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Xu, Lili and Zhang, Yidan and Matsumura, Toshihiro

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Lili Xu[a+], Yidan Zhang[a], and Toshihiro Matsumura[b]

[a] Department of Economics and Trade, Dalian Maritime University, 1 Linghai Road, Dalian, 116026, China; lilixuchonnam@hotmail.com; (L. Xu); zhangyidan@dlmu.edu.cn (Y. Zhang)

[b] Institute of Social Science, the University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo, 1130033, Japan; matsumur@iss.u-tokyo.ac.jp; ORCID:0000-0003-0572-6516

[+] Correspondence: lilixuchonnam@hotmail.com; Tel: +86-411-8472-6698; Fax: +86-411-8472-5667; ORCID: 0000-0001-9660-3145

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Abstract: Price competition is more intense than quantity competition in private oligopolies, wherein all firms are profit maximizers. However, in mixed oligopolies where one state-owned public firm competes with profit-maximizing private firms, price competition may not provide tougher competition than quantity competition. In this study, we introduce common ownership, a distinct feature of recent financial markets, into a mixed oligopoly model and investigate how common ownership affects this ranking. We find that under common ownership, quantity competition is likely to be tougher than price competition. Moreover, we find that common ownership harms welfare regardless of competition mode. Common ownership enhances private firms' profits under Bertrand competition while these may decline under Cournot competition.

Keywords: Cournot model, Bertrand model, common ownership, mixed oligopoly

JEL classification: D43, H42, L13, G32

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1. Introduction

Extant literature shows that competition in oligopoly markets is tougher and thus firms' profits are smaller under price competition than under quantity competition. Under quantity competition, the equilibrium price in homogeneous product markets is proximate to the marginal cost of each firm, but only if the number of firms is sufficiently large. By contrast, under price competition, even a symmetric duopoly yields perfect competition (Vives, 1999).[1] This implies that competition is much more stringent under price competition than quantity competition. In differentiated product markets, price competition leads to lower prices, lower firm profits, higher consumers and total social surplus than quantity competition under moderate conditions. Therefore, antitrust agencies should focus on markets with quantity competition instead of price competition (Shubik and Levitan, 1980; Singh and Vives, 1984; Vickers, 1985).

Ghosh and Mitra (2010) demonstrate that this ranking is reversed in a mixed duopoly in which one state-owned public firm competes with a private firm.[2] Haraguchi and Matsumura (2016) further show that depending on the number of private firms, quantity competition may not be tougher than price competition in mixed oligopolies. This study introduces common ownership among private firms and investigates how the level of common ownership affects the ranking of competition types in mixed oligopolies.

Financial markets have recently witnessed increased participation and involvement of the investment industry (Backus et al., 2021). The growth of financial markets has led to the same set of institutional investors, such as Vanguard, BlackRock, State Street, and Fidelity, holding substantial shares in major listed firms that compete in the same industries (common ownership). For example, BlackRock, Vanguard, and State Street, the top three institutional investors, globally own more than 10% of the shares in listed firms, and are often the largest stockholders of many (Nikkei Market News, 2018/10/24). If these firms are concerned about the interests of their common owners, they are indirectly concerned about other firms' profits. This may lead them to deviate from profit-maximizing behavior. In addition, Moreno and

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[1] For a discussion of an asymmetric duopoly, see Dastidar (1997).

[2] For recent discussions on mixed oligopolies, see Bárcena-Ruiz and Garzón (2020) and Haraguchi and Matsumura (2020).

Petrakis (2022) use a dynamic model to show that all stationary equilibria involve large common investors holding symmetric portfolios, regardless of whether the firms face price or quantity competition.

Common ownership has recently become a subject of discussion, and studies on the issue are diverse. While common ownership softens competition in product or service markets and raises prices, partial ownership by common owners in the same industry may lead firms to internalize industry-wide externalities and improve welfare. López and Vives (2019) find an inverted relationship between the degree of common ownership and welfare. Common ownership internalizes the positive externality of R&D. This welfare-improving effect may dominate the welfare-harming (competition-reducing) effect when the degree of common ownership is small. Sato and Matsumura (2020) investigate a free-entry market and find that common ownership internalizes the business-stealing effect; thus, moderate common ownership may improve welfare. They also show that significant common ownership reduces welfare. Chen et al. (2021) examine a vertically related market and demonstrate that common ownership mitigates the issue of double marginalization. This welfare-improving effect dominates the welfare-harming effect in a downstream market, especially if competition among downstream firms is weak. Hirose and Matsumura (2022) investigate the relationship between common ownership and environmental corporate social responsibility. They show that a moderate degree of common ownership promotes environmental corporate responsibility and reduces emissions, whereas a significant degree of common ownership harms welfare.[3] However, no study has analyzed the relationship between common ownership and the price-quantity comparison in mixed oligopolies.

The remainder of this paper is organized as follows. In Section 2 introduces the model. Section 3 examines the Cournot and Bertrand models. Section 4 compares the results of the Cournot and Bertrand models, and Section 5 concludes the paper. The proofs of the propositions are presented in the Appendix.

2. The model

[3] For a discussion on environmental corporate responsibility, see Poyago-Theotoky and Yong (2019), Lee and Park (2019), Fukuda and Ouchida (2020), Hirose et al. (2020), Bárcena-Ruiz and Sagasta (2022), and Xu et al. (2022).

We consider a mixed oligopoly in which one public firm and n (>1) private firms compete in a differentiated product market. Following Dixit (1979) and Singh and Vives (1984), a representative consumer's utility function is:

$$U(q_0, q_1, ...q_n) = A\sum_{i=0}^{n} q_i - \frac{1}{2}\left(\sum_{i=0}^{n} q_i{}^2 + b\sum_{i=0}^{n}\sum_{i\neq j}^{n} q_i\, q_j\right) + y, \tag{1}$$

where q_i is the output of firm i $(i = 0,1, ..., n)$, and y is the consumption of an outside good. The parameter $b \in (0, 1)$ measures the degree of product substitutability. A higher value of b represents greater substitutability, or a lower degree of product differentiation. The inverse demand function is $p_i = A - q_i - b\sum_{i\neq j} q_j$, where p_i $(i = 0,1, ..., n.)$ denotes the price of firm i's product. We further assume that the cost function of the firm is linear and denoted by $C_i = c_i q_i$, $i = 0,1, ..., n$, where c_i is the marginal cost. Then, the profit function of firm i is given by:

$$\pi_i = (p_i - c_i)q_i. \tag{2}$$

Following recent theoretical literature on common ownership (Lopez and Vives, 2019), we assume that each private firm i $(i \neq 0)$ has the following objective function:

$$V_i = \pi_i + \alpha\sum_{j\neq i,0}^{n} \pi_j, \tag{3}$$

where $\alpha \in (0,1)$ denotes the level of common ownership.

Welfare is defined as the sum of profits and consumer surplus:

$$W = \sum_{i=0}^{n}(p_i - c_i)q_i + A\sum_{i=0}^{n} q_i - \frac{1}{2}(\sum_{i=0}^{n} q_i{}^2 + b\sum_{i=0}^{n}\sum_{i\neq j}^{n} q_i\, q_j) - \sum_{i=0}^{n} p_i q_i. \tag{4}$$

Following the standard assumption in mixed oligopolies (De Fraja and Delbono, 1989), we assume that the public firm's objective function is welfare.

3. The analysis

In this section, we analyze the equilibrium outcomes under Cournot and Bertrand competitions. Define $a_i \equiv A - c_i$. We provide the equilibrium outcomes in each case, where the superscript C (B) denotes equilibrium outcomes under Cournot (Bertrand) competition. We assume that all private firms are symmetric (i.e., we assume that a_i is common for all private firms). Let a denote this common a_i among private firms.

3.1 Cournot competition

First, we consider Cournot competition in a mixed oligopoly wherein firms choose quantity. The first-order conditions for the public and each private firm are, respectively,

$$\frac{\partial W}{\partial q_0} = a_0 - q_0 - b\sum_{i=1}^{n} q_i = 0,$$

$$\frac{\partial V_i}{\partial q_i} = \alpha - 2q_i - bq_0 - b(1+\alpha)\sum_{i\neq j} q_j = 0 \; (i \neq 0). \tag{5}$$

From the first-order conditions, we obtain the following reaction functions for the public and private firms, respectively:

$$R_0^C(q_i) = a_0 - b\sum_{i=1}^{n} q_i,$$

$$R_i^C(q_j) = \frac{\alpha - bq_0 - b(1+\alpha)\sum_{i\neq j} q_j}{2} \; (i \neq 0). \tag{6}$$

These functions lead to the following equilibrium quantities:

$$q_0^C = \frac{a_0(2+b(n-1)(1+\alpha)) - abn}{2-b^2n+b(n-1)(1+\alpha)},$$

$$q_i^C = \frac{\alpha - ab}{2-b^2n+b(n-1)(1+\alpha)} \; (i \neq 0). \tag{7}$$

Intuitively, the output of each private firm decreases with α. When α is larger, each private firm is more concerned with other private firms' profits, and thus, each firm prefers a smaller output to increase the other private firms' profits. As the strategies are strategic substitutes, the smaller the output of each private firm, the higher the output of the public firm. Therefore, the output of the public firm increases with α.

By substituting these equilibrium quantities into the inverse demand functions, we obtain the following equilibrium prices:

$$p_0^C = c_0,$$

$$p_i^C = \frac{A(2-b(1-n(1-b+\alpha)+\alpha)) - a - b(a_0 + a(n-1-bn) + a_0 b(n-1)\alpha)}{2-b(1-n(1-b+\alpha)+\alpha)} \; (i \neq 0). \tag{8}$$

Note that the price of each private firm increases with α, whereas that of the public firm is independent of α.

Substituting these equilibrium quantities into (2) and (4), we obtain the following outcomes:

$$W^C = \frac{\begin{array}{c} a^2n(3-b(1-n(1-b+2\alpha)+2\alpha)) - 2a_0abn(3+b(1-n(1-b+2\alpha)+2\alpha)) + \\ a_0{}^2(4-b^3(n-1)n+4b(n-1)(1+\alpha)+b^2((1+\alpha)^2+n^2(1+\alpha)^2-n(3+2\alpha(2+\alpha)))) \end{array}}{2(2-b^2n+b(n-1)(1+\alpha))^2},$$

$$\pi_i^C = \frac{(a-a_0b)^2(1+b(n-1)\alpha)}{(2-b^2n+b(n-1)(1+\alpha))^2} \; (i \neq 0). \tag{9}$$

From (9), we obtain the following proposition.

Proposition 1.

(i). W^C decreases with α.

(ii). $\frac{\partial \pi_i^C}{\partial \alpha} \gtrless 0$ if $\alpha \lesseqgtr 1 - \frac{nb}{n-1}$ $(i \neq 0)$.

Welfare decreases with the degree of common ownership through these two routes. First, an increase in the degree of common ownership directly reduces the output of each private firm. In imperfectively competitive markets, the output of each private firm is too small from the welfare perspective; thus, common ownership increases inefficiency. Second, common ownership induces production substitution from private to public firms. As De Fraja and Delbono (1989) and Matsumura (1998) show, production substitution in the opposite direction improves welfare. In our model, the best outcome is equal production by all firms, while common ownership raises the difference between the public and each private firm's outputs. This asymmetry further reduces welfare. As both effects reduce welfare, it can be surmised that common ownership harms welfare.

Proposition 1(ii) may produce a nonstandard result. It states that common ownership among private firms may reduce their profits. The basis of Proposition 1(ii) is as follows: an increase in α reduces the output of each private firm because of the competition-restricting effect of common ownership. This raises the price of each private firm's product and increases their profits. However, because the strategies are strategic substitutes, private firms' reduced outputs lead to an increase in the public firm's output, which then reduces private firms' profits. The latter effect dominates the former when α is large. This leads to Proposition 1(ii).

Proposition 1(ii) implies that the relationship between the degree of common ownership and private firms' profits is an inverted U-shaped one. This is a new revelation that does not appear if all firms are symmetric (i.e., all firms are private).

3.2 Bertrand competition

Second, we discuss the Bertrand model in which firms choose price. The demand function is given by $q_i = \frac{A-Ab-(1+b(n-1))p_i+b\sum_{i\neq j}p_j}{(1-b)(1+nb)}$. The first-order conditions for public and private firms are, respectively,

$$\frac{\partial W}{\partial p_0} = \frac{(1+(n-1)b)(c_0-p_0)+b\sum_{i=1}^{n}(p_i-c_i)}{(1-b)(1+nb)} = 0,$$

$$\frac{\partial V_i}{\partial p_i} = \frac{(1-b)A-\big(1+b(n-1)\big)(2p_i-c_i)+b(1+\alpha)\sum_{i\neq j}p_j+bp_0-\alpha b(n-1)c_i}{(1-b)(1+nb)} = 0\ (i\neq 0) \quad . \tag{10}$$

From the first-order conditions, we obtain the following reaction functions for the public and private firms, respectively:

$$R_0^B(p_i) = \frac{b\sum_{i=1}^{n}(p_i-c_i)+(1+(n-1)b)c_0}{1+(n-1)b},$$

$$R_i^B(p_j) = \frac{(1-b)A+b(1+\alpha)\sum_{i\neq j}p_j+(1+\mathrm{b}(\mathrm{n}-1)(\alpha+1))c_i+bp_0}{2(1+b(n-1))} \quad (i\neq 0) \quad . \tag{11}$$

These functions lead to the following equilibrium prices:

$$p_0^B = \frac{b(A(1-b)-c_i)n+\mathrm{c}_0(1+b(n-1))(2+b(n-1)(1-\alpha))}{2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)+2\alpha)+\alpha)},$$

$$p_i^B = \frac{b(\mathrm{c}_0(1+b(n-1))-bc_in)+(1-b+bn)(A-Ab+\mathrm{c}_i+bc_i(n-1+\alpha-n\alpha))}{(1+b(n-1))(2+b(n-1)(1-\alpha))-b^2n} \quad (i\neq 0) \quad . \tag{12}$$

The prices of both public and private firms increase with α. When α is larger, each private firm is more concerned with other private firms' profits, and thus, each firm prefers a higher price to increase the other private firms' profits under Bertrand competition. As the strategies are strategic complements, the higher price of each private firm increases the price of the public firm. Therefore, the price of the public firm also increases with α.

Substituting these equilibrium prices into the demand functions, we obtain the following equilibrium quantities:

$$q_0^B = \frac{(b(n-1)+1)a_0-abn}{(1-b)(1+nb)},$$

$$q_i^B = \frac{(a-a_0b)(1+b(n-1))(1+b(n-1)(1-\alpha))}{(1-b)(1+bn)(2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)+2\alpha)-\alpha))} \quad (i\neq 0) \quad .$$

(13

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Note that the output of the private firm decreases with α whereas that of the public firm is independent of α.

Substituting these equilibrium quantities into (2) and (4), we obtain the following equilibrium outcomes:[4]

$$W^B = \frac{a_0{}^2(1-b)^3(2-b(1-\alpha))^2+n(1-b)^2H_1+n^2(1-b)bH_2+n^3b^2H_3+n^4b^3(1-\alpha)H_4+n^5b^4H_5}{2(1-b)(1+bn)(2-b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)-2\alpha)-\alpha))^2},$$

$$\pi_i^B = \frac{(a-a_0b)^2(1+b(n-1))^2(1+b(n-1)(1-\alpha))}{(1-b)(1+bn)(2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)-2\alpha)-\alpha))^2} \quad (i \neq 0) \quad .$$

(14

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From (14), we obtain the following proposition.

Proposition 2.

(i). W^B decreases with α.

(ii). π_i^B increases with α.

Welfare decreases with α and each private firm's profit increases with α. Proposition 2(i) and 1(i) of the Cournot model are similar in terms of their results and bases. However, the result from Proposition 2(ii) contrasts with 1(ii) under Cournot competition. Under Cournot competition, a larger α induces a smaller output for each private firm, which increases the output of the public firm. The larger the output of the public firm, the lower the private firm's profit. By contrast, under Bertrand competition, a larger α induces a higher price for each private firm, which raises the price of the public firm. A higher price for the public firm

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[4] $H_1 \equiv a^2(1-b(1-\alpha))(3-b(1-\alpha)) - 2a_0ab(1-b(1-\alpha))(3-b(1-\alpha)) + a_0{}^2b(4(4-\alpha) - b(23-2(10-\alpha)\alpha - b(1-\alpha)(7-5\alpha)))$,

$H_2 \equiv a^2(10-4\alpha-b(17-2(8-\alpha)\alpha-2b(1-\alpha)(3-2\alpha))) - 2a_0ab(10-4\alpha-b(17-2(8-\alpha)\alpha-2b(1-\alpha)(3-2\alpha))) + a_0{}^2b(25-(14-\alpha)\alpha-b(44-42\alpha+8\alpha^2)+2b^2(9-\alpha(13-5\alpha)))$,

$H_3 \equiv a^2(12-23b+11b^2-2(1-b)(5-8b)\alpha+(1-6(1-b)b)\alpha^2) - 2a_0ab(12-23b+11b^2-2(1-b)(5-8b)\alpha+(1-6(1-b)b)\alpha^2) + a_0{}^2b(19-3(6-\alpha)\alpha-b(37-4(11-3\alpha)\alpha)+2b^2(9-\alpha(13-5\alpha)))$,

$H_4 \equiv a^2(6-2\alpha-b(6-4\alpha)) + 4a_0ab(b(3-2\alpha)+\alpha-3) + a_0{}^2b(7-3\alpha-b(7-5\alpha))$, and

$H_5 \equiv (1-\alpha)^2(a^2-2a_0ab+a_0{}^2b)$.

further increases the private firm's profit. Therefore, the private firm's profit increases with α.

4. Comparison

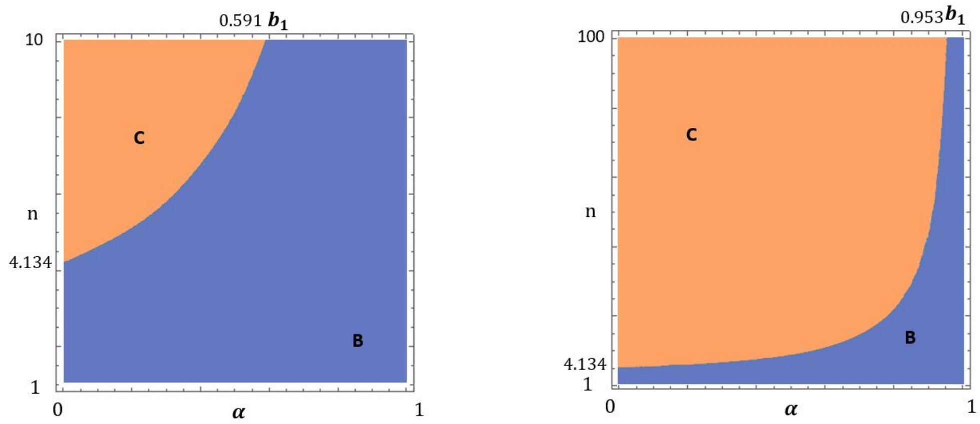
This section compares the results of the Cournot and Bertrand models. First, we compare the profit of each private firm.

Proposition 3.

(i). When $n < 5$, Bertrand competition yields greater private firms' profits (and thus, greater payoff) than Cournot competition.

(ii). When $n \geq 5$, Cournot (Bertrand) competition yields greater profits for private firms than Bertrand (Cournot) competition if α is small (large).

If we do not consider the integer constraint on the number of private firms, we show that Cournot competition can yield greater profits for private firms than Bertrand competition when $n > 4.134$. Figure 1 illustrates this point. In Figure 1, the vertical axis represents *n,* the horizontal axis represents α, and the region labeled *C* indicates that there exists *b* such that Cournot competition yields greater private firms' profits than Bertrand competition does.



$1 < n \leq 10$ $\quad$ $1 < n \leq 100$

Figure 1. Range for which the profit ranking of each private firm can be reversed

In Figures 2 and 3, the vertical and horizontal axes represent α and *b,* respectively. From Figure 2, we can see that the region for which Bertrand competition yields greater profit becomes smaller as n increases, and Bertrand competition always yields greater profits when α is large.

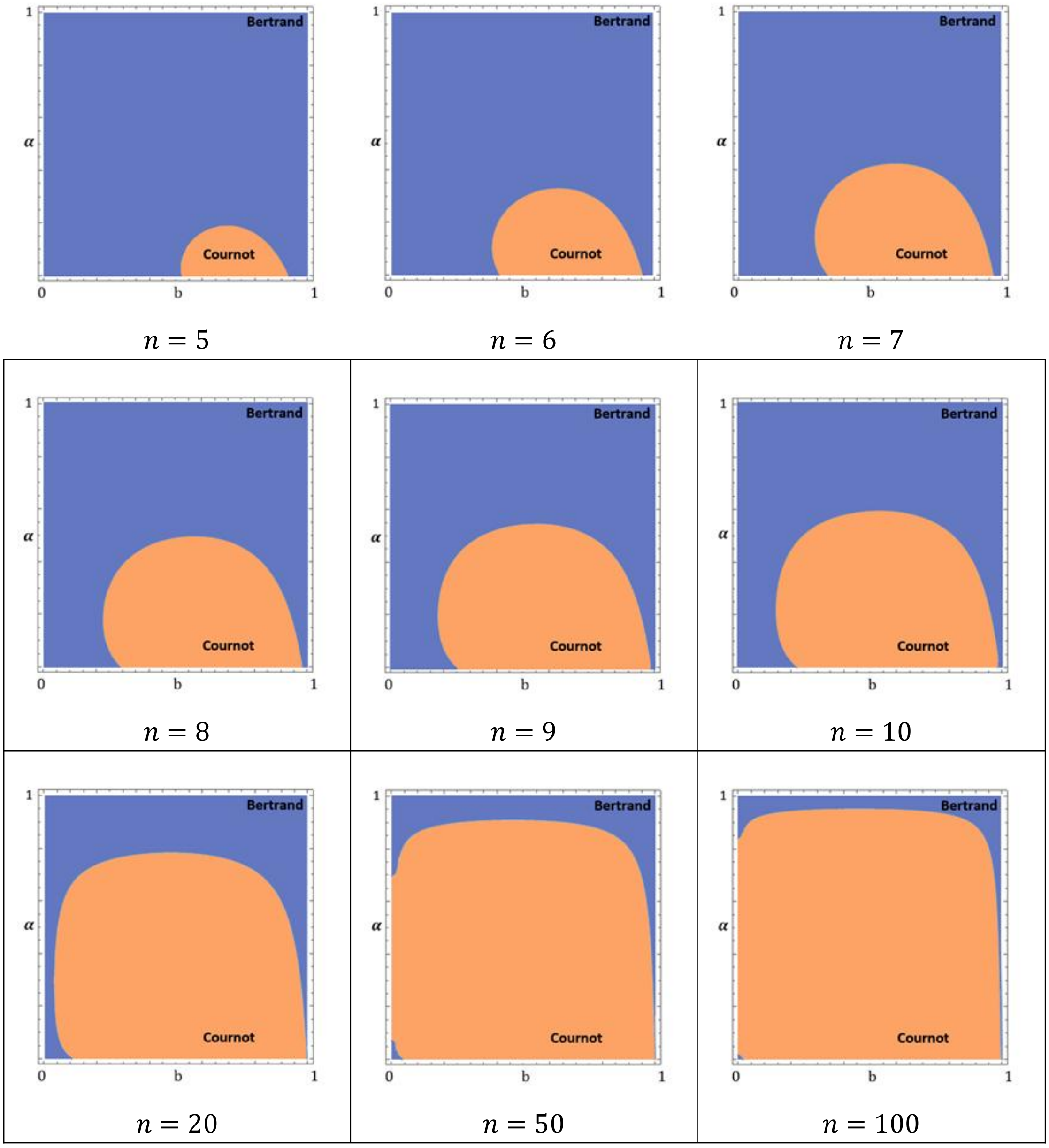


Figure 2. Region for which Cournot competition yields greater private firm's profits

We then compare welfare between the two models. Haraguchi and Matsumura (2016) show that Bertrand competition yields greater welfare than Cournot competition when α is zero. If $\partial W^C/\partial\alpha < \partial W^B/\partial\alpha$ holds, we can directly obtain the welfare ranking. However, $\partial W^C/\partial\alpha > \partial W^B/\partial\alpha$ can hold. Figure 3 shows that $\partial W^C/\partial\alpha < \partial W^B/\partial\alpha$ if α is small and $\partial W^C/\partial\alpha > \partial W^B/\partial\alpha$ if α is large. Therefore, we cannot use the results of Haraguchi and

Matsumura (2016) directly.

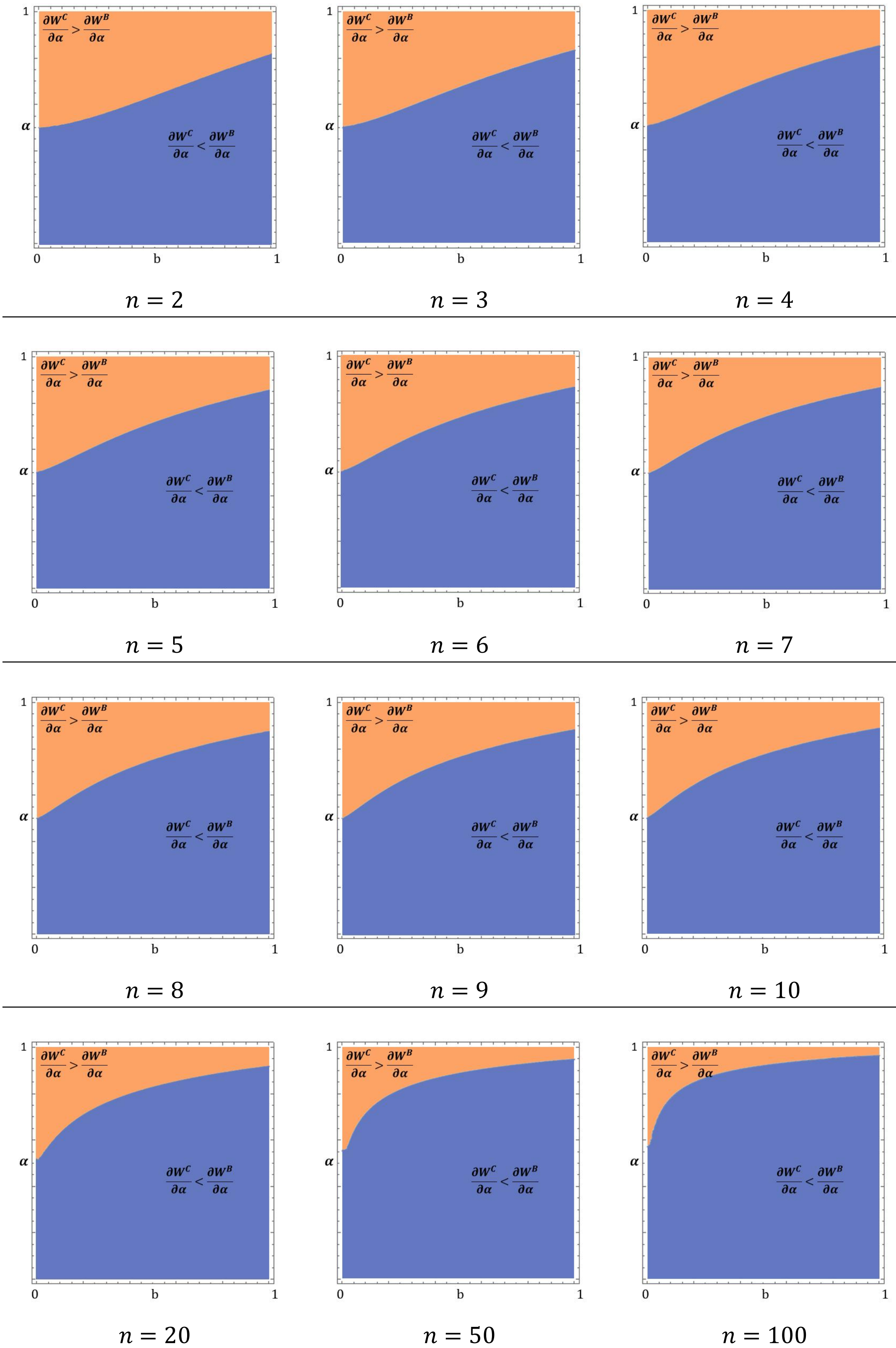


Figure 3. Effect of common ownership on the derivatives

As $W^C - W^B$ is complicated, we fail to conclusively show that it is always negative. Therefore, we substitute $n = 2,3,4, \ldots, 100$ successively and numerically verify that $W^C < W^B$ always holds. This implies Proposition 4.

Proposition 4. Bertrand competition results in higher welfare than Cournot competition if the number of private firms is equal to or smaller than 100.

Figure 3 suggests that the region in which $\partial W^C/\partial\alpha > \partial W^B/\partial\alpha$ becomes smaller as n increases. Thus, Proposition 4 naturally holds for any number of private firms. We check the cases with $n = 200, 500, 1000$, and find that Bertrand competition yields greater welfare than Cournot competition.

5. Concluding remarks

This study examines the effects of common ownership on welfare and private firms' profits under Cournot and Bertrand competitions in a mixed oligopoly. We show that Bertrand competition always results in higher welfare than Cournot competition does, irrespective of common ownership and product differentiation. Furthermore, when the number of private firms is relatively small, Bertrand competition yields higher payoffs for each private firm than Cournot competition. However, when the number of private firms is relatively large and the level of common ownership is low, the opposite result can be found. Our findings suggest that quantity competition is stronger than price competition when the level of common ownership is high.

We also find that regardless of the competition mode, common ownership harms welfare. The relationship between the degree of common ownership and private firms' profits is non-monotonic and has an inverted U-shape. In other words, a moderate degree of common ownership enhances private firms' profits, whereas a significant degree of common ownership reduces them. By contrast, private firms' profits always increase with the degree of common ownership under Bertrand competition.

In this study, we use a model with constant marginal costs. This is a standard assumption in the literature. However, as Xu and Matsumura (2022) show, convex cost functions may play a critical role in oligopolistic markets with nonprofit-maximizing players. However, this extension remains a topic for future research.

Appendix

Proofs

Proof of Proposition 1.

From (9), we obtain

(i). $\frac{\partial W^C}{\partial \alpha} = -\frac{bn(a-a_0b)^2(n-1)(1+b(n-1)\alpha)}{(2-b(1-n(1-b+\alpha)+\alpha))^3} < 0$; and

(ii). $\frac{\partial \pi_i^C}{\partial \alpha} = -\frac{b^2(a-a_0b)^2(n-1)(1-\alpha-n(1-b-\alpha))}{(2-b(1-n(1-b+\alpha)+\alpha))^3} \gtrless 0$ if $\alpha \lessgtr 1-\frac{nb}{n-1}$. Q.E.D.

Proof of Proposition 2.

From (14), we obtain

(i). $\frac{\partial W^B}{\partial \alpha} = -\frac{bn(a-a_0b)^2(1+b(n-1))^2(n-1)}{(2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)-2\alpha)-\alpha))^3} < 0$; and

(ii). $\frac{\partial \pi_i^B}{\partial \alpha} = \frac{b^2(a-a_0b)^2((n-1)(1-\alpha)+b(1-n+n^2(1-\alpha)-\alpha+2n\alpha))}{(1-b)(1+bn)(2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)-2\alpha)-\alpha))^3} > 0$. Q.E.D.

Proof of Proposition 3.

From (9) and (14), we obtain $\pi_i^C - \pi_i^B = \left(\frac{(a-a_0b)^2(1+b(n-1)\alpha)}{\left(2-b^2n+b(n-1)(1+\alpha)\right)^2} - \frac{(a-a_0b)^2(1+b(n-1))^2(1+b(n-1)(1-\alpha))}{(1-b)(1+bn)\left(2-b\left(3-3n-b(1-(3-n)n)-\alpha+\left(b(n-1)^2+n\right)\alpha\right)\right)^2}\right)$. Then, we have $\pi_i^C \lessgtr \pi_i^B$ when $b \lessgtr b_1$ where $0 < b_1 < 1$ when $n \geq 5$. In addition, when $n = 2$, $\pi_i^C - \pi_i^B = \frac{(a-a_0b)^2(1+b\alpha)}{\left(2-2b^2+b(1+\alpha)\right)^2} - \frac{(a-a_0b)^2(1+b)^2(1+b(1-\alpha))}{(1-b)(1+2b)\left(2+b(3-b+\alpha-(2+b)\alpha)\right)^2} < 0$ for all α and b; when $n = 3$, $\pi_i^C - \pi_i^B = \frac{(a-a_0b)^2(1+2b\alpha)}{(2-3b^2+2b(1+\alpha))^2} - \frac{(a-a_0b)^2(1+2b)^2(1+2b(1-\alpha))}{(1-b)(1+3b)(2+b(6+b+\alpha-(3+4b)\alpha))^2} < 0$ for all α and b; when $n = 4$, $\pi_i^C - \pi_i^B = \frac{(a-a_0b)^2(1+3b\alpha)}{(2-4b^2+3b(1+\alpha))^2} - \frac{(a-a_0b)^2(1+3b)^2(1+3b(1-\alpha))}{(1-b)(1+4b)(2+b(9+5b+\alpha-(4+9b)\alpha))^2} < 0$ for all α and b. Finally, $V_i^C - V_i^B = (1+(n-1)\alpha)(\pi_i^C - \pi_i^B)$ and $(1+(n-1)\alpha) > 0$. Therefore, the payoff ranking of each private firm is consistent with the profit ranking. Q.E.D.

Proof of Proposition 4.

We obtain $\frac{\partial W^C}{\partial \alpha} - \frac{\partial W^B}{\partial \alpha} = bn(n-1)(a-a_0b)^2\left(\frac{(1+b(n-1))^2}{\left(2+b(n-1)(3-\alpha)+b^2(1-n(3-n(1-\alpha)-2\alpha)-\alpha)\right)^3} - \right.$

$\left.\frac{(1+b(n-1)\alpha)}{(2-b(1-n(1-b+\alpha)+\alpha))^3}\right)$ and its negativity is checked by substituting $n = 2,3,4,\ldots,100$.

Q.E.D.

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