i EAD_i \qquad (2)$$

Where:

TL_n is the total loss on the portfolio over time t,

LGD_i and EAD_i are the LGD and EAD of the i-th company,

U_i is the Boolean indicator, which takes on a value of 1 if the i-th enterprise defaults within time t, or the value 0 if the i-th enterprise continues to perform in the same period. This indicator is also a function of x, which represents both a Gaussian variable in the macroeconomic context and another scenario in which enterprises in the portfolio operate.

$$x_i = \upsilon K + \tau e_i \qquad (3)$$

Here, K is limited to the macroeconomic aspect of the random variable, while e_i indicates the random idiosyncratic factor of the individual counterparty. υ and τ are the respective multipliers, one of which is often set *a priori*. It should be noted, however, that a certain component of e_i is dependent on K. Therefore:

[3] Losses in the portfolio measured *ex post* with minimal deviations.

$$TL_n = \sum_{i=1}^{n} U_i(K, e_i) LGD_i EAD_i \qquad (4)$$

$U_i(K, e_i)$ replaces PD in the formula to indicate the likelihood that enterprise i will default based on its idiosyncratic factor e_i and on the conditions of its macroeconomic context.[4] Since LGD_i and EAD_i are assumed to be known *a priori*, the e_i and K of banks and enterprises should be maximized.[5] Insofar as these components relate to the bank-enterprise relationship within the economic system, banks need to minimize losses and reduce capital absorption, and enterprises need to obtain credit and do *business* sustainably and profitably.

K is, of course, comprised of even more macroscopic components, including but not limited to monetary policy, international economic influences, and the credibility and trust of the country. As such, in focusing our attention on the dynamics of the bank-enterprise system (and, in turn, households), we address the positive influence, including on the macroeconomic setting, generated by an efficient banking sector that transmits liquidity to the productive complex, allowing it to remain healthy and performing (and, under contrasting conditions, the negative effects produced by the disappearance of this virtuous cycle).[6]

Where Z_1 is the population of the banks and Z_2 is the population of the companies in a given portfolio, $z_1 = z_1(t)$ and $z_2 = z_2(t)$ represent the loans *performing* (or *in bonis*)[7] over the time *t* of the two populations Z_1 and Z_2 respectively:

$$\begin{cases} \dfrac{dz_1}{dt} = z_1 f_1(z_1, z_2) \\ \dfrac{dz_2}{dt} = z_2 f_2(z_1, z_2) \end{cases} \qquad (5)$$

This system of differential equations indicates that the number of *performing* loans within one population depends on the number of *performing* loans in the other:[8] the link between the two populations is intrinsically mutual.

$$\frac{\partial f_1}{\partial z_2} > 0 \quad \text{and} \quad \frac{\partial f_2}{\partial z_1} > 0 \qquad (6)$$

The empirical observations above indicate that a healthy productive sector relies on a healthy banking sector, which strengthens the condition of companies in turn. It is therefore necessary to delve into the dynamics of the bank-enterprise system, accounting for the limits set by the K and e components. First felt by each company within the Z_2 population, these limits impose a maximum value on the logistical trend of z_2 that is always less than the totality of Z_2, as well as

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4 For a more thorough development of this reasoning, see [9].

5 A targeted analysis of the K component, based on the same initial postulate, was conducted by [8].

6 Cf. [18] and [22].

7 The rationale for default levels could be developed in contrasting terms: the final result would be the same.

8 Cf. [10].

procedural and strategic constraints for banks at the time of deliberation and approval.

2. Methodological notes and the Italian dataset

As mentioned, each company generates positive flows for the banking system, which manifest in the following forms:

- the creation of income, part of which is deposited in banks as savings and is thus a form of funding for banks; and
- the payment of intermediation charges (broadly understood), which constitutes income for banks.

As such, we begin with analyses of the complexity and non-linearity of interactions between the population of banks and that of companies, and of the equilibrium points that arise in the dynamic system from these interactions. Through these analyses, we argue that the correlation of the contraction of credit – especially for micro and small enterprises – to the increase in the mortality rate of those same businesses and the deterioration of their existing credit produces “bio-economic” effects. Such effects are reminiscent of the observed exploitation (more precisely, *over-exploitation*) of a system’s resources by the *dominant species* or *player* when the system is inhabited by both populations. The negative consequences that such excessive exploitation by a generic agent may cause for the entire system, including difficulty regenerating the resources, can be empirically deduced.

For the purposes of this paper, we semantically adapt the concept of exploitation for the ‘enterprise’ resource. If, in fact, the reduction of credit leverage below a certain threshold changes the depth and adequacy of bank intervention, the outcome for banks will be deceptively positive and short-lived. This, as the portfolio will see a corresponding reduction in supervisory provisions, increase in available liquidity and more contained risk indicators. However, an excessive restriction of credit, gradually tightened over time, jeopardizes the favourable conditions needed for a resilient production system and business environment, as can be seen from the 2012-2019 Italian dataset below.[9]

[9] Tables in the appendix. Processing based on data from the Bank of Italy, ISTAT and Chambers of Commerce.

Figure 1 – Loans to enterprises in Italy, in million euros

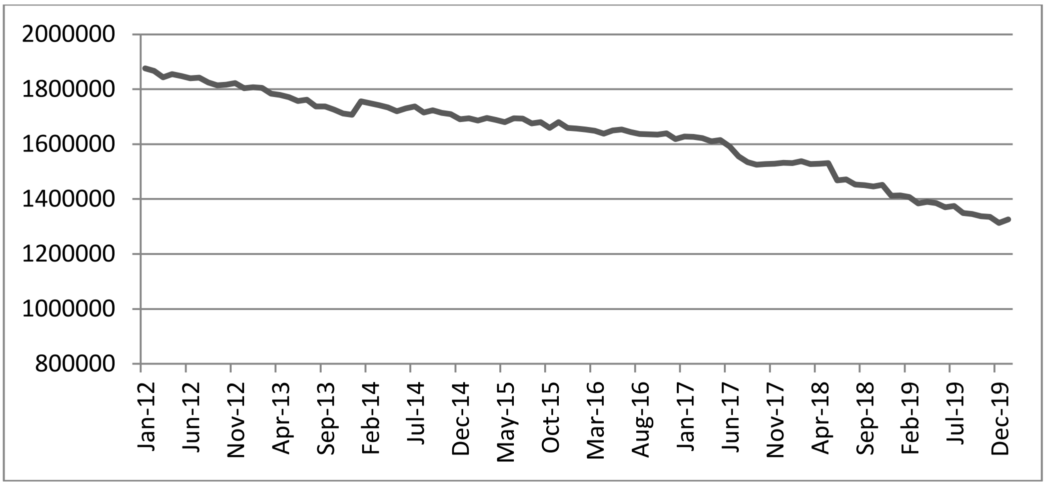


Figure 2 – Net non-performing loans in Italy, in million euros (March 2012 – March 2017)

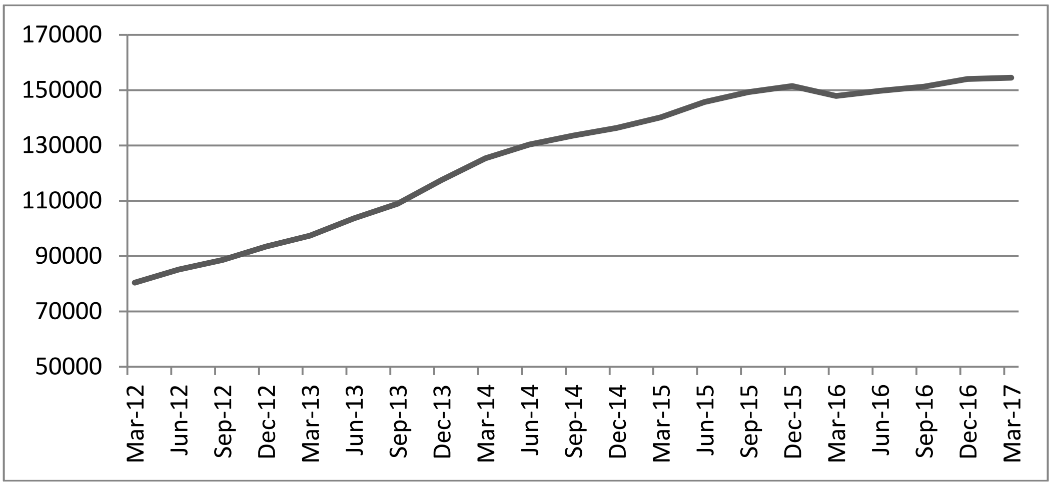


Figure 3 – Total number of Italian enterprises

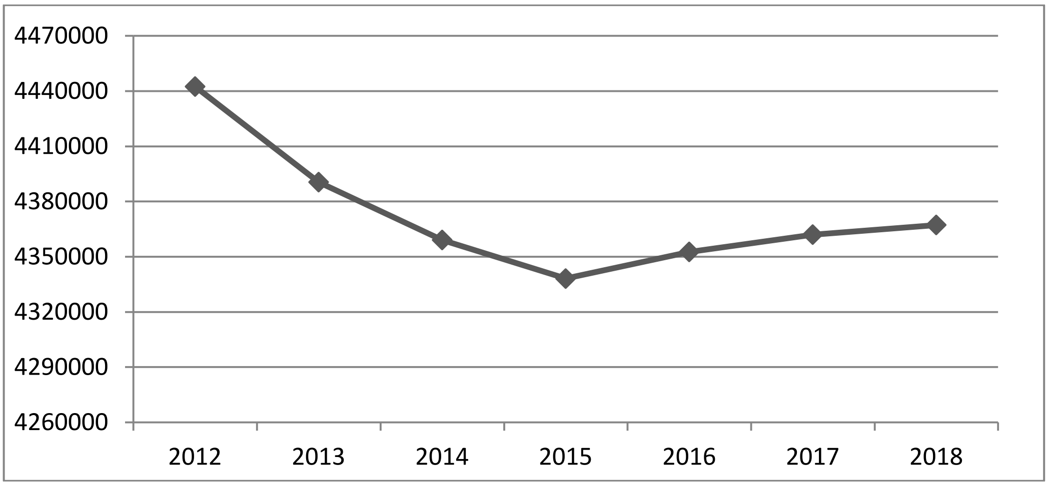
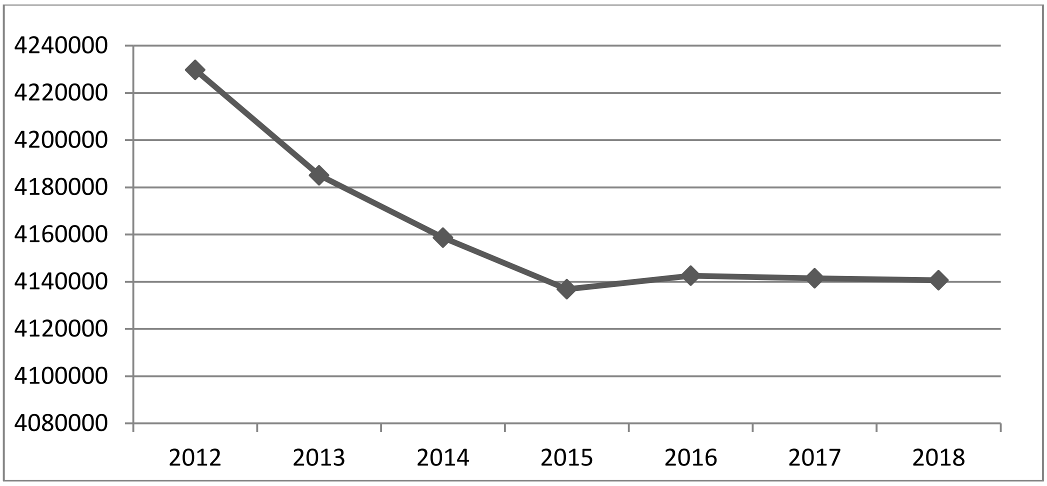


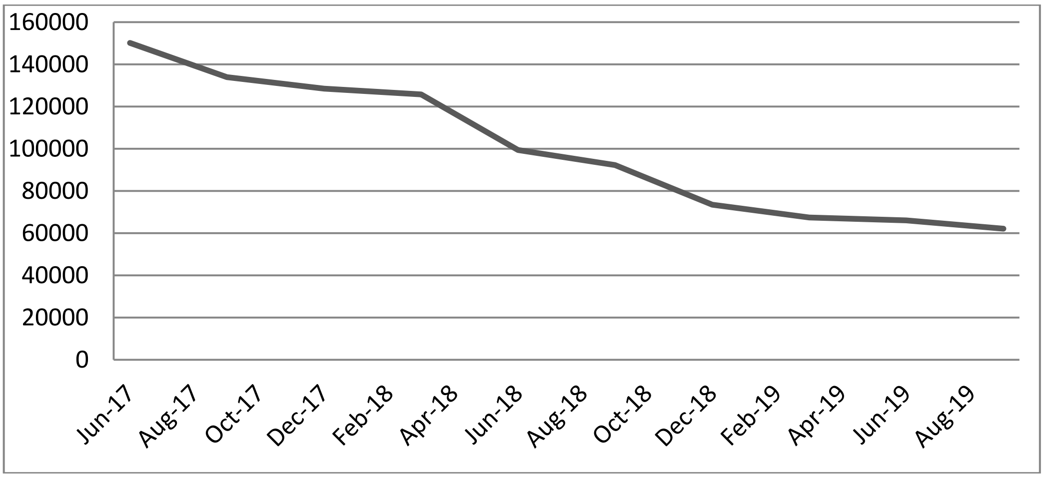
Figure 4 – Number of Italian micro enterprises



The scenario presented above begins with a progressive reduction in the disbursement of loans – assumed to be an independent variable – to which we can reasonably correlate a general increase in net non-performing loans until the first quarter of 2017 (mean Bravais-Pearson correlation coefficient at March 2017 = -0.57). The repercussions of this trend, however, vary according to the size of the business. On the one hand, micro enterprises leave the market more quickly than *new-co* companies of similar size. In fact, in the *dataset* adopted, the correlation between the reduction of credit granted to the production system and the number of SMEs operating on the market comes to +0.73. On the other hand, macro enterprises appear unaffected by this phenomenon. The incidence of the macro-enterprise component corrects the total figure only slightly.

For methodological completeness, we also assess trends in net non-performing loans from March 2017 to December 2019.

Figure 5 – Net non-performing loans in Italy, in million euros (June 2017 – September 2019)



Considered in isolation, the data in Figure 5 suggest a reduction in impaired loans. This trend is, however, due to updated ECB provisions regarding the management of non-performing loans. [12] In March 2017, new guidelines and subsequent addenda both strengthened monitoring procedures and provided incentives for the sale of these loans. These regulatory adjustments led to respective provisions and losses in balance sheets across the banking sector. In its 2018 and 2019 reports, the ABI (Italian Banking Association) [1] disposed of 50-70 billion euros per period. Moreover, even prior to the COVID-19 emergency, it had projected new growth in gross and net non-performing loans in its 2020-2021 two-year outlook.
Based on this logic, the banking sector is seen to be exploiting the banking-enterprise system by continuing to attract business savings while reducing its own risks and risk capital. This, as widely cited in literature, (*ex multis*, cf. [2]) causes shocks to 'enterprise' resources, reducing the quantity of those resources and even leading to extremely critical scenarios. Where such shocks endanger the very stability of the system, banks will inevitably see losses on their balance sheets caused by decreases in funding and higher costs due to deteriorated assets and net losses. [28] Expanding the frame of reference to the whole economic system, it is clear that the harmful effects will have a procyclical impact on the entire nation's macroeconomics since income from salaried employment is also dependent on companies. [4]

3. The evolution of the population of enterprises in relation to available leverage. A first dynamic model

Let us proceed with some dynamic analyses.[10] We indicate with X the periodic stock of Z_2 – in other words, the *metaphorical* quantity of the *renewable* resource, in this case, enterprise, over time.

$$X(t+1) = F\big(X(t)\big) = X(t) + GX(t) - C(t) \qquad (7)$$

In line with the characteristics of the system, time t is understood as the discrete intervals of quarters, half-years or years. $X(t)$ is therefore the number of enterprise resources in each period, while G is the growth rate during that period for each population unit. Indicating the quantity of resources lost ("exploited"), $C(t)$ thus represents the quantity of enterprises that have left the market or defaulted due to lesser banking support.
The temporal evolution of the model is described by the function that provides the law. Where this law determines the evolution of the quantity of the enterprise resource in $t+1$, the quantity known at time t either increases by its natural growth capacity or decreases by the quota lost.
The first deduction we make using this equation concerns the condition of equilibrium when $X(t+1)$ equals $X(t)$: that is to say, when the population remains constant over the period in question, which therefore means that $GX(t)$ is equal to $C(t)$. The proposed law thus accounts for both the bio-economic

[10] The following mathematical analyses is partly inspired by and conducted based on – with the appropriate adaptations to the contents here treated –the contribution of [3].

characteristics (G factor) and the agentic action of the banks (C factor). If the latter is less than the former, there will be an increase in the population of enterprises over the period $t + 1$. Likewise, if the former is less than the latter, there will be a decrease in the population. The intrinsic characteristics of this law are most usefully represented through a periodic growth model. It is worth noting, however, that this performance trend, among other things, is apparent in the n intervals examined in the Italian dataset and can be modelled with a continuous variable. The derivative dX/dt then demonstrates instantaneous evolution.

In the absence of a credit crunch, we can approximate $C(t) = 0$. As such, the population of enterprises will describe a quantitative evolution based solely on the micro- and macro-economic context in which it operates. In this scenario, it is assumed that, in each period, γX new entities enter the market and lX existing entities leave due to "natural causes." Formula (7) can thus be rewritten as follows:

$$X(t+1) = X(t) + \gamma X(t) - lX(t) = (1 + \gamma - l)\, X(t) \qquad (8)$$

Where $G = \gamma - l$ is the population growth rate and the $X(t+1) = \gamma X(t)$ model is linear, they depict a geometric progression due to g.

$$X(1) = gX(0);\;\; X(2) = gX(1) = g^2 X(0);\;\; \dots;\;\; X(t) = g^{t} X(0);\;\; \dots \qquad (9)$$

Should $g < l$ (and therefore $g < 1$), the progression converges to zero exponentially. If $g = l$ the population number remains the same and therefore has a constant value. If instead $g > l$, and therefore $g > 1$, the population growth trend will move exponentially towards infinity.[11]

Determining the mortality rate of enterprises l depends not only on the disappearance of leverage, but also on other micro- and macro-economic causes. For example, the context of the country in which the enterprise exists and the level of competition within that country's enterprise population. To the latter, we can write $l = \lambda X$, where the parameter λ indicates the precise difficulty of remaining in the market relative to the intensity of product competitors present – conversely, it also indicates the ease of exiting the market. Incorporating this, G is then equal to $\gamma - \lambda X$. At $K = \gamma/\lambda$, the line of this linear function passes through 0, such that K indicates carrying capacity, or an equilibrium point with zero growth.

$$X(t+1) = F\big(X(t)\big) = (1 + \gamma)\, X(t) - \lambda X(t)^2 \qquad (10)$$

Translated into a non-linear model, the parabola of the $F(X)$ function intersects with the abscissae at $X = 0$ and $X = (1 + \gamma)/\lambda$. The intersections between the parabola and the bisector $F(X) = X$, that is $X(t+1) = X(t)$, correspond with the system's equilibrium points at $X = 0$ (extinction equilibrium point) and $X = K$ (carrying capacity).

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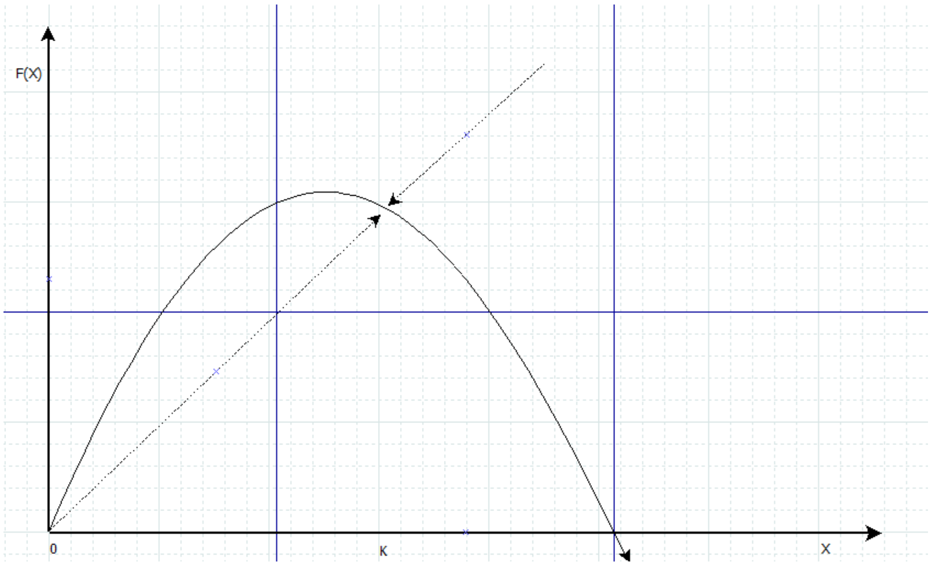
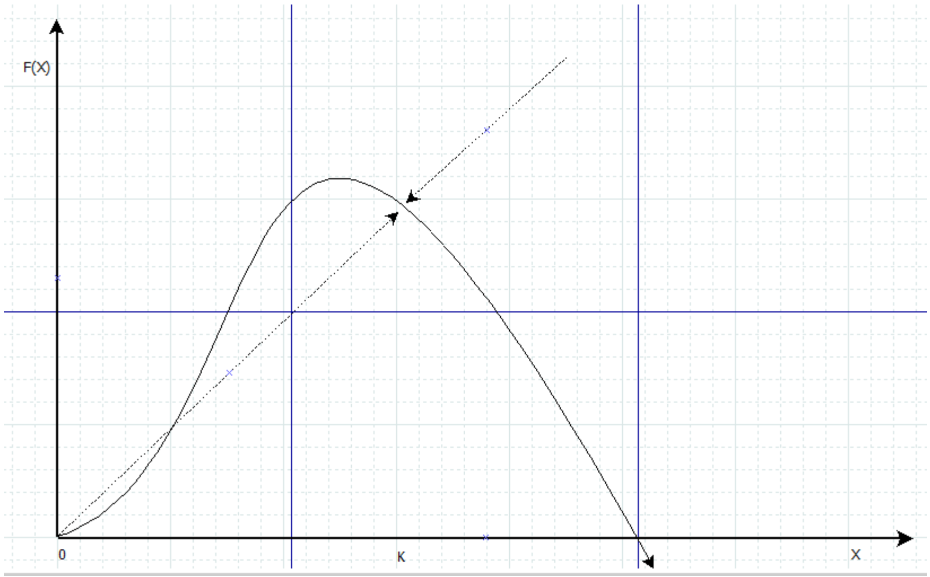
[11] The exponential trend moving toward infinity is, of course, an abstract situation since the finite environment in which companies exist and operate would provoke a change in the function's concavity until its arrest.

Figure 6 - Logistic growth function



Since the latter is a natural equilibrium point, the forces acting on it, especially weaker ones, are more easily compensated for by endogenous automatic processes. This is because $F(X) > X$ and $F(X) < X$ are found to the left and right respectively of the equilibrium point of the logistic growth function. The logic is reversed for $X = 0$ as any spontaneous or stimulated increase of resources would result in an accelerated departure from this equilibrium point in successive time intervals while still tending towards carrying capacity.
These general considerations can be further broken down according to the type of growth function. Assuming a positive starting value with an intrinsic growth rate $r = G(0)$, we reach another intersection with the abscissae at the value $K > 0$ (i.e. $G(K) = 0$), which occurs at the natural equilibrium point. If the function thus described is increasing and continuous – though not necessarily linear – the previous observations remain valid: $F(X) = X\big(1 + G(X)\big)$ is in fact concave, unimodal and not particularly different from the parabola used in the general case.
However, recall that the increase in the number of enterprises is conditioned by the total number of enterprises already existing in the system, especially by those with the same commercial objectives. As such, with recourse to the specific growth function $G(X)$, the population of enterprises will be described by a growth function that achieves only an intermediate maximum point for that population: this situation is called growth *depensation* (Allee effect).

Figure 7 - Growth with depensation function



Despite being characterized by a single maximum in accordance with its unimodality, this curve also undergoes an initial phase of convexity for small values of X before reaching an inflection point that shifts the curve to concavity as X grows. Here, too, we observe two equilibrium points: the fixed extinction point $X = 0$ and the carry capacity point $X = K > 0$. As we shall see later, this change of concavity is particularly relevant in the context of the credit crunch.
Returning to formula (7), we set $C(t) > 0$, with $C(t)$ being the number of companies that exit the market or go into default due to a reduction in credit leverage. The condition of equilibrium is $F\big(X(t)\big) = C(t)$.
Let us now introduce the parameter S. This parameter represents the banking strategy that reduces credit for the productive system to minimize capital allocations and maximize profits, including from non-commercial business (e.g. sovereign bond purchases). Alongside S, we consider the coefficient a, which accounts for the relative 'aggressiveness' with which this credit-crunch strategy is implemented. Thus, we will have $C(t) = aSX(t)$. Taken in full, formula (10) is rewritten as follows:

$$X(t+1) = F\big(X(t)\big) = X(t)\big(1 + \gamma - aS - \lambda X(t)\big) \qquad (11)$$

The $F(X)$ function is now a parabola passing through the origin with the vertical line $X = (1 + \gamma - aS)/(2\lambda)$ as its axis of symmetry. Setting $F(X) = X$, the equilibrium points are $X_0 = 0$ and $K_S = (\gamma - aS)/\lambda$. From a natural equilibrium point with a carrying capacity of $K = K_0 = \gamma/\lambda$ when $S = 0$ in the absence of credit crunch, we discern different scenarios with the increase of S:

- With $aS < \gamma$, the K_S equilibrium point remains positive and stable, whereas the X_0 point is unstable.

- An additional increase of the parameter S sees the value of the positive equilibrium point reduced because K_S is a decreasing function of S.
- If the level of S vis-à-vis the level of the a coefficient increases up to $aS = \gamma$, the two equilibrium points overlap.
- A further increase of S results in a bifurcation in which the K_S equilibrium point becomes negative and unstable and a basin of attraction forms around the point X_0.

In other words, when aS exceeds the value of the parameter γ – the 'demographic rate' of enterprises – the system leads inexorably towards the (theoretical) extinction of the enterprise resource.
Hence, let us assume that the banks have only two extreme strategies available:

a) $aS > \gamma$
b) $aS < \gamma$

The utility functions associated with each of these are outlined below:

Figure 8 – Utility functions of credit-crunch strategies by banks

Strategy

Player

	a	b
Banks	$u(\eta)$	$u(\theta)$
Companies	$u(\nu)$	$u(\xi)$

As set out above, the pay-off for banks will be $\theta > \eta$ (at least over the medium- to long-term); for companies, the pay-off will be $\xi > \nu$ (always). [20] What emerges is a game with a non-cooperative, dominant player (Z_1 – the banks – who decide the strategy) and only one win-win solution in choice b (figure 9), reinforced by the application of the von Neumann-Morgenstern rationality axioms. [8]

Figure 9 – Pay-off of credit-crunch strategies for banks and companies

Strategy

Strategy

↓ Z_1 Z_2→	a	b
a	$u(\eta)$, $u(\nu)$	
b		$u(\theta)$, $u(\xi)$

To formalize the banks' strategy for financing enterprises, particularly concerning the reduction of liquidity, we write $Q = aSK_S$, where Q indicates the net performance of banks in terms of profit and provisions relative to their credit activity:

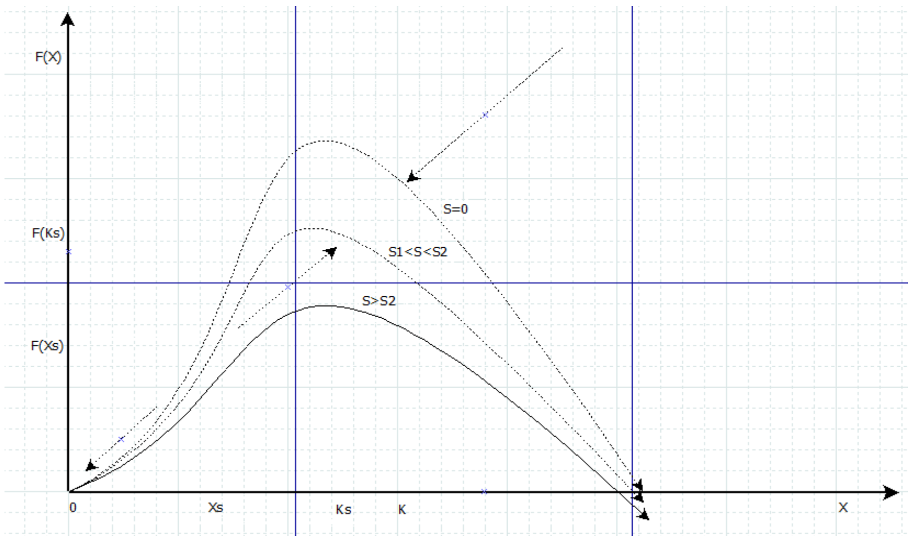
- if $S < \gamma/a$, then $Q = [aS(\gamma - aS)]/\lambda$;
- if $S > \gamma/a$, then $Q = 0$.

The most *remunerative* level of Q that allows for the full preservation of the bank-enterprise system's stability is obtained when $S = \gamma/(2a)$.

If a greater incidence of the restrictive policy represented by aS offers greater advantages in the short term, we begin to understand how the bank-enterprise system, having reached equilibrium, suffers a progressive depletion of the enterprise population and corresponding increase in losses for the banking population over the long term as $S > \gamma/(2a)$. The system will continue along this path until it reaches the unfavourable conditions for $S = \gamma/a$. Should this result in a maximum level of sustainable credit reduction at S_{MSQ} such that $S > S_{MSQ}$, these circumstances would lead to an *over-contraction*, a concept that is very similar to the bio-economic notion of *overexploitation* introduced earlier.

As mentioned above, the growth function of the enterprise population is depensatory, identified as such by the change of concavity in the function F. Alongside credit reduction activities, this characteristic introduces complications that force a new equilibrium X_S that could be located anywhere between the equilibrium points of extinction and carrying capacity (that is to say, $0 < X_S < K_S$). X_S is also unstable and represents a junction between stock values: when $X < X_S$, the system will evolve towards extinction, but when $X > X_S$, the system tends towards the equilibrium point K_S. In other words, X_S is the *survival threshold*. The intersection between this threshold and S establishes the bifurcation value, indicated by S_1.

Figure 10 – Following interactions between growth depensation function and credit reduction



If $S > S_1$, equilibrium is established at $X = 0$ with a survival threshold of $X_S > 0$. As S increases, we observe the two equilibrium points becoming progressively closer to each other. This reduces the value of the enterprise population when a stable equilibrium is reached. At the same time, the rising survival threshold creates greater system vulnerability as enterprises increasingly fall below the threshold and head towards extinction. If credit contraction strategy S becomes even more pronounced, we would observe X_S continuing to grow and K_S continuing to shrink, until they overlap and cancel each other out. This would introduce another bifurcation, S_2, that can only lead to one possible solution: the theoretical *exhaustion* of the enterprise resource.
While the measures adopted by central banks, including monetary policy and injections of liquidity into the productive system, are necessary for avoiding this trap, such actions undermine the survival of the entrepreneurial fabric and the stability of the banking system and economic system as a whole. Yet these measures should always be implemented before the critical value of S_2 is reached. If this scenario is not prevented, the stock $X(t)$ that has fallen below the survival threshold will slip into the range of attraction for the equilibrium point of extinction.
In actuality, such *extreme* effects will only manifest over a very long period and if no containment measures are taken whatsoever. Moreover, the equilibrium point of extinction is asymptotic; there is an asymptote located at a distance equal to radius p from the extinction point. This distance is primarily due to the presence of enterprises, especially the largest, in population Z_2 that can independently make up for the lack of external sources of financial leverage with endogenous means and resources. Furthermore, as introduced in formula (3), the factor $x_i = \alpha K + \beta e_i$ prevents population Z_2 from plummeting to zero. This can be demonstrated with the relation $\Delta x_i \Delta p = m$ where m is parametric. If the distance between the number X of Z_2 and the extinction point is significantly shortened, the contribution of the K (macroeconomic) and e_i (idiosyncratic) factors will proportionally counterbalance the result of the product, leaving the value of m unchanged. This is the resilience of the super-system. Nevertheless, m is still at a critical height and the entire system remains labile.
Should the economy reach a state of hysteresis in which conditions become irreversible, as described above, regulators are induced by the Precautionary Principle to intervene. Such central bank instruments as long-term refinancing operations (LTRO) and main refinancing operations (MRO) that function on a weekly or bimonthly basis (so-called '*overnight*' loans) as well as quantitative easing – with a *bridge*, precisely for banks – are imposed with the intention to restore and/or maintain the value of the stock X above the survival threshold. Reducing S until $X(t)$ is – again – higher than the threshold value X_S, such processes are deemed successful when $S < S_1$.
Returning once more to formula (7), we now see how central banking policy can intervene 'upstream' to avoid the consequences of catastrophe theory that were

characterized in the non-linear system discussed previously. This formula therefore also applies before the implementation of recovery measures, so long as $S_2 < S < S_1$.

We can model control over the credit crunch by observing the levels of company default or exit from the market caused by the reduction of financial leverage and designating $C(t)$ as a constant c:

$$X(t+1) = F\big(X(t)\big) = X(t)\big(1 + \gamma - \lambda X(t)\big) - c \qquad (12)$$

This shifts the parabola of the growth function down by h units. Solving for the equilibrium, through $F(X) = X$, or $\lambda X^2 - \gamma X + c = 0$, we have two solutions:

$$X_c = \frac{r - \sqrt{\gamma^2 - 4c\lambda}}{2\lambda} \quad \text{and} \quad K_c = \frac{r + \sqrt{\gamma^2 - 4c\lambda}}{2\lambda} \qquad (13)$$

When $c < \gamma^2/(4\lambda)$, both solutions are real and positive. Given a moderate reduction of enterprise population c, there will be two points of equilibrium: X_c and K_c, the first of which is unstable and the second stable.

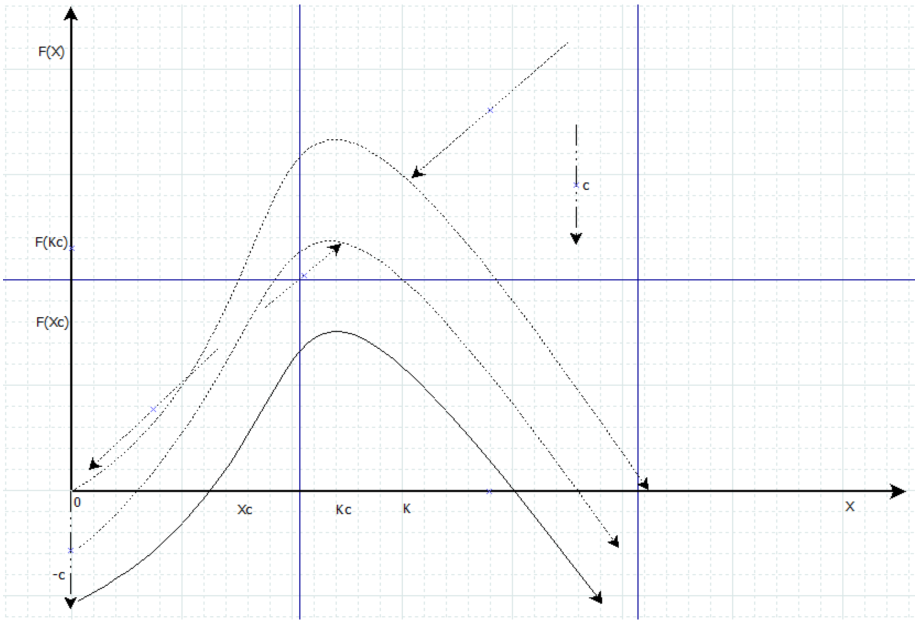
The instability of X_c is due to the changing value of the stock $X(t)$ over time t; when $X(t)$ is located in a right-hand interval of X_c – that is, $X(t)$ is slightly greater than X_c – it will tend in the next interval to move even further away. Where $F(X) > X$ to the right of X_c, then $X(t+1) > X(t)$. If, on the other hand, $X(t)$ were slightly less than X_c and thus in a left-hand interval, then $X(t+1) < X(t)$ and similar departures would be observed.

X_c is therefore also a *survival* threshold value. If the value of the stock $X(t)$ were to fall below point X_c due to an external shock – that being an immediate and idiopathic restriction of the credit granted – the system would begin to move towards negative values and once again towards the (theoretical) extinction of the population Z_2 within a certain $t + n$ period. If an unfavourable contraction does not cause the stock $X(t)$ to fall below the point X_c – although it will reduce it – then $X(t) > X_c$ and the system would be spontaneously attracted towards the stable equilibrium point K_c.

Where K_c is in any case lower than the carrying capacity of the population of enterprises K (equal to γ/λ), it is of note that a credit crunch, even if controlled, would return the system and its population of enterprises to a stable equilibrium of lesser value than would have resulted in the total absence of funding retention strategies. In this scenario, the relative position of the two equilibrium points depends on the height c, the increase of which would reflect an increase in the value of X_c and a decrease in that of K_c. In turn, the natural equilibrium value would decrease, and the system would become more fragile.

Should $c = \gamma^2/(4\lambda)$, X_c and K_c overlap and the parabola becomes tangent to the bisector. Once the value of c is surpassed, an additional point of bifurcation is realized, and we are again in a situation where the only evolution of the model is towards extinction.

Figure 11 – Considering control over the credit crunch



Following the implementation of a restrictive credit strategy by banks, activating similar forms of control according to the loss share threshold of each enterprise resource – and in the absence of structural or exogenous crises – the system is able to achieve a stable equilibrium. This new equilibrium inevitably has a density lower than the carrying capacity of the population observed prior to the contractions of financial leverage. This, too, is on the condition that the stock X does not fall below the threshold X_c in order that the height c does not pass the bifurcation point.

4. Is there an optimal minimum level of disbursed financial leverage? A second dynamic model

To offer an alternate way forwards, we reconsider our introduction, expanding it such that at adequate levels of credit leverage, the population of companies grows by $\frac{dZ_2}{dt} = GZ_2$ with $G > 0$. Taking the abstraction to the opposite extreme, if the population of companies were to disappear as a result of an unsustainable financial squeeze, the banks would go into default: if $Z_2 = 0$, $\frac{dZ_1}{dt} = -HZ_1$ with $H > 0$.

The number of (positive) transactions between banks and businesses is clearly proportional to the product of their *performing* populations. We have said that, in the immediate future,[12] a credit retention strategy produces a positive effect $\alpha Z_2 Z_1$ on banks' balance sheets, while enterprises feel this burden as $-\beta Z_2 Z_1$ in

[12] At least, as long as client enterprises are able to remain in the market and properly repay their quotas of borrowed capital and the related finance charges.

their own numbers. In terms of credit disbursed and received, α and β are positive constants and measure the immediate result of the interaction for the two populations.

From these assumptions, the initial equations are developed formally as:

$$\frac{dZ_2}{dt} = GZ_2 - \beta Z_2 Z_1 \quad \text{and} \quad \frac{dZ_1}{dt} = -HZ_1 + \alpha Z_2 Z_1 \qquad (14)$$

Now that positive starting values have been arbitrarily set (at time t) for the two populations Z_1 and Z_2, the objective is to identify and define the qualitative behaviour of the system's trajectories.[13]

Solving

$$\begin{cases} Z_2(G - \beta Z_1) = 0 \\ Z_1(-H + \alpha Z_2) = 0 \end{cases} \qquad (15)$$

We find the critical points $A(0,0)$ and $B\left(\frac{H}{\alpha}, \frac{G}{\beta}\right)$.

Near point A the corresponding linear system will be:

$$\frac{d}{dt}\begin{pmatrix} Z_2 \\ Z_1 \end{pmatrix} = \begin{pmatrix} G & 0 \\ 0 & -H \end{pmatrix}\begin{pmatrix} Z_2 \\ Z_1 \end{pmatrix} \qquad (16)$$

With the eigenvalues and eigenvectors:

$$\rho_1 = G, \qquad \kappa_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\rho_2 = -H, \qquad \kappa_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

And solution:

$$\begin{pmatrix} Z_2 \\ Z_1 \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{Gt} + C_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-Ht} \qquad (17)$$

From this, it is apparent that point $A(0,0)$ – the origin – is a saddle point and therefore unstable. It is an ordinate and all the different trajectories move away from the critical point.

In the same way, we observe the surroundings of point B, setting

$$Z_2 = \left(\frac{H}{\alpha}\right) + w \quad \text{and} \quad Z_1 = \left(\frac{G}{\beta}\right) + v \qquad (18)$$

Therefore

$$\frac{d}{dt}\begin{pmatrix} w \\ v \end{pmatrix} = \begin{pmatrix} 0 & -\beta H/\alpha \\ \alpha G/\beta & 0 \end{pmatrix}\begin{pmatrix} w \\ v \end{pmatrix} \qquad (19)$$

With (imaginary) eigenvalues $\rho_{1/2} = \pm i\sqrt{GH}$ that demonstrate the existence of a stable centre at $B\left(\frac{H}{\alpha}, \frac{G}{\beta}\right)$.

The system's trajectories will be given by:

$$\frac{d}{dt} = \frac{dv/dt}{dw/dt} = -\frac{\left(\frac{\alpha G}{\beta}\right) w}{\left(\frac{\beta H}{\alpha}\right) v} \qquad (20)$$

[13] An (extensive) application of the Lotka-Volterra equations is proposed. *Ex multis,* cf. [16] and [6].

or:

$$\alpha^2 Gw\,dw + \beta^2 Hv\,dv = 0 \qquad (21)$$

From which:

$$\alpha^2 Gw^2 + \beta^2 Hv^2 = k \qquad (22)$$

With k constant of integration (not negative).
The solutions are formalized as:

$$w = \frac{H}{\alpha} J \cos\left(\sqrt{(GH)}t + \psi\right) \quad \text{and} \quad v = \frac{G}{\beta}\sqrt{\frac{H}{G}} J \sin\left(\sqrt{(GH)}t + \psi\right) \qquad (23)$$

Where the constants J and ψ are determined by the initial conditions. Substituted into (18), we get:

$$Z_2 = \frac{H}{\alpha} + \frac{H}{\alpha} J \cos\left(\sqrt{(GH)}t + \psi\right) \quad \text{and} \quad Z_1 = \frac{G}{\beta} + \frac{G}{\beta}\sqrt{\frac{H}{G}} J \sin\left(\sqrt{(GH)}t + \psi\right) \qquad (24)$$

The trajectories of the resulting linear system are therefore almost elliptical and move anticlockwise. Equations (24) offer a good approximation of the representation.
Let us return to the non-linear system and reduce it to the equation:

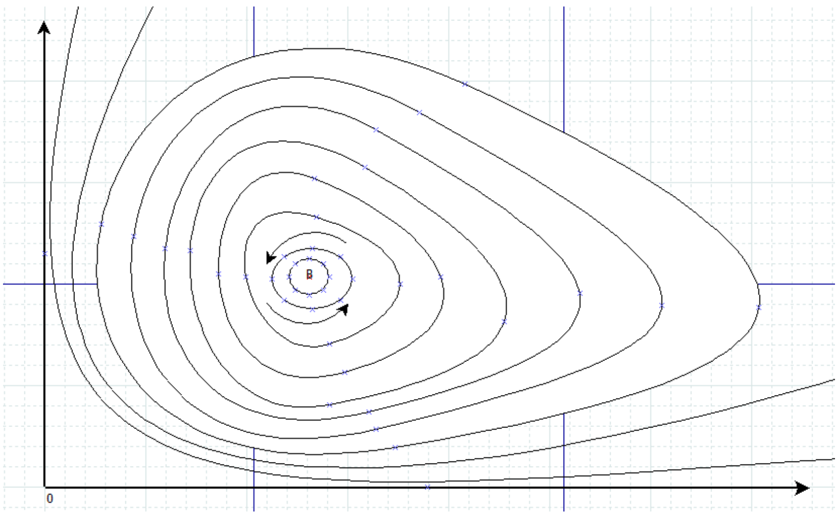
$$\frac{dZ_1}{dZ_2} = \frac{dZ_1/dt}{dZ_2/dt} = -\frac{Z_1(-H + \alpha Z_2)}{Z_2(G - \beta Z_1)} \qquad (25)$$

Which, by separating, leads to the solution:

$$G\ln(Z_{1)} - \beta Z_1 + H\ln(Z_2) - \alpha Z_2 = q \qquad (26)$$

Setting q – which is also a constant of integration – the graph of the equation is still a curve surrounding the critical point $B\left(\frac{H}{\alpha}, \frac{G}{\beta}\right)$, which reaffirms its existence as a centre for the non-linear system.

Figure 12 – Phase representation of the nonlinear system



Depending on the initial conditions, the trajectories around the stable critical point may show slight variations in the abscissa (Z_2) and the ordinate (Z_1), or they may be more pronounced and produce a less elliptical shape. Since Z_2 and Z_1 are periodic functions of t, a non-overlapping oscillatory movement will follow: the curves will intersect at levels of leverage sufficient to support the financial, productive and economic health of enterprises such that their systemic confidence and *performing* presence in the market can be increased.
At these points, banks will have the convenience of restricting credit volumes to increase their short-term performance, causing new mortality in the entrepreneurial fabric and an increase in non-performing loans *in the system*, which will lead them again towards a greater expansion of lending and the reactivation of the cycles.

Figure 13 – Changes in credit disbursed, population of enterprises in the market and number of NPLs

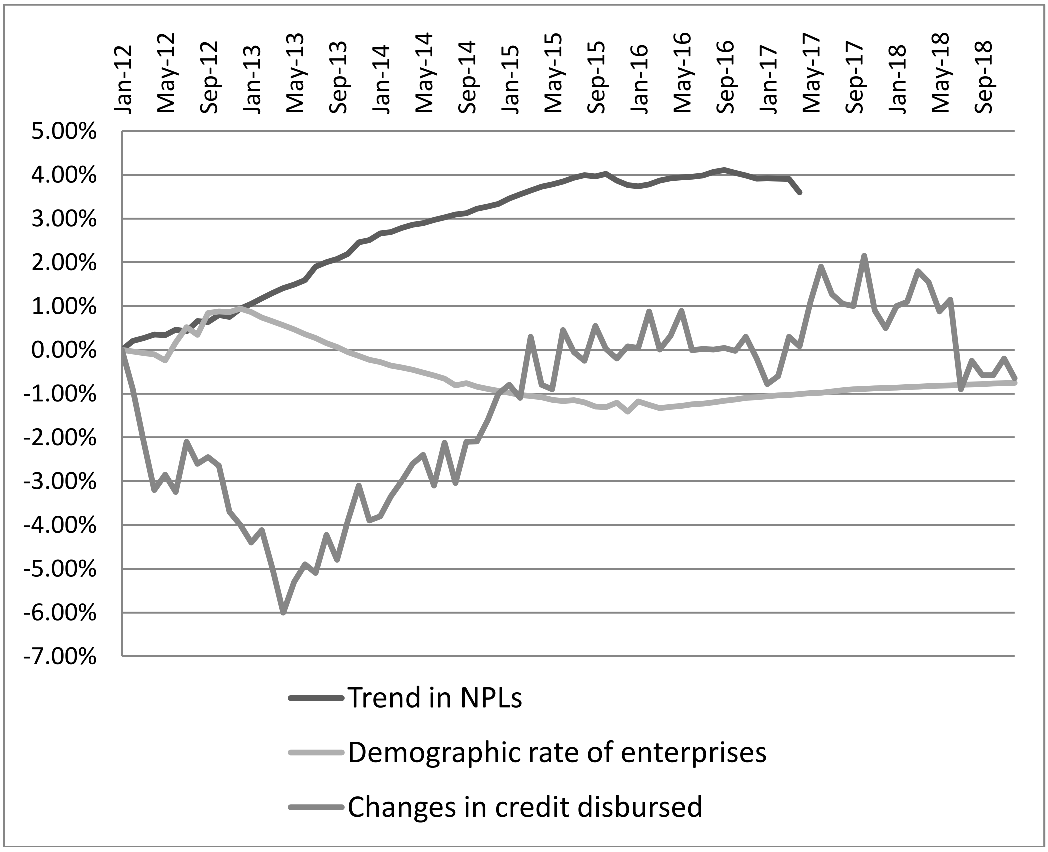


Figure 13 describes this phenomenon empirically.[14] Where our earlier treatments focused on the dynamics of periodic stocks, figure 13 tracks percentage fluctuations in monthly recorded flows in relation to credit disbursed, the demographic rate of the entrepreneurial scene and the number of NPLs in Italy in

[14] Processing based on data from the Bank of Italy, ISTAT and Chambers of Commerce.

the 2012-2018 period. Considering the changes to regulation and bad debts discussed in relation to figures 2 and 5, we present the trend in NPLs only until the first quarter of 2017.
In addition to the time delay naturally caused by the macro-complexity of the objects analysed, the above diagram confirms that the contraction of credit granted by banks to maximise their profit margins by reducing supervisory capital and administrative costs in the short term correlates with a progressive reduction in the presence (*in bonis*) of companies in the market and a general increase in NPLs. Where these trajectories necessitate the recovery of disbursed credit, this action will also see the trends in NPL and enterprise figures reversed.
This model has greater validity for banks that operate mainly in the credit market, since the simultaneous diversification of assets by banks would mitigate the cause-effect mechanism of the proposed model.

5. Position conclusions

The preservation of the bank-enterprise relationship is therefore necessary to maintain the macrosystem. Measures taken to stabilize levels of leverage provided to the productive and entrepreneurial sectors should be consistent with the dynamic models outlined above. At the very least, the recursive phases discussed should be guaranteed through standardized regulations. This is especially significant during fragile and/or partially compromised economic scenarios and when unfavourable economic conditions prevail.
We return to the Italian case study to understand how the government enacted measures to support the economy amid the COVID-19 emergency, considering also how intervention has worked towards the main objective of keeping the bank-enterprise system in working order. It should first be noted that all evidence suggests that the 2020 economic crisis is a result of exogenous forces. It is not caused by internal *pathologies*, as was the case for the 2009-2011 crisis. Understanding this distinction is particularly important for managing and resolving the current crisis. Among the first steps taken, governments should allay concerns about restoring impacted economic and financial aspects by ensuring temporary compensation for the sudden cessation of productive activities.
Coming to a total 750 billion euros – 350 billion as leverage on a nominal 25 billion and another 400 billion also resulting from leverage[15] – the liquidity allocated by the Italian state was not directly injected into the productive system (or to households). Instead of intervening more directly or adopting a ‘helicopter drop’ approach, the Italian government has elected to strengthen public guarantee funds, leaving the banking role of financial intermediation unchanged while simultaneously pushing for more quantitative easing to encourage banks to grant credit.

[15] Our discussion of the merits of macroeconomic arguments for stability, public deficit or European Community loan instruments – bonds or no bonds – does not intend to encourage the State to raise these resources.

Indeed, the recent Decree-Laws No. 18 and No. 23 of 2020 expressly expanded the scope of direct public guarantees to all enterprises, immediately enforceable and with zero weighting on provisions, and also offered free reinsurance measures for credit guarantee consortia, with access and coverage of up to 80-90% for most financing operations and 100% for certain smaller loans.
Implementing the measures modelled above – assessed in part and only as pertinent to this paper – should assist in impeding a credit crunch in Italy, which may certainly manifest given the generalized worsening of creditworthiness following the lockdown, while also preventing certain critical developments that would push the nation's enterprise and bank populations towards default.

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Appendix

Dataset 1*: Loans to enterprises in Italy, in million euros*

Feb-12	1,867,069	**Feb-14**	1,748,482	**Feb-16**	1,653,726	**Feb-18**	1,538,125
Mar-12	1,843,099	**Mar-14**	1,741,682	**Mar-16**	1,648,836	**Mar-18**	1,528,030
Apr-12	1,855,380	**Apr-14**	1,734,242	**Apr-16**	1,638,352	**Apr-18**	1,528,992
May-12	1,848,450	**May-14**	1,719,438	**May-16**	1,650,041	**May-18**	1,531,149
Jun-12	1,839,294	**Jun-14**	1,730,636	**Jun-16**	1,652,764	**Jun-18**	1,468,328
Jul-12	1,842,289	**Jul-14**	1,737,299	**Jul-16**	1,644,173	**Jul-18**	1,471,657
Aug-12	1,824,729	**Aug-14**	1,714,682	**Aug-16**	1,637,576	**Aug-18**	1,453,087
Sep-12	1,814,339	**Sep-14**	1,723,485	**Sep-16**	1,635,688	**Sep-18**	1,449,988
Oct-12	1,816,006	**Oct-14**	1,714,305	**Oct-16**	1,634,588	**Oct-18**	1,445,756
Nov-12	1,822,061	**Nov-14**	1,709,791	**Nov-16**	1,639,756	**Nov-18**	1,452,218
Dec-12	1,803,779	**Dec-14**	1,690,082	**Dec-16**	1,618,884	**Dec-18**	1,412,341
Jan-13	1,807,279	**Jan-15**	1,694,068	**Jan-17**	1,627,127	**Jan-19**	1,412,799
Feb-13	1,804,572	**Feb-15**	1,686,081	**Feb-17**	1,626,021	**Feb-19**	1,407,968
Mar-13	1,784,326	**Mar-15**	1,694,929	**Mar-17**	1,622,248	**Mar-19**	1,383,853
Apr-13	1,779,044	**Apr-15**	1,688,135	**Apr-17**	1,610,564	**Apr-19**	1,389,698
May-13	1,771,135	**May-15**	1,679,991	**May-17**	1,614,943	**May-19**	1,385,884
Jun-13	1,757,391	**Jun-15**	1,694,540	**Jun-17**	1,591,435	**Jun-19**	1,370,106
Jul-13	1,761,553	**Jul-15**	1,693,195	**Jul-17**	1,554,869	**Jul-19**	1,374,303
Aug-13	1,737,363	**Aug-15**	1,675,366	**Aug-17**	1,534,324	**Aug-19**	1,349,538
Sep-13	1,737,088	**Sep-15**	1,679,762	**Sep-17**	1,524,810	**Sep-19**	1,346,075
Oct-13	1,725,214	**Oct-15**	1,659,496	**Oct-17**	1,527,558	**Oct-19**	1,337,474
Nov-13	1,712,062	**Nov-15**	1,680,464	**Nov-17**	1,528,311	**Nov-19**	1,334,656
Dec-13	1,706,800	**Dec-15**	1,659,189	**Dec-17**	1,531,909	**Dec-19**	1,312,601
Jan-14	1,755,443	**Jan-16**	1,656,962	**Jan-18**	1,531,414	**Jan-20**	1,325,769

Dataset 2*: Non-performing loans in Italy, in million euros*

Mar-12	80,372	**Mar-14**	125,350	**Mar-16**	147,868	**Mar-18**	125,784
Jun-12	85,174	**Jun-14**	130,275	**Jun-16**	149,683	**Jun-18**	99,454
Sep-12	88,628	**Sep-14**	133,524	**Sep-16**	151,241	**Sep-18**	92,281
Dec-12	93,420	**Dec-14**	136,323	**Dec-16**	154,034	**Dec-18**	73,545
Mar-13	97,330	**Mar-15**	140,097	**Mar-17**	150,485	**Mar-19**	67,456
Jun-13	103,642	**Jun-15**	145,662	**Jun-17**	150,251	**Jun-19**	66,077
Sep-13	108,895	**Sep-15**	149,286	**Sep-17**	133,973	**Sep-19**	62,205
Dec-13	117,511	**Dec-15**	151,423	**Dec-17**	128,586		

Dataset 3: *Total number of enterprises in Italy*

Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
B: extraction of minerals from quarries and mines	2,451	2,336	2,257	2,186	2,250	2,317	2,332
C: manufacturing	417,306	407,344	396,422	389,317	399,458	404,529	406,508
D: supply of electricity, gas, steam and air conditioning	8,926	10,169	10,459	10,775	10,015	10,041	10,056
E: supply of water, sewerage, waste management and environmental remediation services	8,967	9,121	9,146	9,231	9,060	9,230	9,301
F: construction	572,412	549,846	529,103	511,405	534,824	537,348	537,853
G: wholesale and retail trade, repair of motor vehicles and motorcycles	1,163,413	1,153,640	1,123,134	1,105,227	1,128,117	1,129,702	1,130,596
H: transport and storage	131,755	129,865	125,688	123,625	127,651	127,817	128,172
I: accommodation and food service businesses	307,878	313,207	312,013	315,464	312,000	311,478	312,009
J: information and communications services	97,280	95,989	96,997	98,381	96,933	96,915	97,079
K: financial and insurance service businesses	91,434	93,031	95,209	96,173	93,199	93,341	93,394
L: real estate businesses	235,434	243,564	239,134	238,273	237,137	237,094	237,067
M: professional, scientific and technical businesses	710,017	691,700	705,895	714,934	700,468	700,308	700,406
N: rental and travel agencies, business support services	143,770	139,362	139,898	139,595	139,959	140,415	140,724
P: education	26,890	27,677	29,088	29,566	28,360	28,256	28,304
Q: healthcare and social services	259,400	261,056	277,295	285,231	269,170	269,050	269,191
R: arts, sports, entertainment and amusement businesses	63,054	62,704	64,169	65,022	63,165	63,350	63,404
S: other service businesses	202,065	199,902	203,180	203,680	200,831	200,794	200,857
TOTAL	**4,442,452**	**4,390,513**	**4,359,087**	**4,338,085**	**4,352,597**	**4,361,985**	**4,367,253**

Dataset 4: *Total number of micro enterprises in Italy*

Year	**2012**	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**
B: extraction of minerals from quarries and mines	1,907	1,850	1,775	1,712	1,796	1,795	1,795
C: manufacturing	345,293	338,015	328,486	321,837	330,613	330,526	330,459
D: supply of electricity, gas, steam and air conditioning	8,380	9,610	9,916	10,205	9,448	9,445	9,443
E: supply of water, sewerage, waste management and environmental remediation services	6,485	6,688	6,748	6,816	6,628	6,626	6,625
F: construction	548,709	528,592	509,648	492,388	515,477	515,341	515,237
G: wholesale and retail trade, repair of motor vehicles and motorcycles	1,124,546	1,116,087	1,086,631	1,068,659	1,089,768	1,089,481	1,089,262
H: transport and storage	119,126	117,430	113,241	110,756	114,173	114,143	114,120
I: accommodation and food service businesses	288,119	294,007	292,996	295,706	290,253	290,177	290,119
J: information and communications services	91,274	89,895	91,020	92,279	90,353	90,329	90,311
K: financial and insurance service businesses	88,998	90,637	92,831	93,799	90,799	90,775	90,757
L: real estate businesses	234,738	242,874	238,492	237,637	236,437	236,374	236,327
M: professional, scientific and technical businesses	702,053	683,778	698,154	707,020	691,902	691,720	691,581
N: rental and travel agencies, business support services	132,452	128,082	128,721	128,394	128,327	128,294	128,268
P: education	25,239	25,957	27,351	27,781	26,359	26,352	26,347
Q: healthcare and social services	253,160	254,655	270,894	278,646	262,123	262,054	262,001
R: arts, sports, entertainment and amusement businesses	60,658	60,382	62,001	63,011	60,997	60,981	60,969
S: other service businesses	198,593	196,542	199,755	200,185	197,103	197,051	197,011
TOTAL	**4,229,730**	**4,185,081**	**4,158,660**	**4,136,831**	**4,142,556**	**4,141,465**	**4,140,633**

For COI Statement

Compliance with Ethical Standards.

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Conflict of Interest

The authors declare that they have no conflict of interest.