How does bank equity affect credit creation? Multiplier effects under Basel III regulations

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How does bank equity affect credit creation? Multiplier effects under Basel III regulations⋆,⋆⋆

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Abstract

Both equity and regulation play key roles in determining the ability of banks to create credit. Equity varies endogenously, while regulations are exogenously imposed. This study proposes a banking model to investigate how changes in bank equity due to interest receipts and expenditures affect credit and money creation under Basel III regulations. Four Basel III regulations—the capital adequacy ratio, the leverage ratio, the liquidity coverage ratio, and the net stable funding ratio—are discussed. Their effect on credit creation are demonstrated by the changes that occur in the credit supply in response to the changes in equity arising from interest payments. This study identifies seven regulatory scenarios under these four regulations. In each scenario, there exists a multiplier that relates the change in equity to the resultant change in the credit supply. Correspondingly, there is a multiplier effect on the money supply. This study sheds new light on how bank equity and Basel III regulations affect credit and money creation.

Keywords: Credit creation, Bank equity, Interest payments, Multipliers, Basel III, Balance sheets

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1. Introduction

As Adrian and Shin (2010a, b, 2011, 2014) have pointed out, banks’ equity behaves like a predetermined variable and affects the credit supply. Therefore, it is necessary to clarify the relationship between the amount of equity and the supply of credit and money. However, because the traditional money multiplier model abstracts from bank equity, it sheds no light on this issue. To explore this issue, Adrian and Shin (2010a, b, 2011); Bezemer (2010); Li and Wang (2020); McLeay et al. (2014); Werner (2014b) proposed a banking model based on the balance sheet called the “bank balance sheet model.” In this model, starting with a predetermined amount of equity, banks expand or contract their balance sheets. However, this expansion or contraction will be affected or limited by risk management, bank regulations, etc. Adrian and Shin (2014) demonstrated how risk management constraints, such as Value-at-Risk, determine the relationship between banks’ equity and the credit supply. This paper will explore how bank regulations, such as Basel III regulations, affect the relationship between banks’ equity and their credit or money creation.

To address this issue, this paper develops a bank balance sheet model based on the credit creation theory of banking. This theory demonstrates that banks have the ability to create credit and money (Bezemer, 2010; Li and Wang, 2020; Jakab and Kumhof, 2015; McLeay et al., 2014; Werner, 2014a,b, 2016). The expansion and contraction of balance sheets by banks influences the creation and destruction of credit and money. Using this model, regulations become a regulatory relationship imposed on balance sheet quantities. The credit creation theory of banking suggests that both the equity position and the regulatory relationship will limit the amount of credit and money that banks can create. This is not, however, the end of credit and money creation. Bank equity will change endogenously: it is accumulated from retained earnings (i.e., net interest income). When net interest income causes an increment in equity, banks then
adjust their supply of credit and money in response to the change in their equity. So, the question then becomes: how are changes in the credit and money supply related to changes in equity under Basel III regulations?

This paper focuses on four regulations that were introduced under the Basel III accord. Basel III introduced two capital regulations—the capital adequacy ratio (CAR) (Basel Committee on Banking Supervision, 2011) and the leverage ratio (LR) (Basel Committee on Banking Supervision, 2014a). The CAR and the LR require banks to hold sufficient capital to avoid bank failures caused by adverse shocks. Such shocks include a reduction in bank capital or threats to bank solvency such as a decline in security prices and loan defaults. The CAR is a risk-based capital regulation, while the LR is a non-risk-based capital regulation. The first version of the CAR was introduced by Basel I and updated by Basel II. Basel III further strengthens the CAR by raising the capital quantity and quality requirements. By contrast, the LR is a new capital regulation proposed by Basel III. The LR is aimed at restricting banks’ leverage and acts as a backstop to the CAR. In addition, it has been widely recognized that merely having adequate capital does not ensure the soundness of banks. The liquidity difficulties faced by banks during the 2008 financial crisis emphasized how crucial it is for banks to maintain sufficient liquidity buffers and stable funding sources.

To address this issue, Basel III proposed two liquidity regulations—the liquidity coverage ratio (LCR) (Basel Committee on Banking Supervision, 2013) and the net stable funding ratio (NSFR) (Basel Committee on Banking Supervision, 2014b).

The aforementioned four regulations can be divided into seven regulatory scenarios. In each scenario, a multiplier that relates the change in equity given by net interest income to the change in the credit supply can be used to determine the relationship between bank equity and the credit supply. Similarly, another multiplier gives the relationship between equity and the money supply. Furthermore, there are three main findings associated with each scenario as follows. First, this study answers whether the credit supply increases or decreases when equity increases. Second, this study identifies the determinants of the
multipliers. Third, the multipliers that relate a change in equity to a change in
the money supply are presented.

There is one scenario subject to the CAR. An increase in equity increases
the credit supply. Furthermore, the CAR leads to a multiplier greater than one
and amplifies the change in the equity. Similarly, one scenario is linked to the
LR. These results also hold in the LR scenario.

In contrast to the CAR and LR scenarios, the LCR has four scenarios.
According to Basel Committee on Banking Supervision (2013), the LCR has
two different regulatory regimes—cash inflows greater than or equal to three-
quarters of the cash outflows (labeled regime H) and cash inflows less than
three-quarters of the cash outflows (labeled regime L). The LCR regimes can
switch due to a change in equity. This results in four scenarios as follows: (i)
regime H before and after interest payments (denoted scenario HH-LCR), (ii)
regime L before and after interest payments (denoted scenario LL-LCR), (iii)
regime L before and regime H after interest payments (denoted scenario LH-
LCR), and (iv) regime H before and regime L after interest payments (denoted
scenario HL-LCR). In scenarios HH-LCR, LL-LCR, and LH-LCR, increasing the
equity raises the credit supply. On the contrary, in scenario HL-LCR, increasing
the equity decreases the credit supply.

In scenarios HH-LCR and LH-LCR, the multipliers are exactly equal to one:
the change in the credit supply equals the change in the equity. In scenario
LL-LCR, the multiplier is greater than one and the LCR therefore amplifies
the equity changes. In scenario HL-LCR, the multiplier can be greater or less
than one, and the equity changes are either amplified or contracted. Note
that multipliers can also be linked to bank liquidity. In scenario LL-LCR, the
multiplier effect and amplification are increasing in the liquidity of banks. On
the contrary, in scenario HL-LCR, the multiplier is decreasing in the liquidity of
banks: an increase in bank liquidity either increases the contraction or decreases
the amplification.

Finally, one scenario is associated with the NSFR: an increase in the equity
raises the credit supply. The multiplier ranges from less than one to greater than
one. The NSFR can cause either a contraction or an amplification of the change in equity. This multiplier can also be related to bank liquidity. This is most readily observable by considering the special case in which the minimum NSFR requirement takes the value of one, as Basel III requires. Then, the multiplier is greater than or equal to one and increasing in the liquidity of banks. So the amplification arising from the multiplier effect is also increasing in the liquidity of banks.

Thus far, multiplier effects on the credit supply have been shown. Banks creating or destroying credit implies that they are creating or destroying money at the same time and by the same amount. Thus, for each scenario, the link between the change in equity and the change in the money supply can be obtained.

The main contributions are as follows. (i) This study finds that the micro-prudential regulatory framework established by Basel III can be divided into seven regulatory scenarios. In each regulatory scenario, this study provides the multipliers that relate banks’ equity to their supply of credit and money. (ii) The multipliers clarify the impact of regulations on banks’ ability to create credit and money. (iii) The LCR has four scenarios. There exists one scenario in which an increase in bank equity leads to a reduction in the supply of credit and money.

These results offer three main policy implications. First, my model reveals the roles that the parameters introduced by these regulations play in determining the supply of credit and money. In particular, the supply under the regulations are linked to their stringency. These findings can help policymakers control the volatility of the credit supply due to interest payments by adjusting these regulations. Second, my results concerning the LCR and NSFR provide a better understanding of how bank liquidity influences their supply of credit and money. Third, my results may help policymakers to see how the policy interventions that influence banks’ interest income or expenses affect the supply of credit and money under the LCR.
2. Literature review

My paper belongs to the stream of literature that develops theoretical banking models to examine the effects of bank regulations. In particular, these studies draw on the credit creation theory of banking with bank balance sheet models. Li and Wang (2020); McLeay et al. (2014); Werner (2014a,b, 2016) provide details regarding the credit creation theory of banking. Adrian and Shin (2010a,b, 2011); Li and Wang (2020); McLeay et al. (2014); Werner (2014b) show the foundations of bank balance sheet models by illustrating the expansion and contraction of bank balance sheets with predetermined equity. Furthermore, Adrian and Shin (2014) explore how banks’ risk management affects the supply of credit at a given level of equity.

More closely related to my paper, Li et al. (2017); Xing et al. (2020); Xiong et al. (2020) develop bank balance sheet models to discuss credit and money creation by banks under Basel III regulations. However, their models abstract from interest payments. Without interest payments, their models treat bank equity as a constant and mute the influence of the current amount of credit and money on the following creation of them. Additionally, because interest payments are a key determinant of banks’ cash flows, the models ignore the significant influence of interest payments on the credit and money supply under the LCR.

By contrast, this study extends the bank balance sheet model by incorporating interest payments on credit and deposits and adjustments in bank balance sheets in reaction to changes in equity due to interest payments. In this way, this paper considers the two-way influence between a change in equity and the creation of credit and money. This also enables us to provide a more complete discussion of the LCR (consisting of four scenarios) and obtain a more accurate solution for how the credit and money supply react when subject to the LCR. Moreover, under the credit creation theory of banking, the literature that develops bank balance sheet models to explore the determinants of the credit and money supply is nascent. By including the aforementioned necessary building
blocks, this model intends to provide a benchmark for future research.

Contrary to the credit creation theory of banking, most of the theoretical banking literature builds on the financial intermediation theory of banking. In these papers, banks intermediate funds rather than create credit and money simultaneously. Early papers focus on the effects of the CAR on the credit supply. More recent papers in this strand consider the impact of liquidity regulations or the combination of capital and liquidity regulations on the credit supply. The main findings of these papers are summarized in Table 1.¹

Note that these papers model banks as intermediaries. Rather than bank regulation, it is the supply of deposits that is the major constraint on the supply of loans. These papers feature the effects of intermediation friction and costs on the supply of credit. Unlike these papers, this study turns attention to (i) banks' ability to create credit and money and (ii) how this ability is limited by regulation. With banks' ability to create credit and money, the intermediation friction and costs may not be of first-order importance. Additionally, compared to the literature examining only one LCR regulatory regime, my study considers both of the two LCR regimes and the effects of when the regimes switch due to interest payments.

Another important examination of the effects of Basel III regulations on the credit supply is provided by macroeconomic models. Goodhart et al. (2012, 2013) integrate bank balance sheets into a general equilibrium model. Their model emphasizes the role of the balance sheet in introducing the regulations and presents the dynamics of balance sheet quantities. They reveal that the CAR or LCR reduces risky illiquid mortgage loans and that the LCR increases

¹For a survey of the literature on the examination of the CAR, see Martynova (2015); VanHoose (2007). Recently, several papers use theoretical banking models to exhibit the effects that arise from liquidity regulations, such as the LCR’s impact on interbank rates (Bech and Keister, 2017), the price of the securities qualified as high-quality liquid assets (Fuhrer et al., 2017), the resilience of banks (König, 2015), systemic risk measured by the fraction of banks not able to meet CAR or LCR requirements (Aldasoro and Faia, 2016), and the NSFR’s influence on banks’ debt maturity (Wei et al., 2017).
<table>
<thead>
<tr>
<th>Study</th>
<th>Main Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Francis and Osborne (2009), Furfine (2001), and Stiglitz and Greenwald (2003)</td>
<td>The increase in the stringency of the CAR causes a significant decline in the credit supply.</td>
</tr>
<tr>
<td>Kopecky and VanHoose (2004)</td>
<td>The difference between the credit supply with equity given exogenously and the credit supply with equity determined endogenously are revealed.</td>
</tr>
<tr>
<td>Van den Heuvel (2007)</td>
<td>The decrease in the equity resulting from an increase in deposit rates, reduces the credit supply.</td>
</tr>
<tr>
<td>Hyun and Rhee (2011)</td>
<td>To increase equity ratios under the CAR, banks prefer to reduce loans rather than issue new equity.</td>
</tr>
<tr>
<td>De Nicolo et al. (2014)</td>
<td>First, there exists an inverted U-shaped relationship between the credit supply and the stringency of the CAR. Second, when banks comply with the CAR, the addition of the LCR leads to a significant reduction in lending.</td>
</tr>
<tr>
<td>Balasubramanyan and VanHoose (2013)</td>
<td>Increases in loans and deposits will be caused by a rise in the spread between security and deposit rates or between loan and security rates when banks comply with the LCR.</td>
</tr>
<tr>
<td>Schmaltz et al. (2014)</td>
<td>Banks respond to the joint Basel III regulations (the CAR, LR, LCR, and NSFR) mainly by managing their debt and equity with few changes in loans.</td>
</tr>
<tr>
<td>Birn et al. (2017)</td>
<td>Banks increase their equity to meet the CAR or LR, increase high-quality liquid assets to meet the LCR, and raise the available stable funding factors to meet the NSFR.</td>
</tr>
</tbody>
</table>
riskless liquid short-term loans. Furthermore, the LCR may cause massive bank deleveraging. Macroeconomic Assessment Group (2010a,b) examines the impact of phasing in the CAR, LR, LCR, and NSFR. Implementing these regulations results in decreasing the quantity of loans and increasing loan spreads. Angelini et al. (2015); Basel Committee on Banking Supervision (2010) select several typical macroeconomic models, most commonly dynamic stochastic general equilibrium models, to examine the long-term costs and benefits of implementing the CAR and NSFR. These two papers find that the regulations affect loan spreads rather than loan quantities.

The macroeconomic models that intend to examine bank regulations need to consider the role of banks as creators of credit and money (Jakab and Kumhof, 2015). My model focuses on banks expanding and contracting their balance sheets. Then, the regulatory constraints on such bank behavior limit the supply of credit and money. This description of banks may provide a foundation for integrating bank balance sheets and the creation of credit and money into macroeconomic models. In addition, to examine regulations via macroeconomic models, it is necessary to simplify regulations, especially liquidity regulations. For example, such models abstract from switches within the two LCR regimes according to banks’ cash flow positions.

A vast amount of empirical literature examines the impact of the CAR introduced under Basel I and II on the credit supply. For a survey of this literature, see VanHoose (2006). Most of the relevant literature reports that regulations reduce the credit supply. In recent years, empirical papers pay more attention to the effects of the more stringent capital regulations and the new liquidity regulations introduced under Basel III. Similar to the CAR under Basel I and II, the Basel III’s CAR leads to a reduction in the credit supply (Gropp et al., 2019), increases in loan spreads (Slovik and Cournède, 2011), or a reduction in the credit supply together with an increase in loan rates (Cosimano and Hakura, 2011).

Relative to the examinations of the CAR, there are few efforts to explore the impact of the LCR and the NSFR mainly due to data limitations. King (2013)
finds that when banks are subject to the NSFR, they do not prefer to reduce loans with high returns but experience a decline in net interest margins. Furthermore, Naceur et al. (2018) show that the NSFR has a positive effect on lending. Other efforts investigate the effect of the LCR and NSFR on bank failure (Hong et al., 2014), the LCR on the amplification of sovereign risk (Buschmann and Schmaltz, 2017), and the LCR on term deposit facilities (a monetary policy tool that drains reserves from the banking system) (Rezende et al., 2021). In addition, several important insights into the LCR are derived from discussing two similar liquidity regulations: the Dutch liquidity ratio (DLCR) introduced in 2003, and the UK individual liquidity guidance (ILG) introduced in 2010. Bonner and Eijffinger (2016) find that the DLCR does not significantly affect loan rates. Furthermore, as Bonner (2016) demonstrates, when considering both the DLCR and the CAR, banks intend to reduce loans and increase their demand for government bonds. As for the ILG, Banerjee and Mio (2018) show that it appears to have no significant impact on loan supply or rates.

My theoretical paper complements the aforementioned empirical studies by showing the basic analytical expressions for the credit and money supply. Such expressions are linked to loan and deposit rates and the rules of the regulations.

This paper is organized as follows. Section 3 briefly describes the CAR, LR, LCR, and NSFR. Section 4 presents the model. The multiplier effects on credit and money creation under the CAR are discussed in Section 5.1, under the LR in Section 5.2, under the LCR in Section 5.3, and under the NSFR in Section 5.4. Section 6 concludes. The Appendix presents mathematical details and a glossary of the notation used.

3. A brief description of bank regulations

This section briefly describes the CAR, LR, LCR, and NSFR.

3.1. Capital adequacy ratios

The CAR requires banks to maintain a minimum ratio of capital to total risk-weighted assets. In the Basel III accord, bank capital is classified into three
types according to quality: Common Equity Tier 1 capital, Additional Tier 1 capital, and Tier 2 capital. The sum of Common Equity Tier 1 and Additional Tier 1 capital is Tier 1 capital. The sum of Tier 1 and Tier 2 capital is Total capital. Total risk-weighted assets are calculated by summing the value of each asset multiplied by its risk weight.

Banks must achieve a ratio of Common Equity Tier 1 capital to total risk-weighted assets no lower than 4.5%, Tier 1 capital no lower than 6%, and Total capital no lower than 8%. Denote by \( \text{car} \) the minimum CAR requirement. The CAR can be given by

\[
\frac{\text{Capital}}{\text{Total risk-weighted assets}} \geq \text{car}.
\]

(1)

3.2. Leverage ratios

The LR is introduced by Base III to act as a complement and backstop to the CAR. In contrast to the CAR, the non-risk based LR is independent of risk assessment, and thus the shortcomings of the risk assessment are avoided. The LR can be defined as

\[
\frac{\text{Capital measure}}{\text{Exposure measure}} \geq l_r.
\]

(2)

The capital measure is the Tier 1 capital defined in the CAR. The exposure measure is defined as the sum of on- and off-balance sheet exposures. As the LR requires, all balance sheet assets should be included in the calculation. The minimum LR requirement is denoted by \( l_r \). The minimum LR requirement is 3% under Basel III.

3.3. Liquidity coverage ratios

The LCR requires banks to maintain a sufficient stock of unencumbered high-quality liquid assets to cover the expected net cash outflows in a 30-calendar-day liquidity stress scenario. During these 30 days, regulators and supervisors are expected to take corrective and effective actions to address liquidity problems.
The unencumbered high-quality liquid assets are classified as Level 1 and Level 2 according to their liquidity.\(^2\) Level 1 assets with the highest liquidity include coins, banknotes, and central bank reserves. Level 2 assets have lower liquidity than Level 1 assets. Level 2 assets include corporate debt securities, covered bonds, and residential mortgage-backed securities. The share of Level 2 assets is up to 40% after the required haircuts. Cash outflows are the sum of outstanding balances of liabilities and off-balance sheet commitments to run off or be drawn down in the stress scenario, such as deposit run-offs and interest expenses. Cash inflows include contractual payments to be received by banks, such as principal payments and interest income on loans. The payments received should be multiplied by their inflow percentages. The cash inflows are capped at 75% of total outflows. Thus, net cash outflows for the subsequent 30 calendar days are given by

\[
\text{Net cash outflows for the subsequent 30 calendar days} = \text{Cash outflows} - \min(\text{Cash inflows}, 0.75 \times \text{Cash outflows}). \tag{3}
\]

The LCR is based on the traditional “coverage ratio” liquidity management method. The LCR can be written as follows:

\[
\frac{\text{Unencumbered high-quality liquid assets}}{\text{Net cash outflows for the subsequent 30 calendar days}} \geq \text{lcr}, \tag{4}
\]

where \(\text{lcr}\) is the minimum LCR requirement, which is 100% under Basel III.

### 3.4. Net stable funding ratios

The NSFR is another liquidity regulation for banks under Basel III to complement the LCR. It is designed to reduce maturity mismatches between assets and liabilities. The NSFR requires banks to have a stable funding profile over a one-year horizon, and it is defined as the ratio of the quantity of available stable funding (ASF) to the quantity of required stable funding (RSF).

\(^2\)Furthermore, Level 2 assets consist of Level 2A and 2B assets. According to the LCR rules, the liquidity of Level 2A assets is higher than that of Level 2B assets. For further details, see Basel Committee on Banking Supervision (2013)
The amount of ASF assesses the overall stability of banks’ funding sources. The NSFR assigns an ASF factor to each of the liabilities and capital. The ASF factor depends on the tenor and propensity of withdrawing the funding. The ASF factors vary from 0% to 100%. The more reliable the funding source, the larger the ASF factor assigned to it. For example, the ASF factor for capital takes a value of one. Multiplying capital and liabilities by their ASF factors and summing all the weighted amounts yield the amount of the ASF. On the other hand, the amount of the RSF measures the total liquidity of banks’ assets and off-balance sheet exposures. The NSFR assigns an RSF factor to each of the assets. The RSF factor is based on the tenor and liquidity of the asset. The RSF factors also vary from 0% to 100%, with the higher the liquidity, the smaller the RSF factor. Similarly, the amount of the RSF is the sum of assets weighted by their RSF factors.

Finally, we express the NSFR as follows:

\[
\frac{\text{Total available stable funding}}{\text{Total required stable funding}} \geq \text{nsfr},
\]

where \text{nsfr} denotes the minimum NSFR requirement, 100% under the Basel III accord.

4. The model

We first describe the bank balance sheets and objective functions before and after interest payments. Then the regulations described in Section 3 become the constraints on bank balance sheets. By combining the objective functions and the regulatory constraints, we can obtain the bank’s maximization problems under the regulations. Finally, the solutions for the maximization problems give the supply of credit and money.
4.1. Balance sheets and timeline

There are three dates $t = 0$, $1$, and $2$. Balance sheets and notations at date $t$ are presented in Table 2.\footnote{The balance sheet presents the stock variables. The quantity of a stock variable at date $t$ represents that of the variable at the end of the date $t$. By contrast, interest payments are flow variables. The amount of a flow variable at date $t$ represents that of the variable during the date $t$.} The balance sheet quantities satisfy the balance sheet identity:

$$L_t + S + R = D_t + E_t.$$  (6)

Here, we focus on banks creating loans and money. Securities and reserves are assumed to be constant.

Table 2

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>Loans $L_t$</td>
<td>Deposits $D_t$</td>
</tr>
<tr>
<td>Securities $S$</td>
<td></td>
</tr>
<tr>
<td>Reserves $R$</td>
<td>Equity $E_t$</td>
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Table 3

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<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tr>
<td>$L_0$</td>
<td>$D_0$</td>
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<td>$S$</td>
<td>$S$</td>
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<td>$R$</td>
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<th>Assets</th>
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<td>$L_0$</td>
<td>$D_0 - I + P$</td>
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<tr>
<td>$S$</td>
<td>$S$</td>
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<td>$R$</td>
<td>$E + I - P$</td>
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<tr>
<th>Assets</th>
<th>Liabilities</th>
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<td>$L_2$</td>
<td>$D_2$</td>
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<tr>
<td>$E$</td>
<td></td>
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</tbody>
</table>

Table 3 illustrates the balance sheets in the three dates. On date 0, banks seek to maximize their profits. Bank equity $E_0$ is given by $E$. As shown by the
balance sheet, banks earn interest on loans and securities. On the other hand, banks have to pay interest on deposits. Taking all the income and expenses into account, we obtain the profit on date 0 as

$$\Pi_0 = i_L L_0 + i_S S - i_D D_0,$$

(7)

where $i_L$ is the loan rate, $i_S$ is the security rate, and $i_D$ is the deposit rate. The objective function does not consider intermediation costs, such as adjustment or balance sheet costs. Such a specification allows us to maintain a narrow focus on how the regulations restrict credit and money creation. Rearranging the balance sheet identity in Eq. (6), we have

$$D_t = L_t + S + R - E_t.$$

(8)

Substituting Eq. (8) into Eq. (7), we obtain

$$\Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E.$$

(9)

Thus, banks choose loans to solve

$$\max_{L_0} \Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E,$$

(10)

subject to one of the CAR, LR, LCR, and NSFR constraints at date 0.

At $t = 1$, loans and securities generate interest payments to banks, which increase their equity. Deposits cause interest payments from banks, which decrease their equity. Interest receipt and expenditure are called interest payment shocks. Interest payment shocks consist of interest receipts on loans, $i_L L_0$, interest receipts on securities, $i_S S$, and interest expenditures on deposits, $i_D D_0$. To identify the effects of interest payment shocks, we need to introduce dummy variables. A dummy variable takes a value of one if the interest payment shocks include the corresponding interest receipt or expenditure and zero otherwise. The dummy variable $\sigma_L$ is associated with the interest receipt on loans, $\sigma_S$ with the interest receipt on securities, and $\sigma_D$ with the interest expenditure on deposits. Then the formula for interest payment shocks can be written as

$$\Delta E = E_1 - E = I - P,$$

(11)
where

\[ I = \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S, \]  
(12)
\[ P = \sigma_D \cdot i_D D_0. \]  
(13)

The interest payment shocks change equity to \( E_1 = E + I - P \).

At date 2, banks adjust their loans to maximize their profits. Because \( E_2 = E_1 \), from Eq. (11), the date-2 equity, \( E_2 \), also equals \( E + I - P \). Based on the maximization problem at date 0 in Eq. (10), we obtain the bank’s maximization problem at date 2 as

\[
\max_{L_2} \Pi = (i_L - i_D) L_2 + (i_S - i_D) S - i_D R + i_D (E + I - P),
\]  
(14)

subject to one of the regulatory constraints: the CAR, LR, LCR, and NSFR constraints at date 2.

4.2. Bank regulations
Section 3 briefly describes the rules of the CAR, LR, LCR, and NSFR. Based on the balance sheet in Table 2, this section shows the corresponding regulatory constraints of banks.

*Capital adequacy ratio.* Let \( \gamma_L \) be the risk weight for loans and \( \gamma_S \) be that for securities. Then, the CAR in Eq. (1) can be written as

\[
\frac{E_t}{\gamma_L L_t + \gamma_S S} \geq \text{car}.
\]  
(15)

*Leverage ratio.* The exposure measure equals the sum of all the assets \( L_t + S + R \). Then the definition of the LR in Eq. (2) can be expressed as

\[
\frac{E_t}{L_t + S + R} \geq \text{lr}.
\]  
(16)

*Liquidity coverage ratio.* First, according to the balance sheet illustrated by Table 2, reserves \( R \) and securities \( S \) compose banks’ high-quality liquid assets \( HQLA \). Let \( \chi \) denote the haircut for securities. Thus, we have

\[
HQLA = R + (1 - \chi) S.
\]  
(17)
Second, we turn to the expressions for cash inflows $IF_t$ and cash outflows $OF_t$. The cash inflows are written as

$$IF_t = \kappa(i_L + \mu)L_t,$$

(18)

where $\kappa$ is the inflow percentage, and $\mu$ is the fraction of loans repaid. On the other hand, the outflows are given by

$$OF_t = (i_D + \alpha)D_t,$$

(19)

where $\alpha$ is the run-off rate for deposits.

The LCR has two regulatory regimes associated with the expressions for the net cash outflows in Eq. (3). If $IF_t \geq 0.75OF_t$ ($\kappa(i_L + \mu)L_t \geq 0.75(i_D + \alpha)D_t$), the net cash outflows $NCOF$ are

$$0.25(i_D + \alpha)D_t.$$  

(20)

If $IF_t < 0.75OF_t$ ($\kappa(i_L + \mu)L_t < 0.75(i_D + \alpha)D_t$), $NCOF$ become

$$(i_D + \alpha)D_t - \kappa(i_L + \mu)L_t.$$  

(21)

Finally, the expression for the LCR in Eq. (4) under $IF_t \geq 0.75OF_t$ is

$$\frac{R + (1 - \chi)S}{0.25(i_D + \alpha)D_t} \geq lcr.$$  

(22)

Under $IF_t < 0.75OF_t$, the formula for the LCR becomes

$$\frac{R + (1 - \chi)S}{(i_D + \alpha)D_t - \kappa(i_L + \mu)L_t} \geq lcr.$$  

(23)

**Net stable funding ratio.** According to the rule of the NSFR, the ASF factor for equity takes a value of one. Considering the balance sheet of banks presented by Table 2, we can write the formula for the NSFR in Eq. (5) as

$$\frac{\beta D_t + E_t}{\phi_L L_t + \phi_S S} \geq nsfr,$$

(24)

where $\beta$ is the ASF factor for deposits, $\phi_L$ is the RSF factor for loans, and $\phi_S$ is the RSF factor for securities.
5. The regulatory scenarios

The discussions of the seven regulatory scenarios are based on the balance sheet dynamics and bank regulations. In each scenario, we get the difference $L_2 - L_0$ by comparing the credit supply after interest payment shocks to that before the shocks. The difference reveals the effects of interest payment shocks on the credit supply.

Credit creation drives money creation. Thus the difference between the money supply before and after the shocks, $D_2 - D_0$, is also obtained. This difference presents the effects of interest payment shocks on the money supply.

Here, we will see the main equations to determine $L_2 - L_0$ and $D_2 - D_0$ in each scenario. Basically, the main equations are derived from the first-order conditions of the bank’s maximization problems at date 0 and date 2. All the detailed derivations are relegated to Appendix A for scenario CAR, Appendix B for scenario LR, Appendix C.1 for scenario HH-LCR, Appendix C.2 for scenario LL-LCR, Appendix C.3 for scenario LH-LCR, Appendix C.4 for scenario HL-LCR, and Appendix D for scenario NSFR.

5.1. Capital adequacy ratios

At $t = 0$, the CAR constraint in Eq. (15) and balance sheet identity in Eq. (6) yield the following equations to determine $L_0$ and $D_0$:

\begin{align}
\text{car}(\gamma_L L_0 + \gamma_S S) &= E, \\
L_0 + S + R &= D_0 + E.
\end{align}

At $t = 1$, interest payment shocks $\Delta E$ given by Eq. (11) occur. Then the equity changes to $E_1 = E + I - P$. At $t = 2$, in response to the interest payment shocks, banks adjust their credit supply. The equity $E_2$ is equal to $E_1$. With $E_2$, the equations for determining $L_2$ and $D_2$ are given by

\begin{align}
\text{car}(\gamma_L L_2 + \gamma_S S) &= E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0, \\
L_2 + S + R &= D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0.
\end{align}
In summary, the system of equations to determine $L_0$, $L_2$, $D_0$, and $D_2$ is given in Eqs. (25)-(28). Solving these, we display the solutions in Appendix A. We show $L_2 - L_0$ as follows:

\[
L_2 - L_0 = \sigma_L \cdot \frac{1}{\text{car} \cdot \gamma_L} \cdot i_L L_0 + \sigma_S \cdot \frac{1}{\text{car} \cdot \gamma_L} \cdot i_S S \\
+ \sigma_D \cdot \frac{1}{\text{car} \cdot \gamma_L} \cdot (i_D D_0).
\]

(29)

From Eq. (29), $L_2 - L_0$ can further be linked to the interest payment shocks, as summarized in Proposition 1.

**Proposition 1.** When banks are subject to the CAR, the changes in their credit supply in response to interest payment shocks $\Delta E$ are given by

\[
L_2 - L_0 = \frac{1}{\text{car} \cdot \gamma_L} \cdot \Delta E.
\]

(30)

- The credit supply is increasing in equity.
- Interest payment shocks produce a multiplier effect on the credit supply. The multiplier is

\[
\frac{1}{\text{car} \cdot \gamma_L} \geq 1.
\]

(31)

According to the Basel III rules, \text{car} = 8\% and $\gamma_L \leq 1250\%$. In only a few extreme cases does the risk weight equal the maximum of 1250\%. In general, there is $\gamma_L \ll 1250\%$. Thus, the multiplier is much larger than one. Proposition 1 indicates that banks under the CAR amplify the changes in equity resulting from interest payment shocks. The multiplier is decreasing in \text{car} or $\gamma_L$, either of which represents the stringency of the CAR. An increase in the stringency of the CAR reduces not only the credit supply but also the multiplier effect on the credit supply. This finding supports that Basel III strengthens the CAR to avoid excessive credit expansion.

Additionally, based on the balance sheet identity in Eq. (6), we exhibit the changes in the money supply $D_2 - D_0$:

\[
D_2 - D_0 = L_2 - L_0 - \Delta E = \left( \frac{1}{\text{car} \cdot \gamma_L} - 1 \right) \Delta E,
\]

(32)
which also demonstrates a multiplier effect on the money supply.

Finally, the constraints $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$ yield the following condition:

\[(S + R - E)(\text{car}(\gamma_L - \frac{S}{S + R - E} \cdot \gamma_S) + \frac{E}{S + R - E}) > 0.\]

### 5.2. Leverage ratios

The effects of the LR are analyzed by the same method as in Section 5.1. At date 0, from the LR constraint in Eq. (16) and balance sheet identity in Eq. (6), $L_0$ and $D_0$ are determined by the following equations:

\[
\text{lr}(L_0 + S + R) = E, \tag{33}
\]

\[
L_0 + S + R = D_0 + E. \tag{34}
\]

At $t = 1$, interest payment shocks $\Delta E$ in Eq. (11) take place, which change the equity to $E_1 = E + I - P$. Then, with date-2 equity $E_2 = E_1$, the equations to determine $L_2$ and $D_2$ can be written as

\[
\text{lr}(L_0 + S + R) = E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0, \tag{35}
\]

\[
L_2 + S + R = D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0. \tag{36}
\]

The system of equations to determine $L_0$, $L_2$, $D_0$, and $D_2$ is given in Eqs. (33)-(36). We display the solutions in Appendix B. Then, we show $L_2 - L_0$ as follows:

\[
L_2 - L_0 = \sigma_L \cdot \frac{1}{\text{lr}} \cdot i_L L_0 + \sigma_S \cdot \frac{1}{\text{lr}} \cdot i_S S + \sigma_D \cdot \frac{1}{\text{lr}} \cdot (-i_D D_0). \tag{37}
\]

Then, Proposition 2 follows.

**Proposition 2.** When banks comply with the LR, the response of their credit supply to interest payment shocks $\Delta E$ is

\[
L_2 - L_0 = \frac{1}{\text{lr}} \cdot \Delta E. \tag{38}
\]
• The credit supply increases in equity.

• Interest payment shocks generate a multiplier effect on the credit supply. The multiplier is
\[ \frac{1}{lr} > 1. \] (39)

The minimum LR requirement, \( lr \), is 3%. As Proposition 2 shows, the multiplier is greater than one. The changes in equity arising from interest payment shocks are amplified. The multiplier is decreasing in \( lr \): a strengthening of the LR reduces the multiplier.

Next, we present the changes in the money supply \( D_2 - D_0 \). From the balance sheet identity in Eq. (6), there is a multiplier that relates interest payment shocks to the money supply:
\[ D_2 - D_0 = L_2 - L_0 - \Delta E = (\frac{1}{lr} - 1)\Delta E. \] (40)

In this scenario, loans and deposits must be positive.

5.3. Liquidity coverage ratios

The discussions of the LCR have four scenarios. The reason is that the LCR has two different regimes which correspond to differing LCR constraints. One is given by Eq. (22) under the condition \( IF_0 \geq 0.75OF_0 \), denoted regime H; and the other is given by Eq. (23) under the condition \( IF_0 < 0.75OF_0 \), denoted regime L. Before or after interest payment shocks, the bank is in either regime H or regime L. This leads to four combinations consisting of scenario HH-LCR, scenario LL-LCR, scenario LH-LCR, and scenario HL-LCR. Their conditions are illustrated by Table 4.

In the following sections, we discuss each scenario individually.

5.3.1. Scenario HH-LCR

In scenario HH-LCR, banks are subject to the LCR under (i) \( IF_0 > 0.75OF_0 \) (regime H) and \( IF_2 > 0.75OF_2 \) (regime H) or (ii) \( IF_0 = 0.75OF_0 \) (regime H) and \( IF_2 = 0.75OF_2 \) (regime H).
Table 4
Combinations of LCR regimes. Scenario HH-LCR has two separate conditions: \( \text{IF}_t > 0.75 \text{OF}_t \) for \( t \in \{0, 2\} \) or \( \text{IF}_t = 0.75 \text{OF}_t \) for \( t \in \{0, 2\} \). Moreover, the condition for scenario LH-LCR is \( \text{IF}_0 < 0.75 \text{OF}_0 \) and \( \text{IF}_2 > 0.75 \text{OF}_2 \), and that for scenario HL-LCR is \( \text{IF}_0 > 0.75 \text{OF}_0 \) and \( \text{IF}_2 < 0.75 \text{OF}_2 \). The reason is that \( \text{IF}_2 = 0.75 \text{OF}_2 \) if and only if \( \text{IF}_0 = 0.75 \text{OF}_0 \) (see Appendix C.1 for the proof).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Date 0</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH-LCR</td>
<td>( \text{IF}_0 &gt; 0.75 \text{OF}_0 )</td>
<td>( \text{IF}_2 &gt; 0.75 \text{OF}_2 )</td>
</tr>
<tr>
<td>HH-LCR</td>
<td>( \text{IF}_0 = 0.75 \text{OF}_0 )</td>
<td>( \text{IF}_2 = 0.75 \text{OF}_2 )</td>
</tr>
<tr>
<td>LL-LCR</td>
<td>( \text{IF}_0 &lt; 0.75 \text{OF}_0 )</td>
<td>( \text{IF}_2 &lt; 0.75 \text{OF}_2 )</td>
</tr>
<tr>
<td>LH-LCR</td>
<td>( \text{IF}_0 &lt; 0.75 \text{OF}_0 )</td>
<td>( \text{IF}_2 &gt; 0.75 \text{OF}_2 )</td>
</tr>
<tr>
<td>HL-LCR</td>
<td>( \text{IF}_0 &gt; 0.75 \text{OF}_0 )</td>
<td>( \text{IF}_2 &lt; 0.75 \text{OF}_2 )</td>
</tr>
</tbody>
</table>

The constraints at date 0 and date 2 take the same form as in Eq. (22). From the LCR constraint in Eq. (22) and balance sheet identity in Eq. (6), we have

\[
0.25 \text{lcr}((i_D + \alpha)(L_0 + S + R - E)) = R + (1 - \chi)S \tag{41}
\]

and

\[
L_0 + S + R = D_0 + E \tag{42}
\]

to determine \( L_0 \) and \( D_0 \).

At \( t = 1 \), banks are hit by interest payment shocks \( \Delta E \) in Eq. (11). The equity is changed to \( E_1 = E + I - P \). At date 2, we have \( E_2 = E_1 \). Then the solutions for \( L_2 \) and \( D_2 \) are determined by

\[
0.25 \text{lcr}((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0))) = R + (1 - \chi)S, \tag{43}
\]

\[
L_2 + S + R = D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0. \tag{44}
\]

Finally, \( L_0 \), \( L_2 \), \( D_0 \), and \( D_2 \) are obtained by solving the system of equations given in Eqs. (41)-(44). The solutions are presented in Appendix C.1. The
difference in loans is given by

\[ L_2 - L_0 = \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S + \sigma_D \cdot (-i_D D_0). \] (45)

Eq. (45) yields Proposition 3.

**Proposition 3.** When banks are subject to the LCR under (i) \( IF_0 > 0.75OF_0 \) and \( IF_2 > 0.75OF_2 \) or (ii) \( IF_0 = 0.75OF_0 \) and \( IF_2 = 0.75OF_2 \), interest payment shocks \( \Delta E \) lead to

\[ L_2 - L_0 = \Delta E. \] (46)

- The credit supply is increasing in equity.
- Interest payment shocks do not cause multiplier effects on the credit supply. The multiplier equals one.

Proposition 3 shows a special case of banks responding to interest payment shocks. This is tantamount to banks using profits to finance loans or intermediating funds from shareholders to borrowers.

Moreover, based on the balance sheet identity in Eq. (6), we see no changes in the money supply:

\[ D_2 - D_0 = L_2 - L_0 - \Delta E = 0. \] (47)

5.3.2. Scenario LL-LCR

Now, we turn to the LCR scenario under \( IF_0 < 0.75OF_0 \) (regime L) and \( IF_2 < 0.75OF_2 \) (regime L). In this scenario, the forms of the constraints at \( t = 0 \) and \( t = 2 \) are the same, which are given by Eq. (23).

Using the LCR constraint in Eq. (23), together with the balance sheet identity in Eq. (6), we have \( L_0 \) and \( D_0 \) determined by

\[ lcr((i_D + \alpha)(L_0 + S + R - E) - \kappa(i_L + \mu)L_0) = R + (1 - \chi)S, \] (48)

\[ L_0 + S + R = D_0 + E. \] (49)

At date 1, interest payment shocks \( \Delta E \), given by Eq. (11), take place. Then the equity changes to \( E_1 = E + I - P \). At date 2, with \( E_2 = E_1 \), the equations
to determine $L_2$ and $D_2$ become

$$R + (1 - \chi)S = lcr((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_LL_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)) - \kappa(i_L + \mu)L_2),$$

(50)

$$L_2 + S + R = D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0.$$  

(51)

The solutions for $L_0, L_2, D_0$, and $D_2$ are given by the system of equations in Eqs. (48)-(51). The solutions are shown in Appendix C.2. The impact on the credit supply is given by the changes in loans:

$$L_2 - L_0 = \sigma_L \cdot \frac{i_D + \alpha}{i_D + \alpha - \kappa(i_L + \mu)} \cdot i_L L_0 + \sigma_S \cdot \frac{i_D + \alpha}{i_D + \alpha - \kappa(i_L + \mu)} \cdot i_S S + \sigma_D \cdot \frac{i_D + \alpha}{i_D + \alpha - \kappa(i_L + \mu)} \cdot (-i_D D_0).$$

(52)

From Eq. (52), we have Proposition 4.

**Proposition 4.** When banks are subject to the LCR under $IF_0 < 0.75OF_0$ and $IF_2 < 0.75OF_2$, the changes in their credit supply in response to interest payment shocks $\Delta E$ can be expressed as

$$L_2 - L_0 = \frac{i_D + \alpha}{i_D + \alpha - \kappa(i_L + \mu)} \cdot \Delta E.$$  

(53)

- The credit supply rises if equity increases.

- Interest payment shocks have a multiplier effect on the credit supply. The multiplier is

$$\frac{i_D + \alpha}{i_D + \alpha - \kappa(i_L + \mu)} > 1.$$  

(54)

Proposition 4 demonstrates how banks amplify interest payment shocks. The multiplier is increasing in $\kappa$ and decreasing in $\alpha$. A fall in $\kappa$ or a rise in $\alpha$ increases the stringency of the LCR. Such increases result in a smaller multiplier. Strengthening the LCR reduces the amplification of interest payment shocks. Notably, the multiplier, or the degree of amplification, does not depend on the value of the minimum LCR requirement.
To show further findings, we rearrange the multiplier in Proposition 4 as

$$\frac{1}{1 - \frac{\kappa(i_L + \mu)}{i_D + \alpha}}.$$  \hspace{1cm} (55)

The above expression for the multiplier has the implication concerning the liquidity of banks. To see the implication behind Eq. (55), we define the derivative of cash inflows with respect to loans as the marginal inflow of loans and the derivative of cash outflows with respect to deposits as the marginal outflow of deposits. From Eqs. (18) and (19), we discern that the marginal inflow of loans is $\kappa(i_L + \mu)$, and the marginal outflow of deposits is $i_D + \alpha$. Therefore, $\kappa(i_L + \mu)/(i_D + \alpha)$ is the ratio of the marginal inflow of loans to the marginal outflow of deposits. This ratio indicates the liquidity of banks. A higher $\kappa(i_L + \mu)/(i_D + \alpha)$ means a higher liquidity of banks. As Eq. (55) presents, the multiplier is increasing in the ratio of $\kappa(i_L + \mu)/(i_D + \alpha)$. So an increase in the liquidity increases the value of the multiplier or the amplification of interest payment shocks.

Next, we have the changes in the money supply. As the balance sheet identity in Eq. (6) implies, the changes in the money supply are given by

$$D_2 - D_0 = L_2 - L_0 - \Delta E = \frac{\kappa(i_L + \mu)}{i_D + \alpha - \kappa(i_L + \mu)} \cdot \Delta E.$$  \hspace{1cm} (56)

Eq. (56) shows a multiplier effect on the money supply.

5.3.3. Scenario LH-LCR

Scenario LH-LCR is connected to the LCR under $IF_0 < 0.75OF_0$ (regime L) and $IF_2 > 0.75OF_2$ (regime H). In contrast to scenario HH-LCR and scenario LL-LCR, interest payment shocks change the regime of the LCR. Specifically, the constraint changes from Eq. (23) (regime L) at date 0 to Eq. (22) (regime H) on date 2.

The equations to determine the solutions for loans and deposits at date 0 and date 2 are the combination of those in Section 5.3.2 and Section 5.3.1. Then, the system of equations specified in Eqs. (43), (44), (48) and (49) determines $L_0$, $L_2$, $D_0$, and $D_2$. The solutions are presented in Appendix C.3. The changes in
loans are presented as

\[ L_2 - L_0 = \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S + \sigma_D \cdot (-i_D D_0) - D_0 + \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)}. \]  

(57)

More importantly, from Eq. (57), we obtain the link between the credit supply and the interest payment shocks.

**Proposition 5.** Under the LCR with \( IF_0 < 0.75OF_0 \) and \( IF_2 > 0.75OF_2 \), interest payment shocks \( \Delta E \) cause the changes in the credit supply as

\[ L_2 - L_0 = \Delta E - D_0 + \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)}. \]  

(58)

- The increase in equity increases the credit supply.

- Interest payment shocks do not lead to multiplier effects on the credit supply. The shocks have a multiplier of exactly one.

Proposition 5 presents that Eq. (58) is divided into two groups. One with \( \Delta E \) is caused by the shocks; the other without \( \Delta E \) results from the liquidity condition switching from \( IF_0 < 0.75OF_0 \) to \( IF_2 > 0.75OF_2 \). The group without \( \Delta E \) in Eq. (58) can be decomposed into \( R, S, \) and \( E \), which we present in Eq. (C.28).

Using the balance sheet identity in Eq. (6), we obtain the changes in the money supply as

\[ D_2 - D_0 = L_2 - L_0 - \Delta E = -D_0 + \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)}. \]  

(59)

Eq. (59) shows that \( D_2 - D_0 \) has nothing to do with interest payment shocks \( \Delta E \). This means that the changes in the money supply are independent of the shocks. Eq. (59) is the same as the group without \( \Delta E \) in Eq. (58).

5.3.4. Scenario HL-LCR

Scenario HL-LCR concerns the LCR under \( IF_0 > 0.75OF_0 \) (regime H) and \( IF_2 < 0.75OF_2 \) (regime L). As in scenario LH-LCR, the constraint for scenario
HL-LCR at date 0 is changed by interest payment shocks. In contrast to scenario LH-LCR, scenario HL-LCR begins with the constraint in Eq. (22) and ends with that in Eq. (23).

Repeating the same steps as in Section 5.3.1 and Section 5.3.2 yields the system of equations in Eqs. (41), (42), (50) and (51) to determine \( L_0 \), \( L_2 \), \( D_0 \), and \( D_2 \). The solutions are shown in Appendix C.4. The changes in loans are given by

\[
L_2 - L_0 = -\sigma_L \cdot \frac{i_D + \alpha}{\kappa(i_L + \mu) - (i_D + \alpha)} \cdot i_L L_0 - \sigma_S \cdot \frac{i_D + \alpha}{\kappa(i_L + \mu) - (i_D + \alpha)} \cdot i_S S
- \sigma_D \cdot \frac{i_D + \alpha}{\kappa(i_L + \mu) - (i_D + \alpha)} \cdot (-i_D D_0)
+ \frac{(i_L + \alpha) D_0 - \kappa(i_L + \mu) L_0}{\kappa(i_L + \mu) - (i_D + \alpha)} - \frac{R + (1 - \chi) S}{\text{lcr}(\kappa(i_L + \mu) - (i_D + \alpha))}.
\]

Based on Eq. (60), we get Proposition 6.

**Proposition 6.** When banks are subject to the LCR under \( IF_0 > 0.75OF_0 \) and \( IF_2 < 0.75OF_2 \), interest payment shocks \( \Delta E \) lead to the changes in their credit supply as follows:

\[
L_2 - L_0 = -\frac{i_D + \alpha}{\kappa(i_L + \mu) - (i_D + \alpha)} \cdot \Delta E
+ \frac{(i_D + \alpha) D_0 - \kappa(i_L + \mu) L_0}{\kappa(i_L + \mu) - (i_D + \alpha)} - \frac{R + (1 - \chi) S}{\text{lcr}(\kappa(i_L + \mu) - (i_D + \alpha))}.
\]

- Increases in equity decrease the credit supply.
- Interest payment shocks produce a multiplier effect on the credit supply. The multiplier is

\[
\frac{i_D + \alpha}{\kappa(i_L + \mu) - (i_D + \alpha)}.
\]

Proposition 6 shows that contrary to scenarios HH-LCR, LL-LCR, and LH-LCR, an increase in equity decreases the credit supply. The changes in the credit supply consist of two groups: one with \( \Delta E \) arises from the shocks and the other without \( \Delta E \) results from the switch of the LCR regimes. An alternative expression for the group without \( \Delta E \) in Eq. (61) decomposed into \( R \), \( S \), and...
$E$ is given in Eq. (C.37). This proposition also presents how the values of the multiplier can be greater or less than one, given by

$$\begin{cases} 
> 1 & \text{if } \kappa(i_L + \mu) < 2(i_D + \alpha), \\
= 1 & \text{if } \kappa(i_L + \mu) = 2(i_D + \alpha), \\
< 1 & \text{if } \kappa(i_L + \mu) > 2(i_D + \alpha).
\end{cases}$$

On the one hand, if $\kappa(i_L + \mu) < 2(i_D + \alpha)$, then the changes in the credit supply are greater than the size of interest payment shocks. The LCR amplifies the shocks. On the other hand, if $\kappa(i_L + \mu) > 2(i_D + \alpha)$, then the changes in the credit supply are smaller than the size of interest payment shocks. The LCR absorbs the shocks.

The multiplier is decreasing in $\kappa$ and increasing in $\alpha$. A fall in $\kappa$ or a rise in $\alpha$ increases the stringency of the LCR. The stringency of the LCR increased by decreasing $\kappa$ or increasing $\alpha$ leads to a larger multiplier. Strengthening the LCR either increases the amplification of interest payment shocks if $\kappa(i_L + \mu) < 2(i_D + \alpha)$ or reduces the contraction of interest payment shocks if $\kappa(i_L + \mu) > 2(i_D + \alpha)$. Note that the multiplier is independent of the value of the minimum LCR requirement.

To derive more implications about the multiplier, we rearrange Eq. (62) as

$$\frac{1}{\kappa(i_L + \mu)/(i_D + \alpha) - 1}. \quad (63)$$

This expression offers a link between the multiplier and the liquidity of banks. The link can be obtained by using the ratio of the marginal inflow of loans to the marginal outflow of deposits, $\kappa(i_L + \mu)/(i_D + \alpha)$, which is associated with the liquidity of banks. A rise in the ratio means an increase in the liquidity of banks. As a result, increasing the bank liquidity given by $\kappa(i_L + \mu)/(i_D + \alpha)$ decreases the multiplier. Ultimately, this increase in bank liquidity reduces the amplification of interest payment shocks if $\kappa(i_L + \mu) < 2(i_D + \alpha)$ or increases the contraction of interest payment shocks if $\kappa(i_L + \mu) > 2(i_D + \alpha)$.

According to the balance sheet identity in Eq. (6), the changes in the money
supply are linked to interest payment shocks $\Delta E$ as

$$D_2 - D_0 = L_2 - L_0 - \Delta E = -\frac{\kappa(i_L + \mu)}{\kappa(i_L + \mu) - (i_D + \alpha)} \cdot \Delta E$$

$$+ \frac{(i_D + \alpha)D_0 - \kappa(i_L + \mu)L_0}{\kappa(i_L + \mu) - (i_D + \alpha)} - \frac{R + (1 - \chi)S}{\text{lcr}(\kappa(i_L + \mu) - (i_D + \alpha))}. \tag{64}$$

Eq. (64) suggests a multiplier that relates the changes in equity to the changes in the money supply. And the increase in equity decreases the money supply. Like the changes in the credit supply in Eq. (61), Eq. (64) can be divided into two groups: one with $\Delta E$ and the other without $\Delta E$. The group without $\Delta E$ in Eq. (64) is the same as that in Eq. (61).

5.3.5. Conditions for the scenarios of the LCR

In this section, we show the conditions for the four scenarios in Table 4. They are derived from (i) the combinations of the conditions for the LCR regimes before and after interest payment shocks, as shown in Table 4 and (ii) the conditions for loans and deposits greater than zero. Detailed derivations of the conditions can be found in Appendix C.1 for scenario HH-LCR, in Appendix C.2 for scenario LL-LCR, in Appendix C.3 for scenario LH-LCR, and in Appendix C.4 for scenario HL-LCR. The conditions are summarized in Table 5.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Date 0</th>
<th>Date 2</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>HH</td>
<td>$IF_0 &gt; 0.75OF_0$</td>
<td>$IF_2 &gt; 0.75OF_2$</td>
<td>$\kappa(i_L + \mu) &gt; 0.75(i_D + \alpha)$</td>
</tr>
<tr>
<td>HH</td>
<td>$IF_0 = 0.75OF_0$</td>
<td>$IF_2 = 0.75OF_2$</td>
<td>$\kappa(i_L + \mu) = 0.75(i_D + \alpha)$</td>
</tr>
<tr>
<td>LL</td>
<td>$IF_0 &lt; 0.75OF_0$</td>
<td>$IF_2 &lt; 0.75OF_2$</td>
<td>$\kappa(i_L + \mu) &lt; 0.75(i_D + \alpha)$</td>
</tr>
<tr>
<td>LH</td>
<td>$IF_0 &lt; 0.75OF_0$</td>
<td>$IF_2 &gt; 0.75OF_2$</td>
<td>$\kappa(i_L + \mu) &gt; i_D + \alpha$ and $$(R + S - E)(i_D + \alpha - \frac{(1 - \chi)S + R}{\text{lcr}(R + S - E)}) &gt; 0$$</td>
</tr>
<tr>
<td>HL</td>
<td>$IF_0 &gt; 0.75OF_0$</td>
<td>$IF_2 &lt; 0.75OF_2$</td>
<td>$\kappa(i_L + \mu) &gt; i_D + \alpha$ and $$(R + S - E)(i_D + \alpha - \frac{(1 + 4(i_L - i_D))(1 - \chi)S + R}{\text{lcr}(R + S - E)}) &gt; 0$$</td>
</tr>
</tbody>
</table>

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5.4. Net stable funding ratios

Based on the NSFR constraint in Eq. (24) and balance sheet identity in Eq. (6), we have $L_0$ and $D_0$ determined by

\begin{align}
nsfr(\phi_L L_0 + \phi_S S) &= \beta D_0 + E, \quad (65) \\
L_0 + S + R &= D_0 + E. \quad (66)
\end{align}

At $t=1$, the bank is hit by interest payment shocks $\Delta E$ in Eq. (11). Then the equity is changed to $E_1$. At $t=2$, there is $E_2 = E_1$. Based on the equity $E_2$, banks adjust their balance sheets. From the NSFR constraint in Eq. (24) and balance sheet identity in Eq. (6), $L_2$ and $D_2$ are determined by the following equations:

\begin{align}
nsfr(\phi_L L_2 + \phi_S S) &= \beta D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0, \quad (67) \\
L_2 + S + R &= D_2 + E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0. \quad (68)
\end{align}

In summary, the system of equations to determine $L_0$, $L_2$, $D_0$, and $D_2$ is given in Eqs. (65)-(68). The solutions are shown in Appendix D. The changes in loans are as follows:

\begin{align}
L_2 - L_0 &= \sigma_L \cdot \frac{1-\beta}{nsfr \cdot \phi_L - \beta} \cdot i_L L_0 + \sigma_S \cdot \frac{1-\beta}{nsfr \cdot \phi_L - \beta} \cdot i_S S \\
&\quad + \sigma_D \cdot \frac{1-\beta}{nsfr \cdot \phi_L - \beta} \cdot (-i_D D_0). \quad (69)
\end{align}

In addition, we can prove that $nsfr \cdot \phi_L > \beta$ must hold (see Appendix D). From Eq. (69), we have Proposition 7.

**Proposition 7.** When banks comply with the NSFR, the changes in their credit supply in response to interest payment shocks $\Delta E$ can be expressed as

\begin{align}
L_2 - L_0 &= \frac{1-\beta}{nsfr \cdot \phi_L - \beta} \cdot \Delta E. \quad (70)
\end{align}

- Increases in equity increase the credit supply.
- Interest payment shocks generate a multiplier effect on the credit supply. The multiplier is

\begin{align}
\frac{1-\beta}{nsfr \cdot \phi_L - \beta}. \quad (71)
\end{align}
Proposition 7 has implications as follows. The values of the multiplier are

\[
\begin{align*}
&> 1 & \text{if } nsfr \cdot \phi_L < 1, \\
&= 1 & \text{if } nsfr \cdot \phi_L = 1, \\
<& 1 & \text{if } nsfr \cdot \phi_L > 1.
\end{align*}
\]

First, if \( nsfr \cdot \phi_L < 1 \), the multiplier in Eq. (71) is greater than one. Banks under the NSFR amplify interest payment shocks. Furthermore, the multiplier is decreasing in \( nsfr \cdot \phi_L \) and increasing in \( \beta \). A rise in \( nsfr \cdot \phi_L \) or a fall in \( \beta \) increases the stringency of the NSFR. Thus, a more stringent NSFR with increasing \( nsfr \cdot \phi_L \) or decreasing \( \beta \) leads to a smaller multiplier. The amplification of interest payment shocks is thus reduced. Second, if \( nsfr \cdot \phi_L > 1 \), the multiplier in Eq. (71) is less than one. Banks under the NSFR contract or absorb interest payment shocks. The multiplier is decreasing in \( nsfr \cdot \phi_L \) or \( \beta \). Then, either strengthening the NSFR by increasing \( nsfr \cdot \phi_L \) or loosening the NSFR by increasing \( \beta \) decreases the multiplier. As a result, such adjustments of the NSFR increase the contraction of the shocks.

Another interpretation links the multiplier to the liquidity of banks. To understand this interpretation, it is helpful to discuss a special case in which \( nsfr \) takes the value of one, as required under Basel III. Rearranging Eq. (71), we obtain the multiplier as

\[
\frac{1}{1 - \frac{1 - \phi_L}{1 - \beta}}.
\]

Eq. (72) depends on the ratio \((1 - \phi_L)/(1 - \beta)\). Consider the meanings of the ASF factor for deposits, \( \beta \), and the RSF factor for loans, \( \phi_L \). The ASF factor reflects the stability of deposits and the RSF factor indicates the liquidity of loans. An increase in the stability of deposits raises \( \beta \), and an increase in the liquidity of loans lowers \( \phi_L \). Consequently, the ratio \((1 - \phi_L)/(1 - \beta)\), measures the liquidity of banks. A higher \((1 - \phi_L)/(1 - \beta)\) resulting from a rise in the stability of deposits or the liquidity of loans suggests a more liquid bank. Using such a ratio, we have the following interpretation for the multiplier. As Eq. (72) shows, when the liquidity of banks measured by the ratio increases, the multiplier and thus the amplification increase.
In addition, as the balance sheet identity in Eq. (6) suggests, we have the changes in the money supply in response to interest payment shocks $\Delta E$ as

$$D_2 - D_0 = L_2 - L_0 - \Delta E = \frac{1 - nsfr \cdot \phi_L - \beta}{nsfr \cdot \phi_L - \beta} \cdot \Delta E. \quad (73)$$

That is, the relationship between the changes in the money supply and the changes in equity is determined by a multiplier.

Finally, from the constraints $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$, we obtain

$$(S + R - E)(nsfr \cdot \phi_L - \beta)(\beta - \frac{S}{S + R - E} \cdot nsfr \cdot \phi_S + \frac{E}{S + R - E}) > 0,$$

$$(S + R - E)(nsfr \cdot \phi_L - \beta)(nsfr \cdot \phi_S - \frac{S}{S + R - E} \cdot nsfr \cdot \phi_S + \frac{E}{S + R - E}) > 0.$$

See Appendix D for the detailed derivation of the conditions.

6. Conclusion

This study investigated how the changes in banks’ equity resulting from interest payments affect credit and money creation under Basel III regulations. My model builds on the credit creation theory of banking. This paper discusses four Basel III regulations—the CAR, the LR, the LCR, and the NSFR.

This study determines that the effects of the four Basel III regulations on credit and money creation can be divided into seven regulatory scenarios. Each scenario corresponds to a multiplier that relates the change in equity to the change in the credit supply. The multipliers that determine the credit supply are summarized in Fig. 1. At the same time, by subtracting one from the value of the multipliers in Fig. 1, we can obtain the value of the multipliers that relate the change in equity to the change in the money supply.

These results present a reformulation of the traditional theory of credit and money supply, which is demonstrated by one simple deposit multiplier under a reserve requirement. My results present seven regulatory scenarios under Basel III’s microprudential regulations. Each scenario implies two multipliers that relate bank equity to the credit and money supply. These multipliers enrich our knowledge of credit and money creation under contemporary bank regulation.
<table>
<thead>
<tr>
<th>Regulation</th>
<th>Scenario</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital adequacy ratio</td>
<td>CAR</td>
<td>$\frac{1}{\text{car} \cdot \gamma}$</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>LR</td>
<td>$\frac{1}{\text{lr}}$</td>
</tr>
<tr>
<td>Liquidity coverage ratio</td>
<td>HH-LCR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>LL-LCR</td>
<td>$\frac{i_p + \alpha}{i_p + \alpha - \kappa(i_c + \mu)}$</td>
</tr>
<tr>
<td></td>
<td>LH-LCR</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>HL-LCR</td>
<td>$\frac{i_p + \alpha}{\kappa(i_c + \mu) - (i_p + \alpha)}$</td>
</tr>
<tr>
<td>Net stable funding ratio</td>
<td>NSFR</td>
<td>$\frac{1 - \beta}{\text{nsfr} \cdot \phi_l - \beta}$</td>
</tr>
</tbody>
</table>

**Figure 1**: The multipliers in the seven Basel III regulatory scenarios. The multipliers relate changes in bank equity to changes in the credit supply. The meanings of the symbols can be found in Appendix E.

My results have important policy implications. First, if policymakers intend to influence the supply of credit and money, they should identify banks’ regulatory scenario. Policymakers may employ the most effective policy tools if they recognize the specific regulatory scenario. Under scenarios CAR and LR, equity injections, caused, for example, by the Capital Purchase Program of the Troubled Asset Relief Program, would be the most effective tool for controlling the credit and money supply. Under scenarios LL-LCR and HL-LCR, the multipliers are sensitive to interest rates. Thus, those policy tools that affect the interest rate will also affect banks’ ability to create credit and money. Additionally, such sensitivity calls for attention to the interplay between the monetary policy and the LCR. Second, the multipliers depend on the parameters introduced under regulations. In particular, the links between the multipliers and the stringency of the regulations are demonstrated. These findings indicate how the
amplification or the contraction, as measured by the multipliers, change when policymakers adjust regulations. Third, my discussion offers a way to examine the effectiveness of bank regulations to protect banks from insolvency or liquidity risk and maintain their credit supply. The effectiveness of the CAR and LR in guarding against insolvency risk could be demonstrated by the multipliers under the CAR and LR scenarios. The effectiveness of the LCR and NSFR guarding against liquidity risk could be assessed by the relationship between the credit supply and bank liquidity as presented in the LCR and NSFR scenarios.

A few extensions of this framework that may inform future studies are as follows. The present version of this model ignores some factors that may influence credit creation such as adjustment costs, balance sheet costs, and risk-taking. Therefore, one may extend this model by considering these factors as terms that are dependent on banks’ balance sheet quantities and adding them to the objective function of banks.

CRediT authorship contribution statement

Boyao Li: Conceptualization; Formal analysis; Funding acquisition; Investigation; Methodology; Resources; Software; Validation; Writing - original draft; Writing - review & editing.

Declaration of Competing Interest

None.

Appendix

For each regulation, first, we present the bank’s maximization problems, Lagrangians, and first-order conditions at date 0 and date 2. Second, we show the expressions for \( L_0 \) and \( D_0 \). The solutions for \( L_2 \) and \( D_2 \) are not displayed explicitly. We can obtain \( L_2 \) and \( D_2 \) by letting the dummy variables, \( \sigma_L \), \( \sigma_S \), and \( \sigma_D \), take a value of one and adding \( L_2 - L_0 \) to \( L_0 \) and \( D_2 - D_0 \) to \( D_0 \).
The changes in loans, $L_2 - L_0$, and deposits, $D_2 - D_0$, have been shown in Sections 5.1-5.4. Third, we present the conditions for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$. In addition, for the LCR, we derive the cash flow conditions given in Table 5. The last section provides a glossary of notations.

**Appendix A. Capital adequacy ratio**

At $t = 0$, from the objective function in Eq. (10) and the CAR constraint in Eq. (15), the bank’s maximization problem is

$$\max_{L_0} \Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E$$

subject to

$$\text{car}(\gamma_L L_0 + \gamma_S S) \leq E,$$

and the nonnegativity constraint $L_0 \geq 0$. Let $\lambda_0^C$ be the Lagrangian multiplier for the date-0 CAR constraint. The Lagrangian at date 0 is

$$L_0^C = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E$$

$$+ \lambda_0^C (E - \text{car}(\gamma_L L_0 + \gamma_S S)).$$

The first-order conditions can be written as

$$0 = i_L - i_D + \text{car} \cdot \gamma_L \lambda_0^C, \quad (A.1)$$

$$0 = E - \text{car}(\gamma_L L_0 + \gamma_S S). \quad (A.2)$$

At date 2, we substitute Eqs. (12) and (13) into the objective function in Eq. (14) and the CAR constraint in Eq. (15) to obtain the bank’s problem

$$\max_{L_2} \Pi = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + i_D (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S + \sigma_D \cdot i_D D_0)$$

subject to

$$\text{car}(\gamma_L L_2 + \gamma_S S) \leq E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S + \sigma_D \cdot i_D D_0,$$
and the nonnegativity constraint $L_2 \geq 0$. Similarly, the Lagrangian at date 2 can be expressed as

$$L_2^C = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + i_D(E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0) + \lambda_2^C((E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0) - \text{car}(\gamma_L L_2 + \gamma_S S)),$$

where $\lambda_2^C$ is the Lagrangian multiplier. We have the first-order conditions as

$$0 = i_L - i_D + \text{car} \cdot \gamma_L \lambda_2^C, \quad (A.3)$$

$$0 = E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0 - \text{car}(\gamma_L L_0 + \gamma_S S). \quad (A.4)$$

Then, we show the solutions for $L_0$ and $D_0$.\(^4\) Loans and deposits at date 0 are

$$L_0 = \frac{E - \text{car} \cdot \gamma_S S}{\text{car} \cdot \gamma_L}, \quad (A.5)$$

$$D_0 = (1 - \frac{\gamma_S}{\gamma_L})S + R + (\frac{1}{\text{car} \cdot \gamma_L} - 1)E. \quad (A.6)$$

Based on the solutions, we give the condition for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$. Note that securities, $S$, reserves, $R$, and equity, $E$, are large and of the order of magnitude of $10^Q$. On the contrary, the loan rate, $i_L$, security rate, $i_S$, and deposit rate, $i_D$, are small and of the order of magnitude of $10^{-j}$. In practice, $Q$ and $j$ are greater than zero, and $Q$ is far greater than $j$. From $L_0$, $L_2 - L_0$, $D_0$, and $D_2 - D_0$, we obtain $L_2$ and $D_2$, which consist of terms of the order of $10^Q$ and $10^Q - j$. Retaining only the highest-order terms in $L_2$ and $D_2$, we get the same expressions as $L_0$ and $D_0$. Thus, we only need to consider the constraints $L_0 > 0$ and $D_0 > 0$. From Eqs. (A.5) and (A.6), $L_0 > 0$ and $D_0 > 0$ yield

$$E - \text{car} \cdot \gamma_S S > 0 \quad (A.7)$$

and

$$\text{car} \cdot \gamma_L (S + R - E) - \text{car} \cdot \gamma_S S + E > 0, \quad (A.8)$$

\(^4\) We do not consider the cases in which banks do not lend.
respectively. The CAR constraint in Eq. (15) implies Eq. (A.7) must hold. Finally, the condition for loans and deposits greater than zero is given by Eq. (A.8).

**Appendix B. Leverage ratio**

At $t = 0$, from the objective function in Eq. (10) and the LR constraint in Eq. (16), the bank’s maximization problem is

$$\max \frac{\Pi}{L_0} = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E$$

subject to

$$lr(L_0 + S + R) \leq E,$$

and the nonnegativity constraint $L_0 \geq 0$. Denote by $\lambda^L_0$ the Lagrangian multiplier for the date-0 LR constraint. The Lagrangian at date 0 is

$$\mathcal{L}^L_0 = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E + \lambda^L_0(E - lr(L_0 + S + R)).$$

The first-order conditions can be written as

$$0 = i_L - i_D + lr \cdot \lambda^L_0, \quad (B.1)$$
$$0 = E - lr(L_0 + S + R). \quad (B.2)$$

Similarly, at date 2, from the objective function in Eq. (14) and the LR constraint in Eq. (16), using Eq. (11), we obtain the bank’s problem at date 2:

$$\max \frac{\Pi}{L_2} = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + i_D (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)$$

subject to

$$lr(L_2 + S + R) \leq E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0,$$

and the nonnegativity constraint $L_2 \geq 0$. Denote by $\lambda^L_2$ the Lagrangian multiplier. The Lagrangian at date 2 can be expressed as

$$\mathcal{L}^L_2 = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R$$
$$+ i_D (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)$$
$$+ \lambda^L_2((E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)lr(L_2 + S + R)).$$
We have the first-order conditions as

\[ 0 = i_L - i_D + lr \cdot \lambda_{L2}, \quad \text{(B.3)} \]
\[ 0 = E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0 - lr(L_2 + S + R). \quad \text{(B.4)} \]

Then the solutions for \( L_0 \) and \( D_0 \) are given by

\[ L_0 = \frac{E - lr(R + S)}{lr}, \quad \text{(B.5)} \]
\[ D_0 = \left( \frac{1}{lr} - 1 \right) E. \quad \text{(B.6)} \]

Finally, we provide the condition for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \). Due to the same reason as in the CAR scenario, we only need to consider \( L_0 > 0 \) and \( D_0 > 0 \). The LR constraint in Eq. (16) ensures that Eq. (B.5) greater than zero must hold. And, due to \( 1/lr > 1 \), Eq. (B.6) must be positive.

\section*{Appendix C. Liquidity coverage ratio}

\subsection*{Appendix C.1. Scenario HH-LCR}

At \( t = 0 \), using Eqs. (10) and (22) and substituting for \( D_0 \) from the balance sheet identity in Eq. (8), we have the bank’s problem:

\[
\max_{L_0} \Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E
\]

subject to

\[ 0.25 lcr((i_D + \alpha)(L_0 + S + R - E)) \leq R + (1 - \chi)S, \]

and the nonnegativity constraint \( L_0 \geq 0 \). Denote by \( \lambda_{0HH} \) the Lagrangian multiplier at date 0. The Lagrangian of the problem at date 0 is

\[
\mathcal{L}_{0HH} = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E
\]
\[ + \lambda_{0HH} (R + (1 - \chi)S - 0.25 lcr((i_D + \alpha)(L_0 + S + R - E))). \]

We get the first-order conditions as

\[ 0 = i_L - i_D + 0.25 lcr \cdot \lambda_{0HH}(i_D + \alpha), \quad \text{(C.1)} \]
\[ 0 = R + (1 - \chi)S - 0.25 lcr((i_D + \alpha)(L_0 + S + R - E)). \quad \text{(C.2)} \]
Substitute Eqs. (12) and (13) into Eqs. (14) and (22), together with the balance sheet identity in Eq. (8), to obtain the maximization problem at \( t = 2 \):

\[
\max_{L_2} \Pi = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + \lambda (E + \sigma L \cdot i_L L_0 + \sigma S \cdot i_S S - \sigma D \cdot i_D D_0)
\]

subject to

\[
0.25 \text{ lcr}((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0))) \leq R + (1 - \chi)S,
\]

and the nonnegativity constraint \( L_2 \geq 0 \). The Lagrangian at date 2 is

\[
\mathcal{L}_2 = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + \lambda (E + \sigma L \cdot i_L L_0 + \sigma S \cdot i_S S - \sigma_D \cdot i_D D_0)),
\]

where \( \lambda \) is the Lagrangian multiplier. The first-order conditions can be written as

\[
0 = i_L - i_D + 0.25 \text{ lcr} \cdot \lambda (i_D + \alpha), \tag{C.3}
\]

\[
0 = R + (1 - \chi)S - 0.25 \text{ lcr}((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0))), \tag{C.4}
\]

\[
\lambda = (E + \sigma L \cdot i_L L_0 + \sigma S \cdot i_S S - \sigma_D \cdot i_D D_0)). \tag{C.5}
\]

The solutions for loans and deposits at date 0 are

\[
L_0 = -R - S + E + \frac{4(R + (1 - \chi)S)}{\text{ lcr}(i_D + \alpha)}, \tag{C.6}
\]

\[
D_0 = \frac{4(R + (1 - \chi)S)}{\text{ lcr}(i_D + \alpha)}. \tag{C.7}
\]

Here, we divide the derivation of the condition for this scenario into two steps. The first step shows the condition for \( IF_0 \geq 0.75 OF_0 \) and \( IF_2 \geq 0.75 OF_2 \). The second step yields the condition for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \).

Step 1: At date 0, the condition for \( IF_0 \geq 0.75 OF_0 \) is rearranged as

\[
IF_0 - 0.75 OF_0 \geq 0.
\]

Substitute \( IF_0 \) from Eq. (18) and \( OF_0 \) from Eq. (19) into the above inequality to obtain

\[
\kappa (i_L + \mu) L_0 - 0.75(i_D + \alpha) D_0 \geq 0, \tag{C.8}
\]

39
where \( L_0 \) is given by Eq. (C.6), and \( D_0 \) is in Eq. (C.7). As in the discussion of the CAR, we will use approximations to the conditions associated with the LCR. Like the interest rates \( i_L \), \( i_S \), and \( i_D \), the deposit run-off rate \( \alpha \) and fraction of loans repaid \( \mu \) are also of a small order of magnitude. Without loss of generality, we assume that \( \alpha \) and \( \mu \) are of the order of magnitude of \( 10^{-j} \), the same as that of \( i_L \), \( i_S \), and \( i_D \). In addition, \( lcr \approx 1 \) and \( 0 < \kappa \leq 1 \) are of the order of 1. Then, the terms on the left-hand side of Eq. (C.8) are of the order of \( 10^{Q} \) and \( 10^{Q-j} \). Retaining only the highest-order terms, we have

\[
\frac{4\kappa(i_L + \mu)(R + (1 - \chi)S)}{lcr(i_D + \alpha)} - \frac{3(R + (1 - \chi)S)}{lcr} \geq 0.
\]

This leads to

\[
\kappa(i_L + \mu) \geq 0.75(i_D + \alpha). \tag{C.9}
\]

Next, we turn to the condition for \( IF_2 \geq 0.75OF_2 \). It can be written as

\[
IF_2 - 0.75OF_2 \geq 0.
\]

Substituting Eqs. (18) and (19) into \( IF_2 - 0.75OF_2 \geq 0 \) yields

\[
\kappa(i_L + \mu)L_2 - 0.75(i_D + \alpha)D_2 \geq 0.
\]

The above inequality can be rewritten as

\[
\kappa(i_L + \mu)(L_2 - L_0) - 0.75(i_D + \alpha)(D_2 - D_0)
+ \kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0 \geq 0.
\]

Substituting for \( L_2 - L_0 \) from Eq. (45), \( D_2 - D_0 \) from Eq. (47), \( L_0 \) from Eq. (C.6), and \( D_0 \) from Eq. (C.7), we find the highest order of the terms on the second line is higher than that of those on the first line. Thus, retaining only the highest-order terms yields

\[
IF_2 - 0.75OF_2 = \kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0
= \frac{4\kappa(i_L + \mu)(R + (1 - \chi)S)}{lcr(i_D + \alpha)} - \frac{3(R + (1 - \chi)S)}{lcr} \geq 0, \tag{C.10}
\]

which is the same as the condition for \( IF_0 \geq 0.75OF_0 \) in Eq. (C.9).
Step 2: we show the condition for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \). It is obvious that \( D_0 > 0 \) and \( D_2 > 0 \) must hold. From Eq. (C.6), \( L_0 > 0 \) yields
\[
\frac{lcr(i_D + \alpha)E + (4 - lcr(i_D + \alpha))R + (4(1 - \chi) - lcr(i_D + \alpha))S}{lcr(i_D + \alpha)} > 0. \tag{C.11}
\]
According to the LCR rule, we have \( \chi \leq 0.75 \), which leads to \( 4(1 - \chi) \geq 1 \). In general, there is \( lcr(i_D + \alpha) \leq 1 \); then, \( 4(1 - \chi) - lcr(i_D + \alpha) \geq 0 \) and \( 4 - lcr(i_D + \alpha) > 0 \). Thus, \( L_0 > 0 \) must hold. The terms in \( L_2 \) are of the order of \( 10^{Q+j}, 10^Q, \) and \( 10^{Q-j} \). Retaining only the highest-order terms, we simplify \( L_2 \) to
\[
4(R + (1 - \chi)S) \frac{lcr(i_D + \alpha)}{lcr(i_D + \alpha)}, \tag{C.12}
\]
which must be greater than zero.

Therefore, the condition for scenario HH-LCR is Eq. (C.9) from Step 1:
\[
\kappa(i_L + \mu) \geq 0.75(i_D + \alpha). \tag{C.13}
\]

In addition, the condition for \( IF_0 \geq 0.75OF_0 \) and that for \( IF_2 \geq 0.75OF_2 \) are the same. Therefore, we have \( IF_2 = 0.75OF_2 \) if and only if \( IF_0 = 0.75OF_0 \).

Appendix C.2. Scenario LL-LCR

At date 0, from Eqs. (10) and (23), the bank’s maximization problem can be written as
\[
\max_{L_0} \Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E
\]
subject to
\[
lcr((i_D + \alpha)(L_0 + S + R - E) - \kappa(i_L + \mu)L_0) \leq R + (1 - \chi)S,
\]
and the nonnegativity constraint \( L_0 \geq 0 \). Denote \( \lambda^{LL}_0 \) as the Lagrangian multiplier at date 0. We show the date-0 Lagrangian as
\[
\mathcal{L}^{LL}_0 = (i_L - i_D)L_0 + (i_S - i_D)S - i_D R + i_D E + \lambda^{LL}_0(R + (1 - \chi)S - lcr((i_D + \alpha)(L_0 + S + R - E) - \kappa(i_L + \mu)L_0)).
\]
The first-order conditions are given by

\[ 0 = i_L - i_D + lcr \cdot \lambda_0^{L} (i_D + \alpha - \kappa(i_L + \mu)), \]  
\[ 0 = R + (1 - \chi)S - lcr((i_D + \alpha)(L_0 + S + R - E) - \kappa(i_L + \mu)L_0). \]  

At date 2, we obtain the bank’s problem by substituting Eqs. (12) and (13) into Eqs. (14) and (23) and using the balance sheet identity in Eq. (8). This leads to the following problem:

\( \max_{L_2} \Pi = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + i_D E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0 \)

subject to

\[ lcr((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)) - \kappa(i_L + \mu)L_2) \leq R + (1 - \chi)S, \]

and the nonnegativity constraint \( L_2 \geq 0 \). We write the date-2 Lagrangian as

\[ L_2^{LL} = (i_L - i_D)L_2 + (i_S - i_D)S - i_D R + i_D E + \lambda_2^{L}(R + (1 - \chi)S - lcr((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)) - \kappa(i_L + \mu)L_2)), \]

where \( \lambda_2^{L} \) is the Lagrangian multiplier. The first-order conditions are given by

\[ 0 = i_L - i_D + lcr \cdot \lambda_2^{L} (i_D + \alpha - \kappa(i_L + \mu)), \]  
\[ 0 = R + (1 - \chi)S - lcr((i_D + \alpha)(L_2 + S + R - (E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0)) - \kappa(i_L + \mu)L_2). \]

In this scenario, \( L_0 \) and \( D_0 \) are given by

\[ L_0 = \frac{(1 - lcr(i_D + \alpha))R + (1 - \chi - lcr(i_D + \alpha))S + lcr(i_D + \alpha)E}{lcr(i_D + \alpha - \kappa(i_L + \mu))}, \]
\[ D_0 = \frac{(1 - lcr \cdot \kappa(i_L + \mu))R + (1 - \chi - lcr \cdot \kappa(i_L + \mu))S + lcr \cdot \kappa(i_L + \mu)E}{lcr(i_D + \alpha - \kappa(i_L + \mu))}. \]

As in scenario HH-LCR, we divide the derivation of the condition into two steps. The first step shows the condition for \( IF_0 < 0.75OF_0 \) and \( IF_2 < 0.75OF_2 \). The second step gives the condition for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \).
Step 1: At date 0, the condition for $IF_0 < 0.75OF_0$ can be rewritten as

$$IF_0 - 0.75OF_0 < 0.$$ 

Substitute for $IF_0$ from Eq. (18) and for $OF_0$ from Eq. (19) into the above inequality to obtain

$$\kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0 < 0.$$ 

Plugging Eqs. (C.19) and (C.20) into the left-hand side, we have that the terms on the left-hand side are of the order of $10^Q$ and $10^{Q-j}$. Retaining only the highest-order terms, we obtain

$$\frac{(\kappa(i_L + \mu) - 0.75(i_D + \alpha))(R + (1 - \chi)S)}{lcr(i_D + \alpha - \kappa(i_L + \mu))} < 0. \quad (C.21)$$

At date 2, again, using Eqs. (18) and (19), we have

$$IF_2 - 0.75OF_2 = \kappa(i_L + \mu)L_2 - 0.75(i_D + \alpha)D_2$$

$$= \kappa(i_L + \mu)(L_2 - L_0) - 0.75(i_D + \alpha)(D_2 - D_0)$$

$$+ \kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0 < 0.$$ 

Substituting for $L_2 - L_0$ from Eq. (52) and $D_2 - D_0$ from Eq. (56) into the second line and substituting for $L_0$ from Eq. (C.19) and $D_0$ from Eq. (C.20) into the third line, we see that the highest order of the terms on the third line is higher than that of those on the second line. Retaining only the highest-order terms, we have

$$\kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0$$

$$= \frac{(\kappa(i_L + \mu) - 0.75(i_D + \alpha))(R + (1 - \chi)S)}{lcr(i_D + \alpha - \kappa(i_L + \mu))} < 0. \quad (C.22)$$

Thus, both $IF_0 < 0.75OF_0$ and $IF_2 < 0.75OF_2$ yield the same condition given by Eq. (C.21).

Step 2: we show the condition for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$. The terms in $L_0$ and $D_0$ are of the order of $10^{Q+j}$ and $10^Q$. First, retaining only the
highest-order terms, we simplify $L_0 > 0$ and $D_0 > 0$ to

$$\frac{R + (1 - \chi)S}{lcr(i_D + \alpha - \kappa(i_L + \mu))} > 0,$$

which leads to

$$i_D + \alpha - \kappa(i_L + \mu) > 0. \quad (C.23)$$

Second, the terms in $L_2$ and $D_2$ are of the order of $10^{Q+j}, 10^{Q}$, and $10^{Q-j}$. Retaining only the terms of the order of $10^{Q+j}$, we obtain $L_2 > 0$ and $D_2 > 0$ as

$$\frac{R + (1 - \chi)S}{lcr(i_D + \alpha - \kappa(i_L + \mu))} > 0, \quad (C.24)$$

which is the same condition as that for $L_0 > 0$ and $D_0 > 0$ in Eq. (C.23).

Finally, combining the condition in Eq. (C.21) from Step 1 and the condition in Eq. (C.23) from Step 2 yields the condition for scenario LL-LCR:

$$\kappa(i_L + \mu) < 0.75(i_D + \alpha). \quad (C.25)$$

**Appendix C.3. Scenario LH-LCR**

In this scenario, on date 0, the bank’s problem is the same as that in Section 5.3.2. On date 2, the maximization problem takes the same form as that in Section 5.3.1.

In scenario LH-LCR, the solutions for $L_0$ and $D_0$ are

$$L_0 = \frac{(1 - lcr(i_D + \alpha))R + (1 - \chi - lcr(i_D + \alpha))S + lcr(i_D + \alpha)E}{lcr(i_D + \alpha - \kappa(i_L + \mu))}, \quad (C.26)$$

$$D_0 = \frac{(1 - lcr \cdot \kappa(i_L + \mu))R + (1 - \chi - lcr \cdot \kappa(i_L + \mu))S + lcr \cdot \kappa(i_L + \mu)E}{lcr(i_D + \alpha - \kappa(i_L + \mu))}. \quad (C.27)$$

The solutions at date 0 are the same as those in scenario LL-LCR, given by Eqs. (C.19) and (C.20). Using Eqs. (C.26) and (C.27), we can rewrite the group
without $\Delta E$ in Eq. (58), $-D_0 + (4(R + (1 - \chi)S))/(\text{lcr}(i_D + \alpha))$, as

$$\frac{1}{\text{lcr}(i_D + \alpha)(\kappa(i_L + \mu) - (i_D + \alpha))} \times \left( (4\kappa(i_L + \mu) - 3(i_D + \alpha) - \text{lcr} \cdot \kappa(i_D + \alpha)(i_L + \mu))R \\
+ ((1 - \chi)(4\kappa(i_L + \mu) - 3(i_D + \alpha)) - \text{lcr} \cdot \kappa(i_D + \alpha)(i_L + \mu))S \\
\quad + \text{lcr} \cdot \kappa(i_D + \alpha)(i_L + \mu)E \right), \quad (C.28)$$

which is decomposed into $R$, $S$, and $E$.

The derivation of the conditions is divided into two steps. The first step presents the condition for $IF_0 < 0.75OF_0$ and $IF_2 > 0.75OF_2$. The second gives the condition for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$.

Step 1: At date 0, using Eqs. (18) and (19), we rearrange $IF_0 < 0.75OF_0$ as

$$\kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0 < 0.$$  

Substituting Eqs. (C.26) and (C.27) into the left-hand side of the above inequality, we have that the terms on the left-hand side are of the order of $10^Q$ and $10^Q-j$; retaining only the highest-order terms yields

$$\frac{(R + (1 - \chi)S)(\kappa(i_L + \mu) - 0.75(i_D + \alpha))}{\text{lcr}(i_D + \alpha - \kappa(i_L + \mu))} < 0. \quad (C.29)$$

At date 2, again using Eqs. (18) and (19) yields

$$IF_2 - 0.75OF_2 = \kappa(i_L + \mu)L_2 - 0.75(i_D + \alpha)D_2 \\
= \kappa(i_L + \mu)(L_2 - L_0) - 0.75(i_D + \alpha)(D_2 - D_0) \\
\quad + \kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0.$$  

Substituting for $L_2 - L_0$ from Eq. (57) and $D_2 - D_0$ from Eq. (59) into the second line and substituting for $L_0$ from Eq. (C.26) and $D_0$ from Eq. (C.27) into the third line, we prove that the terms in $IF_2 - 0.75OF_2$ are of the order of $10^Q$, $10^Q-j$, and $10^Q-2j$. Retaining only the highest-order terms leads $IF_2 - 0.75OF_2 > 0$ to

$$\frac{4(\kappa(i_L + \mu) - 0.75(i_D + \alpha))(R + (1 - \chi)S)}{\text{lcr}(i_D + \alpha)} > 0,$$
which implies

\[ \kappa(i_L + \mu) - 0.75(i_D + \alpha) > 0. \]  

(C.30)

Combine Eqs. (C.29) and (C.30) to obtain the condition for \( IF_0 < 0.75OF_0 \) and \( IF_2 > 0.75OF_2 \):

\[ \kappa(i_L + \mu) > i_D + \alpha. \]  

(C.31)

Step 2: First, we show the condition for \( L_0 > 0 \) and \( D_0 > 0 \). From Eqs. (C.26) and (C.27), using \( i_D + \alpha < \kappa(i_L + \mu) \), we have \( L_0 > 0 \) leading to \( D_0 > 0 \). Therefore, we only need to show the condition for \( L_0 > 0 \). Rearranging \( L_0 > 0 \) yields

\[ \frac{lcr(i_D + \alpha)(R + S - E) - (R + (1 - \chi)S)}{\kappa(i_L + \mu) - (i_D + \alpha)} > 0. \]  

(C.32)

The terms in Eq. (C.32) are of the order of \( 10^{Q+j} \) and \( 10^Q \). Since \( \kappa(i_L + \mu) > i_D + \alpha \), the terms of the order of \( 10^{Q+j} \) are negative. Because \( L_0 > 0 \), the highest-order approximation cannot be applied to Eq. (C.32): the terms of the order of both \( 10^{Q+j} \) and \( 10^Q \) should be considered. From Eq. (C.32) and \( \kappa(i_L + \mu) > i_D + \alpha \), the numerator of Eq. (C.32) must be greater than zero.

Rearranging the numerator, we obtain

\[ (R + S - E)(i_D + \alpha - R + (1 - \chi)S) > 0. \]  

(C.33)

Second, we turn to \( L_2 > 0 \) and \( D_2 > 0 \). It is clear that \( D_2 \) must be greater than zero. The terms in \( L_2 \) are of the order of \( 10^{Q+j} \), \( 10^Q \), and \( 10^{Q-j} \). Retaining only the highest-order terms, we simplify \( L_2 \) as

\[ \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)}, \]  

(C.34)

which must be greater than zero. In summary, the condition for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \) is given by Eq. (C.33).

Finally, combining Eqs. (C.31) and (C.33), we prove that the conditions for scenario LH-LCR are

\[ \kappa(i_L + \mu) > i_D + \alpha, \]

\[ (R + S - E)(i_D + \alpha - R + (1 - \chi)S) > 0. \]
Appendix C.4. Scenario HL-LCR

At date 0, the bank’s maximization problem is the same as that in Section 5.3.1. The bank’s problem at date 2 is the same as in Section 5.3.2.

In scenario HL-LCR, $L_0$ and $D_0$ are as follows:

\[
L_0 = -R - S + E + \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)} \tag{C.35}
\]

and

\[
D_0 = \frac{4(R + (1 - \chi)S)}{lcr(i_D + \alpha)}. \tag{C.36}
\]

The solutions at date 0 are the same as those in scenario HH-LCR given by Eqs. (C.6) and (C.7). Based on Eqs. (C.35) and (C.36), we rewrite the group without $\Delta E$ in Eq. (61),

\[
\frac{(i_D + \alpha)D_0 - \kappa(i_L + \mu)L_0}{\kappa(i_L + \mu) - (i_D + \alpha)} - \frac{R + (1 - \chi)S}{lcr(\kappa(i_L + \mu) - (i_D + \alpha))},
\]

as

\[
- \frac{1}{lcr(i_D + \alpha)(\kappa(i_L + \mu) - (i_D + \alpha))} \times \left\{ (4\kappa(i_L + \mu) - 3(i_D + \alpha) - lcr \cdot \kappa(i_D + \alpha)(i_L + \mu))R \right. \\
\left. + ((1 - \chi)(4\kappa(i_L + \mu) - 3(i_D + \alpha)) - lcr \cdot \kappa(i_D + \alpha)(i_L + \mu))S \right. \\
\left. + lcr \cdot \kappa(i_D + \alpha)(i_L + \mu)E \right\}. \tag{C.37}
\]

The expression that is decomposed into $R$, $S$, and $E$ is obtained. Note that Eq. (C.37) in scenario HL-LCR is the negative of Eq. (C.28) in scenario LH-LCR.

As in the above scenarios, the first step presents the condition for $IF_0 > 0.75OF_0$ and $IF_2 < 0.75OF_2$. The second provides the condition for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$.

Step 1: At date 0, using the cash inflows in Eq. (18), cash outflows in Eq. (19), $L_0$ in Eq. (C.35), and $D_0$ in Eq. (C.36), we have that the terms in $IF_0 - 0.75OF_0 > 0$ are of the order of $10^Q$ and $10^{Q-j}$. Then, retaining only the highest-order terms yields

\[
\frac{(R + (1 - \chi)S)(4\kappa(i_L + \mu) - 3(i_D + \alpha))}{lcr(i_D + \alpha)} > 0,
\]

47
which implies
\[ \kappa(i_L + \mu) > 0.75(i_D + \alpha). \]  
\[ \text{(C.38)} \]

At date 2, we also use Eqs. (18) and (19) to obtain
\[
IF_2 - 0.75OF_2 = \kappa(i_L + \mu)L_2 - 0.75(i_D + \alpha)D_2 \\
= \kappa(i_L + \mu)(L_2 - L_0) - 0.75(i_D + \alpha)(D_2 - D_0) \\
+ \kappa(i_L + \mu)L_0 - 0.75(i_D + \alpha)D_0.
\]

Substituting for \( L_2 - L_0 \) from Eq. (60) and \( D_2 - D_0 \) from Eq. (64) into the second line and substituting for \( L_0 \) from Eq. (C.35) and \( D_0 \) from Eq. (C.36) into the third line, we see that the terms in \( IF_2 - 0.75OF_2 \) are of the order of \( 10^Q, 10^{Q-j}, \) and \( 10^{Q-2j} \). Retaining only the highest-order terms, we simplify \( IF_2 - 0.75OF_2 < 0 \) as
\[
\frac{(R + (1 - \chi)S)(\kappa(i_L + \mu) - 0.75(i_D + \alpha))}{i_D + \alpha - \kappa(i_L + \mu)} < 0. \]  
\[ \text{(C.39)} \]

Together with Eq. (C.38), Eq. (C.39) reduces to
\[ \kappa(i_L + \mu) > i_D + \alpha. \]  
\[ \text{(C.40)} \]

Eq. (C.40) is the condition for \( IF_0 > 0.75OF_0 \) and \( IF_2 < 0.75OF_2 \).

Step 2: First, we derive the condition for \( L_0 > 0 \) and \( D_0 > 0 \). From Eq. (C.36), it is obvious that \( D_0 > 0 \). From Eq. (C.35), \( L_0 > 0 \) can be rewritten as
\[
\frac{lcr(i_D + \alpha)E + (4 - lcr(i_D + \alpha))R + (4(1 - \chi) - lcr(i_D + \alpha))S}{lcr(i_D + \alpha)} > 0.
\]
The LCR rule says that \( \chi \leq 0.75; \) thus, \( 4(1 - \chi) \geq 1 \). In general, there is \( lcr(i_D + \alpha) \leq 1 \). Therefore, \( 4(1 - \chi) - lcr(i_D + \alpha) \geq 0 \) and \( 4 - lcr(i_D + \alpha) > 0 \). These imply that \( L_0 > 0 \) must hold. Turning to \( L_2 > 0 \) and \( D_2 > 0 \), the terms in \( L_2 \) and \( D_2 \) are of the order of \( 10^{2+j}, 10^q, \) and \( 10^{q-j} \). Their highest-order terms are negative. Because \( L_2 > 0 \) and \( D_2 > 0 \), the terms of the order of both
and $10^{Q+j}$ and $10^Q$ need to be considered. Thus, $L_2$ is approximated by

$$
\frac{1}{lcr(\kappa(i_L + \mu) - (i_D + \alpha))} \times (R + S - E)(i_D + \alpha - \frac{(1 + 4(i_L - i_D))(R + (1 - \chi)S)}{lcr(R + S - E)}).
$$

Because of $\kappa(i_L + \mu) > i_D + \alpha$, $L_2 > 0$ simplifies to

$$(R + S - E)(i_D + \alpha - \frac{(1 + 4(i_L - i_D))(R + (1 - \chi)S)}{lcr(R + S - E)}) > 0 \quad \text{(C.41)}$$

Similarly, $D_2$ is approximated by

$$
\frac{1}{lcr(\kappa(i_L + \mu) - (i_D + \alpha))} \times (R + S - E)(i_D + \alpha - \frac{\frac{i_D + \alpha}{\kappa(i_L + \mu)} + 4(i_L - i_D))(R + (1 - \chi)S)}{lcr(R + S - E)}).
$$

Because of $\kappa(i_L + \mu) > i_D + \alpha$, $D_2 > 0$ simplifies to

$$(R + S - E)(i_D + \alpha - \frac{\frac{i_D + \alpha}{\kappa(i_L + \mu)} + 4(i_L - i_D))(R + (1 - \chi)S)}{lcr(R + S - E)} > 0 \quad \text{(C.42)}$$

Since $(i_D + \alpha)/\kappa(i_L + \mu) < 1$, the inequality in Eq. (C.41) implies the inequality in Eq. (C.42). Thus, the condition for $L_0 > 0$, $D_0 > 0$, $L_2 > 0$, and $D_2 > 0$ is given by Eq. (C.41).

To summarize, combining Eq. (C.40) from Step 1 and Eq. (C.41) from Step 2, we prove that the conditions for scenario HL-LCR are

- $i_D + \alpha < \kappa(i_L + \mu)$,
- $(R + S - E)(i_D + \alpha - \frac{(1 + 4(i_L - i_D))(R + (1 - \chi)S)}{lcr(R + S - E)}) > 0$.

**Appendix D. Net stable funding ratio**

Based on the objective function in Eq. (10) and NSFR constraint in Eq. (24), the bank’s maximization problem at date 0 is

$$\max_{L_0} \Pi = (i_L - i_D)L_0 + (i_S - i_D)S - i_DR + i_DE$$
subject to

\[ nsfr(\phi_L L_0 + \phi_S S) \leq \beta(L_0 + S + R) + (1 - \beta)E, \]

and the nonnegativity constraint \( L_0 \geq 0 \). The date-0 Lagrangian can be written as

\[
L^N_0 = (i_L - i_D) L_0 + (i_S - i_D) S - i_D R + i_D E + \lambda^N_0 (\beta(L_0 + S + R) + (1 - \beta)E - nsfr(\phi_L L_0 + \phi_S S)),
\]

where \( \lambda^N_0 \) is the Lagrangian multiplier. We get the first-order conditions as

\[
0 = i_L - i_D - \lambda^N_0 (\beta - nsfr \cdot \phi_L), \quad (D.1)
\]
\[
0 = \beta(L_0 + S + R) + (1 - \beta)E - nsfr(\phi_L L_0 + \phi_S S). \quad (D.2)
\]

Substitute Eqs. (12) and (13) into the objective function in Eq. (14) and then use the balance sheet identity in Eq. (8) to obtain the bank’s problem at date 2:

\[
\max_{L_2} \Pi = (i_L - i_D) L_2 + (i_S - i_D) S - i_D R + i_D E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0
\]

subject to

\[ nsfr(\phi_L L_2 + \phi_S S) \leq \beta(L_2 + S + R) + (1 - \beta)(E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0), \]

and the nonnegativity constraint \( L_2 \geq 0 \). Denote by \( \lambda^N_2 \) the Lagrangian multiplier at date 2. We show the date-2 Lagrangian as

\[
L^N_2 = (i_L - i_D) L_2 + (i_S - i_D) S - i_D R + i_D E + \lambda^N_2 (\beta(L_2 + S + R) + (1 - \beta)(E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0) - nsfr(\phi_L L_2 + \phi_S S)).
\]

The first-order conditions are given by

\[
0 = i_L - i_D - \lambda^N_2 (\beta - nsfr \cdot \phi_L), \quad (D.3)
\]
\[
0 = \beta(L_2 + S + R) + (1 - \beta)(E + \sigma_L \cdot i_L L_0 + \sigma_S \cdot i_S S - \sigma_D \cdot i_D D_0) - nsfr(\phi_L L_2 + \phi_S S)). \quad (D.4)
\]

\[ (D.5) \]
The solutions for loans and deposits at date 0 are given by

\[ L_0 = \frac{(\beta - \text{nsfr} \cdot \phi_S)S + \beta R + (1 - \beta)E}{\text{nsfr} \cdot \phi_L - \beta}, \quad (D.6) \]

\[ D_0 = \frac{\text{nsfr}(\phi_L - \phi_S)S + \text{nsfr} \cdot \phi_L R + (1 - \text{nsfr} \cdot \phi_L)E}{\text{nsfr} \cdot \phi_L - \beta}. \quad (D.7) \]

The solution for loans in Eq. (D.6) greater than zero implies that if \( \text{nsfr} \cdot \phi_L < \beta \), then \( \beta > 1 \). According to the NSFR rule, \( \beta > 1 \) indicates that the deposits are stabler than the equity of banks. This is not realistic in practice. So we must have \( \text{nsfr} \cdot \phi_L > \beta \).

Finally, we derive the conditions for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \). Rearranging \( L_0 \) in Eq. (D.6), we obtain the condition for \( L_0 > 0 \) as

\[ (S + R - E)(\text{nsfr} \cdot \phi_L - \beta)(\beta - \frac{S}{S + R - E} \cdot \text{nsfr} \cdot \phi_S + \frac{E}{S + R - E}) > 0. \quad (D.8) \]

Similarly, we rearrange \( D_0 \) in Eq. (D.7) to show the condition for \( D_0 > 0 \) as

\[ (S+R-E)(\text{nsfr} \cdot \phi_L - \beta)(\text{nsfr} \cdot \phi_S - \frac{S}{S + R - E} \cdot \text{nsfr} \cdot \phi_S + \frac{E}{S + R - E}) > 0. \quad (D.9) \]

Then, the terms in \( L_2 \) and \( D_2 \) are of the order of \( 10^Q \) and \( 10^{Q-j} \). Retaining only the highest-order terms, we reduce \( L_2 \) and \( D_2 \) to \( L_0 \) and \( D_0 \), respectively. Therefore the conditions for \( L_2 > 0 \) and \( D_2 > 0 \) are the same as those for \( L_0 > 0 \) and \( D_0 > 0 \). In summary, the conditions for \( L_0 > 0, D_0 > 0, L_2 > 0, \) and \( D_2 > 0 \) are given by Eqs. (D.8) and (D.9).

**Appendix E. Table of notations**

<table>
<thead>
<tr>
<th>Variable or parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Bank balance sheets</td>
<td></td>
</tr>
<tr>
<td>( L )</td>
<td>Loans</td>
</tr>
<tr>
<td>( S )</td>
<td>Securities</td>
</tr>
<tr>
<td>( R )</td>
<td>Reserves</td>
</tr>
</tbody>
</table>

(continued on next page)
<table>
<thead>
<tr>
<th>Variable or parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Deposits</td>
</tr>
<tr>
<td>$E$</td>
<td>Equity</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Profits</td>
</tr>
</tbody>
</table>

Panel B: Interest rates

<table>
<thead>
<tr>
<th>$i_L$</th>
<th>Loan rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_S$</td>
<td>Security rates</td>
</tr>
<tr>
<td>$i_D$</td>
<td>Deposit rates</td>
</tr>
</tbody>
</table>

Panel C: Shocks

<table>
<thead>
<tr>
<th>$I$</th>
<th>Interest receipt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Interest expenditure</td>
</tr>
</tbody>
</table>

Panel D: Dummy variables

<table>
<thead>
<tr>
<th>$\sigma_L$</th>
<th>Dummy variable for interest receipt on loans</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_S$</td>
<td>Dummy variable for interest receipt on securities</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>Dummy variable for interest expenditure on deposits</td>
</tr>
</tbody>
</table>

Panel E: Regulations

<table>
<thead>
<tr>
<th>$car$</th>
<th>Minimum capital adequacy ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_L$</td>
<td>Risk weight for loans</td>
</tr>
<tr>
<td>$\gamma_S$</td>
<td>Risk weight for securities</td>
</tr>
<tr>
<td>$lcr$</td>
<td>Minimum liquidity coverage ratio</td>
</tr>
<tr>
<td>$HQLA$</td>
<td>High-quality liquid assets</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Haircut for securities</td>
</tr>
<tr>
<td>$NCOF$</td>
<td>Net cash outflows</td>
</tr>
<tr>
<td>$OF$</td>
<td>Cash outflows</td>
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<tr>
<td>$\alpha$</td>
<td>Run-off rate for deposits</td>
</tr>
<tr>
<td>$IF$</td>
<td>Cash inflows</td>
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<tr>
<td>$\mu$</td>
<td>Fraction of loans repaid</td>
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<table>
<thead>
<tr>
<th>Variable or parameter</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\kappa$</td>
<td>Inflow rate for repayments</td>
</tr>
<tr>
<td>$nsfr$</td>
<td>Minimum net stable funding ratio</td>
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<tr>
<td>$\beta$</td>
<td>Available stable funding (ASF) factor for deposits</td>
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<tr>
<td>$\phi_L$</td>
<td>Required stable funding (RSF) factor for loans</td>
</tr>
<tr>
<td>$\phi_S$</td>
<td>Required stable funding (RSF) factor for securities</td>
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</tbody>
</table>

References


