On a Regime Switching Illiquid High Volatile Prediction Model for Cryptocurrencies

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September 2022

On a regime switching illiquid high volatile prediction model for cryptocurrencies

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Abstract

Cryptocurrencies are increasingly utilized by investors and financial institutions worldwide. The current paper proposes a prediction model for a cryptocurrency that encompasses three properties observed in the markets for cryptocurrencies—namely high volatility, illiquidity, and regime shifts. By using Ito calculus, we provide a solution for the suggested stochastic differential equation (SDE) along with a proof. Moreover, numerical simulations are performed and are compared to the real data, which seems to capture the dynamics of the price path of a cryptocurrency in the real markets.

Keywords: Stochastic Modeling, cryptocurrencies, illiquid, high volatility, regime switching, CTMC.

1 Introduction

With the fast development of digital finance over the past decade and with the introduction of blockchain technology, there has been an immense expansion in the trading of cryptocurrencies. According to [38] there exists around 10000 cryptocurrencies and the most prominent one with the largest market value is bitcoin, which was initiated in the report [33] written by an unknown author under the pseudonym [33]. Researchers are not in total agreement regarding the usefulness of cryptocurrencies. For example, [38] doubts that cryptocurrencies qualify for even being called currencies. In contrast, [25] believes in cryptocurrencies as the newcomers that will

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reshape the entire financial system, which needs to be reshaped according to him. As of the first week of June 2022, the market capitalization of cryptocurrencies is estimated to exceed 900 billion dollars at [6]. Modeling cryptocurrency prices is then an important issue. Reference [11] provides an interesting presentation of the economics of cryptocurrencies. On the other hand, [5] shows that the market for cryptocurrencies is vulnerable to bubbles and crises. References [9] and [10] demonstrate empirically that a cryptocurrency can function as a very useful hedge. In [39] the author observes that the market of the bitcoin is informationally inefficient. A comprehensive survey on the publications on cryptocurrencies is provided by [17], which shows that the number of publications on the underlying topic has as a strong positive trend since its start in year 2013. The work of [35] provide a detailed survey of numerous publications that deal with the security concerns of cryptocurrencies. Citation [30] suggests an index that can be used for measuring uncertainty in the cryptocurrencies. The basis for this index is the news coverage from the mass-media. Reference [2] investigates empirically the factors that determine the adaptation of cryptocurrencies and blockchains in 137 countries. In [29] a three-factors model for cryptocurrencies is presented. They find that the return of each individual cryptocurrency is explained by the return of the entire market for the cryptocurrencies, the size, and the momentum. Citation [28] finds empirical evidence that the returns of cryptocurrencies are robustly linked to the network factors but not to the production factors.

Since cryptocurrencies are energy intensive, there are naturally environmental effects. This important issue has been investigated by [16] using asymmetric causality tests developed by [18]. It is found that there are negative asymmetrical environmental causal impacts of the demand for major cryptocurrencies. The authors suggest that the policy makers should introduce environmental taxes imposed on cryptocurrency transactions to dampen the damaging effects of the cryptocurrencies on the environment. By applying asymmetric causality tests, the authors of [24] obtain empirical evidence that supports the dynamic interaction and risk transmission between the oil market and bitcoin.

Publication [27] explores the potential portfolio diversification gains between ten main cryptocurrencies, which results in finding that diversifying across these cryptocurrencies lead to better investment outcomes indeed. In [20] the potential portfolio diversification benefits between bitcoin, stocks, bonds, and the US dollar in the global market is explored. The authors find that there are no portfolio diversifications benefits that can be obtained from this cryptocurrency if the portfolio is created by the standard approach pioneered by [32]. However, if the portfolio is created by using [19] approach, which combines risk and return in the optimization problem, including bitcoin in the portfolio results in a higher risk adjusted return. In addition, [23] obtains empirical support for increasing return per unit of risk for investors from the Middle East if they add cryptocurrencies in their portfolios.

Cryptocurrencies are increasingly chosen as financial assets that are included in the investment portfolios by the individual investors and financial institutions worldwide[‡]. Many studies on cryptocurrencies have emerged investigating principally portfolio diversification profits, market effectiveness, hedging, or capturing the data generating process for the volatility of the cryptocurrencies. The main contribution of this paper is to suggest a prediction model for cryptocurrencies. Market observations imply several differences between a cryptocurrency and traditional asset prices. Cryptocurrencies have higher volatility compared to regular assets. Besides, cryptocurrencies are widely less liquid than conventional financial instruments. The paper of [31] examines the relationship between cryptocurrency liquidity, herding behavior, and profitability during extreme price movement periods and it reveals that herding behavior variations have a decreasing magnitude. When building a prediction model for cryptocurrency prices we need to keep in mind that these instruments are different from traditional assets. Cryptocurrency modeling has been studied by a great number of authors in the literature. Many of these works investigated prediction models. In [1] the authors suggest a method for predicting price variations in bitcoin and Ethereum using Twitter data and Google Trends data. Reference [22] provides a review of the research works on predicting cryptocurrency prices from 2010 to 2020.

[‡]Since cryptocurrencies are based on new technology, software development is an integral part of it.

Most of these studies utilize machine learning for price prediction. Other papers investigate the cryptocurrency price volatility and prediction using econometrics, and statistical models on time-series data. As far as we know, no previous research has suggested a prediction model using stochastic differential equations (SDEs) dealing with cryptocurrencies' high fluctuations and illiquidity. In this paper, we propose an SDE to model future trajectories of cryptocurrency values. Our suggested model comprises three of the most significant stylized facts of cryptocurrencies: illiquidity, high volatility, and regime switching (RS).

The paper is organized as the following. Section 2 describes the construction of the model. Section 3 studies the existence, uniqueness and positivity of the SDE solution. In section 4, numerical simulations are conducted and several figures are presented in order to illustrate the performance of the model. The final section expresses the concluding statements.

2 Model formulation

We follow the works of [26] and more recently [15] to construct our cryptocurrencies prediction model. First of all, a filtered probability space and sources of randomness living on it are to be specified. In [26] and [15], only the Brownian motion is generating the randomness. The idea of this paper is to include a second source of randomness that is independent from the Brownian motion namely a continuous time Markov chain-CTMC. Then, the model becomes a regime switching model which can be seen as an expansion of the [15].

2.1 Probability space and sources of randomness

Consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P)$. Let $(B_t)_{t\in[0,T]}$ be a Brownian motion and denote by $(\mathcal{F}_t^B)_{t\in[0,T]}$ the filtration generated by $(B_t)_{t\in[0,T]}$. We assume that there is a second source of randomness living in the probability space. Let $(Z_t)_{t\in[0,T]}$ denote a Markov jump process defined values in a finite state space $\mathcal{S} := \{1, 2, ..., N\}$ and denote by $\mathcal{F}^Z = (\mathcal{F}_t^Z)_{t\in[0,T]} := \sigma(Z_t, 0 \le t \le T)$, which is the natural filtration generated by the Markov process Z under P. The filtration $\mathcal{F}$ is then defined as $\mathcal{F} := \mathcal{F}^B \vee \mathcal{F}^Z$.

2.2 The model

Let $C = (C_t)_{t\in[0,T]}$ be the rate of the cryptocurrency with regard to the US dollar. Assume that μ is the expected return of the underlying cryptocurrency, and σ is its volatility. The price impact factor of the broker is $\lambda(t, C)$ and θ_t represents the number of units of the underlying cryptocurrency that the trader owns at time t. Thus, $\lambda(t, C)d\theta_t$ represents the price impact of the investor's trading. We assume that the cryptocurrency's value is therefore governed by the following stochastic differential equation:

$$\begin{aligned} dC_t &= \mu(Z_t)C_t dt + (\sigma(Z_t)C_t + g(t))\, dB_t + \lambda(t, C_t)C_t d\theta_t, \quad t \in [0, T], \\ C_0 &= x > 0. \end{aligned} \tag{2.1}$$

Where $g(t)$ is originally suggested by [8] and it signifies a function that is deterministic and captures well the impact of increase in the volatility that takes place during a special period. It is also able to embody Sornette's [37] empirical encompassing of the market price index during a crunch via a dissipative harmonic oscillator characterized by a sinusoidal function that is exponentially decreasing. This function is expressed as the following:

$$g(t) = c_1 + c_2 e^{c_3 t} \sin(c_4 t),$$

where $c_i, i = 1, 2, 3, 4$ are real constants. Let θ_t be the number of the underlying cryptocurrency that the trader owns and assume that it satisfies the following process:

$$d\theta_t = \eta_t dt + \zeta_t dB_t, \quad t \in [0, T]. \tag{2.2}$$

The above proposed model can be seen as generalisation of the models considered in [13] on the valuation of options during crisis and in [14], and [15] where the authors investigated option's pricing in illiquid and high volatile situations. The SDE (2.1) expands the previous models by adding regime switching which permits to variate the parameters according to different economic situations.

2.3 Solution existence and uniqueness analysis

The below proposition outlines the existence and uniqueness of the solution of the SDE of our model (2.1).

Proposition 1 *Let* $a_t := \mu(Z_t) + \lambda(t, C_t)\eta_t$, $b_t := \sigma(Z_t) + \lambda(t, C_t)\zeta_t$, *and*

$$\xi_t = \exp\left(\int_0^t \left(a_u - \frac{b_u^2}{2}\right) du + \int_0^t b_u dB_u\right), \quad \xi_0 = 1. \tag{2.3}$$

Then

$$C_t = \left(C_0 - \int_0^t g(u)\big[\sigma(Z_u) + \lambda(t, C_t)\zeta_t\big]\xi_u^{-1} du + \int_0^t g(u)\xi_u^{-1} dB_u\right)\xi_t. \tag{2.4}$$

Proof. We consider the process $(\xi_t)_{t\in[0,T]}$ defined by the SDE

$$d\xi_t = a_t\xi_t dt + b_t\xi_t dB_t \quad \xi_0 = 1, \tag{2.5}$$

where a_t and b_t are defined at the begining of the above proposition. The SDE provides a geometric Brownian motion with solution given by (2.3). We use the variation of constants method. First, assume that

$$C_t = Y_t\xi_{1t}, \qquad Y_0 := C_0. \tag{2.6}$$

Then Y_t can be obtained using the below integration by parts for stochastic processes

$$dY_t = d(\xi_t^{-1}C_t) = \xi_t^{-1}dC_t + C_t d\xi_t^{-1} + [d\xi_t^{-1}, dC_t]. \tag{2.7}$$

The solution of the regime switching model SDE (2.1) given by (2.4) can be obtained after embedding (2.2) into equation (2.1), then calculating $d\xi_t^{-1}$ by using the Ito formula applied to $f(\xi_t) = \frac{1}{\xi_t}$ and then by making use of (2.7), (2.3), and (2.6). $\square$

Remark 1 *The equation (2.4) demonstrates the existence and uniqueness of a solution for the SDE (2.1) but does not guarantee non-negative values.*

- *For the positivity, the below condition is required*

$$C_0 + \int_0^t g(u)\xi_u^{-1} dB_u \geq \int_0^t g(u)\big[\sigma(Z_u) + \lambda(t, C_t)\zeta_t\big]\xi_u^{-1} du. \tag{2.8}$$

  The above condition can be satisfied with a careful choice of g *and* λ.

- *It should be pointed out that choices of g and λ that do not satisfy the condition (2.8) but produce a rate of cryptocurrencies approaching zero or become negatively with very low probability could be considered as reasonable choices. This can be interpreted in the sense that any rate that reaches zero implies that the underlying cryptocurrency does not survive and disappears from the market, which in fact can happen in the real markets.*

3 Numerical results and simulations

3.1 Methodology

Consider the one dimensional stochastic process $X := (X_t)_{t\in[0,T]}$ driven by the following SDE:

$$dZ_t = a(t, Z_t, X_t)dt + b(t, Z_t, X_t)dB_t, \quad Z_0 = z \text{ is a given constant.} \tag{3.1}$$

To obtain a discretized trajectory of Z_t from the SDE (3.1) using Euler-Maruyama scheme, the following steps need to be implemented:

1. simulate ΔB_k as normally distributed random variable $N(0, \Delta t)$
2. simulate X_k the Continuous Time Markov Chain
3. set $\tilde{Z}_0 := Z_0 = z$ and evaluate $\tilde{Z}_{k+1}$ using

$$\tilde{Z}_{k+1} = \tilde{Z}_k + a(k\Delta t, \tilde{Z}_k, X_k)\Delta t + b(k\Delta t, \tilde{Z}_k, X_k)\Delta B_k, \tag{3.2}$$

for $k = 0, \ldots, N-1$. Notice that $\Delta B_k = B_{k+1} - B_k$. We will not use the symbol $\tilde{}$ for discretized version of a given SDE from now on. The application of (3.2) to the model (2.1) gives the system

$$\begin{aligned} C_{k+1} &= C_k + \mu(Z_k)C_k\Delta t + (\sigma(Z_k)C_k + g(k))\,\Delta B_k + \lambda(k, C_k)C_k\Delta\theta_k, \\ \theta_{k+1} &= \theta_k + \eta_k\Delta t + \zeta_k\Delta B_k, \end{aligned} \tag{3.3}$$

where we have discretized the time into M time steps t_k with equal sizes $\Delta t = t_{k+1} - t_k = \frac{T}{M}$, for $k = 0, \ldots, M-1$.

3.2 Algorithm

Our algorithm consists of the following phases:

1. We simulate a trajectory for the Brownian motion: $(B_k)_{k=0,\ldots,M-1}$.

2. We simulate independently a trajectory for the CTMC: $(X_k)_{k=0,\ldots,M-1}$.

3. For $k=1$, (C_0,θ_0) and all the parameters of the model are given, we use (3.3) to calculate (C_1,θ_1),

4. for $k=2$, (C_1,θ_1) and the parameters of the model are all known from the previous step, then apply (3.3) to calculate (C_2,θ_2),

5. repeat the previous two steps to $k=M-1$.

3.3 Illustrations

The above algorithm is implemented by creating a code in Python. It is assumed that the CTMC has 3 possible states, $Z\in\{0,1,2\}$. Here, for the illiquid with high volatility case the parameters have the values: $C_0=10, g(t)=\alpha(Z_t)cos(\pi t/4), \lambda=1.5, \eta_t=t, \zeta_t=sin(\pi t/4), T=40, N=10000$.

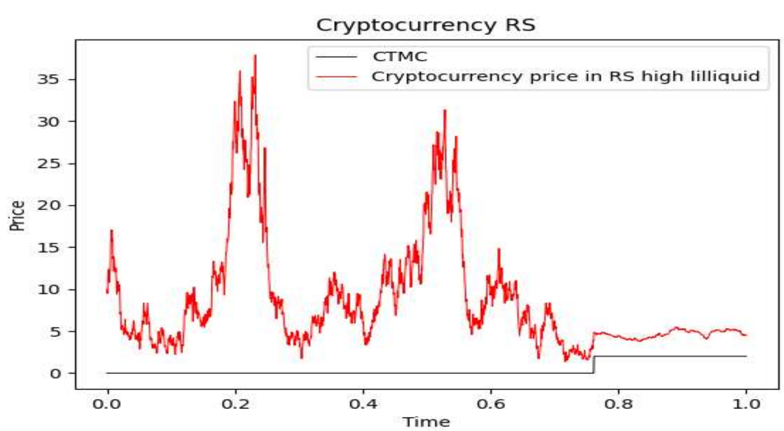

Figure 1: Realizations of the cryptocurrency to dollar value. First run of the simulations.

- State one when $Z_t=0$ corresponds to a bad economic situation for cryptocurrencies. The following parameters values are used for covering this specific situation:

1. expected return $\mu(0) = 0.005$
2. volatility $\sigma(0) = 2.5$,
3. increase of the volatility factor in function g is $\alpha(0) = 10$.

- State two when $Z_t = 1$ represents a normal economic circumstance. This case is dealt with via the following parameters values:

1. expected return $\mu(0) = 0.045$
2. volatility $\sigma(0) = 0.5$,
3. increase of the volatility factor in function g is $\alpha(0) = 1$.

- State three when $Z_t = 2$ depicts a good economic condition. The parameters values for this situation are below

1. expected return $\mu(0) = 0.2$
2. volatility $\sigma(0) = 0.3$,
3. increase of the volatility factor in function g is $\alpha(0) = 0.5$.

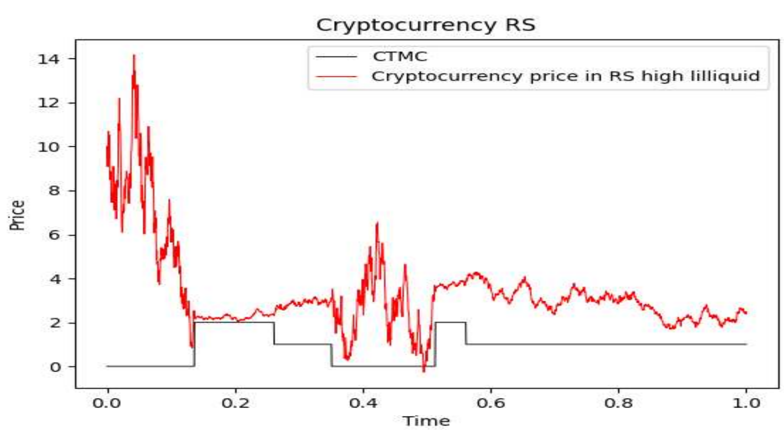

Figure 2: Realizations of the the cryptocurrency to dollar value. Second run of the simulations.

Figure 3 offers the bitcoin to dollar values from November 2021 to August 2022 taken from the website of yahoo finance. We have divided the time period into four parts. These four periods can be seen as four different states for the Bticoin to dollar values.

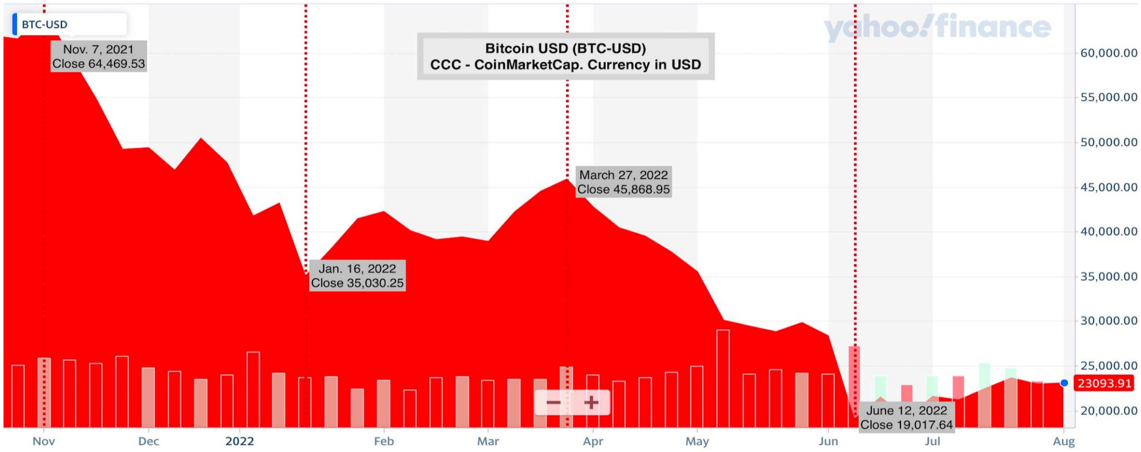

Figure 3: Bitcoin to US dollar as of August 7, 2022 divided into 4 states. Source: https://finance.yahoo.com

Figures 1 and 2 provide two simulations of cryptcourrency to dollar values trajectory in an illiquid market under stress. The graphs seem to accord well with the reality since cryptocurrency prices tend to be exceptionally volatile during certain periods.

4 Conclusions

A cryptocurrency is a digital form of payment that is based on cryptography. Cryptocurrencies are increasingly considered as a serious alternative measure of payment versus the traditional fiat currencies. Despite having their advantages, cryptocurrencies are extremely volatile and risky. They are also characterized by the markets that can suffer from the illiquidity issue. Structural breaks or regime shifts are also taking place in the markets for cryptocurrencies. The current paper provides a model that can be used for predicting the ex-ante path of the exchange rate of the cryptocurrencies. The suggested approach contributes to the existing literature on the topic by considering simultaneously three explicit characteristics of cryptocurrencies—namely— (1) illiquidity, (2) high volatility and (3) regime shifts. A solution for the SDE modelling the exchange rate of the cryptocurrency is provided along with a mathematical proof. Numerical simulations are provided, which can capture the situations in which the dynamism of the cryptocurrency rates with regard to the US dollar operate in real markets. There are massive publications on the cryptocurrencies. However, these publications are mainly empirical dealing with issues like portfolio diversifications benefits, market efficiency, hedging or capturing the data generating process for the volatility of the cryptocurrencies. To our best knowledge, this is the first attempt to suggest a stochastic differential equation for modeling the exchange rate (i.e., the pricing) of the cryptocurrencies that covers the three mentioned characteristics.

References

[1] Abraham, J., Higdon, D., Nelson, J., Ibarra, J. (2018) Cryptocurrency price prediction using tweet volumes and sentiment analysis. *SMU Data Science Review*, 1(3), 1.

[2] Bhimani, A., Hausken, K., and Arif, S. (2022) Do national development factors affect cryptocurrency adoption? *Technological Forecasting and Social Change*, 181, 121739.

[3] Black F, Scholes M. (1973) The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81 (3): 637-654.

[4] Chan L, Lakonishok J. (1995) The behavior of stock prices around institutional trades. *Journal of Finance*, 50: 1147–1174.

[5] Cheah E.T and Fry J. (2015) Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economics Letters*, 130, 32-36.

[6] Coinmarketcap, [online] (2022) Available: https://coinmarketcap.com/charts/. Verified online on July 15, 2022.

[7] Company R, Jodar L, Pintos J-R. (2010) Numerical analysis and computing for option pricing models in illiquid markets. *Mathematical and Computer Modelling*, 52: 1066-1073.

[8] Dibeh G, Harmanani H-M. (2007) Option pricing during post-crash relaxation times. *Physica A.*, 380: 357-365.

[9] Dyhrberg A.H. (2016a) Hedging capabilities of bitcoin. Is it the virtual gold? *Finance Research Letters*, 16, 139-144.

[10] Dyhrberg A.H. (2016b) Bitcoin, gold and the dollar – A GARCH volatility analysis. *Finance Research Letters*,16, 85-92.

[11] Dwyer P.G. (2015) The economics of Bitcoin and similar private digital currencies. *Journal of Financial Stability*, 17, 81-91.

[12] El-Khatib Y, Hatemi-J A. (2013) On option pricing in illiquid markets with jumps. *ISRN Mathematical Analysis*, 1-5. doi:10.1155/2013/567071

[13] El-Khatib Y, Hatemi-J A. (2017) Option valuation and hedging in markets with a crunch. *Journal of Economic Studies*, 44(5): 801-815.

[14] El-Khatib Y, Hatemi-J A. (2019) Option pricing in high volatile markets with illiquidity. *AIP Conference Proceedings 2116*, 110007, https://doi.org/10.1063/1.5114100

[15] El-Khatib, Y., Hatemi-J, A. (2022) Option pricing with illiquidity during a high volatile period. *Mathematical Methods in the Applied Sciences*, 45(5), 3213-3224.

[16] Erdogan, S., Ahmed, M. Y., and Sarkodie, S. A. (2022) Analyzing asymmetric effects of cryptocurrency demand on environmental sustainability. *Environmental Science and Pollution Research*, 29(21), 31723-31733.

[17] Fang, F., Ventre, C., Basios, M., Kanthan, L., Martinez-Rego, D., Wu, F., and Li, L. (2022) Cryptocurrency trading: a comprehensive survey. *Financial Innovation*, 8(1), 1-59.

[18] Hatemi-J, A.(2012) Asymmetric causality tests with an application. *Empirical Economics*, 43(1), 447-456.

[19] Hatemi-J, A. and El-Khatib Y. (2015) Portfolio selection: An Alternative Approach. *Economics Letters*, 135, 141-143.

[20] Hatemi-J, A., Hajji, M. A., Bouri, E., and Gupta, R. (2021) The Benefits of Diversification between Bitcoin, Bonds, Equities and the US Dollar: A Matter of Portfolio Construction. *Asia-Pacific Journal of Operational Research*, 2040024.

[21] Keim D, Madhavan A. (1996) The upstairs market for large-block transactions: analysis and measurement of price effects. *Review of Financial Studies*, 9: 1-36.

[22] Khedr, A. M., Arif, I., El-Bannany, M., Alhashmi, S. M., Sreedharan, M. (2021) Cryptocurrency price prediction using traditional statistical and machine-learning techniques: A survey. *Intelligent Systems in Accounting*, Finance and Management, 28(1), 3-34.

[23] Kumaran S. (2022) Portfolio diversification with cryptocurrencies – Evidence from Middle Eastern stock markets. *Investment Analysts Journal*, 51, 14-34.

[24] Li, Dongxin and Hong, Yanran and Wang, Lu and Xu, Pengfei and Pan, Zhigang (2022) Extreme risk transmission among bitcoin and crude oil markets. *Resources Policy*, 77, 102761.

[25] Lipton A. (2021) Cryptocurrencies change everything. Quantitative Finance, 21, 1257–1262.

[26] Liu H, Yong J. (2005) Option pricing with an illiquid underlying asset market. *Journal of Economic Dynamics and Control*, 29: 2125-2156.

[27] Liu W. (2019) Portfolio diversification across cryptocurrencies. Finance Research Letters, 29, 200-205.

[28] Liu, Y., and Tsyvinski, A. (2021) Risks and returns of cryptocurrency. *Review of Financial Studies*, 34(6), 2689-2727.

[29] Liu, Y., Tsyvinski, A., and Wu, X. (2022) Common risk factors in cryptocurrency. *Journal of Finance*, 77(2), 1133-1177.

[30] Lucey, B. M., Vigne, S. A., Yarovaya, L., and Wang, Y. (2022) The cryptocurrency uncertainty index. *Finance Research Letters*, 45, 102147.

[31] Manahov, V. (2021) Cryptocurrency liquidity during extreme price movements: is there a problem with virtual money? *Quantitative Finance*, 21(2), 341-360.

[32] Markowitz H. (1952) Portfolio selection. *Journal of Finance*, 12(7): 77-91.

[33] Nakamoto, S. (2008) Bitcoin: A peer-to-peer electronic cash system. *Technical Report*. Available online at: https://bitcoin.org/bitcoin.pdf (accessed 5 August 2022).

[34] Protter Ph. (1990) *Stochastic integration and differential equations. A new approach*. Berlin, Springer-Verlag.

[35] Quamara S., and Singh A.K. (2022) A systematic survey on security concerns in cryptocurrencies: State-of-the-art and perspectives. *Computers and Security*, 113, 102548.

[36] Sharpe W-F, Alexander G-J, Bailey J-V. (1999) *Investments.* New Jersey, Prentice Hall.

[37] Sornette D. (2003) *Why Stock Markets Crash: Critical Events in Complex Financial Markets.* Princeton University Press, Princeton, NJ.

[38] Taleb N.N. (2021) Bitcoin, currencies, and fragility. *Quantitative Finance*, 21, 1249–1255.

[39] Urquhart A. (2016) The inefficiency of Bitcoin. *Economics Letters*, 1148, 80-82.