Detection avoidance and deterrence: some paradoxical arithmetics

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Abstract

This paper extends Malik's (1990) analysis to the case where criminals' avoidance efforts and public expenditures in the detection of criminals are strategic complements in the aggregate technology of control of illegal behaviours. In this set up, we show that whenever criminals' avoidance efforts are more sensitive to the frequency than to the severity of sanctions, it is always socially efficient to set the fine at the maximal possible level. However, several paradoxical consequences occur: more repressive policies lead to less arrestations of offenders while more crimes may be committed; at the same time, the society may be closer to the first best number of crimes.

Keywords: deterrence, avoidance activities, optimal enforcement of law.
JEL Classification: D81, K42.

1 Introduction

Malik (1990) has initialy established that criminals' avoidance activities[1] may explain the optimality of less than maximum fines, contradicting the classical result of Becker (1968)[2]. The purpose of this note is to show that if criminals' avoidance expenditures are more sensitive to the frequency than to the severity of punishment, then the beckerian' s view always holds[3], but with paradoxical

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[1] Avoidance expenditures correspond to self-protection activities undertaken by criminals in order to reduce the probability of being caught and punished by an enforcement agent. They comprise installing radar detectors to avoid speeding tickets, lobbying politicians to relax the enforcement of regulations, bribing an enforcement agent to let free a culprit, destroying or covering up incriminating evidence, or investing in long and costly litigations and so on.

[2] See the surveys by Garoupa (1997) and Polinski and Shavell (2000).

[3] In a sense, our paper is in the same spirit as Nielson's (1998) one who challenged Polinski and Shavell (1979)'s argument, showing that if risk averse criminals are more sensitive to the

consequences for enforcement authorities not previously described in the literature. On the one hand, some level of overdeterrence may be socially optimal. On the second, raising the fine may lead to a decrease in the level of deterrence. We will show that this may occur when criminals' avoidance efforts and public expenditures in deterrence are strategic complements in the aggregate technology of detection of illegal activities[4], meaning that the higher public expenditures in crime deterrence, the higher criminals' own avoidance efforts.

Thus, the paper suggests that the *fine tuning* of crime deterrence is far more uncertain than in standard models of law enforcement, adding to Sanchirico's (2006) recent analysis of the social cost of avoidance activities. When criminals' avoidance detection is taken into account, more repressive policies lead to less arrestations and less sanctions of offenders, but may also result in to less deterrence of offenses (more crimes). We show that paradoxically in such cases, the effectiveness of public policies may increase since the economy may be closer to the first best efficient level of deterrence (number of crimes).

For the sake of concreteness, consider the following example of speed limit violations: potential offenders have the opportunity to install radar detectors to avoid speeding tickets, which render them very sensitive to the frequency of police controls on roads. Notice that in such a case, the higher the tickets, the larger the incentives for offenders to buy such detectors. Nevertheless, given that such technology renders them very reactive to speed limit controls and less sensitive to penalties, high tickets are still optimal. At the same time, higher tickets may require more frequent controls on roads in order to maintain a given level of deterrence: this is because the use of radar detectors allows offenders to escape from police detection and sanction, and enforcers must compensate this with more efforts in monitoring riders' behaviour. Putting it differently, it may not be efficient for the enforcers to substitute more frequent controls on roads by higher speeding tickets since riders using radar detectors avoid police controls and detection, thus reducing the effectiveness and efficiency of controls. To summarise, increasing the fine of speeding tickets has at least two different adverse effects when radar detectors are bought by drivers: on the one hand, the observed number of drivers cimmitting speed limit violations who are caught and punished becomes smaller and smaller (police controls become less and less efficient); on the other hand, the effective number of speed limit violations may also become larger and larger (police controls may imply less and less deterrence effects on roads).

The paper analyses the driving force behind these results: criminals' avoidance expenditures always increase when enforcement authorities undertake more repressive policies. Thus, any increase in the maximal fine has a direct (positive) effect on the level of deterrence, plus an indirect and negative effect since

certainty than to the risk of punishment, then maximal fines are still optimal. It seems that there exists some emprirical evidence which is consistent with such an assumption: see Block and Gerety (1995) and Grogger (1991). Becker (1968) and Nielson and Winter (1997) argue that the expected utility assumption does not allow to rationalize these observations.

[4] Malik(1990) assumed that criminals' avoidance efforts are independant of public expenditures in deterrence.

as criminals invest more on avoidance detection, it decreases the effective probability of arrestation. Whether the direct effect is dominating or in contrast is dominated by the other one depends on the sensitivity of the avoidance activities with respect to the probability and the severity of sanctions. Moreover, the higher the cost of avoidance for criminals, the more likely the occurrence of overdeterrence. The rest of the paper is structured as follows. Section 2 describes the basic model and analyses criminals' individual behaviour. Section 3 focuses on the optimal law enforcement issue and presents the main results of the paper. Section 4 briefly concludes.

2 A model of criminals' avoidance activities

2.1 the technology of arrestation and conviction

Let us consider the case where illegal activity allows the criminal to obtain a payment equal to $b > 0$, but imposes an externality cost $D > 0$ on the society. When caught, the offender has to pay a fine $f > 0$. Let us denote by p the level of public expenditures in the monitoring of the criminals' activity, which may be understood as the frequency of public control. Nevertheless, the probability of arrestation and conviction is less than p, since criminals have the opportunity to invest in avoidance activities.

Let the aggregate technology of arrestation and conviction be characterized by the probability function $q = q(p, x)$, x being the criminal's efforts of concealing the illegal activity and avoiding public controls. We also assume that the monetary equivalent of the utility cost of this effort is equal to x. The probability q corresponds to the effective probability of arrestation and sanction, and satisfies:

Assumption 1:
1) $q(p, 0) = p$, $\forall p$ with: $q(0, x) = 0$;
2) $q_p > 0$; $q_{pp} < 0$;
3) $q_x < 0$; $q_{xx} > 0$;
4) $q_{xp} < 0$.

Conditions 1.2 and 1.3 say that i) the probability of criminals' arrestation and convinction increases with public monitoring, but decreases with private avoidance; and that ii) there exist decreasing returns to scale both in avoidance and in public monitoring. Finally, condition 1.4 says that private avoidance and public expenditures in monitoring are strategic complements in the aggregate technology of control. This basic assumption has a major consequence: $e_p^q \equiv \frac{\partial q}{\partial p}\frac{p}{q} \leq 1$, which is straightforward to proove given the concavity of the probability of criminals' arrestation and convinction with respect to public monitoring.

2.2 avoidance activity and criminals' behaviour

The maximum expected benefit obtained by the criminal when he undertakes the illegal activity and makes avoidance efforts is equal to:

$$u \equiv \max_x (b - q(p,x)f - x) \tag{1}$$

Assuming that $q_x(p,0)f + 1 > 0$, the crimiminal's optimal avoidance expenditures are given by the (necessary and sufficient) condition:

$$-q_x(p,x^*)f = 1 \tag{2}$$

as long as $u > 0$. According to (2), the optimal value of expenditures in avoidance activities is set such that the marginal cost of effort compensates the decrease in the expected value of the penalty.

By the implicit function theorem, it is easy to see that $sign\frac{\partial x^*}{\partial p} = sign(-q_{xp})$ and $sign\frac{\partial x^*}{\partial f} = sign(-q_x)$; thus, the optimal x^* is unambiguously increasing with f and p. The value of the illegal benefit b matters only in the sense that it influences the decision to engage or not in the illegal activity and to undertake the avoidance expenditures. There exists a threshold value of the benefit $b^* \equiv q(p,x^*)f + x^* > 0$ such that the illegal activity is performed only if $b > b^*$. All else equal, deterrence occurs as far as $u \leq 0$ or equivalently: $b \leq b^*$. Given that individuals are not erroneously caught, they do not engage in avoidance activities once they do not engage in the illegal activity.

3 Optimal enforcement of the law

3.1 the efficient probability/penalty tradeoff

Let us assume that public authorities do not observe b nor the effort undertaken by the individuals. They just know that b is distributed according to a cumulative distribution function $G(b)$ taking values on $[0,\infty)$, with a density $g(b) > 0$ everywhere. On the other hand, whatever the private benefit for the criminal, the loss for the rest of the society satisfies $D < \infty$. The management costs associated with the monetary penalty are negligeable, but monitoring the criminal activity entails a cost to $m(p)$, satisfying $m' > 0$ and $m'' \geq 0$. The issue for the government is to choose a fine (monetary sanction) f and a probability of control p (leading to the effective probability of detection and convinction $q(p,.) \leq p$), in order to maximize the social welfare function[5]:

$$W = \int_{b^*}^{\infty} (b - D)dG(b) - (1 - G(b^*))x^* - m(p)$$

[5] That criminals' welbeing - their illegal gain - appears in the social welfare is controversial (Levin and Trumbull (1990), Garoupa (1997) and Polinski and Shavell (2000)). In order to tackle the divergence between private gains and the social value of criminal activities, we could introduce a discounting factor on illegal benefits. Another solution would be to minimize the social cost of crime under a constrained level of utility for criminals. Formally, both are equivalent to the conventional approach used in the text.

with x^* satisfying (2), and under the constraint[6] $f \leq F$. The first (integral) term in W corresponds to the expected private benefit associated with the illegal activity (benefit of the criminals minus the external cost). The two other terms are the cost of avoidance activities per criminal, and the cost of monitoring for public authorities. As far as the fine corresponds here to a simple transfer between the (risk neutral) criminal and the government, they do not appear in the social welfare function - they have no value from a social point of view.

The solution to this problem (see Malik (1990)) may be no deterrence (for example, in the case of small values of the external cost of crime and/or large values of the public cost of monitoring), complete deterrence (opposite conditions) or conditional deterrence (if both the external cost of crime and the public monitoring cost are large enough in order to make the controle of criminals socially worth, but not to deter all of them). We focus here on this last case.

The solution with conditional deterrence (b^*, p^*, f^*) is characterized by the first order conditions of maximization:

$$g(b^*)q_p^* f(D - q^* f^*) = (1 - G(b^*))\frac{\partial x^*}{\partial p} + m'(p^*) \tag{3}$$

$$g(b^*)q^*(D - q^* f^*) = (1 - G(b^*))\frac{\partial x^*}{\partial f} + \lambda \tag{4}$$

where $q^* = q(p^*, x^*)$, with $\lambda = 0$ if $f < F$ but $\lambda \geq 0$ otherwise, and $b^* \equiv q^* f^* + x^*$. They are necessary and sufficient as long as function m and/or function q exhibit sufficiently decreasing returns to scale. Let us denote as e_f^x the elasticity of the criminal's effort with respect to the fine, and e_p^x the elasticity of the criminal's effort with respect to public monitoring. The following proposition shows that the values of these elasticities play a key role[7].

Proposition 1 *The solution with conditional deterrence has the following properties:*

i) If $e_f^x \leq e_p^x$, the maximum fine $f^* = F$ is optimal, and the probability p^* must be set as small as possible.

ii) If $e_f^x > e_p^x$, it may be optimal to choose $f^* < F$.

iii) The optimal expected fine is smaller than the external cost of crime ($q^* f^* < D$), and there may exist either over or under deterrence at optimum ($b^* \equiv q^* f^* + x^* \gtreqless D$).

[6] This is a natural assumption since criminals have a disutility from the avoidance effort. Introducing a monetary cost of effort would not significantly alter the results and main conclusions, although it would require us to modify the definition of the maximal fine (*i.e.* $f < F - x$) in order to take into account that wealth and the costly effort would now be perfect substitutes. Thus, the slight modification that would be obtained corresponds to the case where the constraint is binding, inducing a level of deterrence at optimum (*i.e.* $\bar{b} = qF + (1 - q)x$) which is smaller than under the disutility cost of effort assumption.

[7] All the proofs are in the appendix.

Malik's (1990) model allows only case ii). Hence, results i) and iii) deserve more specific discussion.

To illustrate i), assume that initially $f < F$, and consider that enforcement authorities decrease the probability of control and just compensate it by an increase in the fine in order to keep the level of deterrence $b^* = qf + x^*$ constant. Thus, the total effect on social welfare can be written:

$$\Delta W = (1 - G(b))\frac{x^*}{p}\left(e_f^{x^*} \times e_p^q - e_p^{x^*}\right)\Delta p - m'(p)\Delta p$$

When criminals' efforts are more sensitive to the probability than to the severity of the punishment (*i.e.* the fine), then a decrease in the probability of control leads to a decrease in the (private) cost of avoidance activities that more than compensates the increase in cost which is associated with the even higher fine; moreover, it adds to the cost cut associated with the public monitoring activity, thus improving the social welfare.

The second result (iii) means that although the expected fine is always smaller than the external cost of crime at the optimum, the cost of private avoidance activities may be large enough to induce a level of deterrence greater than the external cost of crime, implying an excessive level of deterrence at optimum; however, the opposite result may arise. It is important to notice that whether individual avoidance expenditures are more or less sensitive to the frequency than to the penalty is irrelevant for this last (ambiguous) result.

3.2 on the effectiveness of public policies

The effect pointed out in Sanchirico's (2006) analysis also holds in our model: any increase in public expenditures in deterrence gives incentives to criminals to undertake more avoidance expenditures, which has an adverse effect on the effective probability of arrestation. Here, we add to it, investigating whether the enforcement authority has the opportunity or not to reach a fine tuning of the level of optimal deterrence. First, we have the following result:

Proposition 2 *Consider a solution where* $f^* = F$.

i) An increase in the maximal fine may yield an increase or a decrease in the optimal probability.

ii) Moreover, assume that $F \to 0$ *such that there is a large level of under-deterrence at optimum (*$b^* \to 0$*); then the optimal probability and the maximal fine may be substitutes.*

iii) On the contrary, even if F *is large enough to produce a level of deterrence arbitrarly close to full internalization of the external cost of crime (*$b^* \to D$*), the optimal probability and the maximal fine may be complements.*

Given that criminals' expenditures in avoidance detection are sensitive both to the probability and to the severity of sanction, the general finding i) is hardly a surprise. Nevertheless, what proposition 3 makes clear is that the probability and the fine may be either substitutes or complements whatever the initial level of deterrence, which is in contrast with the usual results obtained in the canonical set up (Garoupa (2001)). The driving force behind these results is that any increase in the maximum level of the fine all else held equal has a direct and positive effect on the level of deterrence, plus an indirect and negative effect; the latter reflects that as the fine increases, criminals invest more on avoidance activities, implying that the effective probability of arrestation and sanction decreases.

Consider for example a situation where the maximal fine is very large, such that the level of deterrence is close to the full internalization of crime ($b^* \rightarrow D$); in such an event, increasing the fine entails a large increase in criminals' avoidance expenditures, and it may be socially worth compensating the induced impact on the effective probability of sanction by an increase in the optimal probability of control. In the opposite case with a low level of the maximal fine ($F \rightarrow 0$, meaning criminals' assets are small), the level of crime deterrence is also very low ($b^* \rightarrow 0$). Thus, the social benefits coming from the increase in the level of deterrence associated with the larger fine may be more than compensated by the decrease in the effective frequency of sanction coming from the increase in the criminal's private cost of avoidance, requiring that the optimal probability be reduced.

The next issue is the impact of monetary sanctions on the level of deterrence at optimum.

Proposition 3 *Consider a solution where $f^* = F$.*

i) An increase in the maximal fine always yields an increase in the threshold of deterrence when the probability and the fine are complements at optimum, whereas the effect is ambiguous in the case where they are substitutes.

ii) If $q_{xp} = 0$, then an increase in the maximal fine always yields an increase in the threshold of deterrence.

iii) If $q_{xp} < 0$, then an increase in the maximal fine may lead to a decrease in the threshold of deterrence.

Once more, this finding is in contrast with the usual results obtained in the canonical set up (Garoupa (2001)), where the expected fine is an increasing function of the maximal fine, irrespectively of the fact that the sanction and the probability are complements rather than substitutes.

First notice that the ambiguity only arises when the maximal fine and the probability of control are substitutes (see i)): whether the direct effect of the increase in the fine dominates or in contrast is dominated by the second one depends on the sensitivity of the avoidance activities with respect to the probability as shown by results ii)[8] and iii). Specifically, the larger the elasticity of

[8] Which corresponds to Malik's assumption.

criminals' avoidance expenditures with the probability, the larger the likelihood that the level of deterrence decreases with the maximal fine (see iii)).

A straightforward consequence of proposition 3 is the following:

Corollary 4 *An increase in the maximal fine always yields a decrease (respectively, an increase) in the level of underdeterrence (overdeterrence) when the probability and the fine are complements at optimum, whereas the effect is ambiguous in the case where they are substitutes.*

4 Final remarks

We have shown the existence of several uncomfortable albeit unavoidable consequences that arise for enforcers when criminals invest in avoidance activities, which adds to Sanchirico's recent analysis. Specificaly, when public invesments in deterrence and criminals' avoidance expenditures are strategic complements in the aggregate technology of crimes detection, we have found a great degree of ambiguity 1/ concerning the optimal tradeoff between the probability and the severity of punishment - excepted when criminals are more sensitive to the risk of control than to the severity of punishment (an effect absent from Malik (1990)'s paper), or 2/ with respect to their complementarity *versus* substitutability, as well as 3/ with respect to the level of deterrence reached since the existence of avoidance investments by criminals may result in underdeterrence as well as in overdeterrence.

Hence, as far as detection avoidance is a by-product of criminal activities, the fine tuning of illegal activities by enforcement authorities appears as a very controversial issue: in fact, we obtain ambiguous prescriptions from a practical point of view. The basic reason is that depending on the technology of detection avoidance available for criminals, they become more or less sensitive to the frequency of punishment relatively to the severity of punishment; and depending on the cost of this detection avoidance, there may exist over or under deterrence.

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APPENDIX

Proof of proposition 2

Let us consider a solution where the optimal fine satisfies $f^* < F$. Substituting (4) into (3) gives:

$$(1 - G(b^*)) \left(q_p^* f^* \frac{\partial x^*}{\partial f} - q^* \frac{\partial x^*}{\partial p} \right) = q^* m'(p^*) \tag{5}$$

where the bracketed term writes: $\frac{q^* x^*}{p} \left(e_f^{x^*} \times e_p^q - e_p^{x^*} \right)$. Hence, there are two cases:

A/ assume that $e_f^{x^*} \times e_p^q - e_p^{x^*} \leq 0$: given that $e_p^q \leq 1$, this condition is satisfied for example (but not uniquely) when for all (p, f) we have $e_f^x \leq e_p^x$; then, the LHS in (5) would be negative, meaning that any positive value for p implies a loss of social welfare: thus $f^* < F$ cannot be optimal. Then assume that $f^* = F$; using (3), p^* must be set as low as possible according to the condition:

$$g(b^*) q_p^* F(D - q^* F) - (1 - G(b^*)) \frac{\partial x^*}{\partial p} = m'(p^*) \tag{6}$$

B/ assume now that $e_f^{x^*} \times e_p^q - e_p^{x^*} > 0$; obviously, this occurs for example (but not uniquely) when $e_p^{x^*} = 0$ (which occurs when $q_{xp} = 0$). More generally, in order to hold, the inequality $e_f^{x^*} \times e_p^q - e_p^{x^*} > 0$ requires (condition necessary but not sufficient) that for all (p, f) we have $e_f^x > e_p^x$. Then it may be optimal to choose $f^* < F$, although, there are cases where it is not.

Finally, since the RHS of (3) is positive, we obtain that $D > q^*f^*$ both when $f^* = F$ or $f^* < F$; hence, there may exist at optimum either over or under deterrence since $b^* = q^*f^* + x^* \gtreqless D$, which depends on the private cost of avoidance borne by criminals.

Proof of proposition 3

It is sufficient to give the proof only for specific cases. For ease of exposition, assume that $q_{xp} = 0$ (which implies that x^* does not depend on p). In contrast, it is easy to verify that when avoidance expenditures are sensitive to public detection, additional terms appear which make more likely that W_{pF} takes a positive sign. This is left to the reader.

Applying the implicit function theorem to (3) with $q_{xp} = 0$, we obtain that $sign\frac{\partial p^*}{\partial F} = signW_{pF}$ where:

$$\begin{aligned} W_{pF} &= g(b^*)q_p^* \left\{ (D - 2q^*F) + (D - q^*F)\frac{g'(b^*)}{g(b^*)}q^*F + \frac{\partial x^*}{\partial F}(-q_x^*F) \right\} \\ &= g(b^*)q_p^* \left\{ \left(\frac{D - 2q^*F}{D - q^*F} - e \right)(D - q^*F) + \frac{\partial x^*}{\partial F}(-q_x^*F) \right\} \end{aligned}$$

with: $e = -\frac{g'(b^*)}{g(b^*)}q^*F$ denoting to the elasticity of the density with respect to the expected fine. Remark that the first bracketed term $\left(\frac{D-2q^*F}{D-q^*F} - e\right)$ is negative soon as the elasticity term satisfies $e \geq 1$. But the last term is always positive. It is easy to see that the condition $e \leq \frac{D-2q^*F}{D-q^*F} \Leftrightarrow q^*F \leq \frac{1-e}{2-e}$ is sufficient (but not necessary) to obtain that the probability and the fine are complementary instruments (*i.e.* $W_{pF} > 0$). On the other hand (in contrast to Garoupa (2001)), the condition $e > 1$ is no longer sufficient to obtain that the probability and the fine are substitutable instruments (*i.e.* $W_{pF} < 0$).

ii) To go a little further, assume that $F \to 0$ (criminals are poor), such that $q^*F \to 0$: hence, as far as criminals entail a small cost of avoidance (in the limit we also obtain: $x^* \to 0$) when they face low sanctions in case of detection, there exists a large degree of underdeterrence at optimum ($b^* \to 0$). As a result, $W_{pF} = g(b^*)q_p^* \{1 - e\} D < 0$ soon as $e > 1$, meaning that both instruments are substitutes near any arbitrary small level of deterrence.

iii) On the contrary, assume that b is uniformaly distributed (hence $g(b) = cst$) and that the maximal possible level of fine is large enough to induce a level of deterrence which is close to full internalization of the cost of crime at optimum ($b^* \to D$); then: $W_{pF} = g(b^*)q_p^* \left\{(x^* - q^*F) + \frac{\partial x^*}{\partial F}(-q_x^*F)\right\} > 0$ as soon as $x^* \geq q^*F$, meaning that when the cost of avoidance is large enough, both instruments are complements even near full internalization of the cost of the crime.

Proof of proposition 4

It is straightforward to show that:

$$\frac{\partial b^*}{\partial F} = q^* + q_p^* F \frac{\partial p^*}{\partial F} = q^* - q_p^* F \frac{W_{pF}}{W_{pp}}$$

As a result, $\frac{\partial b^*}{\partial F}$ is always positive when the probability and the fine are complements at optimum (*i.e.* $-\frac{W_{pF}}{W_{pp}} \geq 0$). This proves i) More generally, we have:

$$sign\left(\frac{\partial b^*}{\partial F}\right) = sign\left(q_p^* F W_{pF} - q^* W_{pp}\right)$$

ii) Given that $\frac{\partial x^*}{\partial p} = 0$ when $q_{xp} = 0$, we obtain:

$$\begin{aligned} q_p^* F W_{pF} - q^* W_{pp} &= g(b^*) q_p^{*2} F(D - q^* F) + q^* m'' \\ &+ g(b^*) F \left(\frac{\partial x^*}{\partial F}\left(-q_x^* F\right) q_p^{*2} + q^*(-q_{pp}^*)(D - q^* F)\right) \end{aligned}$$

which is always positive.

iii) On the other hand, when we assume that $q_{xp} < 0$, additional terms appear which depend on $\frac{\partial x^*}{\partial p}$ (it is easy to verify that some of them are negative), $\frac{\partial^2 x^*}{\partial p \partial F}$ or $\frac{\partial^2 x^*}{\partial p^2}$ (they have an ambiguous sign, which may be solved with more restrictions on the third derivatives of q).