A Household's Preferences Vary Depending on Whether Incomes Are Permanent or Temporary: A Solution to the Time-Inconsistency Problem and Equity-Premium Puzzle

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Abstract

A household's preferences are usually assumed not to vary temporally or depending on the objects to which they are applied, but this assumption is often inconsistent with empirical estimates, for example, with the time-inconsistency problem of the time preference rate and the equity-premium puzzle. I show that these inconsistencies are generated because a household's preferences vary depending on whether they are applied to permanent or temporary incomes. Preferences applied to permanent incomes are anchored to the steady state or a balanced growth path, but those for temporary incomes are not. Hence, the former are fixed and unchanged, but the latter can take various values depending on conditions.

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Keywords: Equity premium; Permanent income; Risk aversion; Temporary income; Time inconsistency; Time preference

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1 INTRODUCTION

A household's preferences are usually assumed not to vary temporally or depending on the objects to which they apply. However, there is little evidence that they really do not vary temporally and depending on objects. For example, the time-variability of the time preference rate (RTP) has been long studied (e.g., Böhm-Bawerk, 1889; Fisher, 1930; Parkin, 1988; Lawrance, 1991; Becker and Mulligan, 1997), and Harashima (2004, 2009[1] 2014, 2016[2], 2019a) showed a mechanism describing why RTP occasionally changes and the consequences of those changes to an economy. Nevertheless, the assumption of constant preferences seems to make sense if it is used in dynamic economic models because this assumption makes the models time-consistent and tractable.

The assumption of invariable preferences, however, generates some important anomalies. That is, empirical estimates of preferences have been often inconsistent with the assumption of invariable preferences, for example, with the time-inconsistency problem of RTP and the equity-premium puzzle for the degree of risk aversion (DRA). The former is the anomaly where experimental estimates of RTP have often shown systematic changes depending on the length of delay in rewards (e.g., with hyperbolic discounting; Strotz, 1955; Pollak, 1968; Laibson, 1997; Barro, 1999; DellaVigna, 2009). The latter is the inconsistency between the average equity risk premium calculated using actual U.S. data and the values of DRA predicted by conventional economic theories (e.g., Mehra and Prescott, 1985; Rietz, 1988; Constantinides, 1990; Brown and Goetzmann, 1995; Kocherlakota, 1996; McGrattan and Prescott, 2003).

These inconsistencies may emerge because the assumption of invariable preferences does not actually hold. That is, a household's RTP and DRA may vary depending on conditions. In this paper, I pursue this possibility and examine a mechanism explaining how a household's preferences can vary.

Being constant means being anchored or fixed to something intrinsically unchangeable. Because the constancy of RTP and DRA is required in dynamic economic models with indefinite time horizons, RTP and DRA seem to be anchored to something that remains constant into the indefinitely far future (i.e., they are at steady state or on a balanced growth path). Hence, in this paper, I first examine the nature of the steady state and balanced growth path, particularly how a household reaches them, following the procedures and models developed in Harashima (2018b[3], 2020a, 2021b, 2022a[4]).

Harashima (2018b, 2020a, 2021b, 2022a) showed that there are two procedures

[1] Harashima (2009) is also available in Japanese as Harashima (2018a).
[2] Harashima (2016) is also available in Japanese as Harashima (2021a).
[3] Harashima (2018b) is also available in Japanese as Harashima (2019b).
[4] Harashima (2022a) is also available in Japanese as Harashima (2022b).

for a household to reach steady state or a balanced growth path: the RTP-based procedure and the MDC (maximum degree of comfortability)-based one. The former is the usually assumed procedure under which a household generates rational expectations and behaves so as to maximize the expected utilities that are discounted with a constant RTP. The latter is an alternative procedure Harashima (2018b, 2020a, 2021b, 2022a) presented. Under this procedure, the capital-wage ratio (CWR) at the "maximum degree of comfortability" (MDC) plays the crucial role, and a household has only to act on its feelings about whether its combination of labor income and capital (wealth) is comfortable or not. The steady state reached under the MDC-based procedure can be interpreted to be equivalent to the one reached under the RTP-based one.

Given the definition of CWR at MDC, the income at MDC is not stochastic, i.e., it is the mean of (potentially stochastically fluctuating) incomes at steady state or of the constantly growing part of incomes on a balanced growth path. The income at MDC therefore is not temporary but permanent. Because of the equivalence between the two procedures, the incomes for which RTP and DRA are applied under the RTP-based procedure to reach steady state or a balanced growth path must also be permanent. Conversely, the values of RTP and DRA that are applied for temporary incomes need not necessarily be constant and equal to those for permanent incomes because they are not used to reach steady state or a balanced growth path. This means that each household's preferences can vary in whether they are applied to permanent or temporary incomes. Nevertheless, it seems to be quite natural that a household possesses different preferences simultaneously because the natures of permanent and temporary incomes seem very different.

In this paper, I examine the difference in mechanisms behind RTP and DRA when they are applied to permanent and temporary incomes. I show that although the RTP and DRA for permanent incomes are constant, the RTP for temporary incomes varies depending on the length of delay in rewards and the DRA for temporary incomes varies depending on the survival period of firms in whose equity a household invests.

2 MDC-BASED PROCEDURE

In this section, I briefly explain the MDC-based procedure following Harashima (2018b, 2020a, 2021b, 2022a).

2.1 Rules to reach a steady state

2.1.1 "Comfortability" of CWR

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$, where $\breve{k}_t$ and $\breve{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ by a household and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i (i = 1, 2, 3, … , M). Each household assesses whether it feels comfortable with its current Γ (i.e., its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which a household is most comfortable with its combination of revenues and assets. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

2.1.2 Rules and steady state

In a homogeneous population (i.e., all households are identical), households act according to the following rules. Even when a population is heterogeneous, the rules are basically the same as those in a homogeneous population, but some revisions are required (see Harashima, 2018b).

2.1.2.1 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.
Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

2.1.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ for the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Also, let $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.

Proof: See Harashima (2018b).

The proposition indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

2.2 Response to endogenous growth

2.2.1 Response to technological progress

Harashima (2018b) showed how a household responds to technological progress under the MDC-based procedure as follows:

(a) If a new version of a product with higher performance at almost the same price as the old version is introduced, a household will buy the new version instead of the old version while its MDC remains unchanged.

(b) If a household's income unexpectedly and permanently increases, the household begins to feel that its current Γ is unexpectedly higher than $\Gamma(\tilde{s})$. However, because of the increase in income, its capital unexpectedly gradually increases, and the household will leave this accumulation of capital as it is until its Γ returns to its $\Gamma(\tilde{s})$.

Technological progress thereby causes the economy to grow through the household's responses via channels (a) and (b).

If technologies are endogenously generated, the quickness of households' responses will have an important effect on growth because firms have to make decisions on investments in new technologies fully considering how households will respond to new technologies. If households respond less quickly, fewer new products with new technologies will be purchased by households in a unit period. Firms therefore will be more cautious about investments in new technologies because they may not obtain sufficient returns from the investments or, even worse, suffer losses. As a result, if households respond less quickly, the speed of technological progress and thereby the growth rate of the economy will be lower. A slower response can be thought of as a more cautious response, and therefore the economic growth rate will depend on a household's degree of cautiousness in responding to new technology.

2.2.2 Degree of cautiousness

In this section, I briefly explain the relation between the degree of cautiousness and the economic (consumption) growth rate on a balanced growth path following Harashima (2020b). Under the MDC-based procedure, households feel the utilities from consumption in a similar manner as they do under the RTP-based procedure. Under the MDC-based procedure, μ is a function of the level of current or future consumption estimated by the household (c_l). It is important to note that the levels of future consumption are guessed values, and the expected μ is not discounted by RTP.

Suppose a usual power utility function such that

$$\mu = \frac{c_l^{1-\delta}}{1-\delta} \qquad \text{if } \delta \neq 1$$
$$\mu = \ln c_l \qquad \text{if } \delta = 1$$

where δ (≥ 0) is a parameter that reflects a household's degree of cautiousness about new technology. Therefore,

$$\frac{\dot{c}_l}{c_l} = -\delta^{-1}\frac{\dot{\upsilon}}{\upsilon}$$

where

$$\upsilon = \frac{d\mu}{dc_l} \ ,$$

and on a balanced growth path,

$$-\frac{\dot{\upsilon}}{\upsilon} = \Upsilon = \text{constant} \ .$$

The value of δ indicates a household's degree of cautiousness about new technology. A higher value indicates that households are more cautious about new technologies. Υ is not constrained by a household's state of mind or preferences but is an exogenous variable that is determined by the existing talent to produce innovations in an economy (Harashima, 2020b). Given a common constant Υ, the economic growth rate and equivalently the speed of technological progress depend on the value of δ. Harashima (2020b) showed that the value of δ under the MDC-based procedure is equivalent to the value of DRA under the RTP-based procedure on balanced growth path. Furthermore, they are equivalent only on balanced growth path.

3 RTP AND DRA USED FOR TEMPORARY PERMANENT INCOMES

3.1 Permanent income

3.1.1 RTP applied for permanent income

As is well known, in Ramsey-type growth models that assume the RTP-based procedure,

$$\theta = \left(\frac{1-\alpha}{\alpha}\right)\frac{w_t}{k_t} \tag{1}$$

holds at steady state, where θ is RTP, w_t and k_t are wage (labor income) and capital per capita in period t, respectively, and α is a parameter in a Cobb-Douglass production function that indicates the labor share. Equation (1) also holds on a balanced growth path in the endogenous growth model presented by Harashima (2018b, 2020a, 2021b, 2022a). Furthermore, equation (1) holds only at steady state or on a balanced growth path, which means that the RTP under the RTP-based procedure is tied with $\Gamma(\tilde{s}_i)$ only when it is used to reach steady state or a balanced growth path.

On the other hand, by the definition of $\Gamma(\tilde{s}_i)$, the values of w_t and k_t at MDC are not stochastic; that is, they indicate the means of (potentially stochastically fluctuating) w_t and k_t. In this sense, they represent permanent income and capital, not temporary ones. Because the MDC- and RTP-based procedures are equivalent to the procedure to reach steady state or a balanced growth path, the incomes for which RTP is applied under the RTP-based procedure to reach steady state or a balanced growth path must also be permanent.

3.1.2 DRA for permanent income

As shown in Section 2.2.2, DRA under the RTP-based procedure is only equivalent (in other words, anchored) to δ under the MDC-based procedure on a balanced growth path. Because a balanced growth path is a constantly growing economy, the value of δ is valid and meaningful only if it is applied to constantly growing values (i.e., permanent, not temporary, incomes). Because the two procedures are equivalent, the incomes for which DRA is applied under the RTP-based procedure to reach steady state and a balanced growth path must also be permanent.

3.2 Temporary income

3.2.1 RTP for temporary income

By their nature, temporary incomes have no permanent effect on steady state and a balanced growth path. Temporary incomes therefore are unrelated to CWR at MDC (i.e., $\Gamma(\tilde{s}_i)$). In other words, they are independent of CWR at MDC. Because the two procedures are equivalent, the value of RTP under the RTP-based procedure (θ) to reach steady state or a balanced growth path is also unrelated to temporary incomes and not anchored to CWR at MDC. Conversely, the value of a household's RTP (θ) that is applied for temporary incomes can take various and different values from that applied to permanent incomes (i.e., $\left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i)$).

3.2.2 DRA for temporary income

As indicated in the previous section, temporary incomes have no permanent effect on a balanced growth path. Therefore, temporary incomes are also unrelated to δ (i.e., they are not anchored to CWR at MDC). Because the two procedures are equivalent, the value of DRA under the RTP procedure to reach steady state or a balanced growth path is also unrelated to temporary incomes. Conversely, the value of a household's DRA (ε) that is applied to temporary incomes can take various and different values from that applied to permanent incomes (i.e., δ).

3.3 RTP and DRA vary depending on uncomfortability

As shown in Section 3.2, a household's RTP and DRA for temporary incomes can take various values, but do they float freely without any anchor?

3.3.1 RTP

Suppose that a household can obtain temporary income (ψ) today, but if it chooses to instead receive it tomorrow, it can obtain a larger amount ($\chi\psi$) (where $\psi > 0$ and $\chi > 1$). If a household is indifferent between ψ today and $\chi\psi$ tomorrow, χ reflects this household's magnitude of time preference for the temporary income.

However, because the income is temporary, the value of χ will reflect not only the conventionally assumed factors of time preference but also an additional factor derived from the nature of temporary income. Unlike permanent income, temporary income generates uncomfortable feelings in a household's mind with regard to MDC because obtaining temporary income means deviating from CWR at MDC (i.e., $\Gamma(\tilde{s}_i)$). Because of anticipatory anxiety, the period of experiencing uncomfortable feelings will be longer as temporary incomes can be obtained a longer period ahead, and therefore with this additional factor (i.e., uncomfortable feelings with regard to MDC), a household will further prefer temporary income today to temporary income tomorrow.

This additional factor works only in the case of temporary income. Hence, because of the nature of equivalence between the RTP and MDC procedures, the value of RTP (θ) under the RTP-based procedure for temporary income, which is not used to reach steady state or a balanced growth path, is not necessarily equivalent to $\left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i)$. Furthermore, it will be usually higher than $\left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i)$ because of the additional factor (i.e., households' uncomfortable feelings).

The effect of the factor of uncomfortable feelings on RTP (θ) can be most simply modeled as

$$\theta = \theta_P + \theta_T \tag{2}$$

where $\theta_P (> 0)$ is a constant regardless of permanent and temporary incomes, $\theta_T (\geq 0)$ is a variable, and $\theta_T = 0$ when all income is permanent. That is, θ_P represents the conventionally assumed factors of time preference that apply for both permanent and temporary incomes, and θ_T represents the additional factor that apply only for temporary income. Equation (2) indicates that RTP for temporary incomes (i.e., $\theta = \theta_P + \theta_T$ for $\theta_T > 0$) is higher than RTP for permanent incomes (i.e., $\theta = \theta_P$ because $\theta_T = 0$ when all income is permanent). θ_P is anchored to $\Gamma(\tilde{s}_i)$ (i.e., equivalent to $\left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i)$), but θ_T is not. This means that equation (1) holds only when θ is applied only to permanent income, that is, when it is used to reach steady state or a balanced growth path.

Note that in equation (2), θ_T is assumed to be a variable because it seems likely that θ_T varies depending on the length of delay. This possibility will be examined in Section 4.1.

3.3.2 DRA

Permanent income does not generate any uncomfortable feeling with regard to MDC because a household can prepare for it and therefore appropriately adjust its consumption in advance so as not to deviate from MDC. However, temporary income always generates uncomfortable feelings in a household's mind because it inevitably makes CWR deviate from MDC. When a household knows that it will obtain temporary income, it will anticipate the uncomfortable feelings that the temporary income will generate. This difference in how households feel about permanent and temporary incomes implies that a household's degree of cautiousness about new technologies (δ) will consist of not only an "intrinsic" fear of contact with the unknown or of uncertainty (the conventionally assumed attitude of risk aversion in economics). It will also consist of the fear of the anticipated uncomfortable feelings generated by its deviation from MDC due to temporary income.

In this sense, the degree of cautiousness (δ) that is used for permanent income only represents the intrinsic fear. In the case of temporary income, however, the degree of cautiousness will not necessarily be equal to δ. Because of the nature of equivalence between the RTP and MDC procedures, the value of DRA (ε) under the RTP-based procedure for temporary income, which is not used to reach steady state or a balanced growth path, is also not necessarily equivalent to δ. Furthermore, it will be higher than δ because of the additional fear of uncomfortable feelings that emerges in a household's mind in the case of temporary income.

The above discussed nature of DRA (ε) can be most simply modeled as

$$\varepsilon = \varepsilon_P + \varepsilon_T \tag{3}$$

where $\varepsilon_P (> 0)$ is a constant regardless of permanent and temporary incomes, $\varepsilon_T (\geq 0)$ is a variable, and $\varepsilon_T = 0$ when incomes are permanent. That is, ε_P represents the intrinsic fear that applies to both permanent and temporary incomes, and ε_T represents the fear of uncomfortable feelings that applies only to temporary income. Equation (3) indicates that the DRA for temporary income (i.e., $\varepsilon = \varepsilon_P + \varepsilon_T$ for $\varepsilon_T > 0$) is higher than DRA for permanent income (i.e., $\varepsilon = \varepsilon_P$ because $\varepsilon_T = 0$ when incomes are permanent). Although ε_P is anchored to CWR at MDC (i.e., ε_P is equivalent to δ), ε_T is not.

Note that in equation (3), ε_T is assumed to be a variable because it seems likely that ε_T varies depending on the probability of a household having uncomfortable feelings. This possibility will be examined in Section 4.2.

4 ANOMALIES OF RTP AND DRA

Experimental estimates of RTP and DRA have been often inconsistent with the values that conventional economic theories predict. Typical examples of such anomalies are the time-inconsistency problem of RTP and the equity-premium puzzle for DRA. In this section, I examine these anomalies following the results in the previous sections.

4.1 Time inconsistency and hyperbolic discounting

4.1.1 Time inconsistency

Many experimental estimates indicate that RTP changes systematically depending on the length of delay in rewards, for example, with hyperbolic discounting. On the other hand, many dynamic economic models rely on the assumption of a constant RTP as the discount factor because this assumption enables the models to be time consistent. The time-inconsistency problem between estimates of RTP and the needed assumption of a constant RTP has been long discussed (e.g., Strotz, 1955; Pollak, 1968; Laibson, 1997; Barro, 1999; DellaVigna, 2009). However, the nature of RTP indicated by equation (2) implies that the primary issue behind the time-inconsistency problem is that the experimental estimates of RTP and the RTPs used in dynamic economic models seem to represent different kinds of RTP.

Experimental research uses temporary income as a reward, and therefore the estimates obtained are RTP values that apply to temporary income. On the other hand, a dynamic economic model can work even if only permanent incomes are assumed (i.e., even if it is a deterministic model and not a stochastic one), which means that a dynamic economic model does not necessarily need to include the RTP for temporary income (see also the literature about stochastic growth models, e.g., Brock and Mirman, 1972). Hence, it is justifiable that a dynamic economic model assumes a constant RTP.

Hence, it will be quite natural that the values of experimental estimates and RTPs used in dynamic economic models differ. This implies that there is no inconsistency between equation (2) and dynamic economic models and that the time-inconsistency problem is a result of a mismatch between experimental and theoretical RTPs.

4.1.2 A hyperbolic model of RTP for temporary incomes

Many experimental estimates indicate that RTP has the nature of hyperbolic discounting (DellaVigna, 2009); that is, RTP changes temporally and systematically. A simple hyperbolic discounting model is as follows:

$$D(t) = \frac{1}{1 + \zeta(t - t_0)} \tag{4}$$

where $D(t)$ is the discount rate in period t, $t \geq t_0$, and ζ is a positive constant. The period $t - t_0$ indicates the delay in obtaining rewards (incomes). If RTP (θ) has the nature indicated by equation (4),

$$\theta(t) = \frac{\zeta}{1 + \zeta(t - t_0)} \ . \tag{5}$$

Equation (5) indicates that the value of RTP decreases as the delay becomes longer (i.e., it is temporally changeable).

As discussed in Section 3.3.1, the magnitude of uncomfortable feelings that stem from temporary income vary depending on the length of delay of obtaining the income. In addition, it seems likely that this magnitude will diminish as the length of delay becomes longer as with hyperbolic discounting.

Considering the above discussed features, RTP for temporary income can be most simply modeled with hyperbolic discounting. Suppose that θ_T has the nature of hyperbolic discounting indicated by equations (4) and (5); therefore,

$$\theta_T(t) = \frac{\zeta}{1 + \zeta(t - t_0)} \ . \tag{6}$$

By equations (2) and (6), RTP (θ) can be modeled as

$$\theta(t) = \theta_P + \theta_T(t) = \theta_P + \frac{\zeta}{1 + \zeta(t - t_0)} \ . \tag{7}$$

Equation (7) indicates that RTP ($\theta(t)$) is a hybrid of exponential and hyperbolic discounting. With data extracted from a household's short-term behavior (e.g., data with regard to temporary income), $\theta(t) = \theta_P + \theta_T(t)$ will be likely approximated by a hyperbolic discounting. On the other hand, with data extracted from a household's long-term behavior (e.g., data with regard to permanent income), $\theta(t) = \theta_P + \theta_T(t)$ will be likely approximated by an exponential discounting because, by equation (7),

$$\lim_{t \to \infty} \theta(t) = \theta_P \ .$$

An important point is that no time-inconsistency problem will arise in dynamic economic models with an indefinite time horizon because the needed RTP in these models is θ_P, which is constant. On the other hand, many experimental estimates of RTP will reflect $\theta_P + \theta_T(t)$ where $\theta_T(t) > 0$ because experiments basically can be conducted only with short time rewards (i.e., temporary incomes); therefore, they will be well approximated by hyperbolic discounting.

Note that there can be other and more complex types of discounting than hyperbolic discounting in which RTP also varies temporally. The discounting with regard to temporary income therefore need not be limited to hyperbolic discounting and may not actually be hyperbolic.

4.2 Equity-premium puzzle

4.2.1 Survival period

The equity-premium puzzle addresses the inconsistency between the average equity risk premium estimated using actual U.S. data and the value of DRA predicted by conventional economic theories (e.g., Mehra and Prescott, 1985; Rietz, 1988; Constantinides, 1990; Brown and Goetzmann, 1995; Kocherlakota, 1996; McGrattan and Prescott, 2003). Before examining this puzzle, I first examine the relation between incomes and the returns on risk-free bonds and equities. Government bonds are often assumed to be risk-free, or at least be the lowest risk bonds. Risk-free or lowest risk in this context means that returns on these bonds are almost certain to be obtained in the future. In this sense, these returns can be seen as part of permanent income.

On the other hand, it is not easy to determine whether returns on risky assets (e.g., equities) belong to the permanent or temporary income categories. If returns on equities could be certainly obtained almost indefinitely, they could be considered to be permanent. Because many equities have existed over a long term in financial markets, they could be viewed, at least in an aggregate sense, as part of permanent income. However, individually, most (probably all) equities (firms) cannot exist indefinitely. In essence, firms come and go from the market. Each firm has a unique finite "survival period", although this period is unknown presently and can only be known *ex post*.

If a household could perfectly anticipate the rise and fall of all firms and correctly replace the failing firms' equities it holds with equities of new and rising firms in advance, sufficient amounts of positive returns would always be obtained indefinitely. However, in actuality, it is impossible for most households to perfectly anticipate these types of movements. Hence, for most households, investments in equities are risky, which implies that the returns on investments in equities simultaneously have the natures of both permanent and temporary incomes.

The idea of the non-existence of the equity-premium puzzle because of finite survival periods is not new. It has been argued that only examining data from equities that have survived for long periods will lead to incorrect conclusions about the equity-premium puzzle (e.g., Brown and Goetzmann, 1995).

4.2.2 Model of different values of DRA for permanent and temporary incomes

As discussed in Section 3.3.2, the value of DRA (ε) applied for temporary income (i.e., $\varepsilon_P + \varepsilon_T$ for $\varepsilon_T > 0$) is higher than that to permanent income (i.e., ε_P). In addition, as discussed in Section 4.2.1, most households perceive that the returns on the lowest-risk bonds are permanent and those on equities have the natures of both permanent and temporary incomes. Therefore, the DRA that is applied for equities under the RTP-based procedure is $\varepsilon_P + \varepsilon_T$ where $\varepsilon_T > 0$, and it is higher than that of the lowest-risk bonds (i.e., ε_P).

In addition, the results in Sections 3.3.2 and 4.2.1 indicate that, as the expected survival period of an equity (firm) becomes shorter, the probability of the firm's bankruptcy in the near future will be higher, and therefore, the probability of a household suffering uncomfortable feelings due to its departure from MDC in the near future will also be higher. This means that the ε_T applied to an equity will be positively correlated to the expected survival period of the equity (firm).

Following this idea, ε_T can be most simply modeled as

$$\varepsilon_T(s) = \frac{\eta}{1 + \eta s} \tag{8}$$

where s indicates the expected survival period of equity (firm) and η is a positive constant. By equations (3) and (8),

$$\varepsilon(s) = \varepsilon_P + \varepsilon_T(s) = \varepsilon_P + \frac{\eta}{1 + \eta s} \ . \tag{9}$$

If a firm in whose equity a household invests is expected to survive for a longer period, the value of DRA that is applied for the investment in this equity ($\varepsilon_P + \varepsilon_T(s)$) becomes smaller, and vice versa. Because ε_P is constant, as indicated in Section 3.3.2, by equation (9),

$$\lim_{s \to \infty} \varepsilon(s) = \varepsilon_P \ .$$

That is, if a firm in whose equity a household invests is expected to survive almost indefinitely, the applied DRA becomes equal to that of permanent income (i.e., the lowest-risk bonds). On the other hand, if the expected survival period is finite, the value of DRA applied to equities is higher than that of the lowest-risk bonds because $s < \infty$ and by equation (9),

$$\varepsilon_P < \varepsilon(s) = \varepsilon_P + \frac{\eta}{1 + \eta s} \ .$$

An important point is that it does not seem to be appropriate to use a unique single value of DRA to examine the equity-premium puzzle because the value of DRA applied to permanent incomes (the lowest-risk bonds) is generally different from and lower than that to temporary incomes (equities).

5 CONCLUDING REMARKS

A household’s preferences are usually assumed not to vary temporally or depending on objects to which they are applied. However, there are many empirical estimates that are inconsistent with this assumption, in particular, the RTP time-inconsistency problem and the DRA equity-premium puzzle. These inconsistencies may emerge because a household’s preferences do actually vary temporally and depend on the objects to which they are applied.

Being constant means being anchored or fixed to something intrinsically unchangeable. Because the constancy of RTP and DRA is particularly required in dynamic economic models with an indefinite time horizon, RTP and DRA seem to be anchored to something that remains the same indefinitely into the future (i.e., the steady state or a balanced growth path).

By the definition of CWR at MDC, incomes at MDC are not stochastic; that is, they are the mean of (potentially stochastically fluctuating) incomes at steady state or constantly growing values on a balanced growth path. The incomes at MDC therefore are not temporary but permanent. Because of the equivalence between the MDC- and RTP-based procedures for reaching steady state or a balanced growth path, the incomes for which constant RTP and DRA are applied under the RTP-based procedure to reach steady state or a balanced growth path must be also permanent. Conversely, the values of RTP and DRA that are applied for temporary income need not necessarily be constant and equal to those of permanent incomes. This means that a household’s preferences can vary depending on in whether they are applied to permanent or temporary incomes.

I showed that, although the RTP and DRA for permanent income are constant, the RTP for temporary income varies depending on the length of delay in rewards and the DRA for temporary income varies depending on the survival period of firm in whose equity a household invests.

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