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WHY YOU SHOULD NEVER USE THE HODRICK-PRESCOTT FILTER: COMMENT

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Abstract. Hamilton (2018) argues that one should never use the Hodrick-Prescott (HP) filter to detrend economic time series and proposes a new regression-based approach. This comment shows that this alternative shares the main drawbacks Hamilton finds in the HP filter: filter-induced dynamics in the estimated cycles and arbitrariness in the choice of a filter-defining parameter. In addition, the Hamilton trend lags the data by construction, leading to peculiar timing properties. Overall, it seems unlikely that the Hamilton filter really improves on the HP filter in practice.

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1. Introduction

In an important paper, Hamilton (2018) argues that one should never use the HP filter, proposed by Hodrick and Prescott (1981, 1997) to decompose a time series into separate trend and cyclical components. Hamilton makes his point in two steps. First, he highlights three drawbacks of the HP filter: (a) It introduces spurious dynamic relations that have no basis in the underlying data-generating process (DGP). (b) The cyclical estimates at the boundaries of the sample are not reliable. (c) Common choices for the smoothing parameter are arbitrary. Second, he proposes an alternative regression-based strategy, since known as the Hamilton filter, that, he argues, extracts plausible cyclical components while eschewing the pitfalls of the HP filter. In Hamilton's view, these elements close the case against the HP filter.

But is the case really closed? Hamilton's dismissal of the HP filter runs counter to widespread practice in academia, policy-making institutions, and the private sector. Limitations of the HP filter have been known for some time (e.g., Nelson and Plosser, 1982; Harvey and Jaeger, 1993; Cogley and Nason, 1995), but people still use it for lack of a clearly superior alternative. Therefore, the main question is whether the Hamilton filter indeed improves on the HP filter. Based on a simple empirical check and a transparent analytical discussion, this comment argues that it does not. This conclusion follows from the Hamilton regression creating the same kind of filter-induced dynamics in cyclical estimates as the HP filter and facing similar arbitrariness in the choice of a filter-defining parameter. Therefore, it is doubtful that the Hamilton filter really outperforms the HP filter in practice.

Consider first the empirical check. Hamilton motivates his criticism of the HP filter from a real-world example: the detrending of consumption and stock prices. Both series resemble random walks, but the cyclical components extracted by the HP filter feature complex dynamics. According to Hamilton, these are artificial patterns that reflect the filter and hide the true properties of the DGP. Surprisingly, Hamilton (2018) does not discuss the properties of the cycles estimated by his alternative approach. This comment fills this gap and finds that the Hamilton cycles extracted from consumption and stock prices exhibit persistence and comovements that closely mirror those found in HP cycles. Therefore, the empirical example Hamilton uses against the HP filter fails to establish the superiority of his preferred strategy: if the HP cyclical dynamics are spurious, as Hamilton argues, then the identical Hamilton dynamics have to be equally biased.

The empirical check also emphasizes the presence of a mechanical delay between the data and the estimated Hamilton trend. This property is not surprising: Hamilton *defines* the trend as an 8-quarter-ahead forecast for quarterly series, so that estimates react to data movements with an automatic two-year gap. Hamilton (2018) does not discuss this timing, but the implied discrepancy between the data and the trend estimate at any point in time can be uncomfortably large when the series is volatile, as is the case for stock prices. It is especially problematic when the trend component has an economic interpretation, for instance as potential output in a trend-cycle decomposition of real GDP. Finally, given the additive trend-cycle decomposition, questioning the timing of trend estimates necessarily leads to doubt cyclical estimates as well.

Second, consider the analytical discussion. When the underlying DGP is a random walk, which happens to be Hamilton's focus, simple computations reveal that the Hamilton cycle exhibits strong serial correlation, is highly predictable from its past, and can predict other variables. These properties explain the outcome of the empirical check and highlight why a backward-looking filter like the Hamilton regression generates similar persistence and predictability patterns in the cyclical component as a two-sided moving average like the HP filter. Again, this property weakens Hamilton's case, for his alternative method does not solve the main issue Hamilton raises against the HP filter.

The analytical discussion also shows how the choice of the forecast horizon in the Hamilton regression shapes the properties of the estimated cyclical components. With longer horizons, the filter extracts more volatile and more persistent cycles from a given DGP. Yet, there is no compelling criterion to select the forecast window. For instance, Hamilton (2018) suggests using 8 periods for quarterly data, on the grounds that a two-year horizon is a "standard benchmark" for business-cycle analysis and that a multiple of 4 helps dealing with seasonal patterns. While the second argument is objective, it is not clear why a two-year forecast should be the benchmark.

In fact, available empirical evidence suggests that a one-year window might more adapted to quarterly macro variables. Indeed, Angeletos, Collard, and Dellas (2020) document that the shock with the largest effect on GDP, consumption, investment, and unemployment at business-cycle frequencies roughly maximizes forecast error variances at the one-year horizon. Thus, using a 4-quarter forecast window in the Hamilton regression would extract from macro variables a stationary component with almost direct cyclical interpretation, which might be of special interest. Given that many macroeconomic variables resemble random walks, this change would roughly halve the volatility of estimated cycles and lowers their persistence compared to Hamilton's benchmark 2-year horizon, a clearly important statistical shift. From a broader perspective, the effect of the forecast horizon on the the estimated Hamilton decomposition and the uncertainty about the most appropriate value resemble the arbitrariness Hamilton finds in the choice of the smoothing parameter for the HP filter.

To summarize, this comment argues that the Hamilton filter cannot constitute a clearly superior alternative to the HP filter. A more balanced assessment is that the two filters provide different views of the data, and that whether one of the two views is more interesting remains an open question. (This idea is from Burnside, 1998.) Until this question is answered, the HP and Hamilton filters should be regarded as complementary tools for business-cycle analysis.

Previous studies offer a critical evaluation of the Hamilton filter. For instance, Schüler (2018) uses spectral methods to show that the Hamilton filter emphasizes cycles with longer duration than typical business cycles and that it mutes shorter cycles, leading to a failure to reproduce the chronology of U.S. business cycles. Hodrick (2020) applies simulation methods to compare the Hamilton filter with alternative detrending strategies, including the HP filter, and finds that the Hamilton filter yields better cyclical estimates for simple models, while the HP filter performs better for complex models. Compared to these studies, this comment focuses more narrowly on the mechanical impact the Hamilton filter has on estimated trends and cycles, which is of particular interest for applied economists.

2. The HP and Hamilton Filters

For completeness, this section provides a brief characterization of the HP and Hamilton filters. More details can be found in the original papers (Hodrick and Prescott, 1981, 1997; Hamilton, 2018).

Both the HP and the Hamilton filter decompose a time series x_t into the sum of two components: $x_t = g_t + v_t$, where g_t is the trend and v_t is the cycle. The difference between the two filters lies in the statistical restrictions used to identify the trend component.

The HP filter defines the trend component as a smooth variable that does not differ much from the observed series. This objective can be formalized by choosing g_t as the solution to the following program:

$$\min_{\{g_t\}_{t=-1}^{T}} \left\{ \sum_{t=1}^{T} (x_t - g_t)^2 + \lambda \sum_{t=1}^{T} \left[(g_t - g_{t-1}) - (g_{t-1} - g_{t-2}) \right]^2 \right\}, \tag{1}$$

where $\lambda \geq 0$ is a smoothing parameter penalizing large changes in the slope of the trend g_t. The estimated HP trend collapses to the original series when there is no smoothness penalty ($\lambda \to 0$), and it corresponds to a linear time trend when the penalty is extreme ($\lambda \to \infty$). The estimated cycle verifies $v_t = x_t - g_t$.

On the other hand, the Hamilton filter defines the trend component as the value that we would expect for the original series at date t, based on its behavior up to date $t-h$. This is formalized using a simple linear regression of x_t on a constant, the realization h periods ago x_{t-h}, and $p-1$ additional lags $x_{t-h-1}, \ldots, x_{t-h-p+1}$. For quarterly time series, Hamilton (2018) suggests using $h = 8$ quarters and $p = 4$ lags, so that the regression has the following form:

$$x_t = b_0 + b_1 x_{t-8} + b_2 x_{t-9} + b_3 x_{t-10} + b_4 x_{t-11} + u_t. \tag{2}$$

The fitted values and residuals from this linear regression correspond to the estimated Hamilton trend and cycle: $g_t = \widehat{x}_t$ and $v_t = \widehat{u}_t$.

3. Cyclical Dynamics of Stock Prices and Consumption

Section III.A in Hamilton (2018) questions the appropriateness of applying the HP filter to detrend typical economic time series. (Unless otherwise specified, all quotes reported in this section are from Hamilton's Section III.A. p. 833.) Hamilton argues that many such series resemble random walks and shows that detrending a random walk with the HP filter generates spurious dynamics, in the sense that the extracted cycle features high persistence, in contrast to the serially uncorrelated innovations of the underlying process. He provides an empirical example, based on stock prices and consumption. This section reexamines this example by submitting the Hamilton filter to the same evaluation as the HP filter.

Figures 1 and 2 below reproduce Hamilton's Figures 2 and 3 using an extended sample. Data definitions and sources are the same as in Hamilton (2018). Stock prices are measured as to 100 times the natural log of the end-of-quarter value for the S&P composite stock price index published by Robert Shiller, available online from `http://www.econ.yale.edu/~shiller/data.htm`. Consumption is measured as 100 times the natural log of real personal consumption expenditures from the U.S. National Income and Product Accounts. The data are quarterly and run from 1950Q1 to 2019Q4.

Figure 1 reports the autocorrelation structure for the first differences of log stock prices and real consumption, as well as their cross-correlations. The top panels show that growth in either series is essentially unpredictable, while the bottom panels indicate that after first differencing neither series has strong predictive power for the other. These features are in line with the idea that both variables resemble random walks.

Figure 2 reports the same statistics for the HP cycles extracted from the two series when the smoothing parameter takes the standard value $\lambda = 1,600$. As emphasized by Hamilton, the cyclical components of stock prices and real consumption display strong persistence, so that they are predictable from their past values. Furthermore, the cross-correlograms exhibit rich autoregressive structures with wave-like patterns, indicating that the two cyclical components forecast each other.

This discrepancy between the properties of the first differences of the data and those of HP cycles embodies Hamilton's claim that the HP filter distorts the series: "The rich dynamics in [the cyclical components] are purely an artifact of the filter itself and tell us nothing about the underlying data-generating process. Filtering takes us from the very clean understanding of the true properties of these series [. . .] to the artificial set of relations [found in the cycles, which] summarize the filter, not the data."

According to Hamilton, two characteristics of the HP filter combine to generate these spurious dynamics. First, because the HP filter is two-sided, the cyclical estimate at each date loads on past, present, and future shocks. It follows that the cyclical component "is both highly predictable (as a result of the dependence on [lagged shocks]) and will in turn predict the future (as a result of dependence on future [shocks])." Second, the coefficients relating the cyclical estimate to the underlying shocks "are determined solely by the value of λ," so that the HP filter effectively imposes dynamics on the data instead of adapting to the specific time series at hand. To overcome these deficiencies, Hamilton designs his detrending method as an estimated backward-looking regression. Because the coefficients $b_0, \ldots, b_4$ in equation (2) are estimated from the data, the filter has the flexibility to adapt to the underlying DGP. Because the regression uses only past information, the estimated cyclical component will not depend on future shocks.

Surprisingly, Hamilton (2018) does not report the autocorrelation function for the cycles extracted from stock prices and real consumption by his alternative approach. Yet, evaluating both filters on the same dataset would be a fair comparison. It would also clarify how moving from the two-sided, calibrated HP filter to the one-sided, estimated Hamilton filter affects the cyclical dynamics extracted from the data. Figure 3 fills this gap. Following Hamilton's recommendation for quarterly series, the filter uses $p = 4$ and $h = 8$, so that the cyclical components are obtained by regressing each series at date t on the four most recent observations available at date $t - 8$.

A striking finding is that the Hamilton cycles display virtually the same dynamic behavior as the HP cycles: the cyclical components are very persistent (the autocorrelations decay slowly toward zero); they have strong forecasting power for each other (the cross-correlations are high at several lags); and there are complex dynamics in cross-correlations that are very similar to those found in HP-filtered series. Focusing on the absolute size of the correlations, there appears to be even *more persistence* and *more cross-variable predictability* in Hamilton cycles than in HP cycles. This is confirmed by the business-cycle statistics reported in Table 1: the first-order autocorrelations of Hamilton-filtered series are 0.89 for stock prices and 0.90 for real consumption, larger than the corresponding values computed from HP-filtered series (0.76 and 0.81).

Of course, the HP and Hamilton cycles extracted from stock prices and real consumption are different. For instance, Table 1 shows that the Hamilton cycles are about twice as volatile as HP cycles. Nevertheless, the contemporaneous correlation between the cyclical components extracted by the two filters from a given variable is high: 0.71 for stock prices and 0.66 for real consumption. Comparing the autocorrelation functions of the cycles also highlights their similar dynamics.

These are surprising results, which weaken Hamilton's case for his alternative to the HP filter. If one accepts Hamilton's view that the serial correlation and predictability found in the HP cycles are artificial, then it is difficult not to draw the same conclusion regarding the same features in the Hamilton cycles. Alternatively, if one is willing to accept the cyclical component from Hamilton's filter, then comparing Figure 2 to 3 would indicate that the HP filter does at least a satisfactory job estimating the cyclical properties of the data. In either case, based on this empirical example, it is unclear why one would choose the Hamilton filter over the HP filter.

Another important property appears in Figure 4, which compares the historical path of log stock prices with the estimated HP and Hamilton trends. (Reporting the same figure for consumption would be less interesting because the data and the trends are more difficult to disentangle visually due to the smoothness of the series.) Unsurprisingly, the HP trend is smooth and lies well within the path of the actual time series, while the Hamilton trend is more volatile because it does not make use of future information. However, a direct consequence of using an 8-quarter-ahead forecast is that the Hamilton trend reacts to economic developments with a mechanical two-year delay. This is especially apparent in the later part of the sample: the trend systematically lags the 1995-2000 rise in stock prices, the burst of the dot-com bubble in 2001-2002, the 2003-2007 rebound, and the 2008-2009 financial crisis by a constant window of 8 quarters.

Few economists would view the red line in Figure 4 as the best possible estimate of the trend in stock prices. It follows from the additivity of the trend-cycle decomposition that the Hamilton cycle cannot be the best possible estimate of the cyclical component. In particular, the mechanical lag in the Hamilton trend amplifies the magnitude of the estimated cycle when the data present sharp movements, as can be seen most clearly during the 1990-2020 period. A more plausible trend component would attenuate the magnitude of these swings, and thus the volatility of estimated cycles.

The lag in Hamilton estimates also matters when the trend has a direct economic interpretation. For instance, imagine applying Hamilton's method to decompose real GDP into potential output (trend) and the output gap (cycle).[1] Then, due to the two-year delay in the Hamilton trend, estimated potential output will rise during most downturns and fall only when the recovery is already ongoing, in a very mechanical fashion. Clearly, the economic plausibility of such estimates would be questionable.

4. Simple Properties of the Hamilton Filter

This section elaborates on the previous empirical example by showing analytically how the Hamilton filter shapes the statistical properties of the cyclical component it estimates. The discussion focuses on the random-walk case for simplicity, but the generalization is obvious.

Let x_t and y_t follow two random walks: $x_t = x_{t-1} + \epsilon_t$, $y_t = y_{t-1} + \eta_t$, with ϵ_t and η_t two white noise processes with variances σ_ϵ^2 and σ_η^2 and covariance $\rho\sigma_\epsilon\sigma_\eta$. For instance, x_t might represent the log of stock prices and y_t the log of real consumption: the two variables have

[1]To be sure, such interpretations of trend-cycle decompositions may be at odds with economic theory, which favors model-based estimation of potential output and the output gap (see, e.g., Justiniano, Primiceri, and Tambalotti, 2013). However, as noted by Christiano, Trabandt, and Walentin (2010), in practice the HP filter is often used for this purpose.

low forecasting power for each other, and a common shock might induce contemporaneous comovement.[2]

Section IV.B in Hamilton (2018) shows that, in population, the cyclical components obtained by applying the Hamilton filter to x_t and y_t are equal to the forecast errors at horizon h:

$$v_t^x = x_t - x_{t-h} = \sum_{j=0}^{h-1} \epsilon_{t-j}, \qquad v_t^y = y_t - y_{t-h} = \sum_{j=0}^{h-1} \eta_{t-j}. \tag{3}$$

Since ϵ_t and η_t are white noise processes, it is straightforward to compute the second moments of these random variables:

$$\mathrm{Var}(v_t^x) = h\sigma_\epsilon^2, \qquad \mathrm{Var}(v_t^y) = h\sigma_\eta^2,$$

$$\mathrm{Corr}(v_t^x, v_{t-j}^x) = \mathrm{Corr}(v_t^y, v_{t-j}^y) = \frac{h-j}{h} \text{ if } j = 0, 1, \ldots, h, \quad = 0 \text{ if } j \geq h+1,$$

$$\mathrm{Corr}(v_t^x, v_{t-j}^y) = \mathrm{Corr}(v_t^y, v_{t-j}^x) = \frac{(h-j)\rho}{h} \text{ if } j = 0, 1, \ldots, h, \quad = 0 \text{ if } j \geq h+1.$$

These expressions highlight three key properties of the Hamilton filter. First, it extracts a persistent cycle out of a random walk. Second, it extracts interrelated cycles out of correlated random walks. Third, the variances, the persistence, and the joint dynamics of the cycles are mechanically determined by the forecast horizon h. All three properties follow from Hamilton's definition of the cycle as a h-step-ahead forecast error: as shown by equation (3), two realizations of v_t^x and v_t^y separated by j periods share $h - j$ common innovations when $j \leq h$, necessarily leading to serial correlation and comovement. While the random-walk setting makes this feature especially apparent given the permanent effect of shocks, similar persistence and predictability would arise from applying the Hamilton filter to more general ARIMA processes.

These properties explain the dynamics found in Hamilton-filtered stock prices and real consumption. Thus, the empirical example in Section 3 represents the normal behavior of the filter and demonstrates that filter-induced dynamics are as present in Hamilton cycles as in HP cycles.

The choice of the forecast horizon h provides another important illustration of filter-induced dynamics. Hamilton (2018, Section IV.C, p. 838) motivates his recommendation of $h = 8$ for quarterly data from two arguments: (i) using a multiple of 4 is useful to purge the estimated cycles from potential seasonal patterns, and (ii) the notion that "a two-year

[2]This bivariate random-walk representation provides a good approximation of the data. Letting x_t denote 100 times the log of stock prices and y_t 100 times the log of real consumption, estimating a simple first-order vector autoregression yields the following parameter values:

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} -0.76 \\ 2.47 \end{bmatrix} + \begin{bmatrix} 0.98 & 0.03 \\ 0.00 & 0.99 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \widehat{\epsilon}_t \\ \widehat{\eta}_t \end{bmatrix}, \quad \mathrm{Var} \begin{bmatrix} \widehat{\epsilon}_t \\ \widehat{\eta}_t \end{bmatrix} = \begin{bmatrix} 51.89 & 0.99 \\ 0.99 & 0.62 \end{bmatrix}.$$

The implied correlation between the innovations is $\widehat{\rho} = 0.17$.

horizon should be the standard benchmark" for business-cycle analysis. If the first argument is objective, the second one seems less empirically or theoretically grounded. For instance, the frequency band typically associated with business cycles ranges from six quarters to eight years (e.g., Stock and Watson, 1999), so that it is not clear why $h = 8$ quarters should be the benchmark choice.

One can search the literature for further guidance. In recent work, Angeletos, Collard, and Dellas (2020) show that the shock which contributes most to the variance of key macroeconomic variables over the standard business-cycle frequency band (6-32 quarters) maximizes the forecast error variance at a one-year horizon in the time domain. This result suggests that setting $h = 4$ quarters would be interesting to detrend quarterly macro series because it would isolate a stationary component highly correlated with the main cyclical factor of the data. Since many macroeconomic variables behave like random walks, the above expressions show that moving from $h = 8$ to $h = 4$ in the Hamilton regression should roughly halve the variance of the estimated cycles and lower their persistence. Hamilton-filtered stock prices and real consumption broadly confirm these implications: with $h = 4$, their respective standard deviations fall to 15.3 and 1.8, about 30% below those reported in Table 1 for $h = 8$, while their autocorrelations drop to 0.76 and 0.71. Clearly, these are very different cyclical estimates, which paint a distinct picture of fluctuations in stock markets and consumption expenditures.

Overall, both the empirical example and analytical discussion emphasize that the population characteristic estimated by the Hamilton filter, the forecast error in the linear regression of the variable at $t+h$ on a constant and p lags, corresponds to a very particular view of the trend-cycle decomposition. In particular, the Hamilton approach has mechanical effects on the timing of the estimates relative to the data, as well as on the magnitude, persistence, and volatility of the estimated cyclical component. In the important random-walk case, these filter-induced dynamics resemble those found in HP cycles when it comes to "artificial" serial correlation and "spurious" predictability. Highlighting these properties of the Hamilton filter to the audience of applied economists is the main purpose of this comment.

5. Conclusion

Hamilton (2018) argues that economists should stop using the HP filter. However, the alternative filter he proposes can be criticized using essentially the same arguments invoked against the HP filter, namely the presence of filter-induced dynamics in the estimated cycles and the relative arbitrariness of a key parameter choice. Furthermore, the trends estimated by the Hamilton approach lag the data by construction. These results cast doubts on Hamilton's claim that his filter will always outperform the HP filter in practice. A more balanced assessment is that the two filters provide different ways to look at the cyclical properties of the data, with no clearly superior choice.

More generally, there is really nothing new or wrong in recognizing that detrending data affects its statistical properties in a way that depends on the chosen approach: using a polynomial time trend, the HP filter, a band-pass filter, or the Hamilton filter to separate the trend from the cycle will necessarily lead to different estimates of the cyclical component. Canova (1998) illustrated this point nicely twenty years ago. As stressed by Burnside (1998), this is not a major issue when the goal is to relate stationary economic models to non-stationary data, since it is always possible to compare filtered real-world data with filtered series from the model. For instance, the widespread software package Dynare (2011) automatically computes moments for HP- and band-pass filtered series simulated from DSGE models, allowing for straightforward comparison between theory and data. It would be useful to also implement the Hamilton filter, providing economists yet another window through which they can compare their models to reality.

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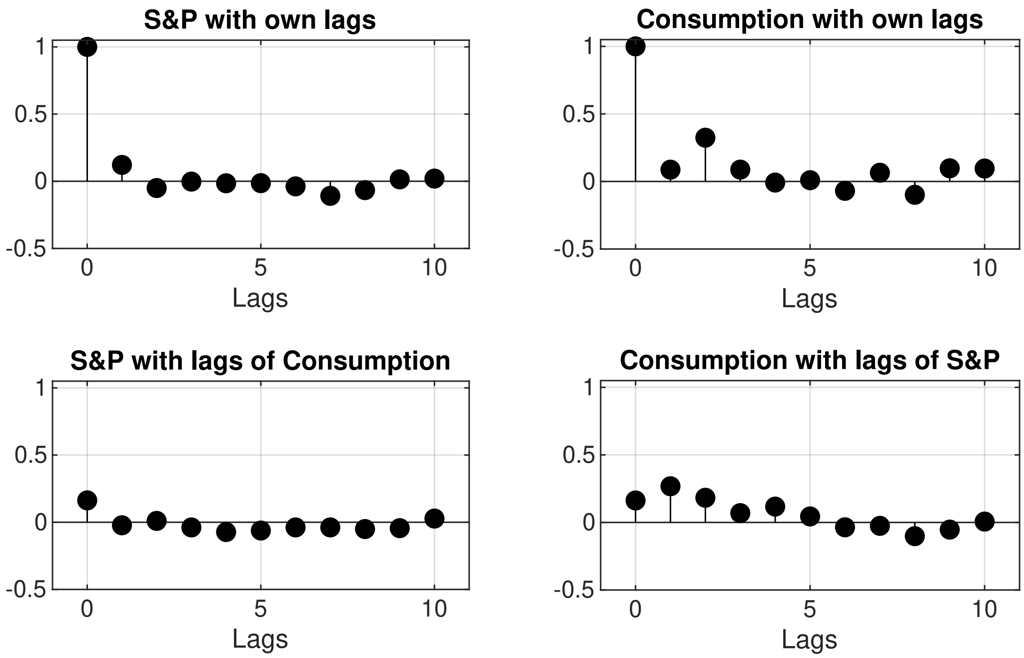
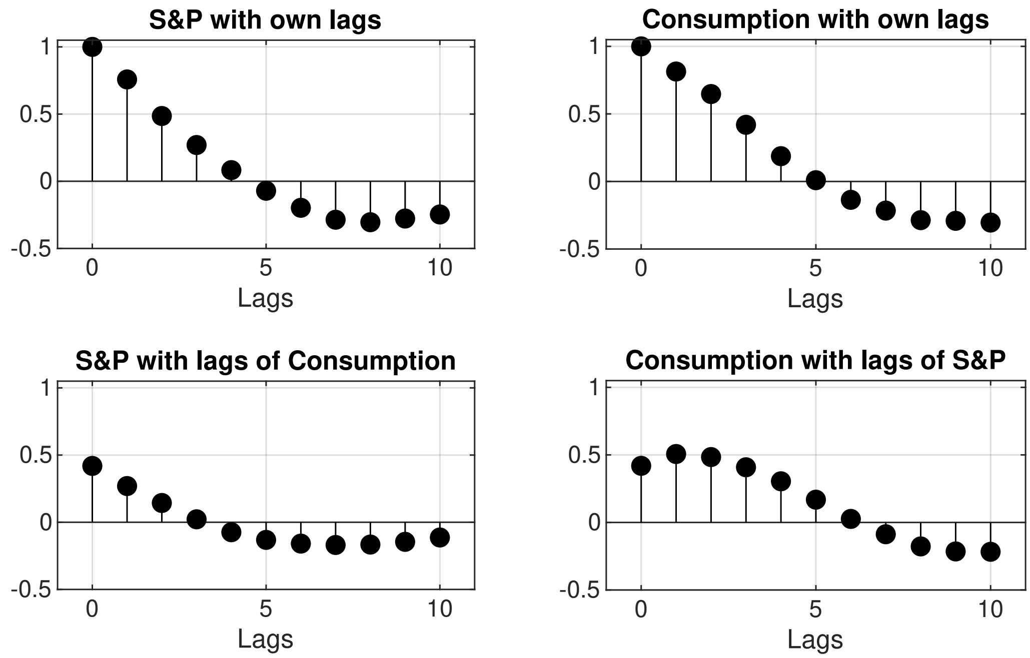
Figure 1. Autocorrelations and cross-correlations for the first differences of log stock prices and real consumption.



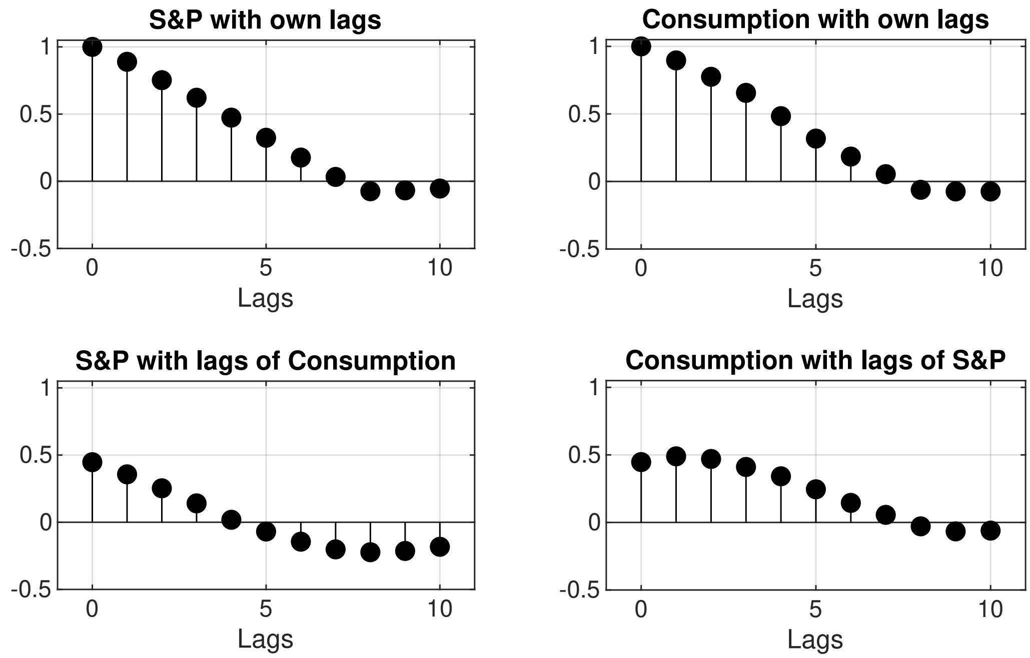
Upper left: Autocorrelations of the first difference of end-of-quarter value for S&P composite. Upper right: Autocorrelations of the first difference of real consumption. Lower panels: Cross-correlations.

Figure 2. Autocorrelations and cross-correlations for HP-filtered stock prices and real consumption.



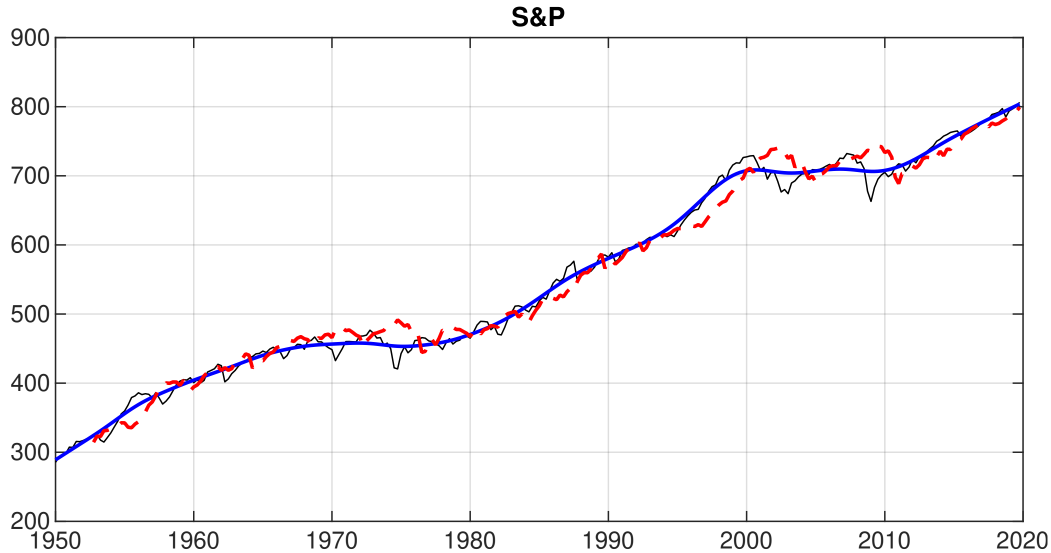
Upper left: Autocorrelations of HP-filtered end-of-quarter value for log S&P composite. Upper right: Autocorrelations of HP-filtered log real consumption. Lower panels: Cross-correlations. Smoothing parameter: $\lambda = 1,600$.

Figure 3. Autocorrelations and cross-correlations for Hamilton-filtered stock prices and real consumption.



Upper left: Autocorrelations of Hamilton-filtered end-of-quarter value for log S&P composite. Upper right: Autocorrelations of Hamilton-filtered log real consumption. Lower panels: Cross-correlations. Regression parameters: $p = 4$ and $h = 8$.

FIGURE 4. HP and Hamilton trends for stock prices.



Thin black line: Data. Thick blue line: HP trend ($\lambda = 1,600$). Dashed red line: Hamilton trend ($p = 4$, $h = 8$).

Table 1. Business-cycle statistics.

Panel A - Volatility and persistence

	Standard deviation	Autocorrelation
Stock prices		
First difference	7.20	0.12
HP cycle	9.99	0.76
Hamilton cycle	20.95	0.89
Real consumption		
First difference	0.81	0.09
HP cycle	1.23	0.81
Hamilton cycle	2.73	0.90

Panel B - Contemporaneous correlation

	Stock prices			Real consumption		
	First diff.	HP filter	Hamilton filter	First diff.	HP filter	Hamilton filter
Stock prices						
First difference	1.00					
HP filter	0.34	1.00				
Hamilton filter	0.31	0.71	1.00			
Real consumption						
First difference	0.18	0.29	0.29	1.00		
HP filter	−.15	0.45	0.33	0.24	1.00	
Hamilton filter	−.03	0.36	0.45	0.40	0.66	1.00

Notes. Stock prices: 100 times the natural log of the end-of-quarter value for the S&P composite stock price index. Consumption: 100 times the natural log of real personal consumption expenditures from the U.S. NIPA. Sample: 1950Q1 to 2019Q4. HP cycles computed with $\lambda = 1,600$; Hamilton cycles computed with $p = 4$ and $h = 8$.