Economic Effects of Covid-19 and Non-Pharmaceutical Interventions: applying a SEIRD-RBC Model to Italy

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Abstract

We study the economic effects generated by the proliferation of the Covid-19 epidemic and the implementation of non-pharmaceutical interventions by developing a SEIRD-RBC model, where the outbreak and policy interventions shape the labor input dynamic. We microfound an Epidemic-Macro model grounded on the RBC tradition, useful for epidemic and economic analysis at business cycle frequency, which is able to reproduce the highly debated health-output trade-off. Assuming a positive approach, we show the potential of our model by matching the epidemic and macroeconomic empirical evidence of the Italian case.

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1 Introduction

The COVID-19 pandemic has affected our lives remarkably over the last two years. In response to the proliferation of Coronavirus, national governments worldwide implemented the widest spectrum of interventions seeking to mitigate the spread of the outbreak. Since then, our everyday life and customs have been turned upside down by the Non-Pharmaceutical Interventions (NPIs).[1] Even though their implementation was crucial to reducing new infections and saving lives, individuals were hit by another effect, namely, the manifestation of a forceful economic recession.

It is in this context that we propose an analysis on the dynamics of the Covid-19 outbreak along with its effects on the economy, developing an original epidemiological-macroeconomic model, which also takes into account the implementation of NPIs.

Since the outbreak of the disease - starting from the basic *Susceptible-Infected-Recovered* (SIR) model formulated by Kermack and McKendrick in 1927 - epidemiologists have developed more complex models with the purpose of matching the time path of Covid-19 epidemic data and to make predictions to guide governments' actions. Accordingly, we extend the SIR model through the inclusion of the exposed (E) and deceased (D) compartments, yielding the well-known SEIRD model.[2]

[1] NPIs - also known as social-distancing measures - concern disruptions and closures of business activities and travel, testing and mask-wearing requirements as well as lock-downs on a portion of the population.

[2] The choice to consider exposed individuals as well, when studying Covid-19 properties, was followed by Atkeson (2020), Cadwell et al. (2021) and Piguillem and Shi (2020), while SEIRD models were adopted by Loli Piccolomini et al. (2020) and Romano et. al (2020). More sophisticated models involve additional epidemic compartments which allow for an infected individual to be asymptomatic or symptomatic (Romano et al., 2020; Giordano et al., 2020), detected or undetected by testing (Giordano et al., 2020) and present mild, severe or critical health conditions (Gatto et al., 2020; Giordano et al., 2020; Noll et al., 2020). However, we decided not to include these extensions as we want to keep the epidemic framework simple.

Moreover, given the wide-ranging NPIs adopted by governments, epidemiologists have modified standard models to take into account also their effects on transition probabilities, thus formulating models with time-varying parameters. Actually, the infection rate is severely affected by those interventions that seek to reduce contacts among individuals. While the majority of the literature captures the effect of interventions by changing the infection rate at discretion (Giordano et al., 2020), defining it as a step-function (Noll et al., 2020) or as a direct function of time (Loli Piccolomini et al., 2020), we follow the idea of Romano et al. (2020), who define it as a direct function of a proxy of the NPIs.[3] The case fatality rate is another parameter that is highly affected by external factors. In particular, the inception of new variants, vaccines and deficiencies in the healthcare system can alter its value (Cadwell et al., 2021; Sadeghi et al., 2020). We consider these externalities indirectly, setting the case fatality rate time-varying, described by a step-function.

Furthermore, the novel disease and NPIs have pushed economists to analyse the generation and exacerbation of epidemic-driven economic shocks. As highlighted by Baldwin and di Mauro (2020), these shocks are triggered by three sources: First of all, the spreading of Covid-19 initially generates a supply-side shock, i.e. a labor supply shock, as ill and deceased workers cannot supply labor; secondly, further negative shocks are triggered by NPIs as these temporary shut down all activities that require a high level of physical contact, impose a lock-down regime on a portion of the population and suspend transport routes through air and sea. Accordingly, NPIs affect both

[3]From the *Google Covid-19 Community Mobility Report*, they construct a mobility function as the weighted average on the mobility data in different places. This parameter gives an idea of how NPIs have modified community movements in specific locations. Differently, we utilise the *Containment and Health* Index (CH^{Index}) - computed by Hale et al. (2021) - and derive a continuous function describing how the CH^{Index} perturbs the infection rate.

the aggregate supply - via a disruption of industrial production - and the aggregate demand - through shrinkage of exports, imports and consumption.[4] In addition, aggregate demand is affected by economic agents' behavioural responses to the epidemic evolution. Demand for goods and services drops as consumers follow saving-for-emergency, wait-and-see and hoarding strategies. Firms reduce investments due to closures of company plants, supply-chain contagions and negative expectations on future economic activity.

Part of the literature has theorised and modelled aggregate demand shortages triggered by aggregate supply plunges, envisaging the recurrence of a supply-demand vicious spiral (Fornaro and Wolf, 2020; Guerrieri et al., 2022). McKibbin and Fernando (2021) have considered multiple exogenous supply and demand shocks, while for Faria-e-Castro (2021), the Covid-19 outbreak generates a demand shock.[5] Ciccarone et al. (2021) and Gagnon et al. (2020) set up an analysis on the intergenerational costs and benefits of pandemics and lockdowns, employing a life-cycle macroeconomic scheme.[6]

Other researchers have focused the analysis on economic as well as health issues, developing a new category of models that can be defined as SIR-Macro/Health models, where epidemic dynamics are combined with the macroeconomic setup (Alvarez et al., 2021; Eichenbaum et al., 2021; Gonzalez-Eiras and Nieptal, 2020; Piguillem and Shi, 2020). These studies have in common the formulation of a maximisation problem that takes into account

[4]This is since ill consumers are prevented from access to stores due to quarantine restrictions. As a result, the demand for non-essential consumption goods falls.

[5]For instance, the economic effects of Covid-19 epidemic are modelled as a permanent fall in the growth rate of labor productivity (Fornaro and Wolf, 2020), a one-period reduction in the labor supply (Guerrieri et al., 2022), a simultaneous reduction in labor supply, a rise in the cost of doing business, shifts in consumer preferences and increase in equity risk premia on companies and countries (Fernando and McKibbin, 2021) and as a negative shock to the marginal utility of consumption (Faria-e-Castro, 2021).

[6]Both papers introduce the outbreak as an exogenous shock to the mortality rate of the elderly.

a health-output trade-off, where the point of view of a Social Planner who seeks to find the optimal lock-down or social distancing policies is assumed.[7] Differently from the aforementioned literature, we employ an economic model grounded on the *Real Business Cycle* tradition. To link the Macro framework with the SIR framework, we revive the Hansen's version of the RBC model (Hansen, 1985), which introduces a distinction between the intensive and extensive margin of labor.[8] Starting from this specification, we modify the model in order to let the SEIRD block determines exogenously the path of the extensive margin, while economic agents derive optimally the actual worked hours. Additionally, to overcome the manifestation of excessive consumption smoothing, we introduce one heterogeneity among households: In our economy, households are distinguished between *Optimising* and *Rule-of-Thumb* as described, for example, in Galì et al. (2007).

Our approach diverges from the step-up of the SIR-Macro literature, inasmuch as our model is microfounded along the lines of the business cycle tradition. Basically, the SIR-Macro literature employs health-related models, in which the relevant economic trade-offs are shaped by the health status of the economic agents and by the transition probabilities between epidemic compartments. This comes at the cost of giving up the canonical behavioural equations of economic agents, namely, Euler equations are absent from the analysis. Additionally, we do not follow a normative approach, but a positive one, since we aim to build a model that can replicate the empirical evidence of Italy.

Three versions of the model are presented, each featuring a labor supply shock. In the simplest version, Model 0 (M_0), the possibility to work is restricted by

[7] The Social Planner manages a trade-off as individuals, on one hand, benefit from the mitigation of the virus, and, on the other, worsen economically due to disruptions in activities.

[8] where their product defines the actual worked hours.

the harmfulness of the disease. Hence, in the first version, only the epidemic shock is activated. In Model 1 (M_1), we show how the implementation and, successively, the removal of NPIs produce multiple epidemic waves. Even though interventions almost neutralise the epidemic shock, these end up generating a further negative shock on labor supply. In the last version, Model 2 (M_2), we follow a quantitative approach, namely, the epidemic and NPIs shocks are modelled in order to replicate both the epidemiological and business cycle empirical evidence of Italy in the period 24/02/20 - 24/02/22.
There are three main results of this study. First of all, we developed a model for epidemic and economic analysis which is stunningly simple. Actually, M_0 can be seen as a toy model and a starting point for more complex analyses, which seek to capture further aspects. The possibility of reproducing epidemic waves - as done in M_1 - is an example. Secondly, by comparing M_0 with M_1, it is possible to observe the presence of the health-output trade-off, discussed in the literature. In detail, when the government has to decide whether to intervene with NPIs and choose their strictness, it bears a trade-off inasmuch as interventions reduce the number of infections and save lives in the face of a more forceful and long-lasting economic recession. Finally, M_2, our most sophisticated version of the model, is able to match suitably the empirical evidence of Italy, reproducing the path of the five waves of infections and the quarterly conjunctural growth rate of macroeconomic variables.
The paper is organised as follows: in section 2, we described the epidemiological and macroeconomic frameworks on which our model is grounded. From their combination, we obtain the simplest version of our model, M_0. In section 3, the model is modified, allowing for the presence of NPIs. Here, two model versions are illustrated, namely, M_1 and M_2. In section 4, the model is calibrated on epidemic and economic data of Italy and in section 5, results

of the three versions are displayed. Section 6 summarises and concludes.

2 The Epidemiological-Macroeconomic Model

We employed a discrete-time deterministic SEIRD model, where individuals are separated into groups - or compartments - depending on their disease status and can move from one compartment to another as the infectious virus spreads among the population.

The economy is featured by a RBC model, whose standard version is modified by two specifications. We revive the Hansen's version of the RBC model (Hansen, 1985) - making, however, some adjustments - and introduce one heterogeneity among households: they are distinguished between *Optimising* and *Rule-of-Thumb* as described in Galì et al. (2007).

2.1 The Epidemiological Framework

In a canonical discrete-time SEIRD model [9] the population is subdivided among five compartments:

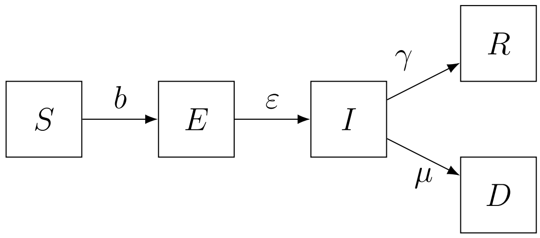
- Susceptible (S), where individuals have not been infected with the disease and are exposed to the risk of infection,
- Exposed (E), composed of individuals who have been infected but cannot transmit the disease due to the incubation period,[10]
- Infected (I), composed of individuals who have been infected with the disease and are capable of spreading it to those in the susceptible compartment,

[9] The model is deterministic, allowing us to exactly predict the path of epidemic variables throughout the progression of the infectious disease.

[10] The period between exposure and onset of clinical symptoms is called the incubation period. In this amount of time, the virus cannot be transmitted. In addition, in this model we assume the absence of asymptomatic individuals. Hence, exposed individuals can only be in the pre-symptomatic phase. For models with asymptomatic individuals, see Giordano et al. (2020) and Romano et al. (2020).

- Recovered (R), where individuals have been infected and then recovered from the disease through immunisation,
- Deceased (D), composed of individuals who have deceased due to the infectious illness.

The dynamic of the population can be represented by the following sequence:



Since we are mainly interested in short-run analysis, we consider a closed population. Accordingly, this model does not take into account births, migration and deaths which are not related to the disease.

The rate at which susceptible individuals become exposed depends on the number of susceptible and infected individuals and their effective contact rate b. This parameter - also known in the literature as the infection rate - accounts for the transmissibility of the disease as well as the mean number of contacts per individual. [11] Consequently, the whole number of susceptible individuals contracting the disease from infectious individuals per unit of time is given by the product bS_tI_t. In other words, this term represents the number of newly exposed individuals in time t. Then, only a fraction ε of exposed individuals become infected in each period, with ε^{-1} representing the incubation period. The rate at which infected individuals move into the recovered compartment depends on the amount of time during which an individual is contagious, which is captured by the recovery rate γ. Hence, γ^{-1} represents the average time of being infected. Finally, the rate at which

[11] The model assumes homogeneous mixing of the population, meaning that all individuals in the population have an equal probability of making contact with one another. However, this does not reflect human social structures, where contact occurs within limited networks.

infected individuals decease is indicated with μ, known as the case fatality rate.

In the epidemiological field, the infectiousness of a disease - namely its transmissibility within a population - is described by the *basic reproduction number*, indicated with R_0. This parameter characterises the initial propagation of an infectious virus and, in a SEIRD model without births and non-disease-related deaths, is computed as the ratio of the infection rate and the sum of the recovery and case fatality rate, $R_0 = b\left(\gamma + \mu\right)^{-1}$.[12]

From the definition of the basic reproduction number, it is possible to write the infection rate as:

$$b = R_0(\gamma + \mu) \tag{1}$$

Summing up, the epidemiological model is described by the following first order non-linear difference equations:

$$S_{t+1} - S_t = -b\frac{S_t I_t}{P} \tag{2}$$

$$E_{t+1} - E_t = b\frac{S_t I_t}{P} - \varepsilon E_t \tag{3}$$

$$I_{t+1} - I_t = \varepsilon E_t - \gamma I_t - \mu I_t \tag{4}$$

$$R_{t+1} - R_t = \gamma I_t \tag{5}$$

$$D_{t+1} - D_t = \mu I_t \tag{6}$$

where population is defined as:

$$P = S_t + E_t + I_t + R_t + D_t$$

[12]The basic reproduction number is the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible. In general, an infectious disease spreads within a susceptible population if R_0 is greater than one. Under this condition, the number of infected individuals increases exponentially over time. Conversely, if $R_0 < 1$, the virus does not spread as the number of infected agents converges monotonically to zero, while with $R_0 = 1$ the infection remains constant.

with the initial condition $S_0 > 0$ and $I_0 > 0$. Additionally, it is imposed that:

$$S_t, E_t, I_t, R_t, D_t \geq 0$$
$$S_t + E_t + I_t \leq P$$

2.2 Linking the SEIRD and RBC Blocks

An epidemic can be seen as an external real shock that perturbs the equilibrium of the economic system. In detail, it damages the health of individuals, compromising their ability and propensity to work and produce until recovery is achieved. Therefore, we decide to take into account this external phenomenon by modelling it as a labor supply shock. After having established which epidemic compartments are still able to work, the SEIRD model deterministically derives the path of the available labor force over time. As result, macroeconomic variables display short-run variations until long-run equilibria are reached again.

We assume that the labor force is equal to the population net of infected and deceased individuals. As a matter of fact, we are supposing that infected individuals are too ill to work, while exposed individuals can still supply labor since they are in a pre-symptomatic phase. Hence, in the absence of NPIs, the labor force - which is equal to the employment level in a RBC model - is given by:

$$N_t = S_t + E_t + R_t \tag{7}$$

From equation (7) it is possible to define the employment rate:

$$n_t = \frac{N_t}{P - D_t} \tag{8}$$

We stress that n_t is the variable that generates a bridge between the two frameworks. As long as the infectious disease is in circulation, the employment rate will change, modifying economic agents' behavioural responses.

2.3 The Macroeconomic Framework

In this study, we employ a RBC model considering the specification of Hansen (1985), where the economy is characterised by *indivisible labor*.[13] From this original version, we borrow the separation between extensive and intensive margin and the shape of the utility function, making, however, some adjustments. In our model, the extensive margin is not a control variable for households and firms, but it is determined by the epidemiological model.[14] The extensive margin can be interpreted as the employment rate n_t. The intensive margin represents the worked hours (h_t^0) and, differently from Hansen's version, is not constant but time-dependent. At each time, it adjusts to variations of the extensive margin and of the actual worked hours (h_t) in order to guarantee the following relation:

$$h_t = n_t h_t^0 \tag{9}$$

where variables are in per capita terms. Hence, the product of the two margins represents the actual worked hours in each time. As shown successively, h_t is derived optimally through the simultaneous solution of households and firms'

[13] In detail, households get to choose the probability of working, but, once in the labor market, they work for a fixed amount of hours. Hansen (1985) introduces into the analysis a distinction between the intensive and the extensive margin of labor, yielding - thanks to the lotteries setup - a utility function linear in labor which holds for any value of the elasticity of labor.

[14] This assumption is particularly suitable for the case of Italy, where the government has introduced a ban on dismissals on 17th March 2020 through the *Cura Italia* decree-law, whose validity has been extended until nowadays.

maximisation problems.

We assume that households' utility function is logarithmic in consumption and still linear in labor as in Hansen (1985), even though the coefficient that multiplies actual worked hours, v_t, is no longer constant. Accordingly, it is represented by the following equation:[15]

$$U(c_t, h_t; h_t^0) = \log(c_t) - v_t h_t \tag{10}$$

Hereafter we refer to v_t as the Hansen's variable, which is defined by:

$$v_t = -\frac{\psi}{h_t^0}\left[\frac{(1-h_t^0)^{1-\theta} - 1}{1-\theta}\right] \tag{11}$$

where ψ and θ are the households' relative preference for labor and the inverse of the elasticity of labor, respectively.

Furthermore, we assume that the economy is characterised by homogeneous firms and heterogeneous households, namely, Optimising (*Opt*) and Rule-of-Thumb (*RoT*) households in the sense of Galì et al. (2007). In particular, *RoT* consumers are financially constrained so that they do not save and invest; they are assumed to consume their current income fully. Labor is supplied by both households, while capital is provided only by *Opt* households. Firms employ the two production factors in the production process, remunerating labor (h_t) with the real wage (w_t) and capital (k_t) with the real interest rate (r_t). Markets are perfectly competitive and complete.

From the solution of the maximisation problem of *RoT* and *Opt* households and firms,[16] we obtain the following equations, i.e. the Euler equation of *Opt* households, the *modified* Hansen labor supply of *RoT* and *Opt* households

[15] The derivation of the utility function is presented in appendix A.

[16] See appendix B.

and the capital and labor demand of firms:

$$\frac{1}{c_t^p} = \beta E\left[\frac{1+r_{t+1}}{c_{t+1}^p}\right] \tag{12}$$

$$w_t = v_t^p c_t^p \tag{13}$$

$$w_t = v_t^r c_t^r \tag{14}$$

$$r_t = (1-\phi)\frac{y_t}{k_t} - \delta \tag{15}$$

$$w_t = \phi\frac{y_t}{h_t} \tag{16}$$

where variables with the superscript p and r refer to *Opt* and RoT households, respectively. Households' time preference is indicated with β, while δ and ϕ are the deprecation rate of capital and the labor share of production, respectively. Aggregation is computed through a weighted average of the corresponding variables for each household type. Formally:

$$h_t^0 = (1-\lambda)h_t^{0,p} + \lambda h_t^{0,r} \tag{17}$$

$$c_t = (1-\lambda)c_t^p + \lambda c_t^r \tag{18}$$

$$i_t = (1-\lambda)i_t^p \tag{19}$$

$$k_t = (1-\lambda)k_t^p \tag{20}$$

where $(1-\lambda)$ and λ are respectively the share of optimising and rule-of-thumb households in the economy.

Combining the SEIRD with the RBC block, we derive the simplest version of our model - labeled as Model 0 (M_0) - where no NPIs are established.[17] In this way, it is possible to observe and analyse the plain effect of the epidemic on the economy. Accordingly, M_0 may be seen as a counterfactual scenario,

[17]In appendix C and D the equilibrium equations of the RBC block and the steady state of the model are illustrated , respectively.

which illustrates epidemic and economic outcomes when the government decides not to intervene to hinder the outbreak from spreading.
Calibration and results of Model 0 are exhibited in paragraph 4.1 and 5, respectively.

3 Non-Pharmaceutical Interventions

In this paragraph, M_0 is modified allowing for the presence of NPIs.
Two versions of the basic model are illustrated. In Model 1 (M_1), NPIs are introduced following a qualitative approach, where it is shown how the implementation of government interventions may generate epidemic waves and a forceful output downturn. In Model 2 (M_2), we assume a quantitative approach, modelling the effects of NPIs on transition probabilities more rigorously in order to match the epidemic and business cycle evidence of Italy.

3.1 Qualitative Approach, Model 1

In M_0, the SEIRD block produces only one peak, as shown in Figure 1. Thus, the outbreak dies out as soon as the first wave ends. This occurs since, following basic epidemic models, we have employed fixed parameters. To obtain multiple waves - which indicate repressions and revivals of the virus - the infection rate must change, reducing and augmenting over time (Atkeson, 2020). Actually, NPIs - such as school and workplace closures, lockdowns, face coverings... - diminish contacts among individuals and thus lessen the infection rate. As these measures are relaxed, contacts rise again, which augments inevitably the infection rate. Consequently, to model the impact of NPIs, we take inspiration from Noll et al. (2020) and introduce a time-dependent infection rate in equations (2) and (3), defined by:

$$b_t = bf(\eta_t) \tag{21}$$

where b is the constant infection rate in the absence of NPIs[18] and $f(\eta_t)$ describes a generic (decreasing) function of NPIs, labeled with η_t. For the sake of simplicity, we assume this function to be equal to:

$$f(\eta_t) = (1 - \kappa\eta_t)$$

with $0 \leq \eta_t \leq 1$ and $0 \leq \kappa \leq 1$.[19] The former can be interpreted as the NPIs rate, which measures the strictness of interventions (1 = strictest), and is described by a step-function, as shown in paragraph 4.2; the latter represents an efficacy coefficient of the overall interventions. As κ tends to 1, the higher is the reduction effect on the infection rate.

Furthermore, to model the negative supply shock generated by NPIs, we modify equation (7), assuming that the labor force is defined by:

$$N_t = (S_t + E_t + R_t)(1 - g(\eta_t)) \tag{22}$$

where $g(\eta_t)$ is a function of the NPIs which represent the fraction of labor force that is effectively kept out from the production process. Its functional form is given by:

$$g(\eta_t) = \alpha\eta_t^2$$

[18] See equation (1).

[19] We take this idea from Alavarez et al. (2020). Accordingly, it is possible to assume a limited effect of NPIs since individuals subject to restrictions might still be subject to some contact and thus infect and be infected.

with $0 \leq \alpha \leq 1$. Hence, the labor force is not only reduced by infected and deceased individuals, but also by the share of individuals ($g(\eta_t)$) who are subjected to restrictions. Parameter α, in the same way as κ, represents an efficacy coefficient, which captures the effectiveness of NPIs in reducing labor supply.[20] As the efficacy coefficient converges to one, NPIs negative effects on labor force augment, thus generating a higher plunge in output. The manifestation of a health-output trade-off depends on the values assigned to α. Actually, as the latter assumes smaller values, NPIs reduce less labor force, preserving, however, the same effect on the infection rate.
Calibration and results of Model 1 are presented in paragraphs 4.2 and 5, respectively.

3.2 Quantitative Approach, Model 2

In order to replicate the empirical evidence of Italy, we need to take into account further aspects. When the analysis is focused on an extended period, external factors may change the properties of the spreading epidemic, i.e. transition probabilities. In this circumstance, it is necessary to adopt time-varying parameters to include the effects of these external factors.[21] To this

[20] Actually, even under restrictions, some labourers can still supply work. It is easy to get this idea by thinking of the extraordinary implementation of remote - or smart - working displayed worldwide from March 2020, which allowed households to work even under lockdown requirements. Nonetheless, not all sectors of the economy can operate remotely, such as the manufacturing sector and, for this reason, NPIs still generate a significant negative shock on labor supply.

[21] In particular, the infection rate is remarkably reduced by self-protective behaviours and government actions - i.e. non-pharmaceutical interventions - which aim at diminishing contacts among individuals (Giordano et al., 2020; Loli Piccolomini et al., 2020; Noll et al., 2020; Romano et al., 2020; Sadeghi et al., 2020; Zhang et al., 2021). In addition, the infection rate can be scaled down through the production and distribution of vaccines that immunise the susceptible population and can be moved up by more transmissible virus' variants (Angeli et al., 2022; Caldwell et al., 2021 and Zhang et al., 2021). The incubation period is disease-specific and thus virus variants may present slightly different incubation rates. The recovery rate depends not only on the nature of the epidemic disease but also on the available medical resources and the efficiency of the healthcare system (Giordano et

aim, we impose a time-dependent infection rate (b_t) and case fatality rate (μ_t), which modify equations (2)-(3) and (6) of the canonical SEIRD model. The time-varying case fatality rate is described by a step-function, to be illustrated in paragraph 4.3. As for $f(\eta_t)$ in equation (21), we borrow from reality a proxy of NPIs, namely, a measure that assigns a numerical value to restrictions adopted in Italy. We employ the *Containment and Health* Index (CH^{Index}) - which is one of the efforts of the OxCGRT[22] project - elaborated by Hale et al. (2021). More specifically, this index is a composite measure based on thirteen policy response indicators including school closures, workplace closures, travel bans, testing policy, contact tracing, face coverings, and vaccine policy re-scaled to a value from 0 to 100 (100 = strictest). The functional form of $f(\eta_t)$ is given by:

$$f(\eta_t) = \Big(1 - \kappa_{1,t}\eta_t - \kappa_{2,t}\eta_t^2\Big) \tag{23}$$

where

$$\eta_t = \frac{CH_t^{Index}}{100}$$

with $-1 \leq \kappa_{i,t} \leq 0$, $i = 1, 2$. As before, η_t can be interpreted as the NPIs rate. Conversely, the two coefficients - which multiply, respectively, η_t and its square - are time-varying, reflecting the fact that the effectiveness of NPIs in reducing the infection rate changes over time.[23] Moreover, not all the NPIs impede workers from supplying labor. For instance, face-covering

al., 2020; Romano et al., 2020 and Sadeghi et al., 2020). Finally, the case fatality rate can be pushed up by the inception of new variants and deficiencies in the healthcare system (Caldwell et al., 2021 and Sadeghi et al., 2020).

[22]The Oxford Coronavirus Government Response Tracker.

[23]When considering an extended period, it is difficult to assume that NPIs maintain the same effect on the infection rate, as other factors perturb their impact. Thus, by imposing time-varying efficacy parameters we are assuming that equal policy interventions, implemented at different times, can have differential effects on reducing the infection rate.

requirements, testing policies and other health measures allow the supply of labor if workers comply with them. Differently, lock-downs, workplace closures and travel restrictions lessen the possibility to work. For this reason, to model the impact of restrictions on N_t we do not use the CH^{Index}, as it contains interventions that do not affect labor, but we build our own index, labeled as *Modified Stringency* Index (MS^{Index}). This is obtained starting from the dataset collected by the OxCGRT project and following their methodology for calculating indices.[24] In detail, the new index is constituted by five indicators, i.e. workplaces closures, public transport disruptions, stay-at-home requirements, restrictions on internal movements and international travel.[25]

The definition of labor force given by equation (7) changes, becoming:

$$N_t = (S_t + E_t + R_t)(1 - g(\chi_t))$$

where

$$g(\chi_t) = \alpha_t \chi_t^2 \quad \text{and} \quad \chi_t = \frac{MS_t^{Index}}{100}$$

with $0 \leq \alpha_t \leq 1$ and $0 \leq \chi_t \leq 1$. Contrary to M_1, coefficient α_t is not constant over time, but it varies according to the step-function depicted in paragraph 4.3.[26]

Calibration and results of Model 2 are conveyed in paragraph 4.3 and 5, respectively.

[24] A full description is available at `https://github.com/OxCGRT/covid-policy-tracker/blob/master/documentation/index_methodology.md`.

[25] Referring to the code-book for OxCGRT, our index is computed using only indicators C2, C5, C6, C7 and C8. Conversely, the *Stringency* Index - computed by the Oxford team - contains also C1, C3, C4 and H1. It is for this reason that we rename our index as *Modified Stringency* Index.

[26] In the first phase of the Covid-19 outbreak, policy interventions were more disruptive than in the following phases, since production processes were not designed to work under restrictions.

4 Model Calibration

We calibrate the model on Italian data. Our analysis covers a time period that goes from 24/02/2020 to 24/02/2022 (732 days), in which Italy faced a sequence of five waves of Covid-19. Since epidemiological models usually present a daily frequency, we decided to maintain it, calibrating macroeconomic parameters on a daily basis.[27] Epidemic data is taken from the dataset of the Italian Civil Protection Department, available in the GitHub repository.[28]

4.1 Model 0

The basic reproduction number is derived from Romano et al. (2020), equal to 3.8, which is in the range of values estimated by Gatto et al. (2020). Like Romano et al. (2020), we set the incubation period equal to 3 days, lying in the range of values obtained by Guan et al. (2020). The number of days to recover is equal to 10, which is in line with values employed by Angeli et al. (2022) and close to the value assumed by Piguillem and Shi (2020) and Ferguson et al. (2020). The case fatality rate is set equal to 2%, which is within the value range used by Loli Piccolomini et al. (2020) and Piguillem and Shi (2020). Lastly, from equation (1) we compute the value of the infection rate, equal to 0.456, which is in line with values assigned by the literature (Giordano et al., 2020; Loli Piccolomini et al., 2020; Romano et al., (2020)). Table 1 summarises our choice of parameters.

[27]This is also the choice of Alvarez et al (2021), Gonzalez-Eiras and Niepelt (2020) and Piguillem and Shi (2020).

[28]Find data at `https://github.com/pcm-dpc/COVID-19`.

Table 1: SEIRD block Calibration

Definition	Parameter	Value	Reference
Population	P	$59,641,488$	[20]
Initial Infections	I_1	221	[24]
Initial Recoveries	R_1	1	[24]
Initial Deceases	D_1	7	[24]
Basic Reproduction Number	R_0	3.8	[13, 28]
Incubation Rate	ε	1/3	[16, 28]
Recovery Rate	γ	1/10	[2, 9, 26]
Infection Rate	b	0.456	[14, 22, 28]
Case Fatality Rate	μ	0.02	[22, 26]

Macroeconomic parameters assume standard values, converted on a daily basis. The daily time preference β and the daily depreciation rate δ are set to match a yearly real interest rate equal to 4.04% and a yearly depreciation rate equal to 10%. The calibration of the RBC model is displayed in table 2.

Table 2: RBC block Calibration

Definition	Parameter	Value
Daily Time Preference	β	0.9992
Daily Depreciation Rate	δ	$2.74e^{-4}$
Labor Share of Output	ϕ	0.64
Inverse of the Elasticity of Labor	θ	3
RoT Households' Relative Preference for Labor	ψ^r	1.6
Opt Households' Relative Preference for Labor	ψ^p	0.8843
Share of RoT Households in the economy	λ	0.8

In order to make comparisons across the three versions of the model, we maintain the same calibration for common parameters. Hence, θ and λ - given the wide range of values provided by the literature - are calibrated by adopting a sensitivity analysis to increase the matching of M_2 with real data. Differently, the two households' relative preferences for labor are determined by the following equations, imposing that in steady state $\bar{h}^{0,p} = \bar{h}^{0,r} = 1/3$ and $\bar{n} = 1$:

$$\psi^r = \left[\frac{1-\theta}{1-\left(1-\bar{h}^{0,r}\right)^{1-\theta}}\right]\frac{1}{\bar{n}}$$
$$\psi^p = \frac{1-\theta}{1+z}\left[\frac{1-\theta}{1-\left(1-\bar{h}^{0,p}\right)^{1-\theta}}\right]\frac{1}{\bar{n}}$$

4.2 Model 1

In Model 1, the variable η_t and parameters κ and α are introduced.
The NPIs rate assumes values according to the following step-function:

$$\eta_t = \begin{cases} 0 & \text{if } t \leq 60 \\ 0.8 & \text{if } 60 < t \leq 160 \\ 0 & \text{if } t > 160 \end{cases}$$

Thus, in the first 60 days, the government decides not to intervene, allowing the virus to spread across the population. Successively, strict NPIs are imposed for 100 days, which are removed from day 160 until the end of simulation.

As shown in paragraph 5, we conduct three simulations for M_1 in which α assumes different values, while κ remains constant. Table 3 summarises our choice of parameters.[29]

[29] Increasing values of α imply a higher degree of NPIs efficacy in reducing labor supply.

Table 3: Efficacy Coefficients

Simulation	κ	α
1	0.8	0.3
2	0.8	0.5
3	0.8	0.7

4.3 Model 2

In Model 2, we employ time-dependent infection and case fatality rates. The former is described by the combination of equations (21) and (23), while a step-function is employed to set values of the latter. Time-varying coefficients $\kappa_{i,t}$ $(i = 1, 2)$ and variable μ_t are estimated by minimising a simple mean of the *sum of squared errors* (SSE) of the share of infections and deceases. The minimisation problem is described by: [30]

$$\begin{aligned} \min_{\{\kappa_{1,t},\kappa_{2,t},\mu_t\}_{t=1}^{T}} \quad & \frac{1}{2}\left[\sum_{t=1}^{T}\left(\frac{I_t^{Real}-I_t}{P}\right)^2+\sum_{t=1}^{T}\left(\frac{D_t^{Real}-D_t}{P}\right)^2\right] \\ \text{s.t.} \quad & b_t = b\big(1-\kappa_{1,t}\eta_t-\kappa_{2,t}\eta_t^2\big) \\ & S_{t+1}-S_t = -b_t\frac{S_tI_t}{P} \\ & E_{t+1}-E_t = b_t\frac{S_tI_t}{P}-\varepsilon E_t \\ & I_{t+1}-I_t = \varepsilon E_t-\gamma I_t-\mu_tI_t \\ & R_{t+1}-R_t = \gamma I_t \\ & D_{t+1}-D_t = \mu_tI_t \end{aligned} \tag{24}$$

[30] It is solved using the built-in function *fmincon* of Matlab, giving as starting values 1 for κ_1, κ_2 and 0 for μ_0, and imposing as lower and upper bond $0 \leq \kappa_{i,t} \leq 1$ $(i = 1, 2)$ and $0 \leq \mu_t \leq 1$. The SSE of the share of infections and deceased is computed with data from the Italian Civil Protection Department (I_t^{Real}, D_t^{Real}) and data obtained from the model (I_t, D_t).

where $T = 732$. The period in which the problem is solved was divided into five sub-intervals, allowing estimated parameters to vary, and, thus, assuming the form of the following step-functions:

$$\mu_t = \begin{cases} \mu_1 & \text{if } t_0 < t \leq t_1 \\ \mu_2 & \text{if } t_1 < t \leq t_2 \\ \mu_3 & \text{if } t_2 < t \leq t_3 \\ \mu_4 & \text{if } t_3 < t \leq t_4 \\ \mu_5 & \text{if } t > t_4 \end{cases} \qquad \kappa_{i,t} = \begin{cases} \kappa_{i,1} & \text{if } t_0 < t \leq t_1 \\ \kappa_{i,2} & \text{if } t_1 < t \leq t_2 \\ \kappa_{i,3} & \text{if } t_2 < t \leq t_3 \\ \kappa_{i,4} & \text{if } t_3 < t \leq t_4 \\ \kappa_{i,5} & \text{if } t > t_4 \end{cases}$$

with $i = 1, 2$ and where t_0 corresponds to day one (24/02/20) in which $\mu_t = \mu$ and $\kappa_{i,t} = 0$.[31] The length of intervals is related to the time period of the five waves exhibited in Italy. Table 4 reports values of time-dependent parameters, obtained from the solution of problem (24).

Table 4: Calibration of μ_t and $\kappa_{i,t}$ $(i = 1, 2)$

Period (Date)	Interval (Day)	μ_t (%)	$\kappa_{1,t}$	$\kappa_{2,t}$
25/02/20 – 21/07/20	2 – 149	0.812	0.3114	0.9855
22/07/20 – 19/02/21	150 – 362	0.091	0.9978	0.0423
20/02/21 – 12/07/21	363 – 505	0.053	0.1441	0.9945
13/07/21 – 22/10/21	506 – 607	0.026	0.4152	0.7544
23/10/21 – 24/02/22	608 – 732	0.038	0.0005	0.8744

Moreover, in our model the presence of NPIs has an immediate impact on the infection rate. This means that as the government raises the strictness of interventions, b_t declines promptly. Nevertheless, it has been experienced in

[31] In the first period, we are imposing the absence of NPIs.

reality that the effect of non-pharmaceutical interventions is displayed with a certain temporal lag. Therefore, to capture this aspect we assume a delay of 33 days in the impact of NPIs on the infection rate, which is determined in order to optimise the matching between real and artificial data on infections and deceased.[32] Conversely, effects on the labor force are observed instantly, since, for instance, the decision to restrict travel and/or to lock-down households has an immediate impact on the possibility to supply labor. Therefore, in our model measures contained in the MS^{Index} have a direct impact.

Finally, parameter α_t - representing the effectiveness of interventions in producing a negative shock on the labor force - is calibrated in order to match the conjunctural quarterly growth rate of actual worked hours in Italy. Accordingly, its values are depicted in table 5.

Table 5: Calibration of α_t

Period	Value
1	0
$2-128$	0.65
$129-220$	0.20
$221-402$	0.15
$403-732$	0.05

The fact that α_t assumes a value of 0.65 in the first part of simulation and then declines progressively means that, in the first months of the Covid-19 epidemic, policy interventions are more effective in reducing labor supply, while, successively, their negative effects weaken.[33]

[32]In practice, we impose that on 24^{th} February 2020 the CH_t^{Index} assumes the value exhibited on 22^{nd} January 2020, which is equal to 0.

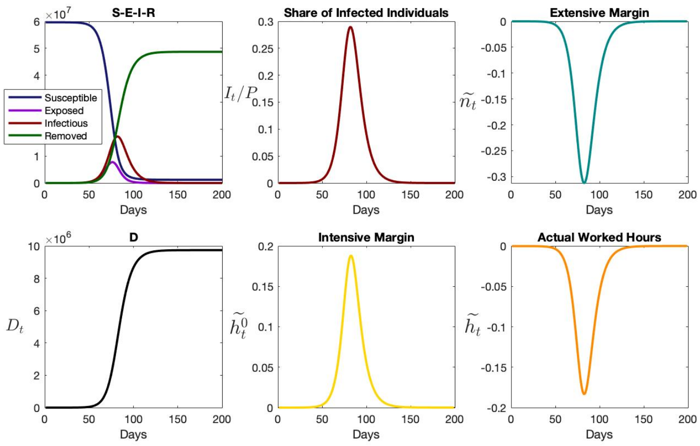
[33]See note 26.

5 Results

This section reports and comments on results obtained from simple numerical simulations, where the initial and final steady states are computed on 24/02/2020 and 24/02/2022, respectively. The evolution of the pandemic is the driver of our deterministic simulations, which can also be affected (in M1 and M2) by the implementation of NPIs and by the possible evolution of the time-dependent coefficients discussed in section 4.

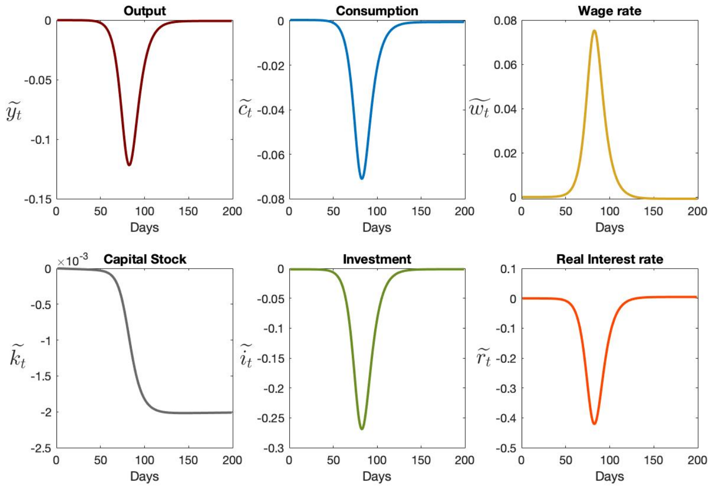
Figures 1 and 2 display the dynamics of the SEIRD and RBC blocks produced by M_0, where for each macroeconomic variable the relative deviation from the steady state ($\tilde{x}_t$) is conveyed.

Figure 1: SEIRD Model and IRFs of Labor Related Variables (M_0)



Note: The top-left panel shows the time path of susceptible, exposed, infected and removed individuals, while the bottom-left panel depicts the time path of deceased individuals. The top-centre panel displays the time path of the share of infected individuals. The bottom-centre, top-right and bottom-right panels report, respectively, the relative deviation from steady state of the intensive margin, extensive margin and actual worked hours. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

Figure 2: IRFs of the Main Macroeconomic Variables (M_0)



Note: The top-left and bottom-left panels show, respectively, the relative deviation from steady state of output and capital stock. The top-centre and bottom-centre panels display, respectively, the relative deviation from steady state of consumption and investments. The top-right and bottom-right panels exhibit, respectively, the relative deviation from steady state of the wage rate and the real interest rate. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

Given our calibration and the absence of NPIs, the epidemic ends within 238 days[34], in which the share of infections exhibits a maximum of 29% on day 82. The share of deceased reaches a value of 16.3% from day 157 until the end of simulation.

As the virus kicks in, labor force reduces due to ill workers, or in other words, the outbreak generates a fall in the endowment of the labor input. Consequently, the extensive margin n_t reduces by 31.3% on day 84. Simultaneously, the intensive margin h_t^0 increases by 18.8% on day 84. This may be since disease-free members of the representative household are willing to work more hours in order to compensate for the lower income yielded by ill members. Likewise, firms want to compensate for the reduction of the extensive margin, which otherwise will generate a sharp plunge in production. Notwithstanding

[34]On day 239, the number of infections is lower than one.

this, the shrinkage of the extensive margin drowns out the rise of the intensive margin, resulting in a tumble of the actual worked hours h_t by 18.3% on day 84.

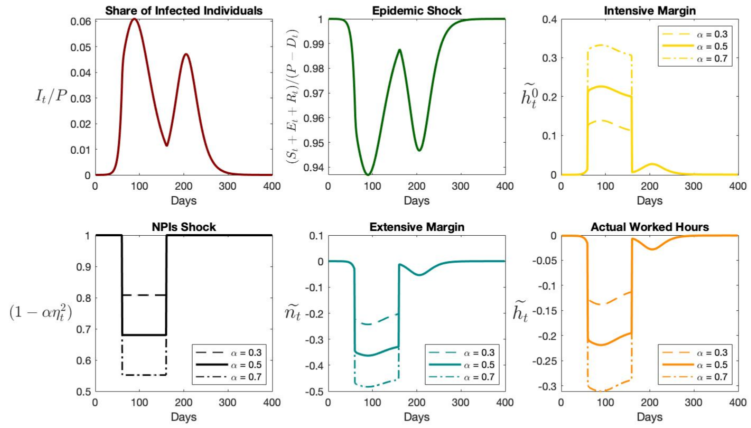
Moreover, the plunge in actual worked hours undoubtedly generates a recessive effect on output, which falls by 12.2% on day 84. Households' income resources are reduced, therefore, consumption and investment decline. However, households are averse to deep consumption reductions since they prefer to smooth consumption over time. Once their income drops, households are willing to reduce savings, i.e. investments, on behalf of consumption. Nevertheless, since in this economy we assume the presence of RoT households (accounting for 80% of households), the manifestation of the so-called consumption smoothing is less effective. As a result, consumption and investments fall, respectively, by 7.1% and 26.9% on day 84. Successively, less investment today generates a lower capital accumulation tomorrow, determining a fall in the capital stock over time.[35] Considering the reduction in labor input, firms have to adjust the level of capital stock. Therefore, capital demand moves backward. This shift more than compensates for the reduction in savings supply as shown by the real interest rate fall of 42.1%. Conversely, the increase in the hourly wage rate w_t - equal to 7.5% on day 84 - implicates that the fall in labor supply is higher than the drop in labor demand. Finally, macroeconomic variables converge to their steady state value as soon as the epidemic is expelled from the population.

In figures 3 and 4 we report the outcome produced by Model 1 in which we allow for the presence of NPIs. To analyse the economic impact of government interventions we ran three simulations in which NPIs assume a different

[35]From day 149, capital stock increases. However, due to the small value of investments, it augments slightly taking a long time before reaching the steady state again, which occurs after the time frame of our simulation.

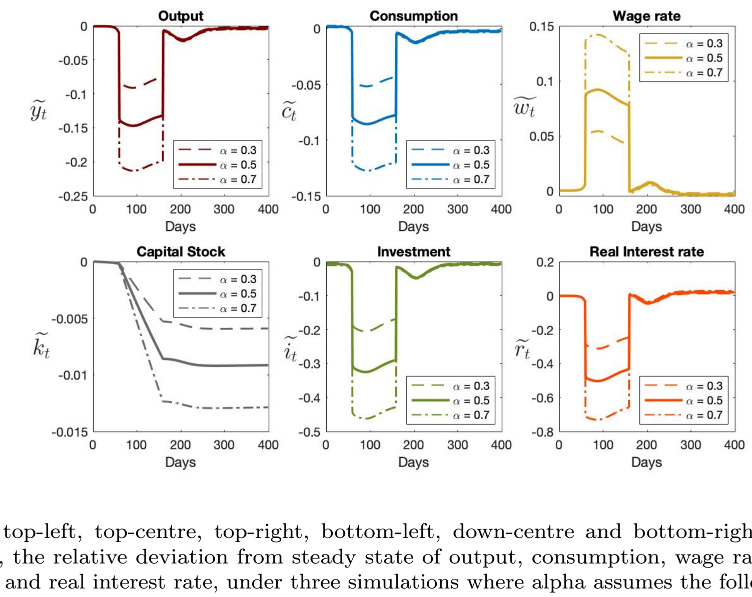
efficacy (α) in reducing the labor force.

Figure 3: IRFs of Labor Related Variables (M_1)



Note: The top-left and top-centre panels shows the time path of the share of infected individuals and the epidemic shock. The bottom-left panel exhibits the time path of the NPIs shock and the top-right, bottom-centre and bottom-right panels display, respectively, the relative deviation from steady state of the intensive margin, extensive margin and actual worked hours, under three simulations where alpha assumes the following values: 0.3 (dashed line), 0.5 (solid line) and 0.7 (dashdot line). Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

Figure 4: IRFs of the Main Macroeconomic Variables (M_1)



Note: The top-left, top-centre, top-right, bottom-left, down-centre and bottom-right panels display, respectively, the relative deviation from steady state of output, consumption, wage rate, capital stock, investments and real interest rate, under three simulations where alpha assumes the following values: 0.3 (dashed line), 0.5 (solid line) and 0.7 (dashdot line). Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

It is possible to observe that by implementing measures that reduce the infection rate, the government is able to lessen the spread of the outbreak, which infects 6.1% of the population on day 88 and then starts to decline. However, the complete removal of NPIs on day 161 enables the virus to be transmitted more rapidly, producing a surge of infections and the manifestation of a second wave. After reaching a maximum of 4.7% on day 205, the share of infections declines and the epidemic dies after 488 days.

To highlight how shocks generated by the epidemic and NPIs affect overall the extensive margin, we re-wright it by merging equations (8) and (22), to obtain:

$$n_t = \left(\frac{S_t + E_t + R_t}{P - D_t}\right)(1 - \alpha\eta_t^2)$$

where the first term in the bracket on the right-hand side represents the effect of the *Epidemic Shock* and the second term captures the employment effect of the *NPIs Shock*. As shown in figure 3 the impact of NPIs is displayed as soon as η_t is modified, making the extensive margin plunge or rocket. By contrast, when the NPIs rate is maintained constant, it is possible to appreciate the gradual effect of the epidemic shock.

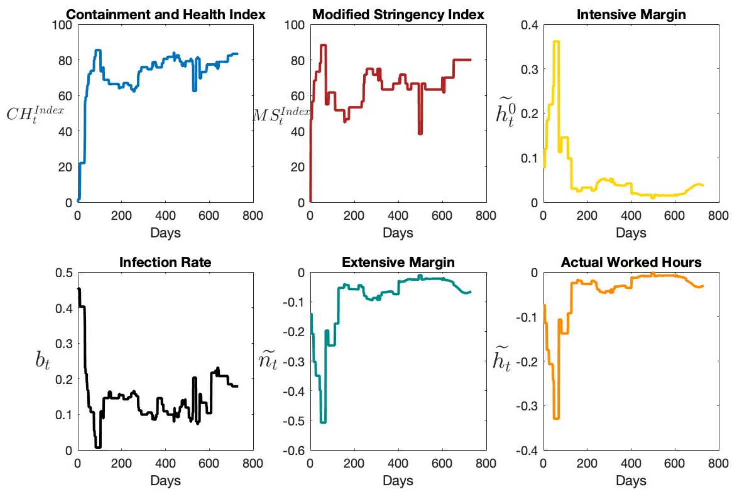
We observe that n_t reduces by 24.3-48.3% at the peak (day 91), given $0.3 \leq \alpha \leq 0.7$. In addition, it reaches a second bottom on day 207, falling by 5.3%, which is generated by the manifestation of a second epidemic wave. As already highlighted in M_0, other macroeconomic variables are affected by variations in the extensive margin, thus exhibiting two minima as well.[36]

From the comparison of M_0 and M_1 a trade-off between health and output arises. Its severity is conditional on the value of the efficacy coefficient α.

[36] At the first minimum, actual worked hours, output, consumption and investments tumble, respectively, by 13.8-31.1%, 9.1-21.3%, 5.2-12.8% and 20.6-46.2%, while at the second minimum reduce by 2.9%, 2.3%, 1.3% and 5.0%.

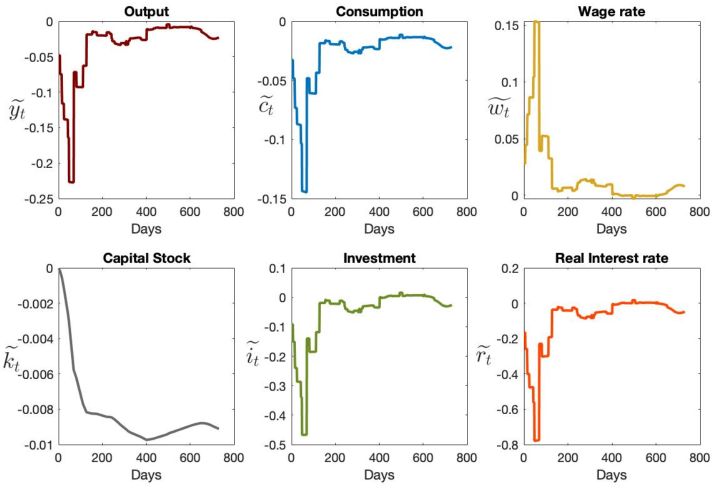
According to our simulations, when NPIs are not implemented, the epidemic runs out in 238 days, hitting the population strenuously, namely, infecting 17,3 million individuals at the peak and taking 9.7 million lives overall. The epidemic shock knocks off output by 12.2% at the minimum. Conversely, in the scenario in which the government intervenes with NPIs, the outbreak lasts for a longer period, conveying multiple waves. However, it infects a lower portion of the population (3.6 million at the first peak and 2.8 million at the second peak) and causes the death of fewer individuals (8.8 million). Notwithstanding this, when $\alpha > 0.415$, the NPIs shock, which adds to the remaining epidemic shock, generates a more forceful and long-lasting recession. Figures 5 and 6 display the results of Model 2, where parameters are calibrated in order to match the empirical evidence of Italy in the analysed period.

Figure 5: IRFs of Labor Related Variables (M_2)



Note: The top-left, top-centre and bottom-left panels show, respectively, the time path of the *Containment and Health* Index, *Modified Stringency* Index and the infection rate. The top-right, bottom-centre and bottom-right panels exhibit the relative deviation from steady state of the intensive margin, extensive margin. and actual worked hours. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

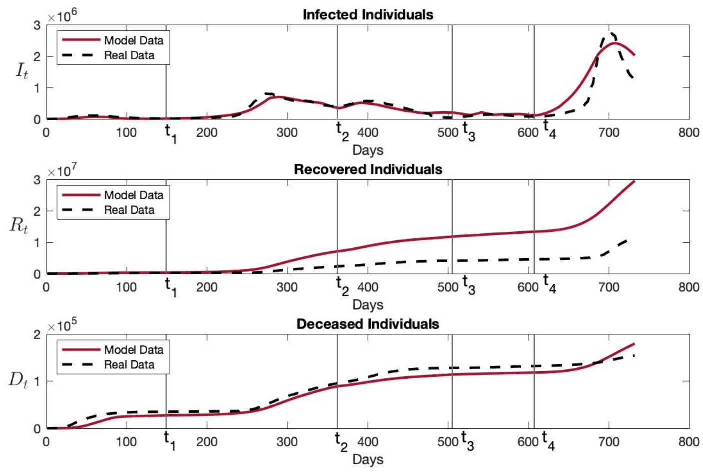
Figure 6: IRFs of the Main Macroeconomic Variables (M_2)



Note: Top-left, top-centre, top-right, bottom-left, bottom-centre and bottom-right panels display, respectively, the relative deviation from steady state of output, consumption, wage rate, capital stock, investments and real interest rate. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

First of all, it is interesting to observe the path of the infection rate over time (figure 5), which is affected by the *Containment and Health* Index and $k_{i,t}$ $(i = 1, 2)$, according to our function $f(\eta_t)$. As the strictness of non-pharmaceutical interventions augments, b_t reduces slackening the propagation of the virus among individuals. Conversely, when the strictness of interventions softens, the infection rate increases. The presence of rises and declines in the index replicates the interchange of suppression phases and weakening periods adopted by the Italian government. It is possible to also appreciate the effect of the *Modified Stringency* Index on the extensive margin, which results as being the main shock on n_t as NPIs almost neutralise the epidemic shock. Accordingly, the extensive margin falls by 50.8% on day 63. As explained for M_1, the presence of several bottom points - exhibited by macroeconomic variables - highlights the presence of multiple epidemic waves, where infections rise, reach a maximum and, then, decline. Differently from the outcome of the two previous models, in M_2 macroeconomic variables do not come back to the initial steady state at the end of simulation, since on day 732 the number of infections is still considerable.

Figure 7: The Epidemiological Empirical Evidence of Italy vs Model 2



The three panels report, respectively, the time path of infected, recovered and deceased individuals derived from simulation (red lines) and real data (dashed black lines). Vertical grey lines are drawn in correspondence of t_i, with $i = \{1, 2, 3, 4\}$, which highlight intervals of the five epidemic waves. Source: Authors, developed with with Dynare-4.6.1 and Matlab-R2018a.

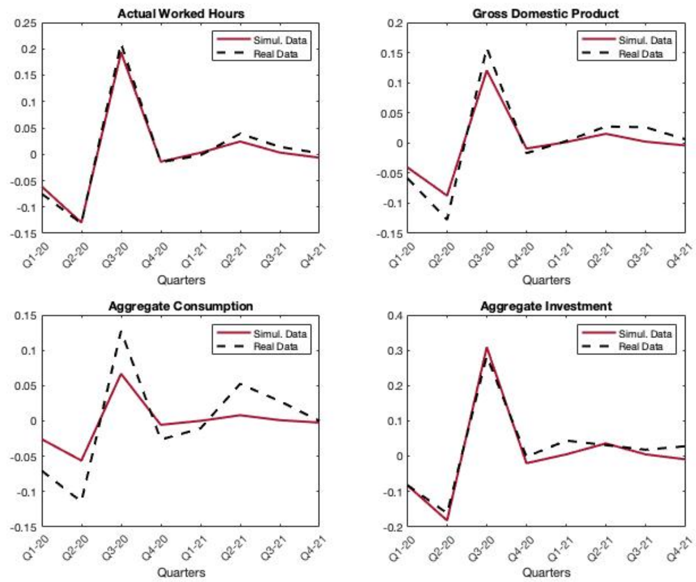
We observe in figure 7 that the employment of a time-varying infection rate allows us to replicate approximately the five different waves of Covid-19 infections, which characterised the evolution of the epidemic in Italy in the period 24/02/20 - 24/02/22. In the same way, the path of deceased is closely matched thanks to the variation of the case fatality rate.[37] However, our model overestimates recoveries starting from day 200.[38]

[37]In appendix E, table E.1 illustrates the goodness-of-fit of data on infections and deceased.

[38]To reduce the overestimation, we could change problem (24) by adding the SSE of the share of recoveries in the objective function. However, we saw in simulation that this procedure, on one hand, eliminates the overestimation of recoveries, but, on the other, worsens the fit of infections, yielding an underestimation. Differently, the overestimation may be reduced if we set the recovery rate γ time-varying, as done for μ, allowing γ to vary in the five sub-intervals.

We conclude this paragraph by reporting in figure 8 a comparison between the quarterly conjunctural growth rate of actual worked hours, output, consumption and investment computed from M_2 and those obtained from real data[39], covering a time period that goes from Q1-2020 to Q4-2021.[40]

Figure 8: The Macroeconomic Empirical Evidence of Italy vs Model 2



Note: the four panels report, respectively, the quarterly conjunctural growth rate of actual worked hours, output, consumption and investment computed from simulation (red lines) and real data (dashed black lines). To conduct a fair comparison, we have converted the frequency of model data from daily to quarterly and, since variables in our model are in per capita terms, each measure was transformed in absolute terms, multiplying it by $(P - D_t)$. Source: Authors, developed with Dynare-4.6.1 and Matlab-R2018a.

It is possible to observe that our model is able to match quantitatively - with a suitable degree of accuracy - the quarterly conjunctural growth rates of Italian economic aggregates. Even though aggregate consumption exhibits smaller volatility, its path replicates the oscillations reported by the empirical evidence. As explained before, this is due to the consumption smoothing of

[39]Real data is taken from Istat, where aggregate consumption is compared with the final household's spending and aggregate investment with gross fixed investment.

[40]Since our simulation starts from 24/02/20, to compute growth rates in Q1-2020 we assume that each variable is in steady steady state from 01/01/2020 until 24/02/2020.

Opt households.

The goodness-of-fit of the model relies also on the implementation of the time-varying coefficient α_t. This is necessary inasmuch as the reintroduction of NPIs to hinder new waves of Covid-19 did not have the same negative impact on the economy as the implementation of first interventions. Actually, in the first months of 2020, the outbreak along with policy interventions sneaked up on economic agents, yielding severe recessive effects. However, as the government lingered on the implementation of NPIs, firms and households adapted, weakening the negative impact of interventions. This may explain why a lower fall of economic aggregates is associated with equal values of MS_t^{Index}, starting from the third quarter of 2020.

6 Conclusions

This paper proposes a unified framework for analysing the economic effects generated by the proliferation of an epidemic and the implementation of non-pharmaceutical interventions. Such framework is based on direct integration of a slightly modified version of the RBC model a la Hansen (with financially constrained households) with a SEIRD epidemiological model.

We depart from the standard SIR-Macro set-up that typically employs health-related models in which the health status and the transition probabilities between epidemic compartments shape the relevant agents' decisions; conversely, we propose a simplified and direct integration in which the SEIRD block together with the NPIs determine exogenously the path of the extensive margin of the labor input (in line with RBC spirit), while economic agents derive optimally the actual worked hours and the intertemporal allocations of consumption and asset accumulation.

We present three versions of our model in order to highlight different aspects. M_0 points out how the epidemic shock alters the equilibrium of macroeconomic variables when the government decides not to interfere with the spread of the virus. In M_1, NPIs are introduced; since these interventions not only reduce the infection rate but also have a negative impact on the labor force, NPIs shock adds to the remaining epidemic shock producing a forceful economic recession and the occurrence of new waves of infections and deceased when NPIs are relaxed. This eventuality lays the foundation for a debate on the presence of a health-output trade-off faced by the government when choosing whether to intervene to hinder the spread of the outbreak and save lives at the cost of a more forceful and long-lasting economic recession. In M_2, we bring our model to data; we calibrate the NPIs on the basis of proxies such as the *Containment and Health* Index and the *Modified Stringency* Index. Thanks to time-varying infection and case fatality rates and efficacy coefficients (i.e. $\kappa_{1,t}$, $\kappa_{2,t}$ and α_t), the model is able to replicate the time path of infections and deceased as well as quarterly conjunctural variations of the main macroeconomic variables, exhibited in Italy between 24/02/20 and 24/02/22.

We believe that the strength of our analysis lies in its simplicity and capacity to replicate the epidemic and business cycle evidence of the Italian economy, in which employment variations have been strictly related to the harmfulness of the epidemic and NPIs introduced by the government and not to the decisions of agents. This is particularly true for the Italian case where the government has introduced a long-lasting ban on dismissals. Further, because of its parsimony, it is straightforward to integrate our setup into more structured business-cycle models in order to study a much broader set of issues.

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Appendix

A Derivation of the Utility Function

We consider a utility function logarithmic in consumption (c_t) and CRRA in leisure ($1 - h_t^0$). Following Hansen (1985), we apply the lotteries setup, deriving:

$$U = \log(c_t) + \left[n_t \psi \frac{(1 - h_t^0)^{1-\theta} - 1}{1 - \theta} + (1 - n_t) \psi \frac{1^{1-\theta} - 1}{1 - \theta} \right]$$

Hence, leisure is multiplied by the respective probability of being employed and unemployed. Rearranging the equation and substituting $n_t = h_t (h_t^0)^{-1}$, we obtain:

$$U = \log(c_t) + \frac{h_t}{h_t^0} \psi \left[\frac{(1 - h_t^0)^{1-\theta} - 1}{1 - \theta} - \frac{1^{1-\theta} - 1}{1 - \theta} \right] + \psi \frac{1^{1-\theta} - 1}{1 - \theta}$$

Omitting constant terms - as they do not affect the household's optimal choice - we get equation (10):

$$U = \log(c_t) - v_t h_t$$

where

$$v_t = -\frac{\psi}{h_t^0} \left[\frac{(1 - h_t^0)^{1-\theta} - 1}{1 - \theta} \right] \geq 0$$

B Maximisation Problems

***Opt* households' maximisation problem**. As in the basic model, households maximise their expected utility - defined, here, by equation (10) - subject to the budget constraint and the low of motion of capital. Thus, the problem is described by:

$$\max_{\left\{c_t^p, h_t^p, k_{t+1}^p\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_t \left[\log c_t^p - v_t^p h_t^p\right]$$

$$\text{s.t.} \quad c_t^p + i_t^p = w_t h_t^p + (\delta + r_t) k_t^p$$

$$k_{t+1}^p = (1-\delta) k_t^p + i_t^p$$

with

$$v_t^p = -\frac{\psi^p}{h_t^{0,p}} \left[\frac{(1-h_t^{0,p})^{1-\theta} - 1}{1-\theta}\right]$$

By applying the Lagrange multiplayer approach, we derive the first-order conditions (FOCs). Combining them, we obtain equations (12) and (13).

RoT households' maximisation problem. In this case, the optimisation problem is different from that of *Opt* households, as the low of motion of capital does not enter as a constraint and the budget constraint is modified. Hence, the problem is described by:

$$\max_{\left\{c_t^r, h_t^r\right\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_t \left[\log c_t^r - v_t^r h_t^r\right]$$

$$\text{s.t.} \quad c_t^r = w_t h_t^r$$

with

$$v_t^r = -\frac{\psi^r}{h_t^{0,r}} \left[\frac{(1-h_t^{0,r})^{1-\theta} - 1}{1-\theta}\right]$$

As done before, we compute the FOC of the problem, yielding equation (14).

Firms' maximisation problem. Firms maximise profits by combining capital stock and labor through a Cobb-Douglas production function. Accordingly, the problem is defined as:

$$\max_{h_t, k_t} \quad y_t - w_t h_t - (r_t + \delta) k_t$$
$$\text{s.t.} \quad y_t = k_t^{1-\phi} h_t^{\phi}$$

From the FOCs of the problem, we determine equations (15) and (16).

C Equilibrium of the RBC Block

The equilibrium of the RBC model is described by the following system of 17 equations:

$$
\begin{aligned}
\frac{1}{c_t^p} &= \beta E\left[\frac{1+r_{t+1}}{c_{t+1}^p}\right] \\
w_t &= v_t^p c_t^p \\
c_t^p + i_t^p &= w_t h_t^p + (\delta + r_t) k_t^p \\
k_{t+1}^p &= (1-\delta) k_t^p + i_t^p \\
v_t^p &= -\frac{\psi^p}{h_t^{0,p}}\left[\frac{(1-h_t^{0,p})^{1-\theta} - 1}{1-\theta}\right] \\
w_t &= v_t^r c_t^r \\
c_t^r &= w_t h_t^r \\
v_t^r &= -\frac{\psi^r}{h_t^{0,r}}\left[\frac{(1-h_t^{0,r})^{1-\theta} - 1}{1-\theta}\right] \\
h_t^0 &= (1-\lambda) h_t^{0,p} + \lambda h_t^{0,r} \\
c_t &= (1-\lambda) c_t^p + \lambda c_t^r \\
i_t &= (1-\lambda) i_t^p \\
k_t &= (1-\lambda) k_t^p \\
n_t &= \frac{N_t}{P - D_t} \\
h_t &= h_t^0 n_t \\
y_t &= k_t^{1-\phi} h_t^{\phi} \\
r_t &= (1-\phi)\frac{y_t}{k_t} - \delta \\
w_t &= \phi \frac{y_t}{h_t}
\end{aligned}
$$

D Steady State of the Model

Differently from a short-run macroeconomic model, in an epidemic model the initial steady state is different from the final steady state inasmuch as individuals move permanently from the susceptible compartment to the recovered and deceased compartments. Therefore, in the first period:

$$S_1 = P - (E_1 + I_1 + R_1 + D_1)$$

while with $t \to \infty$:

$$\begin{aligned} &S_t \to 0, \quad E_t \to 0 \\ &I_t \to 0, \quad (R_t + D_t) \to P \end{aligned}$$

Even though the macroeconomic model is driven by epidemic variables, initial and final steady states (s.s.) coincide, as time converges to infinity.
From equation (7), we derive the s.s. labor force:[41]

$$\bar{N} = \bar{S} + \bar{E} + \bar{R}$$

where $\bar{\chi} = 0$. From equation (8), we derive the s.s. extensive margin:[42]

$$\bar{n} = \frac{\bar{N}}{P - \bar{D}}$$

Combining the budget constraint of *Opt* Households with equation (13) and making some substitutions, we derive the s.s. intensive margin of *Opt*

[41] $N_1 = P - (I_1 + D_1)$ and $\lim_{t \to \infty} N_t = P$.
[42] $n_1 = 1 - \frac{I_1}{P - D_1} \approx 1$ and $\lim_{t \to \infty} n_t = 1$.

Households:

$$\bar{h}^{0,p} = 1 - \left(1 - \frac{1-\theta}{\psi^p(1+z)}\frac{1}{\bar{n}}\right)^{\frac{1}{1-\theta}}$$

where

$$z = \frac{f}{\phi(1-\lambda)}\left(\frac{1-\phi}{f+\delta}\right) \quad \text{and} \quad f = \frac{1}{\beta} - 1$$

Combining the budget constraint of *RoT* Households with equation (14), we derive the s.s. intensive margin of *RoT* Households:

$$\bar{h}^{0,r} = 1 - \left(1 - \frac{1-\theta}{\psi^r}\frac{1}{\bar{n}}\right)^{\frac{1}{1-\theta}}$$

From the definition of Hansen's variable for *Opt* Households, we derive its steady state value:

$$\bar{v}^p = -\frac{\psi^p}{\bar{h}^{0,p}}\left[\frac{(1-\bar{h}^{0,p})^{1-\theta} - 1}{1-\theta}\right]$$

From the definition of the Hansen's variable for *RoT* Households, we derive its steady state value:

$$\bar{v}^r = -\frac{\psi^r}{\bar{h}^{0,r}}\left[\frac{(1-\bar{h}^{0,r})^{1-\theta} - 1}{1-\theta}\right]$$

From equation (17), we derive the s.s. aggregate intensive margin:

$$\bar{h}^0 = (1-\lambda)\bar{h}^{0,p} + \lambda\bar{h}^{0,r}$$

From equation (9), we derive the s.s. aggregate actual worked hours:

$$\bar{h} = \bar{n}\bar{h}^0$$

Combining the production function with the s.s. output-capital ratio[43], we derive the s.s. aggregate capital stock:

$$\bar{k} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1}{\phi}} \bar{h}$$

From equation (20), we derive the s.s. capital stock of *Opt* Households:

$$\bar{k}^p = \frac{\bar{k}}{1-\lambda}$$

Combining the law of motion of capital stock for *Opt* Households with the s.s. aggregate capital stock, we derive the s.s. aggregate investment level:

$$\bar{\imath} = \delta \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1}{\phi}} \bar{h}$$

From equation (19), we derive the s.s. investment level of *Opt* Households:

$$\bar{\imath}^p = \frac{\bar{\imath}}{1-\lambda}$$

Combining the production function with the s.s. aggregate capital stock, we derive the s.s. aggregate output:

$$\bar{y} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1-\phi}{\phi}} \bar{h}$$

Combining equation (16) with the s.s. aggregate output, we derive the s.s. wage rate:

$$\bar{w} = \left(\frac{1-\phi}{f+\delta}\right)^{\frac{1-\phi}{\phi}}$$

[43] It is obtained combining the s.s. Euler equation with the s.s. capital demand and it is equal to $\frac{\bar{y}}{\bar{k}} = \frac{f+\delta}{1-\phi}$

From equation (12), we derive the s.s. real interest rate:

$$\bar{r} = \frac{1}{\beta} - 1$$

From equation (13), we derive the s.s. consumption of *Opt* Households:

$$\bar{c}^p = \frac{\bar{w}}{\bar{v}^p}$$

From equation (14), we derive the s.s. consumption of *RoT* Households:

$$\bar{c}^r = \frac{\bar{w}}{\bar{v}^r}$$

From equation (18), we derive the s.s. aggregate consumption:

$$\bar{c} = (1-\lambda)\bar{c}^p + \lambda\bar{c}^r$$

E Goodness-of-Fit of Data

To evaluate the goodness-of-fit of simulated data (infections and deceased), we compute the *Root Mean Squared Error* (RMSE), expressed by:

$$RMSE = \sqrt{\frac{\sum_t^T \left(X_t^{Real} - X_t\right)^2}{T - t + 1}}$$

where X_t^{Real} and X_t refer, respectively, to real and simulated data of a single epidemic compartment among infected and deceased.

Results are reported in table E.1, where goodness-of-fit measures are computed for each epidemic wave interval.

Table E.1: Goodness-of-Fit of Data on Infections and Deceases

Period	RMSE		RMSE/P	
$t - T$	Infections (e^{+5})	Deceased (e^{+4})	Infections (%)	Deceased (%)
$1 - 149$	0.308	0.941	0.05	0.016
$150 - 362$	0.777	0.710	0.13	0.012
$363 - 505$	0.870	1.372	0.15	0.023
$506 - 607$	0.634	1.338	0.11	0.022
$608 - 732$	4.689	1.248	0.79	0.021