Persistence and Volatility Spillovers of Bitcoin price to Gold and Silver prices

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Abstract

The paper investigates persistence, returns and volatility spillovers from the bitcoin market to the gold and silver markets using daily datasets from 2 January 2018 to 31 July 2020 by employing the fractional persistence framework. The results show strong price persistence with bitcoin posing the highest volatility persistence, while silver poses the lowest volatility persistence. The results of multivariate GARCH modelling, using the CCC-VARMA-GARCH model and other lower variants indicate the impossibility of returns spillover between the bitcoin and gold (or silver) market, while there exist bi-directional volatility spillovers. Appropriate portfolio management and hedging strategies render towards the end of the paper require more gold and silver investments in the portfolio of bitcoin to fully have the diversification advantage and reduce risk to the minimum without reducing the expected returns of their portfolio.

Keywords: Bitcoin; Commodity markets; CCC-VARMA-GARCH model; Volatility spillovers; Portfolio management

JEL Classification: C22

1. Introduction

Among precious metals, gold and silver are used for industrial or investment purposes due to their high economic values. These precious metals, especially gold, serves as a reserve instrument for central banks. It is important to state that instrument like gold possesses high transaction volume and significant cash flow in the global market. Gold is one of the top-most marketed products worldwide and serves as a wealth storage tool, especially during economic and political instability (Aggarwal and Lucey, 2007; Batten et al., 2010; Lucey et al., 2013; Yaya et al. 2016; Gil-Alana et al., 2017; Yaya et al., 2021a). Historically, the function of gold and silver are very similar, and a long-run relationship exists between the two metals (Wahab et al., 1994; Ciner et al. 2013; Gokmenoglu and Fazlollahi, 2015; Pierdzioch et al., 2015; Auer, 2016; Pierdzioch et al., 2016; Zhu et al., 2016; Schweikert, 2018). Both gold and silver are used for jewellery, and are often traded as investment assets, even though silver possesses little industrial usage but it is more commodity-driven than gold (Yaya, Vo and Olayinka, 2021).

As an alternative investment, cryptocurrency was introduced through the introduction of Bitcoin by Nakamoto (2008). As of today, of about 9900 cryptocurrencies, bitcoin is the most innovative digital currency.[1] The currency had experienced ups and downs price swings but has not failed to keep drawing attention to all parts of society about its investment option. Following Nakamoto (2008), Bitcoin is described as peer-to-peer electronic cash that achieves its decentralization anonymously and transparently at the time of its creation. A few years back, bitcoin was labelled the “New Gold” by some social network agencies, and other data providers (Chuen, 2015; Dwyer, 2015; Bariviera et al., 2017; Elendner et al., 2018; Härdle et al., 2018). According to Wu and Pandey (2014), Bitcoin is less useful as a currency, but it can play an essential role in enhancing the efficiency of an investor’s portfolio. Bitcoin is known to drive prices of other cryptocurrencies and generally, markets of these crypto coins are high volatility and inefficient (Babatunde et al., 2021; Yaya, et al., 2019; Yaya et al., 2021b; Yaya et al., 2022).

A few studies have considered Bitcoin and precious metals such as Gold and Silver (Bouoiyour and Selmi, 2015; Dyhrberg, 2016; Bouri et al., 2017; Corbet et al., 2018; Klein et

[1] https://finance.yahoo.com/cryptocurrencies/

al.. 2018; Shahzada et al., 2019). Dyhrberg (2015a, 2015b; 2016) adopted the Asymmetric Generalized Autoregressive Conditional Heteroscedasticity (AGARCH) methodology to explore Bitcoin hedging capabilities and concluded that Bitcoin possesses close hedging abilities as Gold. According to Dwyer (2015), Bitcoin volatility returns is higher than in gold and foreign exchanges. Bouoiyour and Selmi (2015) adopted the optimal-GARCH model to explore the relationship between precious metals and Bitcoin prices with high fluctuations in financial markets and found that these commodities are not fixed over time. Also, it is noted that Bitcoin serves as a weak safe-haven in the short run, and as a hedge in the long run.

Eryigit (2017) investigated the short-term and long-term effects of gold prices on precious metals such as palladium, silver and platinum, and energy prices such as crude oil and gasoline. The author employed a vector autoregressive (VAR) model to explore the short-term interaction between prices of gold and metals and the results of the VAR analysis showed that gold has a short-term dependence on silver prices. Zhu et al. (2017) examined the influence of some economic factors such as gold price on the Bitcoin price using the Vector Error Correction (VEC) model. They concluded that gold price has the least impact among other factors considered. Liu and Su (2018) examined the dynamic causality between the returns of gold and silver in China market using the rolling window bootstrap approach. The results show that gold has positive and negative impacts on silver in multiple sub-periods. Klein et al. (2018) implemented the Baba-Engle-Kroner-Kraft (BEKK-GARCH) model on Bitcoin and Gold as well as other assets and found Gold to play an important role with flight-to-quality in times of financial market distress, while Bitcoin behaved differently.

The current paper investigates price persistence and volatility spillovers of Bitcoin price to Gold and Silver prices using the Multivariate GARCH framework. Specifically, the paper applied the Constant Conditional Correlation – Vector Autoregressive Moving Average - GARCH (CCC-VARMA-GARCH) model which allowed for constant correlations in conditional volatility and cross-markets spillovers of returns and volatility to be investigated. As pre-analysis, volatility persistence is examined using the fractional persistence approach and volatility observed is decomposed using the VARMA structure in the multivariate volatility model. Based on the findings, appropriate design and hedging strategies for the management of investments of Bitcoin with Gold and Silver are presented.

Section 2 of the paper reviews the relevant literature on the topic while section 3 presents the time series analysis methods. Section 4 presents the data and findings on CCC-MGARCH and CCC-VARMA-GARCH models, while Section 5 presents a strategy to manage portfolios of assets using optimal weights and hedge ratios. Section 6 concludes the paper.

2. Methodology

2.1. Long memory process and fractional persistence technique

Granger and Joyeux (1980) and Hosking (1981) define the long memory process in both time and frequency domain approaches for a stationary time series process $\tilde{X}_t$ with an autocovariance function $\tilde{\gamma}(k) = Cov(\tilde{X}_t, \tilde{X}_{t+k}) = E(\tilde{X}_t, \tilde{X}_{t+k})$ as,

$$\tilde{\gamma}(k) \rightarrow c_{\gamma} k^{2d-1}, \text{ as } k \rightarrow \infty \tag{1}$$

where k is the time lag with respect to time t, and the autocorrelation $\tilde{\gamma}(k)$ is a slowly hyperbolic decreasing function with respect to time t. The autocorrelations are also not summable, i.e.:

$$\sum_{k=-\infty}^{\infty} \tilde{\gamma}(k) = \infty. \tag{2}$$

In the frequency domain, the spectral density of $\tilde{X}_t$ is defined as:

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \tilde{\gamma}(k) e^{ik\omega} \tag{3}$$

where ω denotes the Fourier frequency. From (3), it can be shown that

$$f(\omega) = c_f \omega^{d-1} \text{ as } \; d \to 0 \tag{4}$$

where c_f is a positive constant. Equation (4) implies that the spectral density will tend to infinity as the frequency approaches zero frequency.

According to the literature, there are different estimation techniques to estimate and test the fractional persistence parameter (Geweke and Porter-Hudak, 1983; Fox and Taqqu, 1986; Kunsch, 1987; Robinson, 1995a, b; Ooms and Hassler, 1997 and others). The local Whittle estimation and log-periodogram regression methods are semi-parametric estimation approaches to fractional integration. The local Whittle's method for estimating *d* is based on a frequency domain using the Whittle function as in Kunsch (1987); Beran (1994) and Robinson (1995a). Other extensions of the local Whittle estimator are found in Shimotsu and Phillips (2005) and Shimotsu (2010). The estimator by Robinson (1995a) is often implemented in most software packages. Geweke and Porter-Hudak (1983) developed the log-periodogram method to obtain an estimate of the fractional differencing parameter. This method was modified in Robinson (1995b).

Robinson (1995a) developed the local Whittle semi-parametric estimator. In the frequency domain, the authors defined,

$$I\left(\tilde{\lambda}_k\right) \sim e^{f\left(\tilde{\lambda}_k\right)^{-1}} \tag{5}$$

with $\tilde{\theta} = \left(\tilde{C}, d\right)$ and at zero frequency,

$$L\left(\tilde{C}, d\right) = \left[\log \tilde{C} - 2d \ln\left(\tilde{\lambda}_k\right) + \frac{I\left(\tilde{\lambda}_k\right)}{\tilde{C}\tilde{\lambda}_k^{-2d}}\right] \tag{6}$$

The GSE of *d* is obtained as follows by minimizing the likelihood function in equation (6)

$$\hat{d} = \arg\min\left(\ln\left\{\frac{1}{m}\sum_{k=1}^{m}\left[\frac{I\left(\tilde{\lambda}_k\right)}{\tilde{\lambda}_k^{-2d}}\right]\right\} - \frac{2d}{m}\sum_{k=1}^{m}\ln\left(\tilde{\lambda}_k\right)\right) \tag{7}$$

Robinson (1995b) proves consistency for $d \in (-0.5,0.5)$. Though, the log-periodogram regression is popularly employed, it is important to note that its consistency is less than that of WSE at the nonstationary range. The log-periodogram regression is defined as:

$$\ln\left(I_{\tilde{X}}\left(\tilde{\lambda}_k\right)\right) = \beta_0 + \beta_1 \ln\left(\tilde{\lambda}_k\right) + \eta_k,\ k = 1,2,..m \tag{8}$$

where $I_{\tilde{X}}\left(\tilde{\lambda}_k\right)$ is the periodogram of the time series $\tilde{X}_t$ and η_k is assumed to be i.i.d. Thus, using the least square estimator, the differencing parameter is obtained as follows:

$$\hat{d}_1 = -\hat{\beta}_1 \tag{9}$$

The estimator is asymptotically normal and corresponds to the theoretical standard error $\frac{\pi}{\sqrt{2\pi m}}$.

2.2. Multivariate volatility modelling

Multivariate volatility models are popularly adopted in finance to capture both volatility clustering and contemporaneous correlation of asset return vectors. Following Bollerslev (1990) and Chevallier (2011), the CCC-GARCH model is defined as follows:

$$r_t = \Phi + \Theta x_t + \eta_t, \quad \eta_t = H_t^{1/2} v_t \tag{10}$$

where r_t is a $m \times 1$ vector of response variable; Φ is a $k \times 1$ vector of constant terms, Θ is a $m \times k$ matrix of parameters; x_t is a $k \times 1$ vector of predictors; $H_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix H_t; η_t is the identically and independently distributed error process that is heteroscedastic defined such that v_t is $m \times 1$ vector of innovations that are normally, independently and identically distributed.

The conditional covariance matrix is specified as follows:

$$H_t = D_t^{1/2} R D_t^{1/2} = \rho_{ij} \sqrt{h_{it} h_{jt}},\ i \neq j \tag{11}$$

where D_t is a diagonal matrix with elements as conditional variances, R is a matrix of unconditional correlations of the standardized residuals ρ_{ii} $(i = 1,...,m)$ as elements:

$$R = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{12} & 1 & \cdots & \rho_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m} & \rho_{2m} & \cdots & 1 \end{pmatrix} \tag{12}$$

and h_t is defined as any univariate GARCH (1,1) model,

$$h_t = \omega + \alpha\eta_{t-1} + \beta h_{t-1} \tag{13}$$

where $\omega > 0, \alpha \geq 0$ and $\beta \geq 0$ are the model parameters conditioned in order to realize covariance stationary mean reverting conditional variances of shocks in the return series. Since the shocks revert themselves to more stable states, we obtain the persistence of volatility and half-life for each conditional variance series as:

$$\text{Persistence} = \alpha + \beta \tag{14}$$

$$\text{Half-life} = \ln(0.5)/\ln(\alpha + \beta) \tag{15}$$

For the purpose of modelling, the parameters of the CCC-GARCH model are estimated using the maximum likelihood estimator and the unconcentrated log-likelihood function is based on the multivariate normal distribution for observation *t*. The log-likelihood function is given in

$$l_t = -0.5m\ln(2\pi) - 0.5\ln\{\det(R)\} - \ln\{\det(D_t^{1/2})\} - 0.5\tilde{\eta}_t R^{-1}\tilde{\eta}_t' \tag{16}$$

where $\tilde{\eta}_t = D_t^{-1/2}\eta_t$ is a $m \times 1$ vector of standardized residuals, $\eta_t = r_t - \Theta x_t$. Assume that v_t follow a multivariate *t* distribution with degrees of freedom greater than 2, then the unconcentrated log-likelihood function for observation *t* is

$$l_t = \ln\Gamma\left(\frac{df+m}{2}\right) - \ln\Gamma\left(\frac{df}{2}\right) - \frac{m}{2}\ln\{(df-2)\pi\} - 0.5\ln\{\det(R)\} - \ln\{\det(D_t^{1/2})\} - \\ -\left(\frac{df+m}{2}\right)\ln\left(1+\frac{\tilde{\eta}_t R^{-1}\tilde{\eta}_t'}{df-2}\right) \quad (17)$$

The correlation matrix R can be concentrated out of equations. (17) and (18) by defining the (i, j) the element of R as

$$\rho_{ij} = \frac{\sum_{t=1}^{T}\tilde{\eta}_{it}\tilde{\eta}_{jt}}{\sqrt{\sum_{t=1}^{T}\tilde{\eta}_{it}^2}\sqrt{\sum_{t=1}^{T}\tilde{\eta}_{jt}^2}} \quad (18)$$

We obtained the starting values for the parameters in the mean equations and the initial residuals $\hat{\eta}_t$ using the least squares estimator. The starting values for the parameters in the variance equations are obtained by a procedure proposed by Gourieroux and Monfort (1997).

3. Data, Empirical analysis and Discussion

Daily prices of Bitcoin, gold and silver were analyzed in this paper. The datasets span the period 2 January 2018 to 31 July 2020. These were retrieved from the Federal Reserve Bank of St Louis Economic Database (FRED) at https://fred.stlouisfed.org. Bitcoin is priced in US dollars per coin while gold and silver are priced in US dollars per troy. Plots of the price series are given in Figure 1, while plots of the log-returns series are given in Figure 2. Both figures clearly show possible co-movements of assets since 2018 as they both picked during the Covid-19 crash around March 2020 and prices are increasing astronomically. Having obtained the log-returns based on the formula

$$r_t = \log\left(P_t / P_{t-1}\right) \quad (19)$$

where r_t is the log-returns series and P_t and P_{t-1} are the current day and previous day prices of the commodity, respectively. We have plots of the returns series in Figure 1. There are volatility spikes that indicate possible co-movements between Bitcoin and gold returns, and between Bitcoin and silver returns.

INSERT FIGURE 1 ABOUT HERE

INSERT FIGURE 2 ABOUT HERE

Table 1 summarizes the descriptive statistics of the variables in terms of price and log returns. The Bitcoin market and Silver market prices both have the lowest and highest values, respectively. We also observed that only Bitcoin have a negative mean return while Gold and Silver possess a small positive mean return. On average Bitcoin has the highest price, followed by Gold. Gold market return and Bitcoin market price both have the lowest and highest standard deviation, respectively. Skewness, Kurtosis, and subsequently, the Jarque-Bera test result in Table 1 are employed to examine if the time series data are distributed normally. All the variables are positively skewed except the bitcoin market and Silver market return. The kurtosis values show that the distributions of the series are too peaked. This result is further supported by the Jarque–Bera (JB) test statistics. The statistics are significant at the 5% level, which shows that all the series are not normally distributed.

INSERT TABLE 1 ABOUT HERE

Table 2 presents the results of serial correlations and conditional heteroscedasticity tests on the returns series. The Ljung-Box test was conducted on the returns and squared returns series for lags 1 and 5 and the results indicated the significance of serial correlations in these three returns series. To confirm heteroscedasticity, the ARCH LM test of Engle (1982) was conducted and we found the strong significance of the ARCH effect in gold and silver returns while Bitcoin returns are only significant at a 10% level.

INSERT TABLE 2 ABOUT HERE

Table 3 presents the results of price series persistence, P_t; returns series, r_t; volatility persistence using absolute and squared returns series ($|r_t|$ and r_t^2). The log-periodogram and Whittel semi-parametric estimators were applied to estimate the long memory and fractional persistence values for three periodogram points $T^{0.6}$, $T^{0.7}$ and $T^{0.8}$ for periodogram $T^{0.6}$, persistence results show that Bitcoin price persists most compared to gold and silver prices in both fractional persistence estimators, whereas the results are contradictory in other periodogram points. In the log-returns, the randomness of returns (random walk) is observed in the three markets and these markets are in their state of market efficiency. in terms of volatility persistence, we observe persistence in absolute and squared returns for Bitcoin to be the highest while silver is the lowest. Thus, volatility persists more in Bitcoin than in gold and silver prices, while silver volatility persists for the shortest period among the three assets.

INSERT TABLE 3 ABOUT HERE

The results obtained from the sign and size bias tests of Engle and Ng (1993) for asymmetry in returns and the conditional correlation (CCC) test of Engle and Sheppard (2001) are presented in Table 4. In the three returns series, Bitcoin, Gold and Silver, none of the sign and size bias tests is significant implying that the symmetric multivariate GARCH model is suitable to model the relationship from Bitcoin to Gold returns and from Bitcoin to Silver returns. The CCC test also indicates constancy of correlation as the null of the CCC test against dynamic correlations (DCC) is significantly unrejected in the two relationships: Bitcoin-Gold spillovers and Bitcoin-Silver spillovers.

INSERT TABLE 4 ABOUT HERE

We present the results of the CCC-GARCH models for Bitcoin-Gold and Bitcoin-Silver relationships. Recall, in (11), Φ and x_t are $k\times 1$ vectors of constant terms variables, respectively, such that in the bivariate settings, each matrix is of dimension 2×1, that is

$\Phi = (\phi_{bitcoin} \quad \phi_i)'$ where $i = \{gold, silver\}$. The matrix Θ, the $m \times k$ matrix of parameters is a $k \times 1$ vector of products. In the case of bivariable, Θ is a 2×2 matrix:

$$\Theta = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix} = \begin{pmatrix} \theta_{bitcoin} & \theta_{bitcoin,i} \\ \theta_{i,bitcoin} & \theta_i \end{pmatrix} \tag{20}$$

where $\theta_{bitcoin}$ and θ_i are the autoregressive parameters of order 1 for Bitcoin and Gold (or Silver) price returns, respectively; and $\theta_{bitcoin,i}$ measures the returns spillover from the Gold (Silver) market to the Bitcoin market, and also $\theta_{i,bitcoin}$ measures the returns spillover from the Bitcoin market. to the Gold (Silver) market. If both spillover parameters are significant, it implies a bidirectional spillover effect in the two asset markets.

The results of CCC-GARCH models for Bitcoin-Gold and Bitcoin-Silver relationships are presented in Table 5. From the mean equation of the Bitcoin-Gold relationship, only the AR(1) parameter for Bitcoin return is significant while the spillover parameters, $\theta_{bitcoin,i}$ and $\theta_{i,bitcoin}$. Thus, there is no spillover from Bitcoin to Gold price, and neither is any return spillover from Gold to Bitcoin returns. A similar result is obtained in the case of the Bitcoin-Silver relationship. By looking at the results of the variance equation, all GARCH parameters are highly significant. For the Bitcoin-Gold relationship, the volatility persistence of Bitcoin in the Bitcoin-Gold portfolio is 0.8806 while that of Silver in the same portfolio is 0.9522. Thus, the persistence of volatility on Bitcoin in this portfolio is lesser than that of Gold, and the corresponding half-lives of these shocks are 5.45 and 14.15, respectively. Similarly to the Bitcoin-Silver relationship, the volatility persistence of Bitcoin in the Bitcoin-Silver portfolio is 0.8904 while that of Silver in the portfolio is 0.9321. Thus, the half-lives of Bitcoin are shorter compared to that of Silver. Post estimation diagnostics reported in the results table showed that the two models are adequate in modelling returns and volatility spillovers in the portfolio pairs.

INSERT TABLE 5 ABOUT HERE

The fact that there is no returns spillover in Bitcoin-Gold and Bitcoin-Silver portfolios but volatility spillovers suggested further probe into the volatility/shocks forms. The MGARCH model in (13) is modified to allow for volatility spillovers as in the CCC-VARMA-GARCH model of McAleer et al. (2009). The VARMA-GARCH component of the model allows for the conditional variance of Bitcoin not to depend only on its own past conditional variance and shocks, but also on those of Gold (Silver) market returns. The same explanation holds for the conditional variance of the Gold (Silver) market. The bivariate model is given in (21).

$$\begin{aligned} h_{bitcoin,t} &= \omega + \alpha_{bitcoin}\eta_{bitcoin,t-1} + \alpha_{bitcoin,i}\eta_{i,t-1} + \beta_{bitcoin,i}h_{bitcoin,t-1} + \beta_{bitcoin,i}h_{i,t-1} \\ h_{i,t} &= \omega + \alpha_{i}\eta_{i,t-1} + \alpha_{i,bitcoin}\eta_{bitcoin,t-1} + \beta_{i,bitcoin}h_{i,t-1} + \beta_{i,bitcoin}h_{bitcoin,t-1} \end{aligned} \tag{21}$$

where the shock spillover effects from Gold (Silver) are captured by the parameter $\alpha_{bitcoin,i}$, and $\alpha_{i,bitcoin}$ captures the shock spillover effect of Bitcoin to Gold (Silver). The cross-market conditional volatility is also captured in the model, which $\beta_{bitcoin,i}$ captures the effect of the conditional volatility of Gold (Silver) on Bitcoin, and $\beta_{i,bitcoin}$ captures the effect of the conditional volatility of Bitcoin on Gold (Silver). Thus, the transmission of shocks from one market to another is quantified using the VARMA-GARCH models.

In Table 6, the results of mean equations for CCC-VARMA-GARCH models for the two relationships (Bitcoin-Gold and Bitcoin-Silver) agree with that of CCC-GARCH models as returns spillover parameters are not significant. In the Bitcoin-Gold relationship, the parameter $\alpha_{bitcoin,i}$ is insignificant implying that there is no shocks spillover from the Gold market to the Bitcoin market. Similarly, $\alpha_{i,bitcoin}$ parameter is not significant meaning that there is no shock spillover from the Bitcoin market to the Gold market. By looking at volatility spillovers, the parameter $\beta_{bitcoin,i}$ is 398.83 and it is significant implying that there is a sharp positive volatility spillover from the Gold to the Bitcoin market. This estimate is positive, implying that as shocks increase in one market, shocks also increase in the other market. The parameter $\beta_{i,bitcoin}$ is -0.1369 and is also significant implying its spillover from the Bitcoin market to the Gold market meaning an inverse relationship in the transmission of shocks spillovers. The conditional volatility spillover in the Bitcoin-Gold portfolio of assets is bidirectional since both $\beta_{bitcoin,i}$ and $\beta_{i,bitcoin}$ are significant. In the Bitcoin-Silver portfolio, there are significant bidirectional shocks and volatility spillovers. In this portfolio, $\alpha_{bitcoin,i}$ and $\alpha_{i,bitcoin}$ are negative meaning inverse relationships in shock transmission. Similarly, $\beta_{i,bitcoin}$ is negative while $\beta_{bitcoin,i}$ is positive. These inverse relationships show that the Bitcoin market transmits shocks or volatility inversely compared to gold and silver. here, both constant correlations for Bitcoin-Gold and Bitcoin-Silver models are significant, where the correlation for Bitcoin-Silver is negative.

INSERT TABLE 6 ABOUT HERE

Post estimation results in Table 6 show the adequacy of the CCC-VARMA-GARCH models for Bitcoin-Gold and Bitcoin-Silver relationships. Also, by comparing the Akaike (AIC) and Swartz (SBC) information criteria of CCC-VARMA-GARCH models in Table 6 with those of CCC-GARCH models in Table 5, we found CCC-VARMA-GARCH models for the two portfolios more relevant in the management of investment in these assets.

4. Bitcoin asset management in the portfolio of Gold and Silver

This section presents portfolio design and hedging strategies for the management of Bitcoin assets in the portfolio of Gold and Silver. The approach documented in Kroner and Sultan (1993), Kroner and Ng (1998) and Arouri et al. (2011) is used. The approach sets to minimize the risk without minimizing the expected returns. Their estimator uses the estimates of the conditional variances and covariances in an hedge ratio portfolio of Bitcoin and Gold (Silver) assets. The interest of the investor is to minimize the risk of his Bitcoin-Gold (Silver) portfolio without reducing the expected returns. The portfolio weight formula is,

$$w_{bitcoin,i,t} = \frac{\sigma_{i,t}^2 - \sigma_{bitcoin,i,t}^2}{\sigma_{bitcoin,t}^2 - 2\sigma_{bitcoin,i,t}^2 + \sigma_{i,t}^2} \tag{22}$$

where $w_{bitcoin,i,t} = \begin{cases} 0, & if\ w_{bitcoin,i,t} < 0 \\ w_{bitcoin,i,t}, & if\ w_{bitcoin,i,t} \leq 1 \\ 1, & if\ w_{bitcoin,i,t} > 1 \end{cases}$ and $w_{bitcoin,i,t}$ is the weight of Bitcoin in a $1 Bitcoin-Gold (or Bitcoin-Silver) portfolio at time *t*, $\sigma_{bitcoin,i,t}^2$ is the conditional covariance between the Bitcoin price and Gold (Silver) price, $\sigma_{i,t}^2$ is the conditional variance for Gold (Silver) price and $\sigma_{bitcoin,t}^2$ is the conditional variance for Bitcoin price. The optimal weight of Gold (Silver) in the Bitcoin-Gold (or Bitcoin-Silver) market portfolio can be evaluated as $1 - w_{bitcoin,i,t}$. Kroner and Sultan (1993) present risk-minimizing hedge ratios between Bitcoin and Gold (Silver), and the formula is,

$$\beta_{bitcoin,i,t} = \frac{\sigma_{bitcoin,i,t}^2}{\sigma_{i,t}^2} \tag{23}$$

and the low ratio $\beta_{bitcoin,i,t}$ implies that Bitcoin price risk can be hedged by taking a short position in Gold (Silver) markets.

Table 7 presents the optimal weight of Bitcoin in $1 of the Bitcoin-Gold (or Bitcoin-Silver) portfolio and the corresponding hedge ratio. These were 2.41 and 8.57%, respectively.

Conversely, the weights of Gold and Silver in the portfolio were 97.59 and 91.43%, respectively. This means that, for every $1 investment in Bitcoin-Gold and Bitcoin-Silver portfolios, 2.41cents and 8.57cents should be invested in Bitcoin. So, for Gold and Silver allocation in the Bitcoin mixed portfolios, more investments are expected for Gold and Silver, that is at 97.59 cents and 91.43cents, respectively. Regarding the hedge ratios, 3.06% of Gold is needed to shorten $1 long in Bitcoin, while 0.58% of Silver is to be added to the Bitcoin-Silver portfolio in a $1 long in Bitcoin.

INSERT TABLE 7 ABOUT HERE

The results here are not surprising since Bitcoin has higher volatility persistence compared to Gold and Silver (see Table 3). Thus, more investments in Gold and Silver are needed. Investors are therefore advised to hold more Gold and Silver in theory portfolios in order to have a diversification advantage and minimize the risk without lowering the expected returns of their portfolio.

5. Conclusions

Due to the popularity of Bitcoin as a new “gold”, the present paper examines returns and volatility persistence and spillovers of Bitcoin prices to gold and silver prices using daily series spanning from 2 January 2018 to 31 July 2020. The fractional persistence framework, using log-periodogram and local Whittle estimators are first applied to the price, returns and volatility proxies and the results show strong persistence in prices as the Bitcoin market poses the highest volatility persistence, followed by the gold market. The results of the CCC-GARCH and CCC-VARMA-GARCH models show the impossibility of return spillovers from the Bitcoin market to the gold and silver market, and neither of the two commodity markets can spill their price returns over to the Bitcoin market. Results of the variance components of both MGARCH models show significant market volatility and cross-market volatility and conditional correlations. That is, there are shocks and volatility spillovers from the Bitcoin market to the gold or silver markets, and these are bi-directional. Thus, based on the performance of Bitcoin in the analysis, the assets can still perform as a “safe haven”.

Due to volatility spillovers in the paired portfolios of Bitcoin-Gold and Bitcoin-Silver, portfolio and hedging strategies are designed for the management of investments in the paired portfolios. We find that more investments in Gold and Silver are needed in the portfolio of Bitcoin in order to fully have the diversification advantage and minimize the risk without lowering the expected returns of their portfolio. To further validate our findings, the dynamic connectedness approach used in Adekoya et al. (2022) and tail risk dependence, comovement with the predictability of Tiwari et al. (2022) can be employed to check if there exist bi-directional spillovers between gold (silver) and bitcoin. This is left to curious readers.

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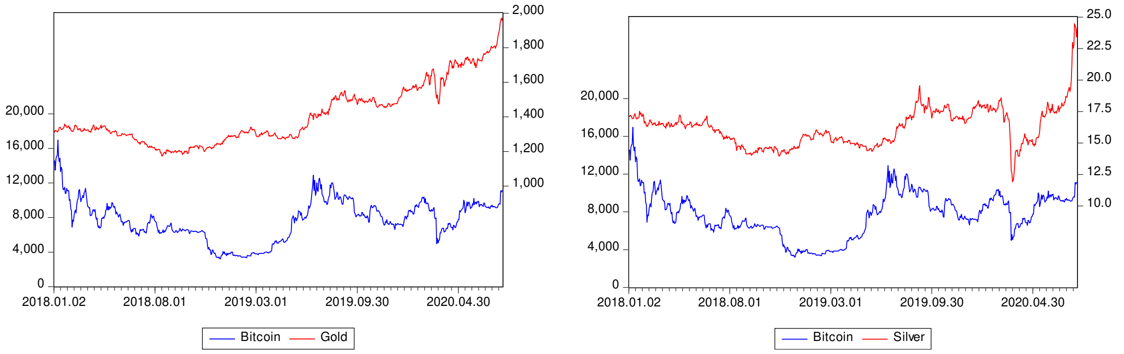

Figure 1: Comovements of Bitcoin price with Gold and Silver prices

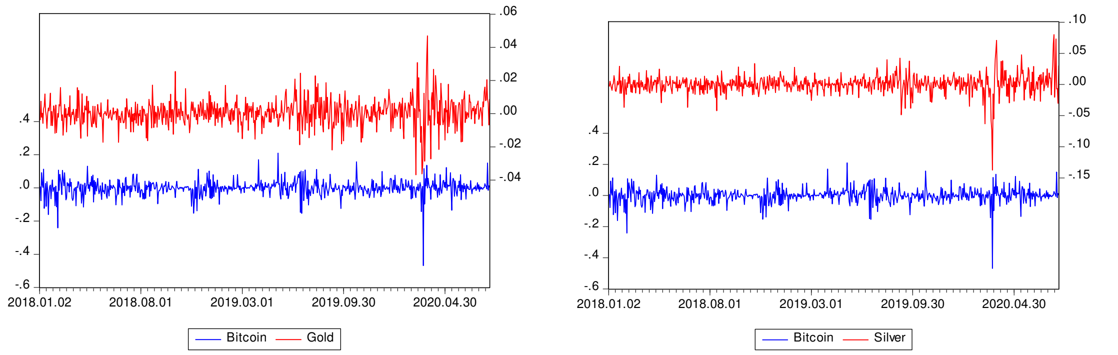

Figure 2: Comovements of Bitcoin price returns with Gold and Silver price returns

Table 1: Descriptive Statistics on Prices and Log-returns

	Bitcoin market		Gold market		Silver market	
Statistics	***P*t**	**rt**	***P*t**	**rt**	***P*t**	**rt**
Mean	7713.381	-0.00039	1409.655	0.000608	16.21276	0.000514
Median	7819.355	0.001850	1332.705	0.000618	16.29850	0.000732
Maximum	16960.01	0.209941	1974.870	0.046875	24.47600	0.080148
Minimum	3195.710	-0.46863	1173.890	-0.03688	11.92200	-0.13734
Std. Dev.	2387.161	0.048639	176.2417	0.008078	1.559288	0.015359
Skewness	0.131933	-1.4304	0.872098	0.080752	1.264661	-0.84602
Kurtosis	3.145375	17.65112	2.897782	7.071959	7.780771	16.67059
Jarque-Bera	2.5186	6183.8***	84.71***	460.84***	811.78***	5265.51***
Prob	0.2839	0.0000	0.0000	0.0000	0.0000	0.0000

Note, *** indicates significance at 5% level.

Table 2: Conditional Heteroscedasticity and Autocorrelation Tests on Log-returns, r_t

Statistics	Bitcoin market	Gold market	Silver market
LB(1)	4.906 (0.027)**	1.854 (0.173)	18.420 (0.000)***
LB(5)	13.714 (0.018)**	16.072 (0.007)**	34.514 (0.000)***
$LB^2(1)$	1.358 (0.244)	35.252 (0.000)***	51.259 (0.000)***
$LB^2(5)$	11.368 (0.045)**	126.870 (0.000)***	115.720 (0.000)***
ARCH LM(1)	1.348 (0.246)	36.883 (0.000)***	55.064 (0.000)***
ARCH LM(5)	2.017 (0.074)*	15.916 (0.000)***	16.840 (0.000)***

In parentheses are computed rejection probability errors, and ***, ** and * indicate significance at 1, 5 and 10% levels.

Table 3: Fractional integration estimates based on Whittle semi-parametric and Log-periodogram regression

	Bandwidth	P_t		r_t	
Series	m.	Whittle Semi-parametric	Log-Periodogram	Whittle Semi-parametric	Log-Periodogram
Bitcoin market	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.9170** 1.0795** 1.0031**	0.9988** 1.0968** 1.0476**	-0.0326 -0.0648 0.0458	0.0801 0.0316 0.0805
Gold market	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.8629** 0.9052** 0.9449**	0.8701** 0.9420** 0.9662**	0.0013 -0.0198 0.1319**	-0.0231 0.0200 0.1553**
Silver market	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.7440** 0.9182** 1.0320**	0.7462** 0.9621** 1.0525**	0.0024 0.0544 0.0489	0.0600 0.1143 0.0998
		$\|r_t\|$		r_t^2	
Series	m.	Whittle Semi-parametric	Log-Periodogram	Whittle Semi-parametric	Log-Periodogram
Bitcoin	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.4897** 0.3039** 0.2296**	0.5606** 0.3767** 0.2387**	0.5713** 0.2917** 0.2206**	0.5519** 0.3189** 0.2140**
Gold	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.3122** 0.2973** 0.2871**	0.4133** 0.3531** 0.3310**	0.2499** 0.2623** 0.2682**	0.3217** 0.2682** 0.3141**
Silver	$T^{0.6}$ $T^{0.7}$ $T^{0.8}$	0.2287** 0.2913** 0.2085**	0.1969 0.2427** 0.2505**	0.0400 0.1180** 0.0900**	0.0135 0.0837 0.0725

Note: total sample T is 667 and the three periodogram points (bandwidths), $T^{0.6}$, $T^{0.7}$ and $T^{0.8}$ are 49, 94 and 181, respectively. These are bandwidths with stable estimates of *d*
** indicates significant estimates at the 5% level.

Table 4: Asymmetry test and CCC test

	Bitcoin market	Gold market	Silver market
Sign Bias test	0.8691 (0.3851)	0.5753 (0.5653)	0.6073 (0.5438)
Negative Size Bias test	0.3599 (0.7191)	0.1920 (0.8478)	0.9717 (0.3316)
Positive Size Bias test	0.0897 (0.9285)	0.1635 (0.8701)	0.4870 (0.6264)
Joint bias test	0.9199 (0.8206)	0.7775 (0.8548)	1.4771 (0.6876)
Engle Sheppard CCC χ_2^2 test	----	0.7938 (0.4618)	0.9185 (0.1700)

In parentheses are computed rejection probability errors and tests were conducted at a 5% significant level.

Table 5. Results of CCC-GARCH models

Parameters	Bitcoin-Gold	Bitcoin-Silver
Mean Equation		
$\phi_{bitcoin}$	-3.81E-04 (0.6157)	-5.05E-04 (0.4681)
ϕ_i	-0.0521 (0.3790)	-0.0654 (0.2427)
$\theta_{bitcoin}$	0.6989 (0.0000)***	0.5726 (0.0000)***
θ_i	1.45E-04 (0.2027)	6.75E-05 (0.7246)
$\theta_{bitcoin,i}$	-4.64E-03 (0.9172)	-0.0422 (0.3849)
$\theta_{i,bitcoin}$	3.05E-03 (0.5854)	-1.3411 (0.8911)
Variance Equation		
$\omega_{bitcoin}$	6.40E-05 (0.0000)***	6.09E-05 (0.0000)***
$\alpha_{bitcoin}$	0.1847 (0.0000)***	0.2108 (0.0000)***
$\beta_{bitcoin}$	0.6959 (0.0000)***	0.6796 (0.0000)***
ω_i	5.32E-07 (0.0000)***	2.89E-06 (0.0000)***
α_i	0.0822 (0.0000)***	0.1681 (0.0000)***
β_i	0.8700 (0.0000)***	0.7640 (0.0000)***
ρ	0.0163 (0.6788)	0.0234 (0.0303)***
$\hat{\alpha}_{bitcoin} + \hat{\beta}_{bitcoin}$	0.8806	0.8904
$\hat{\alpha}_i + \hat{\beta}_i$	0.9522	0.9321
HL (bitcoin)	5.45	5.97
HL (*i*)	14.15	9.86
Post Estimation diag.		
AIC	-13.599	-12.492
SBC	-13.511	-12.404
Ljung-Box $Q(2)_{bitcoin}$	1.5664 (0.4569)	1.8431 (0.3979)
Ljung-Box $Q(5)_{bitcoin}$	3.9808 (0.5522)	4.0963 (0.5356)
McLeod-Li$(2)_{bitcoin}$	0.9968 (0.6075)	1.2374 (0.5386)
McLeod-Li$(5)_{bitcoin}$	6.0652 (0.2999)	7.3366 (0.1968)
Ljung-Box $Q(2)_i$	1.7636 (0.4140)	4.0471 (0.1322)
Ljung-Box $Q(5)_i$	7.5560 (0.1825)	4.7468 (0.4476)
McLeod-Li$(2)_i$	2.4387 (0.2954)	2.9651 (0.2271)
McLeod-Li$(5)_i$	3.5475 (0.6162)	7.1875 (0.2071)

Table 6. Results of CCC-VARMA-GARCH models

Parameters	Bitcoin-Gold	Bitcoin-Silver
Mean Equation		
$\phi_{bitcoin}$	-0.0002 (0.7789)	-4.8E-04 (0.3823)
ϕ_{i}	-0.0495 (0.3333)	-0.0613 (0.1497)
$\theta_{bitcoin}$	0.2855 (0.7789)	0.5720 (0.0000)***
θ_{i}	0.0002 (0.1687)	2.2E-05 (0.9048)
$\theta_{bitcoin,i}$	-0.0179 (0.6867)	-0.0630 (0.1844)
$\theta_{i,bitcoin}$	0.0021 (0.6846)	0.0007 (0.9396)
Variance Equation		
$\omega_{bitcoin}$	7.0E-06 (0.0025)***	6.2E-05 (0.0000)***
$\alpha_{bitcoin}$	0.1116 (0.0000)***	0.2104 (0.0000)***
$\alpha_{bitcoin,i}$	0.2061 (0.2667)	-0.4706 (0.0000)***
$\beta_{bitcoin}$	0.5621 (0.0000)***	0.6989 (0.0000)***
$\beta_{bitcoin,i}$	398.83 (0.0000)***	49.7289 (0.0004)***
ω_{i}	1.0E-06 (0.0000)***	2.0E-06 (0.0000)***
α_{i}	0.1013 (0.0000)***	0.1537 (0.0000)***
$\alpha_{i,bitcoin}$	0.0060 (0.1378)	-0.0174 (0.0592)***
β_{i}	0.8263 (0.0000)***	0.7835 (0.0000)***
$\beta_{i,bitcoin}$	-0.1369 (0.0000)***	-1.9969 (0.0000)***
ρ	0.0050 (0.0000)***	-0.0020 (0.0000)***
Post Estimation diag.		
AIC	-13.624	-12.527
SBC	-13.565	-12.412
Ljung-Box $Q(2)_{bitcoin}$	3.1519 (0.2068)	2.3617 (0.3070)
Ljung-Box $Q(5)_{bitcoin}$	5.5548 (0.3520)	4.0084 (0.5482)
McLeod-Li$(2)_{bitcoin}$	1.0121 (0.6029)	0.8370 (0.6580)
McLeod-Li$(5)_{bitcoin}$	1.7713 (0.8798)	4.3681 (0.4977)
Ljung-Box $Q(2)_{i}$	1.4660 (0.4805)	5.3482 (0.0690)
Ljung-Box $Q(5)_{i}$	7.6458 (0.1769)	6.0388 (0.3025)
McLeod-Li$(2)_{i}$	1.8625 (0.3941)	4.5361 (0.1035)
McLeod-Li$(5)_{i}$	3.2366 (0.6636)	8.5998 (0.1261)

Table 7. Estimates of Optimal portfolio weight and hedge ratio

	Bitcoin-Gold	**Bitcoin-Silver**
$w_{bitcoin,i,t}$	0.0241	0.0857
$1-w_{bitcoin,i,t}$	0.9759	0.9143
$\beta_{bitcoin,i,t}$	0.0306	-0.0058