Backward-bending Labor Supply and Urban Location

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Abstract

This paper is an attempt to combine a labor supply model with a housing location model. We focus on the trade-off between the hours of work, commute times and leisure time as well as the trade-off between the consumption of a good, housing space, and leisure time. We show that both labor supply and urban location choice are inverted U-shaped in relation to the wage rate. These results are empirically shown by using Japanese data on the hours of work and commute times by household income class and on the number of households by income class.

Keywords: labor supply; leisure time; residential location; U-shaped relationship.

JEL classification: J22; R21; R31.

1 Introduction

Within the OECD countries, Japanese works hours are among the longest — Japanese males work 10 hours longer per week than Americans, while Japanese females work 7 hours longer (Kuroda, 2010). The Japanese Ministry of Health, Labour and Welfare promotes work-life balance, which is the amount of time you spend doing your job

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(hours of work) compared with the amount of time you spend with your family and doing things you enjoy (leisure time).

To focus on the trade-off between the hours of work and leisure time, Section 2 starts from the standard model presented in labor economics textbooks. Here, a representative worker maximizes the consumption of a good and leisure time given the income and time constraints. We show that as the wage rate increases, labor supply, as measured by the hours of work, first increases and then decreases.

There are many empirical studies on the relationship between labor supply and the wage rate. Alesina et al. (2005) have collected a wide range of estimates on the Marshallian labor supply elasticity over seventy years, mostly from the Handbook of Labor Economics. The signs of the estimates for men and women are positive in 20 studies, zero in 1 study, and negative in 9 studies. Keane (2011) also summarizes a wide range of estimates on the Marshallian labor supply elasticity for men and women. They are positive in 18 studies and negative in 8 studies. This implies that the slope of a labor supply curve is positive in some labor markets whereas it is negative, i.e., backward bending in other labor markets.

From a world point of view, GDP per capita, which acts as a surrogate for the wage rate has steadily increased since WWII as a result of technological progress (https://data.oecd.org/gdp/gross-domestic-product-gdp.htm). On the other hand, the total hours worked has steadily decreased since WWII (https://data.oecd.org/emp/hours-worked.htm). From these two trends, one might surmise that the labor supply curve is on the backward-bending segment.

However, this is not true because the labor force has also been changing during this long-term period. The share of the elderly population aged 65 and over has been steadily increasing since WWII among others (https://data.oecd.org/pop/elderly-population.htm).[1] As elderly people are likely to prefer leisure time more than the consumption of goods, an aging population is the main reason for declining the hours of work.

We avoid structural changes in the labor force by using cross-sectional data instead

[1] The Japanese GDP per capita has steadily increased since WWII (https://www.esri.cao.go.jp/jp/sna/otoiawase/faq/qa2.html) while the total hours worked has continued to decrease since 1960 (https://www.jil.go.jp/kokunai/statistics/timeseries/html/g0501_02.html).

of time-series data. Thus, Section 2 investigates the relationship between the hours of work (labor supply) and household income (wage rate) by examining the trade-off between leisure time and the consumption of a good.

Section 3 goes further by taking the urban space into account. The trade-off is now between leisure time, consumption of a good, and housing space. We then explore the relationship between the hours of work (labor supply), household income (wage rate), and housing location. This is an attempt to *combine a labor supply model in labor economics with a housing location model in urban economics.* Our primary objective is to analyze how a rise in the household income affects the leisure time, consumptions of a good and housing space, and commute time.[2] To put it briefly, we want to study if richer households spend more leisure time, consume more goods and more housing space, and/or spend more time commuting. In a nutshell, this study shows that both labor supply and location choice have an inverted U-shaped curve when mapped with the wage rate.

In Section 4, we check if there exists an equilibrium by focusing on the three income classes and conducting numerical simulations. We also examine the choice of leisure time and consumption of a good and housing by different income classes.

In Section 5, we check these theoretical results and demonstrate their validity using data on the spatial distributions of households by income class of the three largest metropolitan areas (MA): Tokyo, Osaka, and Nagoya MAs. Section 6 summarizes the study.

2 Labor supply and wage

2.1 The textbook model

It is well-known in labor economics textbooks that as the wage rate rises, the hours of work initially increase, and then decrease. This is simply shown by the use of the

[2] It is empirically shown by Abe (2011) and Iwata and Tamada (2014) that the commute time affects labor supply as well as leisure time of women in Japan.

following quadratic utility function (Spiegel and Templeman, 2004)[3]

$$U(t_l, z) = az - \frac{b}{2}z^2 + t_l, \qquad a, b > 0,$$

the budget constraint

$$wt_w = z$$

and the time constraint

$$T = t_l + t_w,$$

where z is the consumption of the composite good as the numeraire, w is the hourly wage, t_l is the leisure time, t_w is the hours of work, and T is the total fixed time.

Utility maximization with the two constraints easily yields the labor supply

$$t_w = \frac{aw - 1}{bw^2} \tag{1}$$

as a function of the wage rate w. This is an inverted U-shaped curve on w and t_w as coordinates, or a backward-bending curve on t_w and w as coordinates. This implies that as the wage rate rises, the hours of work initially increase as long as the wage rate is low $w < 2/a$; then decrease when the wage rate gets high $w > 2/a$. Put differently, leisure time is an inferior good for low-wage earners, whereas it is a normal good for high-wage earners. The upward-sloping segment of the labor supply curve implies that substitution effects are stronger initially; the backward-bending segment implies that income effects dominate eventually (Borjas, 2020).

2.2 Empirical evidence

As stated in the introduction, the labor supply appears to increase with the wage rate in some studies, whereas the opposite occurs in other studies. That is, both substitution and income effects can be present depending on the conditions of the labor market.

If we use nationwide data, the range of the wage distribution would be wide enough for the labor supply to increase with the wage rate for low wage earners, and decrease with the wage rate for high wage earners, i.e., a backward bending labor supply curve.

[3] We do not employ commonly used utility functions such as CES, which do not yield such a backward-bending curve (Barzel and McDonald, 1973).

We test this by looking at the cross-sectional relationship between labor supply and income class using the data from *The Survey on Time Use and Leisure Activities* (conducted every five years in Japan) for the period 1991-2021. The data includes the hours of work and commute times by household income class, which covers a wide range of the income distribution. See the Appendix for data conversion from income ranges to average incomes.

We compute the real income by dividing the nominal income by the consumer price index and pooling the data for all years.[4] The number of observations is 84 (= 12 income classes across 7 survey years). Figure 1 displays the relationship between the wage rate and labor supply. The wage rate is represented by the household income I (vertical axis), and while the labor supply is measured by the on-duty hours t_d (horizontal axis), which is the sum of the hours of work (t_w) and commute times (t_c). Since the wage range is wide enough, we observe a backward-bending labor supply curve.

In order to statistically test if the supply curve is backward-bending, we conduct a U-test developed by Lind and Mehlum (2010). Specifically, we swap the vertical and horizontal axes and fit the following quadratic curve:

$$t = \alpha + \beta I + \gamma I^2,$$

where α, β, and γ are regression parameters. The OLS estimate is shown in the second column of Table 1.

Table 1: U-tests between time t_d, t_w, t_c and household income I

[4] If we replace the real income by the nominal income, the overall results below are almost the same.

dependent variable	on-duty hours t_d	hours of work t_w	commute time t_c
α	139**	133**	6.27**
	(7.97)	(7.70)	(7.29)
β	0.345**	0.297**	0.0475**
	(9.22)	(8.04)	(25.9)
γ	−0.000104**	−0.0000898**	−0.0000144**
	(−7.42)	(−6.46)	(−209)
slope at minimum I	0.330**	0.285**	0.0455**
	(9.29)	(8.10)	(26.1)
slope at maximum I	−0.197**	−0.169**	−0.0276**
	(−5.01)	(−4.36)	(−14.3)
R^2 adjusted	0.549	0.480	0.906

t-values are in parentheses.

** significant at the 0.01 level.

Since the estimate γ is negative, this function exhibits an inverted U-shaped curve. It is the backward-bending blue curve shown in Figure 1 with the two axes swapped. The necessary and sufficient conditions for the backward-bending labor supply curve, i.e., the inverted U-shaped relationship between I and t_d, are that (i) the slope at the minimum income is significantly positive, and (ii) the slope at the maximum income is significantly negative. Since both are significant at the 0.01 level, we conclude that the labor supply curve is backward-bending. This is consistent with labor economics textbooks — as the wage rises, workers initially increase, and then decrease their hours of work.

Swapping the two axes, the inverted U-shaped relationship between I and t_w (the third column) is statistically significant and is drawn in the orange curve, while the inverted U-shaped relationship between I and t_c (the fourth column) is also statistically significant[5] and is represented in the green curve in Figure 1. Since $t_d \equiv t_w + t_c$, the horizontal sum of the two curves is equal to the blue curve. When the household income changes, workers adjust both their hours of work (orange curve) and commute times (green curve). However, the orange curve varies much more than the

[5] The inverted U-shaped relationship between the wage rate and commute time is shown by Iwata and Tamada (2014) using the microdata of married women.

green curve because it is easier for workers to adjust their hours of work than their commute time, which is associated with housing relocation.

Furthermore, the inverted U-shaped relationship between the commute time and household income may imply that workers change their locations according to their income levels — workers initially relocate farther away from the city center seeking larger space for housing, but then move closer to the city center for longer leisure time. Testing this hypothesis is the main purpose of the remainder of this study.

3 Labor supply in urban space

Next, we investigate the impact of wage rise on the labor supply in an urban spatial context. The wage rise affects not only the time allocation of leisure and work, but also the consumption of housing space and the composite good. The latter decision is not straightforward. For example, as the wage rate rises, workers want to increase their consumption of housing spaces as well as that of the composite good. However, consuming a large housing space means that workers have to relocate from the central city to a suburb, which raises the commute time and consequently reduces the leisure time. We deal with the housing space and location decision, by extending the textbook model to add urban spaces based on the traditional urban economic model à la Alonso (1964).

We consider a monocentric city with the central business district (CBD) located at the city center $x = 0$ and k commuter railways radiating from the CBD to the suburbs. Assume that the inhabitable area is, say, a 30-minute walk from each railway station. Then, the inhabitable area (i.e., supply of housing space) is constant regardless of the distance from the CBD, x, so that the city is in a linear space.[6]

There are n residents living in the city. A representative city resident commutes to

[6] There are 174 municipalities whose distance to Tokyo Station is less than 50km. We classify them into $d = 1, \ldots, 10$ rings, where ring d is between $5(d-1)$ km and $5d$ km from the CBD. We calculate the total inhabitable area in each ring. Then, we regress the total inhabitable area in 2000 on the average distance from the CBD and check significance of the regression coefficient. We find it to be positive, but insignificant. This result remains the same if the total inhabitable area is replaced with the total area for building levied as property tax in 2000. Thus, we can confirm the linear city assumption.

the CBD[7] and maximizes her utility with respect to the housing space s, leisure time t_l, composite good z (which is assumed to be a numeraire), and location x, which is the distance from the CBD:

$$\max_{z,s,t_l,x} U = U(t_l, z, s). \tag{2}$$

Following Nelson (1977), Fujita (1989) and Tabuchi (2019), each resident's total time T is fixed and distributed between leisure time t_l, working time t_w, and commute time $t_c = t(x)$. That is, the time constraint is given by

$$T = t_l + t_w + t(x). \tag{3}$$

The trade-off between leisure and commute times implies a marginal disutility from longer commutes. In addition, the representative resident is subject to the usual income constraint:

$$wt_w = z + rs + \tau(x), \tag{4a}$$

where r is the housing rent and $\tau(x)$ is the pecuniary commuting cost for distance x. This means that each resident incurs not only the pecuniary commuting cost $\tau(x)$, but also the opportunity commuting cost $wt(x)$ of time. We assume that both costs increase with the distance x from the CBD: $\tau'(x) > 0$ and $t'(x) > 0$.

Substituting the constraints (3) and (4a) into the utility (2) yields

$$U = U\left(t_l, w\left[T - t_l - t(x)\right] - rs - \tau(x), s\right),$$

which is maximized with respect to t_l, s, and x. The first-order conditions are

$$\begin{aligned}
\frac{\partial u}{\partial t_l} - \frac{\partial u}{\partial z}w &= 0 && (5)\\
\frac{\partial u}{\partial s} - \frac{\partial u}{\partial z}r &= 0 && (6)\\
\frac{\partial u}{\partial s}s' + \frac{\partial u}{\partial t_l}t_l' - \frac{\partial u}{\partial z}\left[wt_l' + wt'(x) + r's + rs' + \tau'(x)\right] &= 0,
\end{aligned}$$

where the prime denotes the derivative with respect to the commute distance x. Solving them leads to the rent gradient

$$r' = -\frac{t'(x)w + \tau'(x)}{s}. \tag{7}$$

[7]Trains accounted for 53% of commuter transport in Tokyo MA in 2008, with passenger cars at 24% and bicycles at 10% according to the Tokyo MA Transport Planning Council (2012, p. 24).

Because the time and pecuniary costs of commuting increase with distance, the rent gradient r' is always negative. Thus, the housing rent decreases with distance x from the CBD.

Differentiating the rent gradient with the wage rate yields

$$\frac{\partial r'}{\partial w} \gtreqless 0 \quad \Leftrightarrow \quad w \lesseqgtr \overline{w} \equiv \frac{\varepsilon \tau'(x)}{(1-\varepsilon)\, t'(x)}, \tag{8}$$

where ε is the elasticity of space s with respect to the wage rate w. If ε is constant for all w, there are three possibilities depending on the wage range.

1. When the wages of all workers are lower than $\overline{w}$, the richer locate farther away from the CBD.

2. When the wages of all workers are higher than $\overline{w}$, the poorer locate farther away from the CBD.

3. When the wages of some workers are lower than $\overline{w}$, while those of others are higher than $\overline{w}$, the middle class is located in the suburbs. Both the richer and the poorer locate closer to the CBD, such that the richest and the poorest collocate are collocated in the CBD.

Detroit may be an example of case 1, while Paris may be an example of case 2 (Brueckner, Thisse, and Zenou, 1999). Nevertheless, case 3 would be more likely in many cities. In fact, case 3 is consistent with Figure 1.

3.1 Specific utility function

To give the model a concrete exposition as in Section 2, it is useful to choose a specific utility function as

$$u(t_l, z, s) = a_z z - \frac{b_z}{2} z^2 + a_s s - \frac{b_s}{2} s^2 + t_l, \qquad a_z, b_z, a_s, b_s > 0. \tag{9}$$

Solving (3) and (4a) for t_l and t_w, plugging them into (9), and maximizing it for z and s yields the optimal consumptions:

$$z^* = \frac{a_z w - 1}{b_z w}, \qquad s^* = \frac{a_s w - r}{b_s w}. \tag{10}$$

It is clearly shown that the composite good z^* is a normal good because it increases with the wage rate w. The housing space s^* is also a normal good, particularly if the housing rent r is fixed. We can also compute the optimal time allocations:

$$\begin{aligned} t_l^* &= \frac{b_s b_z [T - t(x)] w^2 - \left(a_z b_s + a_s b_z r + b_s b_z r^2 \right) w + b_s + b_z r^2}{b_s b_z w^2} \\ t_w^* &= \frac{[a_z b_s + a_s b_z r + b_s b_z \tau(x)] w - b_s - b_z r^2}{b_s b_z w^2} \end{aligned} . \tag{11}$$

Substituting s^* in (10) into (7), we get the rent gradient:

$$r' = -\frac{b_s \left[t'(x) w + \tau'(x) \right] w}{a_s w - r(x)}. \tag{12}$$

Noting that the city is linear, we integrate both sides and obtain two solutions of $r(x)$; one of which is the relevant equilibrium housing rent at location x:

$$r(x) = a_s w - \sqrt{\left[a_s^2 + 2 b_s t(x) \right] w^2 + 2 b_s \tau(x) w - 2C}, \tag{13}$$

where C is an integral constant. Plugging s^* in (10) yields the equilibrium housing space at location x:

$$s(x) = \frac{1}{b_s w} \sqrt{\left[a_s^2 + 2 b_s t(x) \right] w^2 + 2 b_s \tau(x) w - 2C}. \tag{14}$$

Substituting (13) into (11), we can show that

$$\frac{dt_l^*}{dx} = -\frac{\left[t'(x) w + \tau'(x) \right] r(x)}{\left[a_s w - r(x) \right] w} < 0, \qquad \frac{dt_w^*}{dx} = -\frac{\left[a_s w - 2r(x) \right] t'(x) - \tau'(x) r(x)}{\left[a_s w - r(x) \right] w} \lesseqgtr 0, \tag{15}$$

where each denominator is positive because s^* in (10) is positive. The former negative sign of dt_l^*/dx implies that the leisure time t_l^* decreases as one moves away from the city center. This suggests that workers living in the suburbs have little time to spare for leisure. On the other hand, the latter sign of dt_w^*/dx can be positive or negative since the sign of $a_s w - 2r(x)$ is indeterminate. It is negative when the wage rate w is high, implying that the working hours are shorter for residents with longer commute times. However, it is positive when the wage rate w is low, implying that working hours are longer as they move away from the city center. This is because low-wage workers work longer in order to reside in a larger space for housing in spite of the longer commute time. In sum, the relationship between the wage rate and working hours is not obvious in an urban spatial context.

3.2 Spatial distribution of income classes

When there are multiple income classes in a city, their housing rents intersect at the border as illustrated in Figure 2. We denote their wages by w_1 and w_2, and assume $w_1 > w_2$. The difference in the rent gradient (12) at intersection $x = x_{12}$ is

$$r'|_{w=w_1} - r'|_{w=w_2} = \frac{b_s (w_1 - w_2)}{(a_s w_1 - r)(a_s w_2 - r)} \left[-a_s t'(x_{12}) w_1 w_2 + t'(x_{12}) r (w_1 + w_2) + \tau'(x_{12}) r \right]. \tag{16}$$

Because the fraction in the RHS of (16) is positive, the sign of the bracketed term determines the rent gradient differential.

Suppose that there are many income classes and that w_1 and w_2 are close enough. When both wages are low, the bracketed term is positive; thus, $r'|_{w=w_1} < r'|_{w=w_2} < 0$ holds. Since the rent gradient of income class 1 is steeper, the poorer locate closer to the CBD than the richer as can be seen from Figure 2. On the other hand, when both wages are high, the richer locate farther away from the poor.

The sign of (16) equals zero when the bracketed term equals zero. This happens only at the city border $\overline{x} = x_{12}$. Setting $\overline{w} \equiv w_1 = w_2$ and $r(\overline{x}) = \overline{r}$, and solving the bracketed term equal to 0, we have the threshold wage

$$\overline{w} = \frac{\overline{r}}{a_s} \left[1 + \sqrt{1 + \frac{a_s \tau'(\overline{x})}{\overline{r} t'(\overline{x})}} \right], \tag{17}$$

where $\overline{r}$ is the opportunity cost of the land, or the agricultural land rent at the city border $\overline{x}$. Thus, we have shown the following.

Proposition 1 *As the wage rate rises, urban residents locate farther away from the CBD as long as $w < \overline{w}$. However, when $w > \overline{w}$, urban residents locate closer to the CBD.*

This represents the inverted U-shaped relationship between the wage rate w and location x, which is *a spatial version of backward-bending labor supply.* The threshold wage $\overline{w}$ in (17) does not depend on the city size, and is common for all cities. This implies that the spatial version of the backward-bending labor supply would be observed in any size of cities.

As shown in labor economics textbooks, the composite good z^* is a normal good and the leisure time t_l^* is an inferior good for low wage earners, while it is a normal good for high wage earners. On the other hand, the housing space s^* is somewhat different. From (10), we get

$$\frac{\partial s^*}{\partial w} = \frac{-\frac{\partial r}{\partial w}w + r}{b_s w^2}. \tag{18}$$

As long as the wage rate is low $w < \overline{w}$, $\partial r/\partial w$ is negative, so that (18) is positive. This means that the housing space s^* is a normal good. However, when the wage rate is high $w > \overline{w}$, $\partial r/\partial w$ is positive, so that (18) can be negative. That is, the housing space s^* can be an inferior good. This implies that as the wage gets higher, workers relocate from the suburbs to the city center in order to increase their leisure time t_l^*, which is a normal good, at the expense of the housing space s^*, which is now an inferior good. Hence, a more central location is chosen by higher-wage earners to enjoy enough leisure time by saving the commute time to their workplace. It is also chosen by low-wage earners to save on the commute cost to their workplace, which is a non-negligible amount for this group. On the other hand, the suburbs are chosen by middle-wage earners so that they can continue residing in large housing.

Finally, we gain further insights by conducting numerical simulations for three income classes in the next section.

Three income classes

Suppose there are three income classes: class A located nearest to the CBD, class C located farthest from the CBD, and class B who are in between. Class A is 1, class B is 2, and class C is 3 in Figure 2. Since the population density at x is $k/s(x)$, the three population constraints are given by

$$\begin{aligned} n_A &= \int_0^{x_{AB}} \frac{k}{s(x)} dx \bigg|_{w=w_A,\ c=C_A} \\ n_B &= \int_{x_{AB}}^{x_{BC}} \frac{k}{s(x)} dx \bigg|_{w=w_B,\ c=C_B} \\ n_C &= \int_{x_{BC}}^{\overline{x}} \frac{k}{s(x)} dx \bigg|_{w=w_C,\ c=C_C}, \end{aligned}$$

where n_c is the population of class c and $s(x)$ is given by (14). The three boundary conditions are

$$\begin{aligned} r(x_{AB})|_{w=w_A,C=C_A} &= r(x_{AB})|_{w=w_B,\, c=C_B} \\ r(x_{BC})|_{w=w_B,C=C_B} &= r(x_{BC})|_{w=w_C,\, c=C_C} \\ r(\overline{x})|_{w=w_C,\, c=C_C} &= \overline{r}. \end{aligned}$$

These three population constraints and the three boundary conditions determine six unknowns x^*_{AB}, x^*_{BC}, $\overline{x}^*$, C^*_A, C^*_B, and C^*_C, which determine all the other variables.

Given the parameter values of $n_A = n_B = n_C = a_s = b_z = c_z = 1$, $a_z = 2$, $b_s = \overline{r} = 1/10$, $T = k = 10$, $t(x) = x$, and $\tau(x) = x$, we compute an equilibrium numerically given the several wage rates (w_A, w_B, w_C). As a result, three examples may arise.

(i) When all the wages are low such as $(w_A, w_B, w_C) = (0.58, 0.60, 0.62)$, we get $(x^*_{AB}, x^*_{BC}, \overline{x}^*) = (0.136, 0.566, 1.274)$. Since $w_A < w_B < w_C$, the poor locate nearer to the CBD, the rich reside in the suburbs, and the middle are located in between.

(ii) When the wages are intermediate such as $(w_A, w_B, w_C) = (0.8, 1.4, 1.2)$, we get $(x^*_{AB}, x^*_{BC}, \overline{x}^*) = (0.188, 0.873, 1.698)$. Since $w_A < w_C < w_B$, the poor locate nearer to the CBD, the middle class reside in the suburbs, while the rich are located in between.

(iii) When the wages are high such as $(w_A, w_B, w_C) = (4, 3, 2)$, we get $(x^*_{AB}, x^*_{BC}, \overline{x}^*) = (0.738, 1.538, 2.413)$. Since $w_C < w_B < w_A$, the poor are located in the suburbs, the rich reside close to the CBD, while the middle class are locate in between.

Among these three examples, we focus on example (ii). This is consistent with Proposition 1 in that the location has an inverted U-shaped relationship with the wage rate. The time allocations by location in example (ii) are numerically computed and drawn as a function of distance x from the CBD in Figure 3.

As expected from (15), the hours of work are not necessarily monotonic in terms of the distance from the CBD even within the same income class. The hours of work of the middle class is inverted U-shaped in relation to the distance from the CBD although it is hard to see this from Figure 3. The hours of work of the poor increase with the distance from the CBD, whereas those of the rich decrease with the distance. The housing space $s(x)$ of the poor is small. However, if they earn a higher income, they will work more, so that they can reside in larger housing like the middle class. This is possible by relocating farther from the CBD.

Nevertheless, the on-duty hours, which is the sum of the hours of work and commute time, monotonically increase with the distance from the CBD within and across income classes. As the mirror image of the on-duty hours, the leisure time monotonically decreases with the distance from the CBD. This is similar to what we have shown in the first inequality of (15) within the same income class. This is reflected by the fact that the increasing commute time reduces the leisure time.

Next, we examine the time allocations in Figure 3 from a lifecycle point of view. When the workers are young, they are relatively poor, so that they are located near the CBD with short commutes and long leisure times. When they become middle-aged, they become middle class with longer commutes and shorter leisure times, i.e., the middle class work hard and reside in larger houses in the suburbs. Finally, when they become senior, they are rich with long/medium leisure times and short/medium commutes. Thus, the longitudinal labor supply is inverted U-shaped with respect to the wage rate and the longitudinal location choice is inverted U-shaped with respect to the distance from the CBD. In sum, *both labor supply and location choice are backward-bending* from a lifecycle point of view.

As for the longitudinal consumption of the composite good and housing space, we observe that as the household income rises, the consumption of the composite good z increases. That is, it is a normal good, which corresponds to $\partial s^*/\partial w > 0$ in (18). However, as the household income rises, the consumption of housing space s initially increases by locating farther away from the city center, and then decreases as they locate closer to the city center, which corresponds to $\partial s^*/\partial w < 0$ in (18). Therefore, the housing space is initially a normal good, whereas it later changes to an inferior good as the rich value their leisure time more than their housing space.

4 Empirical Analysis

Alonso's (1964) rent theory, stated above, assumes that access to the workplace, x, is the only location characteristic. However, in reality, there are diverse location characteristics such as access to parks, retail stores and public services, and neighborhood environments. If access to the workplace is the only location characteristic, then the income classes are sorted as shown in Figure 2, where the different income classes never collocate. On the other hand, if we take these other location characteristics into account, then the income classes may be illustrated as shown in Figure 4, where the different income classes are collocated everywhere. The former corresponds to a deterministic utility approach, while the latter is a random utility approach a la McFadden (1974). We obviously take the latter approach in our empirical analysis.

In this section, we show Proposition 1 by using the data from *the Housing and Land Survey* (conducted every five years in Japan) for the period 1978-2003.[8] This data is the number of households by income class in the three largest MAs of Tokyo, Osaka, and Nagoya and by five rings (0-10, 10-20, 20-30, 30-40, 40-50km) of distance from the city center. Thus, the data shows the spatial distribution of households by income class.

Let n_{cd} be the number of households with income class c in the ring d. The number of household income classes are between 5 to 9 depending on the survey years, e.g., 7 classes in 1973 and 9 classes in 2003 as follows:

class c	1	2	3	4	5	6	7	8	9
Income in 1973	$0-50$	$50-100$	$\cdots$	$\cdots$	$\cdots$	$300-500$	$500-$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Income in 2003	$0-100$	$100-200$	$\cdots$	$\cdots$	$\cdots$	$500-700$	$700-1000$	$700-1000$	$700-1000$

where the household income is in thousand yen. The Tokyo MA is divided into five rings as:

ring d	1	2	3	4	5
Distance from Tokyo Station in km	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$

Since the spatial distribution of the different income classes are illustrated in Figure 4, we calculate the mean distance from the CBD by each income class, and conduct t-tests to test if the mean location of one group is closer to the CBD than that of another group. The null hypothesis is that there is no significant difference in the mean locations across two groups.

[8]Unfortunately, there is no such data after 2003.

Assuming that the household density is uniform within each ring, the sample mean distance from the CBD of income class c is given by

$$m_c = \frac{\sum_{d=1}^{5} (10d - 5)\, n_{cd}}{\sum_{d=1}^{5} n_{cd}}$$

and the sample variance of income class c is given by

$$s_c^2 = \frac{\sum_d (10d - 5 - m_c)^2\, n_{cd}}{\sum_d n_{cd} - 1}.$$

The t-value between income classes c_1 and c_2 is

$$t_{c_1 c_2} = \frac{m_{c_1} - m_{c_2}}{\sqrt{\frac{s_{c_1}^2}{\sum_d n_{c_1 d}} + \frac{s_{c_2}^2}{\sum_d n_{c_2 d}}}}.$$

The results of the three MAs from 1973 to 2003 are listed as follows. It is revealed that all absolute values of the t-values are larger than the critical t-values with a significance level of 0.05, which rejects each null hypothesis. That is, all classes c can be strictly sorted by their distance from the CBD. For example, the mean distances are $m_1 < m_2 < m_5 < m_3 < m_4$ in the first column of the Tokyo MA in 1978. This shows that the lowest income class $c = 1$ is located near the CBD, the second highest income class $c = 4$ is located near the city edge, and the highest income class $c = 5$ is located in between.

Tokyo MA: mean distance from the CBD	year
$m_7 < m_1 < m_2 < m_6 < m_3 < m_5 < m_4$	1973
$m_1 < m_2 < m_5 < m_3 < m_4$	1978
$m_2 < m_1 < m_3 < m_7 < m_4 < m_5 < m_6$	1983
$m_2 < m_1 < m_3 < m_8 < m_4 < m_5 < m_7 < m_6$	1988
$m_3 < m_2 < m_1 < m_9 < m_4 < m_5 < m_6 < m_8 < m_7$	1993
$m_9 < m_2 < m_1 < m_3 < m_4 < m_8 < m_5 < m_7 < m_6$	1998
$m_9 < m_8 < m_1 < m_2 < m_3 < m_4 < m_7 < m_5 < m_6$	2003

Nagoya MA: mean distance from the CBD	year
$m_7 < m_6 < m_5 < m_4 < m_3 < m_2 < m_1$	1973
$m_1 < m_5 < m_2 < m_4 < m_3$	1978
$m_1 < m_7 < m_2 < m_3 < m_6 < m_4 < m_5$	1983
$m_1 < m_2 < m_8 < m_3 < m_4 < m_7 < m_5 < m_6$	1988
$m_2 < m_1 < m_9 < m_3 < m_4 < m_5 < m_8 < m_6 < m_7$	1993
$m_9 < m_1 < m_2 < m_3 < m_4 < m_8 < m_5 < m_7 < m_6$	1998
$m_1 < m_9 < m_2 < m_3 < m_8 < m_4 < m_5 < m_6 < m_7$	2003

Osaka MA: mean distance from the CBD	year
$m_2 < m_7 < m_3 < m_1 < m_6 < m_4 < m_5$	1973
$m_2 < m_3 < m_1 < m_4 < m_5$	1978
$m_2 < m_3 < m_1 < m_4 < m_5 < m_7 < m_6$	1983
$m_3 < m_4 < m_2 < m_5 < m_8 < m_1 < m_6 < m_7$	1988
$m_3 < m_4 < m_5 < m_2 < m_1 < m_9 < m_6 < m_7 < m_8$	1993
$m_2 < m_3 < m_4 < m_9 < m_1 < m_5 < m_8 < m_6 < m_7$	1998
$m_9 < m_1 < m_2 < m_3 < m_4 < m_5 < m_8 < m_6 < m_7$	2003

These results of the Tokyo MA in 1978 are illustrated in the second left of Figure 5a indicating that as the income rises, the lower-half income classes $\{1, 2, 3, 4\}$ relocate farther away from the CBD, whereas the upper-half income classes $\{4, 5\}$ relocate closer to the CBD.

The location by income class in the other years of the three MAs are illustrated in Figures 5a, 5b, and 5c, exhibiting similar backward-bending location patterns by income, particularly in the Tokyo and Nagoya MAs. Hence, we confirm Proposition 1 that the poor and rich are located close to the CBD, while the middle are located near the suburbs. To be more precise, as income rises, workers initially increase their labor supply, which consists of the hours of work and commute time, and locate far away from the CBD; they later decrease their labor supply and relocate closer to the CBD. Thus, *labor supply is backward-bending temporally, while location choice is backward-bending spatially.*

5 Conclusion

We have considered the trade-off between the hours of work, commute times and leisure time in this study. Based on the standard model used in labor economics textbooks, we have extended the model by incorporating urban space, so that there is another trade-off between leisure time, the consumption of a good, and housing space. This is an attempt to combine a labor supply model with a housing location model.

Since different income groups coexist in an urban economy, we have analyzed how the richer households spend more leisure time, consume more good, spend more housing space, and/or more time commuting. Then, we have shown that as the household income rises, the labor supply initially increases and then decreases as stated in labor economics textbooks. We have also shown that as the household income rises, the household location is initially closer to the CBD, then in the suburbs, and finally close to the CBD again. In sum, we have shown that *both labor supply and urban location choice are inverted U-shaped in relation to the wage rate.*

These analytical results are empirically shown by using data on the hours of work and commute times by household income class and on the number of households by income class in the three largest MAs.

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Appendix: Income class data

The data on the hours of work and commute times by income class are taken from *The Survey on Time Use and Leisure Activities*, (taken every five years from 1991 to 2021), where the income class is

class c	1	2	3	4	5	6
household income	$0-100$	$100-200$	$200-300$	$300-400$	$400-500$	$500-600$
class c	7	8	9	10	11	12
household income	$600-700$	$700-800$	$800-900$	$900-1000$	$1000-1500$	$1500-$

To observe the relationship between the household income and time allocation, we convert such income ranges to average incomes in the following manner. We utilize *the National Survey of Family Income, Consumption and Wealth* for 1999, 2004, 2011, and 2014, because it includes the average income for each income class. The income range of the lowest class is $0-200$ and its average income is 139.8.[9] Since the income range of the lowest class is $0-100$, we estimate its average income as $100/200*139.8 = 69.9$. Likewise, the income range of the highest class table is $2000-$ and its average income is 2921.[10] Since the income range of the highest class in the above table is $1500-$, we estimate its average income as $1500/2000 * 2921 = 2191$. For the rest of the income classes 2 to 11, we take a middle point income on each income range, which is close to each average income in *the National Survey of Family Income, Consumption and Wealth*.

[9] If we use the lowest or highest average income from the four years rather than the average income of the four years, the overall results remain the same.

[10] If we use the lowest or highest average income from the four years rather than the average income of the four years, the overall results remain the same.

Figure 1: The relationship between the on-duty hours t_d and household real income I

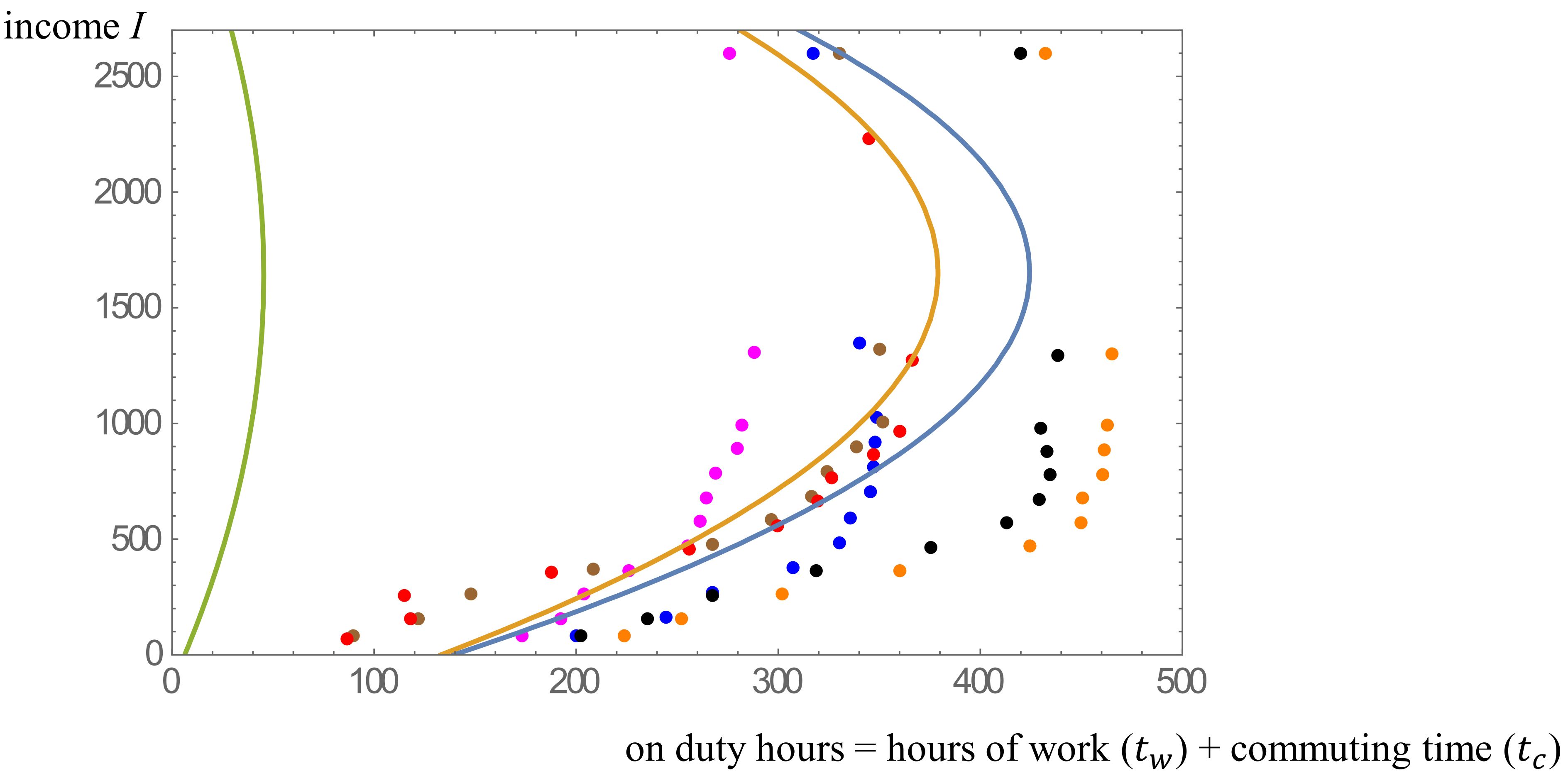

Figure 2: Bid rent of income groups with deterministic utility

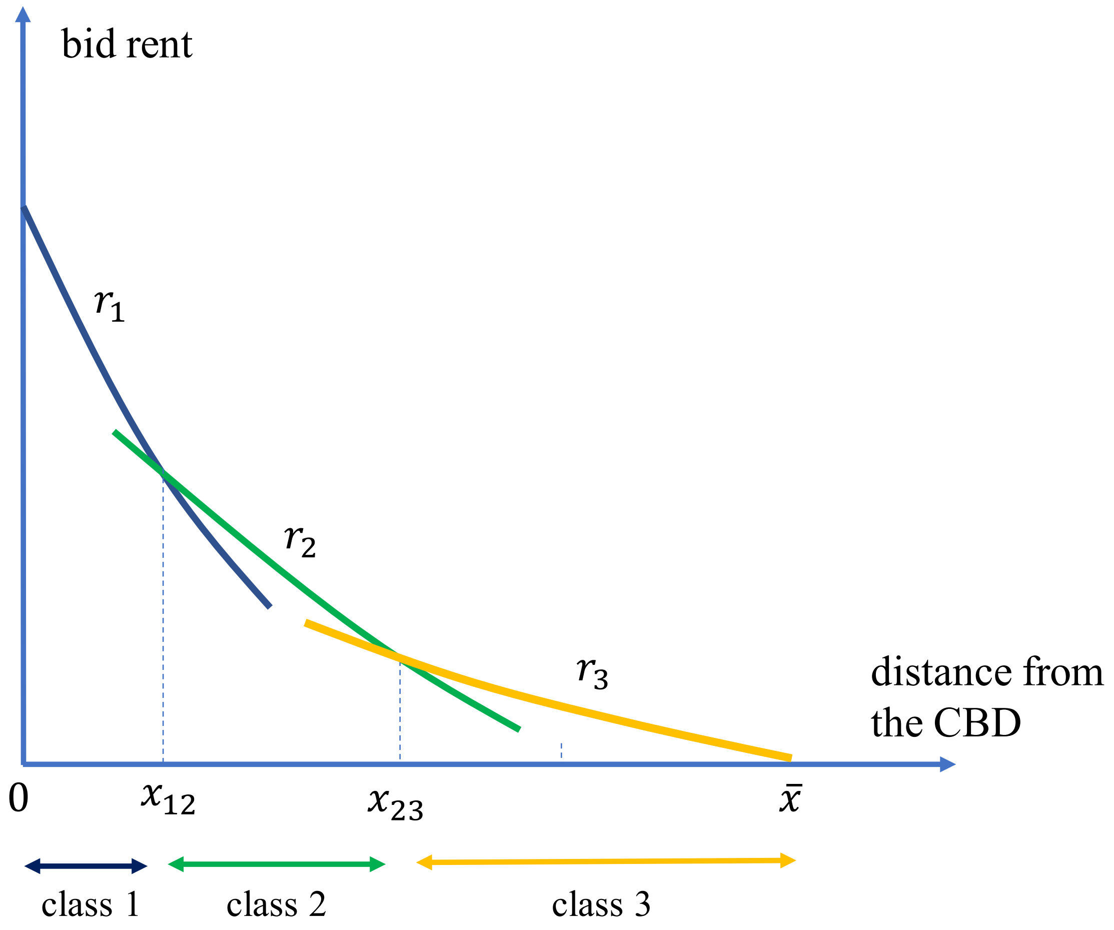

Figure 3: Spatial distribution of time allocation by different income groups

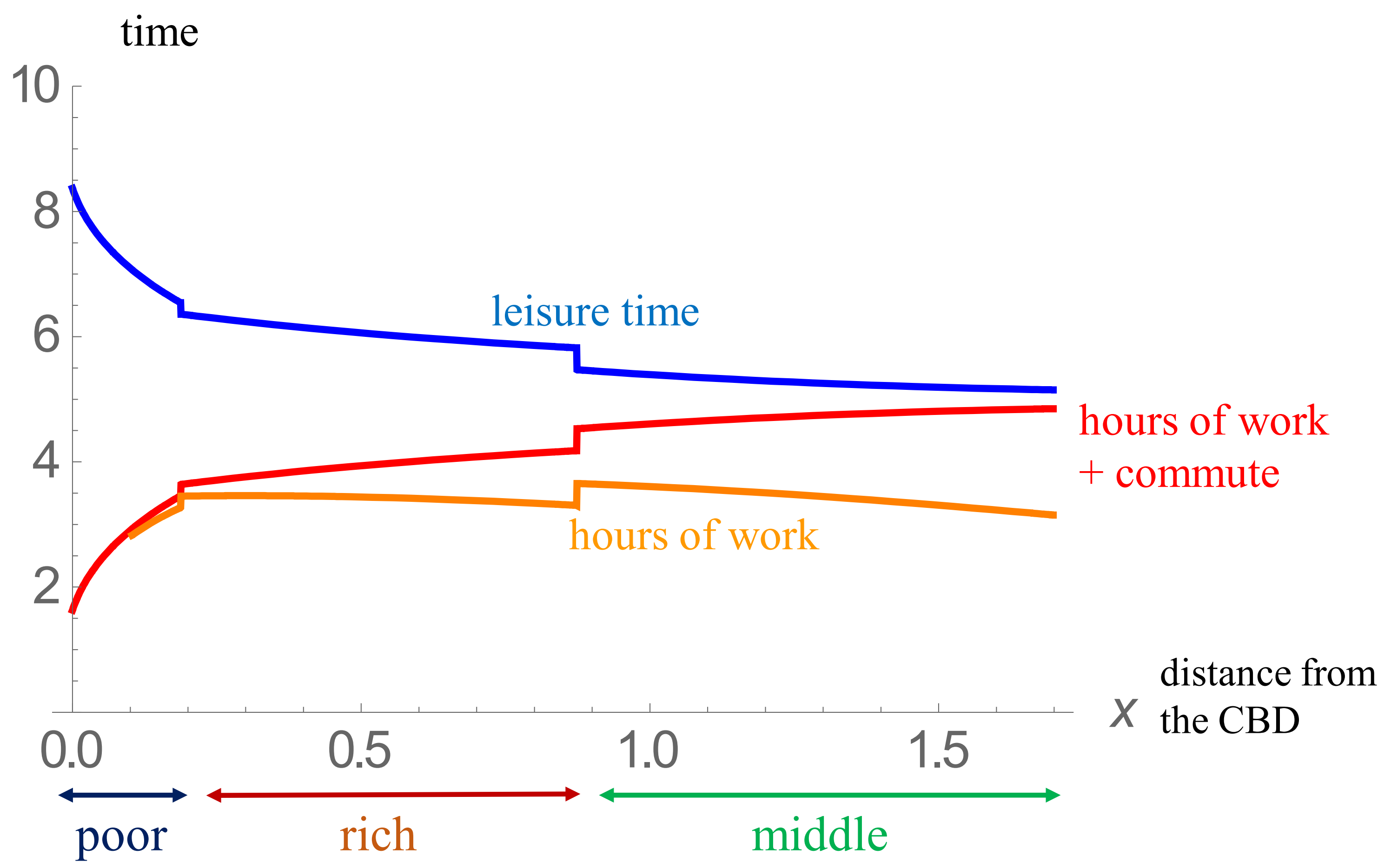

Figure 4: Bid rent of income groups with random utility

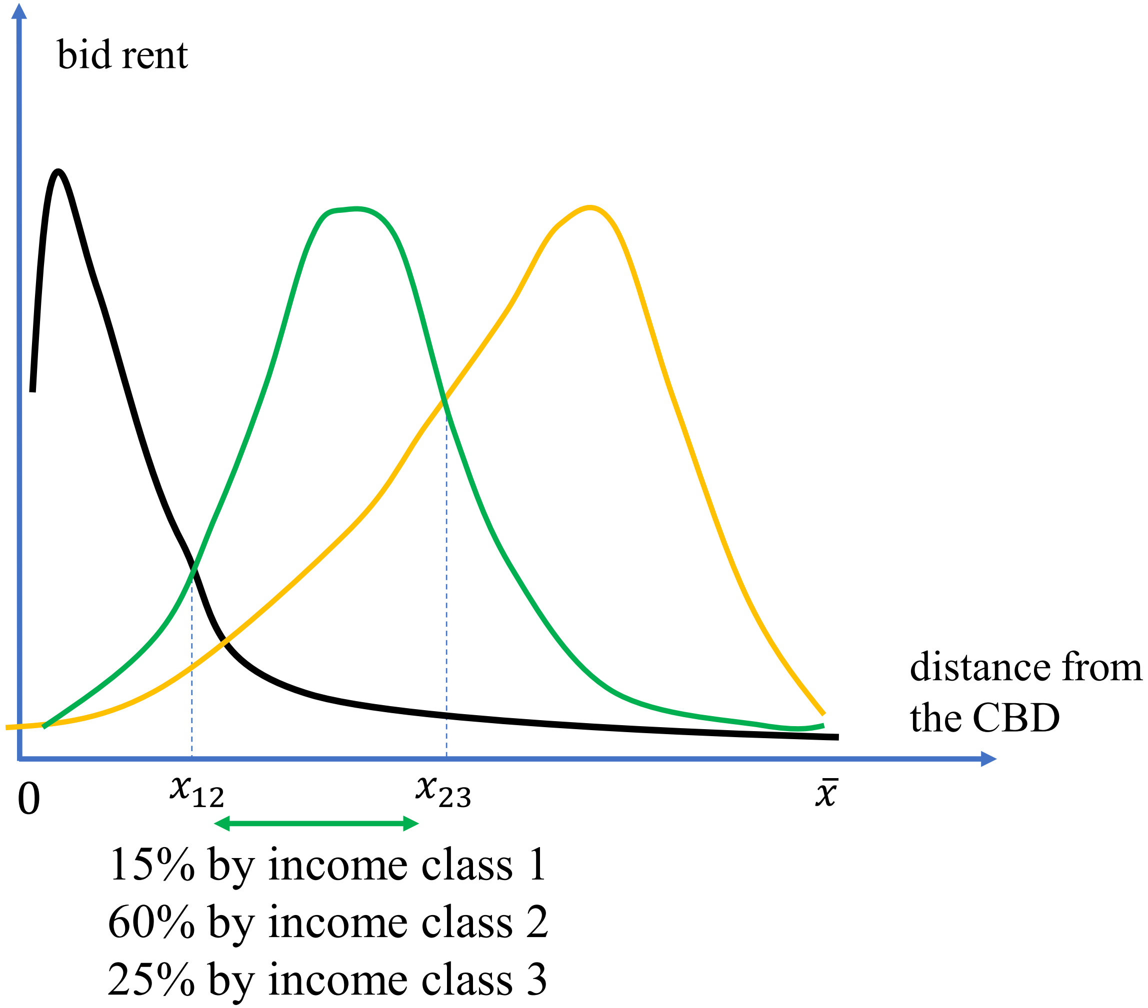

Figure 5a: Spatial distribution of different income groups in Tokyo MA

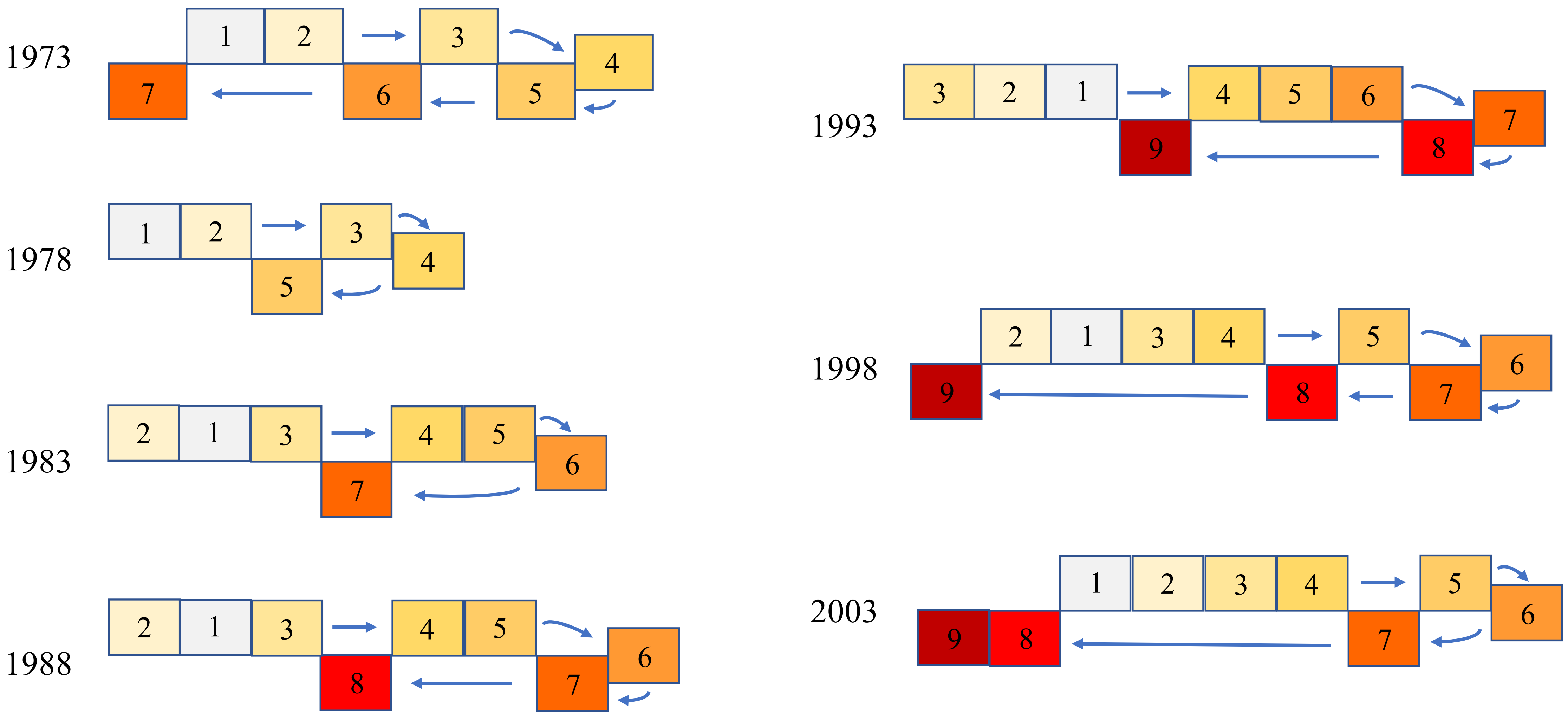

Figure 5b: Spatial distribution of different income groups in Nagoya MA

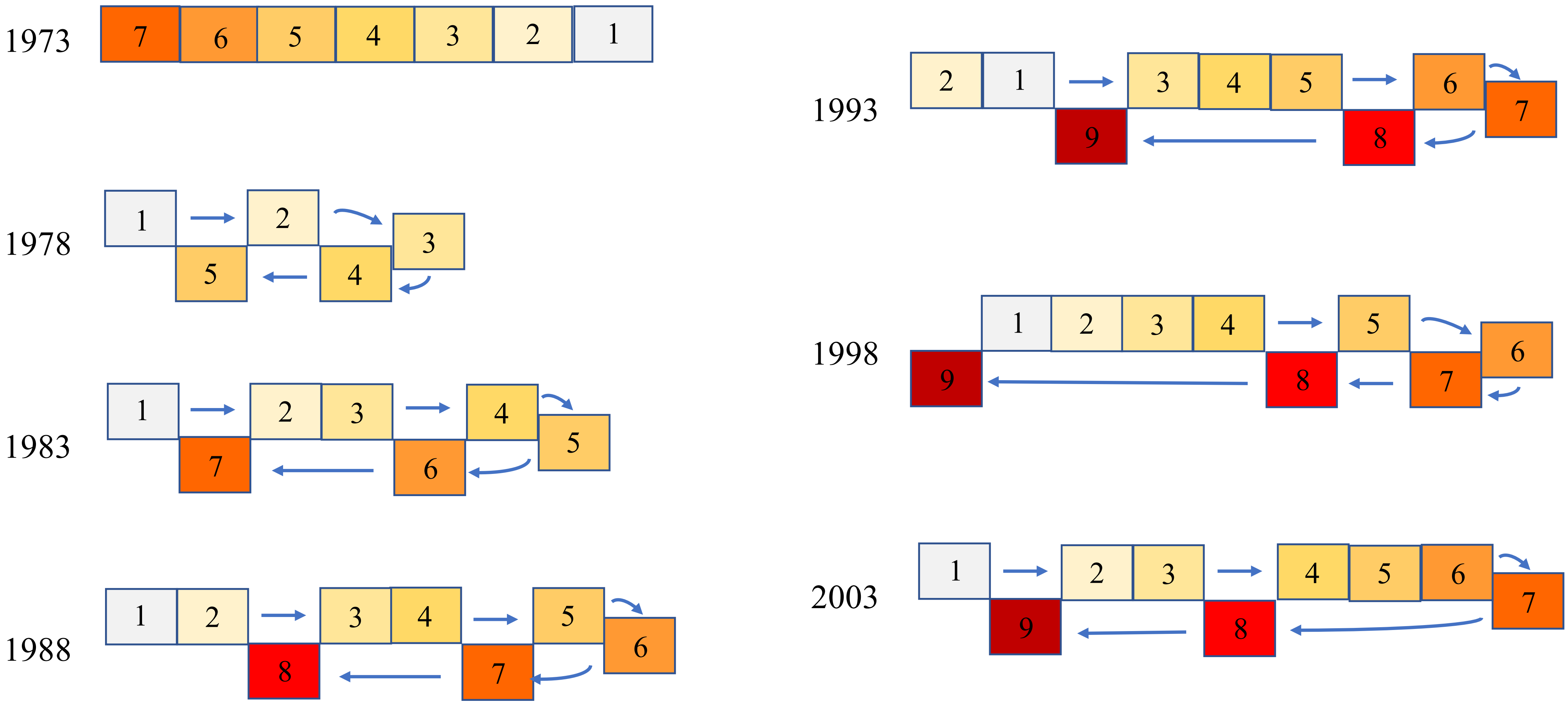

Figure 5c: Spatial distribution of different income groups in Osaka MA