Which is Better, Childcare Support or Public Capital Investment?

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Abstract In recent years, the birthrate in developed countries has been declining. The government has been providing financial support to households raising children by increasing childcare allowances and offering free childcare as part of its policy to reduce the birthrate. However, the total fertility rate in Japan continues to be at an all-time low, and the effects of these policies are unclear. This analysis will examine whether the direct allocation of funds to households as described above is beneficial to the declining birthrate and growth or whether the government's indirect increase in public capital can affect the birthrate by boosting labor productivity and expanding income. This analysis shows that an increased share of public capital investment brings higher economic growth. The growth rate will be maximized if all tax revenue is allocated to public capital investment.

Keywords: Public capital investment・Childcare support・Income tax・Economic growth

JEL classification: D91・E62・O41

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Introduction

The number of children born in Japan continues to decrease. The total fertility rate was 1.36 in 2019, the lowest level to date, as indicated by the Japanese Ministry of Health, Labor, and Welfare (MHLW). The Cabinet Office insists that Japan has been declining birth rates for many years, resulting in what is referred to as an "ultra-declining birth rate society." The demographic trends are such that, by 2050, one in 2.5 people will be elderly (aged 65 or older). Viewing life in the long term, workers should determine their spending based on their estimated lifetime income. According to the overlapping generations (OLG) model proposed by Diamond (1965), an individual's lifetime income is assumed to consist of earnings received in two periods: their working period and their later life. Individuals make decisions from a lifetime perspective while adhering to budgetary constraints. Becker (1981) and Becker and Lewis (1973) showed that the number of children in developed countries would decline; at first glance, this is seemingly a contradiction, considering that children are positive to societies. However, this results from the fact that childcare costs are proportional in scale to their quantity multiplied by their quality. In this study, models are established based on a neoclassical theory that suggests that growth in capital boosts gross domestic product (GDP) and leads to a greater growth rate for the nation. The main portion of this study utilizes Romer's endogenous growth model (1986) to introduce the public capital models proposed by Barro (1990), Barro and Sala-i-Martin (1992), Futagami et al. (1993), Turnovsky (1997), Yakita (2008), and Maebayashi (2013). These models indicate that public capital stock boosts labor productivity. Investment in public capital is financed by levying income taxes (on labor and capital income). Yakita (2008) used a birth rate internalization model that considers two public expenditures: public capital investment and public capital maintenance. Maebayashi (2013) showed the dynamics of the private-public capital ratio and confirmed the existence of steady-state and global stability. Furthermore, the author analyzed the optimal tax revenue allocation between expenditure on public capital investment and public pension subsidies under a pay-as-you-go pension system. The study concluded that it was clear that the best policy for growth is to allocate all financial resources to public capital investment; however, from a social-welfare perspective, the optimal tax revenue allocation rate depends on the magnitude of the social discount rate.

This study analyzes the policy trade-off between public capital investment and childcare support and the effects on the growth rate under government budget constraints. The government sources revenue only from income taxes on labor and capital. Furthermore, as an important point of this study, the child-rearing support policy should be subsidized for the direct opportunity cost to workers (we will call this case "Case A".), or the child should be regarded as a normal consumer good rather than a capital good. A subsidy policy on the price should be implemented. (And let us call this case "Case B.) The point is that comparisons are made and explicitly derived from the effect of policy on growth in these cases. In both cases, we prove the existence of a steady state and confirm that the economy converges to a steady state globally and stably. We show that all variables: public capital, private capital, and GDP, grow simultaneously on the balanced growth path (BGP). Second, we analyze the effect of increasing the share of public capital investment on growth under constant tax revenue and use a numerical example. We find that this growth is positive. Also, the elasticity of an increase in the relative share of public capital investment ratio on private-public capital and the labor share of GDP is considered. In the first case, the sign of this elasticity is positive, implying that the additional increase in public capital is pushing up private capital more than that increase. Clearly states that this is driving economic growth. In the second case, the sign turned out to be negative, and the absolute elasticity value is less than one for a marginal increase in public capital investment. The sign of effect on the relative value is negative. It means that the effect of increasing the wage rate due to the increase in public capital does not contribute much to the increase in savings. The reason for this was very clear. First, this depends on the shape of the utility function, as shown in the linear logarithm. This function means savings depend only on income in the first period, not the interest rate. In other words, in this change in interest rate, the substitution effect and the income effect cancel each other out, and the effect on savings against changes in the interest rate becomes zero. The second thing to think about is that governmental childcare support measures do not contribute to an increase in the labor force. In this analysis, the social security system's public pension and long-term care insurance systems are not considered, so there is no externality to the parent generation. Therefore, the incentive for parents to have children is related to them being considered consumer goods rather than capital goods from an economic point of view. We constructed here using the Diamond model (1965), a two-period OLG model. We introduce public capital stock to construct a model that has labor-augmented production technology. Therefore, whether to have children depends on the preferred rate for children as general consumer goods and consumption in the second period, so-called how much deposit is required the second period because there is no public pension system in this model, Also, whether to leave for the second term depends greatly on the preference rate.

The remainder of this paper is organized as follows. The next section presents the model and its (private and public) capital dynamics. The global stability of the dynamics in the steady state is then confirmed. The effects of governmental increases in income tax and public capital investment shares in the steady state are analyzed. The final section concludes the paper.

2. Model

We analyze childcare subsidies by distinguishing between subsidies of the same fixed amount for each household and income-proportional subsidies that depend on the household's wage income. The current childcare fee in Japan is proportional to the inhabitant tax, which is proportional to income.

Case A: the childcare cost is regarded as an opportunity cost

2.1 Individuals

The two-period OLG model presented by Diamond (1965) is considered a fully competitive market. A homogeneous individual has assumed utility from consumption in the working and later periods of life and selects the number of children they have. We consider a child a consumer rather than a capital good, and there is a public pension p in the second period as risk aversion insurance. Individuals supply labor inelastically in only the first period, and it is assumed that every individual has one unit of labor to supply to the labor market. Individuals allocate income for consumption, saving, and childcare costs in the first period. The individual consumes all income, including saving and interest, in the first period, with no bequests in the second period. A logarithmic, linear utility function and lifetime budget constraint, which must hold in order for the economy to be sustainable in the long term, are specified as follows:

$$max. \;\; u_t = logc_t + \rho logd_{t+1} + \varepsilon logn_t \tag{1}$$

$$s.t \quad w_t\,(1-\tau)[1-n_t(z-h_t)] = c_t + \frac{d_{t+1}}{r_{t+1}(1-\tau)} \tag{2}$$

$$c_t^* = \frac{(1-\tau)w_t}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]} \tag{3}$$

$$n_t^* = \frac{\varepsilon}{[1+\varepsilon+\rho r_{t+1}(1-\tau)](z-h_t)} \tag{4}$$

$$d_{t+1}^* = \frac{\rho r_{t+1} w_t (1-\tau)^2}{[1+\varepsilon+\rho r_{t+1}(1-\tau)]} \tag{5}$$

$$s_t^* = \frac{\rho w_t r_{t+1}(1-\tau)^2}{(1+\varepsilon)} \tag{6}$$

where time preference, child preference, childcare cost, childcare support, and income tax (wage income and capital income) are denoted as $\rho \in (0,1)$, $\varepsilon > 0$, $z \in (0,1)$, $h_t \in (0,1)$, $z \geq h_t$, and $n_t \geq 1$, respectively.

2.2 Production

A Cobb-Douglas production technology in which labor increases with public capital investment, as in Romer (1986), is used. It is assumed that there are many firms in a goods market, and these firms have access to the same technology. The inputs are the private capital stock and labor. The production function of firm i is specified as follows:

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha} \tag{7}$$

$$A_t = \frac{G_t}{L_t} \tag{8}$$

$$Y_t = K_t^{\alpha} {G_t}^{1-\alpha} = \left(\frac{K_t}{G_t}\right)^{\alpha} G_t = x_t^{\alpha} G_t \tag{9}$$

We assume a perfectly competitive market and solve the profit maximization problem as follows:

$$(1-\alpha)\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} {A_t}^{1-\alpha} = w_t \tag{10}$$

$$\alpha\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} {A_t}^{1-\alpha} = r_t \tag{11}$$

From (10) and (11), the private capital-labor ratio will become the same value as in $K_{ti}/L_{ti} = K_t/L_t$. Also, $\sum_{i=1}^{\infty} L_{it} = L_t\, , and\ \sum_{i=1}^{\infty} K_{it} = K_t$ can be derived, where L_t and K_t denote the total labor supply and private capital, respectively. By defining a new variable, $x = \frac{K}{G}$, to be the ratio of private and public capital, (10) and (11) can be rewritten as the following equations:

$$(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\alpha} \frac{G_t}{L_t} = (1-\alpha)x_t^{\alpha}\frac{G_t}{L_t} = w_t \tag{12}$$

$$\alpha\left(\frac{K_t}{G_t}\right)^{\alpha-1} = \alpha x_t^{\alpha-1} = r_t \tag{13}$$

The above equations mean the interest rate is independent of the number of laborers, and the wage rate is decreasing.

2.3 Government

The government taxes income and divides tax revenues between public capital investment, $E > 0$, and childcare support, $H > 0$. The share of spending on public capital investment and the income tax rate is denoted $\varphi \in [0,1], \tau \in [0,1)$. The depreciation rate of public and private capital is 0. The government budget constraint is shown in the following equations:

$$E_t + H_t = \tau Y_t = \tau x_t^\alpha G_t \tag{14}$$

$$E_t = G_{t+1} - G_t = \varphi\tau Y_t = \varphi\tau x_t^\alpha G_t \tag{15}$$

$$w_t h_t n_t N_t = (1-\varphi)\tau Y_t = (1-\varphi)\tau x_t^\alpha G_t \tag{16}$$

The per-capita childcare support is determined as the following equation (the value of which will be constant):

$$h_t = \frac{(1-\varphi)(1-\varepsilon z)\tau}{\varepsilon[1-(1-\varphi)\tau]} \tag{17}$$

By using equations (4), (13), and equation (17), we represent the growth rate of the labor force in the period t is rewritten as follows:

$$g_t^L = \frac{L_{t+1}}{L_t} = \frac{N_{t+1}}{N_t} = \frac{n_t N_t}{N_t} = n_t = \frac{\varepsilon^2[1-(1-\varphi)\tau]}{[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)][\varepsilon z-(1-\varphi)\tau]} \tag{18}$$

2.4 Equilibrium

There are three markets, and we consider only the capital market by Walras' law. The equilibrium condition is as follows:

$$s_t L_t = K_{t+1} \tag{19}$$

We substitute the optimal savings (6) for the equilibrium condition (19) and substitute for the wage rate (12) and the interest rate (13). These allow us the private capital stock's dynamic equation (20) as follows:

$$g_t^K = \frac{K_{t+1}}{K_t} = \frac{(1-\alpha)(1-\tau)^2\rho\alpha}{(1+\varepsilon)(x_t x_{t+1})^{1-\alpha}} \tag{20}$$

Next, we derive the differential equation of public capital stock by using the equation (15) as follows:

$$g_t^G = \frac{G_{t+1}}{G_t} = \varphi\tau x_t^\alpha + 1 \tag{21}$$

2.5 Dynamics

Let x be the relative size of the private capital stock and the public capital stock. Using the equation (20) and (21), the difference equation for x is shown as follows:

$$x_{t+1} = \left[\left(\varphi\tau + \frac{1}{x_t^{\alpha}}\right)\frac{(1+\varepsilon)}{(1-\alpha)(1-\tau)^2\rho\alpha}\right]^{\frac{1}{\alpha-2}} \tag{22}$$

To graphically show the relationship between x_t and x_{t+1}, the equation (22) is differentiated by x_t.

$$\frac{dx_{t+1}}{dx_t} = \frac{(1+\varepsilon)}{\rho x_t^{\alpha+1}(1-\alpha)(2-\alpha)(1-\tau)^2}\left[\frac{\varphi\tau(1+\varepsilon)(\varphi\tau + x_t^{-\alpha})}{\rho\alpha(1-\alpha)(1-\tau)^2}\right]^{\frac{3-\alpha}{\alpha-2}} > 0 \tag{23}$$

$$\frac{dx_{t+1}{}^2}{d^2x_t} = \frac{-\alpha(1+\alpha)(1+\varepsilon)}{\rho\alpha x_t^{\alpha+1}(1-\alpha)(2-\alpha)(1-\tau)^2}\left[\frac{(1+\varepsilon)(\varphi\tau + x_t^{-\alpha})}{\rho\alpha(1-\alpha)(1-\tau)^2}\right]^{\frac{3-\alpha}{\alpha-2}}$$

$$-\frac{(1+\alpha)(3-\alpha)(1+\varepsilon)^2}{\rho^2 x_t^{2\alpha+2}(\alpha-2)^2(1-\alpha)^3(1-\tau)^4}\left[\frac{(1+\varepsilon)(\varphi\tau + x_t^{-\alpha})}{\rho\alpha(1-\alpha)(1-\tau)^2}\right]^{\frac{5-2\alpha}{\alpha-2}} < 0 \tag{24}$$

We indicate the value of x in the i period as x_i. When i approaches 0, in which i represents the period, 0,1,2,…, the value of x is zero in equation (23) $\lim_{i\to 0}\frac{dx_{t+1}}{dx_t} \cong \infty$. Furthermore, when x_t approaches infinity, $\lim_{i\to\infty}\frac{dx_{t+1}}{dx_t} \cong 0$. These results indicate that the shape of the curve is shown as Fig.1. Where x^* indicates the steady state value, which is derived by the equation (22), which is derived by the equation (25). We indicate the steady state value of x as follows:

$$x^* = \left[\frac{(1-\alpha)(1-\tau)^2\rho\alpha}{(1+\varepsilon)\varphi\tau}\right]^{\frac{1}{2(1-\alpha)}} \tag{25}$$

where we can see that the expenditure share on the public capital investment decreases with x^*. "a rising childcare opportunity cost effect" overcomes "a boost wage effect."

Proposition 1.

We can find a unique steady state relative capital stock x^*in which has a balanced growth path in which private capital, public capital stock, and GDP grow at the same rate; it is shown as follow: $Y_{t+1}/Y_t = K_{t+1}/K_t = G_{t+1}/G_t$. The equation (25) indicates the steady-state value and is globally stable.

Next, we observe the effect of increasing the public capital stock share on the growth rate

(26)

at a steady state. Then, we differentiate the equation (21), of course, through the equation (20) derivative by φ to analyze qualitatively.

$$\psi = \frac{\partial g_t^G}{\partial \varphi} = \tau (x^*)^\alpha = \tau \left[\frac{(1-\alpha)(1-\tau)^2 \rho \alpha}{(1+\varepsilon)\varphi\tau} \right]^{\frac{\alpha}{2(1-\alpha)}} > 0$$

Because of the positive sign of the equation (26), from the perspective of the optimal value of the policy, the value of share φ must be 1. The growth rate is maximized by allocating all income tax revenues to public capital investment. Furthermore, an increase in public capital investment boosts the public capital stock scale in this period and next. It led to a higher wage rate and savings, which included the effect of rising income in both periods through higher private and public capital stock. In the aspect of government, two kinds of tax revenue increase: wage income tax and capital income tax, where these rates are assumed as the same rate $0 < \tau < 1$ by simplicity, and it raises both public and private capital stock. We must consider another one; we say the "dilution effect" here. When a government increases the public capital investment share, an individual's income will decrease by raising the opportunity cost of raising children. The equation (26) indicates that the "income effect" overcomes the "delusion effect".

Proposition 2.

In cases, the cost of childcare increases in proportion to labor income, the allocation of tax revenues to spending is preferable from a growth rate perspective if public capital investment is increased more.

Case B: the childcare cost is as same as a nominal goods price

Unlike A, we assume here that the cost of childcare is considered the price of normal materials. Therefore, the childcare cost does not depend on the wage rate.

2.6 Individuals

As in Case A, individual utility depends on consumption in the first and second periods and the number of children. The difference is that the cost of having children is not assumed to be an opportunity cost in the form of income but rather a cost as a constant price, similar to that of consumption goods. The utility function and budget constraint are as follows;

$$max.\ \ u_t = logc_t + \rho logd_{t+1} + \varepsilon logn_t \tag{27}$$

(28)

$$s.t \quad w_t\,(1-\tau) = c_t + (z-h_t)n_t + \frac{d_{t+1}}{r_{t+1}(1-\tau)}$$

The optimal solution can be shown as follows;

$$c_t^* = \frac{(1-\tau)w_t}{(1+\varepsilon+\rho)} \tag{29}$$

$$n_t^* = \frac{\varepsilon w_t(1-\tau)}{(z-h_t)(1+\varepsilon+\rho)} \tag{30}$$

$$d_{t+1}^* = \frac{\rho w_t r_{t+1}(1-\tau)^2}{(1+\varepsilon+\rho)} \tag{31}$$

$$s_t^* = \frac{\rho w_t(1-\tau)}{(1+\varepsilon+\rho)} \tag{32}$$

$$K_{t+1} = s_t L_t = \frac{\rho w_t(1-\tau)L_t}{(1+\varepsilon+\rho)} \tag{33}$$

Derive the dynamic equation for capital using the equilibrium equation for the capital market (19).

$$\frac{K_{t+1}}{K_t} = \frac{\rho(1-\tau)(1-\alpha)}{(1+\varepsilon+\rho)x_t^{1-\alpha}} \tag{34}$$

where the equation (21) and (34) can derive the dynamic equation in x.

$$\frac{x_{t+1}}{x_t} = \frac{\frac{K_{t+1}}{K_t}}{\frac{G_{t+1}}{G_t}} = \frac{\rho(1-\tau)(1-\alpha)}{(\varphi\tau x_t^{\alpha}+1)(1+\varepsilon+\rho)x_t^{1-\alpha}} \tag{35}$$

$$x_{t+1} = \frac{\rho(1-\tau)(1-\alpha)x_t^{\alpha}}{(\varphi\tau x_t^{\alpha}+1)(1+\varepsilon+\rho)} \tag{36}$$

Using the equation (36), differentiate it by x_t to analyze the stability of x.

$$\frac{dx_{t+1}}{dx_t} = \frac{\alpha\rho(1-\tau)(1-\alpha)x_t^{\alpha-1}}{(1+\varepsilon+\rho)(\varphi\tau x_t^{\alpha}+1)^2} > 0 \tag{37}$$

(38)

$$\frac{dx_{t+1}{}^2}{d^2x_t} = -\frac{(\varphi\tau x_t^\alpha + 1)^2\alpha\rho(1-\tau)(1-\alpha)^2x_t^{\alpha-2} + 2\alpha\varphi\tau x_t^{\alpha-1}(\varphi\tau x_t^\alpha + 1)}{(1+\varepsilon+\rho)(\varphi\tau x_t^\alpha + 1)^4} < 0$$

Here, we indicate the value of x in the i period as x_i. When i approaches 0, in which i represents the period, 0,1,2,…, the value of x is zero in equation (37) $\lim_{i\to 0}\frac{dx_{t+1}}{dx_t} \cong \infty$. Furthermore, when x_t approaches infinity, $\lim_{i\to\infty}\frac{dx_{t+1}}{dx_t} \cong 0$.These results indicate the shape of the curve shown in figure 1. The above derives a steady state in which the economy converges because the stability condition has been satisfied. In the steady state, since $x_{t+1} = x_t = x^*$ is hold, the equation (36) is rewritten as follows:

$$[\varphi\tau x^* + (x^*)^{1-\alpha}] = \rho(1+\varepsilon+\rho)(1-\tau)(1-\alpha) \tag{39}$$

As in case A, we can derive the effect of increasing the public capital investment share in the growth. Furthermore, to quantitatively derive the effect on the growth rate, the numerical value of each parameter is clarified as $(\alpha, \varepsilon, \rho, \tau, x, z, \varphi) = (0.5, 0.05, 0.95, 0.3, 3.2, 0.06, 0.83)$. We will discuss the specific quantification of each parameter. Here, extreme numerical examples such as $\varepsilon = 0.05\ and\ \rho = 0.95$ are presented. The reasons for this are that the engine of growth in this model is a public capital investment; therefore, in order to connect the wage rate and interest rate pushed up by public capital investment to higher growth, it is necessary to supply more labor time, that is, lower the opportunity cost for childcare, or raise the preference rate for future consumption. The value of labor income share α is derived by ILO (2020), in which the average world value is presented as 50% of the front half of the stage. First, substituting each parameter for the equation (39), the following univariate nonlinear equation is derived.

$$f(x) = 0.249x^* + (x^*)^{\frac{1}{2}} - 0.665 = 0 \tag{40}$$

Since equation (40) is a nonlinear equation, it is impossible to derive it explicitly. Therefore, using the Newton-Raphson method and restricting the solution to a certain range, the solution is 0.3375. Furthermore, we differentiate the equation (21), which indicates the dynamic equation of public capital stock by its public expenditure share, φ.

$$\Gamma(\varphi) = \frac{dg_t^G}{d\varphi} = \tau(x^*)^\alpha > 0 \tag{41}$$

Proposition 3.

Even if children are considered a normal good, i.e., the cost of child care is assumed to be a constant amount, the growth rate will increase if the allocation of tax revenue expenditures increases public capital investment.

3.Comparering Case A and B.

3.1 Comparison of incremental growth rates

We substitute x^{*} =0.3375, $\tau = 0.3, \alpha = 0.5$ for the equation (41). Its value equals $\Gamma = 0.174$. The specific number of equation (26) is also derived by introducing a parameter to compare cases A and B. It can be derived as $\psi = 0.1335$. In other words, when the cost of child care increases in proportion to labor income, the allocation of tax revenues to public capital investment would result in a larger increase in the growth rate. The policy to increase the growth rate (the number of laborers) is better in case A than case B, in which a government would rather pile up the public capital stock than decrease childcare costs per child. It means that a government is better off pushing up the labor income relative to the childcare cost than cutting childcare costs per child.

Proposition 4.
Whether childcare costs are assumed to increase in proportion to labor income or held constant, an increase in the share of public capital investment in allocating government expenditures in tax revenues will increase the growth rate. However, the former case is larger in terms of the increase.

Having derived a positive relationship between the share of spending on public capital investment and the growth rate in both cases, we now observe the response of the incremental growth rate to changes in the share. We first observe the variation of $\Gamma(\varphi)$ in equation (41) concerning the change in φ for case A. As a result, the graph in Appendix can be shown.

3.2 The effects on the number of children

We have seen that increasing public capital accumulation contributes to higher economic growth in both cases, but we now turn our attention to equation (4) and equation (30) to see how the fertility rate is affected by using the equation (12), (13) and (17). We also use the government budget constraint in the case of B to show the childcare support per capita.

(42)

$$h_t n_t N_t = (1-\varphi)\tau x_t^{\alpha} G_t \leftrightarrow h_t = \frac{(1-\varphi)\tau x_t^{\alpha} G_t}{n_t N_t} = \frac{(1-\varphi)\tau x_t^{\alpha} G_t}{n_t N_t}$$

$$n_t^* = \frac{\varepsilon^2[1-(1-\varphi)\tau]}{[1+\varepsilon+\rho\alpha x_{t+1}^{\alpha-1}(1-\tau)][z\varepsilon-(1-\varphi)\tau]}, \quad \frac{\partial n_t^*}{\partial \varphi} > 0 \tag{43}$$

$$n_t^* = \frac{(1-\varphi)\tau x_t^{\alpha} G_t}{zN_t} + \frac{(1-\alpha)(1-\tau)\varepsilon x_t^{\alpha} G_t}{(1+\varepsilon+\rho)zL_t}, \quad \frac{\partial n_t^*}{\partial \varphi} < 0 \tag{44}$$

From equations (43) and (44), the number of children expands in Case A due to the expansion of the public capital stock because the income-raising effect outweighs the accompanying increase in the cost of childcare. On the other hand, the population is decreasing in Case B, but the effect is the same as in Case A, indicating that increased income is large enough to cancel out the effect of increased child care. The results of this analysis suggest the importance of flexible policies in each economic situation. Two points to observe are, first, whether income-dependent educational disparities are occurring in the current economy. If the education gap is large, decreasing the share of government spending on education subsidies will increase the growth rate and fertility rate. Second, if education inequality is almost none or minimal, a reduction in education subsidies will increase the growth rate but decrease the number of children.

Concluding remarks

This study focused on the relative value of private-public capital in the presence of a childcare support policy. First, the global stability of economic growth and the unique steady state to which economic convergence is clarified. In a steady state, the economy is on a balanced growth path in which private capital, public capital, and GDP grow simultaneously. Second, the effect of increasing the share of public capital investment on the steady-state growth rate was analyzed and was found to depend on the absolute value of the elasticity of increasing the share of public capital investment relative to capital value or the labor share of GDP. More specifically, a smaller absolute elasticity value and larger labor share of GDP were more likely to result in a positive growth rate. The magnitude of the effect that increases public capital exceeds the effect of raising private capital; thus, a larger increase in income is needed to increase savings.

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Appendix

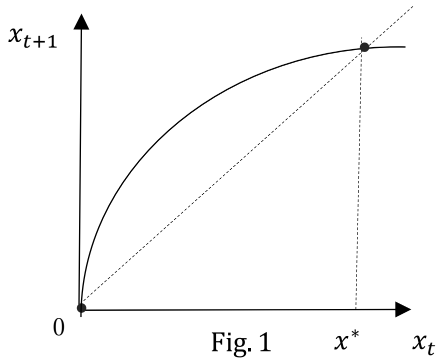

Fig. 1