Personal income distribution and the endogeneity of the demand regime

Tonni, Lorenzo

University of Rome La Sapienza

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Lorenzo Tonni*

Department of Economics and Law

Sapienza University of Rome

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Abstract

This paper deals with two intrinsically linked issues: the endogeneity of the demand regime and the personal distribution impact on aggregate demand. By assuming that saving is a function of personal rather than functional income distribution, an increase of the labor share is effective in boosting consumption and aggregate demand, not per se, but only as long as it reduces personal inequality. As the labor share increases, both the demand regime type – the *sign* of the slope of the demand schedule - and its strength- the *size* of the slope of the demand schedule - can endogenously change. Concerning the former, there can be a threshold value for the wage share beyond which there is a shift from wage-led to profit-led demand. The analysis shows that, unlike most Kaleckian models, profit inequality is just as important as wage inequality in determining the demand regime type and its strength.

Keywords: Personal distribution, functional distribution, wage-led, profit-led, endogenous demand regime.

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1 Introduction

One of the key features of the Kaleckian model of growth, both in its original version (Rowtorn, 1981 and Dutt, 1984) and in its subsequent evolutions (Amadeo, 1987 and Taylor, 1990), is a saving function based on the functional distribution of income. Both in the former simplified versions where workers do not save at all and in the latter more advanced versions where a propensity to save out of wages is added, class-based distribution is fundamental in generating a wage-led demand regime. Other variants of the model, starting from the contribution of Bhaduri and Marglin (1990) and Kurz (1990), modify the other model's equation, the investment function, including the profit share to take into account the profitability impact on investment decisions. Blecker (1989) introduces into the analysis the possible negative effects on the trade balance of an increase in the wage share in an open economy. With these evolutions, the positive effect on consumption of a lower profit share could be offset by the negative effect on investment and net exports. Depending on which of these opposite effects prevails, the demand will be wage-led or profit-led. However, in all these models, the consequence of adopting a saving function based on the functional distribution of income is that the demand will be either universally wage-led or profit-led. As already pointed out by Nikiforos (2016), this conclusion is problematic, as an economy would reach its maximum capacity utilization rate either with a labor share tending towards one or zero, depending on whether it is wage-led or profit-led.

The introduction of an endogenous demand regime that bounds the "distribution-ledness" of an economy has been the object of different contributions. In all of them the non-linearity in the demand schedule comes, explicitly or not, from a changing propensity to invest and/or to save in the labor share level. In Palley (2013) both mechanisms are at work, and a non-linear demand schedule is justified, on the one hand, with the presence of neoclassical capital stock adjustment costs and, on the other, with redistribution towards top-tier income households produced by a profit share increase. Only the saving channel is present in Palley (2015), who introduces an endogenous wage bill split between workers and managers that in turn makes the demand regime endogenous: the increased economic activity following a pro-capital redistribution in a profit-led regime can trigger a redistribution of the wage bill towards workers if managers are a fixed normal cost. The redistribution generates a fall in the aggregate saving rate because workers' propensity to save is lower than that of managers. Both channels are present in Nikiforos (2016), who explicitly assumes that the propensity to invest and the propensity to save change in response to the variations in the labor share. These two mechanisms, together with an unstable distribution of income influenced by class power, "distribution-ledness" of the economy and lagged effects, generate endogenous changes between wage-led and profit-led periods.

However, since a universally wage/profit-led demand is a direct consequence of a saving function based on functional income distribution, this paper proceeds by simply shifting from functional to personal distribution as the key distributional variable determining the saving rate. This perspective shift is not merely instrumental in generating a non-monotonic demand schedule. It seems more plausible that the individual propensity to save depends on the individual position in total income ranking, i.e., on personal income distribution[1], rather than on the type of income earned - wage or profit. Figure 1 - similar to the one reported in Carvalho and Rezai (2016) - shows that the saving rate in the US economy is an increasing function of income quintiles[2].

[Figure 1]

As pointed out by Ranaldi (2021) and Ranaldi and Milanovic (2021), if the economy is populated by a majority of low-income workers who earn only labor income and a small number of high-income capitalists who earn only profit income, then functional income distribution can be a good proxy for personal income distribution. However, this simplification - which could be reasonable for nineteenth-century early capitalism - does not necessarily hold in advanced economies. Indeed, the lower the compositional inequality - i.e., how the composition of income in capital and labor varies across the income distribution - the less changes in functional income distribution will be transmitted to inter-personal income inequality[3]. Two main factors differentiate an advanced economy from a 'classic capitalism' economy. Primarily, in modern economies, there is a sizable within-class inequality, from the part-time worker's low wage to the very high top manager pay, and from the small business profit income to the major shareholders of large retailer and tech companies. Secondly, the lower compositional inequality sometimes makes the class concept difficult to handle. Even though the majority of people earn most of their income either from labor or from capital, a certain number of people draw their income from both income sources. Although it typically represents a small part of their income, it is common for workers to draw income from interests and dividends paid on their savings. Another more prominent case is that of the self-employed, as their income is partly imputed to labor and partly to capital in official statistics. This paper, therefore, attempts to answer the following question. To what extent does a change in the functional income distribution affect the personal income distribution and aggregate demand?

The paper proceeds as follows. The saving function is microfounded following Carvalho and Rezai (2016): aggregating individual saving decisions, where individual consumption depends on the income deviation from the median individual, yields a positive relation between the aggregate saving rate and the Gini index. The Gini index is then decomposed, following Lerman and Yitzhaki (1985), as a function of the Gini indices of wages and profits and of the functional distribution. From this decomposition, it derives that wage inequality and profit inequality play a symmetric, though opposite, role in determining the demand regime type - the *sign* of the slope of the demand schedule - and its strength - the *size* of the slope of the demand schedule. Moreover, since saving is a function of the personal income distribution rather than the functional one, raising the wage share is effective in reducing the aggregate saving rate, not per se, but only as long as it reduces personal inequality. As the labor share increases, depending on how it affects the personal distribution, the demand regime type and strength can endogenously change. In particular, there can be a threshold value of the wage share beyond which a further increase raises inequality rather than reducing it. This generates a shift from wage-led to profit-led demand. For this reason, when the saving rate is determined by personal distribution rather than functional distribution, a non-monotonic propensity to save in the labor share level can arise naturally, generating a non-monotonic demand schedule, without the need to make additional assumptions[4].

Another approach to introduce personal inequality within the Kaleckian model, adopted - among others[5] - by Lavoie (1996), Palley (2014 and 2015) and Tavani and Vasudevan (2014), consists in introducing an unproductive managerial class, which shares wage income with workers. Inequality between these two classes of wage earners is a proxy for personal inequality. This approach has two major shortcomings. Firstly, it deals only with wage inequality, neglecting the other side of the coin: profit inequality. Secondly, it measures only the "between" inequality, as inequality within workers, managers and capitalists is not considered. These downsides may be applied to a similar approach, taken by Palley (2017a) and (2017b), which, however, realistically considers that both workers and capitalists earn both capital and labor income. Carvalho and Rezai (2016) show that aggregating workers' individual saving functions where saving depends on the deviation between the individual's income and the median income yields a positive relationship between the aggregate saving rate and the Gini index. Although this approach considers within class inequality, personal income inequality is still limited to inequality within wage earners as the distribution of profit income is not taken into account.

The view of consumption upheld by this paper, where the main determinant of individual saving rate differences is the individual position in income ranking, is linked to the strand of literature, which draws primarily on Veblen's (1899) "conspicuous consumption" and Duesenberry's (1949) "relative income hypothesis"[6], that sees consumption demand mainly as a social phenomenon: individuals primarily consume to keep up with a certain social standard of well-being, reflected in a given consumption level and, thus, saving is mainly determined as a residual component once those needs have been attained. Within this setup, the determination of the consumption level targeted by individuals plays a critical role: depending on how this target is formulated, an *increasing* or *decreasing* relationship between the saving rate and personal inequality arises. The "expenditure cascades" hypothesis proposed by Frank et al. (2014) predicts that, through a sort of trickle-down consumption mechanism, an increase in inequality within a group makes its saving rate decrease. According to this idea, individuals try to emulate the consumption behavior of those just above them in the income rank. Consequently, an increase in income and consumption of those at the top of income distribution, generates a reduction of saving rates along all the distribution in the attempt "to keep up with the Joneses". Along this line, the contributions of Setterfield and Kim (2016), Kapeller and Schütz (2015) and Kapeller et al. (2018) show that, if a redistribution from labor to capital is coupled with an expenditure cascade effect, it can boost demand through a *consumption-driven profit-led regime*. On the contrary, the individual consumption function of Carvalho and Rezai (2016) embeds a *positive* relationship between inequality and the aggregate saving rate. Each individual compares her consumption not with the income group immediately above in the income rank but with the median individual of the whole distribution. Instead of a continuum of consumption levels targeted by people, there is only one target for all, which should be seen as a threshold of social satisfaction rather than a target. This lack of consensus in the literature about the sign of the effect of personal inequality on the saving rate is pointed out by Prante (2018). The paper relies on Carvhalo and Rezai's (2016) saving function but remains agnostic about the sign of this effect. If we see the use of the functional income distribution in all the traditional Kaleckian and neo-Kaleckian models as a simple proxy for the personal distribution, it can be said that this literature falls under the Carvalho and Rezai category, as it predicts a drop in the aggregate saving rate following a redistribution from capital to wage. This paper can be led to this last approach, as its main goal is bounding the “distribution-ledness” of traditional neo-Kaleckian models, in which wage-led aggregate demand is, in principle, possible through the positive effect on consumption of a pro-labor redistribution. Nevertheless, as we will see later, relying on a saving function where the aggregate saving rate is a decreasing function of personal inequality does not significantly alter the main findings of this article.

The paper assumes - as done by Dutt (2016) and others in the literature - that the saving rate is a function of personal rather than functional income distribution. The novelty lies in showing how this is sufficient to generate a set of relevant results. First, endogenous utilization regime changes may arise naturally without the need for further assumptions regarding the investment function. Second, even if such regime change does not occur, it shows that the utilization regime’s degree of wage or profit-ledness - its “strength” - may not be constant. Koher (2018) and Prante (2019) already presented the possibility of endogenous distribution-ledness change. However, while in their models, the wage-led regime exhibits increasing marginal returns and the profit-led regime decreasing marginal returns, in this model, both regimes exhibit decreasing marginal returns under plausible parameter values. Third, it shows that in determining the regime *type* and its *strength* a symmetric, though opposite, role is played by wage and profit inequality, despite the role of the latter being neglected in the literature. Lastly, it offers a more general and comprehensive way of dealing with personal distribution, as any distribution can be virtually represented within the model.

The paper is organized as follows. Section 2 discusses the general model and its properties. As the model’s features are strictly dependent on the particular distribution assumed for individual wages and profits, in Section 3, four particular distributions, among the infinite possibilities, are simulated better to understand the characteristics and implications of the model. This exercise is similar in spirit to the one conducted by Milanovic (2018) but extended to the whole macroeconomic context. Section 4 analyzes what happens if the saving function embeds a decreasing relationship between personal inequality and the aggregate saving rate. Section 5 shows that model results are robust even if investment demand is a function of the profit share as in Badhuri and Marglin (1990) and Kurz (1990). Section 6 concludes.

2 The Model

Investment is described by a standard Kaleckian investment function:

$$g^i = \gamma + \gamma_u(u - u_n) \tag{1}$$

where g^i is the investment to capital ratio, γ is the growth rate of autonomous investment, u is the rate of capacity utilization, u_n is the normal capacity utilization rate, and γ_u is the sensitivity of investment to the deviation of capacity utilization from its normal rate. The choice not to include any distributional variables in the investment function comes from focusing on consumption demand as the transmission channel from distribution to aggregate demand. Although the wage-led/profit-led distinction is traditionally linked to the sensitivity of the investment function to the functional distribution, in this model - as we will see - this change is not necessary to have a profit-led demand regime. The saving function is described in the following way:

$$g^s = s\frac{u}{v} \tag{2}$$

Where g^s is the saving to capital ratio, v is the capital to full capacity output ratio, and s, the average propensity to save, is microfounded as follows. Each individual income Y_i is equal to the sum of labor income and capital income:

$$Y_i = w_i \omega Y + p_i(1 - \omega)Y \tag{3}$$

Where ω is the labor share, i.e., the relative share of output paid as compensation to employees. $1-\omega$ is the share paid to capital and w_i and p_i are, respectively, the portion of total wage mass and total profit mass earned by individual i. These individual shares are assumed to be constant over time[7]. Note that - as in the standard Kaleckian model - all profits are distributed to households, and there are no retained profits. This assumption will be relaxed in section 2.1.

Individuals take their saving decisions as follows:

$$S_i = a_0 Y_i + a_1(Y_i - Y_m) \qquad a_0, a_1 \geq 0 \tag{4}$$

Saving S_i depends partly on individual income Y_i, through the coefficient a_0, and partly on its deviation from the median income Y_m, through the coefficient a_1, which is indeed a measure of how much saving decisions are affected by inter-personal distribution. The more affluent the individual is, the higher his saving rate. Aggregating the individual saving functions and assuming a Pareto distribution for income, Carvalho and Rezai (2016) show that the aggregate saving rate can be written as:

$$s = a_0 + a_1 \left(1 - 4^{\frac{G_y}{1+G_y}} \frac{1 - G_y}{1 + G_y}\right) \tag{5}$$

Where G_y is the Gini index of total income. See Appendix A for the derivation process. This equation - as shown in Appendix B - states that a unique positive relationship links the saving rate and personal inequality: every decrease in personal inequality always reduces the saving rate, boosting aggregate demand. Note how, if there is no income inequality ($G_y = 0$), all individuals save the same portion of their income, and everyone has a saving rate equal to a_0. There is no difference between the individual and the aggregate saving rate. At the opposite end, if personal inequality is at its maximum ($G_y = 1$), then the aggregate saving rate equals the sum of a_0 and a_1.

The personal and functional income distribution are linked following the Lerman and Yitzhaki (1985) Gini index decomposition[8] (equation 6). This decomposition is chosen because it is one of the few to link personal and functional income distribution and because, among those, it is the one that fits better within the model. Indeed, in Atkinson (2009) personal inequality is expressed in terms of income variance rather than the Gini index. In Ranaldi (2022) the Gini index is not a function of the Gini indices of wages and profits. The decomposition reads:

$$G_y = \rho_w G_w \omega + \rho_p G_p (1 - \omega) \tag{6}$$

Where G_y is the Gini index for total income, G_w and G_p are the Gini indices for wages and profits, respectively. G_w measures the inequality in wage distribution across the whole population (and not only across wage earners). It is equal to zero when all people earn the same wage level and 1 when one individual earns all the wage mass. Analogously G_p measures the inequality in the profit distribution. ω is the labor share, and its complement $1 - \omega$ is the profit share. Finally, ρ_w and ρ_p are the Gini correlation coefficients for wages and profits, respectively[9]. ρ_w measures the correlation between the individual's level of monetary wage and his or her position in the income ranking. It is defined as follows:

$$\rho_w = \frac{cov(W, f(Y))}{cov(W, f(W))} = \frac{\sum(W_j - \overline{W})(f(Y_j) - \overline{f}(Y))}{\sum(W_i - \overline{W})(f(W_i) - \overline{f}(W))} \tag{7}$$

Starting from the numerator, W_j and Y_j are the wage and total income of individual j, respectively. $\overline{W}$ is the average wage in the population. $f(Y_j)$ is the cumulative distribution of income at Y_j, i.e., individuals are sorted in ascending order by *income*, and $f(Y_j)$ thus represents the percentage of individuals with income less than or equal to Y_j. $\overline{f}(Y)$ is the mean value of $f(Y_j)$, equal to 0.5. Similarly, in the denominator, W_i is the wage of individual i. The only difference is that $f(W_i)$ is the cumulative distribution of *wages* at W_i, i.e., individuals are sorted in ascending order according to their wages and not income, and $f(W_i)$ thus represents the percentage of individuals with wages less than or equal to W_i. Again, $\overline{f}(W)$ is the mean value of $f(W_i)$.

ρ_w equals 1 (-1) when the wage level is an increasing (decreasing) function of income ranking. No individual at the same time has a lower wage than someone else but outranks him or her in the income ranking. Thus, the wage ranking and the income ranking overlap perfectly. In contrast, ρ_w is positive (negative), but smaller (greater) than 1 (-1), when it is only *on average* an increasing (decreasing) function of income ranking. In other words, there is at least one individual who at the same time has a lower salary than someone else but outranks him or her in the income ranking. The more this occurs, the lower the correlation coefficient is. However, negative values are unlikely in a real economy, as higher wage values are unlikely to be associated with lower income ranking positions on average. Lastly, ρ_w equals zero when there is no correlation between monetary wage and income ranking.

Likewise, ρ_p is a measure of the correlation between the individual profit income and the income ranking and a positive (negative) value indicates that capital income is *on average* an increasing (decreasing) function of the income ranking.

$$\rho_p = \frac{cov(P, f(Y))}{cov(P, f(P))} = \frac{\sum(P_j - \overline{P})(f_j(Y) - \overline{f}(Y))}{\sum(P_i - \overline{P})(f_i(P) - \overline{f}(P))} \tag{8}$$

Note that, while G_w and G_p are exogenous parameters (do not depend on ω), ρ_w and ρ_p are a function of ω. As ω increases, the income of all those who derive most of their income from labor grows. As these people surpass in the income ranking other individuals whose income is more capital intensive, the correlation between wage and income ranking (ρ_w) rises, and that between profit income and income ranking (ρ_p) decreases. Appendix C discusses the sign of the partial derivatives of the two correlation coefficients and shows that:

$$\frac{\partial \rho_w}{\partial \omega} \geq 0 \text{ and } \frac{\partial \rho_p}{\partial \omega} \leq 0 \ \forall \ \omega \tag{9}$$

In particular, ρ_w increases every time an individual a surpasses in the income ranking an individual b whose individual share w_i of total labor income is smaller than a, otherwise it stays constant. Likewise, ρ_p decreases every time an individual a surpasses in the income ranking an individual b whose individual share p_i of total capital income is higher than a, otherwise it stays constant.

Therefore, equation (6) can be rewritten as:

$$G_y = \rho_w(\omega)G_w\omega + \rho_p(\omega)G_p(1-\omega) \tag{10}$$

According to equations (6) and (10), the functional distribution of income is not the only distributional variable that can affect personal distribution and demand. Indeed, provided ρ_w and ρ_p are greater than zero, every reduction in wage and/or profit inequality reduces personal inequality and, through equation (5), stimulates aggregate demand.

Equating equation (1) to equation (2) we get the equilibrium rate of capacity utilization:

$$u = \frac{(\gamma - \gamma_u u_n)v}{s(\omega, G_w, G_p) - \gamma_u v} \tag{11}$$

As in all Kaleckian models, to have a positive value for u, it must be assumed that $\gamma - \gamma_u u_n > 0$ and that the Keynesian stability condition holds, which in our case means that $s > \gamma_u v$. We can now turn to the core question of the paper: the impact of functional distribution on aggregate demand. Taking the partial derivative of equation 11 with respect to ω, we get:

$$\frac{\partial u}{\partial \omega} = -\frac{(\gamma - \gamma_u u_n)v}{[s(\omega, G_w, G_p) - \gamma_u v]^2}\frac{\partial s}{\partial \omega} \tag{12}$$

The sign of the partial derivative, and the demand regime, clearly depends only on the impact of ω on the saving rate ($\frac{\partial s}{\partial \omega}$). Accordingly, aggregate demand is wage-led (profit-led) if the saving rate decreases (increases) following a rise in the wage share. In turn - as shown in Appendix B - the sign of the relationship between functional distribution and saving rate ($\frac{\partial s}{\partial w}$) depends only on the effect of changes in the wage share on personal inequality ($\frac{\partial G_y}{\partial \omega}$). If an increase in wage share leads to a reduction in personal inequality ($\frac{\partial G_y}{\partial \omega} < 0$), the saving rate falls, and the economy is wage-led ($\frac{\partial u}{\partial \omega} > 0$). Vice versa, if an increase in wage share leads to a rise in personal inequality ($\frac{\partial G_y}{\partial \omega} > 0$), the saving rate increases, and the economy is profit-led ($\frac{\partial u}{\partial \omega} < 0$).

Therefore, to analyze the demand regime type, it is sufficient to study the conditions under which an increase in the labor share leads to a higher or lower personal inequality. The following condition describes the reaction of personal distribution to changes in functional distribution:

$$\frac{\partial G_y}{\partial \omega} = G_w \left[\rho_w + \frac{\partial \rho_w}{\partial \omega}\omega\right] - G_p \left[\rho_p - \frac{\partial \rho_p}{\partial \omega}(1-\omega)\right] \tag{13}$$

What equation (13) tells us is that the *type* of demand regime - the *sign* of the slope of the demand schedule in the (u, ω) plane - and its *strength* - the *size* of the slope of the demand schedule - depend: on the level of the labor share (ω), on the initial value of parameters ρ_w and ρ_p, on their variation ($\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$) and on the inequality of wages (G_w) and profits (G_p) distributions. The impact of an increase in the wage share on personal inequality and aggregate demand depends on the action of three different forces. Firstly, it depends on the difference between ρ_w and ρ_p. Given that both coefficients are positive in a real economy, the greater the correlation between profit and income ranking compared to that between wages and income ranking, the more likely the net effect of an ω increase on G_y will tend to be negative. A ρ_p greater than ρ_w increases the probability that those who derive most of their income from wages (profits) are concentrated on average at the bottom (top) of the income distribution. In such a situation, increasing the wage share means redistributing from the top down, thus reducing personal inequality and stimulating aggregate demand. A second element is the difference between the Gini indices of wages and profits. The more unequal the distribution of profits relative to wages, the more the net effect of an increase in ω on G_y is likely to be negative. The reason is similar to the difference between the two correlation coefficients. A G_p greater than G_w increases the probability that those who derive most of their income from wages (profits) are concentrated at the bottom (top) of the income distribution. Indeed, provided that the ρ_p and ρ_w coefficients are positive, the higher the inequality index of a factor income, the more it tends to be concentrated in the hands of a few at the top of the distribution. Again, in such a situation, increasing the wage share means redistributing from top to bottom, thus reducing personal inequality and stimulating aggregate demand. Hence, an important finding derived in equation (13) is that profits inequality play a symmetric, though opposite, role as wage inequality in determining the regime type and its strength, despite being neglected in the literature. Finally, the last factor is the induced change in the correlation coefficients ($\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$). This effect always operates by reducing the effectiveness of functional income redistribution. Suppose that initially, the difference between the correlation coefficients and the Gini indices is such that the net effect of an increase in wage share on personal inequality is negative. The rise in ρ_w and the reduction in ρ_p operate by weakening the negative effect on inequality. The consequence is that the marginal impact on personal inequality and demand of an increase in the wage share is decreasing. The change in the two coefficients and the associated decreasing marginal effect on inequality may be such that the sign of the relationship between wage share and personal inequality (and demand) reverses, and a further increase in the wage share increases inequality rather than reducing it. The same considerations apply if, initially, the difference between the correlation coefficients and the Gini indices is such that the net effect of an increase in the profit share on personal inequality is negative.

To better grasp the intuition behind equation (13), suppose that initially, those who derive most of their income from wages are concentrated at the bottom of the income distribution, and those who derive most of their income from profit are, on average, concentrated at the top. In this situation, increasing the wage share means redistributing from the top down, thus reducing personal inequality and stimulating aggregate demand. However, the more the wage share increases - and the redistributive process continues - the less sharp will be the concentration at the bottom (top) end of the distribution of those whose income is more wage (profit) intensive. In other words, increasing the wage share has decreasing marginal effectiveness in reducing personal inequality and rising aggregate demand. This effect is reflected in equation (13) by the change in correlation coefficients ($\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$). As the labor share increases, it is in principle possible to reach a point where those who derive most of their income from wages will be, on average, concentrated at the top of the income distribution, and those who derive most of their income from profit will be on average concentrated at the bottom. From this point on, further increases in the wage share will raise inequality and reduce aggregate demand rather than increase it since we would be redistributing from the bottom up. In other words, there could be a wage share level that, once passed, would trigger an endogenous regime shift from wage-led to profit-led.

In the particular case in which, as ω increases, there is no change in income ranking, $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are both equal to zero (see Appendix C) and equation (13) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w \rho_w - G_p \rho_p \tag{14}$$

Therefore, as long as the income ranking does not change, the first derivative is constant and no regime change can occur, be it initially wage or profit-led.

We can analyze the non-monotonicity of the relationship between wage share and aggregate demand - whose intuition we saw just above - analytically. Equating to zero equation (13) and solving for ω we obtain the critical value for ω, i.e., that value that once passed shifts the economy from a wage-led to a profit-led regime:

$$\omega^* = \frac{\rho_p G_p - \rho_w G_w - \frac{\partial \rho_p}{\partial \omega} G_p}{\frac{\partial \rho_w}{\partial \omega} G_w - \frac{\partial \rho_p}{\partial \omega} G_p} \tag{15}$$

Note that if the income ranking never changes, for instance because those who earn a higher wage have a higher profit income too, $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are equal to zero and ω^* does not exist.

The economy is wage-led for wage share levels smaller than the critical value. On the contrary, the economy is profit-led for wage share levels higher than the critical value (see Appendix D for the proof). In other terms:

$$\text{If } \omega < \omega^* \Rightarrow \frac{\partial u}{\partial \omega} > 0 \Rightarrow \text{ wage-led} \tag{16}$$

$$\text{If } \omega > \omega^* \Rightarrow \frac{\partial u}{\partial \omega} < 0 \Rightarrow \text{ profit-led} \tag{17}$$

Summing up, depending on the distribution of wages and profits, there can be an ω^* such that the regime is wage-led if $\omega < \omega^*$ and the regime is profit-led if $\omega > \omega^*$[10]. Provided the conditions for which $0 < \omega^* < 1$ hold and that ρ_w and ρ_p are positive, an increase (decrease) in G_w tends to lower (raise) ω^*, while an increase (decrease) in G_p tends to lower (raise) ω^*. In other words, the higher wage inequality and the lower profit inequality, the smaller the space for re-distributional policies toward wages with the aim of increasing the level of economic activity, as the boundary beyond which demand turns profit-led becomes more tightened. The intuition is similar to that of equation (13). As wage (profit) inequality increases, wages (profits) tend to concentrate higher in the income distribution, thus lowering the point beyond which an increase in wage (profit) share redistributes from the bottom up rather than the other way around.

Even if the change in the functional distribution of income is not such to make the *sign* of the slope of the demand schedule change, it does not mean that the *size* of the slope does not change at all. In other words, even if, as ω increases, a shift from a wage-led to a profit-led demand regime does not occur, it does not mean that the *wage-ledness* of the economy remains the same. However, analytically analyzing the linearity of the *distribution-ledness* of the two demand regimes - and hence the second derivative of equation (11) - is not straightforward. However, if there is a continuous change in income ranking and ω^* does exist, then there is a neighborhood of ω^* in which the second derivative is negative. To put it another way, if equation (11) is continuously derivable and has a point of maximum, then its second derivative must be negative in a neighborhood of this point. This implies that, as we get close to such ω^* from the wage-led area, the effectiveness of redistributional policies towards wages to stimulate aggregate demand is decreasing in ω, as we have seen when discussing equation (13). The same is true for a profit share increase in a profit-led regime: as we get close to ω^* from the profit-led area, the effectiveness of redistributional policies toward capital to stimulate aggregate demand is decreasing in $1-\omega$.

2.1 Introducing retained profits

Let d be the percentage of distributed profits so that $0 < d < 1$. Personal inequality is now computed not on the economy's total income but only on income that accrues to households, i.e., on the total income net of retained profits. Equation (6) becomes:

$$G_y = \rho_w G_w \overline{\omega} + \rho_p G_p (1 - \overline{\omega}) \tag{18}$$

where $\overline{\omega}$ and $1 - \overline{\omega}$ are the wage and profit share of total disposable income. Rearranging the two shares in terms of the usual wage and profit shares, we obtain:

$$G_y = \rho_w G_w \frac{1}{1+\left(\frac{1-\omega}{\omega}\right)d} + \rho_w G_p \left[1 - \frac{1}{1+\left(\frac{1-\omega}{\omega}\right)d}\right] \tag{19}$$

Note that if $d = 1$ this condition is nothing but the Gini decomposition of Equation (6), that is, therefore, a special case when all profits are distributed to households.

The equilibrium rate of capacity utilization depends now also on d, since, as stated in equation (5), s is a positive function of G_y, which in turn is a function of d, ω, G_w and G_p (equation 19). Equation (11) becomes:

$$u = \frac{(\gamma - \gamma_u u_n)v}{s(d, \omega, G_w, G_p) - \gamma_u v} \tag{20}$$

We can now focus on the impact of distributed profits on aggregate demand:

$$\frac{\partial u}{\partial d} = \frac{-(\gamma - \gamma_u u_n)v}{(s - \gamma_u v)^2} \frac{\partial s}{\partial d} \tag{21}$$

The partial derivative depends only on the response of the saving rate to changes in d.

$$\frac{\partial s}{\partial d} = \frac{\partial s}{\partial G_y} \frac{\partial G_y}{\partial d} \tag{22}$$

Which in turn depends exclusively on $\frac{\partial G_y}{\partial d}$, since $\frac{\partial s}{\partial G_y}$ is always positive, as already pointed out in the discussion regarding equation (5). The main point of interest is, thus, how the percentage of distributed profits affects personal inequality.

$$\frac{\partial G_y}{\partial d} = \frac{(\omega - 1)\omega(\rho_w G_w - \rho_p G_p)}{[d(1-\omega)+\omega]^2} \tag{23}$$

The sign of the partial derivative clearly depends on $\rho_w G_w - \rho_p G_p$. A reasoning similar to that made for equation (13) applies here. If profits concentrate mainly in the upper (lower) part of the distribution, then a rise in d increases (decreases) total inequality (G_y). Indeed, the higher (lower) is $\rho_p G_p$ relative to $\rho_w G_w$, the more profits tend to concentrate in the upper (lower) tail of the distribution, and an increase in distributed profits would mean further increasing (reducing) their income, raising (lowering) inequality.

$$\text{If } \rho_w G_w < \rho_p G_p \Rightarrow \frac{\partial G_y}{\partial d} > 0 \Rightarrow \frac{\partial u}{\partial d} < 0 \tag{24}$$

$$\text{If } \rho_w G_w > \rho_p G_p \Rightarrow \frac{\partial G_y}{\partial d} < 0 \Rightarrow \frac{\partial u}{\partial d} > 0 \tag{25}$$

In other words, the percentage of distributed profits amplifies the impact of profits distribution on inequality and aggregate demand.

3 Some distributional examples

To better understand the properties of the model, I simulate in this section four particular distributions among the infinite possibilities[11]. For simplicity, the following simulations are based on the standard model without distinguishing between distributed and not distributed profits. This exercise is similar in spirit to the one conducted by Milanovic (2018), but the distributional part is integrated within the macroeconomic context. Firstly, I simulate a 'classical' income distribution, as it might have been in nineteenth-century capitalism. The population is divided into two groups: 70 % of people earn only labor income and the remaining 30 % draw their income only from capital. There is perfect equality in wages and profit earned within those two groups. Thus, this distribution is characterized - to employ the terminology of Ranaldi (2021) - by a very high compositional inequality. In statistical terms, there can be 'between' inequality in this distribution, but there is always 'within' equality. This does not imply that G_w and G_p are equal to 0 because the Gini indices are computed on the whole population, not on subgroups. Starting from the bottom-left panel of Figure (2), it can be noted that ω^* exists and is equal to 0.7. For wage share values smaller than 0.7, rising the wage share to stimulate aggregate demand is an effective policy, as it increases capacity utilization. This is no more true for values beyond 0.7, for which, on the contrary, an effective policy would be increasing the profit share. Note that the reaction of aggregate demand to changes in the functional distribution (bottom-left panel) strictly follows the pattern of the response of personal inequality to changes in functional distribution (upper-left panel): the economy is wage-led (profit-led) only as long as an increase in the labor share reduces (increases) personal inequality. In the two right panels, it can be noted that to the left of ω^*, ρ_w and ρ_p are equal to -1 and 1, respectively, and constant. The two values reflect the fact that, up to ω^*, wage (profit) income is a decreasing (increasing) function of the income rank, as all wage (profit) earners lie in the bottom (upper) part of the distribution. The situation reverses beyond

ω^*, when ρ_w and ρ_p become equal to 1 and -1, respectively. The two parameters are constant up to ω^* and beyond it because as ω increases, there is no change in the income ranking. The only exception is when ω is equal to 0.7, the point at which all the wage earners simultaneously surpass in the income rank all the profit earners.

[Figure 2]

In the second distribution (Figure 3), wage and profit levels are correlated, as it could be in a population only made of self-employed autonomous workers, where their stock of wealth is proportional to their wage, assuming a uniform rate of return on capital. All people earn both labor and capital income. Thus, there are no more subgroups here, but the more an individual earns a high wage, the higher his or her capital income. The parameters ρ_w and ρ_p equal 1 for every ω (right panels) since both the wage and the profit level are an increasing function of the income ranking. In this case the demand regime *type* and its *strength* depend only on which between G_w and G_p is greater. Indeed equation (13) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w - G_p \tag{26}$$

In this example, the economy is always profit-led since it is assumed that $G_w > G_p$. Moreover, ω^* in this case does not exist, since $\frac{\partial \rho_w}{\partial \omega}$ and $\frac{\partial \rho_p}{\partial \omega}$ are always zero and the denominator of equation (15) is consequently null. These characteristics can be noted looking at the two left panels: an increase in the wage share always raises personal inequality (upper-left panel), leading to a lower capacity utilization (bottom-left panel). A redistribution policy from wage to capital is always effective in stimulating aggregate demand. Summing up, the economy is always wage-led (profit-led) if $G_w < G_p$ ($G_w > G_p$), and a threshold value for ω beyond which there is a regime change does not exist.

[Figure 3]

The third distribution (Figure 4) is characterized by profit equality[12] and wage inequality: again, all people have both labor and capital income, and everyone earns the same profits, but they earn different wages. It is a sort of economy of cooperatives where everyone owns the same stock of capital, but skills and wages are different. G_p and ρ_p are equal to 0 (bottom-right panel), as profit income is equally distributed and uncorrelated with income ranking. Instead, ρ_w is always equal to 1 (upper-right panel), being wages an increasing function of income ranking. Equation (13) reduces to:

$$\frac{\partial G_y}{\partial \omega} = G_w \tag{27}$$

Since profits are the income source equally distributed across the population, an increase in labor share in total income increases total inequality by an amount equal to the Gini index for wages. As in the previous example, ω^* does not exist since the denominator of equation (15) is null. These characteristics can be noted looking at the two left panels: an increase in the wage share always raises personal inequality - equal to G_y - (upper-left panel), which in turn leads to a lower capacity utilization (bottom-left panel). A redistribution policy from wage to capital is effective in stimulating aggregate demand. The opposite occurs with wages equally distributed across the population and profits unequally distributed.

[Figure 4]

We have so far investigated particular income distributions; they are helpful to understand model properties but are far from providing a realistic representation of the economy. The fourth and last distribution example (Figure 5) tries to reproduce some traits of income distribution in a real economy. Two Pareto distributions randomly generate individual quotas of wage and profit shares. There is, thus, a continuum of individuals who always differ in income composition and income levels. Differently from previous distributions, as ω changes, there is a continuous change in the income ranking, and the parameters ρ_w and ρ_p are never constant (right panels). These two parameters are never negative despite changing as the labor share increases. This reflects that both wage and profit are always *on average* an increasing function of the income rank. From the two left panels, it is evident that ω^* exists and is equal to 0.67. Therefore, a redistributive policy from capital towards wage is effective to stimulate aggregate demand up to a wage share level equal to 0.67; further increases in the wage share beyond this value reduce capacity utilization. Moreover, it is worth noting that the second derivative of the curve in the bottom-left panel is always negative. This implies that, to the left of ω^*, the effectiveness of re-distributional policies towards wage is decreasing in ω and, to the right of ω^*, the effectiveness of re-distributional policies toward capital is decreasing in $1-\omega$. Summing up, redistributing from capital to wage reduces personal inequality (upper-left panel) up to ω equal to 0.67; beyond this value, the redistribution increases inequality rather than reducing it. This pattern, through the positive relationship between personal inequality and the aggregate saving rate of equation (5), is transmitted to the demand schedule (bottom-left panel). Initially positively sloped in the (ω, u) plane, the demand schedule continuously rotates downward, eventually becoming negatively sloped once outpassed ω^*.

[Figure 5]

Functional income distribution is not the only distributional variable that can affect aggregate demand in this model, and the last distributional example can be used to analyze also the impact of changes in inequality among wages and profits. As already pointed out when discussing equation (15), provided ρ_w and ρ_p are positive, as it is likely to be in a real economy, a reduction in wage inequality shifts ω^* to the right, increasing the span of ω over which the economy is wage-led (Figure 6). On the contrary, a reduction of profit inequality shifts ω^* to the left, reducing the span of ω over which the economy is wage-led (Figure 7).

[Figure 6]

[Figure 7]

Moreover, from equation (6), it is clear that provided ρ_w and ρ_p are positive, a decrease in wage and/or profit inequality always reduces overall inequality, which in turn stimulates aggregate demand. This is evident in Figures (6) and (7), where, following a reduction in G_w or G_p, the new curve always lies above the old one. Thus, as opposed to policies that aim to change the functional distribution, reducing inequality within wages and profits always stimulates aggregate demand under plausible parameter values.

Lastly, it is left to the reader to imagine how the curves behave in the case where the economy is characterized by perfect income composition equality. In this case - as mentioned in the introduction - a change in the functional distribution has no impact on personal inequality and, therefore, on aggregate demand. It follows, for example, that a society with a low level of compositional inequality is better shielded from structural changes leading to greater automation of the production process.

4 The 'expenditure cascades' case

What we have seen so far was based on a positive relationship between inequality and the aggregate saving rate. Since, as pointed out by Prante (2018), this is a sensitive assumption, the purpose of this section is to show that the main findings of this paper hold even if an inverse relationship links the aggregate saving rate and inequality. To this aim, suppose that, in place of equation (5), we have[13]:

$$s = a_0 - a_1 G_y \tag{5b}$$

Inserting (5b) in (2) we get the new saving function. The rest of the model is the same and it would be redundant to repeat it here. There are only two other differences. The first regards equation (38) in Appendix B, which now becomes:

$$\frac{\partial s}{\partial G_y} = -a_1 < 0 \tag{28}$$

This condition implies that, differently from the model of sections 2 and 3, a reduction (increase) in inequality increases (reduce) the aggregate saving rate. The second difference, stemming from equation (28), is that now the economy is wage-led (profit-led) if an increase in the labor share raises (lowers) inequality rather than reducing it. Therefore, as in Section 2, it is sufficient to study the conditions under which an increase of the labor share leads to a higher or lower personal inequality: these conditions are the same stated in equation (13) to (15). In other words, the determinants of ω^* are the same; the only difference is that now the economy is profit-led to the left of ω^* and wage-led to the right. This happens because a reduction in inequality - which as before occurs to the *left* of ω^* - now increases the saving rate. Likewise, an increase in inequality - which as before occurs to the *right* of ω^* - now decreases the saving rate. The only difference with the model of Section 2 concerns the span of ω over which the demand schedule is wage-led or profit-led, which results reversed. However, all the results of Section 2 - the possibility of an endogenous change in the regime *type* and its *strength* and the role of profit and wage inequality in determining them - are preserved.

The differences with the model of Sections 2 and 3 can be better understood by simulating the distribution 4 of Section 3 again.

[Figure 8]

The only difference with Figure 5 concerns the demand schedule in the (u, ω) plane, which now results reversed.

5 The case of a 'profit share-augmented' investment function

So far, we have seen that a profit-led regime is possible even without assuming that investment demand is a function of the profit share. Nevertheless, what would happen if the investment function is à la Badhuri and Marglin? In such a case, in place of equation (1), we would have:

$$g^i = \gamma + \gamma_u(u - u_n) + \gamma_p(1 - \omega) \tag{1b}$$

Where $\gamma_p \geq 0$. Equating this investment function to the saving equation (2) yields the following equilibrium capacity utilization rate:

$$u = \frac{[\gamma + \gamma_p(1 - \omega) - \gamma_u u_n]\, v}{s(\omega, G_w, G_p) - \gamma_u v} \tag{29}$$

The difference with equation (11) is that now the profit share has an impact not only on the denominator via the aggregate saving rate but also on the numerator through the parameter γ_p. The next natural step would be to compute ω^*. However, since this is not as straightforward as with the original investment function, I will only simulate here what happens to ω^* when γ_p varies to avoid overburdening the paper. To do this, I use an income distribution similar to the one in figure (5) and analyze how the demand schedule changes in the (u, ω) plane as the sensitivity of the investment to the profit share (γ_p) increases. In other words, we can see how the bottom-left panel of figure (5) changes. Since the variations in this parameter do not affect the other three panels of figure (5) we can leave them out and focus only on the corresponding bottom-left panel (Figure 9).

[Figure 9]

As the sensitivity of investment to the profit share increases, on the one hand, this stimulates the aggregate demand, as the new curve always lies above the old one. On the other hand, ω^* shifts leftwards, meaning that the span of the wage share over which the economy is wage-led reduces. Note that when $\gamma_p = 0$, we are back to the baseline model of section (2). The effect of an increase in the γ_p parameter on both the *aggregate demand* level and the *distribution-ledness* of the economy is similar to the effect of a reduction in profit inequality (Figure 7). However, the transmission mechanism is entirely different. Indeed, a change in profit inequality affects aggregate demand through the saving rate (consumption demand channel), while a change in sensitivity of investment to the profit share affects investment demand.

6 Conclusion and considerations

The paper assumes - as done by others in the literature - that the saving rate is a function of personal, rather than functional income distribution. This means that individuals make their consumption decisions based on their position in the income rank and not on their income type. The paper shows how a series of significant results stems from this assumption: (i) wage and profit inequality play a symmetric, though opposite, role in determining the utilization regime *type* and its *strength* - the degree of wage or profit-ledness of the demand regime; (ii) as the labor share increases, depending on the distribution of wages and profits, there may be an endogenous regime change, i.e. there can be a threshold value of the wage share where the economy shifts from a wage-led to a profit-led utilization regime. (iii) Even without passing such a threshold, as the labor (profit) share increases, the effectiveness of redistributional policies - i.e. the degree of wage or profit-ledness of the utilization regime - is decreasing in the wage (profit) share level under plausible parameter values. (iv) The percentage of distributed profits amplifies the impact of profits distribution on inequality and aggregate demand.

These results hold even if the saving function is of the "expenditure cascade" type rather than a la Carvalho and Rezai (2016), i.e., the relationship between personal inequality and the saving rate is negative rather than positive. The only difference concerns the span of the wage share over which the demand schedule is wage-led or profit-led, which results reversed. Also, the results are robust even considering the investment demand as a function of the profit share. There are other points affecting our results that are worth discussing. Obviously, different results would be possible if the *distribution schedule* was not simply assumed flat as we did in the paper. Another point concerns the assumption that individual shares of total wealth are constant. As the savings flows do not sum up to the individual stock of wealth over time, this confines the model to a short to medium-run perspective. In a long-run framework, as highlighted by Ederer and Rehm (2019) and Palley (2017), individual shares of total wealth must be endogenized or, put it differently, wealth distribution must be taken into account. Further research along this line is required to extend the model to a long-run perspective.

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Appendices

A

The aggregate saving function can be derived similarly to Carvalho and Rezai (2016). Assuming that income distribution follows a Pareto type I distribution, the mean μ and the median z of a Pareto distribution are defined as:

$$\mu = \frac{x\alpha}{\alpha - 1} \text{ and } z = 2^{\frac{1}{\alpha}} \tag{30}$$

Where x is the Pareto index.

Individual saving decisions are described by Equation (4) here repeated for convenience:

$$S_i = a_0 Y_i + a_1(Y_i - Y_m) \tag{4}$$

The aggregate saving is equal to the sum of all individual savings:

$$S = \int [a_0 Y_i + a_1(Y_i - Y_m)] f(Y)dY = a_0\mu + a_1(\mu - z) = \left[a_0 + a_1\left(1 - \frac{z}{\mu}\right)\right]\mu \tag{31}$$

With a constant coefficients production function it is true that $\mu L = \frac{uK}{v}$, or normalizing L to one ($L \equiv 1$):

$$\mu = \frac{uK}{v} \tag{32}$$

From equation (30) $x = \mu\frac{\alpha-1}{\alpha}$ or, substituting equation (32), $x = \frac{\alpha-1}{\alpha}\frac{uK}{v}$. Therefore, equation (32) can be rewritten as:

$$\mu = \frac{\alpha - 1}{\alpha}\frac{uK}{v}\frac{\alpha}{\alpha - 1} \tag{33}$$

Substituting equation (33) in equation (31):

$$S = \left[a_0 + a_1\left(1 - \frac{2^{\frac{1}{\alpha}}(\alpha - 1)}{\alpha}\right)\right]\frac{uK}{v} \tag{34}$$

Dividing by $Y = \frac{uK}{v}$ we obtain the saving function in terms of the saving rate:

$$s = \left[a_0 + a_1 \left(1 - \frac{2^{\frac{1}{\alpha}} (\alpha - 1)}{\alpha} \right) \right] \tag{35}$$

Bearing in mind that the Gini index of a Pareto distribution is defined as $G_y = \frac{1}{2\alpha - 1}$, equation (35) can be rearranged as:

$$s = a_0 + a_1 \left(1 - 4^{\frac{G_y}{1+G_y}} \frac{1 - G_y}{1 + G_y} \right) \tag{5}$$

B

The partial derivative of equation (11) with respect to ω is:

$$\frac{\partial u}{\partial \omega} = \frac{-(\gamma - \gamma_u u_n)v}{[s - \gamma_u v]^2} \frac{\partial s}{\partial \omega} \tag{36}$$

Where:

$$\frac{\partial s}{\partial \omega} = \frac{\partial s}{\partial G_y} \frac{\partial G_y}{\partial \omega} \tag{37}$$

The first term is always positive, indeed:

$$\frac{\partial s}{\partial G_y} = a_1 4^{G_y/(1-G_y)} \left[\frac{2(1+G_y) - (1-G_y)\ln 4}{(1+G_y)^3} \right] > 0 \tag{38}$$

Therefore, the sign of $\frac{\partial u}{\partial \omega}$ depends only on $\frac{\partial G_y}{\partial \omega}$.

C

Let's define:

$$\rho_w = \frac{cov(W, f(Y))}{cov(W, f(W))} = \frac{\sum(W_j - \overline{W})(f_j(Y) - \overline{f}(Y))}{\sum(W_i - \overline{W})(f_i(W) - \overline{f}(W))} = \frac{A}{B} \tag{39}$$

The first partial derivative of ρ_w with respect to ω is:

$$\frac{\partial \rho_w}{\partial \omega} = \frac{\frac{\partial A}{\partial \omega} B - \frac{\partial B}{\partial \omega} A}{B^2} \tag{40}$$

Where B is always greater than zero, $\frac{\partial B}{\partial \omega}$ is always positive and constant, A can be either positive or negative, and $\frac{\partial A}{\partial \omega}$ is smaller than zero if A is negative, and greater than zero if A is positive. The sign of equation (40) depends on the sign of the numerator, which will always be greater or equal to zero, as can be noted in figure (10) and (11). In those two figures, random values[14] of w_i and p_i are generated from a Pareto distribution with different combinations of the parameter α[15]. For each of them, the responses of ρ_w and ρ_p to changes in ω are plotted on the vertical axis.

[Figure 10]

[Figure 11]

In particular, as ω increases, if at least one individual surpasses in the income ranking someone else with a lower individual share w_i of total wage mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} > \frac{A}{B} \Rightarrow \frac{\partial \rho_w}{\partial \omega} > 0 \tag{41}$$

If, instead, the increase in labor remuneration is not sufficient to make at least one individual better off than someone else with a lower individual share w_i of total wage mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} = \frac{A}{B} \Rightarrow \frac{\partial \rho_w}{\partial \omega} = 0 \tag{42}$$

Analogous reasoning applies to ρ_p. By expressing ρ_p in the same form as equation (39), as ω increases, if at least one individual surpasses in the income ranking someone else with a higher individual share p_i of total profit mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} < \frac{A}{B} \Rightarrow \frac{\partial \rho_p}{\partial \omega} < 0 \tag{43}$$

While, if the increase in labor remuneration is not sufficient to make at least one individual better-off than someone else with a higher individual share p_i of total profit mass, then:

$$\frac{\frac{\partial A}{\partial \omega}}{\frac{\partial B}{\partial \omega}} = \frac{A}{B} \Rightarrow \frac{\partial \rho_p}{\partial \omega} = 0 \tag{44}$$

This behavior of parameters ρ_w and ρ_p can be better understood in the following example, where the change in income ranking is not continuous given the small number of individuals.

C.1 Example with a three persons economy

Suppose three individuals populate the economy. This makes it easier to understand what happens to ρ_w and ρ_p when ω varies. Each of the three earns the following shares w_i of the total wage mass ωY:

- $w_1 = 0.3$
- $w_2 = 0.7$
- $w_3 = 0$

And the following shares p_i of the total profit mass $(1-\omega)Y$:

- $p_1 = 0.1$
- $p_2 = 0.3$
- $p_3 = 0.6$

Thus, individuals sorted by w_i in ascending order are $i = (3, 1, 2)$, while individuals sorted by p_i in ascending order are $i = (1, 2, 3)$. In figure (12) the level of individual monetary incomes, ρ_w and ρ_p are plotted. As we have stated in equations 42 and (44), as ω increases, ρ_w and ρ_p

remain constant unless there is a change in the income ranking. This happens at $\omega = 0.29$ where individual 2 surpasses individual 3 in the income ranking and at $\omega = 0.62$ where individual 1 surpasses individual 3. In those two points $\frac{\partial \rho_w}{\partial \omega} > 0$ and $\frac{\partial \rho_p}{\partial \omega} < 0$ as stated in equations (41) and (41).

In brief, as ω increases, ρ_w increases every time an individual a surpasses in the income ranking an individual b whose individual share w_i of total labor income is smaller than a, otherwise it stays constant. Likewise, ρ_p reduces every time an individual a surpasses in the income ranking an individual b whose individual share p_i of total capital income is higher than a. Otherwise, it stays constant.

[Figure 12]

D

Substituting $\omega = \omega^* - \varepsilon$ in equation (13), where ω^* is defined as in equation (15), $\varepsilon > 0$, and $\frac{\partial \rho_p}{\partial \omega} \leq 0$ and $\frac{\partial \rho_w}{\partial \omega} \geq 0$, we obtain:

$$\frac{\partial G_y}{\partial \omega} = G_p \frac{\partial \rho_p}{\partial \omega} \varepsilon - G_w \frac{\partial \rho_w}{\partial \omega} \varepsilon \leq 0 \tag{45}$$

Which along with equations (36), (37) and (38) proves equation (16).

Likewise, substituting $\omega = \omega^* + \varepsilon$ in equation (13), we obtain:

$$\frac{\partial G_y}{\partial \omega} = G_w \frac{\partial \rho_w}{\partial \omega} \varepsilon - G_p \frac{\partial \rho_p}{\partial \omega} \varepsilon \geq 0 \tag{46}$$

Which along with equations (36), (37) and (38) proves equation (17).

E Figures

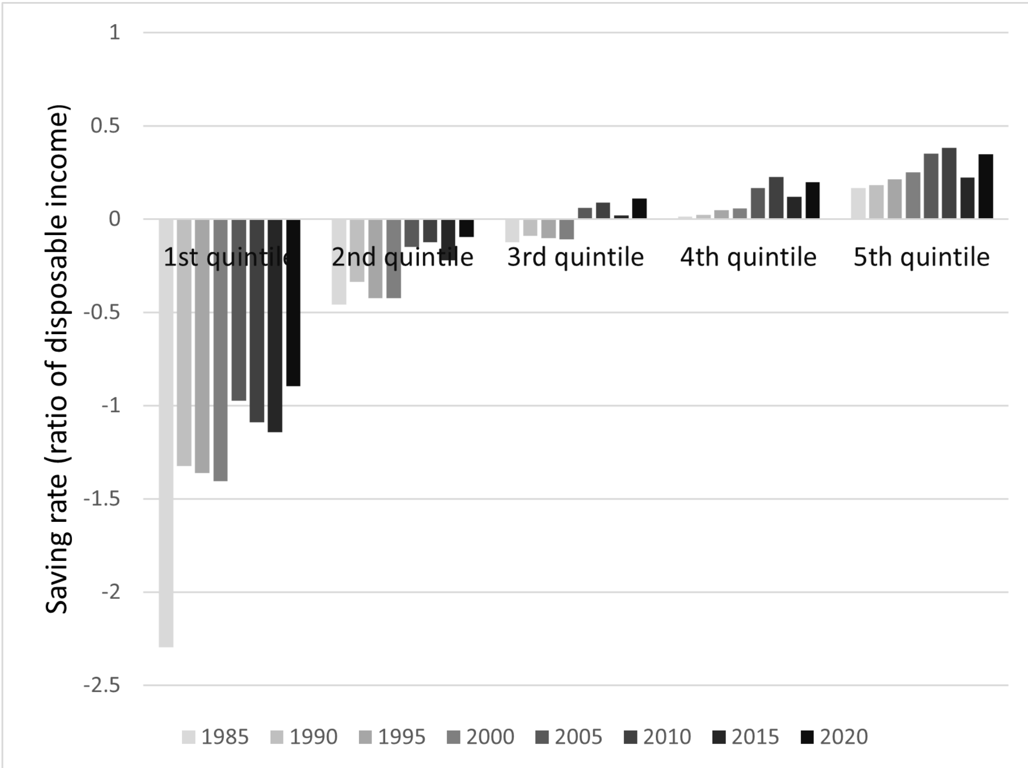

Figure 1: Saving rates across income quintiles in the US

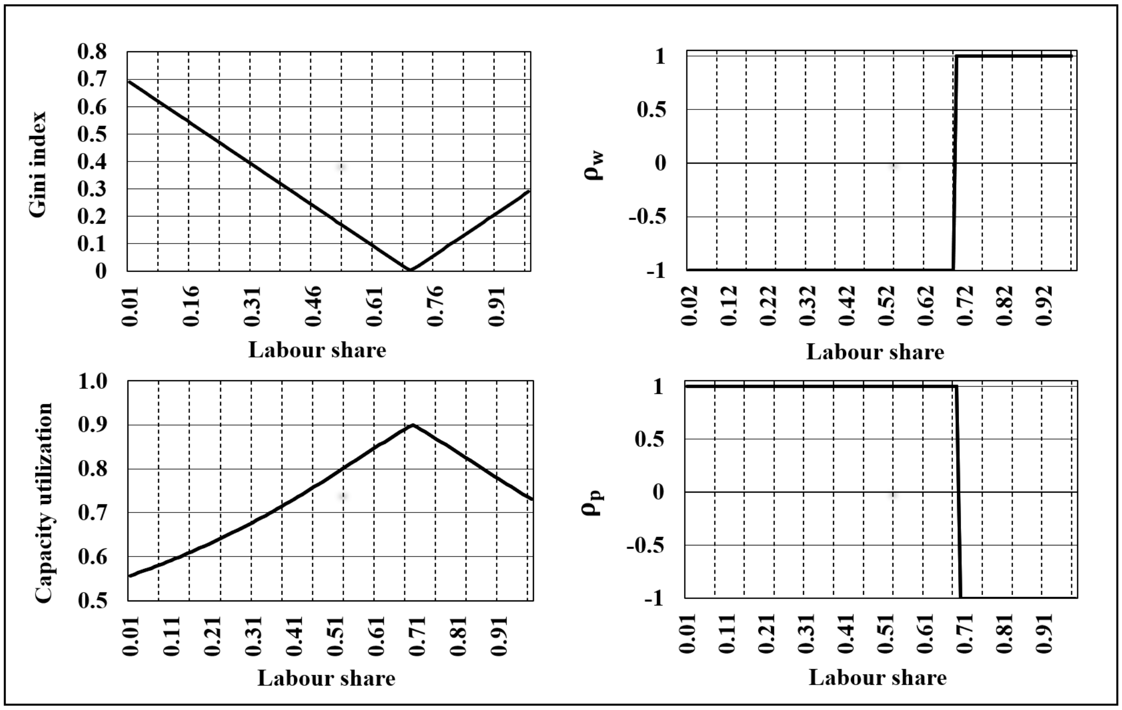

Figure 2: Distribution 1

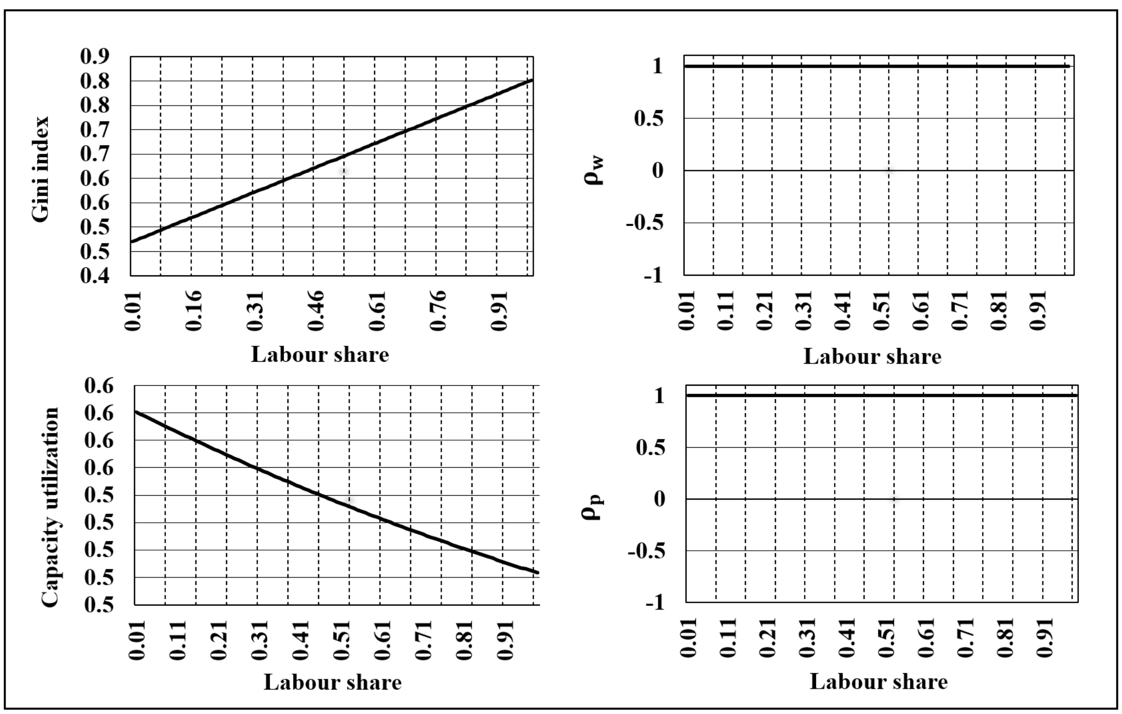

Figure 3: Distribution 2

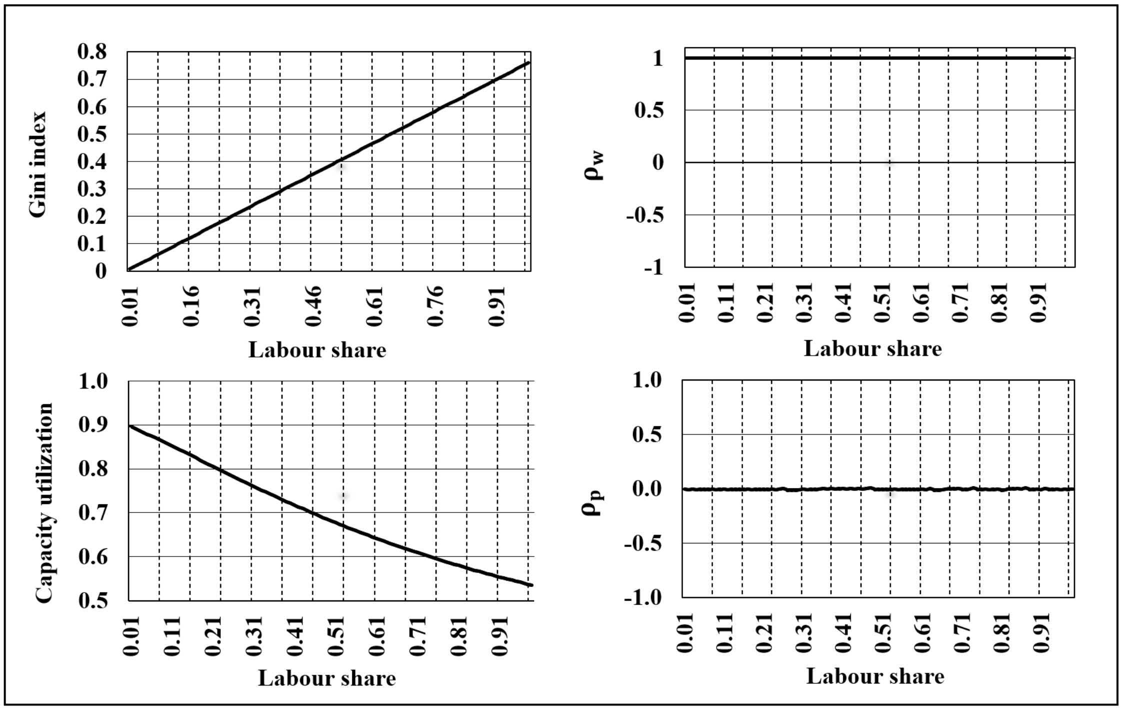

Figure 4: Distribution 3

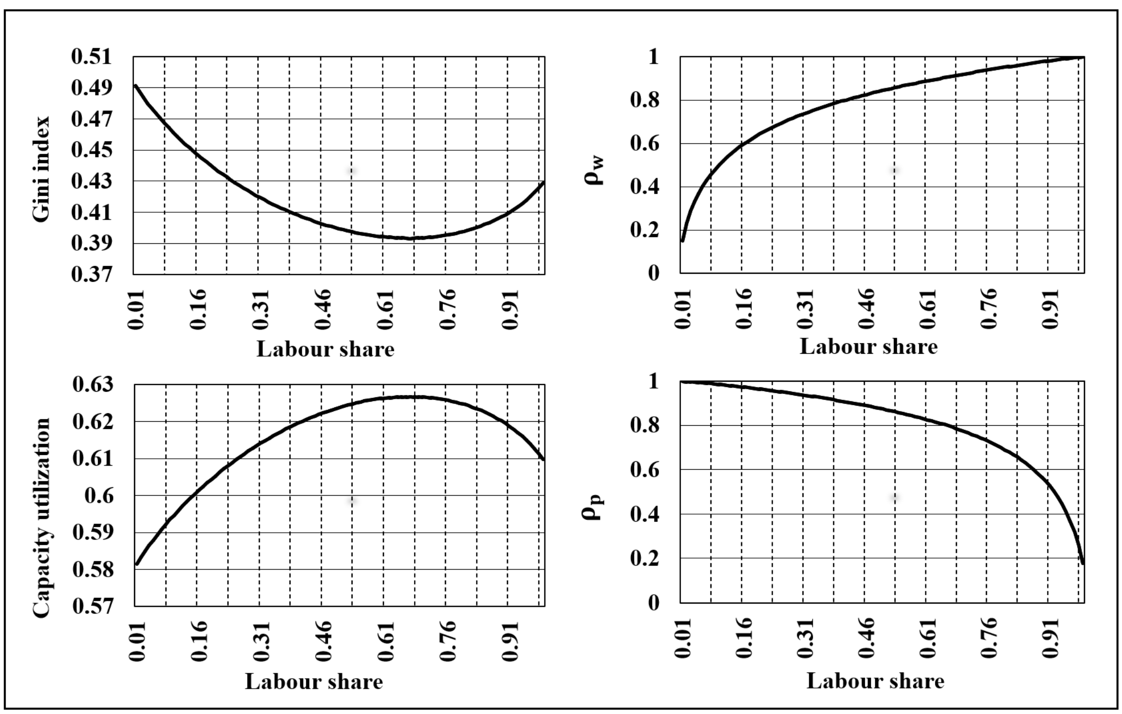

Figure 5: Distribution 4

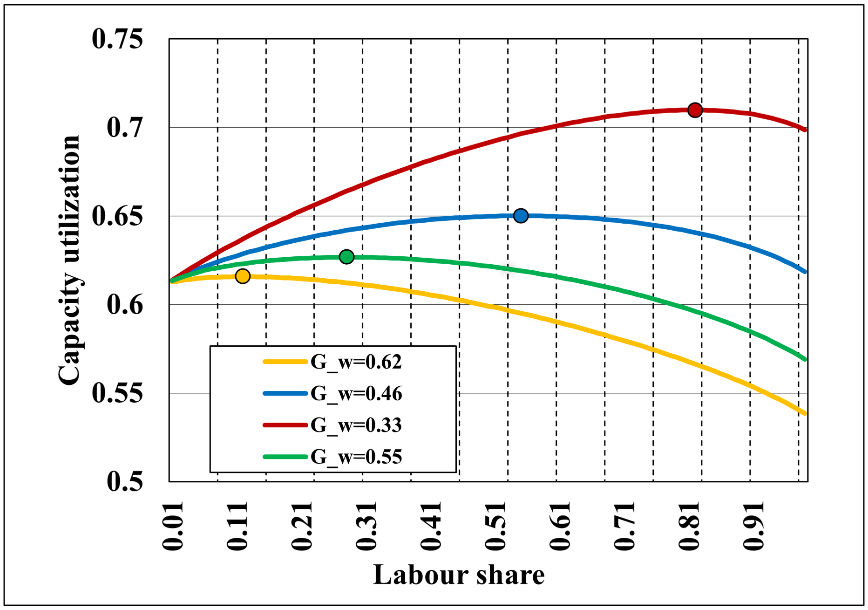

Figure 6: G_w

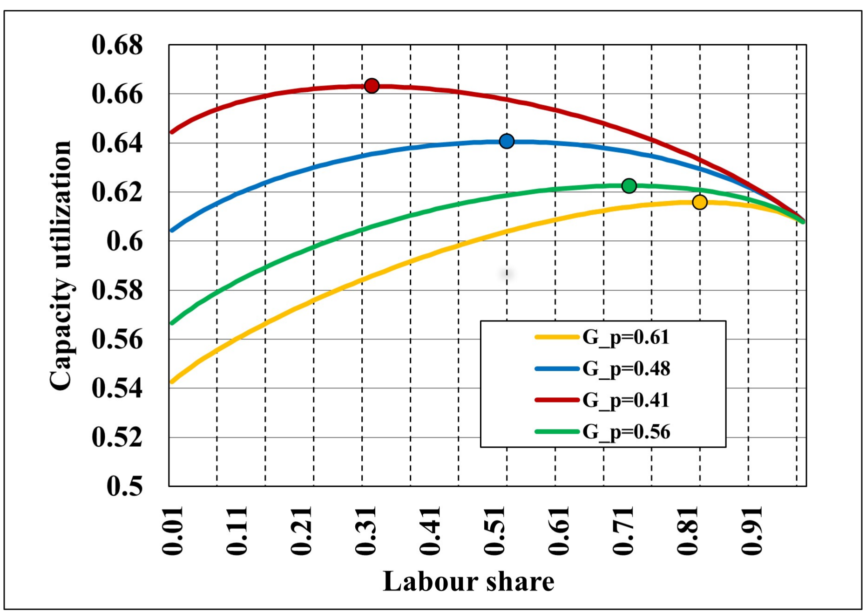

Figure 7: G_p

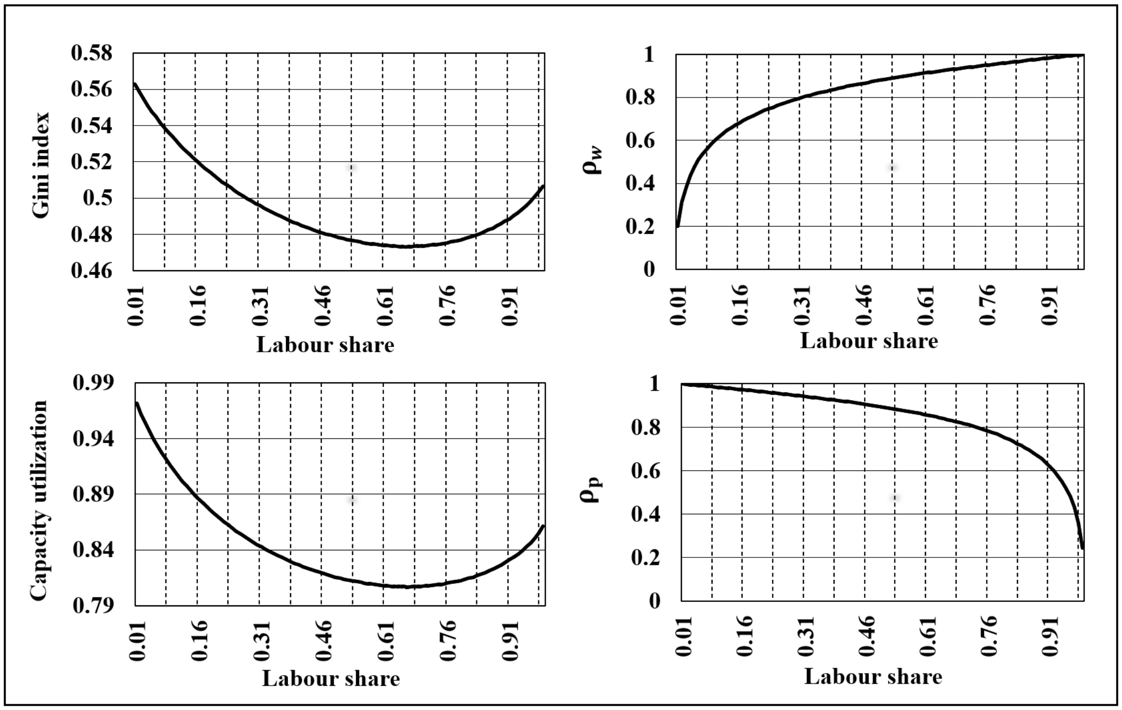

Figure 8: The 'expenditure cascade' case

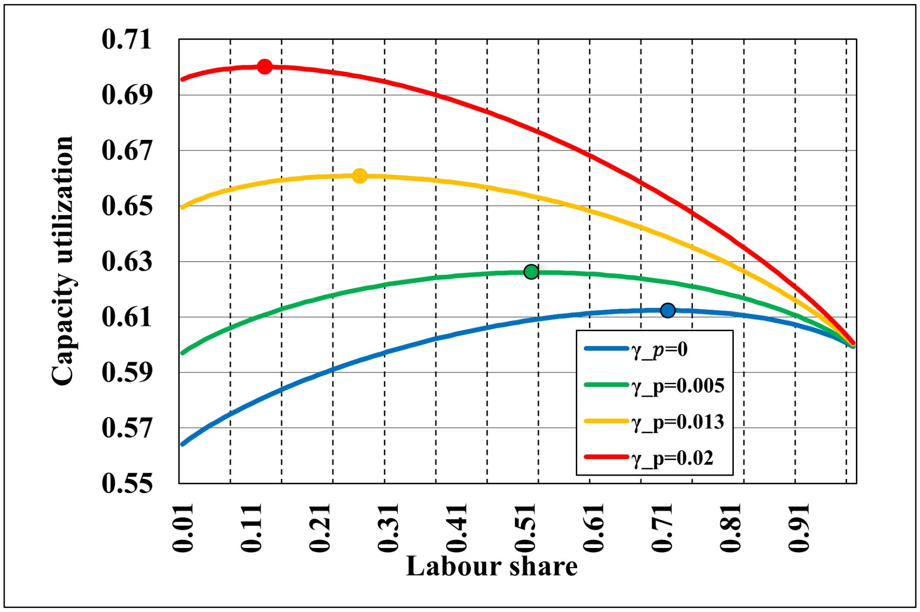

Figure 9: The case of a 'profit share-augmented' investment function

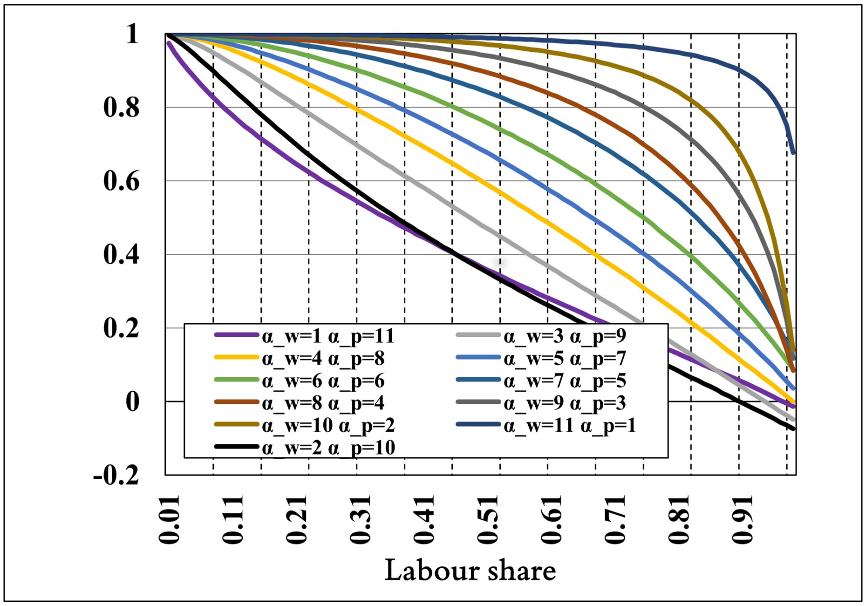

Figure 10: ρ_w

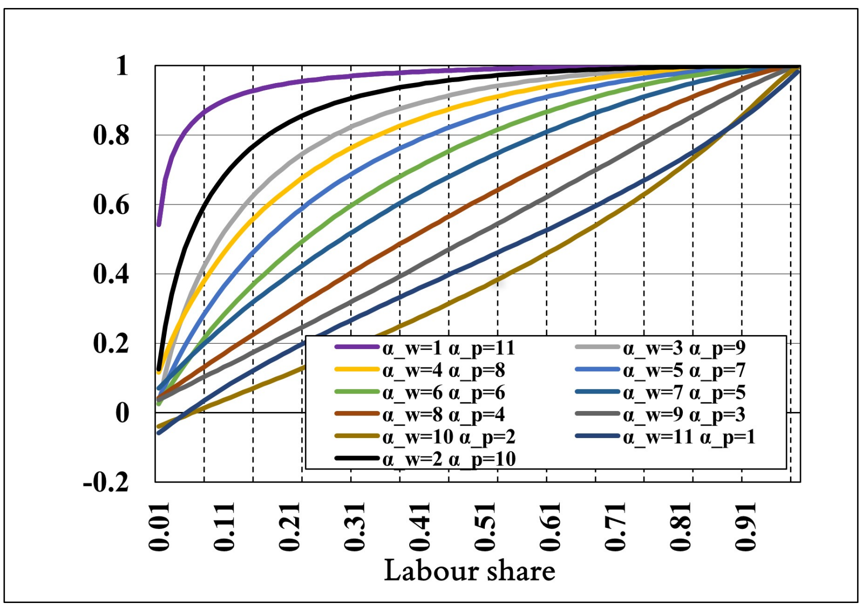

Figure 11: ρ_p

Figure 12: ρ_w and ρ_p in a three persons economy

Notes

[1] Dynan et al. (2004)) provide empirical evidence of the positive relationship between saving rates and lifetime income - based on data from the Panel Study of Income Dynamics (PSID), the Survey of Consumer Finances (SCF) and the Consumer Expenditure Survey (CEX) - along with an extensive examination of empirical and theoretical debate on the issue. Evidence based on recent data from Consumer Expenditure Survey can also be found in Carvalho and Rezai (2016).

[2]Data come from the Consumer Expenditure Survey and are available on the Bureau of Labor Statistics website.

[3]See Milanovic (2018) for an empirical analysis of the strength of the association between functional and personal income distribution.

[4]Obviously, this does not mean that there are not other mechanisms that contribute to the endogeneity of the demand regime.

[5]For an extensive review of personal inequality in Kaleckian models see Hein (2018).

[6]For an extensive review of this strand of literature see Trezzini (2005).

[7]This assumption implies that the flows of savings do not sum up to wealth over time; hence, the model must be intended in a short to medium-run perspective, as for a long-run analysis the assumption that p_i is constant must be released. In the long-run $\dot{p_i} = \frac{S_i}{K} - gp_i$, where K is the stock of wealth/capital. In other words, the wealth share is constant when the individual saves exactly the amount that corresponds to his share in the increase in total wealth; see Ederer and Rehm (2019) and Palley (2017a). This issue is common to all contributions that, on the one hand, have wage earners with a positive saving rate and, on the other hand, do not consider wealth distribution.

[8]In general, for K income sources, the Gini index can be decomposed as $G = \sum_{k=1}^{K} \rho_k G_k S_k$, where s_k is the share of source k in total income.

[9]See Milanovic (2018) for empirical estimation of Gini correlation coefficients for several economies.

[10]Note that, as we will see in Section 4, this result is reversed if the saving function embodies an 'expenditure cascade' mechanism. If the relationship between personal inequality and the saving rate is negative rather than positive as it is in equation (5), the demand regime is profit-led for $\omega < \omega^*$ and wage-led for $\omega > \omega^*$.

[11]All the following simulations are based on an economy populated by 1000 individuals and the following parameters calibration: $u_n = 0.7$, $\gamma = 0.12$, $\gamma_u = 0.05$, $v = 2$, $a_0 = 0.3$ and $a_1 = 0.2$.

[12]Actually, profits are not strictly equal for all but are generated by a distribution with a very low dispersion which yields negligible individual differences. This modification is necessary since a profit income strictly equal for everyone would have made ρ_p - as well as G_y and u - impossible to compute, as the denominator of Equation (8) would have been null.

[13]This equation, differently from equation (5), is not microfounded. This is because aggregating individual consumption functions where each individual targets a different consumption level as in Frank et al. (2010), is not so straightforward.

[14]1000 values are generated, which corresponds to an economy populated by 1000 individuals.

[15]When income follows a Pareto distribution the Gini index has a closed-form solution. In this case: $G_w = \frac{1}{2\alpha_w - 1}$ and $G_p = \frac{1}{2\alpha_p - 1}$.