The Uncertainty of Fairness: a Game Theory Analysis for a Debt Mutualization Scheme in the Euro Area

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Abstract

This paper aims to briefly present the fairness approach in game theory and its potential application. Fairness means that players consider not only personal payoffs but also others' payoffs and beliefs regarding their actions. In this context, we distinguish two approaches, one based on the material payoff and the other on beliefs. We adopt the fairness approach in proposing three games for studying the strategic interaction between a hypothetical country and the European Union in proposing a debt mutualization scheme. We find that the optimal debt quota to share with the European Union is 50%; concerning the moral hazard problem, commitment to structural reforms for countries with high public debt leads to the best equilibrium, that can be preserved following an incentive strategy.

Keywords: Game Theory, Fairness Approach, Debt Mutualization, Euro Area

JEL Codes: H63, C7

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1 Introduction

The classical approach to game theory analyses situations of strategic interaction where players are subject to certain rules and know what payoffs they can achieve. Strategic interaction means that one player's payoff depends on the other's choices. The game aims to find a solution, such as an outcome that the literature identifies with the notion of Nash Equilibrium. A Nash Equilibrium is a situation in which none has an incentive to move. Exploiting the fixed point theorem (Banach, Browers, and Kakutani), the existence of Nash Equilibrium has been proved (Glicksberg [1952]) and this opens the track for additional refinements of the concept of Nash Equilibrium. Some of the greatest contributes is due to Harsanyi, Aumann and Gibbons (Harsanyi [1973], Harsanyi et al. [1988], Aumann [1987] Gibbons et al. [1992]), whose studies introduced the concept of player's beliefs about the other player actions. Basically, Harsanyi applied the Bayesian approach to the players' decision-making process in non-cooperative games of incomplete information. Starting from the Harsanyi approach, epistemic game theory further enlarges the Bayesian analysis to consider one player's beliefs about the other players' beliefs.

Within this context, we find several experiments that try to understand how people behave in strategic interaction (Thaler [1988] and Kilgour and Zagare [1991]), usually repeated. Players do not care only about their own payoffs, but also about the others' and how they behave with them. More specifically, experiments want to consider altruism, threats, retaliation or opportunistic behaviors departing from Selten [1990] analysis[1]. Two main approaches deal with this theory, called "fairness approach"[2]: one considers the payoffs of both players and the other the players' belief about the other players' beliefs, i.e. "payoff driven" (or "social preferences" Fehr and Schmidt [2001]) approach and "intention driven approach".

Concerning the former, Fehr and Schmidt [1999] provides a more complete description of this phenomenon and theorizes it without relaxing the rationality assumption. They model self-interest people by adding a fraction who cares about fairness in the game. Fairness is defined as "self-centered inequity aversion", meaning that people are willing to give up some material payoffs to get more equitable outcomes. An additional hypothesis introduced in this theory concerns the heterogeneity of preferences that interacts with the economic environment in which people are asked to make choices.

We refer to this approach as "payoff driven" (Fehr and Schmidt [1999]). The payoff function is built upon rationality and a component of fairness and reciprocity. Levine [2006] presents this approach by writing that people care intrinsically about fairness, with regards

[1] We consider the psychological aspect of the player who tries to be in the shoes of the other, hence not only how she/he responds to the opponent player's previous actions.

[2] For an exhaustive literature review, see Fehr and Schmidt [2001].

to how the game is played and to payoffs distribution. In fact, evidence of the aversion against disadvantageous inequality is provided by Loewenstein et al. [1989], while Fehr and Schmidt [1999] add also aversion to advantageous inequality in their approach.

Considering the latter, the greatest contribution comes from Geanakoplos et al. [1989] (GPS hereinafter) who introduce the concept of psychological game in the context of sequential rationality. With psychological games, they want to consider a player's belief about the other player's beliefs regarding his/her actions. Players' utility function depends on "summarized" hierarchically ordered beliefs about the others. GPS further demonstrate that with this approach backward induction is not applicable and Perfect Psychological Equilibria do not always exist. On the contrary Subgame Perfect Nash Equilibria do. We refer to this modus operandi as "intention-driven approach".

Moreover, in Rabin [1993] we find a mix of the two approaches. He extends GPS psychological game by making the players' utility function depending on material payoff [3] and on what he calls a "fairness function". A fairness function is a function that considers the best, the worst, the fair, and the minimum payoff. Rabin defines the concept of "fairness equilibrium" when the payoffs are mutual-min or mutual-max. The underlying idea is that players play for maximizing or minimizing the other players' payoffs depending on their beliefs about the other's behavior.

Placed in this framework, we want to sustain the topic of debt mutualization in the Euro Area and risk sharing based on game theory. While we are writing, the debate about debt mutualization in the Eurozone has gained importance again. The "Great Shutdown" (Wolf [2020]) due to the global pandemia of Coronavirus will substitute the Great Recession as the worst crisis after WWII. The European Union is struggling in finding conspicuous resources for sustaining the economy and debt mutualization in the form of a redeemable fund or the common balance sheet of the European Commission seems to be the most welcome solution (European Council Press Release of April 23, 2020 euc). We support the idea of a redeemable fund as Parello and Visco [2012] and Cioffi et al. [2019]. We distinguish between core countries (i.e. virtuous countries in terms of public debt like Germany or The Netherlands), thus countries that would seem to have no gain from debt mutualization and risk sharing, and peripheral countries (i.e. countries with high public debt like Italy, Spain, and Greece) which should profit from risk sharing.

We present three games to show the advantages of debt mutualization for both types of countries. The first one follows the "payoff-driven" approach and it is based on the ultimatum game. We find that the solution of this game (i.e. the quotas to be repaid by each country) strictly depends on the cost of taking advantage of the mutualization and

[3] Fehr and Schmidt [1999] took the inspiration for their "payoff driven" approach from Rabin [1993].

the risk of insolvency of a single country. The second one adopts the "intention-driven" approach and it is a refinement of the Prisoner's Dilemma with sympathy coefficient to deal with the moral hazard problem arising by the core countries. We find that a non-cooperative behavior of the core countries together with a strong commitment of the peripherals leads to the optimal equilibrium, whereas if both are free to choose their actions, cheating is the most preferred action. The third game is a Gift-Exchange game (Akerlof [1982]) that considers both material payoffs and players' beliefs and it allows us to demonstrate that the equilibrium found in the first game is also a Subgame Perfect Nash Equilibrium in the traditional sense of Selten [1990].
The paper continues as follow: in Section 2 we discuss the theory behind the two approaches, Section 3 presents the three games and Section 4 concludes.

2 Theoretical Model

2.1 Payoff driven approach

Fehr and Schmidt [1999] model a utility function in which equitable outcomes are preferred to inequitable outcomes. Inequity is defined as a situation in which a player is either worse off or better off in material terms than other players. They also add that advantageous inequity is preferred to disadvantageous inequity.

The utility function is linear in the payoff x and in the inequality aversion. In a n-players game, the utility of player i is the following

$$U_i(x) = x_i - \alpha_i \frac{1}{n-1} \sum_{j \neq i} \max\{x_j - x_i, 0\} - \beta_i \frac{1}{n-1} \sum_{j \neq i} \max\{x_i - x_j, 0\} \tag{1}$$

Where the index $i \in [i, \ldots, n]$ represents the generic player and the vector of monetary payoffs is defined by $x = x_1, \ldots, x_n$. The parameter β_i reflects the fact that the subject dislikes advantageous inequality. This parameter is bounded $0 \leq \beta_i \leq 1$. The parameter $\alpha_i \geq \beta_i$ rules out the possibility that people prefer disadvantageous inequality.

In this model, the parameter α_i is not upper-bounded because advantageous inequity is preferred to disadvantageous inequity. Player i would pay 1 dollar to reduce the advantage of player j by more than 1 dollar, but he would not pay 1 dollar to reduce his own advantage by 1 dollar or more [4].

The disutility from inequality is averaged by dividing the sum by $n-1$. The number of players n does not affect the relative impact of inequity aversion on the total payoff. This kind of utility is defined as "self-centered inequity aversion". It is "self-centered"

[4]Fehr and Schmidt [1999] also provide a numerical example for this interpretation.

because, when comparing the payoffs to determine inequity, player i does not care about the differences within the payoffs of the other players: he/she only compares his/her own payoff with the payoff of the others[5].

If we focus on a game with only 2 players ($n = 2$), the utility of player $i \in [1, 2]$ is:

$$U_i(x) = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} \qquad (2)$$

Where the preference parameters are constrained as in the previous n-players game: $0 \leq \beta_i \leq 1$ and $\alpha_i \geq \beta_i$. Figure 1 represents the utility $U_i(x_j|x_i)$ of player i as a

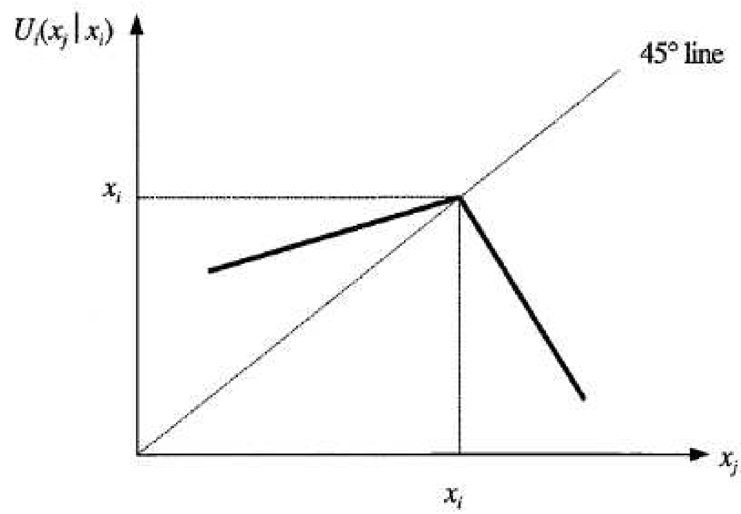

Figure 1: Utility of player i in a 2x2 game with inequity aversion (Fehr and Schmidt [1999])

function of his opponent payoff x_j, given his own payoff x_i. We can see that the utility of player i is maximized when the final payoffs are distributed in a fair way, i.e. when $x_j = x_i$, along the 45 degrees line.

Moreover, the disutility of disadvantageous inequity is greater than the disutility of advantageous inequity because the slope of the function evaluated at $x_j > x_i$ is greater than the slope of the function evaluated at $x_j < x_i$. This means that the marginal disutility given by disadvantageous inequity is greater than the marginal disutility given by advanta-

[5]Fehr and Schmidt [1999] prove also that the final outcome does not change when there are many or only a few subjects who exhibit strong inequity aversion. It remains unchanged whether they know or not the preference parameters or the payoffs of the other players.

geous inequity. Formally: $|U_i^{'}(x_j > x_i)| > |U_i^{'}(x_j < x_i)|$. In levels, this figure shows that $U_i(x_j|x_i = x_j) > U_i(x_j|x_i > x_j) > U_i(x_j|x_i < x_j)$.

2.2 Incentive driven approach

GPS designed psychological form games to model feelings and belief-dependent emotions like anger or surprise. Based on Selten [1990] approach of subgame perfection and on Kreps and Wilson [1982] sequential rationality, they extend the Bayesian analysis of Gibbons et al. [1992] for letting players payoffs depend on what everybody thinks. Namely, players' utility function depends on prior knowledge (in Bayesian sense) and the equilibrium strategy profile is common knowledge. This enables GPS to summarize all players' beliefs of every order in a single profile. But to be clearer, we proceed in steps by first explaining the normal form game and secondly the extensive psychological game.

Normal form definition of the game The formal definition of the game is based on $N = 1, \ldots, n$ set of players and on A, the non-empty set of actions for player i. For any set X, $\Delta(X)$ is the subset of probability measure on X, $\Sigma_\tau = \Delta(A_\tau)$ is the set of mixed strategy for player i. The strategy profile $\sigma \in \Sigma = x_{i\in N}\Sigma_i$ and describes the probability distribution P_σ over $A : x_{i\in N}A_i$. The beliefs of a player, in the first order, represent a probability measure over the product of the other player's mixed strategy set $B_i^1 = \Delta\Sigma_{-i}$ where $\Sigma_{-i} = x_{i\neq j}\Sigma_j$. Since the probability set belongs to the Euclidean Space, the set of higher-order beliefs can be represented as the product of the previous orders (topology property[6]), such that for $k \geq 1$

$$
\begin{aligned}
&B_i^{k+1} := \Delta(\Sigma_{-i} \times B_{-i}^1 \times \ldots \times B_{-i}^k) \\
&B_i := x_{k=1}^{\infty} B_i^k
\end{aligned}
$$

This formal definition allows for correlation among beliefs of all orders, enabling to compute the marginal beliefs, i.e. a coherent restriction of higher-order beliefs that coincide with the first. $\hat{B}_i(0)$ is the set of player's i coherent beliefs that is common knowledge since all players are assumed to be rational. Thus, $\bar{B}$ is the set of collectively coherent beliefs. Given this summary of higher order beliefs, we can define player i utility function as $\bar{u}_i : \hat{B}_i \times A \to \Re$, which depends on outcome and beliefs; the player maximizes the expected utility in Von Neumann and Morgenstern sense such that $u_i(b_i, \sigma) := \Sigma_{t\in A}P_\sigma(t)\bar{u}_i(b, t)$. Thus we can define a Normal Form Psychological Game as $G = (A_i, \ldots, A_n; u_i, ..., u_n)$ which consists of an action set A and utility function $u_i : \bar{B}_1 \times \Sigma \to \Re$ for all players whose Nash Equilibrium is a pair $(\hat{b}, \hat{\sigma}) \in \bar{B} \times \Sigma$ such that

[6] A topological space is a set endowed with a structure, called a topology, which allows defining continuous deformation of subspaces, of the Euclidean spaces. The open sets of the Euclidean topology on $\Re^n$ are given by (arbitrary) unions of the open balls $B_r(p)$ defined as $B_r(p) := x \in \Re|d(p, x) < r$, $\forall r > 0$ and $\forall p \in \Re^n$, where d is the Euclidean metric.

- $\hat{b} = \beta(\hat{\sigma})$ where $\hat{\sigma}$ is the equilibrium profile and β the profile beliefs;
- $\forall i \in N$ and $\sigma_i \in \Sigma_i u_i(\hat{b}_i, (\sigma, \hat{\sigma}_{-i})) \leq u_i(\hat{b}_i, \hat{\sigma})$

By considering the summary of beliefs, which turns into a summary of utility function for player i $\omega_i(\sigma, \tau) := u_i(\beta_i(\sigma), \tau), \omega : \Sigma \times \Sigma \rightarrow \Re$ GPS demonstrate that ω is continuous, thus exploiting Kakutani fixed point theorem (Glicksberg [1952]), they also prove the existence of the Psychological Nash Equilibrium. GPS also model a sort of "disappointment function", however it is better to consider the "fairness function" of Rabin [1993] that we discuss in Section 2.3.

Extensive psychological game Once we have defined the normal form game, now we move to the extensive form of the game. The extensive form representation is important since it allows us to see that the payoffs of the terminal nodes are endogenously determined, thus ruling out backward induction. The extensive form game is defined as $F(N, v, <, m, \rho, \Pi, A)$, where N is the set of players, v represents the finite vertices with a partial order $<$ in which there is a path to the successor non-terminal node and $A(v)$ is the set of actions available, m is a function for each non-terminal node which specifies the action chosen according to a probability distribution $\rho(v)$, $\Pi(v)$ is the information set on non-terminal nodes. Players do not know in which vertex they are (as in a game of incomplete information) but they know when they are on the move.

The initial beliefs of the extensive form game $\Sigma(h)$ are the same as in the normal form, hence the utility function is defined for each strategy profile $\sigma \in \Sigma$ which induces a probability distribution P_σ over the terminal node as $u_i(b_i, \sigma) := \Sigma_{t \in T} P_\sigma(t) \bar{u}_t(b_t, t)$. Finally, the Nash Equilibrium of the extensive psychological game $\Gamma := (F, u_{i \in \aleph})$ is $u_i(\hat{b}_i, (\sigma_i, \hat{\sigma}_{-i}) \leq u_i(\hat{b}_i, \hat{\sigma})$[7]. The pair $(\hat{b}, \hat{\sigma}) \in \bar{B} \times \Sigma$ is a Subgame Perfect Psychological Equilibrium of Γ if it is a Psychological Nash Equilibrium of Γ and $\hat{\sigma}$ is Subgame Perfect Nash Equilibrium of $\Gamma(\hat{b})$ in traditional sense. Once again the existence can be demonstrated through the fixed point theorem by the finding that best correspondence $BR_h^\varepsilon : \Sigma \rightarrow \Sigma_i^\varepsilon(h)$ is upper semi-continuous, compact, and convex. As we stated before, backward induction is not available since terminal nodes depend on the player's beliefs about the other player's actions and beliefs; the payoffs are different whether the beliefs come true or not. In Figure 2, we can appreciate the difference between a Psychological game, whose payoffs depend on the other players' beliefs (i.e. $\tilde{r}, \tilde{q}$), and a standard perfect information game, where backward induction is available. In Section 3, we will discuss this approach to a Prisoner's Dilemma game with sympathetic coefficient.

[7] Recall that σ is common knowledge.

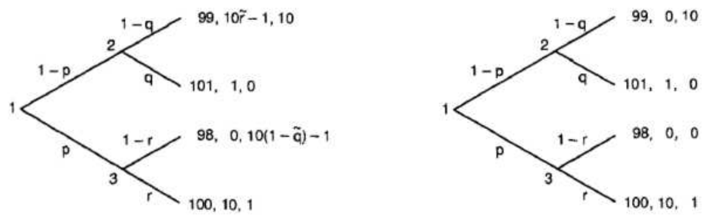

Figure 2: Differences between an extensive psychological game and a standard extensive game of perfect information Geanakoplos et al. [1989]

2.3 Fairness into game theory

The subsequent work of Rabin [1993] wants to introduce a sort of feelings feedback loop, in the sense that people want to be kind to those that were nice to them and want to hurt those who bitrate them. Rabin considers the following facts:

1. People are happy to donate to those who were kind to them;
2. People are willing to sacrifice their wealth for revenge[8];
3. Both behaviors have high material costs.

Hence, Rabin [1993] does not only consider personal beliefs about the other but also material payoffs, summarizing the previous works of Geanakoplos et al. [1989] and Fehr and Schmidt [1999]. In particular, he bases his game on the assumption that each person tries to maximize (minimize) the other material payoff producing an outcome that is a mutual-max (mutual-min). The results of this kind of game hold if:

- Any Nash Equilibrium that is either a mutual-max or a mutual-min, it is also a fairness equilibrium;
- If payoffs are small, roughly the outcome is a fairness equilibrium if and only if it is a mutual-max or a mutual-min;
- If payoffs are high the outcome is a fairness equilibrium if and only if is a Nash Equilibrium

Fairness model In this model, the psychological game is derived from the material game. The role of expectations concerns the other players' payoffs and beliefs. The model is built as follows: consider a 2×2 player normal form game with mixed strategy set S_1 and S_2

[8] see Levine [1998] for further descriptions of this behavior.

derived from pure strategy A_1 and A_2. $\pi_i : S_1 \times S_2 \rightarrow \Re$ is player i material payoff. Each player subjective expected utility depends on the strategy chosen $a_1 \in A_1$ and $a_2 \in A_2$, $b_1 \in B_1$ and $b_2 \in B_2$ which are respectively player 2 beliefs about the choice of player 1 and player 1 beliefs about the choice of player 2, $c_1 \in S_1$ and $c_2 \in S_2$ where c_1 is player 1 beliefs about b_1 and c_2 is player 2 beliefs about b_2. Player i is choosing $(\pi_i(a_i, b_j); \pi_j(b_j, a_i))$ from the set of feasible payoff $\Pi(b_i)$ for maximising his/her expected utility which depends on a "fairness function", a sort of player i measure of kindness. Let's define $\pi_i^h(b_i)$ and $\pi_i^l(b_i)$ player i highest and lowest material payoffs respectively among the point on the Pareto Efficient frontier in $\Pi(b_i)$; $\pi_i^e(b_i) = [\pi_j^h(b_i) + \pi_j^l(b_i)]/2$ is the equitable payoff (i.e. 50% split of the material payoff), $\pi_i^{min}(b_i)$ is the worst possible payoff in $\Pi(b_i)$, thus the fairness function[9] is

$$f_i(a_i, b_i) = \frac{\pi_i(b_i, a_i) - \pi_j^e(b_j)}{\pi_j^h(b_j) - \pi_i^{min}(b_i)} \tag{3}$$

where $f(.) > 0$ player i is giving j more than the equitable payoff and if $f(.) < 0$ i is giving less. Consequently, it is possible to model player i beliefs about how kind player j is being to him as $\tilde{f}_i(b_j, c_i) = \frac{\pi_i(b_i, c_i) - \pi_j^e(c_j)}{\pi_j^h(c_j) - \pi_i^{min}(c_i)}$. If $\tilde{f}(.) < 0$ also $f(.) < 0$, meaning that player behaviour is sensitive to scale material payoffs.

Thus, each player chooses a to maximize his/her expected utility $u_i(a_i, b_j, c_i)$ according to $u_i(a_i, b_j, c_i) = \pi_i(a_i, b_j) + \tilde{f}_i(b_j, c_i)[1 + f(a_i, b_j)]$ and we can characterise the fairness equilibria as: $(a_1, a_2) \in (S_1, S_2)$ is a fairness equilibrium for $i = 1, 2$ and $j \neq i$ if

$$a_i \in argmax_{a \in S_i} U_i(a_i, b_j, c_i) \tag{4}$$

$$c_i = b_j = a_i \tag{5}$$

Fairness equilibrium can rule out strict Nash Equilibrium as in the Dare - Chicken game, as in Table 2.3.

	DARE	CHICKEN
DARE	-2X, -2X	2X, 0
CHICKEN	0, 2X	X, X

Table 1: Dare-Chicken game for countries interaction Rabin [1993]

This game is usually exploited in political science to study the strategic interaction between two countries; each country hopes to "dare" the other and not be dared, however, both of them are afraid of the outcome (D, D) which is the Nash Equilibrium. For small X, it is inconsistent with the idea of fairness equilibrium. This is why in Section 3 we do not

[9]Fairness function might be discontinuous if $\pi_j^h(c/b_j) = \pi_i^{min}(c/b_i)$, however this can be solved by changing the functional form as Rabin [1993] shows in his Appendix.

consider this game for studying the interaction among core countries and peripheral.

Finally, we can conclude that $G(X) \in g$ is the game corresponding to a given value of X; we can impose restrictions on X to find the fairness equilibrium, such as looking for a situation where players try to maximize (minimize) the others' payoff. As material payoffs become larger, player behavior is dominated by self-interest, such that if (a_1, a_2) is a strict Nash Equilibrium for games in g, there exists $\bar{X}$ for which for all $X > \bar{X}(a_1, a_2)$ is a fairness equilibrium in $G(X)$. In Section 3 we will analyze a Gift-Exchange game in which the fairness function is continuous, thus by Folk theorem, we can assure that the fairness equilibrium is also a Subgame Perfect Nash Equilibrium.

3 Game theory for debt mutualization scheme

3.1 Ultimatum game for debt sharing quota (payoff driven approach)

In this section, we propose a typical ultimatum game (as the standard one proposed by Thaler [1988] analyzed by Levine [2006]) adapted to a new economic bargaining problem. We consider the case of a peripheral country, country i that belongs to the European Union and proposes to share a quota s of its public debt with the rest of the Union. We suppose (without loss of generality) that the country has to repay a fraction $1 - s$ of its total public debt and the other fraction s has to be repaid by the rest of the Union[10]. The 2 players have the same utility function presented in the theoretical section. The game is built on the following hypothesis:

- Player 1 is the generic country i that belongs to the European Union;
- Player 2 is the rest of the Union (all the other countries and European Institutions);
- The total debt of the country i is normalized to 1;
- The country fully repays a fraction $1 - s$ of its debt, whereas the rest of the Union repays the remaining fraction s;
- The utilities of the players have a "self-centered inequity aversion" form:

$$U_1(s) = s - \alpha_1 \max\{(1-s) - s, 0\} - \beta_1 \max\{s - (1-s), 0\}$$
$$U_2(s) = (1-s) - \alpha_2 \max\{s - (1-s), 0\} - \beta_2 \max\{(1-s) - s, 0\}$$

- Advantageous inequality is preferred to disadvantageous inequality: $\alpha_i > \beta_i$
- The disadvantageous inequality parameter is bounded $0 \leq \beta_i \leq 1$

[10]We do not consider interests on public debt, since we do not introduce a temporal dimension in our problem. This could be a further extension of this game.

Before finding the solution of the game, we discuss the interpretation of the utility functions adopted in this game in the cases of an unfair redistribution of the debt to be repaid. If the country i has to repay more than the 50% of its total debt ($s < 1/2$ and $(1-s) > s$), the parameter α_1 reflects the cost that the country assigns to not exploit a fair mutualization of the game (i.e. a lower rating and a higher interest rate to be paid because of potential economic instability).

Nevertheless, if the country i has to repay less than 50% of the debt ($s > 1/2$ and $(1-s) < s$), the parameter β_1 represents the cost of "abusing" the mutualization of the debt. This could be due to the fear of future constraints imposed by the Union, such as debt restructuring or a limit on public expenditure (austerity measures).

We focus now on the rest of the Union and we provide an interpretation that is in line with the model developed by Canofari et al. [2019][11]. How can the Union benefit from a share of country i debt and an equitable redistribution of it? The lower debt of country i is a benefit for the Union; country i would have more room to finance public expenditure, thus increasing its GDP via the fiscal multiplier [12]. Thus, this will pave the way for a positive externality to the Union, ensured by the growing economic prosperity and by the stability of country i. Moreover, the fraction s is subtracted from the utility of the Union, since this represents a cost. If $s < 1/2$ and $1-s > s$, most of the debt has to be repaid by country i. The coefficient α_2 reflects the cost of the negative externality caused by the economic instability of the peripheral country because of its increasing default risk.

Finally, if $s > 1/2$ and $1-s < s$, the Union has to repay more than one-half of the debt and the coefficient 2 reflects the cost of a monetary loss in the case of insolvency of the country. Having this in mind, we have reason to assume that $\alpha_i \geq \beta_i$ for every player. Now we can start solving the game.

- In the description of the theoretical model, we focused on the "first best solution": the utility of both players is maximized when the debt is distributed fairly, i.e. when $s = 1-s$. Thus, for $s = 1/2$, the players get the same level of utility. Formally $u_1(s = 1/2) = u_2(s = 1/2) = 1/2$

- If $s < 1/2$, the utilities reduce to:

$$U_1(s) = s - \alpha_1(1-2s) \tag{6}$$

$$U_2(s) = (1-s) - \beta_2(1-2s) \tag{7}$$

  Country i proposes to share a quota lower than one-half of its public debt. Player 2 accepts if its utility is positive, thus if $u_2(s) = (1-s) - \beta_2(1-2s) \geq 0$.

[11] They show that the instability of peripheral countries represents a negative externality for the Union.

[12] *"The government should pay people to dig holes in the ground and then fill them up."* (J. M. Keynes, The General Theory, 1936)

This inequality is true for every $\beta_2 \leq \frac{1-s}{1-2s}$. For $s = 0$, the parameter has to be $\beta_2 \leq 1$ (which is true by assumption). For every $0 < s < 1/2$, the coefficient β_2 has to be lower than any number greater than 1. Formally, $\beta_2 \leq c$ with $c > 1$. This case is also true by assumption since the parameter is upper bounded ($\beta_2 \leq 1$).
Thus, this inequality is always true, meaning that the Union will always accept to repay a share lower than 50% of the debt of player 1, even if the utility of country i would not be maximized in this case ($u_1(s < 1/2) < u_1(s = 1/2)$).

- If $s > 1/2$, the utilities become:

$$U_1(s) = s - \beta_1(2s - 1) \tag{8}$$
$$U_2(s) = (1 - s) - \alpha_2(2s - 1) \tag{9}$$

The Union will accept if its utility is positive, thus when $u_2(s) = (1 - s) - \alpha_2(2s - 1) \geq 0$. This inequality is true for every value of s such that $s \leq s^* = \frac{1+\alpha_2}{1+2\alpha_2}$. Core countries will accept to repay a share greater than 50% of the debt of country i if this share is lower than a given threshold (which is a function of the parameter α_2). The behavior of the Union changes when α_2 changes. We can consider 2 limiting cases.

1. The lower bound of α_2 is 0. Thus, for $\alpha_2 = 0$ (i.e. the monetary loss in the case of insolvency of country i is negligible for the Union), player 2 will accept to repay the debt up to a share $s \leq s^* = 1$, namely the Union will repay all the debt.

2. The parameter α_2 has no upper bound; when it tends to infinity, player 2 will accept to repay the debt up to one-half of it (so, for $s > 1/2$, it will not accept).

Formally, the Union will accept if $s \leq s^* = \frac{1+\alpha_2}{1+2\alpha_2} = 1/2$. The economic meaning is the following: if the Union attaches an infinite cost to the monetary loss in the case of insolvency of country i, it will not share a quota greater than one-half of the debt. We now consider under which condition player 1 prefers a fair distribution, rather than an advantageous unequal one. This happens when $u_1(s = 1/2) \geq u_1(s > 1/2)$. Substituting the value of the utility in this inequality yields $1/2 \geq s^* - \beta_1(2s^* - 1)$. The inequality holds for every $\beta_1 \geq 1/2$.

We can prove that player 1 will always prefer a fair distribution, rather than a disadvantageous unequal one, meaning that it will never propose to share a quota $s < 1/2$ (even though player 2 will always accept in this case). We analyze the case in which $u_1(s = 1/2) \geq u_1(s < 1/2)$. For this inequality to hold, the condition $1/2 \geq s - \alpha_1(1 - 2s)$ has to be satisfied. This will happen for every $\alpha_1 \geq -1/2$, which is true by assumption. Thus, for player 1 it is better to share the 50% of its debt with the rest of the Union, rather than a lower quota. Hence, the solution to the game is the following:

- for $1/2 \leq \beta_1 \leq 1$, player 1 proposes $s = 1/2$ and player 2 accepts;

- for $0 \leq \beta_1 < 1/2$, the quotas are $(s^*, 1-s^*)$, with $s^* \leq \frac{1+\alpha_2}{1+2\alpha_2}$ and $1-s^* > \frac{\alpha_2}{1+2\alpha_2}$.

The economic interpretation of this game is quite interesting. In fact, the agreement on the quotas depends on the parameters β_1 and α_2. If the cost of "abusing" the mutualization is too high, country i will never offer to share a quota greater than the 50% of its debt and the Union will always accept. We can suppose that the Union will impose some constraints on the country (i.e. an austerity regime) if it proposes to share a quota greater than one-half of its debt. Hence, country i chooses the "fair agreement". However, if this cost is not perceived as too high, country i can ask for a quota greater than 50% of its debt and the Union will accept subject to how costly the risk of a loss is evaluated. If this risk is negligible, it may also accept to pay all the debt; whereas, if the risk is important, the Union will accept only a quota which is almost one-half of the debt.

3.2 Dynamic prisoner's dilemma with sympathy parameter for moral hazard (incentive driven approach)

This game is presented as a finite horizon prisoner's dilemma in which player 2 becomes sympathetic if player 1 plays unexpectedly cooperatively. Player 2 is endowed with α, a sympathy factor that increases (decreases) at the end of each period by an amount $k \geq 0$, the sympathy coefficient, that is proportional to the difference of what player 2 expects player 1 to do and what player 1 does. Player 1 and player 2 are defined as manipulator (since it can affect payoffs of player 2) and manipulated respectively, thus we can make player 1 the Union (since it can impose rules on countries) and player 2 being country i. The two players can cooperate, thus agreeing on debt quota and the structural reform in favor of economic growth for country i, or defect where for the Union means imposing austerity measure while for country i means breaking the Maastricht parameters. In any case, country i would never accept austerity measures. The game is depicted in Table 2.

UNION COUNTRY i	COOPERATE	DEFECT
COOPERATE	10, 10 + α	0, 11
DEFECT	11, α	1, 1

Table 2: Dynamic Prisoner's Dilemma game with sympathy coefficient Geanakoplos et al. [1989]

The rules of the game are the following:

- If player 1 unexpectedly chooses to cooperate, in the next period player 2 payoff will be augmented by $\alpha + k(1-p)$, where p is the probability assigned to each action;
- If player 1 unexpectedly chooses defect, the next period player 2 payoff will be reduced by $\alpha - kp$;
- If the action of player 1 was expected, there are no changes in player 2 payoff;

- The game lasts T periods, where $G_k^T(\alpha)$ is the first play and $G_k^1(\alpha)$ is the last round.

For our game, assume that the stock α represents the utility gain from spread reduction when country i is compliant with the structural reforms (i.e. cooperate) and $\alpha = 0$ at the beginning of the game; $k = 2$, which represent a sort magnifying effect of the behavior of player 1 and $p = 1/2$, thus the probability of the Union to be cooperative or not is equal; $T = 3$ to deal with a medium-term horizon. The game begins in $G_2^3(0)$, where if player 1 chooses to cooperate unexpectedly, the payoff of player 2 increases to 2 while if the move was expected it remains unchanged, obviously $G_2^2(2)$ is better than $G_2^2(0)$. If player 1 would have wanted to play d in period 3, this would represent a contradiction. This is why we do not have any equilibrium in pure strategies, and we have to randomize around period 3 choice of player 1 to defect. We can identify two situations, one in which country i has shared a quota of its debt bigger or equal to 50%, but it feels confident enough to choose either cooperate or defect for obtaining favorable conditions; in the second one country i has still a quota of debt shared bigger or equal to 50% and to contain the spread, it commits to pursuing structural reform for growth respecting the Maastricht parameters. Call these two alternatives as discretionality and commitment for country i.

Discretionality In period 3, country i feels brave and decides to play defect while the Union can relax or not the austerity measure with probability 1/2. If the Union plays cooperate, the payoff of player 2 would be 1, such as the value of α which makes country i indifferent. Thus, there will not be further cooperation and each player receives a payoff of 2, which is a Subgame Perfect Psychological Equilibrium. If instead player 1 chooses to defect, the payoff will be $G_2^2(-1)$ and the game continues. In period 2, country i plays again defect to make its threat more credible posing the risk of instability above the Union. On the other side, the Union tries to make up country i mind by being cooperative, hence by adopting a sort of wait-and-see approach by saying "go on threatening the Union if you are brave enough". Indeed, by doing so the Union wishes to manipulate the action of country i making its value of $\alpha = 1$. The value of α cannot be raised anymore and for sure the Union will play a defect (i.e. imposing austerity measures) in the last period. However, when $\alpha = 1$ country i has nothing to lose and it will randomize around its actions in the last period by choosing to cooperate only with 20% probability.

Finally, the expected payoff for the Union is 3, while for country i is 13. This result is controversial since it is better to be manipulated than manipulator and it can be interpreted as follow: if the Union does not clarify its position at the beginning of the game because some countries are in favor of austerity measure for reducing public debt while others prefer structural reforms in favor of growth for reducing the debt/GDP ratio through an increase in the denominator, country i will be tempted to defect. By responding defect to defect, country i strengthens its position as a threat to European Union stability. When the Union tries to accommodate country i willing is too late since country i at this point is indifferent and we end up with a sort of Chicken game. Even if the outcome of country i is the highest, the previous game and the next game demonstrate that is not sustainable. Hence, in this first equilibrium set-up, we can conclude that when country i is cocky (remember the case of Greece) non-cooperative behavior of the Union incentives moral hazard as a punishment, leading to a suboptimal equilibrium.

Commitment Suppose instead that country i has a strong intention to follow the rules and it commits to always implement structural reforms respecting Maastricht Parameter. It commits to avoiding the equilibrium described in the discretionality case if the payoff reached in period 2 is $G_2^2(1)$. The Union still randomizes its actions in period 3, if it chooses c then it will always defect leading to a suboptimal situation. Indeed, if it chooses d in period 3, the game will end with a payoff of $(13, 12)$ that is Subgame Perfect Psychological Equilibrium. The meaning of this result can be interpreted as the optimal solution of a debt mutualization scheme. If the Union pretends austerity measures from a country that is already committed to certain reforms, they both gain. The Union plays the role of supervisor, while the country really implements structural reforms for repaying the debt knowing that, if it fails, it would be bridled by the austerity measures it wanted to avoid. With the next game, we will see an incentive scheme to pursue this goal.

3.3 Gift-Exchanging game as a mix of the two approaches

This game adapts the game of Akerlof [1982] for modeling the relationships among workers and firms for studying the interaction between the infamous country i and the Union. In this game we make a further assumption: we assume that the European Union creates a redeemable fund for buying the quota of shared debt by issuing risk-free asset[13]. Now, consider that country i (as in the prisoner's dilemma) could decide to cooperate or defect namely, to put a high effort H in repaying its debt or low L, such that $l \in H, L$. If $l = H$, the Union would obtain a benefit R and country i a disutility of γ, while if $l = L$, the Union has no gain and country i no disutility. The material payoffs are

- $\pi_c = b^{1/2} - \gamma$ if $l = H$; $b^{1/2}$ if $l = L$
- $\pi_{eu} = (R - b)^{1/2} - \gamma$ if $l = H$ and $b < R$; 0 if $l = L$ and $b > R, R > 0$

where b is the benefit level deriving from spread reduction due to debt convergence, γ is the disutility generated by the quota of the primary surplus to destinate to interest payment on non-mutualized debt, R is the gain in stability for the Eurozone, namely the increase in the demand for the risk-free asset issued by the debt fund. It is quite clear that with the usual game theory approach the solution of the game would be the nasty Nash Equilibrium $l = L$ and $b = 0$. Since it is a mutual-min, it is also a fairness equilibrium. However, a

[13] see Parello and Visco [2012] and Cioffi et al. [2019].

better solution could be achieved if we set equitable material payoff as $\frac{R^{1/2}}{2} - \gamma$ if $l = H$ and $\frac{R^{1/2}}{2}$ if $l = L$. We start from the Union side. To compute the utility function, that is

$$u_i(a_i, b_j, c_i) = \pi_i(a_i, b_j) + \tilde{f}_i(b_j, c_i)[1 + f(a_i, b_j)] \tag{10}$$

we need to define the fairness function and the believed fairness function, respectively

$$f_{EU} = \frac{b}{R}^{1/2} - \frac{1}{2} \tag{11}$$

$$f^*_{EU} = \left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2} \tag{12}$$

thus the European Union's utility function is

$$U_{EU} = (R-b)^{1/2} + \frac{1}{2}[\frac{1}{2} + \frac{b}{R}^{1/2}] \tag{13}$$

By maximizing the utility with respect to b, we obtain the optimal value for $b* = \frac{R}{1+4R}$. Let's now focus on the utility of country i, for which we distinguish the case in which $l = H$ or $l = L$; the two functions are

$$U_i(l = H) = b^{1/2} - \gamma + \left\{\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2}\right\}\frac{1}{2} \tag{14}$$

$$U_i(l = L) = b^{1/2} + \left\{\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2}\right\}\frac{1}{2} \tag{15}$$

Country i would put a high effort only if the relative utility is higher than the one associated with lower effort. We find the value $R^* \leq \frac{1}{4}\left[\frac{1}{(1/2+\gamma)^{1/2}} - 1\right]$.

The two values (b^*, R^*) are a mutual-max thus, representing a fairness equilibrium. Moreover, the fairness function is continuous in its domain hence, we can invoke the proof of Geanakoplos et al. [1989] to demonstrate that it is also a Nash Equilibrium. If we consider this game as an extension of the Prisoner's dilemma game we see in Section 3.2, we can also use the Folk Theorem to show that the results achieved are a Subgame Psychological Perfect Equilibrium.

The implications of this result are very important, in fact, we can say that country i and the European Union will cooperate if neither is too tempted by a material concern to cheat. The value of R depends on the disutility of country i; as γ becomes bigger, R reduces, making country i, very tempted to cheat, both because the quota of interest payment is too high and because the Union has no further gain in stability. For not cheating, country i should get a satisfactory reward but this implies a cost for the Union, thus for $\gamma \geq 1/2$ there is no gain for the Union no matter how R is small. The European Union will abandon the fund as well as country i. Fehr and Schmidt [2001] suggest completing contracts for reaching a better outcome (i.e. bonus contract instead of incentive contract).

The result is in line with the one obtained in the first game, if country i has to repay more than 50% of the interest on its debt it has no incentive to join the mutualization scheme as the European Union has no gain in stability. We find further sustain to our results in Kilgour and Zagare [1991], who showed that the lower credibility of a threat "can be offset by the increasing costs of guessing wrong", such as for γ approaching 1/2. Considering also that when there is an asymmetry of credibility, the threat of the lowest credible player (i.e. country i) can become a deterrent by increasing the cost of retaliation (before becoming negligible). We find again the threshold value in $\gamma = 1/2$.

4 Conclusion

In this paper, we present in Section 2 a short version of the literature behind the fairness approach in game theory. This approach aims to include in games the inner aspect of the decision-making process of human beings. One aspect considers the material payoff of all players and the feelings they fire up in them, the other one inspects how players behave subject to what the other players believe they are supposed to do, and the third one tries to balance the two previous methods by linking the psychological satisfaction to material payoff.

In Section 3 we adapt typical games to the analysis of the advantages and disadvantages of the debt mutualization scheme within the Euro Area. This idea runs through European economists since 2012, facing favors or strong veto. Due to the Coronavirus crisis, the debate about a debt-sharing scheme has been placed in the first line once again. With our three games, we have first demonstrated that the optimal quota for debt sharing is 50% of each country willing to join the scheme; the debt mutualization will provide the Union with gain in systemic stability. Secondly, we showed that an opportunistic behavior of both the European Union and the generic country i willing to join the scheme leads to a suboptimal equilibrium. The commitment of both the two to follow the rules conducts to the best result. Thirdly, we proved that this equilibrium can be maintained only with an incentive scheme that rewards both players and at the same time implies high costs for departing from the optimal solution.

Nevertheless, we are aware of the fact that our assumptions (i.e. symmetric games) made the calculations easy and that they can be relaxed in further extensions. Additional studies may determine how to evaluate the various coefficients that we used in our games, and how to make them vary across the European Union and with respect to the infamous country i. We suggest that they might depend on the country's riskiness due to the level of debt/GDP. Another extension can be made on the utility function of the fairness game, by considering Rawl's idea of maximum altruism as in Charness and Rabin [2000]. This could be an amazing challenge since this topic is one of the most studied by scholars and policymakers nowadays.

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A Appendix

A.1 Optimal value of b^*

$$
\begin{aligned}
&U_{EU} = (R-b)^{1/2} + \frac{1}{2}\left[\frac{1}{2} + \frac{b}{R}^{1/2}\right] \\
&\frac{\partial U_{EU}}{\partial b} = -\frac{1}{2}(R-b)^{-1/2} + \frac{1}{4}R^{-1}\frac{b}{R}^{-1/2} = 0 \\
&\left[\frac{1}{2}(R-b)^{-1/2}\right]^2 = \left[\frac{1}{4}(Rb)^{-1/2}\right]^2 \\
&R - b = 4Rb \\
&b^* = \frac{R}{1+4R}
\end{aligned}
$$

A.2 Optimal value of R^*

$$
U_i(l=H) = b^{1/2} - \gamma + \left\{\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2}\right\}(\frac{1}{2}) \geq
$$

$$
U_i(l=L) = b^{1/2} + \left\{\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2}\right\}(\frac{1}{2})
$$

$$
\begin{aligned}
&-\gamma + \frac{1}{2}\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{4} \geq -\frac{1}{2}\left[\frac{1}{1+4R}\right]^{1/2} + \frac{1}{4} \\
&\left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2} - \gamma \geq 0 \\
&\left[\frac{1}{1+4R}\right] \geq (\frac{1}{2} + \gamma)^2 \\
&R* \leq \frac{1}{4}\left[\frac{1}{(1/2+\gamma)^2} - 1\right]
\end{aligned}
$$

A.3 Continuity of fairness function

Remember that the functions have the following form

$$
\begin{aligned}
f_{EU} &= \frac{b}{R}^{1/2} - \frac{1}{2} \\
f^*_{EU} &= \left[\frac{1}{1+4R}\right]^{1/2} - \frac{1}{2}
\end{aligned}
$$

with $b \geq 0$ and $R > 0$ by assumption. Consider first $f*$, it would present a discontinuity for $R = 1/4$ but it is not in the domain, thus the function is continuous in its domain. Consider now $f(b, R) = \frac{b}{R}^{1/2}\frac{1}{2}$, the function is discontinuous in the origin. Applying the line passing through a point formula $y = m(x - x_0) + y_0$, in the origin $y = mx$. Therefore, we take the limit for $x \to 0$ $\lim_{x \to 0} f(x, mx) = \frac{x}{mx}$ we have the indeterminate formula $0/0$ and applying De L'Hopital rule we have $\lim_{x \to 0} 1/m = 1/m$, since it depends on m there is discontinuity in 0. But since R is never equal to 0 the function is defined in all its domain $\Re_0^+$. Hence, the function is continuous.