Relative-performance delegation destabilizes upstream collusion

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Abstract: This paper analyzes upstream firms’ collusive sustainability when downstream firms adopt the relative-performance delegation in an infinitely repeated Cournot or Bertrand game. We find that relative-performance delegation makes managers act more aggressive and upstream collusion more difficult to sustain compared to sales-revenue delegation. The driving force is that downstream relative-performance delegation makes more profits for the deviated firm. This result holds regardless of the competition modes.

Keywords: Relative-performance delegation; Upstream collusion; Vertically related market; Competition modes

JEL classification: D21; D43; L13; L21

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Abstract: In this note, we analyze upstream firms’ collusive sustainability when downstream firms adopt the relative-performance delegation in an infinitely repeated Cournot or Bertrand game. We find that relative-performance delegation makes managers act more aggressive and upstream collusion more difficult to sustain compared to sales-revenue delegation. The driving force is that downstream relative-performance delegation makes more profits for the deviated firm. This result holds regardless of the competition modes.

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1. Introduction
In modern enterprises, the owner delegates the decision on output, marketing and R&D to his manager. Managerial delegation is often observed as a very common phenomenon in firms that face an oligopolistic market structure. The strategic delegation literature is built on the observation that, under strategic interdependence, delegation of decision making and accompanying actions can serve as commitments that influence competitive interactions with rivals and lead to beneficial outcomes. Starting with the sales-revenue delegation presented by Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987), the literature on strategic managerial delegation in oligopoly has been enriched by many studies. Alternative delegation contracts, like the relative-performance delegation is widely used in practice, in which the managers’ incentive contract is based not only on own firms profits but also on the rival’s relative profits. Salas-Fumas (1992), and Miller and Pazgal (2001, 2002) argue that relative-performance delegation is more popular delegation contract in business world. Miller and Pazgal (2001) further demonstrate the equivalence result in that if the owners are able to manipulate their managers’ behavior under the relative-performance contracts, the equilibrium outcome is the same, regardless of the competition modes. It is clear that managers are more aggressive under relative-performance delegation than sales-revenue delegation. Sengual et al. (2012) review the strategic delegation literature and provide a theoretical framework that integrates this perspective into management research.

In a two-tier related market having considered input-price commitment and delegation decision together, Wang (2010, 2020) proves that input price–delegation–quantity competition order coupled with relative-performance delegation is a dominant strategy for downstream rivals vis-a-vis market-share delegation and sales-revenue delegation. Since the outcomes create highest quantity and lowest price in a Cournot product market with relative-performance delegation, it lessens the double-marginalization problem in such a vertically separated industry. Guigou et al. (2011) follow the approach developed by Lambertini and Trombetta (2002) showing that collusion under relative-performance delegation is always harder to sustain than sales-
revenue delegation. But, no works have been done on the sustainability of upstream collusion under downstream relative-performance delegation. It motivates us to illustrate the comparison of stability conditions for collusion (i.e., critical discount factors) with managers and without managers in downstream firms, and when managerial incentives are alternatively based upon comparative performance or a mix of profits and sales revenues.¹

In a seminal paper, Deneckere (1983) in a repeated Cournot and Bertrand duopolies with linear demand and symmetric marginal costs, derives some basic results on the ability to maximize profits jointly. ² Since then, there are more papers investigating collusive effects from different perspectives. Lambertini and Trombetta (2002), Spagnolo (2005), and Matsumura and Matsushima (2012) examine the implication of managerial delegation on collusive stability in horizontal market with or without product differentiation, but are not in the context of vertically related markets. Even though Nocke and White (2007), Normann (2009) and Piccolo and Reisinger (2011) focus on tacit collusion in the vertical case, they do not consider managerial incentives. Bian et al. (2013) is the first paper considering how sales-revenue delegation influences the sustainability of upstream collusion under downstream Cournot and Bertrand competition. Wang and Wang (2021) extend Bian et al. (2013) in two directions: (1) confining to full collusion, they revisit the Bertrand result for close substitutes and find in contrast the possibility that downstream managerial delegation impedes upstream collusion; (2) the results in both Cournot and Bertrand models are sensitive to the case of partial collusion. We study the sustainably of upstream collusion under relative-performance delegation and show that relative-performance delegation makes upstream collusion more difficult to sustain compared to sales-revenue delegation.

Considering cross ownership among firms in a vertical related market without

¹ In this paper, delegation based upon comparative performance is less keen to yield collusive wholesale pricing than contracts based upon a mix of profits and revenues or a mix of profits and sales (as in vickers, 1985). It is because these two contracts are in fact one, as proven by Lambertini and Trombetta (2002).
² More works on the collusion behavior in the Cournot or/and Bertrand infinitely repeated game can be found in Abreu (1988), Chang (1991), Lambertini and Sasaki (2001), and Østerdal (2003), among many others.
downstream delegation, Charistos et al. (2022) show that symmetric passive forward ownership without input price discrimination hinders the sustainability of upstream collusion. In this paper, we analyze upstream firms’ collusive sustainability when downstream firms adopt the relative-performance delegation. We need to clarify the difference between exogeneous cross ownership (the investor’s shareholding interest) and endogenous relative-performance delegation (market competition between owner’s concern of firm’s profit).

We show that relative-performance delegation and product substitutability intensifies downstream market competition. More intensified downstream competition has two opposing effects on manufacturers' incentives to stay or deviate from tacit collusion. First, it makes punishment more severe. Second, deviation from upstream collusion becomes more profitable. The overall effect is to impede upstream collusion.

This paper is organized as follows. Basic model is provided in Section 2. The analyses of upstream collusion under quantity and price competition are provided in Section 3 and 4. Section 5 concludes the paper.

2. The Model

We assume in a vertically related market that there exists two upstream manufacturers (indexed by \( U1 \) and \( U2 \)) producing homogenous intermediate goods and selling to both retailers (indexed by \( D1 \) and \( D2 \)). Moreover, retailer \( i \) has a two-level governance structure consisting of an owner \( i \) and a manager \( i \) \((i = 1,2)\). For simplicity, we suppose that both managers’ opportunity cost and manufacturers’ marginal cost are zero.

The representative consumer’s utility function is given by:

\[
U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2} (q_1^2 + 2\gamma q_1 q_2 + q_2^2) + m \tag{1}
\]

where \( a \) is the market scale, \( q_1 \) and \( q_2 \) represent two retailer’s output. The parameter \( \gamma \in (0,1) \) measures the degree of the product differentiation, and \( m \) denotes composite goods.

\[3\] Sun and Wang (2022) analyze the impact of vertical cross-ownership with input price discrimination on social welfare. Under forward cross-ownership, a higher degree of product differentiation and a higher degree of cross-ownership raise industry profit, consumer surplus, and social welfare.
The inverse demand function is derived as follows:

\[ p_i(q_i, q_j) = a - q_i - \gamma q_j, \quad i, j = 1, 2, \quad i \neq j \]  

(2)

We assume that both owners have two alternative incentive schemes: (i) the relative-performance scheme under which manager \( i \)'s reward is given by: 

\[ M_i = [(1 - \lambda_i)\pi_i + \lambda_i(\pi_i - \pi_j)] = (\pi_i - \lambda_i\pi_j), \]

where \( \lambda_i \in [0,1] \) is the weight that owner \( i \) puts on the relative performance, \( \pi_i \) and \( \pi_j \) respectively denote retailer \( i \)'s and retailer \( j \)'s profits; (ii) the pure-profit scheme under which manager \( i \)'s salary is \( \pi_i \).

We consider an infinitely repeated game where a multi-stage game occurs in each period. The timing of multi-stage game is that manufacturer \( U_i \) (\( i = 1, 2 \)) in the upstream market simultaneously chooses its own wholesale price \( w_i \) (\( i = 1, 2 \)) in the first stage; owner \( i \) sets its incentive variable \( \lambda_i \) (\( i = 1, 2 \)) in the second stage; in the last stage, manager \( i \) competes in the downstream market by choosing the quantity \( q_i \) or the price \( p_i \) (\( i = 1, 2 \)). For the pure-profit scheme, it degenerates into a two-stage game: in the first stage, the manufacturers set the wholesale price simultaneously, and the managers of both retailers choose the quantity or price in the last stage.

3. Upstream Collusion: Cournot Competition

We first focus on the link between relative-performance delegation and sustainability of upstream collusion under Cournot competition. We apply backward induction to solve for the subgame perfect Nash equilibrium (SPNE). The following process replicates the analysis in Miller and Pazgal (2001).

In the third stage, each manager chooses his quantity to maximize his reward:

\[ \max_{q_i} M_{i, RP} = (\pi_i - \lambda_i \pi_j) = [(p_i - w_i)q_i - \lambda_i(p_j - w_j)q_j], i, j = 1, 2, i \neq j, \]  

(3)

where the superscript \( C \) and the subscript \( RP \) represent quantity competition in the downstream market and the relative-performance scheme, respectively.

Substituting (2) into (3) and solving the first-order conditions (FOCs) from (3), we

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Delbono and Lambertini (2020) show that managerial delegation based upon relative performance may generate collusive outcomes observationally equivalent to those typically associated with repeated games or cross ownership.
have:

\[ q_i(\lambda_i, \lambda_j) = \frac{2(a-w_i) - \gamma(1-\lambda_i)(a-w_j)}{4-\gamma^2(1-\lambda_i)(1-\lambda_j)}, \quad i, j = 1, 2, \quad i \neq j. \quad (4) \]

Differentiating with respect to \( \lambda_i \) and \( \lambda_j \), we have:

\[ \frac{\partial q_i}{\partial \lambda_i} = \frac{2\gamma(-2w_j + \gamma w_i(1-\lambda_j)+a(2-\gamma(1-\lambda_j)))}{(4-\gamma^2-\gamma^2(1-\lambda_j)+\lambda_j)^2} > 0 \]

\[ \frac{\partial q_i}{\partial \lambda_j} = \frac{\gamma^2(2(\alpha-w_1)+\gamma(\alpha-w_2)(1-\lambda_1))(1-\lambda_1)}{(4-\gamma^2(1-\lambda_1)(1-\lambda_2))^2} < 0 \]

The incentive variables will increase its own output and reduce the rival output.

In the second stage, owner \( i \) sets the incentive parameter simultaneously to maximize his own profit. So, owner \( i \)'s maximization problem is given by:

\[ \max_{\lambda_i} \Pi_{i,RP}^C = (p_i(\lambda_i, \lambda_j) - w_i)q_i(\lambda_i, \lambda_j), \quad i = 1, 2. \quad (5) \]

Substituting (2) and (4) into (5) and solving the FOCs from (5) yield:

\[ \lambda_i(w_i, w_j) = \frac{\gamma(\alpha-w_i-a\gamma+w_j)}{2\alpha-2w_j-\gamma(\alpha-w_i+a\gamma-w_j)}, \quad i, j = 1, 2, \quad i \neq j. \quad (6) \]

In the first stage, there are two choices for manufacturer \( U_i \) (\( i = 1, 2 \)): (i) to set the wholesale price \( w_i \) non-cooperatively to maximize its own profit \( \pi_i \); (ii) to collude in the upstream market so that they choose the wholesale price together to maximize the total profits \( w_1q_1 + w_2q_2 \).

In the first case, there is a non-cooperative game, and manufacturer \( i \)'s problem is:

\[ \max_{w_i} \pi_{UI} = w_i q_i(w_i, w_j) \quad (7) \]

Plugging (4) and (6) back into (7) and solving the system of FOCs, we have:

\[ w_{i,RP}^C = \frac{a(2-\gamma-\gamma^2)}{4-\gamma-2\gamma^2} \]

\[ \pi_{UI,RP}^C = \frac{a^2(1-\gamma)^2(2-\gamma)^2}{4(1+\gamma)(4-\gamma-2\gamma^2)^2} \quad (8) \]

where the superscript \( N \) denotes Nash equilibrium.

Given \( \lambda_i = \lambda_j = 0 \), we have the input price without delegation \( w_i = \alpha - \frac{2a}{4-\gamma} \).

Compared the delegation with the input price, we find that

\[ w_{i,RP}^C - w_i = -\frac{a\gamma^3}{(4-\gamma)(4-\gamma-2\gamma^2)} < 0 \]

Relative-performance delegation will increase the total output of the downstream
firms and lower the input price of the upstream firm.

In the second case, two manufacturers form a cartel to set wholesale price together for the maximization of the joint profit:

\[
Max_{w_1, w_2} \pi_{U_1} + \pi_{U_2} = w_1 q_1(w_1, w_2) + w_2 q_2(w_1, w_2) \tag{9}
\]

Substituting (4) and (6) into (9) and taking differentiation, we obtain:

\[
w_{iR}^C = \frac{a^2}{2}
\]

\[
\pi_{U_iR}^C = \frac{a^2(2+\gamma)}{16(1+\gamma)}
\tag{10}
\]

where the superscript \(C\) denotes collusion occurring in the upstream market.

However, another situation should also be taken into consideration, where manufacturer \(i\) deviates privately from the cartel in some period and the other manufacturer does not notice this behavior until the next period. Under this circumstance, manufacturer \(i\) will maximize its own profit \(\pi_{U_i}\) while the other still follows the agreement stipulated in the cartel:

\[
Max_{w_i} \pi_{U_i} = w_i q_i(w_i, w_j) \quad s. t. \quad w_j = \frac{a}{2}, \quad i, j = 1, 2 \quad i \neq j. \tag{11}
\]

Substituting (4) and (6) into (11) leads to the following results:

\[
w_{iR}^D = \frac{a(4-\gamma-2\gamma^2)}{4(2-\gamma^2)}
\]

\[
\pi_{U_iR}^D = \frac{a^2(4-\gamma-2\gamma^2)^2}{64(2-3\gamma^2+\gamma^4)}
\tag{12}
\]

where the superscript \(D\) represents the deviation from collusion.

The following analysis relies on Friedman’s (1971) folk theorem based on grim trigger strategies (i.e., the perpetual Nash reversion after any unilateral deviation from the collusive path). Manufacturer \(i\) is faced with a trigger strategy: it chooses collusion (i.e., \(w_i = \frac{a}{2}\)) at period 1, and at the \(t^{th}\) \((t > 1)\) period still sticks to collusion if both manufacturers set wholesale price \(w_i = \frac{a}{2}\) in all the previous periods; otherwise, it uses the Nash equilibrium outcome (i.e., \(w_i = \frac{a(2-\gamma-\gamma^2)}{4-2\gamma^2}\)) as punishment in the \(t^{th}\) and all the subsequent periods. Let \(\delta\) be the discount factor of each manufacturer, which measures the manufacturers’ patience or how much importance they attach to the future.

Both manufacturers will then apply the trigger strategy for higher profits, i.e., they will
not deviate from the cartel and upstream collusion will inevitably appear if and only if the following inequality holds:

$$\frac{\pi_D - \pi_C}{\pi_D - \pi_N} \leq \frac{\pi_C}{\pi_D - \pi_N}$$  \hspace{1cm} (13)

where for upstream firms $\pi_D$: deviating profit; $\pi_C$: cooperative profit; $\pi_N$: Nash-punishment profit.

We then compare three payoffs (deviating, cooperative and punishment) in the model with delegation to those in the model without delegation, to see how delegation affects the cutoff $\frac{\pi_D - \pi_C}{\pi_D - \pi_N}$.

Solving Ineq. (13) yields the threshold $\delta_{RP}$ as follows.\(^6\) The upstream market will fall into collusion if and only if each manufacturer’s $\delta^*$ satisfies $\delta^* \geq \delta_{RP}$.

$$\delta_{RP} = \frac{(4-\gamma-2\gamma^2)^2}{32-16\gamma-31\gamma^2+8\gamma^3+8\gamma^4}$$  \hspace{1cm} (14)

$$\frac{d\delta_{RP}}{d\gamma} = \frac{4\gamma(2+\gamma)^2(4-\gamma-2\gamma^2)}{(32-16\gamma-31\gamma^2+8\gamma^3+8\gamma^4)^2} > 0$$  \hspace{1cm} (15)

When it comes to the pure-profit scheme, as shown in Bian et al. (2013), the critical $\delta_P$ is:

$$\delta_P = \frac{(4-\gamma)^2}{32-16\gamma+\gamma^2}$$  \hspace{1cm} (16)

$$\frac{d\delta_P}{d\gamma} = \frac{8(4-\gamma)\gamma}{(32-16\gamma+\gamma^2)^2} > 0$$  \hspace{1cm} (17)

where the subscript $P$ denotes the pure-profit scheme.

Furthermore, Bian et al. (2013) investigate downstream sales-revenue delegation, which is denoted by the subscript $SR$. They calculate the critical value $\delta_{SR}$ and find $\delta_{SR} > \delta_P$.

$$\delta_{SR} = \frac{(4-\gamma-\gamma^2)^2}{32-16\gamma-15\gamma^2+4\gamma^3+2\gamma^4}$$  \hspace{1cm} (18)

$$\frac{d\delta_{SR}}{d\gamma} = \frac{2\gamma(4+\gamma)^2(4-\gamma-\gamma^2)}{(32-16\gamma-15\gamma^2+4\gamma^3+2\gamma^4)^2} > 0$$  \hspace{1cm} (19)

We obtain three partial derivatives with respect of $\gamma$ in Ineq. (15), (17) and (19), and find that higher product differentiation will hinder upstream collusion. The same

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\(^5\) Similar to our setting, Charistos et al. (2022) consider a homogeneous Cournot duopoly with competing vertical chains, where each upstream firm has symmetric passive ownership over its downstream exclusive client. They show that passive forward ownership hinders upstream collusion.

\(^6\) Notice that $\lim_{\gamma \to 1} \delta_{RP} = 1$. 

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reasoning was provided by Deneckere (1983) for downstream firm’s collusion without any types of managerial delegation: sustaining collusion becomes more difficult with an increasing degree of substitutability.

By comparing (14), (16) and (18), upstream firms are more difficult to sustain collusion in relative-performance scheme than in downstream profit, and sales-revenue incentive schemes. (See Figure 1 for depiction).

We have the following Proposition 1.

**Proposition 1:** In a quantity competition market, the relative-performance delegation impedes upstream collusion, i.e., \( \delta_{RP}^C > \delta_P^C > \delta_{SR}^C \).

**Proof:**

\[
\delta_{RP}^C - \delta_{SR}^C = \frac{\gamma^4(\theta-2\gamma-3\gamma^2)}{(32-16\gamma-15\gamma^2+4\gamma^3+2\gamma^4)(32-16\gamma-31\gamma^2+8\gamma^3+8\gamma^4)} > 0.
\]

![Figure 1. The comparison of \( \delta^C \) between three schemes](image)

The main reasoning is that when the owners symmetrically adopt relative-performance delegation, the managers will behave more aggressively when the firms compete in quantities. Upstream firms will then have more incentive to deviate by lowering input prices. With an increase in the degree of product differentiation (the
products becomes more homogenous), sustaining upstream collusion becomes more difficult. Same as Miller and Pazgal (2001), relative-performance delegation intensifies downstream quantity competition and has an augmented quantity-enhancing effect, similar to Bian et al. (2013) and Wang and Wang (2021).

4. Upstream Collusion: Bertrand Competition

We then want to compare upstream collusive stability in Cournot and Bertrand model.\(^{7}\)

The demand function is expressed as follows:

\[
q_1 = \frac{(1-\gamma)a-p_1+\gamma p_2}{1-\gamma^2} \quad (20)
\]

\[
q_2 = \frac{(1-\gamma)a-p_2+\gamma p_1}{1-\gamma^2} \quad (21)
\]

In the third stage, each manager chooses price to maximize his reward:

From the FOC, we have

\[
p_1 = \frac{w_2(2+\gamma^2(1-\lambda_1))\lambda_2+\gamma w_2(1+\lambda_1)+a(2-\gamma-\gamma^2-(1-\gamma)\gamma \lambda_1)}{4-\gamma^2+\gamma^2(\lambda_1(1-\lambda_2)+\lambda_2)} \quad (22)
\]

\[
p_2 = \frac{w_2(2+\gamma^2\lambda_1(1-\lambda_2))\gamma w_2(1+\lambda_2)+a(2-\gamma-\gamma^2-(1-\gamma)\gamma \lambda_2)}{4-\gamma^2+\gamma^2(\lambda_1(1-\lambda_2)+\lambda_2)} \quad (23)
\]

In the second stage, owner \(i\) sets the incentive parameter simultaneously to maximize his own profit. Owner \(i\)’s maximization problem is given by:

\[
Max_{\lambda_i} \Pi_{i,RP}^B = (p_i - w_i)q_i \quad i,1,2 \quad (24)
\]

Solving the FOCs from (25) yields

\[
\lambda_i = \frac{\gamma(a-w_i)}{2w_j-a(2-\gamma)-\gamma w_j} \quad i,j = 1,2, i \neq j \quad (25)
\]

where the superscript \(B\) denotes Bertrand competition.

In the next part, we adopt the same process as in the Cournot model to find the results of three cases.

In the first case, we have

\[
w_1 = \frac{a(2-\gamma-\gamma^2)}{4-\gamma-2\gamma^2}
\]

\[
w_2 = \frac{a(2-\gamma-\gamma^2)}{4-\gamma-2\gamma^2}
\]

\(^{7}\) Wang and Wang (2021) show that the results in both Cournot and Bertrand models are sensitive to the introduction of partial collusion.
\[ w_{iR}^{BP} = \frac{\alpha(2-\gamma-\gamma^2)}{4-\gamma-2\gamma^2} \]  
\[ \pi_{U_{iR}}^{BP} = \frac{\alpha^2(1-\gamma)(2+\gamma)^2(2-\gamma^2)}{4(1+\gamma)(4-\gamma-2\gamma^2)^2} \]  

(26)

Note that (26) are identical with (8), we do not need to compute the second and the third cases since all the computations are exactly the same as that obtained under Cournot competition. We must point out that \( q_t(w_i, w_j) \) are the same under Cournot and Bertrand modes. So, Cournot and Bertrand yield the same results for all three cases. It is a direct consequence of Miller and Pazgal’s (2001) main result, namely, that managerial contracts based upon comparative performance yield the same profits irrespective of whether competition takes place in prices, quantities, or a mix thereof. This is not the case if delegation contract relies on a combination of profits and sale-revenues, as examined in Bian et al. (2013).

We then have Proposition 2.

**Proposition 2:** The relative-performance delegation in a downstream market impedes upstream collusion at the same degree regardless of the competition mode in downstream market.

Proposition 2 supports one important proposition put forth by Miller and Pazgal (2001, 2005) that if the owners are able to manipulate their managers’ behavior via relative-performance, the equilibrium outcome is the same regardless of the choice of quantity setting or price setting in a horizontal differentiated market. We further demonstrate that upstream firms’ collusion sustainability encounters the same obstacle regardless of the choice of quantity setting or price setting when downstream firms adopt the relative-performance delegation in an infinitely repeated Cournot or Bertrand game with trigger strategy punishment.

**5. Concluding Remarks**

This paper analyzes upstream firms’ collusion sustainability when downstream firms adopt the relative-performance delegation in an infinitely repeated game with trigger
strategy punishment. We demonstrate that the relative-performance delegation makes more difficult for upstream collusion comparing with sales-revenue delegation. We further point out that in a vertically related market with differentiated products, upstream firms’ collusion sustainability encounters the same obstacle regardless of the choice of competition modes when downstream firms adopt the relative-performance delegation.
Reference


