# Maximum profit ensured for industry sustainability

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10 May 2022

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## Abstract

This article tries to calculate a maximum profit from sale items of an industry. This study has considered three inputs, such as capital, labor, and raw materials and other inputs for the mathematical analysis of the production procedures of the industry. In the present competitive global economy policy, to survive strongly, there is no alternate of sustainable economy. For the survival of an industry, profit maximization policy is vital. To acquire maximum profit, the production unit of the industry must be run in an efficient way. In this study an attempt has been taken to maximize the profit of an industry using Lagrange multiplier technique by applying necessary and sufficient conditions.

**Keywords:** Lagrange multiplier; profit maximization; Hesian matrix

## 1. Introduction

In economics, profit is depended on easily available necessary inputs, their marginal proclivities, factor shares in total output and degree of returns to scale [Khatun & Afroze, 2016]. If the production of an industry becomes at maximum level for each input and also if it maintains the rules of prevention of environment pollution, then the industry is considered as sustainable [Roy et al., 2021].

In this study we have tried to show mathematical calculations in some details. We have also introduced theorems where necessary to make the article meaningful. We have used both necessary and sufficient conditions to determine the maximum profit and verify it.

In any industry the profit is defined by profit function and this industry tries to minimize its cost of production to obtain maximum profit [Onalan & Basegmez, 2018].

## 2. Literature Review

Two American scholars, mathematician Charles W. Cobb (1875-1949) and economist Paul H. Douglas (1892-1976) in 1928, for the first time, efficiently have derived the functional distribution of income between capital and labor [Cobb & Douglas, 1928]. Two American researchers, mathematician John V. Baxley and economist John C. Moorhouse, have considered implicit functions and have used an example by generating meaningful economic behavior. They have also discussed aspects of production functions with sufficient mathematical techniques [Baxley & Moorhouse, 1984].

Pahlaj Moolio and his coworkers have analyzed the Cobb-Douglas production function to maximize an output subject to a budget constraint [Moolio et al., 2009]. Famous mathematician Jamal Nazrul Islam and his coauthors have examined the utility maximization and output maximization strategies for multiple constraints and non-linear constraints respectively [Islam et al., 2010a,b]. Lia Roy and her coauthors have tried to present cost minimization techniques of a running industry. Their mathematical modeling displays meaningful economic behavior for the sustainability of an industry [Roy et al., 2021]. Haradhan Kumar Mohajan in a series of papers has tried to flourish optimization policy for the mathematical economic modeling affectionate readers [Mohajan, 2017a, 2021a,b; Mohajan & Mohajan, 2022a,b].

Ajoy Kumar Dey shows a wise reason that profits can be maximized by increasing per unit revenue, decreasing unit cost or a mix of both. According to him, profit maximization is the process by which a firm determines the price and output level that returns the highest profit [Dey, 2007]. Abhishek Tripathi has focused on the profit maximization and value maximization theory from the application point of view [Tripathi, 2019]. Maria-Florina Balcan and her coworkers have provided more complex mechanisms typically have higher average profit over the samples than simpler mechanisms, but more samples are required to ensure that average profit nearly matches expected profit [Balcan et al., 2021].

## 3. Methodology of the Study

Methodology in a research is considered as the guidelines to do a genuine research efficiently and accurately [Legesse, 2014]. Research can be classified into three main categories as [Creswell, 2011]: 1) quantitative research, 2) qualitative research, and 3) mixed method research. Each of these methods plays important roles in modern natural and social science research areas [Mohajan, 2018, 2020]. We have used the Cobb-Douglas production function, $P = f\left(A, B, C\right) = KA^{\alpha} B^{\beta} C^{\gamma}$, for the mathematical analysis to gain maximum profit, where the symbols have their usual meanings. We have tried to find the maximum profit with necessary and sufficient conditions. In the modeling we have provided the mathematical calculations with some details.

In this article we have used secondary data that are consulted from both published and unpublished data sources, such as world famous reputed journals, e-journals, books of renowned authors, handbooks, e-books, theses, valuable conference proceedings, various publications of national and international organizations, information on internet, etc. [Mohajan, 2022a,b,c]. Reliability and validity display the morality of a genuine research, and in our study we are sincere to maintain them as long as possible [Mohajan, 2017b, 2018, 2020].

## 4. Objective of the Study

The key objective of this study is to determine maximum profit of an industry. The other specific objectives are as follows:

- to show mathematical calculations more accurately, and
- to display economic analysis with necessary and sufficient conditions.

## 5. Economic Analysis of Profit Function

Let us consider an industry wants to make maximum profit using its available resources and facilities. Let it has *A* amount of capital, *B* amount of labor, and *C* amount of raw materials (virgin and recycled) and other inputs, where *A*, *B*, and *C* are considered exogenous variables. The industry must acquire maximum profit for its sustainable environment. The objective of the industry is to obtain maximum profit from its sales products [Mohajan, 2021a,b],

$$P = f\left(A, B, C\right) = KA^{\alpha}B^{\beta}C^{\gamma}, \tag{1}$$

where *K* is the efficiency parameter. The expression $KA^{\alpha}B^{\beta}C^{\gamma}$ is considered as Cobb-Douglas production function [Cobb & Douglass, 1928]. Here $\alpha$ indicates the output of elasticity of capital measures the percentage change in *P* for 1% change in *A*, while *B* and *C* are held constants; similar expressions for $\beta$, and $\gamma$ that must satisfy the following inequality [Moolio et al., 2009; Roy et al., 2021]:

$$0 < \alpha, \beta, \gamma < 1. \tag{2}$$

Also we have the budget constraint of the industry,

$$Z = aA + bB + cC, \tag{3}$$

where *a* is rate of interest/services of capital per unit of capital *A*; *b* is the wage rate per unit of labor *B*; and *c* is the cost per unit of raw materials and other inputs *C*; and *f* is a suitable profit function. Now we introduce a single Lagrange multiplier $\lambda$, as a device to combine (1) and (3), and define the Lagrangian function *u*, in a 4-dimensional unconstrained problem as;

$$u\left(A, B, C, \lambda\right) = KA^{\alpha}B^{\beta}C^{\gamma} + \lambda\left(Z - aA - bB - cC\right). \tag{4}$$

Here we consider, for maximum profit, the corresponding quantities of *A*, *B*, *C*, and $\lambda$ are $A^*, B^*, C^*$, and $\lambda^*$, respectively. First order conditions for maximum profit are [Mohajan et al., 2013; Mohajan, 2021a],

$$u_{\lambda} = Z - aA - bB - cC = 0, \tag{5a}$$

$$u_A = \alpha K A^{\alpha-1} B^{\beta} C^{\gamma} - a\lambda = 0, \tag{5b}$$

$$u_B = \beta K A^{\alpha} B^{\beta-1} C^{\gamma} - b\lambda = 0, \tag{5c}$$

$$u_C = \gamma K A^{\alpha} B^{\beta} C^{\gamma-1} - c\lambda = 0, \tag{5d}$$

where $u_\lambda = \frac{\partial u}{\partial \lambda}$, etc. are partial derivatives. From (5a) we get, $Z = aA + bB + cC$, budget constraint as of (3).

From (5b) we get,

$$aA = \frac{\alpha K A^{\alpha} B^{\beta} C^{\gamma}}{\lambda}. \tag{6a}$$

From (5c) we get,

$$bB = \frac{\beta K A^{\alpha} B^{\beta} C^{\gamma}}{\lambda}. \tag{6b}$$

From (5c) we get,

$$cC = \frac{\gamma K A^{\alpha} B^{\beta} C^{\gamma}}{\lambda}. \tag{6c}$$

Using (6a,b,c) in (3) we get,

$$Z = \frac{K A^{\alpha} B^{\beta} C^{\gamma}}{\lambda}(\alpha + \beta + \gamma) = \frac{K A^{\alpha} B^{\beta} C^{\gamma} \Delta}{\lambda}. \tag{7}$$

where $\Delta = \alpha + \beta + \gamma$.

**Theorem 1:** Maximum profit of the industry is, $P = \frac{K \alpha^{\alpha} \beta^{\beta} \gamma^{\gamma} Z^{\Delta}}{a^{\alpha} b^{\beta} c^{\gamma} \Delta^{\Delta}}$.

**Proof:** From (6a) and (7) we get,

$$A^* = \frac{\alpha Z}{a \Delta}. \tag{8a}$$

Now (6b) and (7) gives,

$$B^* = \frac{\beta Z}{b \Delta}. \tag{8b}$$

Now (6c) and (7) gives,

$$C^* = \frac{\gamma Z}{c\Delta}. \quad \text{(8c)}$$

Using (8a,b,c) in (6a) we get,

$$\lambda^* = \frac{\alpha K\left(\frac{\alpha Z}{a\Delta}\right)^{\alpha}\left(\frac{\beta Z}{b\Delta}\right)^{\beta}\left(\frac{\gamma Z}{c\Delta}\right)^{\gamma}}{\frac{\alpha Z}{\Delta}} = \frac{K\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}Z^{\Delta-1}}{a^{\alpha}b^{\beta}c^{\gamma}\Delta^{\Delta-1}}. \quad \text{(9)}$$

Using (8a,b,c) in (1) we get,

$$P = K\left(\frac{\alpha Z}{a\Delta}\right)^{\alpha}\left(\frac{\beta Z}{b\Delta}\right)^{\beta}\left(\frac{\gamma Z}{c\Delta}\right)^{\gamma} = \frac{K\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}Z^{\Delta}}{a^{\alpha}b^{\beta}c^{\gamma}\Delta^{\Delta}}. \text{ Q.E.D.} \quad \text{(10)}$$

The relation (10) is the profit function in terms of $A, B > 0$, $a, b, c > 0$, $\alpha, \beta, \gamma > 0$, and $\Delta > 0$. Using all the known parameters in (10) we can calculate maximum profit of the industry. Now we consider special case for some fixed values of $\alpha$, $\beta$, and $\gamma$. Let us consider a lemma in the support of theorem 1 [Mohajan, 2021b].

**Lemma 1:** If $\alpha = \beta = \gamma = \frac{1}{2}$, then profit of the industry, $P = \frac{KZ^{\frac{3}{2}}\left(\frac{1}{3}\right)^{\frac{3}{2}}}{(abc)^{\frac{1}{2}}}$.

**Proof:** Let us consider, $\alpha = \beta = \gamma = \frac{1}{2}$ then, $\Delta = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$. From (10) we get the profit of the industry as,

$$P = \frac{KZ^{\frac{3}{2}}\left(\frac{1}{2}\right)^{\frac{3}{2}}}{(abc)^{\frac{1}{2}}\left(\frac{3}{2}\right)^{\frac{3}{2}}} = \frac{KZ^{\frac{3}{2}}\left(\frac{1}{3}\right)^{\frac{3}{2}}}{(abc)^{\frac{1}{2}}}. \text{ Q.E.D.} \quad \text{(11)}$$

**Arithmetic Analysis:** We consider, $K = 0.05$, $Z = \$1,000$, $a = b = c = 4$, then from (11) we get the profit of the industry as,

$$P = 0.05 \times (1000)^{\frac{3}{2}} \left(\frac{1}{12}\right)^{\frac{3}{2}} = \$38 . \qquad (12)$$

In relation (12) we have provided a profit by using parameters through random analysis. Any industry can use its parameters from production procedures and calculate profit accurately.

Now we will try to develop the economic model with sufficient conditions of optimization. Hence, we will operate the model with the second-order partial differentiation. The following Theorem 2 will provide the maximum production.

**Theorem 2:** Let us consider the determinant of the Hessian matrix,

$$|H| = \begin{vmatrix} 0 & -Z_A & -Z_B & -Z_C \\ -Z_A & u_{AA} & u_{AB} & u_{AC} \\ -Z_B & u_{BA} & u_{BB} & u_{BC} \\ -Z_C & u_{CA} & u_{CB} & u_{CC} \end{vmatrix} . \qquad (13)$$

For maximum profit, $|H| < 0$ [Islam et al., 2010a,b; Moolio et al., 2009].

**Proof:** Given, the determinant of the Hessian matrix,

$$|H| = \begin{vmatrix} 0 & -Z_A & -Z_B & -Z_C \\ -Z_A & u_{AA} & u_{AB} & u_{AC} \\ -Z_B & u_{BA} & u_{BB} & u_{BC} \\ -Z_C & u_{CA} & u_{CB} & u_{CC} \end{vmatrix} .$$

$$= Z_A \begin{vmatrix} -Z_A & u_{AB} & u_{AC} \\ -Z_B & u_{BB} & u_{BC} \\ -Z_C & u_{CB} & u_{CC} \end{vmatrix} - Z_B \begin{vmatrix} -Z_A & u_{AA} & u_{AC} \\ -Z_B & u_{BA} & u_{BC} \\ -Z_C & u_{CA} & u_{CC} \end{vmatrix} + Z_C \begin{vmatrix} -Z_A & u_{AA} & u_{AB} \\ -Z_B & u_{BA} & u_{BB} \\ -Z_C & u_{CA} & u_{CB} \end{vmatrix}$$

$$= Z_A\left\{-Z_A\begin{vmatrix}u_{BB} & u_{BC}\\ u_{CB} & u_{CC}\end{vmatrix} - u_{AB}\begin{vmatrix}-Z_B & u_{BC}\\ -Z_C & u_{CC}\end{vmatrix} + u_{AC}\begin{vmatrix}-Z_B & u_{BB}\\ -Z_C & u_{BC}\end{vmatrix}\right\}$$

$$-Z_B\left\{-Z_A\begin{vmatrix}u_{BA} & u_{BC}\\ u_{CA} & u_{CC}\end{vmatrix} - u_{AA}\begin{vmatrix}-Z_B & u_{BC}\\ -Z_C & u_{CC}\end{vmatrix} + u_{AC}\begin{vmatrix}-Z_B & u_{BA}\\ -Z_C & u_{CA}\end{vmatrix}\right\}$$

$$+Z_C\left\{-Z_A\begin{vmatrix}u_{BA} & u_{BB}\\ u_{CA} & u_{CB}\end{vmatrix} - u_{AA}\begin{vmatrix}-Z_B & u_{BB}\\ -Z_C & u_{CB}\end{vmatrix} + u_{AB}\begin{vmatrix}-Z_B & u_{BA}\\ -Z_C & u_{CA}\end{vmatrix}\right\}$$

$$= -Z_A^2 u_{BB}u_{CC} + Z_A^2 u_{BC}^2 + Z_A Z_B u_{AB}u_{CC} - Z_A Z_C u_{AB}u_{BC} - Z_A Z_B u_{AC}u_{BC} + Z_A Z_C u_{AC}u_{BB}$$
$$+ Z_A Z_B u_{AB}u_{CC} - Z_A Z_B u_{BC}u_{CA} - Z_B^2 u_{AA}u_{CC} + Z_B Z_C u_{AA}u_{BC} + Z_B^2 u_{AC}^2 - Z_B Z_C u_{AC}u_{AB}$$
$$- Z_A Z_C u_{AB}u_{BC} + Z_A Z_C u_{AC}u_{BB} + Z_B Z_C u_{AA}u_{BC} - Z_C^2 u_{AA}u_{BB} - Z_B Z_C u_{AB}u_{AC} + Z_C^2 u_{AB}^2$$

$$= -Z_A^2 u_{BB}u_{CC} + Z_A^2 u_{BC}^2 + 2Z_A Z_B u_{AB}u_{CC} - 2Z_A Z_C u_{AB}u_{BC} - 2Z_A Z_B u_{AC}u_{BC} + 2Z_A Z_C u_{AC}u_{BB}$$
$$- Z_B^2 u_{AA}u_{CC} + 2Z_B Z_C u_{AA}u_{BC} + Z_B^2 u_{AC}^2 - 2Z_B Z_C u_{AC}u_{AB} - Z_C^2 u_{AA}u_{BB} + Z_C^2 u_{AB}^2 . \quad (14)$$

First-order partial differentiations of (3) give,

$$Z_A = a,\ Z_B = b,\ \text{and}\ Z_C = c. \quad (15)$$

Second-order and cross partial derivatives of (5b-d) give,

$$u_{AA} = \alpha(\alpha-1)KA^{\alpha-2}B^{\beta}C^{\gamma},$$
$$u_{BB} = \beta(\beta-1)KA^{\alpha}B^{\beta-2}C^{\gamma},$$
$$u_{CC} = \gamma(\gamma-1)A^{\alpha}B^{\beta}C^{\gamma-2},$$
$$u_{AB} = u_{BA} = \alpha\beta KA^{\alpha-1}B^{\beta-1}C^{\gamma}, \quad (16)$$
$$u_{AC} = u_{CA} = \alpha\gamma KA^{\alpha-1}B^{\beta}C^{\gamma-1},$$
$$u_{BC} = u_{CB} = \beta\gamma A^{\alpha}B^{\beta-1}C^{\gamma-1}.$$

Using the required values from (15) and (16) in (14) we get,

$$|H| = -\beta(\beta-1)\gamma(\gamma-1)K^2a^2A^{2\alpha}B^{2\beta-2}C^{2\gamma-2} + \beta^2\gamma^2K^2a^2A^{2\alpha}B^{2\beta-2}C^{2\gamma-2}$$
$$+ 2\alpha\beta\gamma(\gamma-1)K^2abA^{2\alpha-1}B^{2\beta-1}C^{2\gamma-2} - 2\alpha\beta^2\lambda K^2acA^{2\alpha-1}B^{2\beta-2}C^{2\gamma-1}$$
$$+ 2\alpha\beta(\gamma-1)\gamma K^2acA^{2\alpha-1}B^{2\beta-2}C^{2\gamma-1} - 2\alpha\beta\gamma^2K^2abA^{2\alpha-1}B^{2\beta-1}C^{2\gamma-2}$$
$$- \alpha(\alpha-1)\gamma(\gamma-1)K^2a^2A^{2\alpha-2}B^{2\beta}C^{2\gamma-2} + 2\alpha(\alpha-1)\beta\gamma K^2bcA^{2\alpha-2}B^{2\beta-1}C^{2\gamma-1}$$
$$+ \alpha^2\gamma^2K^2b^2A^{2\alpha-2}B^{2\beta}C^{2\gamma-2} - 2\alpha^2\beta\gamma K^2bcA^{2\alpha-2}B^{2\beta-1}C^{2\gamma-1}$$
$$- \alpha(\alpha-1)\beta(\beta-1)K^2c^2A^{2\alpha-2}B^{2\beta-2}C^{2\gamma} + \alpha^2\beta^2K^2c^2A^{2\alpha-2}B^{2\beta-2}C^{2\gamma}$$

$$= K^2 A^{2\alpha} B^{2\beta} C^{2\gamma} \left\{ \frac{-\beta(\beta-1)\gamma(\gamma-1)a^2}{B^2C^2} + \frac{\beta^2\gamma^2 a^2}{B^2C^2} + \frac{2\alpha\beta\gamma(\gamma-1)ab}{ABC^2} - \frac{2\alpha\beta^2\gamma bc}{AB^2C} - \frac{2\alpha\beta\gamma^2 ab}{ABC^2} \right.$$

$$+ \frac{2\alpha\beta(\beta-1)\gamma ac}{AB^2C} - \frac{\alpha(\alpha-1)\gamma(\gamma-1)b^2}{A^2C^2} + \frac{2\alpha(\alpha-1)\beta\gamma bc}{A^2BC} + \frac{\alpha^2\gamma^2 b^2}{A^2C^2} - \frac{2\alpha^2\beta\gamma bc}{A^2BC}$$

$$\left. - \frac{\alpha(\alpha-1)\beta(\beta-1)c^2}{A^2C^2} + \frac{\alpha^2\beta^2c^2}{A^2C^2} \right\}$$

$$= K^2 A^{2\alpha} B^{2\beta} C^{2\gamma} \left\{ \frac{a^2\beta^2\gamma}{B^2C^2} + \frac{a^2\beta\gamma^2}{B^2C^2} - \frac{a^2\beta\gamma}{B^2C^2} - \frac{2ab\alpha\beta\gamma}{ABC^2} - \frac{2ac\alpha\beta\gamma}{AB^2C} + \frac{b^2\alpha^2\gamma}{A^2C^2} + \frac{b^2\alpha\gamma}{A^2C^2} - \frac{b^2\alpha\gamma}{A^2C^2} \right.$$

$$\left. - \frac{2bc\alpha\beta\gamma}{A^2BC} + \frac{c^2\alpha^2\beta}{A^2B^2} + \frac{c^2\alpha\beta}{A^2B^2} - \frac{c^2\alpha\beta}{A^2B^2} \right\}. \qquad (17)$$

Now putting the values of $A$, $B$, and $C$ from (8a,b,c) in equation (17) we get,

$$|H| = K^2 \left(\frac{\alpha Z}{a\Delta}\right)^{2\alpha} \left(\frac{\beta Z}{b\Delta}\right)^{2\beta} \left(\frac{\gamma Z}{c\Delta}\right)^{2\gamma} \left\{ a^2\beta^2\gamma \left(\frac{b\Delta}{\beta Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2 + a^2\beta\gamma^2 \left(\frac{b\Delta}{\beta Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2 \right.$$

$$- a^2\beta\gamma \left(\frac{b\Delta}{\beta Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2 - 2ab\alpha\beta\gamma \left(\frac{a\Delta}{\alpha Z}\right)\left(\frac{b\Delta}{\beta Z}\right)\left(\frac{c\Delta}{\gamma Z}\right)^2 - 2ac\alpha\beta\gamma \left(\frac{a\Delta}{\alpha Z}\right)\left(\frac{b\Delta}{\beta Z}\right)^2\left(\frac{c\Delta}{\gamma Z}\right)$$

$$+ b^2\alpha^2\gamma \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2 + b^2\alpha\gamma^2 \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2 - b^2\alpha\gamma \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{c\Delta}{\gamma Z}\right)^2$$

$$- 2bc\alpha\beta\gamma \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{b\Delta}{\beta Z}\right)\left(\frac{c\Delta}{\gamma Z}\right) + c^2\alpha^2\beta \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{b\Delta}{\beta Z}\right)^2 + c^2\alpha\beta^2 \left(\frac{b\Delta}{\alpha Z}\right)^2 \left(\frac{b\Delta}{\beta Z}\right)^2$$

$$\left. - c^2\alpha\beta \left(\frac{a\Delta}{\alpha Z}\right)^2 \left(\frac{b\Delta}{\beta Z}\right)^2 \right\}$$

$$= -K^2 \left(\frac{\alpha Z}{a\Delta}\right)^{2\alpha} \left(\frac{\beta Z}{b\Delta}\right)^{2\beta} \left(\frac{\gamma Z}{c\Delta}\right)^{2\gamma} \frac{a^2b^2c^2\Delta^4}{Z^4} \left(\frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma} + \frac{1}{\alpha\beta}\right)$$

$$= -K^2 \left(\frac{\alpha Z}{a\Delta}\right)^{2\alpha} \left(\frac{\beta Z}{b\Delta}\right)^{2\beta} \left(\frac{\gamma Z}{c\Delta}\right)^{2\gamma} \frac{a^2b^2c^2\Delta^4}{Z^4} \frac{\Delta}{\alpha\beta\gamma}$$

$$= -K^2 \left(\frac{\alpha^{2\alpha}\beta^{2\beta}\gamma^{2\gamma}Z^{2\Delta}}{a^{2\alpha}b^{2\beta}c^{2\gamma}\Delta^{2\Delta}}\right)\left(\frac{a^2b^2c^2\Delta^5}{\alpha\beta\gamma Z^4}\right). \qquad (18)$$

Since $K > 0$, $\alpha, \beta, \gamma > 0$, and $a, b, c$ are the rates of inputs $A$, $B$, and $C$, respectively and hence all are positive; while $Z$ is budget, which will never be negative, therefore, $|H| < 0$. Q.E.D.

**Arithmetic Analysis:** Let us consider, $\alpha=\beta=\gamma=\frac{1}{2}$ then, $\Delta=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$. We consider, $K=0.05$, $Z=\$1{,}000$, $a=b=c=4$, then (18) becomes [Mohajan, 2021a,b],

$$|H|=-(0.05)^2\left(\frac{\left(\frac{1}{2}\right)^3(1000)^3}{4^3\left(\frac{3}{2}\right)^3}\right)\left(\frac{2^{12}\times\left(\frac{3}{2}\right)^5}{\frac{1}{8}\times(1000)^4}\right)=-0.0045<0\,.$$

Since Hesian matrix is negative; hence, profit of the industry is of course maximum.

## 6. Conclusions

In this study we have tried to discuss the profit function of a running competitive industry. We have realized that to gain maximum profit an industry must be careful in every steps of its operation. Mathematical modeling in economics, for example, profit function plays an important role in modern economics for the development of global financial structure. Thinking for the common readers, throughout the paper, we have presented mathematical calculations in some details.

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