Seigniorage Channel and Monetary Effectiveness in Flexible Price Economy

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Abstract

Replacing government lump-sum transfers in the household budget by the seigniorage channel in a modified RBC economy, this paper finds that the monetary shocks can impact the real economy effectively in the Neoclassical flexible price condition. The mechanism of the effectiveness is the resource reallocation triggered by monetary shocks. There are three outstanding characteristics of the seigniorage channeled monetary economy—integrality, interactive pricing, and Pareto optimality of the unique equilibrium—all of which are found to be incompatible with the existing dynamic general equilibrium monetary economics. Many vexing issues in macroeconomics are clarified through the lens of the seigniorage channeled monetary economy, including the price puzzle, missing of the liquidity effect, cause of the hump in the impulse-response curves, nonneutrality of growth rate of money and inflation, origin of the money market interest rate, choosing of reactive monetary policy rule, and negative movement of hours under a positive technology shock. The simulation shows that the seigniorage channeled monetary economy matches the empirical results in the literature well.

Keywords

Effectiveness of Monetary Shock, Seigniorage Channel, Flexible Price, Nonneutrality of Inflation, Liquidity Effect, Price Puzzle, Monetary Transmission Mechanism, Money Market Interest Rate, Reactive Monetary Policy, Tax, Public Goods, Neoclassical Macroeconomics

E13, E3, E4, E52, E6

1. Introduction

To construct a monetary economic model and use it to clearly explain the complex monetary and macroeconomic phenomena in the real world is a long standing pursuit of economists. In the literature, there are two main approaches to embedding money into and searching for the effective role of monetary shocks in the quantitative intertemporal optimization general equilibrium context. The first, from the Neoclassical school, was launched by Sidrauski (1967), Brock (1974), and Fischer (1979), among other writers, who adopted the MIU treatment, as well as by Lucas (1982), Svensson (1985), and Lucas and Stokey (1987), among other researchers, who adopted the CIA treatment. Unfortunately, after finding that monetary shocks cannot impact the real economy effectively based on an RBC economy with the CIA treatment, Cooley and Hansen (1989) concluded that "if money does have a major effect on the cyclical properties of the real economy, it must be through channels that we have not explored here" (p735). Similar results were obtained in the flexible price DSGE economy with the money-in-utility treatment; see chapter 2 of Walsh (2017) for reference. Subsequently,

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the search for the effective role of monetary shocks within the real economy in the Neoclassical flexible price condition became dormant. [2]

The failure of the first approach, to some extent, triggered the flourishing of the other: Dynamic New Keynesian macroeconomics. Yun (1996), Woodford (2003), Christiano, Eichenbaum, and Evans (2005), and many other researchers of the New Keynesian School, who sought to obtain the monetary effectiveness result in the context of rational expectations intertemporal optimization, suggested constructing a monetary economy with a sticky price mechanism. Although this school does obtain the monetary effectiveness result and is the dominant macroeconomic framework nowadays, one of the main problems with the sticky price theory is that when a part of the firms set the product price in the model economy, the product price as a whole is not derived from the first order. In other words, the pricing arrangement of the New Keynesian School is ad hoc. For other critiques of this theory, see the comprehensive critique of Chari, Kehoe, and McGratten (2009), the forward guidance puzzle of Del Negro, Giannoni, Patterson (2015), the comment in the textbook of Romer (2019), among others.

There are other questionable points with both the existing Neoclassical and New Keynesian monetary economics, which are all fundamental in economic sense:

1. The single side pricing. It is well accepted that product pricing is an interactive action between both the demand side and the supply side. However, in existing Neoclassical monetary economics, the product price is decided unilaterally by the demand side, that is, by the household, and in the New Keynesian macroeconomics, it is the supply side, that is, the firms that get the chance of pricing, makes the product price decision unilaterally.

2. Three problems in the treatment of money. (1) Both the existing Neoclassical and New Keynesian monetary economics adopt the MIU or CIA approaches in embedding money into the model economy. However, both MIU and CIA are artificial assumptions, which are in want of a solid economic reasoning. (2) There is no role for money to play in the model economy under these two approaches. In particular, as medium of exchange, money takes essential role in transactions. Unfortunately, there is no transaction process in these models, which implies the transaction mediating role of money is not necessary. In subsection 3.2 of this paper, readers will see the inconsistency between MIU/CIA and the equation of exchange. (3) The government lump-sum transfer mechanism adopted in the existing Neoclassical and New Keynesian monetary economics makes money issuance a resource income rather than an expense of the household, which is starkly inconsistent to the fact that money issuance is a seigniorage. This point will be clear in section 2.

3. Confusion between money market interest rate and rate of return on capital. The Fisher relation, that is, a gross nominal interest rate equals the product of the gross expected inflation and the gross real interest rate, is a crucial part of both the existing Neoclassical and New Keynesian monetary economics. In these models, the monetary authority affects the economy by adjusting the nominal interest rate, which, in turn, affects the economy with the help of the Fisher relation. The confusion here is that the nominal interest rate in the Fisher equation is basically the rate of return of capital,

[2] The New Monetarism (see Williamson and Wright (2010) and the reference mentioned there) is another line of study regarding money in a flexible price condition. However, this theory adopts the search and matching approach rather the general equilibrium approach. A common characteristic shared by the present paper and New Monetarism is their emphasis on the transaction side of the economy, which is absent in the mainstream dynamic general equilibrium monetary theories; this will become clear in section 3.

but the nominal interest rate controlled by the monetary authority is an interest rate of the money market, and the latter one is starkly different with the former one. Here is the explanation: Firstly, with the well accepted Cobb-Douglas production function of homogenous one, we can get the nominal rate of return of capital R^K of period t as, $R_t^K = \frac{E_tP_{t+1}(1-\delta)K_t+\alpha E_tP_{t+1}E_tY_{t+1}}{P_tK_t}$, where P, K, Y are price of product, capital stock, and output, respectively, α is the share of capital in the production, δ is the depreciation rate of capital, E is the expectation operator, the subscript is the time indicator. The numerator in the right side of the equation is the expected capital value of the next period, which consists of two parts: the expected value of depreciated capital and the expected rent income of capital, the denominator is the capital value of the present period. Similarly, we can get the real rate of return of capital, r^K , as, $r_t^K = \frac{(1-\delta)K_t+\alpha E_tY_{t+1}}{K_t}$. With both the nominal and real rate of capital, we get the Fisher relation, $R_t^K = \Pi_t^e r_t^K$, where $\Pi^e_t=E_tP_{t+1}/P_t$ is the expected inflation rate. Note that it is the rate of return on capital, rather than a money market interest rate, that is adopted in the Fisher equation. In the literature of both the existing Neoclassical and New Keynesian monetary economics, the interest rate of bond, R^B, is more frequently adopted in the place of R^K. It is not difficult to get the equivalence between these two rates. Secondly, to get the equality between interest rate of money, R, and R^B, which is necessary because only R could be controlled by the monetary authority, money has been treated as an asset in the existing Neoclassical and New Keynesian literature, and the equality result between these two rates is got correspondingly, see chapter 2 of Woodford (2003) for reference. However, from the fact of the time when gold was medium of exchange, we know money was a liquidity instrument, it doesn't generate any interest, and on the contrary, household had to pay seigniorage to the gold miners to get the instrument of payment. This fact has not been changed nowadays, and we see that the central banks generally don't pay interest to the commercial banks for the base money, and the commercial banks generally don't pay interest to the demand deposit.[3] So, the equality between R and R^B is dubious, and we need a different way to get an understanding of the money market interest.

4. Suboptimal equilibrium. Both the existing Neoclassical and New Keynesian monetary economics admit and accept the non-Pareto optimum of the equilibrium of an economy with money and tax. However, intuition tells us that the economy, including money and tax, which is created by nature, should be a perfect object, that is to say, its equilibrium should be Pareto optimal, at least to the benchmark case.

This paper fixes the above basic problems of the existing Neoclassical and New Keynesian monetary economics by developing a new economy named seigniorage channeled monetary economy, briefly SCME. It insists on the Neoclassical tradition, that is, it maintains the flexibility of prices, and proposes a new way, that is, the seigniorage channel, to introduce money into an RBC economy.[4]

[3] Some central banks and commercial banks do pay some interest to the base money and demand deposit respectively, but the interest rates of these debts are much lower than the respective interest rates of the assets in their respective balance sheets.

[4] Though the founders of the RBC theory believed that business cycles were caused by real shocks and denied the role of monetary shocks, the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983) indeed provided a solid base to study money and monetary shocks in a formal quantitative dynamic stochastic general equilibrium circumstance.

The readers will find that SCME is an interactive pricing economy with transaction process, it nests the RBC economy as a special case, monetary shock can sharply affect the real variables in flexible price condition, and the unique equilibrium of this economy is Pareto optimal.

One of the main reforms of SCME is the modification of the household budget of the existing theories. As mentioned above, it is well understood that the way in which money entered the economy when gold acted as the medium of exchange was that gold miners used gold to purchase goods they needed; in other words, to obtain money, a household had to abandon some of its output. This resource occupation nature of money issuance has not substantially changed under the present central bank monetary regime, where the central bank becomes the seigniorage gatherer. This fact reveals that seigniorage deserves special attention when treating the household budget constraint. As shown in the following sections, when this observation is operationalized in a modified household budget constraint that replacing the government lump-sum transfer by the seigniorage channel in an RBC economy, the monetary shock has a strong real effect in the flexible price condition. We will see output move sharply following a monetary shock, and the effect is persistent and hump-shaped.

In section 2, we discuss the dubious design in the budget of existing Neoclassical monetary economics: the government lump-sum transfers, and find that it is the cause of the monetary ineffectiveness result of this theory. In addition, the lump-sum transfer design helps to get the equality between R and R^B in the existing Neoclassical and New Keynesian monetary economics.

Section 3 studies the seigniorage channeled monetary economy with an exogenous monetary aggregate rule, where the effectiveness of monetary shocks under flexible price conditions is obtained. At the beginning of this section, a modified version of the RBC economy developed by King, Plosser, and Rebelo (1988) is provided, and this is the base of the SCME of this paper. A shocking characteristic of this modified RBC economy with taxation is that its unique equilibrium could be a Pareto optimum, a characteristic that will be maintained in SCME. In addition to the modification of the budget as have been mentioned above, the incorporation of transaction equations is another essential reform of SCME. It makes the economy complete and integrated; that is, SCME is a whole one, which means the major processes in real world economy, transaction, production, resource allocation, are all realized in the model economy. In contrast, the readers will see that the transaction part is lacking and impossible in existing Neoclassical and New Keynesian monetary economics because the cash-in-advance treatment and the money-in-utility treatment are both incompatible with the equation of exchange. The latter will be derived from the first orders and is closely related to the transactions equations. Furthermore, it will be shown that the involvement of the transaction part makes product pricing an interactive behavior between the supply side and the demand side of the economy, which, as mentioned before, is again lacking in existing dynamic general equilibrium monetary economics. Two additional issues about SCME are discussed in detail in this section. The first is mechanism of the monetary effectiveness. In SCME, the monetary shock takes effect by influencing resource allocation through the seigniorage channel. The second additional issue is the steady-state analysis, which studies the nonneutrality of the growth rate of money and inflation, including the optimal rate of inflation and the cost of inflation.

Section 4 studies the interest rate version of SCME. In the literature, the nominal interest rate depends on a pseudo bond that eventually degenerates to zero due to the representative household design, which implies that one cannot lend to himself or herself in equilibrium. In contrast, a money market interest rate is involved in the economy in a new way. Specifically, we find a money market interest rate, which equals the expected growth rate of money, is inherent in the seigniorage channel. The interest rate version of SCME is subsequently obtained. In addition, three more issues are discussed: a) the absence of the liquidity effect in the monetary aggregate rule economy; b) the time lag in monetary policy implementation, which is found to be the cause of the hump in the impulse-response curves; and c) the price puzzle in the interest rate rule economy. The last subsection studies the reactive interest rate rule. An approach for choosing reactive monetary policy is developed, and the negative movement of the hours under a positive technology shock is obtained when output is included in the reactive interest rate rule. The simulation and related results of a reactive interest rate rule economy are provided in detail, which reproduce many of the findings in the empirical studies in the literature and show SCME is promising in replicating the real world economy.

Section 5 concludes this paper by summarizing the findings and initiatives of this paper. The matters need attention and directions for further studies are mentioned as well.

2. The Problem in the Budget Constraint of the Present Monetary Economics

The government lump-sum transfers in the household budget of the existing Neoclassical monetary economics is the mechanism that prevents the effectiveness of monetary shocks. The well-accepted budget constraint of both MIU and CIA is

$$C_t + \mathrm{X}_t + \frac{M_t}{P_t} = Y_t + \Xi_t + \frac{M_{t-1}}{P_t} \qquad \text{E2.1}$$

where C, X, M, P, Y, and Ξ are consumption, investment, monetary aggregate, product price, output, and government lump-sum transfers.[5] The government transfer mechanism discussed here is the requirement in the literature that, in equilibrium,

$$\Xi_t = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} \qquad \text{E2.2}$$

With E2.2, the budget constraint, E2.1, becomes

$$C_t + X_t = Y_t \qquad \text{E2.3}$$

E2.3 means that under the government lump-sum transfer mechanism E2.2, the real resource allocation is not directly connected with money. With the traditional Neoclassical production function, which also lacks a position for money, it is obvious that money does not exist in either the resource allocation or the resource production process in this economy. Consequently, the channel for a monetary shock to affect real variables is substantively eliminated.

The above comment can be verified mathematically. Generally, we can only obtain the solution of the dynamic model numerically; however, we can understand the basic idea analytically and clearly

[5] The complete form of the budget in the literature includes bonds, B, and the gross interest rate on bonds, R^B, which is

$$C_t + \mathrm{X}_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = Y_t + \Xi_t + \frac{M_{t-1}}{P_t} + \frac{R^B_{t-1} B_{t-1}}{P_t}$$

However, because B_t is set to be zero in equilibrium in the literature, we can discuss the monetary issue of MIU and CIA with the simplified version of the budget, E2.1. Because this budget is also used in New Keynesian models, the discussion here is applicable to New Keynesian macroeconomics.

with simple cases. Here, we use the one-factor production CIA economy as an example; the MIU economy can be analyzed similarly. Assume that labor, N, is the only production factor. That is, the production function is $Y_t = {N_t}^{1-\alpha}$, where $0 \le \alpha < 1$ is the parameter and $0 < N_t \le 1$. The population of the economy is assumed to be constant. Additionally, assume that the monetary aggregate policy is $M_t = Z_t^M \Theta^t \overline{M}$. Z^M is the exogenous monetary supply shock, Θ is the exogenous gross growth rate of the monetary aggregate, and $\overline{M}$ is the exogenously given quantity of money. Let the present-period utility function be $U_t = \mathrm{U}(C_t, J_t)$, where $J_t = 1 - N_t$ is the leisure enjoyed by the household. The permanent utility from period t is correspondently $UU_t = \sum_{i=0}^{\infty} \beta^i E_t U_{t+i}$. β is the subjective discount rate, and E is the expectation operator. For simplicity, assume that $\mathrm{U}(C_t, 1 - N_t) = \Phi(C_t) + \Psi(N_t)$. With the undetermined coefficient method, we can obtain the relation between output and the monetary shock of this CIA economy as

$$\widehat{Y_t} = -\frac{1-\rho^M}{\Phi_{CC}C-(1-\rho^M)+(2-\frac{\beta}{\Theta}\rho^M)\frac{\Psi_{NN}N^{1+\alpha}-\alpha(1-\alpha)\frac{\Phi_C}{2-\frac{\beta}{\Theta}}}{(1-\alpha)^2}}\widehat{Z_t^M} \qquad \text{E2.4}$$

where a variable with a ^ denotes the percentage deviation of the variable from its steady state and a variable without a time indicator represents its steady-state value. Additionally, assume that $\widehat{Z_t^M} = \rho^M \widehat{Z_{t-1}^M} + \varepsilon_t^M$, where $0 \le \rho^M < 1$, and $\varepsilon^M_t \sim N(0, \sigma_M^2)$ is a white noise process.

When $\Phi_C > 0$, $\Phi_{CC} < 0$, $\Psi_{NN} < 0$, which are held in the literature, together with the well-accepted values of the parameters, the denominator on the right-hand side of E2.4 is negative. Let the value of the coefficient of $\widehat{Z_t^M}$ in E2.4 be Ⓩ. We can see that the value of Ⓩ ranges from approximately 0.2 to 0 as ρ^M spans from 0 to 1. Because ρ^M is close to 1 in the literature, Ⓩ is a small positive number that is much close to 0, from which we obtain the weak relation between output and the monetary shock. The numerical solution of the more complicated MIU and CIA economies provides similar results.

There is still another crucial problem with the government transfer arrangement. From E2.1, when $M_t > M_{t-1}$, which is generally held, the money issuance activity becomes an income for the household. This is starkly inconsistent to the economic truth that seigniorage is an expense for the household.

3. SCME with Exogenous Monetary Aggregate Rule

Before diving into the study, let us prepare the economy on which the SCME is based. It is the RBC economy in King, Plosser, and Rebelo (1988) with the following modifications:

a. Let the household consume both private goods, C, and public goods, G. Then, the period utility function takes the following form:

$$U_t = \mathrm{U}(C_t, G_t, J_t) \qquad \text{E3.1}$$

b. Assume that the public goods are transformed from taxes by the government sector and that, simply,

$$G_t = T_t \qquad \text{E3.2}$$

c. With the flat-rate income tax rate τ, it is not difficult to obtain

$$T_t = \tau Y_t \qquad \text{E3.3}$$

Consequently, the budget constraint of the household in this modified version of the RBC economy is

$$C_t + X_t + T_t = Y_t \qquad \text{E3.4}$$

With the above modifications, the only intertemporal optimizer in this economy, the household, is subject to two constraints: the budget constraint E3.4 and the public goods constraint E3.2. When the production and utility functions meet specific requirements, we can obtain a unique equilibrium for this version of the RBC economy. An outstanding characteristic of this modified economy is that when public goods are transformed from taxes as in E3.2 and enter the utility of the household as in E3.1, taxation is consumed by household in the form of public goods, and it no longer distorts the economy. Consequently, the unique equilibrium of this RBC economy with taxation is Pareto optimal. In contrast, in the traditional treatment, taxes introduce distortion because households cannot consume the portion of output constituting government spending. With this modified version of the RBC economy, we begin the study of SCME.

3.1 The Model

In this economy, the representative household, which owns the firm and makes intertemporal consumption and investment decisions, weighs consumption, public goods, and leisure in its utility. The government in this economy includes both the monetary and fiscal authorities. The government is not a utility maximizer; it simply implements monetary and fiscal policies according to the rules and provides public goods, such as national defense and monetary services, to the household. The arrangement of this economy is stated in parts A, B, and C below.

A. Monetary Issues and Transactions

a. *Money Issuance Mechanism*. In the present real world economy, the utility-maximizing commercial banks obtain base money from and pay seigniorage to the central bank. In the meantime, the commercial banks create money through lending to the firm and collect the corresponding interest from the loan. This money issuance mechanism, which is called the central bank regime, is described simply in Figure 1, where the simplified balance sheets of the central bank, the commercial bank, and the firm are presented. M^B and L^B are base money and loan of the central bank, respectively, and M and L are monetary aggregate and loan to the firm, respectively. In addition, we have $L^B_t \equiv M^B_t$, and $L_t \equiv M_t$ in the money issuance process.

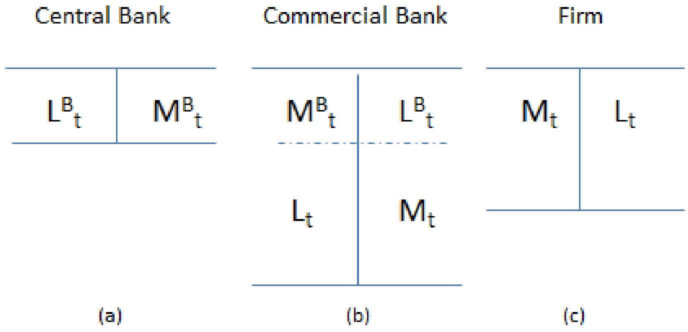


Figure 1

In this paper, to make things simple, we adopt a condensed version of the above monetary regime, where the whole monetary system, that is, the central bank and the commercial bank, is treated as a part of the government and the seigniorage is collected according to the monetary aggregate. This condensed monetary regime is described in Figure 2, and we have $L_t \equiv M_t$ again. This way of money issuance can also be regarded as an imitation of an ancient economy where gold is medium of exchange, gold mine is held by the government, and government can control the growth rate of the quantity of gold. The difference is that it is the note issued by the government rather than the physical gold is accepted as money here.

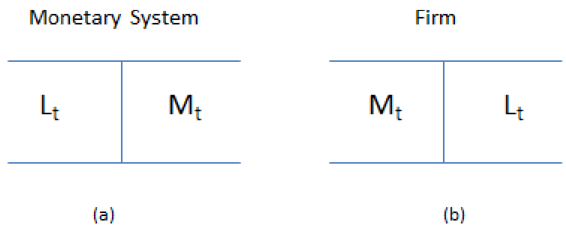


Figure 2

b. *Monetary Policy*. We adopt a simple exogenous monetary aggregate rule in this section:

$$M_t = Z_t^M \Theta^t \overline{M} \qquad \text{E3.5}$$

where Θ is the constant gross growth rate of money and $\overline{M}$ is the amount of the monetary aggregate, which is exogenously given. The log form of the monetary shock Z^M is a stationary first-order autoregressive process:

$$\ln Z_t^M = (1-\rho^M)\ln Z^M + \rho^M \ln Z_{t-1}^M \qquad \text{E3.6}$$

where $0<\rho^M<1$. The steady state of Z^M is set to be unity. The white noise shock, $\varepsilon^M_t \sim N(0, \sigma_M^2)$, is added to the log-linear form of E3.6, so we have $\widehat{Z_t^M} = \rho^M \widehat{Z_{t-1}^M} + \varepsilon_t^M$. At the beginning of each period, ε^M is realized.

As will be clear from the transaction process we will discuss soon, the firm holding M_{t-1} at the end of period t-1, the newly issued money in period **t** is $M_t - M_{t-1,}$ and M_t is the stock of money used in mediating the transactions in period **t**.

c. *Renting*. The firm rents labor and capital from the household, and the household receives money from the firm, which will be used to purchase products later in the same period. Therefore, we have

$$W_t^K K_{t-1} = P_t^M M_t^K \qquad \text{E3.7}$$

$$W_t^N N_t = P_t^M M_t^N \qquad \text{E3.8}$$

where K and N are capital and working hours and W^K and W^N are their respective rent rates. As numeraire, the price of money, P^M_t is constant and normalized to unity, so we have, $P^M_t \equiv 1$. Note we will not show P^M or P^M_t again in the rest of the paper. In this paper, we do not scrutinize the velocity of money, ω, and assume it to be constant and normalize it to be unity. Since we are studying the quarterly economy in this paper, the annual velocity of money is 4.M^K and M^N are the money used in the rental of capital and labor hours, respectively, and we have

$$M_t^K + M_t^N = M_t \qquad \text{E3.9}$$

which means that information is complete, and the quantity of money is known to everyone in the economy.

d. *Purchasing*. After production, the household purchases products with money. Thus, we have

$$M_t^K = P_t Y_t^K \qquad \text{E3.10}$$

$$M_t^N = P_t Y_t^N \qquad \text{E3.11}$$

where Y^K and Y^N are products purchased with M^K and M^N, respectively, and P is the price of output Y.

E3.7-E3.11 show that all transactions are mediated by M_t in this economy, and this is the case in all the models in this paper. In this economy, after the transactions, M_t is held by the firm at the end of period t.

B. Supply Side of the Economy

e. *Production Function*. The production function in this paper is the standard constant returns to scale Cobb–Douglass function:

$$Y_t = Z_t^T K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha} \qquad \text{E3.12}$$

where α is the share of capital in production, and the growth rate of technology is exogenously given. That is, A_t/A_{t-1} is set to be a constant, Γ.

Similar to the monetary shock, the log form of the technology shock, Z^T, is a stationary first-order autoregressive process:

$$\ln Z_t^T = (1-\rho^T)\ln Z^T + \rho^T \ln Z_{t-1}^T \qquad \text{E3.13}$$

where $0<\rho^T<1$. The steady state of Z^T is set to be unity. The white noise process, $\varepsilon^T_t \sim N(0, \sigma_T^2)$, is added to the log-linear form of E3.13, so we have $\widehat{Z_t^T} = \rho^T \widehat{Z_{t-1}^T} + \varepsilon_t^T$. At the beginning of each period, ε^T is realized. ε^T and ε^M are independent of one another in this paper.

f. *Firm Profit Maximization*. The firm maximizes its profit, D, with

$$\mathrm{D}_t = Y_t - Y_t^K - Y_t^N \tag{E3.14}$$

From the maximization process of the firm, we have

$$\alpha P_t Y_t = W_t^K K_{t-1} \tag{E3.15}$$

$$(1-\alpha) P_t Y_t = W_t^N N_t \tag{E3.16}$$

In addition, we can obtain $Y_t^K = \alpha Y_t, Y_t^N = (1-\alpha)Y_t, and\ \mathrm{D}_t = 0$ from E3.7, E3.8, E3.10, E3.11, E3.15, and E3.16.

g. *Equation of Exchange*. From E3.7-E3.11 and E3.15-E3.16, we can obtain the equation of exchange:

$$M_t = P_t Y_t \tag{E3.17}$$

Compared with the well-accepted equation of exchange, that is, $M_t \omega_t = P_t Y_t$, the velocity of money in period t, ω_t, is set to be unity in this paper. E3.17 can be regarded as the supply curve of the SCME of this subsection.

C. Demand Side of the Economy

h. *Budget Constraint with Seigniorage and Taxation*. As to the seigniorage, S, we have

$$S_t = \frac{M_t - M_{t-1}}{M_t} Y_t \tag{E3.18}$$

Here is the explanation about E3.18: 1. According to the money issuance arrangement described above, M_t is consist of two parts, $M_t - M_{t-1}$, which is the new money issued in period t, and M_{t-1}, which is the old money inherited from the last period. The purchasing power of these two parts of money is the same. From the transaction equations E3.7-E3.11, the total product of period t, that is, Y_t , is mediated by M_t . So similar to that in the gold standard regime, the new issued money, M_t-M_{t-1}, obtain its share of product, S_t, that is, the seigniorage of period t. 2. Under the representative agency arrangement of the economy, household is the owner of the firm, and the firm is just a production instrument of the household, so the seigniorage is paid by the household with its income. The income of the household, that is, $Y_t^K + Y_t^N + D_t$, equals Y_t, which is obvious from parts A and B of this subsection.

Concerning the tax, T, the simple flat-rate income tax is adopted in this paper[6], and we have

$$T_t = \tau Y_t \tag{E3.19}$$

where $\boldsymbol{\tau}$ is the tax rate.[7] Together with the seigniorage, we can obtain the total tax, TT, of this economy as

$$TT_t = T_t + S_t \tag{E3.20}$$

Now, we can obtain the representative household's budget constraint as,

[6] There could be other tax arrangements, such as a tax on consumption, investment, and /or capital or some combination thereof.

[7] Here, we assume that the fiscal authority directly collects products, as a tax, from the household. It is not difficult to let the household pay the tax in monetary form and the fiscal authority purchase the equivalent quantity of products from the firm.

$$C_t + X_t + TT_t + \frac{M_t}{P_t} = Y_t + \frac{L_t}{P_t} \quad \text{E3.21'}$$

with the left-hand side representing household's resource allocation, and the right-hand side his/her resource input. In contrast to the traditional budget, E2.1, the government lump sum transfer mechanism is replaced by the seigniorage channel in the new budget E3.21'. Replace TT_t with E3.18, E3.19, and 3.20, and note that $L_t \equiv M_t$ under the money issuance mechanism, the budget constraint can be restated as,

$$C_t + X_t = (\frac{M_{t-1}}{M_t} - \tau)Y_t \quad \text{E3.21}$$

i. *Public Goods*. In this economy, the household consumes public goods, which are created by the government sector with the tax. In this paper, we simply let

$$G_t = TT_t. \quad \text{E3.22'}$$

With E3.18, E3.19, and E3.20, this simple public goods transformation function can be represented as

$$G_t = (\tau + 1 - \frac{M_{t-1}}{M_t})Y_t \quad \text{E3.22}$$

Note that the monetary system, like national defense, is treated as a public good.[8]

j. *Capital Reproduction*. The capital is held by the household and its reproduction is canonical:

$$K_t = (1-\delta)K_{t-1} + X_t \quad \text{E3.23}$$

where δ is the depreciation rate of capital.

k. *Utility Function*. Assume the periodical utility of household to be

$$U_t = \mathrm{U}(\frac{C_t}{A_t}, \frac{G_t}{A_t}, J_t) \quad \text{E3.24}$$

where J_t=1-N_t is the leisure in period **t**. That is, the maximum number of hours is normalized to unity. E3.24 means that the household enjoys private goods, C_t, public goods, G_t, and leisure. The modification of C_t and G_t with the growth factor A_t means that the household cares about growth-adjusted consumption and public goods. This treatment helps easily obtain the detrended form of the economy. The gross growth rate of the population in this paper is assumed to be unity.

The permanent utility of households starting from period t, UU_t, is

$$UU_t = max \sum_{i=0}^{\infty} \beta^i E_t U_{t+i} \quad \text{E3.25}$$

where β is the subjective utility discount rate of the household and E is the expectation operator.

3.2 The Unique Pareto Optimal Equilibrium and Comparison of SCME with MIU and CIA

[8] By the way, we can get the budget of the government as,

$$G_t + \frac{L_t}{P_t} = TT_t + \frac{M_t}{P_t}$$

with the left side its resource output and the right side its resource input. Note the government is not a utility maximizer, it just runs the policies according to monetary and fiscal rules and provides public goods according to E3.22'.

This subsection discusses the equilibrium and compares the SCME with existing dynamic general equilibrium monetary economies, and the incompatibility between the cash-in-advance treatment/the money-in-utility treatment and the equation of exchange is considered in the comparison.

E3.5-E3.25 consist of the basic arrangement of the SCME of this section. In this economy, the household maximizes its permanent utility, E3.25, subject to the budget constraint, E3.21, and the public goods constraint, E3.22. In addition, the equation of exchange is already known by the household at the moment of decision, and it represents the household's third constraint. Let $\dot{Y}_t = \frac{Y_t}{A_t}, \dot{C}_t = \frac{C_t}{A_t}, \dot{X}_t = \frac{X_t}{A_t}, \dot{G}_t = \frac{G_t}{A_t}, \dot{S}_t = \frac{S_t}{A_t}, \dot{T}_t = \frac{T_t}{A_t}, \dot{TT}_t = \frac{TT_t}{A_t}, \dot{K_{t-1}} = \frac{K_{t-1}}{A_t}, \dot{M}_t = \frac{M_t}{\Theta^t}, \dot{P}_t = \frac{P_t}{\frac{\Theta^t}{A_t}}$,

where a letter with a · above it stands for its detrended form, we can obtain the detrended form of the utility function, the constraints of the household, the production function, and monetary policy as follows:

$$UU_t = max \sum_{i=0}^{\infty} \beta^i \mathrm{U}(\dot{C_{t+\iota}}, \dot{G_{t+\iota}}, J_t) \quad \text{E3.26}$$

$$\dot{C}_t + \Gamma \dot{K}_t = \left(\frac{\dot{M_{t-1}}}{\Theta \dot{M}_t} - \tau\right) \dot{Y}_t + (1-\delta)\dot{K_{t-1}} \quad \text{E3.27}$$

$$\dot{G}_t = (\tau + 1 - \frac{\dot{M_{t-1}}}{\Theta \dot{M}_t})\dot{Y}_t \quad \text{E3.28}$$

$$\dot{M}_t = \dot{P}_t \dot{Y}_t \quad \text{E3.29}$$

$$\dot{Y}_t = Z_t^T \dot{K_{t-1}}^{\alpha} N_t^{1-\alpha} \quad \text{E3.30}$$

$$\dot{M}_t = Z_t^M \overline{M} \quad \text{E3.31}$$

E3.26-E3.31 is a recursive dynamic optimization problem with two endogenous state variables, $\dot{K_{t-1}}$ and $\dot{M_{t-1}}$, and two exogenous shocks, Z^T_t and Z^M_t, and we can obtain the Bellman equation subject to the budget constraint, E3.27, public goods constraint, E3.28, and equation of exchange constraint, E3.29. With Theorems 9.6-9.11 of Stokey, Lucas, and Prescott (1989), we get the following main result of this paper:

Proposition: In the economy described by E3.26-E3.31, if the period utility function E3.26 is continuous, bounded, strictly increasing, strictly concave, and continuously differentiable, there is a unique continuous, concave, differentiable function solving the dynamic programming problem, and the optimal policy function is single-valued.

Note that when the firm's optimal behavior is embedded in the exchange equation, which consists of a constraint on the household, the household needs to choose a point in the equation of exchange curve with the monetary aggregate exogenously given. In other words, when the household problem is uniquely solved in the Proposition, the optimal problem of the firm is simultaneously and uniquely determined because the equation of exchange, which includes the information needed to solve the firm problem, is included in the household problem as a constraint. Therefore, we can define the equilibrium and establish the uniqueness of the equilibrium of the SCME in this subsection according to the following Definition:

Definition: The exogenous monetary policy E3.31, the detrended form of the transaction equations E3.7-E3.11, the optimal behavior of the firm E3.14-E.3.16, and the unique optimal intertemporal solution of the household, that is, the Proposition, constitute the unique equilibrium of the SCME of this subsection, in which both the optimal requirements of the supply side, the firm, and the demand side, the household, are met simultaneously.

Compared with the existing Neoclassical and New Keynesian monetary economics, three significant characteristics of the SCME are worth mentioning:

a. The involvement of the transaction equations, E3.7-E3.11, makes SCME a complete and integrated system that consists of all processes of the real-world economy: transaction, production, allocation, and consumption. In contrast, the transaction part is lacking in existing dynamic general equilibrium monetary economics. Furthermore, it is factually impossible to embed the transaction part into existing theories. The reason is that in the CIA economy, when the goods are divided into cash goods, the transaction of which needs to be mediated by money, and credit goods, the transaction of which does not need to be mediated by money, the cash-in-advance constraint, $M_t \leq P_tC_t$, is directly inconsistent with the equation of exchange, $M_t = P_tY_t$, which is the outcome of the transaction process and firm behavior, except the case of $C_t = Y_t$, which is obviously unreasonable. In the MIU economy, the magnitude of the real balance, M_t/P_t, in the utility seems too large in scale if the transaction equations are involved because, from the transaction equations and the behavior of the firm, we obtain the equation of exchange, and from the latter, we obtain $M_t/P_t = Y_t$. A real balance at the magnitude of Y_t is too large compared with the magnitude of C_t in the utility. Factually, from E3.18, we know S_t equals $\frac{M_t - M_{t-1}}{M_t} Y_t$, which is much less than Y_t . In steady state, $S = \frac{\Theta - 1}{\Theta} Y$, where Θ is about 1.015 in the economic history of the US, which means $S \approx 0.015Y$ in steady state. Note that S actually enters the utility as a part of G in SCME.
b. Concerning the pricing mechanism, the involvement of the transaction equations in the SCME makes the decision of the product pricing an interactive activity between the firm and the household. Both the decision of the firm and the household are based on the transaction process. In contrast, the product price is solely decided by the demand side in the existing Neoclassical monetary economics and solely decided by the supply side in New Keynesian economics. From the incompatibility between the cash-in-advance treatment/the money-in-utility treatment and the equation of exchange we have above, interactive pricing is impossible in the existing theories.
c. As the equilibrium of an RBC economy with taxation can be a Pareto optimum with the treatment that public goods enter household utility, as noted at the beginning of this section, when the service of the monetary system, which is transformed from seigniorage, is treated as a part of the public goods as in this section, the unique equilibrium of the SCME in the above Definition is again a Pareto optimal equilibrium. Note that although there is cost, that is, seigniorage, in operating the monetary system, this cost is not a distortion or a deadweight loss. This cost is used to produce the monetary service, which is ultimately consumed by the household. All resources are enjoyed by the household in this economy.

Interestingly, the RBC economy is a special case of the SCME of this subsection. With $\Theta=1$, which means that seigniorage degenerates to zero, and the monetary shock neglected, we obtain the modified version of the RBC economy described at the beginning of this section.

3.3 Simulation

To simulate the SCME, we need a concrete form of the utility function and parameter values. To ensure robustness, well-accepted functional form and parameter values are adopted in this paper. In particular, we assume the following utility function[9]:

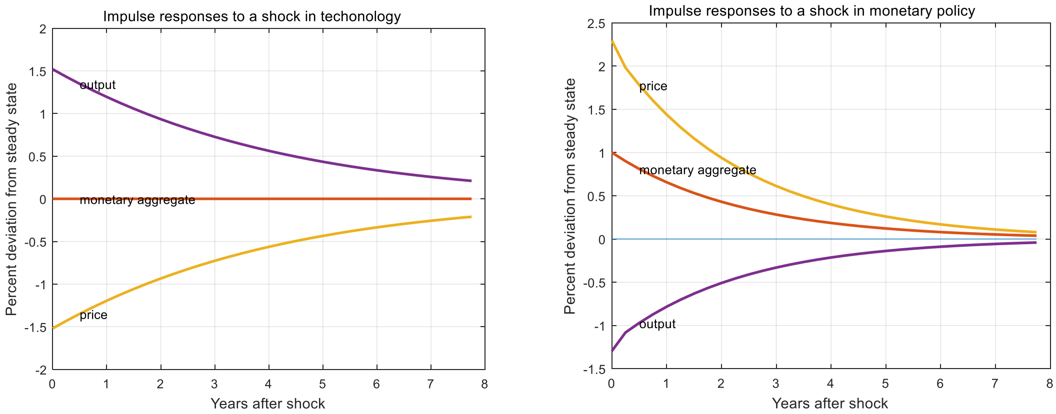
$$U_t = \frac{(\dot{C_t}^{\chi}\,\dot{G_t}^{1-\chi})^{1-\eta}}{1-\eta} + \xi\,\frac{(1-N_t)^{1-\eta N}}{1-\eta N} \qquad \text{E3.32}$$

where χ, η and ηN are the respective coefficients, and ξ is the balance parameter, which will help in obtaining a reasonable steady-state value for hours in the simulation. Concerning the value of the parameters, let α=0.36, β=0.97, δ=0.025, Θ=1.015, Γ=1.0075, τ=0.17, η=0.5, ηN=0.5, and χ=0.8. The value of χ is close to the relative value between C and G in steady state, and this value can also be obtained from a model similar to that in this subsection with τ treated as a variable. The subjective discount rate, β, which is 0.97, is adopted to ensure that the steady-state C/Y, X/Y, T/Y, and S/Y ratios are close to those in the everyday economy, which are 0.64, 0.18, 0.17, and 0.015, respectively, in this model. The value of ξ is set to ensure that the steady-state value of hours is 1/3. Regarding the parameters in the shocks, let ρ^T=0.9, ρ^M=0.9, σ^T=0.7%, and σ^M=0.7%, which are extensively adopted in the literature.

The whole system of the simulated model is provided in Appendix A. By log-linearizing the model around its steady state, Figure 2 provides the impulse response of output and price of this SCME under a one-percent positive technology shock and monetary shock.[10] Panel (a) shows that the responses of output and price are approximately plus and minus 1.5 percent, respectively in the first period of a technology shock, and Panel (b) shows that the responses are approximately minus 1.3 percent in output and plus 2.3 percent in price in the first period of the monetary shock, and the monetary effectiveness is persistent. Note that the amount of the movements of output and price is consistent with the log-linear form of the equation of exchange E3.29, $\widehat{M} = \widehat{P} + \widehat{Y}$. During period t, $\widehat{M} = 0$ in the technology shock case and $\widehat{M} = 1$ in the monetary shock case, which are shown in Panels (a) and (b) of Figure 3, respectively. The negative output effect following a positive monetary aggregate shock is consistent with the findings of Eichenbaum (1992) and Leeper and Gordon (1992). Further simulation results of this economy will be provided in the next subsection when the mechanism of monetary effectiveness is discussed.

9 Similar results can be obtained using logarithmic utility.

10 The toolkit of Uhlig (1999) is used in simulation in this paper.



(a) (b)

Figure 3

3.4 Mechanism of Monetary Effectiveness in SCME

The strong monetary effectiveness and its persistence in the SCME originate from the seigniorage function E3.18, through which M_t is essentially embedded into the budget E3.21. E3.21 explicitly shows that a monetary shock can lead to resource reallocation. This is much different from the MIU and CIA economies as noted in section 2, in which the government lump-sum transfer mechanism impedes the ability of money to affect resource allocation.

In particular, from the detrended form of the seigniorage function E3.18, we can obtain the following log-linear form:

$$\widehat{SOY_t} = \frac{1}{\Theta-1}(\widehat{\dot M_t} - \widehat{\dot M_{t-1}}) \qquad \text{E3.33}$$

where $SOY_t = \frac{\dot S_t}{\dot Y_t}$. E3.33 means that a one-percent positive movement in $\dot M$ in period t, which is triggered by a one-percent monetary shock Z^M, see E3.31, will lead to a 66.67 percent change in $\frac{\dot S_t}{\dot Y_t}$ when Θ=1.015. From the detrended from of the budget constraint E3.21, we can obtain

$$\frac{\dot C}{\dot Y}\widehat{COY_t} + \frac{\dot X}{\dot Y}\widehat{XOY_t} + \frac{S}{\dot Y}\widehat{SOY_t} = 0 \qquad \text{E3.34}$$

where $COY_t = \frac{\dot C_t}{\dot Y_t}, XOY_t = \frac{\dot X_t}{\dot Y_t}$. With $\frac{\dot C}{\dot Y} = 0.64, \frac{\dot X}{\dot Y} = 0.18, \text{and } \frac{\dot S}{\dot Y} = 0.015$ in the above simulated economy, a 66.67 percent movement in SOY will lead to an approximately 1 percent change in the last term on the left-hand side of E3.34, which means that a one-percent monetary shock will trigger an approximately minus 1 percent movement in both COY and XOY. This implies a significant impact of a monetary shock on the real variables of output, consumption, and investment.

The above analysis is elucidated when we use a simple case, from which we can obtain an analytical solution. In particular, let us study the case of a one-factor production function with no growth, as we did in reviewing the MIU and CIA economies in section 2. Here, let capital be the only factor, and we have $Y_t = K_{t-1}^{\alpha}$ where α=0.36. In addition, we neglect taxes and public goods, which means that the period utility function is $U_t = \mathrm{U}(C_t)$, and we have the budget constraint as $C_t + K_t + S_t =$

Y_t, where δ=1 is assumed. There are no changes in the seigniorage or monetary policy functions. That is, E3.18 and E3.5 are maintained. The equation of exchange in this case is $M_t = \alpha P_t Y_t$. In this simpler SCME, the representative household is subject to the budget constraint and the equation of exchange constraint.

When there is a one-percent positive movement in the monetary shock, there will be a 66.7 percent increase in $\widehat{S_t}$ from the following log-linear form of the seigniorage function when Θ=1.015:

$$(1-\frac{1}{\Theta})\widehat{S_t} = (1-\frac{1}{\Theta})\widehat{Y_t} - \frac{1}{\Theta}(\widehat{\dot{M}_{t-1}} - \widehat{\dot{M}_t}) \qquad \text{E3.35}$$

Note that we have $\widehat{Y_t} = 0$ from the production function. In addition, from the budget constraint, we have

$$C\widehat{C_t} + \mathrm{K}\widehat{K_t} + S\widehat{S_t} = Y\widehat{Y_t} \qquad \text{E3.36}$$

which means that if the household does not change its capital investment, that is, it keeps $\widehat{K_t} = 0$, then there will be a -1.51 percent decrease in $\widehat{C_t}$. According to the utility function, a decrease in consumption means reduced utility. However, from E3.36, it is possible to improve utility if the household decreases capital investment, that is, it lets $\widehat{K_t} < 0$. The result depends on the comparison between two opposite effects induced by the decrease in capital investment: the improvement in consumption in period t and the possible worsening of consumption in the following periods; the latter is possible because the decline in capital investment in this period will lead to reduced production in the next period. We can obtain the exact result with the undetermined coefficient method. Although we can obtain the analytical form of the recursive equilibrium laws between each variable and the endogenous state variables and the exogenous monetary shock by pencil and paper, it remains very complex even in this simple case. The strategy we adopt here is to provide the movement equation for capital investment solved by a personal computer directly below, where $U_t = \frac{{C_t}^{1-\eta}}{1-\eta}$, η=0.5, and ρ^M=0 are adopted:

$$\widehat{K_t} = 0.22 * \widehat{K_{t-1}} + 0.62 * \widehat{M_{t-1}} - 0.54 * \widehat{Z_t^M} \qquad \text{E3.37}$$

E3.37 means that given a one-percent positive movement in Z^M_t, the household's optimal choice is to decrease capital investment by 0.54 percent from its steady-state level. This decrease in $\widehat{K_t}$ implies, from E3.36, a 1.24 percent decrease in $\widehat{C_t}$, which will lead to greater utility compared with the 1.51 percent decrease in $\widehat{C_t}$ in the benchmark case with $\widehat{K_t} = 0$. The decreased $\widehat{K_t}$ leads to a 0.19 percent decrease in output in the next period, which is evident from the production function. Figure 3 below shows the variations in the main variables in this economy. The economy quickly returns to its steady state because the monetary shock and the seigniorage effect disappear quickly in the case of ρ^M=0. An interesting point here is that because of the decrease in seigniorage in period t+1, capital investment increases rather than decreases, as expected above.

Compared with the technology shock, which influences the economy through resource production, this simple case shows that monetary shock takes effect through resource reallocation.

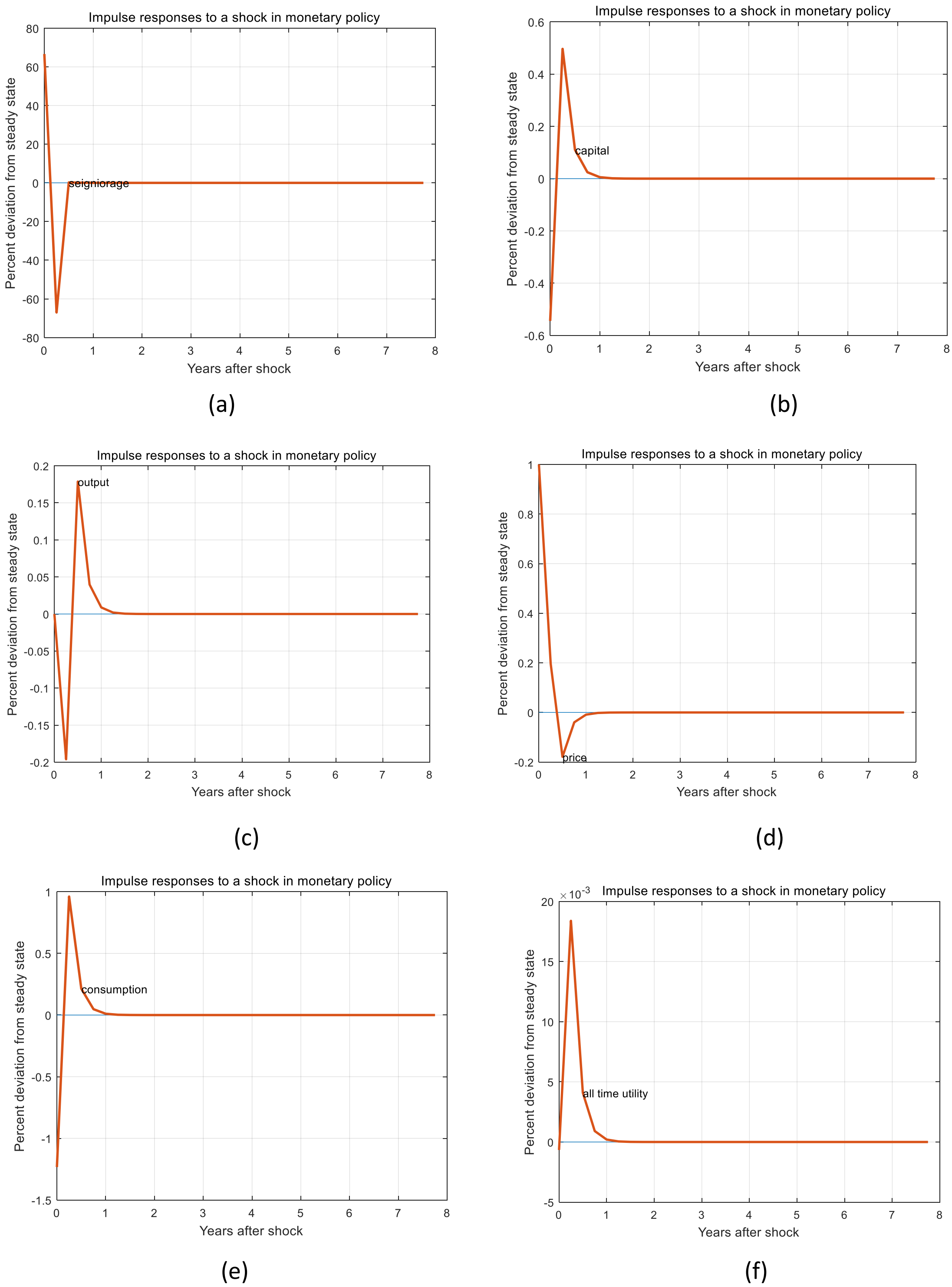


Figure 4

In the above simple case, we can obtain the precise values of the impact of the monetary shock on all variables by hand. For complicated models such as that presented in subsection 3.1, briefly, for Model 3.1, the mechanism is similar, but we can only obtain the result numerically. In Model 3.1, as shown in Figure 5 below, a one-percent positive monetary shock leads to a 66.7 percent increase in seigniorage in period **t**. It is the best response for a household, in period t, to increase consumption by 1.7 percent, decrease investment by 18 percent, which implies a capital decrease of approximately 0.5 percent, and decreases hours worked by approximately 2.2 percent. As a result, the household's permanent utility in period t increases by approximately 0.1 percent. In the resource reallocation process, the total tax increases by approximately 4 percent in the first period. The movements of output and price are shown in Panel (b) of Figure 3.

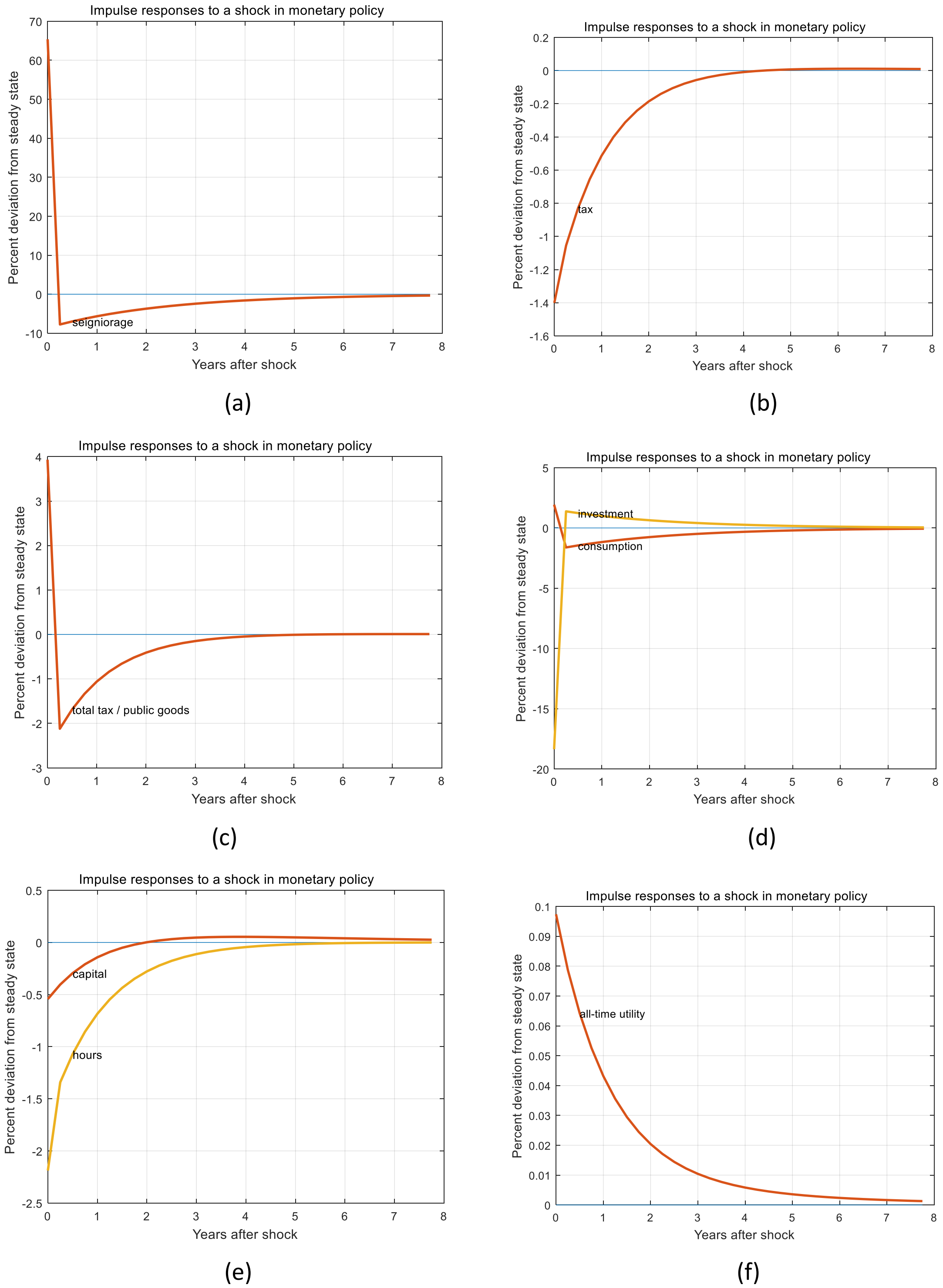


Figure 5

3.5 Steady-state Analysis: Nonneutrality of Growth Rate of Money and Inflation

Above, we studied the impact of a monetary shock on the economy. Now, let us consider the impact of money in the steady state, which has been a long-standing topic in monetary economics. Compared with the neutrality and superneutrality results in some of the studies of traditional Neoclassical monetary economics, in the SCME, the change in the monetary aggregate in steady state is neutral. Nevertheless, the change in the growth rate of money and the inflation rate is nonneutral.

The neutrality of the change in the monetary aggregate is evident because it is the growth rate of money, rather than the stock of the monetary aggregate, that appears in the steady state of the budget constraint E3.27 and the public goods constraint E3.28. From the steady state of the equation of exchange, the change in the stock of monetary aggregate will be entirely absorbed by the change in the price level.

The appearance of Θ in the steady state leads to the nonneutrality result of the growth rate of money in the SCME. Furthermore, dividing the equation of exchange for period t by that for period t-1, $\frac{M_t}{M_{t-1}} = \frac{P_t}{P_{t-1}}\frac{Y_t}{Y_{t-1}}$, we can obtain, in steady state,

$$\Theta = \Pi\Gamma \qquad \text{E3.38}$$

where Π and Γ are the steady-state value of gross inflation and the gross economic growth rate, respectively. When replacing Θ in the steady-state equations of the household with E3.38, the nonneutrality of inflation can be obtained.

Although it is difficult to obtain the analytical solution of the relation between output and inflation in complicated economies such as Model 3.1, we can obtain the idea of quantifying the nonneutrality of money growth rate and inflation with the simple economy described in Subsection 3.4 above. From its steady-state we obtain

$$\mathrm{Y} = (\frac{\Theta}{\beta^2\alpha})^{\frac{\alpha}{\alpha-1}} = (\frac{\Pi\Gamma}{\beta^2\alpha})^{\frac{\alpha}{\alpha-1}} \qquad \text{E3.39}$$

E3.39 is the output-money growth rate curve and output-inflation curve of the simple economy. To save space, we study the latter here, and abbreviate it as YIC. The study of the optimal rate of inflation and inflation cost is based on it.
Regarding the optimal rate of inflation, from E3.39, we obtain

$$\frac{dY}{d\Pi} = \frac{\alpha}{\alpha-1}(\frac{\Pi\Gamma}{\beta^2\alpha})^{\frac{\alpha}{\alpha-1}}\frac{\Gamma}{\beta^2\alpha} \qquad \text{E3.40}$$

With the commonly accepted parameter values, it is obvious that $\frac{dY}{d\Pi} < 0$ in E3.40, which means that the more severe the inflation is, the lower the steady-state output. However, there is a lower bound to the range of inflation. From the steady-state form of the seigniorage function E3.18, $\mathrm{S} = \left(1 - \frac{1}{\Theta}\right)\mathrm{Y}$, we know that Θ≥1 is required to ensure that the seigniorage is positive. Together with E3.39, we can obtain that the lower bound of gross inflation is $\frac{1}{\Gamma}$, which is actually deflation when Γ>1. When Γ is set at 1.0075 in the quarterly term as in this paper, the lower bound of gross inflation is 0.9926. When $\Theta \geq 1$, E3.40 implies that the optimal rate of inflation is the point $\Pi = \frac{1}{\Gamma}$, that is, $\Theta = 1$.

The mechanism of the nonneutrality of inflation is the same as that in the monetary shock case discussed in the above subsection: the rise of inflation, triggered by the increase in the growth rate of the monetary aggregate, leads, according to the steady-state of the seigniorage function, to an increase in the seigniorage-output ratio and triggers the resource reallocation process that leads to utility maximization.

In addition, we can obtain the output cost of inflation, briefly YCOI, from $\left|\frac{\mathrm{d}\ln Y}{\mathrm{d}\ln \Pi}\right| = \left|\frac{\Pi}{Y}\frac{dY}{d\Pi}\right|$.

Panels (a) and (b) of Figure 6 below show the YIC and YCOI of Model 3.1, respectively, where the output cost of inflation runs from approximately 1.4 to 2.6 when gross inflation moves from $\frac{1}{\Gamma}$ to 2.

The comparison of the optimal rate of inflation and cost of inflation with those in existing empirical studies will be provided in the next section when the interest rate is involved.

Similarly, we can obtain the relation between each other variable and inflation. For example, the relation between utility and inflation, UIC, and the utility cost of inflation, UCOI, of Model 3.1 are provided in panels (c) and (d) of Figure 6, respectively.

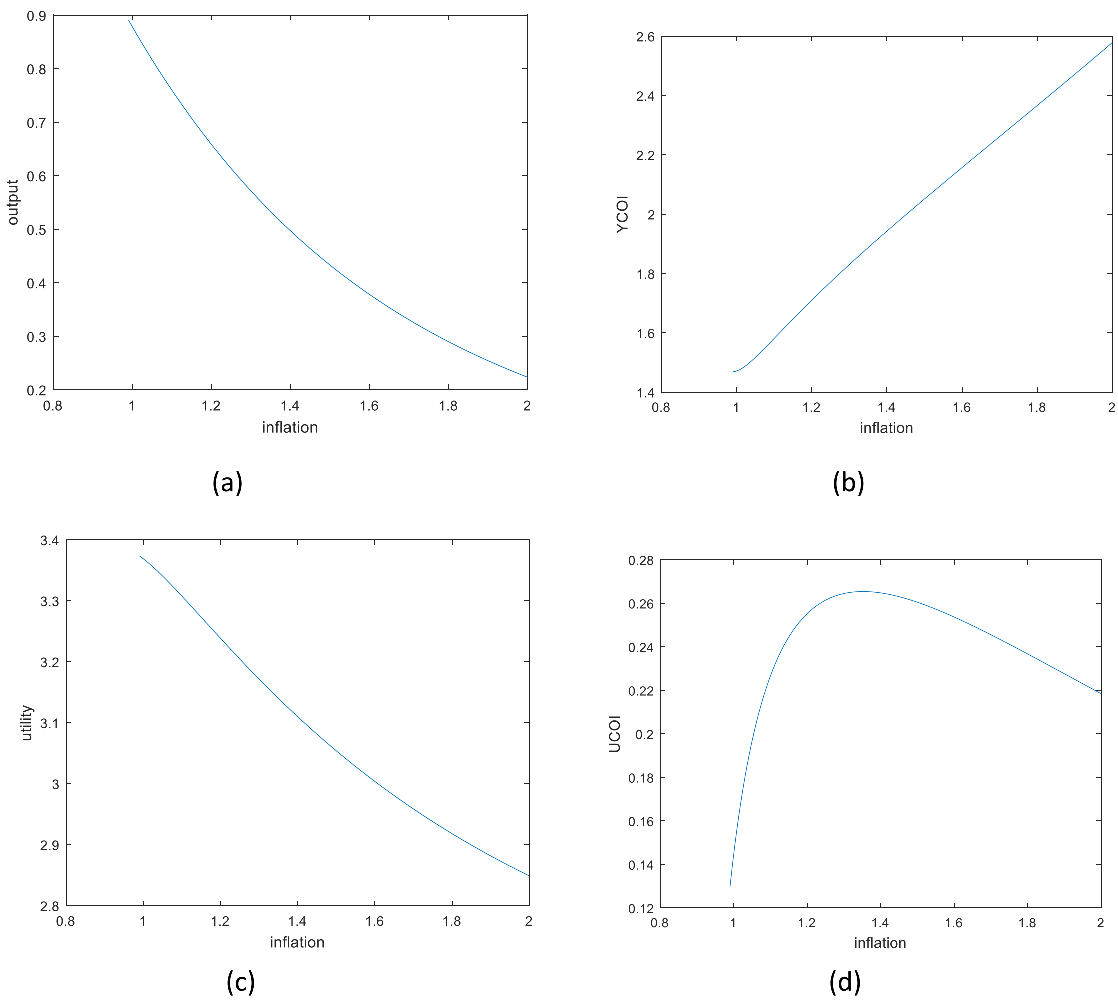


Figure 6

4. SCME with Interest Rate Rule

Since it is the interest rate, rather than the monetary aggregate, that central banks use in monetary policy implementation, it is necessary to involve the interest rate in the economy. As mentioned in the introduction section and footnote 5, the canonical approach of applying the nominal interest rate in the literature is to introduce a pseudo bond in the budget constraint, which is accompanied by an interest rate. However, it is unacceptable for the quantity of bonds to be zero in equilibrium, which is inconsistent with reality, and the monetary authority does not directly control the bond rate in the real world economy. Therefore, this traditional way of treating nominal interest rates is dubious. Here, we find a new way to introduce money market interest rate into the economy and study the SCME with the new interest rate.

4.1 Origin of Money Market Interest Rate, Missing of Liquidity Effect, and the Humps

Let the monetary authority collect the seigniorage with a money market interest rate R_t, which simultaneously means the seigniorage on issuing M_t will be collected in the next period. Similar to E3.18, we have,

$$E_t S_{t+1} = \frac{R_t M_t - M_t}{E_t M_{t+1}} E_t Y_{t+1} \qquad \text{E4.1}$$

This new way of seigniorage collecting should not change the amount of seigniorage collected as in the old way, that is,

$$E_t S_{t+1} = \frac{E_t M_{t+1} - M_t}{E_t M_{t+1}} E_t Y_{t+1} \qquad \text{E4.2}$$

Correspondingly, we have, $\frac{R_t M_t - M_t}{E_t M_{t+1}} = \frac{E_t M_{t+1} - M_t}{E_t M_{t+1}}$, from which we get,

$$R_t = \frac{E_t M_{t+1}}{M_t} \qquad \text{E4.3}$$

Strikingly, this means that the gross interest rate of money equals its expected gross growth rate.

This money market interest rate is strange at first glance. It may seem that monetary authority can arbitrarily control the interest rate through money issuance. However, this is generally not true.
The explanation is as follows: although money issuance is the everyday work of a monetary authority, money is issued according to a specific rule, that is, monetary policy, which could be regarded as a restriction on the behavior of the central bank. In addition, as shown in Section 3, there is an optimal rate of inflation, which means that keeping the inflation rate at an acceptable level is an essential requirement of the household. From the equation of exchange and E4.3, we have

$$\Pi_t^e = \frac{R_t}{\Gamma_t^e} \qquad \text{E4.4}$$

E4.4 means that the expected inflation rate, Π^e_t, which equals $E_t P_{t+1}/P_t$, is closely connected with R_t and the expected output growth rate, Γ^e_t, which equals $E_t Y_{t+1}/Y_t$. Because the economy's growth rate is generally stable, E4.4 shows that the relationship between inflation and the interest rate is close and direct. Monetary authorities have to consider this close and direct relation when issuing money. Accordingly, the money market interest rate, E4.3, is a reasonable object. However, supposing that these two conditions, that is, the monetary policy rule and an acceptable inflation rate of the household, are not heeded by the monetary authority, it is possible that the money market interest rate becomes irrational. Unfortunately, we do observe repeated periods of excessive inflation in economic history, some of which are indeed triggered by mistakes in conducting monetary policy.

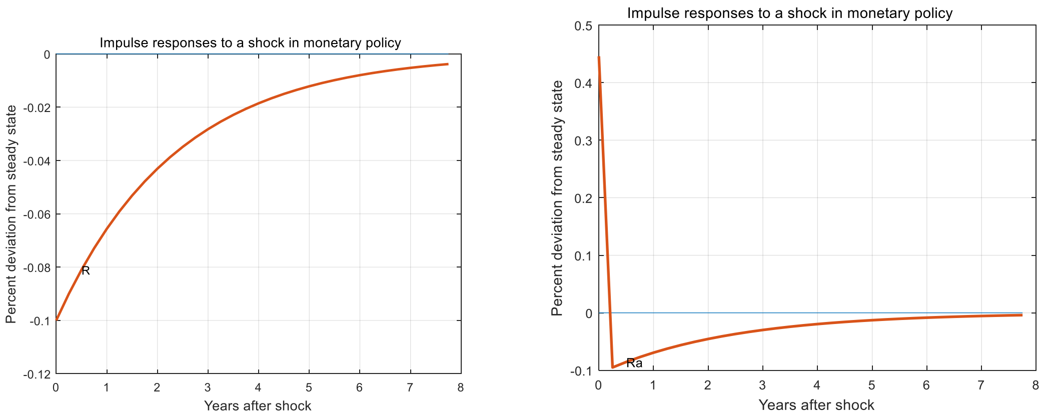
Before turning to our study of the interest rate economy, let us take a detour to explain the liquidity effect puzzle with this new money market interest rate. When E4.3 is introduced into Model 3.1, panel (a) of Figure 7 shows that a positive monetary aggregate shock will decrease the interest rate in the model economy. This is consistent with the liquidity effect, which means that an increased growth rate of money leads to a decrease in the interest rate but different from the empirical findings in Eichenbaum (1992) and Leeper and Gordon (1992), where the money market interest rate increases under a positive monetary aggregate shock. However, when the monetary transmission process is considered, the money market interest rate increases in the early stage(s) under a positive monetary shock. Specifically, let the following actual monetary aggregate, M^a of E4.5 be the quantity of money that mediates the transactions, which means **n** periods are needed to implement the monetary policy:

$$M_t^a = \frac{M_t + M_{t-1} + \cdots + M_{t-n+1}}{n} \qquad \text{E4.5}$$

Correspondingly, we can obtain the actual money market interest rate of period t, R^a_t, by the weighted average, that is, $R_t^a = \frac{R_t M_t + R_{t-1} M_{t-1} + \cdots + R_{t-n+1} M_{t-n+1}}{M_t + M_{t-1} + \cdots + M_{t-n+1}}$, which can be simplified to

$$R_t^a = \frac{E_t M_{t+1}^a}{M_t^a} \qquad \text{E4.6}$$

Panel (b) of Figure 7 shows that the actual money market interest rate of E4.6, in the case of n=2, does increase in the first period of a positive monetary aggregate shock in Model 3.1.



(a) (b)

Figure 7

In addition, this treatment of monetary policy transmission leads to a hump in the response of the nominal and real variables under a monetary shock. The cases of n=2 and n=6 with Model 3.1 are shown below in Figure 8, where the chosen variables are price, output, M^a, and R^a. Note that the movement of price and output of the case of n=1 is shown in panel (b) of Figure 3.

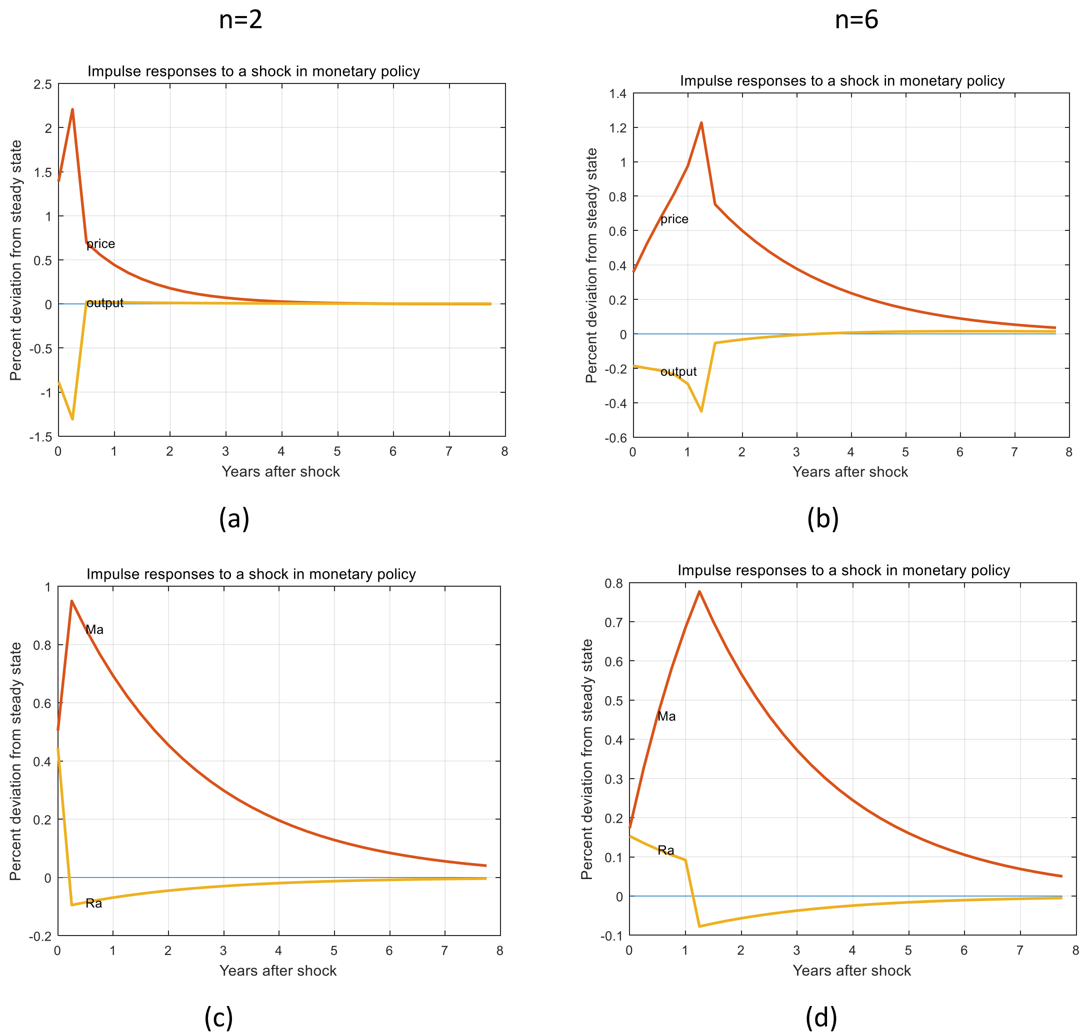


Figure 8

4.2 SCME with Exogenous Interest Rate Rule and Price Puzzle

Equipped with the money market interest rate, we can study the SCME with the interest rate rule. To obtain such an economy, we need the interest rate version of the seigniorage function and equation of exchange. Note that it is convenient to deal with inflation rather than price level in an economy with interest rate rule.

First, with E4.3, we can obtain the new expression for the seigniorage at period t as

$$S_t = \frac{R_{t-1}M_{t-1} - M_{t-1}}{M_t} Y_t \qquad \text{E4.7}$$

When $M_t = E_{t-1}M_t$, that is, the realized monetary aggregate in period t equals the expected one, which means that there is no operational error in the monetary aggregate issuing process when the interest rate is taken as the monetary policy instrument, E4.7 is the same as E3.18. It is possible to introduce a new shock, Z^{MM}, to introduce operational error into the monetary aggregate issuing process. However, we will not discuss this case in this paper. The assumption of $M_t = E_{t-1}M_t$ is retained in this paper. Consequently, we obtain the interest rate version of the seigniorage function from E4.7 as

$$S_t = (1 - \frac{1}{R_{t-1}})Y_t \qquad \text{E4.8}$$

With E4.8, we can obtain the detrended form of the budget constraint and public goods constraint of the interest rate SCME, respectively, as

$$\dot{C}_t + \Gamma\dot{K}_t = \left(\frac{1}{R_{t-1}} - \tau\right)\dot{Y}_t + (1 - \delta)\dot{K_{t-1}} \qquad \text{E4.9}$$

$$\dot{G}_t = (\tau + 1 - \frac{1}{R_{t-1}})\dot{Y}_t \qquad \text{E4.10}$$

Second, we have already obtained the interest rate version of the equation of exchange, that is, E4.4, which can be expressed as

$$R_t = \Pi_t^e \Gamma_t^e \qquad \text{E4.11}$$

In addition, we need an interest rate rule. Here, we adopt the following simple exogenous interest rate rule:

$$R_t = Z_t^M \bar{R} \qquad \text{E4.12}$$

which means that the money market interest rate is set with two elements, a constant rate $\bar{R}$ and the monetary shock Z^M. Note that the value of $\bar{R}$ is the same as Θ in Model 3.1 because, according to E4.3, these two parameters are identical.

All other things are the same as in Model 3.1, and we obtain the interest rate SCME.
The impulse response of the main variables in this interest rate rule economy following a monetary shock is shown in Figure 9 below. The utility function and parameter values are the same as those in Model 3.1. The only change is that Θ in Model 3.1 is replaced by $\bar{R}$. As shown in Figure 9, the effectiveness of the monetary shock and its persistence are obtained again. Note that, with a one-percent increase in the money market interest rate, output increases sharply from its steady state. (This result will be changed in the reactive interest rate rule case studied in the next subsection.) The mechanism is the same as that of Model 3.1, namely, the seigniorage effect. Note that as shown in Figure 9, the price puzzle, which means the actual inflation increases following a contractionary interest rate shock, appears in this interest rate rule seigniorage, which is close to the findings in Sims (1992) and Eichenbaum (1992). The price puzzle can be easily explained in SCME since we can obtain $\Pi_t = \frac{R_{t-1}}{\Gamma_t}$, which shows the intimate relation between actual inflation and interest rate. The entire system of this exogenous interest rate rule economy is provided in Appendix B.

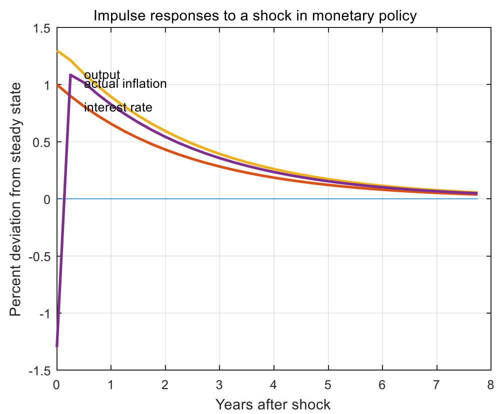
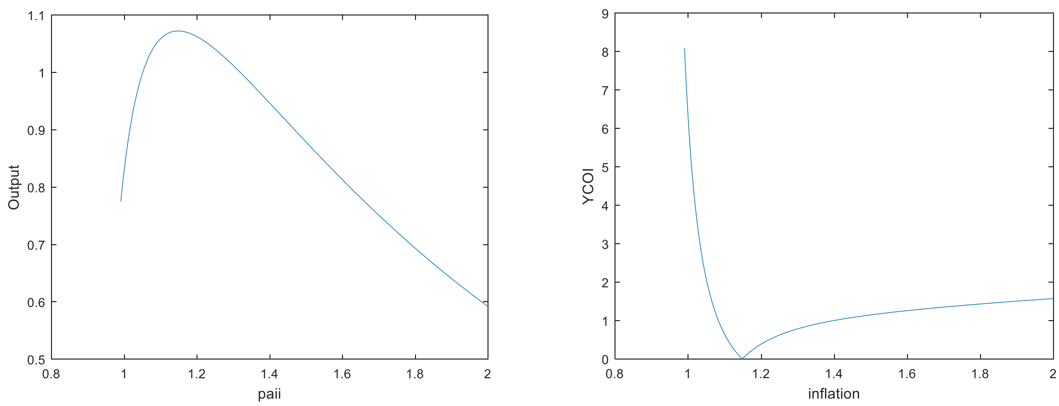


Figure 9

The steady-state YIC and YCOI of this economy, shown below in Figure 10, are obtained in the same way as in Subsection 3.5. From the equivalence between Θ and $\overline{R}$, the optimal inflation rate of Model 3.1 is consistent with the Friedman rule, that is, the nominal interest rate needs to be 0. However, panel (a) of Figure 10 shows that the optimal inflation rate of the economy in this subsection differs from the Friedman rule, which implies that the optimal inflation rate depends on the specific economic settings. Regarding the cost of inflation, in the literature, as Gillman (1995) noted, the welfare cost of inflation for the United States ranges from 0.85 percent to 3 percent of real GNP per percent increase in the nominal interest rate above zero. As shown in panel (b) of Figure 6 and panel (b) of Figure 10, the cost of inflation in Model 3.1 is consistent with the results in the literature. Nevertheless, this cost could be much higher in Model 4.2, that is, the model presented in this subsection, when the net inflation is in single digits.



(a) (b)

Figure 10

4.3 Reactive Interest Rate Rule and Choice of Monetary Policy

Since the monetary shock effectively triggers the movement of real variables, it is a natural idea to manipulate it to influence the economy, especially to counter the fluctuation in the economy. In this subsection, the reactive interest rate rule is embedded into SCME, and an approach for selecting monetary policy is developed. The negative movement of hours under a positive technology shock occurs in this flexible price economy when the reactive interest rate rule includes output as a factor. The simulation and relative results of a 3-period monetary policy transmission SCME are provided in detail, which reproduce many of the empirical findings in the literature and show SCME is promising in replicating the real world economy.

A. Choice of Reactive Interest Rate Rule

Assume, instead of E4.12, that monetary policy is reactive as follows:

$$R_t = Z_t^M \bar{R}\, \Pi^{\frac{\Pi_t^e}{\Pi}-1} \qquad \text{E4.13}$$

where Π is the steady-state value of inflation, Π $(\cdot)$ is a monotone function that responds to the expected inflation gap, and Π is a parameter. E4.13 means that the money market interest rate will be adjusted according to the expected inflation gap. Note that the monetary authority, in this new circumstance, still runs according to the rule, namely, it is still not a utility optimizer.

Compared with Model 4.2, there is no change in the transaction equations, the firm's behavior, or the exchange equation in this new economy. However, the behavior of the household needs some modification because E4.13 means that the household can impact the money market interest rate through Π^e_t. Correspondently, the endogenous monetary policy can be treated as a new constraint in the household's decision.

Equipped with the reactive monetary policy, it is time to concretely implement the SCME. The main problem encountered here is how to decide the value of the new parameter Π, that is, the choice of the policy rule. Note that Π is the only parameter in the reactive policy economy that is different from that in Model 4.2. When applying the Taylor rule (1998), that is, when setting $\ln \Pi = 1.5$, the simulated economy fluctuates violently, with the standard deviation of output being more than 4%, and some of the statistical relationships disappear. For example, the money market interest rate decreases in response to a positive monetary shock, and there is no price puzzle. Therefore, we need to find a way to locate the value of Π in E4.13, or $\ln \Pi$ when we consider the log-linear form.

Here, we begin with the following two principles in locating the value of $\ln \Pi$:

Principle 1: Model 4.2 should be the benchmark for the reactive interest rate economy with rule E4.13, briefly Model 4.3A.

Principle 2: Model 4.3A should be close to the benchmark Model 4.2. In particular, the steady-state values of Model 4.3A should be the same or close to those of Model 4.2.

When comparing the difference between the two economies carefully, especially the steady state values, we find that the difference between them is not unmanageable, and the essential differences lie mainly in the following three aspects:

Difference 1: The policy rule, in log-linear form

Model 4.2 $$\widehat{R_t} = \widehat{Z_t^M} \quad \text{E4.14}$$

Model 4.3A $$\widehat{R_t} = \widehat{Z_t^M} + \ln \textcircled{\Pi}\ \widehat{\Pi_t^e} \quad \text{E4.15}$$

Difference 2: The steady-state Y/K ratio[11]

$$\frac{Y}{K}^{Model4.2} = \frac{\textcircled{1}+\textcircled{3}+\textcircled{6}}{\textcircled{2}-(\textcircled{4}-\textcircled{5})} \quad \text{E4.16}$$

$$\frac{Y}{K}^{Model4.3A} = \frac{\textcircled{1}+\textcircled{3}+\textcircled{6}\,\Phi}{\textcircled{2}-(\textcircled{4}-\textcircled{5})\,\Phi} \quad \text{E4.17}$$

where $\Phi = \dfrac{\ln \textcircled{\Pi}}{\ln \textcircled{\Pi} - 1}$

Difference 3: The steady-state hours[12]

$$N^{Model4.2} = \frac{\left(\frac{(\textcircled{7}+\textcircled{8})\textcircled{9}}{\xi}\right)^{\frac{1}{\eta}}}{1+\left(\frac{(\textcircled{7}+\textcircled{8})\textcircled{9}}{\xi}\right)^{\frac{1}{\eta}}} \quad \text{E4.18}$$

$$N^{Model4.3A} = \frac{\left(\frac{(\textcircled{7}+\textcircled{8}\,\Phi)\textcircled{9}}{\xi}\right)^{\frac{1}{\eta}}}{1+\left(\frac{(\textcircled{7}+\textcircled{8}\,\Phi)\textcircled{9}}{\xi}\right)^{\frac{1}{\eta}}} \quad \text{E4.19}$$

Difference 1 implies that the value of $\ln \textcircled{\Pi}$ needs be small to make the reactive policy rule close to the benchmark rule.

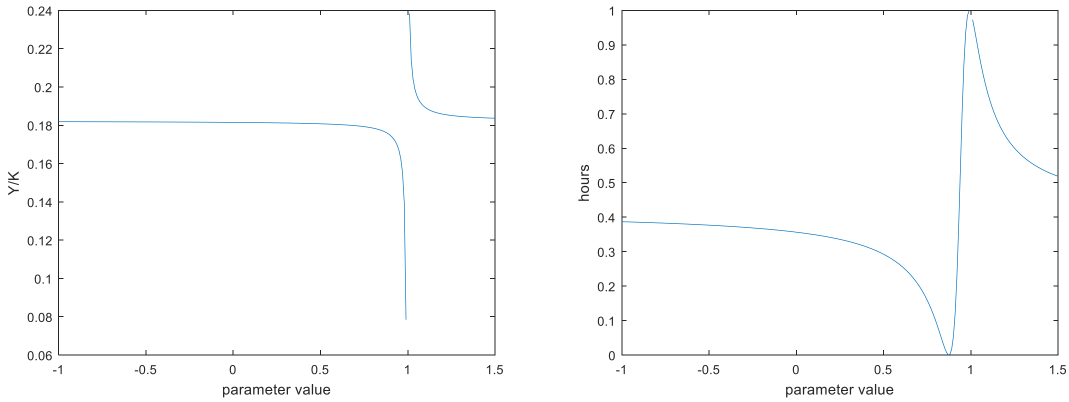
11 $\textcircled{1} = \chi(1-\frac{\beta}{\Gamma}(1-\delta))$; $\textcircled{2} = \frac{\beta}{\Gamma}(\frac{1}{R}-\tau)\alpha$; $\textcircled{3} = \frac{\beta}{\Gamma}(1-\chi)(\Gamma-(1-\delta))\alpha$;

$\textcircled{4} = (1-\beta)\beta\chi\frac{\alpha}{R\Gamma}$; $\textcircled{5} = (1-\beta)\beta(1-\chi)\frac{\frac{1}{R}-\tau}{\tau+1-\frac{1}{R}}\frac{\alpha}{R\Gamma}$; $\textcircled{6} = (1-\beta)\beta(1-\chi)\frac{\Gamma-(1-\delta)}{\tau+1-\frac{1}{R}}\frac{\alpha}{R\Gamma}$

12 $\textcircled{7} = \left(\frac{1}{R}-\tau\right)\chi + (\tau+1-\frac{1}{R})(1-\chi)\frac{\left(\frac{1}{R}-\tau\right)\frac{Y}{K}-(\Gamma-(1-\delta))}{(\tau+1-\frac{1}{R})\frac{Y}{K}}$; $\textcircled{8} = \frac{\beta}{\Gamma}(\chi-(1-\chi)\frac{\left(\frac{1}{R}-\tau\right)\frac{Y}{K}-(\Gamma-(1-\delta))}{(\tau+1-\frac{1}{R})\frac{Y}{K}})$;

$\textcircled{9} = \frac{\textcircled{10}}{\left(\frac{1}{R}-\tau\right)-\frac{(\Gamma-(1-\delta))}{Y/K}}\left(\frac{1}{Y/K}^{\frac{\alpha}{1-\alpha}}\right)^{1-\eta}(1-\alpha)$; $\textcircled{10} = \left(\left(\left(\frac{1}{R}-\tau\right)-\frac{(\Gamma-(1-\delta))}{Y/K}\right)^{\chi}\left(\tau+1-\frac{1}{R}\right)^{1-\chi}\right)^{(1-\eta)}$

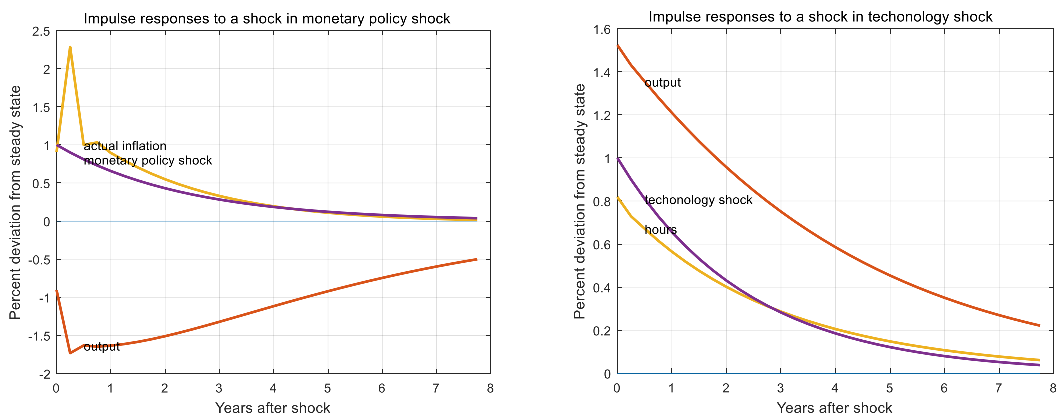
From Difference 2 we have the following: a. $\frac{Y}{K}^{Model4.3A}$ is much closer to $\frac{Y}{K}^{Model4.2}$, and b. The value of $\frac{Y}{K}^{Model4.2}$ is almost stable when the value of $\ln \Pi$ is small, which is shown in panel (a) of Figure 11. Note that the value of the same parameter in Model 4.3A is set to be equal to that in Model 4.2 in the comparison.
E4.19 is depicted in Panel (b) of Figure 11, which shows that the steady-state hours of Model 4.3A is sensitive to the value of $\ln \Pi$, and the value should be small to make the steady-state value of hours of Model 4.3A close to 1/3, the steady-state value in Model 4.2.



(a) (b)

Figure 11

With the above analysis and experiments on Model 4.3A, we can conclude that the value of $\ln \Pi$ should be small under the settings of the model. Figure 12 provides some impulse-response curves of Model 4.3A with $\ln \Pi = 0.2$.



(a) (b)

Figure 12

B. A Simulated Reactive Interest Rate Rule SCME

Based on the study above, we provide in detail the simulation and relative results of a SCME with the following reactive interest rate policy:

$$R_t = Z_t^M \bar{R}\, \Pi^{\frac{\Pi_t^e}{\Pi}-1}\, Y^{\frac{Y_t}{A_t Y}-1} \qquad \text{E4.20}$$

E4.20 means that the money market interest rate responds to both the expected inflation gap and output gap, where Π and Y are the respective parameters. The log-linear form of E4.20 is

$$\widehat{R_t} = \widehat{Z_t^M} + \ln \Pi\, \widehat{\Pi_t^e} + \ln Y\, \widehat{Y_t} \qquad \text{E4.21}$$

In particular, the rule used in the simulation below is

$$\widehat{R_t} = \widehat{Z_t^M} + 0.2\widehat{\Pi_t^e} + 0.2\widehat{Y_t} \qquad \text{E4.22}$$

that is, $\ln \Pi = 0.2$ and $\ln Y = 0.2$. The value of $\ln Y$ can be obtained in a similar way as we study the value of $\ln \Pi$.

In addition, to make the model economy closer to the real-world economy, we consider the case of 3-period monetary policy transmission and from E4.5, E4.6, and E4.3, we have:

$$M_t^a = \frac{M_t + M_{t-1} + M_{t-2}}{3} \qquad \text{E4.23}$$

$$R_t^a = \frac{R_{t-2}(R_t R_{t-1} + R_{t-1} + 1)}{R_{t-1} R_{t-2} + R_{t-2} + 1} \qquad \text{E4.24}$$

Note that, in this case, R^a in E4.24 is the interest rate entering into the seigniorage function E4.8, and the interest rate version of the exchange equation becomes

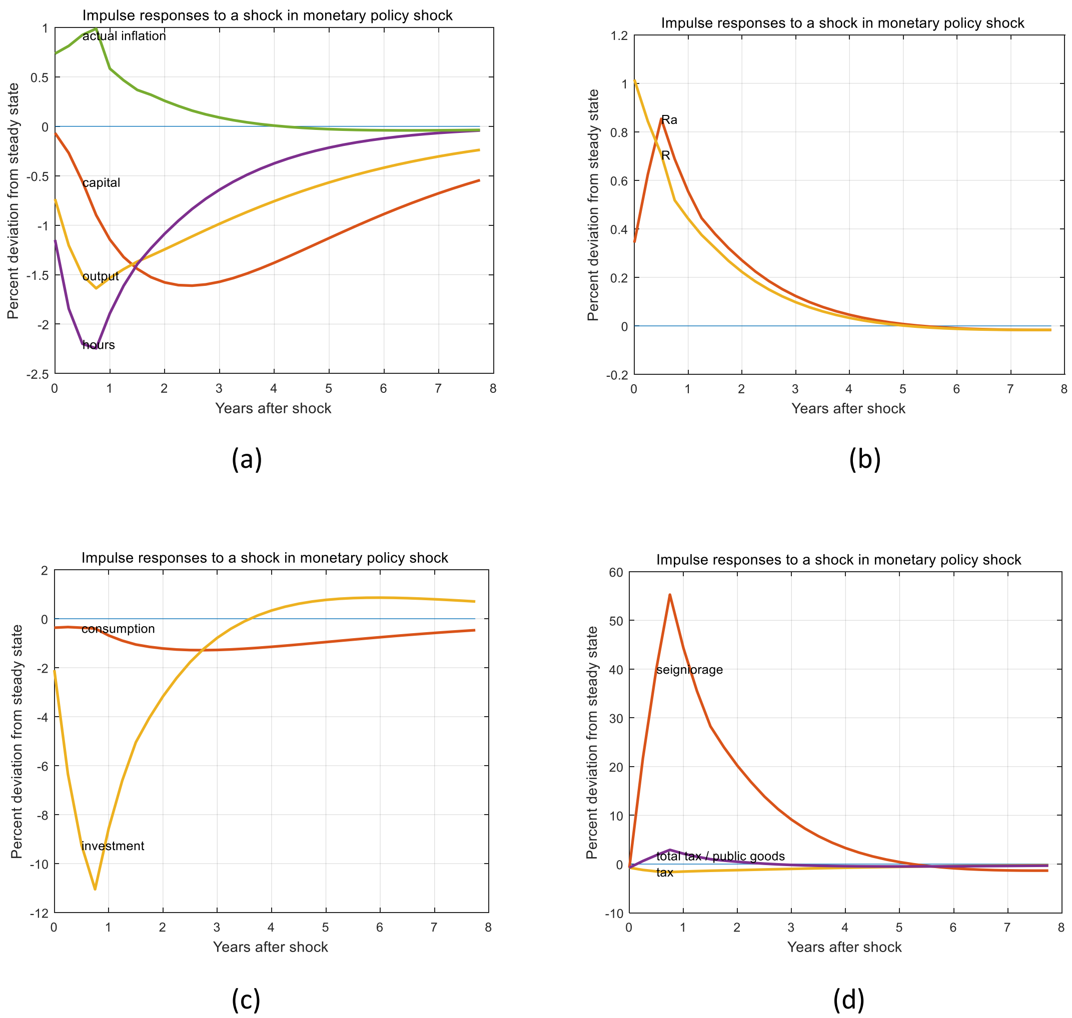
$$R_t^a = \Pi_t^e \Gamma_t^e \qquad \text{E4.25}$$

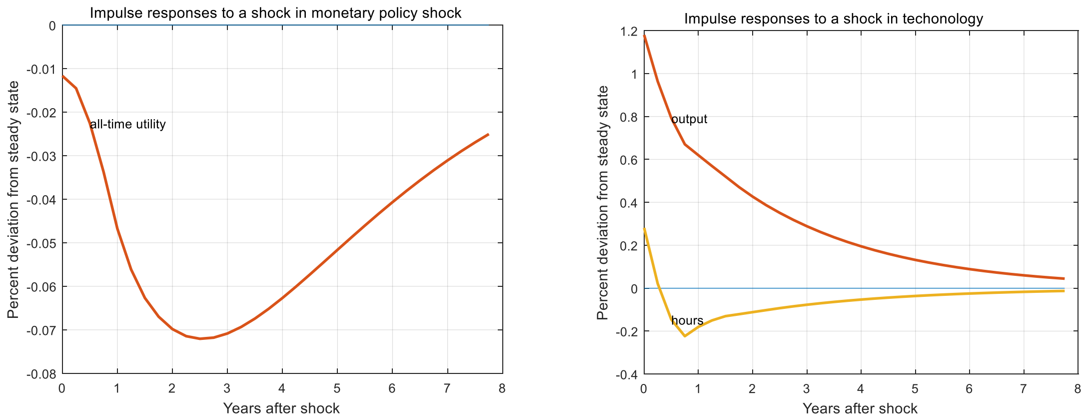
The simulated economy is called Model 4.3B, and the entire system is provided in Appendix C. With the commonly used parameter values of this paper, the steady-state values of C/Y, X/Y, T/Y, S/Y, and Y/K of this model economy are 0.6359, 0.1793, 0.17, 0.0148, and 0.1813, respectively , and we have R=1.015, Γ=1.0075, Π=R/Γ=1.0074, N=1/3.

Model 4.3B is compared with the empirical results in the literature. Ramey (2016) summarized some of the main results from the literature on the impact of a monetary shock on output, which spans from -0.6% to -5% under a 100 basis point fund rate peak. The timing of the trough spans from 8 months to 8 quarters. In addition, the majority of the studies reported a 4%-10% 1 year - 5 years ahead forecast error variance of output explained by the monetary shock. Romer and Romer (2004) and Coibion (2012) are exceptions who reported major and moderate parts of the variance coming from the monetary shock, respectively.

Figure 13 provides the impulse-response curves of the variables in Model 4.3B under a one-percent positive interest rate shock (except panel (f), which is the technology shock case), where monetary effectiveness and its persistence, price puzzle, and the humps are all obtained. Panel (a) shows that output decreases by approximately 1.6 percent in a trough in this model economy when the peak of

R^a is approximately plus 0.85 percent, as shown in panel (b). With the 3-period implementation process, the timing of the trough is at about the third quarter, which could be adjusted if we change the transmission periods. Panel (b) shows that the actual money market interest rate, R^a, increases in early periods, which imitates the step-by-step raising of the funds rate by the Board of the Federal Reserve in a contractionary interest rate policy operation. As shown in panel (e), the permanent utility of the household decreased slightly. Compared with Panel (b) of Figure 12, an interesting aspect of this economy is that hours decrease, as Panel (f) shows, under a positive technology shock when output gap is included in the reactive interest rate rule, which is consistent to the empirical studies of Gali (1999) and Basu, Fernald and Kimball (2006), among others.

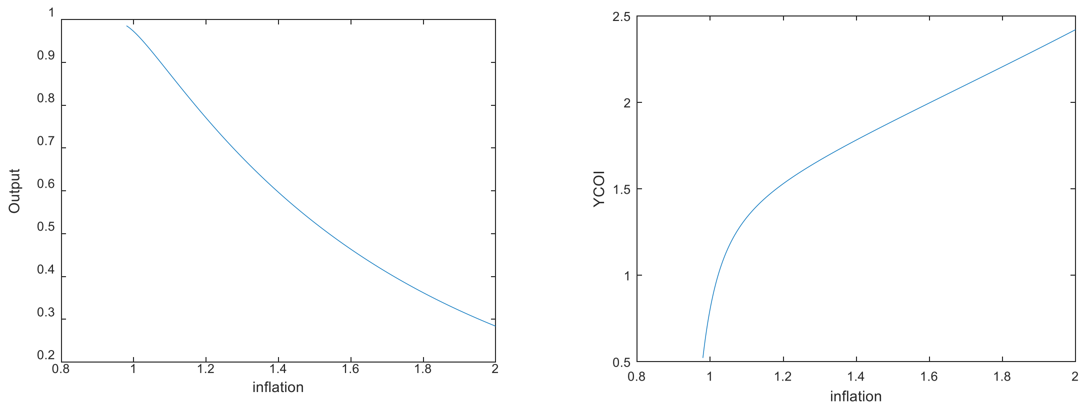




(e) (f)

Figure 13

The YIC and YCOI of Model 4.3B are provided in panels (a) and (b) of Figure 14. The optimal rate of inflation is consistent with the Friedman rule here.



(a) (b)

Figure 14

Table 1 provides the standard deviation and cross-correlation of the main variables in this economy. The standard output variation is 1.69%.

Regarding variance decomposition, 76% of the 8-quarter-ahead forecast error variance for output is explained by the monetary shock in this SCME, which is reported in Table 2.

Table 1: Standard Deviation and Cross-correlation of Model4.3B

	SD%	cross-correlation of output with:										
		-5	-4	-3	-2	-1	0	1	2	3	4	5
Ra	0.76	0.26	0.18	0.04	-0.17	-0.40	-0.59	-0.64	-0.56	-0.43	-0.29	-0.19
output	1.69	0.00	0.15	0.35	0.58	0.81	1	0.81	0.58	0.35	0.15	0.00
actual inflation	1.35	0.29	0.27	0.19	0.06	-0.09	-0.60	-0.61	-0.53	-0.41	-0.29	-0.20
consumption	0.93	0.65	0.72	0.72	0.67	0.59	0.52	0.29	0.09	-0.08	-0.23	-0.33
investment	10.45	-0.28	-0.14	0.10	0.38	0.66	0.85	0.75	0.58	0.41	0.27	0.15
capital	1.15	0.62	0.72	0.79	0.78	0.69	0.52	0.28	0.06	-0.11	-0.24	-0.33
hours	2.10	-0.21	-0.08	0.10	0.34	0.59	0.80	0.77	0.64	0.46	0.29	0.15
seigniorage	49.50	0.18	0.05	-0.16	-0.39	-0.57	-0.62	-0.55	-0.42	-0.29	-0.18	-0.10
tax	1.69	0.00	0.15	0.35	0.58	0.81	1	0.81	0.58	0.35	0.15	0.00
total tax	3.24	0.22	0.13	-0.03	-0.20	-0.31	-0.27	-0.28	-0.24	-0.19	-0.15	-0.12
public goods	3.24	0.22	0.13	-0.03	-0.20	-0.31	-0.27	-0.28	-0.24	-0.19	-0.15	-0.12

Table 2: Percentage Variance Due to Technology Shock and Monetary Policy Shock

	1 quarter ahead		4 quarters ahead		8 quarters ahead		20 quarters ahead	
	tech-shock	mp-shock	tech-shock	mp-shock	tech-shock	mp-shock	tech-shock	mp-shock
Ra	0.0800	0.9200	0.0905	0.9095	0.1044	0.8956	0.1231	0.8769
output	0.7204	0.2796	0.3288	0.6712	0.2351	0.7649	0.1790	0.8210
actual inflation	0.7204	0.2796	0.3693	0.6307	0.3445	0.6555	0.3548	0.6452
consumption	0.4516	0.5484	0.5862	0.4138	0.2541	0.7459	0.0895	0.9105
investment	0.8084	0.1916	0.0910	0.9090	0.0579	0.9421	0.0553	0.9447
capital	0.8084	0.1916	0.1225	0.8775	0.0360	0.9640	0.0156	0.9844
hours	0.0566	0.9434	0.0102	0.9898	0.0097	0.9903	0.0103	0.9897
seigniorage	0.7204	0.2796	0.1017	0.8983	0.1178	0.8822	0.1428	0.8572
tax	0.7204	0.2796	0.3288	0.6712	0.2351	0.7649	0.1790	0.8210
total tax	0.7204	0.2796	0.4696	0.5304	0.5014	0.4986	0.5581	0.4419
public goods	0.7204	0.2796	0.4696	0.5304	0.5014	0.4986	0.5581	0.4419

5. Conclusion

SCME, which is based on RBC model, is a monetary and macroeconomic platform that monetary shocks can effectively impact real variables in the flexible price condition, and the effectiveness is persistent and hump-shaped. With SCME, we obtain many new understandings of the operations of the real-world economy in this paper, which include:

(i) The mechanism for the effectiveness and persistence is the resource reallocation triggered by the variation of seigniorage, which, in turn, is triggered by the monetary shock.

(ii) The humps in the impulse response of the real and nominal variables are caused by the monetary transmission process, which also causes the absence of the liquidity effect.

(iii) In the steady state, the quantity of monetary aggregate is neutral, but the growth rate of money and inflation are nonneutral to the real economy. The YIC and YCOI are derived. The optimal rate of inflation depends on the setting of the economy, which deserves further study.

(iv) The price puzzle, that is, the increase in price levels under contractive interest rate policy shock, emerges in the SCME.

(v) A quantitative method for the choice of reactive monetary policy is developed. The decrease in hours under positive technology shocks is found when the interest rate rule is is reactive to the output gap.

(vi) The pricing is interactive in SCME.

(vii) Money and taxes are not sources of distortion for the economy, and the unique equilibrium of the SCME is Pareto optimal.

The following innovations are the pillars of the findings in this paper:

(1) Differentiating between taxes and public goods, that is, E3.1-E3.2. This modifies the utility and budget of the economy and makes the unique equilibrium of the economy with taxation a Pareto optimum. In addition, it provides the basis for studying seigniorage in the economy.

(2) The budget constraint with seigniorage channel, that is, E3.21. This provides a position for money in the economic system. Money and monetary policy can now substantially contribute to the economic rebalancing process under flexible prices.

(3) The integration of the transaction side into the economy, that is, E3.7-E3.11. In addition to helping handle the complex money and tax affairs in the economy, which clarify the operations of the complicated model economy, the inclusion of the transaction side makes the model economy a whole, which includes all the processes as those in the real-world economy. Furthermore, the transaction equations help obtain the equation of exchange and establish the interactive pricing mechanism.

(4) The way money market interest rate is defined, that is, E4.3. This makes it possible to study the interest rate rule economy when there are no bonds, and it retains the close relation between the monetary aggregate and interest rate rule economy.

In the meantime, we find that some treatments in existing dynamic general equilibrium monetary theory are dubious, including the government lump sum transfer mechanism, the cash-in-advance treatment, the money-in-utility treatment, the equality between money market interest rate and the interest rate of bond, the absence of the transaction side of the economy, the single-side product pricing, and the view that an economy with taxes and/or money is non-Pareto optimal.

Undoubtedly, there are some matters needing attention with the model economy of this paper. The main purpose of this paper is to put forward the SCME platform, so, many important aspects of the real world economy are simplified. The treatment of the monetary system is one of these issues. The whole monetary system is treated as a part of the government and the seigniorage is collected according to the monetary aggregate in this paper. It's necessary to implement the money issuance mechanism described in Figure 1 in future study to make the model economy close to the real world, which means a utility-maximizing commercial bank will be involved. In addition, many issues, such as the velocity of money, fiscal policy, monetary policy choice, the Friedman rule, and the Taylor rule are needed to be scrutinized in-depth. Furthermore, to replicate the real-world economy, the following two works are necessary: (a) Embedding credit and asset price into SCME. (b) Incorporating foreign exchange and international trade to make SCME open.

Appendix[13]

A. Detrended Form of Model 3.1

$$UU_t = max \sum_{i=0}^{\infty} \beta^i \left(\frac{\left(E_t C_{t+i}{}^{\chi} E_t G_{t+i}{}^{1-\chi}\right)^{1-\eta}}{1-\eta} + \xi \frac{(1-E_t N_{t+i})^{1-\eta N}}{1-\eta N} \right) \qquad \text{A.A.1}$$

$$Y_t = Z_t^T K_{t-1}^{\alpha} N_t{}^{1-\alpha} \qquad \text{A.A.2}$$

[13] All variables are in detrended form in the Appendix.

$$\mathrm{M_t} = Z_t^M \bar{M} \quad \text{A.A.3}$$
$$W_t^K K_{t-1} = M_t^K \quad \text{A.A.4}$$
$$W_t^N N_t = M_t^N \quad \text{A.A.5}$$
$$M_t^K + M_t^N = M_t \quad \text{A.A.6}$$
$$M_t^K = P_t Y_t^K \quad \text{A.A.7}$$
$$M_t^N = P_t Y_t^N \quad \text{A.A.8}$$
$$U_t^F = max(Y_t - Y_t^K - Y_t^N) \quad \text{A.A.9}$$
$$\alpha P_t Y_t = W_t^K K_{t-1} \quad \text{A.A.10}$$
$$(1-\alpha) P_t Y_t = W_t^N N_t \quad \text{A.A.11}$$
$$M_t = P_t Y_t \quad \text{A.A.12}$$
$$S_t = (1 - \frac{M_{t-1}}{\Theta M_t}) Y_t \quad \text{A.A.13}$$
$$T_t = \tau Y_t \quad \text{A.A.14}$$
$$TT_t = T_t + S_t \quad \text{A.A.15}$$
$$C_t + X_t + TT_t = Y_t \quad \text{A.A.16}$$
$$\Gamma K_t = (1-\delta) K_{t-1} + X_t \quad \text{A.A.17}$$
$$G_t = TT_t \quad \text{A.A.18}$$
$$\Lambda_t^C = \chi \frac{(C_t^{\chi} G_t^{1-\chi})^{1-\eta}}{C_t} \quad \text{A.A.19}$$
$$\Lambda_t^G = (1-\chi) \frac{(C_t^{\chi} G_t^{1-\chi})^{1-\eta}}{G_t} \quad \text{A.A.20}$$
$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{M_t}{\Theta E_t M_{t+1}} - \tau \right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1-\delta) \right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{M_t}{\Theta E_t M_{t+1}} \right) \alpha \frac{E_t Y_{t+1}}{K_t} -$$
$$\frac{\beta}{\Gamma} E_t \Lambda_{t+1}^{\mathrm{E}} E_t P_{t+1} \alpha \frac{E_t Y_{t+1}}{K_t} = 0 \quad \text{A.A.21}$$
$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{M_{t-1}}{\Theta M_t} - \tau \right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{M_{t-1}}{\Theta M_t} \right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^{\mathrm{E}} P_t (1-\alpha) \frac{Y_t}{N_t}$$
$$\text{A.A.22}$$
$$\Lambda_t^{\mathrm{E}} + \Lambda_t^C \frac{M_{t-1}}{\Theta M_t^2} Y_t - \beta E_t \Lambda_{t+1}^C \frac{1}{\Theta E_t M_{t+1}} E_t Y_{t+1} - \Lambda_t^G \frac{M_{t-1}}{\Theta M_t^2} Y_t + \beta E_t \Lambda_{t+1}^G \frac{1}{\Theta E_t M_{t+1}} E_t Y_{t+1} = 0 \quad \text{A.A.23}$$
$$\ln Z_t^M = (1-\rho^M) \ln Z^M + \rho^M \ln Z_{t-1}^M \quad \text{A.A.24}$$
$$\ln Z_t^T = (1-\rho^T) \ln Z^T + \rho^T \ln Z_{t-1}^T \quad \text{A.A.25}$$

Λ^C, Λ^G, and Λ^E are the Lagrange multipliers of the budget constraint, A.A.16, the public goods constraint, A.A.18, and the equation of exchange constraint, A.A.12, respectively. A.A.19-A.A.23 are first orders on C_t, G_t, K_t, N_t, and M_t, respectively.

B. Detrended Form of Model 4.2

A.A.1 A.B.1

A.A.2 A.B.2

$$\mathrm{R_t} = Z_t^M \bar{R} \quad \text{A.B.3}$$

A.A.4 A.B.4

A.A.5 A.B.5

A.A.6 A.B.6

A.A.7 A.B.7

A.A.8 A.B.8

A.A.9 A.B.9

A.A.10 A.B.10

A.A.11 A.B.11

A.A.12 A.B.12

$$R_t = \frac{E_t M_{t+1}}{M_t} \quad \text{A.B.13}$$

$$R_t = \Pi_t^e \Gamma_t^e \quad \text{A.B.14}$$

$$S_t = (1 - \frac{1}{R_{t-1}}) Y_t \quad \text{A.B.15}$$

A.A.14 A.B.16

A.A.15 A.B.17

A.A.16 A.B.18

A.A.17 A.B.19

A.A.18 A.B.20

A.A.19 A.B.21

A.A.20 A.B.22

$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{1}{R_t} - \tau \right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1-\delta) \right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{1}{R_t} \right) \alpha \frac{E_t Y_{t+1}}{K_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} +$$

$$\beta E_t \Lambda_{t+1}^{\mathrm{E}} E_t \Pi_{t+1}^e \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} = 0 \quad \text{A.B.23}$$

$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{1}{R_{t-1}} - \tau \right) (1-\alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{1}{R_{t-1}} \right) (1-\alpha) \frac{Y_t}{N_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \Gamma \frac{E_t Y_{t+1}}{Y_t} \frac{1-\alpha}{N_t}$$

A.B.24

$$\Lambda_t^{\mathrm{E}} + \beta E_t \Lambda_{t+1}^C \frac{E_t Y_{t+1}}{R_t^2} - \beta E_t \Lambda_{t+1}^G \frac{E_t Y_{t+1}}{R_t^2} = 0 \quad \text{A.B.25}$$

A.A.24 A.B.26

A.A.25 A.B.27

Λ^{E} is the Lagrange multiplier of the constraint on the interest rate form of the equation of exchange, A.B.14. A.B.25 is the first order on R_t.

C. Detrended Form of Model 4.3B

A.A.1 A.C.1

A.A.2 A.C.2

$$R_t = Z_t^M \bar{R} \Phi^{\frac{\Pi_t^e}{\Pi} - 1} \Phi^{\frac{Y_t}{\mathrm{Y}} - 1} \quad \text{A.C.3}$$

A.A.4 A.C.4

A.A.5 A.C.5

A.A.6 A.C.6

A.A.7 A.C.7

A.A.8 A.C.8

A.A.9 A.C.9

A.A.10 A.C.10

A.A.11 A.C.11

A.A.12 A.C.12

A.B.13 A.C.13

$$M_t^a = \frac{M_t + M_{t-1} + M_{t-2}}{3} \qquad \text{A.C.14}$$

$$R_t^a = \frac{R_{t-2}(R_t R_{t-1} + R_{t-1} + 1)}{R_{t-1} R_{t-2} + R_{t-2} + 1} \qquad \text{A.C.15}$$

$$R_t^a = \Pi_t^e \Gamma_t^e \qquad \text{A.C.16}$$

$$S_t = (1 - \frac{1}{R_{t-1}^a}) Y_t \qquad \text{A.C.17}$$

A.A.14 A.C.18

A.A.15 A.C.19

A.A.16 A.C.20

A.A.17 A.C.21

A.A.18 A.C.22

A.A.19 A.C.23

A.A.20 A.C.24

$$\Lambda_t^C - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^C \left(\left(\frac{1}{R_t^a} - \tau \right) \alpha \frac{E_t Y_{t+1}}{K_t} + (1 - \delta) \right) - \frac{\beta}{\Gamma} E_t \Lambda_{t+1}^G \left(\tau + 1 - \frac{1}{R_t^a} \right) \alpha \frac{E_t Y_{t+1}}{K_t} -$$

$$\Lambda_t^{\mathrm{E}} \Pi_t^e \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} + \beta E_t \Lambda_{t+1}^{\mathrm{E}} E_t \Pi_{t+1}^e \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} - \frac{1}{\Gamma} \Lambda_t^{\mathrm{P}} R_t \ln \varpi \frac{E_t Y_{t+1}}{Y_t} \frac{\alpha}{K_t} +$$

$$\frac{\beta}{\Gamma} E_t \Lambda_{t+1}^{\mathrm{P}} E_t R_{t+1} \ln \varpi \frac{E_t Y_{t+2}}{E_t Y_{t+1}} \frac{\alpha}{K_t} = 0 \qquad \text{A.C.25}$$

$$\frac{\xi}{(1-N_t)^{\eta N}} = \Lambda_t^C \left(\frac{1}{R_{t-1}^a} - \tau \right) (1 - \alpha) \frac{Y_t}{N_t} + \Lambda_t^G \left(\tau + 1 - \frac{1}{R_{t-1}^a} \right) (1 - \alpha) \frac{Y_t}{N_t} - \Lambda_t^{\mathrm{E}} \Pi_t^e \Gamma \frac{E_t Y_{t+1}}{Y_t} \frac{1-\alpha}{N_t} -$$

$$\Lambda_t^{\mathrm{P}} R_t \ln \varpi \frac{E_t Y_{t+1}}{Y_t} \frac{(1-\alpha)}{N_t} \qquad \text{A.C.26}$$

$$\Lambda_t^{\mathrm{E}} \widetilde{R_t^a} + \beta E_t \Lambda_{t+1}^{\mathrm{E}} \widetilde{E_t R_{t+1}^a} + \beta^2 E_t \Lambda_{t+2}^{\mathrm{E}} \widetilde{E_t R_{t+2}^a} + \Lambda_t^{\mathrm{P}} + \beta E_t \Lambda_{t+1}^C \frac{\widetilde{R_t^a}}{(R_t^a)^2} E_t Y_{t+1} + \beta^2 E_t \Lambda_{t+2}^C \frac{\widetilde{E_t R_{t+1}^a}}{(E_t R_{t+1}^a)^2} E_t Y_{t+2} +$$

$$\beta^3 E_t \Lambda_{t+3}^C \frac{\widetilde{E_t R_{t+2}^a}}{(E_t R_{t+2}^a)^2} E_t Y_{t+3} - \beta E_t \Lambda_{t+1}^G \frac{\widetilde{R_t^a}}{(R_t^a)^2} E_t Y_{t+1} - \beta^2 E_t \Lambda_{t+2}^G \frac{\widetilde{E_t R_{t+1}^a}}{(E_t R_{t+1}^a)^2} E_t Y_{t+2} -$$

$$\beta^3 E_t \Lambda_{t+3}^G \frac{\widetilde{E_t R_{t+2}^a}}{(E_t R_{t+2}^a)^2} E_t Y_{t+3} = 0 \qquad \text{A.C.27}$$

$$\Lambda_t^{\mathrm{E}} \frac{\Gamma E_t Y_{t+1}}{Y_t} + \Lambda_t^{\mathrm{P}} R_t \ln \varpi \frac{1}{\Pi} = 0 \qquad \text{A.C.28}$$

A.A.24 A.C.29

A.A.25 A.C.30

Λ^{P} is the Lagrange multiplier of the monetary policy constraint, A.C.3. A.C.28 is first order on Π^e_t. A term with a ~ symbol above it stands for its derivative on R_t.

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