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Reconciling resource economics and ecological
economics: the economics of sustainability and
resilience

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Abstract

Cross disciplinary dialogue between economics and ecology has within economics centered on the two subdisciplines of bioeconomics and ecological economics. This division in economics reflects the division in ecology between population and systems ecologists. Recent developments in ecology are aimed at a more integrated approach to ecological research. One example of such an approach is that of models based on thermodynamic reaction networks. By applying the “Law of Mass Action” to biochemical descriptions of ecological networks, it is possible to reformulate ecological systems models as population dynamic models, which can then be embedded within a bioeconomic model framework. Analysis of bioeconomic models far from thermodynamic equilibrium is then possible from within either a steady-state or ergodic framework. The Glansdorff-Prigogine or other related stability criteria from non-equilibrium thermodynamics may then be applied to the study of bioeconomic systems.

1 Introduction

Recent interest in ecological economics has led to the recognition that intertemporal resource allocation problems of natural resource economics and the problem of externalities studies by environmental economics are not the only environmental challenges faced by economics. The physical basis of economic system cannot be ignored by economists if we are to meet these challenges. The research agenda of ecological economics goes some way towards meeting the environmental challenge. Nevertheless, the speed with which ecological economics has grown and the recognition of the inadequacies of existing models has led to a neglect of some aspects of existing models that are related to the research agenda of ecological economics in interesting ways.

Bioeconomic models that have been used in fisheries and forestry management and more recently range management have tended to neglect systemic aspects such as energy use and the thermodynamic underpinnings of the physical world. This does not, however, mean that such models are incompatible with the thermodynamic approach advocated by ecological economists.

In a sense many existing economic models already account for thermodynamic laws, including, the entropy law. The steady-state assumption of deterministic renewable resource models may in fact be viewed in this way.

An area that has until recently been neglected is that of adapting network thermodynamic concepts to economic analysis in the search for a unified paradigm for an “economics of natural systems, resources and the environment” (Beard, 1994) The problem of managing natural systems, resources and the environment is largely one which involves the optimal control of thermodynamic networks. Solving optimal control problems is unlikely to suit all economists and others may Choose to draw on ideas from landscape ecology. nevertheless, this technique has been part of the standard toolbox of resource economics including bioeconomics since their inception. If ecosystems are defined in such a way as to include both populations and material and energy flows, ie. as thermodynamic networks, then

the management of such ecosystems will need to be based upon the optimal control of thermodynamic networks.

Such an approach is highly data intensive as not only population and socioeconomic data are needed but also biochemical data. Nevertheless a number of empirical ecological studies have already been carried out that integrate both the population and the systems approach. This fact leads one to suspect that the addition of economic analysis may not be far away. Note that chemical reaction rate constraints may be obtained via experimental work, necessitating the involvement of experimental and systems ecologists. Other data needed would involve the linkages between systemic and population variables. In this area there is increasing cooperation between systems and population ecologists. The use of network diagrams as a fieldwork tool by human ecologists and ecological anthropologists already points the way towards the adoption of this approach by social scientists (Ellen, 1982).

The data problems that such an approach will face can be solved if populations-community ecologists, systems ecologists and resource economists work together.

Previous work on holistic modelling has also been in a similar vein to the approach taken in this paper albeit not explicitly taking a network thermodynamic approach. Thus Jeffries (1979) work on the stability of holistic ecosystem models, defined as models with a hierarchy of energy states, or trophic levels, is despite the lack of detailed thermodynamic analysis, and early example of the type of approach I am drawing on in this paper (Jeffries, 1979).

2 Thermodynamic Network Models: Integrating Systems Ecology and Population Ecology

There has been a general dissatisfaction in ecology with the split between the functional-systems paradigm. To this end there have been attempts at overcoming the division between the two schools. These two schools have been identified by Reiners as the first and second complementary models of ecology (Reiners, 1985, p. 62). The basic axiom of the first school is, according to Reiners, the second law of thermodynamics. Whereas the basic axiom of the second school, is that “chemical stoichiometry of organisms”. The two main streams of thought that have aimed at developing a unified ecological paradigm and landscape ecology. These fields are prime candidates for what Reiners has termed a third complementary model (Reiners, 1985, p. 70). It should be noted that both of these approaches are not necessarily incompatible with each other.

These newer developments within ecology have important implications for both ecological economics and bioeconomics as they point the way to a possible future integration of bioeconomics and ecological economics.

Network thermodynamics dates in this origins back to Lotka who originally formulated the population dynamic model, which later became known as the Lotka-Volterra model,

as a model of the dynamics of (autocatalytic) chemical reactions (Lotka, 1954).

This would imply that network thermodynamics predates the use of energy flow analysis which dates to Lindeman (1942).

Mikulecky (1991) on the other hand traces its origins to Kirchhoff's (1976) work on network and graph theory (mikulecky, 1991, p. 78).

The development of mathematical systems theory primarily within electrical and electronic engineering and the study of thermodynamic properties of electrical and electronic networks then lead to the realization that the graph theoretic technique of systems engineering could be coupled with energy flow chart models (systems ecology) and population dynamic models (population ecology) to develop an integrated model of ecosystem dynamics which combined certain features of both systems and population ecology.

Reiners has attempted to develop, albeit informally, an axiomatic system that captures the essence of this approach, the formal application of this system by theoretical ecologists to thermodynamic networks has as yet not been carried out.

Note that at the macroscopic level, it is a relatively simple matter to determine the change in the enthalpy generated by a reaction given certain simplifying assumptions, such as constant temperature, pressure (1 atmosphere at sea level), volume, etc. These are relatively simple to determine empirically. In practice however the analysis of ecosystem dynamics in thermodynamic reaction networks would remain at the kinetic (microscopic) level. It suffices to note that aggregation to the macroscopic thermodynamic level is possible given appropriate simplifying assumptions. The microscopic or kinetic level will in practice involve the species population as the basic unit rather than attempting to model the internal biochemistry of individual organisms. Material and energy flows between individual organisms may in principle be modelled in relatively more detail. It should however be noted, that the level of abstraction involved is purely an issue of modelling parsimony and does not have an impact on the ability to apply thermodynamic concepts to the analysis. This approach is highly reductionistic and some, eg. Rosenberg (1994), would object by denying that a reductionist approach is at all possible within the biological and social sciences. The ability to reduce biology to physics is however at the heart of both the systems ecology and ecological economics research programmes.

The opposite view to Rosenberg is taken by Wicken who has attempted on numerous occasions to reconcile thermodynamics with evolutionary theory (Wicken, 1986 and 1987). The essence of Wicken's argument is that irreversible thermodynamic processes drive evolution. This was, according to Wicken, first recognized by Lotka (Wicken, 1986, p. 263). In Lotka's own words:

The laws of the chemical dynamics of a structured system of the linking described will be precisely those laws, or at least a very important section of those laws, which govern the evolution of a system comprising living organisms (Lotka, 1954).

In the final analysis the lack of a thermodynamic foundation for evolutionary theory is the fundamental reason behind the split between systems and population ecology and consequently between ecological economics and bioeconomics

Brown attributes the lack of such a foundation and the consequent methodological split in ecology to the state of mathematical and physical knowledge at the time Lotka was writing (Brown, 1995, p 23). Thus, mathematics and physics were at that time only able to offer limited insights into complex ecological system. Brown is optimistic that things have changed but also cautions against “unreasonable expectations” (Brown, 1995, pp. 23-24).

For the ecological economist the advantage of this approach is that a network thermodynamic constraints into the model at every stage of the modelling process. Furthermore, thermodynamics principles are not used as analogies but are incorporated in a natural manner.

The thermodynamic constraints on any model can be incorporated at a number of stages in its development. Often, thermodynamic constraints on models are ignored, sometimes with disastrous consequences. Network thermodynamics requires that these constraints be incorporated throughout the modelling process (Mikulecky, 1991, p. 72).

Thus, taking a network thermodynamic approach leads one to actually “do thermodynamics” rather than use thermodynamic ic analogies.

Another branch of this literature has emphasized the link between thermodynamic networks and population genetics rather than population dynamics. This work dates back to Eigen (1971) and has primarily been of application to molecular genetics and evolutionary population genetics. This area is known as replicator dynamics and is based primarily on what has become known as the quasi-species model. The evolutionary approach of this school has lead to an interest in the role of mutations in catalytic networks that may have application to the study of issues such as biodiversity, weed invasions, etc (Stadler and Nuno, 1994).

An advantage of the network approach is that network models are already wide widespread in empirical work amongst field ecologists, human ecologists and anthropologists. Thus if ecological economists were to adopt a network approach this would facilitate interdisciplinary links by allowing them to incorporate fieldwork from other disciplines into the modelling enterprise.

3 Network Languages

The modelling of ecosystems using networks requires the use of a specific formal description system or language. A number of these are available ranging from directed graphs to bond graphs and energy flow diagrams. Each of these systems have advantages and disadvantages and have given used with varying degrees of success in ecosystem modelling. bond graphs in particular are highly compatible with thermodynamic concepts.

The use of techniques such as graph theory has been common in dynamic systems theory for some time as methods used for system identification (Siljak, 1977). In ecology network models have been suggested as a possible way to integrate the systems and population ecology approaches to ecosystem modelling.

Familiarity with network modelling techniques can lead to the rapid development of systems models in a field setting and if done properly allows the development of an algebraic model representation to which the usual economic optimization techniques such as linear programming, dynamic programming and optimal control theory may be applied.

The following list (Mikulecky, 1991) provides an overview of network languages that have been used within the network thermodynamic paradigm by ecologists, space restrictions prevent the presentation of detailed features of individual languages.

- compartment models
- signal flow graphs
- bond graphs
- energy circuits
- equivalent circuits
- Hill/King-Altman diagrams
- metabolic maps
- state and transition models (Markov chains)

All of these representation techniques may be more generally formulated as directed graphs. In fact according to Mikulecky (1991) graph theory provides the basis for network thermodynamics (Mikulecky, 1991, p. 78).

In addition Mikulecky claims that network thermodynamics encompasses dynamic systems models, chemical kinetics and both equilibrium and disequilibrium thermodynamics.

The use of bond graphs originated in electronic circuit modelling, but they have been applied to ecosystem modelling by Schnakenberg (1981) and Birkett (1994).

Energy and nutrient flow diagrams (energy circuits) were first used by Odum (1971) but have also been used by human ecologists to model energy flows in ecosystems, including economic and social subsystems (Ellen, 1982).

State and transition models are currently in vogue in range ecology but they have also been used in forestry and in economic optimal stocking models in range management. For an overview see the special edition of *Tropical Grasslands on state and transition models* and especially the Scanlan (1994). Because state and transition models involve the use of Markov chains they can be represented using graph theoretic techniques. This can be

done by representing the states of the system as nodes of a graph and the transitions between states as branches between nodes. These transition branches are then weighted with the probability of transition from one state to another.

Markov chain transition matrices are a special case of incidence (Boolean) matrices which allow one to represent a graph theoretic model of a network in matrix form. Mikulecky (1984) and Siljak provide an overview of the use of incidence matrices in modelling dynamic systems.

If one models an ecosystem as a food web, with each pathway in the web reflecting a simple chemical reaction, then food webs can be expressed in the form of chemical reaction mechanisms, ie. as a thermodynamic reaction network.

Consider a simple autotroph-heterotroph system. Then primary production of the autotroph may be represented by an autocatalytic chemical reaction.



where a resource A is converted by an autotroph population X into more autotroph. This simple chemical reaction mechanism is not appropriate for a heterotroph. Heterotroph production is catalytic:



where E is an enzyme and not a resource. Nevertheless autocatalysis will often be assumed for reasons of simplicity.

The law of mass action implies that the first of the above systems may be written as a system of differential equations of the form

$$\frac{dx}{dt} = \gamma AX - \gamma' X^2 \quad (3.3)$$

The change in the specific entropy of this system is then given by

$$\dot{S} = \frac{1}{T} \sum_{\rho} A_{\rho} \frac{dN_{\rho}}{dt} \quad (3.4)$$

where

$$A_{\rho} = - \sum_i \nu_{i\rho} \mu_i \quad (3.5)$$

is the affinity of the reaction ρ and $i = X$, A and $\frac{dN_{\rho}}{dt} = \frac{dx}{dt}$. Note that the index ρ refers to the number of state equations in the reaction network and i to the number of terms in each reaction. μ_i is the chemical potential per mole.

Under certain simplifying assumptions local entropy production may be written as,

$$\sum_{\alpha} X_{\alpha} J_{\alpha} \geq 0 \quad (3.6)$$

or in matrix form

$$XJ \geq 0, \quad (3.7)$$

J_α is a flow variable or rate of a chemical reaction and X_α is a “generalized thermodynamic force” (Glansdorff et al., 1971, p. 18). In chemical or kinetic reactions $X_\alpha = A_\rho T^{-1}$.

According to Glansdorff and Prigogine entropy production in this sense possesses two aspects, thermodynamic aspect via the forces and a kinetic aspect via the flows (Glansdorff et al., 1971, p.19). This leads the way to the possibility of unifying population and systems ecology on the basis of generalized thermodynamics.

In thermodynamic equilibrium $\dot{S} = 0$ otherwise, ie. in disequilibrium entropy production will be greater than zero. This leads to the reaction affinity A_ρ vanishing in equilibrium because the sum of the product of the stoichiometric coefficients and the chemical potentials μ_i cancel out.

The introduction of system boundaries into such network models, e.g. ecotone boundaries, organizational and institutional boundaries etc., allows the application of Podolinsky’s principle. Which states that a system is only then sustainable when $\dot{S} \leq 0$. Note that the total change in entropy consists of the sum of the change in the “within” system and “between” system entropy,

$$dS = d_i S + d_e S \geq 0, \quad (3.8)$$

where $d_i S = 0$ for a reversible process and $d_i S > 0$ for an irreversible process.

Podolinsky’s principle combined with the second law of thermodynamics implies that $dS = 0$. For reversible processes this implies that the entropy flow between the system and its environment is zero, which amounts to a requirement of energetic self-sufficiency. The reality is that most biological processes will, at an aggregate level, be subject to irreversibly. Thus, for $d_i S > 0$, $d_e S < 0$ or a net import of energy must hold for Podolinsky’s principle to hold. An example of this is the application of fertilizer to agricultural production processes that are characterized by thermodynamic irreversibly.

The fundamental theorems of network thermodynamics that are likely to be of interest to ecological economists are Kirchhoff’s law and Tellegen’s theorem (Mikulecky, 1991). Kirchhoff’s (flow) law states that all flows are equal between nodes of a network.

$$\begin{aligned} AJ &= 0 \text{ Kirchhoff's current law.} \\ X &= A^T c \text{ Kirchhoff's voltage law.} \end{aligned} \quad (3.9)$$

where A is the incidence matrix of the network and c is the density of biomass in a given compartment of volume $c = \frac{n}{V}$ or the density of biomass in a given compartment of volume V . From these laws collectively known as Kirchhoff’s flow law Tellegens theorem may be derived (Mikulecky, p. 166, 1984).

Tellegen’s theorem states that resource flows and forces (eg. differences in populations in two pools) are orthogonal to each other in any linear thermodynamic network and amounts to an assumption concerning thermodynamic equilibrium.

4 A Simple Example: The Lotka -Volterra Model

In the following we shall consider two cases, that of an equilibrium system in the thermodynamic sense and that of a disequilibrium system, again in the thermodynamic senses.

To do this I follow Schnakenberg's (1977) analysis of the Lotka-Volterra system from a reaction kinetic perspective (Schnakenberg, 1981). Given the following autocatalytic process:



where A is some external source of energy, e.g. water, light and nutrients and the last equation describes the death of the predators.

By applying the law of mass action the following set of reaction rates may be defined,

$$\begin{aligned} J_1 &= \gamma_1 AX - \gamma'_1 X^2, \\ J_2 &= \gamma_2 XY - \gamma'_2 Y^2, \\ J_3 &= \gamma_3 Y - \gamma'_3 B, \end{aligned} \quad (4.2)$$

where the γ_i are the forward reaction rates and γ'_i , are the reverse reactions rates. The population dynamics of the system may now be written as,

$$\begin{aligned} \dot{x} &= J_1 - J_2 \\ \dot{y} &= J_2 - J_3. \end{aligned} \quad (4.3)$$

Substituting J_1 , J_2 and J_3 from equation ??, into these gives,

$$\begin{aligned} \dot{x} &= \gamma_1 AX - \gamma'_2 X^2 - \gamma_2 XY + \gamma'_2 Y^2, \\ \dot{y} &= \gamma_2 XY - \gamma'_2 Y^2 - \gamma_3 Y + \gamma'_3 B. \end{aligned} \quad (4.4)$$

According to Schnakenberg (1981) such a system may be interpreted as “and extreme non-equilibrium situation” by setting the reverse fluxes $\gamma'_i = 0$ (Schnakenberg, 1981, p. 23).

$$\begin{aligned} \dot{x} &= \gamma_1 AX - \gamma_2 XY \\ \dot{y} &= \gamma_2 XY - \gamma_3 Y. \end{aligned} \quad (4.5)$$

Thus for $a = \gamma_1 A$, $b = \gamma_2 = c$ and $d = \gamma_3$, the original Lotka-Volterra system may be interpreted as a system far from thermodynamic equilibrium.

This is interesting because it means that thermodynamic disequilibrium systems are likely to be similar to analyze than ecosystems in thermodynamic equilibrium.

Using network thermodynamics the calculation of the entropy production associated with a given system is straightforward. further, by analysing the management of ecosystems using control theory and calculating the associated levels of entropy production some statement regarding the sustainability of the system may be made that is based on thermodynamic laws.

5 Optimal Control of Thermodynamic Networks

Consider the following problem,

$$\max_v \int_0^\infty u(x, y, v) e^{-rt} dt \quad (5.1)$$

subject to

$$\begin{aligned} \dot{x} &= ax - bxy \\ \dot{y} &= cxy - vdf \end{aligned} \quad (5.2)$$

where $y = fv$. Note that the control variable v impacts on the state variable x via a differential term \dot{y} . Control problems of this type are known as indirect control problems (Lefschetz, 1965).

This problem may also be formulated as a stochastic control problem by appending a stochastic term to the kinetic equations,

$$\begin{aligned} dx &= [ax - bxy]dt + \sigma^2 x^2 dW_x(t), \\ dy &= [cxfv - dev]dt + \sigma^2 (fv)^2 dW_y(t), \end{aligned} \quad (5.3)$$

where $W_x(t)$ and $W_y(t)$ are white noise terms. The solution v^* to the problem may then be found via the Hamilton-Hacobi-Bellman equation.

$$0 = J_t + \max_v \{u(x, y, v)e^{-rt} + J_x[ax - bxfv] + J_y[cxfv - dev] + J_{xx}\sigma^2 x^2 + J_{yy}\sigma^2 f^2 v\} \quad (5.4)$$

Explicit solution of this although analytically possible will often require numerical techniques. I will not examine this here in anymore details because the emphasise in this paper is on stability issues as these are intimately linked to the problem of sustainability.

In certain areas of ecology, notably range ecology, there is currently considerable debate concerning the disequilibrium nature of ecosystems. The use of the term disequilibrium in range ecology, although often not clearly specified, in one interpretation may be understood to mean that such ecosystems are not in thermodynamic equilibrium and does not necessarily mean that the system does not possess a steady-state. In fact the application of thermodynamic stability criteria such as the Glansdorff-Prigogine criteria assumes implicitly the existence of a steady-state the stability of which is then evaluated.

In a stochastic setting the concept of a steady state finds its analogue in the stationary or ergodicity (long run average behaviour) of stochastic processes. Steady-state conditions for stochastic control systems may then be derived by means of the Fokker Planck or Kolmogorov forward equation (See below).

6 Stability Criteria: Holling resilience and Glansdorff-Prigogine Stability

In analysing the stability of dynamic ecosystems, two approaches may be taken depending on whether the underlying system is linear or non linear. In linear systems, the

eigenvalues of the characteristic equation are analysed to determine local stability of the system. Global stability is however another issue. In non-linear systems it is this issue of global stability that is then central to the analysis.

In a discussion of the ecological economics of sustainability Common and Perrings (1992) have used ecosystemic stability as a sustainability criteria. In distinguishing between Holing-sustainability and Holling resilience they are actually drawing on the systems theoretic concepts of local and global sustainability. According to Common and Perrings a system may be said to be Holling resilient (globally stable) when the probability density function of the systems parameters is a Lyapunov function (common and Perrings, 1992). The problem with this definition is that a probability density function cannot be a Lyapunov function. The reasons for this I will discuss in detail later in this paper. Instead an alternative definition of global stability is needed that may be used to define Holling resilience.

The stability of non-linear thermodynamic systems is generally analysed by using Lyapunov's direct or second method. If ecosystems are viewed as thermodynamic systems then sustainability criteria also need to be analysed from this perspective. The Glansdorff-Prigogine criteria for thermodynamic stability of a steady-state is defined in terms of the entropy function. In the following I will primarily follow Schnakenbergs treatment of the Flansdorff-Prigogine criteria.

Thus consider the entropy function $S(\bar{x})$, where \bar{x}^* is a vector of steady-state values of state (enable) variables. Recall from above that the variation in total entropy is,

$$dS = dS_i + dS_e \quad (6.1)$$

and that

$$dS = \frac{1}{T} \sum_{\rho} A_{\rho} dN_{\rho} \quad (6.2)$$

where $dN_{\rho} = \sum_i \frac{dN_{i\rho}}{dt}$. $dS = 0$ is a necessary but not a sufficient condition for maximal entropy, ie. thermodynamic equilibrium. To obtain a sufficient condition one requires a second order condition.

Expanding dS as a second order term gives

$$d^2S = -\frac{1}{T} \sum_i d\mu_i dN_{\rho} < 0 \text{ global stability} \quad (6.3)$$

or

$$d^2S = -\sum_{\alpha} \partial X_{\alpha} \partial J_{\alpha} < 0 \text{ local stability} \quad (6.4)$$

Taking the time derivation of this and rewriting gives the Lyapunov function $L = -\frac{1}{2}(d^2S)^e$ with the properties $L \geq 0$ and $\frac{dL}{dt} \leq 0$. The latter may then be considered to be a direct consequence of the second law of thermodynamics.

Note that the above argument holds for a system in thermodynamic equilibrium. In

disequilibrium systems a more general treatment is required.

In thermodynamic disequilibrium the above derivation is reversed. Glansdorff and Prigogine refer to this as a “complementarity principle” (Glansdorff and Prigogine, 1971, p. 66). The main contribution of Glansdorff and Prigogine to the literature involves extending the above approach to the case of convective and dissipative processes. This is done by replacing the specific entropy S by a linear transform

$$z = S - \frac{1}{2}T_0^{-1}v^2 \quad (6.5)$$

where v is the velocity of particles in the system. From this one can determine d^2z and given $Z = \int \rho z dV$ derive the global stability conditions:

$$\begin{aligned} d^2Z &< 0 \\ \partial_t d^2Z &\geq 0 \end{aligned} \quad (6.6)$$

From a system at rest (in the sense of mechanics) the above condition is equivalent to the equilibrium condition because the velocity of particles v is zero and $Z = S$.

This criteria may then be applied to a thermodynamic reaction network. Note that Podolinsky’s principle really refers to some subsystem of the whole. Thus podolinsky’s principle is a systems ecological counterpart to Paine’s and Watt’s conjecture in population dynamics (Siljak, 1978, p. 294):

- Can the total community be stable given an unstable trophic level? (Paine’s conjecture)
- Can the total community be unstable given a stable trophic level? (Watt’s conjecture)

Both Paine and Watt’s conjectures may therefore be reformulated in the light of both Podolinsky’s principle and the Glansdorff-Prigogine Stability criteria. What is the relationship between Podolinsky’ principle and the Glansdorff-Prifofine criteria?

One answer to this question is indicated by the distinction made by Flansdorff and Prigogine between local and global entropy production. In the treatment given above I have only considered local entropy production.

$$\sum_{\alpha} X_{\alpha} J_{\alpha} \geq 0 \quad (6.7)$$

From this one may derive global entropy production by integrating over the volume of the different compartments.

$$\int \sum_{\alpha} X_{\alpha} J_{\alpha} dV \geq 0 \quad (6.8)$$

Thus the whole system may well be in disequilibrium although a given subsystem may be in equilibrium. The latter may be seen as a criteria of sustainability on subsystems

of a given ecosystem. This is Podolinsky's principle, Global application of Podolinsky's principle would however only make sense given an open system. This could be realised by using a spatial stepping stone model with imports and exports of energy and matter between the ecosystem being modelled and it's environment.

The thermodynamic equilibrium $S(\bar{x}^*) = 0$, whereas in non-equilibrium thermodynamics $S(\bar{x}^*) \geq 0$. Thus for a deterministic network if one can define an entropy function for the system and apply the Glansdorff-Prigogine criteria, then one has found a definition of Holling resilience that might be used as a sustainability criteria, ie. a system may be said to be holling resilient if it is stable in the sense of Glansdorff and Prigogine.

The problem which Common and Perrings were trying to solve however, was that of the stability of an uncertain resource system. In the presence of uncertainty the equivalence of the Glansdorff- Prigogine criteria and Holling resilience breaks down. Instead, an alternative approach to stability has been suggested by Siljak (1978). This approach involves applying optimal stochastic control techniques to the analysis of uncertain resource systems and then using the differential operation as a Lyapunov function.

The Glansdorff-Prigogine criterion has been criticized by Lavenda (1978) (Lavenda, 1978, pp.112-115). Lavenda's Primary objection to the Glansdorff-prigogine criteria of thermodynamic stability is that the second variation of specific entropy used by Glansdorff and Prigogine is not a Lyapunov function in the strict sense as it refers to the equations of motion of the linearly perturbed original system of kinetic equations. In a sense this is analogous to the approach taken by Perrings and Common in defining Holling resilience. A second objection to this approach and this has implications for both the Glansdorff-Prigogine and the Perring-Common (understood in its widest sense) approaches, is that an appropriate Lyapunov function may not always exist (Lavenda, 1978, p. 114). Finally, the existence of such a function and its association with the second law is only postulated and never proven.

In light of this, another alternative which is in the spirit of Common and Perrings original idea but is not based on the idea of a Lyapunov function is to use the Fokker-Planck equation (Itô, p. 369, 1982). In our example, the Lotka-Volterra model, delivers two Fokker-Planck equations (alternatively a vector valued Fokker-Planck equation could have been derived).

$$\begin{aligned}\frac{\partial p(x)}{\partial t} &= \left(-\frac{\partial}{\partial x}[ax - bxfv] + \frac{1}{2} \frac{\partial^2}{\partial y^2}[\sigma^2(ev)^2] \right) p(x) \\ \frac{\partial p(y)}{\partial t} &= \left(-\frac{\partial}{\partial y}[cxfv - dev] + \frac{1}{2} \frac{\partial^2}{\partial y^2}[\sigma^2(ev)^2] \right) p(y)\end{aligned}\tag{6.9}$$

Perrings and Common introduced the concept of Holling resilience in terms of structural stability of an ecological economic (including bioeconomic) system. The approach taken by Perring and Commons is based on applying Lyapunov's direct method to the analysis of the stability of ecological economic systems. Perrings and Common examine a system of the following type

$$\dot{\bar{x}} = f(\bar{x}, \varepsilon)\tag{6.10}$$

where ε is some random parameter. They argue that if the probability density function of ε is a Lyapunov function then this system is stable and call stability in this sense Holling resilience.

The danger of this approach is that if ε is random then \dot{x} may not exist because $x(t)$ may be everywhere non-differentiable with respect to time. It is for this reason that stochastic calculus was developed.

A second problem is that the probability density function of ε will not normally be a Lyapunov function. Intuitively, this can be seen if one conceives of a Lyapunov function as some bowl shaped function which places a bound on perturbations of the system around the steady-state. A more detailed critique would require replacing the above function by a stochastic differential equation and then analysing whether the probability density function is a Lyapunov function or not.

Note that a function L is a Lyapunov function if;

$$\begin{aligned} L(\bar{x}) &> 0 \\ \frac{\partial L}{\partial t} &\leq 0 \end{aligned} \tag{6.11}$$

In order to see whether a probability sensitivity function is a Lyapunov function or not we need to see whether the second of these conditions is fulfilled or not.

To do this note that the dynamics of a probability density function are defined by the Fokker-Planck or Kolomogorov forward equation,

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial x}[a(x, t)p] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[b(x, t)p] \tag{6.12}$$

where $a(x, t)$ is the drift term and $b(x, t)$ the diffusion term.

This can be done by evaluating the Fokker-Planck equation for the zero drift case, as this is analogous to the existence of a steady-state in the Perrings-Common model, with all perturbations around this being noise induced. If one standardises the diffusion term to one, then without loss of generality the Perrings-Common model implies that the second partial derivative of the Gaussian density with respect to the random variable of interest must be everywhere negative. This is clearly not the case as one may see by inspection of a normal curve. It is easy to see that this will also not be the case for a wide range of common distributions.

According to Perrings and Common this would imply that the system under study is not Holling resilient. The implication is however much stronger, it is that virtually no system can be Holling resilient according to their criteria. The concept as defined by Perrings and Common consequently loses its usefulness.

An alternative way of capturing the idea of Holling resilience, at least for deterministic systems, is to analyse the structural stability of the system via Peixoto's theorem.

Theorem 6.1. *Peixoto* If f is a once continuously differentiable vector field defined on a compact two dimensional differentiable manifold, if and only if

1. the number of critical points and cycles is finite and hyperbolic.
2. there are no trajectories connecting saddle points.
3. the non-wandering set consists of limit cycles and critical points only.

Imposing the Holling resilience criteria of Common and Perrings on the Fokker-Planck equation, so that $\frac{\partial p}{\partial t} \leq 0$ and $p(t) > 0$ would then following the argument of Common and Perrings amount to imposing a condition of sustainability on the system. Note, that if $\frac{\partial p}{\partial t}$ then the system is not in a steady-state, therefore the density function cannot be a Lyapunov function. What implications this is likely to have for entropy production is however an issue that goes beyond the present paper.

7 Conclusion: Integrating Ecological Economics and Bioeconomics

The split within both ecology and economics between different schools is unfortunate. Within economics in particular the split into arguably four different schools:

- environmental economics
- natural resource economics including bioeconomics
- ecological economics
- sustainable development

is not conducive to ecologically and economically informed dialogue on environmental issues. The dangers have been recognized within ecology and attempts are being made to address this issue as a number of recent publications show. The spirit of Norgaards (1989) plea in the first issue of ecological economics is still very current. However, there is perhaps a need to go beyond methodological pluralism towards attempting to integrate alternative perspectives into a common framework. In this paper I have sketched one possible means of integrating the approach taken by ecological economists with that taken by bioeconomists. This approach is based on developments that have been occurring within ecology over a period of years but have largely been confirmed to theoretical and methodological debate within ecology.

A network thermodynamic approach leads to the application of thermodynamic principles to bioeconomic models, whereby thermodynamic stability and a contemporary analogue of Podolinsky's principle, based on it's reinterpretation in the light of the thermodynamics of irreversible processes, become measures of the ecological performance of the bioeconomic system in a similar manner to the role Pareto efficiency has played in welfare economics.

Unfortunately the analysis is at present confined to autocatalytic networks. The literature on replicator dynamics which has involved linking evolutionary and thermodynamic theory has analyzed catalytic networks, however embedding catalytic reaction networks within a bioeconomic framework is still fraught with difficulties.

The assumption of autocatalysis is a simplifying assumption that in some ways is unacceptable to empirical modellers. The problem however is how one can embed a catalytic network (ie. enzyme driven) within a bioeconomic control theoretic framework, ie. how does one define the control set? The answer to this question is still unclear.

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References

- [1] Beard, R.M., 1994. Optimal Stochastic Control of Renewable but Irreversible Resources. Agricultural Economics Discussion Paper 2/94, University of Queensland, St. Lucia.
- [2] Birkett, S.H., 1994. On the interconnection of ecological system models. *Ecological Modelling*, 72: 152-174.
- [3] Brown, J.H., 1995. Organisms and species as complex adaptive systems: linking the biology of populations with the physics of ecosystems. In: Jones, C.G. and Lawton, J.H. (Editors). *Linking Species and Ecosystems*. Chapman and Hall, New York PP. 387.
- [4] Common, M. and Perrings, C., 1992. Towards and ecological economics of sustainability. *Ecological Economics* 6: 7-34.
- [5] Eigen, M., 1971. Selforganization of matter and the evolution of biological macromolecules. *Die Naturewissenschaften*, 58: 465-523.
- [6] Ellen, R., 1982. *Environment, Subsistence and System: The Ecology of Small-Scale Social Formations*. Cambridge University press, Cambridge, pp. 324.
- [7] Georgescu-Roegen, N., 1971. *The Entropy Law and the Economic Process*, Harvard University Press, Cambridge, Massachusetts, pp. 457.
- [8] Glansdorff, P. and Prigogine, I., 1971. *Thermodynamic Theory of Structure, Stability and Fluctuations*. Wiley, pp. 306.

- [9] Itô, K., 1982. Diffusion Processes. In: Kotz, S., Johnson, M.L. and Read, C.B. (Editors), 1982. Encyclopedia of Statistical Sciences. Wiley, New York.
- [10] Jeffries, C., 1979. Stability of Holistic Ecosystem Models. In: Halfon, E. (Editor), 1979. Theoretical Systems Ecology: Advances and Case Studies. Academic Press, New York, pp. 516.
- [11] Lavenda, B.H., 1978. Thermodynamics of Irreversible Processes. MacMillan, London, pp. 182.
- [12] Lefschetz, S., 1965. Stability of Nonlinear Control Systems. Academic press, New York pp. 150.
- [13] Lotka, A.J., 1956, Elements of Mathematical Biology. Dover, New York, pp. 465.
- [14] Martinez-Alier, J., 1987. Ecological Economics: Energy, Environment and Society. Blackwell, Oxford, pp. 287.
- [15] Mikulecky, D.C., 1984. Network thermodynamics: a simulation and modeling method based on the extension of thermodynamic thinking into the realm of highly organized systems. *Mathematical Biosciences* 72: 157-179.
- [16] Mikulecky, D.C., 1991. Network thermodynamics: a unifying approach to dynamic nonlinear living systems. In: Higashi, M. and Burns, T.P. (Editors), *Theoretical Studies of Ecosystems: The network Perspective*. Cambridge university Press, Cambridge, pp. 364.
- [17] Norgaard, R.B., 1989. The case for methodological pluralism. *Ecological Economics*, 1: 37-57.
- [18] Odum, E.P., 1971. *Fundamentals of Ecology*. Saunders College Publishing, Philadelphia, pp. 574.
- [19] Perko, L., 1991. *Differential Equations and Dynamical Systems*. Springer-Verlag, New York, pp. 407.
- [20] Reiners, W.A.. Complementary models for ecosystems. *The American Naturalist*, 127: 59-73.
- [21] Rosenberg, A., 1994. *Instrumental Biology or the Disunity of Science*: University of Chicago Press, Chicago, pp. 193.
- [22] Scanlan, J.C., 1994. The use of state and transition models for predicting vegetation change in rangelands. *Tropical Grasslands*, 28: 229-240.
- [23] Schnakenberg, J., 1981. *Thermodynamic Network Analysis of Biological Systems*. Springer-Verlag, Berlin, pp. 149.
- [24] Siljak, D., 1978. *Large-Scale Dynamic Systems*. North-Holland, New York, pp. 416.
- [25] Stadler, P.F., 1994. The Influence of Mutation on Autocatalytic Reaction Networks. *Mathematical Biosciences*, 122: 127-160.

- [26] Wicken, J.S., 1986. Evolutionary self-organization and entropic dissipation in biological and socioeconomic systems. *Journal of Social and Biological Structures*, 9: 261-273.
- [27] Wicken, J.S., 1987. *Evolution, Thermodynamics and Information: Extending the Darwinian Program*. Oxford university Press, New York, pp. 243.