Market power and separating equilibrium in job market signaling

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Abstract

The job market signaling model in Spence (1973) deals with a situation of asymmetric information. Workers vary in their productivity. A worker is privately informed of his productivity but the firms are not informed. Spence (1973) shows that in competitive markets, costly education may signal productivity – workers of different productivities take up education to varying degrees, thereby resulting in a separating equilibrium where firms can infer a worker's productivity from his education choice. The importance of a separating equilibrium is that it resolves the informational asymmetry. In this paper, I enquire into the relationship between the firm's market power in the labour market (the market that is the source of asymmetric information) and the existence of separating equilibria. I show that a separating equilibrium exists, and therefore the informational asymmetry may be resolved, if and only if the market power of the firm in the labour market is not above a particular threshold.

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1 Introduction

Since the pioneering work of Akerlof (1973), it is well established that asymmetric information may cause in mis-allocation of resources thereby resulting in inefficient market outcomes. For instance, if the labour inputs (or workers) have differing productivities which is known privately to the worker but not to the employers (or the firms), then one may witness inefficient market outcomes whereby the low-productivity workers are employed by the firms when they should not be, or the high-productivity workers are not employed when they should be. The reason for this inefficiency is that the firms cannot distinguish between the different types of workers. In this context, Spence (1973) – the seminal paper on signaling – demonstrates that, in competitive markets, this may be partly remedied by the prospect of individuals signaling their productivity by the means of a costly observable signal, say education. If education is more costly for a high-productivity worker than a low-productivity worker, then high-productivity workers may obtain a higher level of education. This may then result in a separating equilibrium where the firms are able to separate the worker types by observing their education level.

The importance of a separating equilibrium is that it resolves the informational asymmetry. In view of this, this paper deals with the very basic issue of existence of separating equilibria in the above-mentioned context of workers with privately known and differing productivities on the one hand, and uninformed firms on the other hand. The question is, in the presence of costly and observable education, does the existence of a separating equilibrium depend on the market power of the firms in the labour market? Spence (1973) demonstrates the possibility of a separating equilibrium when the labour market is competitive. On the other hand, I show that if the firm is a monopsonist in the input market – company towns serve as an example of such a situation – then a separating equilibrium does not exist. Since perfect competition and monopsony represent two extremes of market power in the input market, I generalise and extend this to establish that a separating equilibrium exists if and only if a firm's market power is below a particular threshold.

The intuition behind this relationship is as follows. For simplicity, I consider the case where the workers are either employed by the firms, or they pursue an outside self-employment option, and the productivity of the high-type worker is higher than his self-employment income. This implies that it is efficient for this worker to be employed at the firm. In a separating equilibrium, different worker types choose different levels of education – hence, at least one worker type obtains a positive level of education. If a separating equilibrium exists when a firm is a monopsonist, then the firm, by the virtue of being able to separate the worker types by observing the education levels, drives down the wage of each type of worker to his outside option (i.e. the corresponding self-employment income level). However, education being costly, any worker type which chooses a positive level of education is better off by not obtaining education and pursuing self-employment instead. Thus, a separating equilibrium does not exist when a firm is a monopsonist.

On the other hand, it is possible to support a separating equilibrium when the firm's market power in the input market is sufficiently low. In these cases, a separating equilibrium is sustained by the fact that, in such an equilibrium, a wedge appears between the outside option (or, self-employment income) of a high-productivity worker and the wage offered to him, and the magnitude of this wedge is decreasing in a firm's market power. This provides the high-productivity worker the incentive to acquire a positive level of education in return for a wage that is higher than his outside option. If education is less costly for a high-productivity worker than a low-productivity worker, and a firm's market power is sufficiently low so that the size of the afore-mentioned wedge is sufficiently large, then a high-productivity worker would be willing to take up a sufficiently high level of education which a low-productivity worker would not find beneficial to mimic. This creates the conditions for the existence of a separating equilibrium.

The paper closest to the current one is Jeong (2019) which also studies existence of separating equilibria in the presence of imperfect market power. However, in Jeong (2019), the workers are differentiated along two dimensions: their productivity (as in the current paper), and their preference between firms (unlike the current paper). In contrast, the premise of the current paper that workers vary only in their productivity, and not along any other dimension, makes for a cleaner identification of the effect of market power on the existence and nature of separating equilibria in the canonical job market signaling model.

2 The Framework

A firm produces an output using a constant returns to scale technology. The firm is a price-taker in the output market, and the price of the output is normalised to unity. Labour is the only factor of production, and the firm is a monopsonist in the labour market. Labour input (or a worker) may be of two types. When employed at the firm, a worker has a productivity of θ_H with probability $\lambda \in (0, 1)$, and a productivity of θ_L with the complementary probability $1 - \lambda$, where $\theta_H > \theta_L > 0$. A worker either pursues self-employment, or works for the firm. The self-employment income of a worker with productivity θ_H (similarly, θ_L) is r_H (similarly, r_L), and this is common knowledge. I assume $r_H > r_L$, i.e. if a worker is more productive at the firm, then he also obtains a higher self-employment income. A worker knows his own productivty/type, and seeks to maximise his utility through his employment choice. In this section, the utility of a worker is equal to his income, and so, he chooses to work for the firm whenever the firm's wage is at least as much as the self-employment income.

I will now define two benchmark cases – the full information case, where the firm also knows about the productivity of the worker, and the asymmetric information case, where the firm only knows that a worker has high-productivity with probability λ.

The structure of the interaction between a worker and the firm is as follows. In the first stage, a worker's productivity is revealed to him by nature. In the second stage, the firm makes a wage offer to the worker with the objective of maximising expected profit. In the full information benchmark, the firm knows about the worker's productivity at the time of making the wage offer, while in the asymmetric information benchmark, it only knows about the probability with which the worker has high-productivity. In the third stage, the worker either accepts or rejects the firm's wage offer; in case of the latter, he pursues self-employment. If the worker accepts the wage, then the firm receives the corresponding profit (i.e. productivity less wage); otherwise, the firm receives zero profit.

I assume that $\theta_H > r_H$ so that it is efficient for a high-productivity worker to work for the firm. For simplicity of exposition, I also assume $\theta_L \geq r_L$, thereby implying that it is also efficient for the low-productivity worker to work for the firm. The results that I obtain are qualitatively same for the case $\theta_L < r_L$ where the efficient outcome is that only for the high-productivity worker to be employed at the firm.

It is obvious that in the full information scenario, the firm will offer a wage of r_H to a high-productivity worker, and r_L to the low-productivity worker. So, a worker, irrespective of his productivity, always accepts the firm's wage. Depending on whether the worker has high-productivity or low-productivity, the firm's profit is $\theta_H - r_H$ or $\theta_L - r_L$. Clearly, this allocation of workers is efficient.

In the asymmetric information situation, the firm cannot distinguish one type of worker from another. So, it is compelled to offer the same wage w to each type of worker – a necessary and sufficient condition for both types of worker to opt for employment at the firm is $w \geq r_H$. Hence, if the firm sets $w \geq r_H$, it obtains an expected profit of $[\lambda\,\theta_H + (1-\lambda)\theta_L] - w$. On the other hand, if the firm sets $w \in [r_L, r_H)$, it attracts only the low-productivity worker and obtains an expected profit of $(1-\lambda)(\theta_L - w)$. Finally, a wage lower than r_L yields zero profit for the firm.

Since the only uncertainty is about the worker's type, this game reduces a decision-problem under uncertainty for the firm where the firm's objective is to choose the wage to maximise expected profit. Now, a wage that is either lower than r_L, or in between r_L and r_H, yields a profit that is never higher than the profit obtained by setting $w = r_L$. Similarly, a wage that is higher than r_H yields a profit that is less than the profit obtained by setting $w = r_H$. It follows that the profit-maximising firm will either set $w = r_H$ or $w = r_L$. It is also easily verified that the firm sets $w = r_H$ if $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$ and $w = r_L$ otherwise. A high-productivity (low-productivity) worker accepts the wage if and only if it is at least as much as r_H (r_L). This describes the asymmetric information equilibrium.

It follows that the allocation of labour is efficient if and only if $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$. On the other hand, when $\lambda < \frac{r_H - r_L}{\theta_H - r_L}$, only a low-productivity worker works in the firm; the high-productivity worker engages in self-employment even though his productivity at the firm exceeds his self-employment income. I summarise the above in the next lemma.

Lemma 1. *The equilibrium of the asymmetric information benchmark when the firm is a monopsonist is as follows:*
(i) *the firm sets $w = r_H$ if $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$, and $w = r_L$ otherwise.*
(ii) *A high-productivity (similarly, low-productivity) worker accepts the firm's wage offer if and only if it is at least as much as r_H (similarly, r_L).*

3 Monopsony in the Labour Market and Signaling

Suppose it is possible for workers to obtain education, the quantity of which is assumed to be a continuous variable that is observable to all. Education does not have any impact on a worker's self-employment income or his productivity at the firm. The constant marginal cost of attaining education level e for the high-productivity worker and the low-productivity worker equals c_H and c_L, respectively, where $c_H < c_L$. The utility of a worker who obtains level of education e equals his income less the cost of education.

Now, with the possibility attaining education, the structure of the education signalling game is as follows. In the first stage, nature chooses the type of the worker. The probability that he has high-productivity is $\lambda \in (0,1)$. The worker is privately informed of his type, and he chooses the amount of education e which is observable to all. Next, in the second stage, on observing e, the firm forms its belief $\mu(e) \in [0,1]$ that the worker with education level e has high-productivity. The firm, taking its belief into consideration, then sets a wage schedule $w(e)$ with the objective of maximising profit. Finally, in the last stage, with the objective of maximising his own utility, the worker chooses either to accept the firm's offer, or be self-employed. This determines labour allocation and the market outcome.

I examine the game's *perfect Bayesian equilibria*, which are defined by three conditions:
(i) given the firm's wage schedule, the choice of education and employment maximises the utility of each type of worker,
(ii) the firm's belief is derived from the worker's choice of education by using Bayes' rule whenever possible, and,
(iii) given its own belief $\mu(e)$ and the worker's education strategy, the firm's wage schedule maximises its expected profit.

An equilibrium is said to be separating (pooling) if the two types of workers choose different levels (the same level) of education. Thus, in a separating equilibrium, the level of education fully reveals the type of a worker, while in a pooling equilibrium, the firm has no more information than in the asymmetric information benchmark.

The question I analyse is, does a separating equilibrium exist when the firm is a monopsonist in the input market?

In Proposition 1 – the formal statement and proof is in the appendix while the informal statement is presented below – I state that, when the firm is a monopsonist, the unique perfect Bayesian Nash equilibrium of this game is pooling in nature. This equilibrium is described as follows. Both a low-productivity worker and a high productivity worker obtain zero education in the first stage. Hence, in the second stage, on observing a worker with zero education, the firm's belief that the worker has high-productivity equals the ex-ante probability, i.e. $\mu(e) = \lambda$. As a result, this game becomes equivalent to the asymmetric information benchmark, due to which the perfect Bayesian equilibrium also coincides with the asymmetric information equilibrium of the previous sub-section.

Informal statement of Proposition 1. *The unique perfect Bayesian equilibrium of the signaling game with education, when the firm is a monopsonist in the labour market, is pooling in nature, and the description of the equilibrium is as follows:*
The worker's strategy. *A worker, irrespective of his productivity, chooses zero education. He accepts the firm's wage if it is at least as much as his self-employment income.*
The firm's wage offer. ***(i)*** *If $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$, then the firm offers a wage of r_H which is accepted by the worker irrespective of his productivity.*
(ii) *If $\lambda < \frac{r_H - r_L}{\theta_H - r_L}$, then the firm offers a wage of r_L which is accepted only by the low-*

productivity worker.

The intuition behind the equilibrium is as follows. In the third stage of the game, a worker chooses the firm over self-employment if and only if the utility from the former is at least as much as the utility from the latter. As a result, and as argued earlier, in the second stage, a profit-maximising monopsonist will either choose a wage of r_H or r_L. I will now reason that this implies no worker will attain a positive level of education in the first stage, thus implying the nonexistence of a separating equilibrium.

Firstly, a high-productivity worker can always obtain a utility of r_H by choosing zero education and self-employment. So, he will acquire a positive level of education only if he accepts the firm's wage offer, and he will, in turn, accept the firm's wage only if the net utility from doing so is at least as much as r_H. However, since the firm never offers a wage in excess of r_H, the high-productivity worker will never obtain a positive level of education.

Secondly, for the same reason, a low-productivity worker will never acquire education only to accept a wage less than or equal to r_L. Since r_H is the only wage in excess of r_L that is offered by the firm, it follows that if a worker acquires positive education, then it must be a low-productivity worker who does so in the hope of receiving the wage r_H. But, because, as argued above, a high-productivity worker never obtains education, the monopsonist will infer that a worker with positive education must have low-productivity. So, the wage corresponding to this positive level of education will be reduced to r_L thereby providing no incentive for a low-productivity worker to obtain education.

Finally, if both workers obtain zero education, then the game and its equilibrium become equivalent to the asymmetric information benchmark.

4 Imperfect Market Power in the Labour Market and Signaling

In this section, the firm possess some degree of market power in the input market without necessarily being a monopsonist. In order to tractably analyse whether education can now signal productivity, I use a reduced form approach to model the firm's exogenously given market power in the labour market by the means of a parameter $\gamma \in [0, 1]$.

Prior to explaining the manner in which γ reflects the firm's market power, I first define the surplus generated by allocating a worker to firm production rather than in self-employment as the difference between his productivity and self-employment income. So, if a high (similarly, low) productivity worker is employed at the firm, then the surplus equals

$\theta_H - r_H$ (similarly, $\theta_L - r_L$). When the firm has market power γ, then at least $(1-\gamma)$ share of the surplus accrues to the worker. Thus, if the firm's belief that a particular worker has high productivity is μ, then the firm's wage must be at least as much as $\mu[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu)[r_L + (1-\gamma)(\theta_L - r_L)]$.

Now, when $\gamma = 0$, and if the firm believes that a worker has high-productivity (similarly, low-productivity) with probability unity, then the worker receives a wage of θ_H (similarly, θ_L) that equals his productivity. Thus, this represents the situation where the firm does not have any market power in the labour market. On other hand, when $\gamma = 1$, and if the firm believes that a worker has high-productivity (similarly, low-productivity) with probability unity, then the worker receives a wage of r_H (similarly, r_L) that equals his outside option. This represents the situation where the firm is a monopsonist in the labour market. Furthermore, as γ increases from zero to unity, the wage received by the worker decreases. As a result, γ functions as a measure of the firm's market power in the labour market. Specifically, the firm's market power is increasing in γ, and this is reflected in the firm's wage being a monotonically decreasing function of γ.

Apart from the feature of imperfect market power, the structure of the signaling game remains the same. In the informal statement of Proposition 3 below (the formal statement and proof is in the appendix), I state that a perfect Bayesian *separating* equilibrium exists if and only if the firm's market power in the labour market (read, γ) is not too high.

Informal statement of Proposition 2. *In the signaling game where the firm has imperfect market power in the labour market, there exists a perfect Bayesian equilibrium that is separating in nature if and only if the firm's market power is below a threshold.*

The intuition behind this result is as follows. Recall that when the firm has complete market power, i.e. it is a monopsonist, and it is able to perfectly distinguish between the worker types (for instance, in a separating equilibrium), then the firm's wage offer to a worker is exactly equal to his income from self-employment. So, acquiring education only reduces the utility below what he would be able to obtain by choosing no education and self-employment. However, if a firm's market power is not perfect (i.e. $\gamma < 1$), then, whenever the firm is able to perfectly identify a worker with high-productivity, the wage offered to him exceeds his self-employment income. Now, if education is instrumental in the attainment of this higher wage (by enabling the firm to distinguish the high-productivity worker), then the high-productivity worker has an incentive to obtain education. In fact, the amount of education that makes him indifferent between the firm's wage in return for that amount of education on the one hand, and zero education along with self-employment income on the other hand, is decreasing in the firm's measure of market power γ.

At the same time, if the firm's market power is not perfect but still high enough, then a low-productivity worker is also willing to obtain highest amount of education that the high-productivity worker is willing to obtain for the higher wage. Thus, to deter this, the level of education that a high-productivity worker is willing to obtain has to be high enough; this is possible only if the corresponding wage is high enough, which, in turn, is possible only if the market power is below a threshold (i.e. γ is low enough).

5 Conclusion

I analyse the relationship between market power of a firm in the labour market and the existence of a separating equilibrium in the canonical job market signaling game in Spence (1973). The importance of a separating equilibrium is that starting from a situation of asymmetric information, it makes possible the resolution of the uncertainty arising out of this asymmetry. In the specific case of job market signaling, in a separating equilibrium, workers of differing productivities are able to separate themselves by differential use of the signal (in this case, education). In this context, I show that the degree of the firm's market power in the labour market – the market that is the source of the informational asymmetry – has a bearing on the existence of a separating equilibrium. Specifically, a separating equilibrium exists if and only if the firm's market power is not too high.

Appendix

Proposition 1. *When workers may attain education to signal productivity, and the firm is a monopsonist in the labour market, then the unique perfect Bayesian equilibrium of the signalling game is a pooling equilibrium. In this equilibrium:*

(i) *each type of worker chooses zero education in the first stage, and, in the last stage, a high-productivity (low-productivity) worker accepts the firm's wage if it is at least equal to* r_H (r_L)*, and rejects it otherwise.*

(ii) *the firm's belief and wage schedule is as follows:*

- **(a)** *for* $e = 0$: $\mu(0) = \lambda$; *if* $\mu(e) = \lambda < \frac{r_H - r_L}{\theta_H - r_L}$ *then* $w(0) = r_L$, *and* $w(0) = r_H$ *otherwise,*
- **(b)** *for any* $e \in (0, \frac{r_H - r_L}{c_L})$: $\mu(e) \in [0, \frac{r_H - r_L}{\theta_H - r_L}]$; $w(e) = r_L$,
- **(c)** *for any* $e \geq \frac{r_H - r_L}{c_L}$: $\mu(e) \in [0, 1]$; *if* $\mu(e) < \frac{r_H - r_L}{\theta_H - r_L}$ *then* $w(e) = r_L$, *and* $w(e) = r_H$ *otherwise.*

Proof. In Step 1, and Step 2, I rule out the existence of a separating equilibrium, and a pooling equilibrium with positive level of education, respectively; in Step 3, I prove that the strategies described in the proposition constitute the unique perfect Bayesian equilibrium.

Step 1. I will establish the non-existence of a separating equilibrium through a proof by contradiction. So, suppose there exists a separating equilibrium where a high-productivity worker and a low-productivity worker attain education level e_H and e_L, respectively, with $e_H \neq e_L$, and the belief of a firm is $\mu(e_H) = 1$ and $\mu(e_L) = 0$. Since $e_H \neq e_L$, it must be that $e_H > 0$ or $e_L > 0$ (or both).

Now, in the last stage of the game, a high-productivity worker (similarly, low-productivity) worker will accept the firm's wage offer if and only if it is at least as much as r_H (similarly, r_L). The firm will then set $w(e_H) = r_H$ and $w(e_L) = r_L$. In a separating equilibrium, this wage of $w(e_H) = r_H$ and $w(e_L) = r_L$ is accepted by a high-productivity worker and a low-productivity worker who then obtain a utility of $r_H - c_H e_H$ and $r_L - c_L e_L$, respectively. I will establish a contradiction by showing that a worker who attains a positive level of education in the first stage has a utility improving deviation.

Now, in a separating equilibrium, it must be that $e_H > 0$ or $e_L > 0$. If $e_H > 0$ (similarly, $e_L > 0$), then a high-productivity (similarly, low-productivity) worker can increase his utility by choosing zero education. This reason is that – irrespective of $\mu(0)$ and $w(0)$ – he can always obtain a utility of r_H (similarly, r_L) by choosing zero education and self-employment. Hence, this represents a utility-improving deviation for any worker type who attains a positive level of education. Thus, a separating equilibrium does not exist.

Step 2. I will argue by contradiction that there cannot exist a pooling equilibrium where both types of workers choose the same positive level of education $e > 0$. So, suppose such a pooling equilibrium exists. In any such equilibrium, in the last stage of the game, a high-productivity (low-productivity) worker accepts the firm's wage if it is at least as much as r_H (r_L). In the second stage of the game, the firm cannot distinguish the type of the worker; so, it must offer a uniform wage w. Profit-maximisation implies, as in Lemma 1, that the firm's wage offer is either $w = r_H$ when $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$, or $w = r_L$ when $\lambda < \frac{r_H - r_L}{\theta_H - r_L}$.

When $w = r_H$ (similarly, $w = r_L$), a high-productivity worker, who receives a utility equal to $r_H - c_H e$ (similarly, $r_L - c_L e$), has a utility-improving deviation; as in Step 1, choosing zero education and self-employment gives him a higher utility of r_H (similarly, r_L). So, there exists a type of worker who increases his utility by deviating from the education level $e > 0$. Thus, a pooling equilibrium with positive level of education cannot exist.

Step 3. I will prove that the strategies mentioned in the proposition constitute the unique perfect Bayesian equilibrium by showing that: firstly, given the firm's wage schedule, the worker does not have any other strategy that would increase his utility; secondly, given the worker's strategy, the firm's beliefs are derived from worker's strategy using Bayes' rule whenever possible, and the firm's wage schedule maximises its expected profit.

Optimality of the worker's strategy. It is obvious that in the last stage, a high-productivity (low-productivity) worker's strategy of accepting the firm's wage if it is at least equal to r_H (r_L) is optimal. So, I focus on the choice of education in the first stage.

Now, the highest wage offered by the firm is equal to r_H. So, if a high-productivity worker obtains positive level of education, his net utility is less than r_H. On the other hand, if the worker chooses zero education, then he can always guarantee himself a utility of r_H by pursuing self-employment. Hence, given the firm's wage schedule, choosing zero education maximises his utility.

Similarly, a low-productivity worker can always obtain a utility of r_L by choosing zero education and self-employment. So, in view of the firm's wage schedule, a positive level of education *may* may improve utility but only if he is able to receive a wage of r_H. If he chooses $e \in (0, \frac{r_H - r_L}{c_L})$, then $w(e) = r_L$; so, this choice of education does not improve utility. Here, it is also easily verified that if, for any $e \in (0, \frac{r_H - r_L}{c_L})$, $w(e) = r_H$ were to hold instead, then a low-productivity worker could increase his utility by choosing education e and obtaining wage r_H in return. Hence, $w(e) = r_L$ for all $e \in (0, \frac{r_H - r_L}{c_L})$ is both necessary and sufficient to eliminate a utility-improving deviation with an education level in this interval. Next, if a low-productivity worker chooses $e \geq \frac{r_H - r_L}{c_L}$, then corresponding utility $r_H - c_L\, e$ never exceeds r_L. Thus, this does not improve utility either. Hence, given the firm's wage schedule, choosing zero education maximises his utility.

The firm's belief and wage schedule. Firstly, if both a low productivity worker and a high productivity worker choose zero education, then $\mu(0) = \lambda$ follows from Bayes' rule. Hence, as in Lemma 1, when the firm observes zero education, it maximises expected profit by setting $w(0) = r_H$ when $\lambda \geq \frac{r_H - r_L}{\theta_H - r_L}$, and $w(0) = r_L$ when $\lambda < \frac{r_H - r_L}{\theta_H - r_L}$.

Secondly, consider education level $e \in (0, \frac{r_H - r_L}{c_L})$. According to the worker's strategy, a level of education in this interval is never chosen. So, $\mu(e) \in [0, \frac{r_H - r_L}{\theta_H - r_L}]$ is consistent with Bayes' rule. A profit-maximising monopsonist will set the wage equal to either r_H or r_L – as explained earlier, this is because the profit from any other wage never dominates the profit obtained by setting wage equal to either r_H or r_L. The expected profit from setting wage equal to r_H and r_L is $\mu(e)(\theta_H - r_H) + (1 - \mu(e))(\theta_L - r_H)$ and $(1 - \mu(e))(\theta_L - r_L)$, respectively. It can be verified that $w(e) = r_L$ maximises expected profit whenever $\mu(e) < \frac{r_H - r_L}{\theta_H - r_L}$.

Finally, if the firm observes education $e \geq \frac{r_H - r_L}{c_L}$, then $\mu(e) \in [0,1]$ is consistent with Bayes' rule, and the profit-maximising wage corresponding to each value of $\mu(e)$ in the interval $[0,1]$ follows from the preceding discussion. ■

Proposition 2. *Suppose that $c_L(\theta_H - r_H) - c_H(\theta_H - \theta_L) > 0$. Then, there exists $\gamma^* \in (0,1)$ such that a separating equilibrium exists if and only if $\gamma \leq \gamma^*$. In any such equilibrium:*
(i) *a low-productivity worker chooses education $e_L = 0$ in the first stage*
(ii) *the high-productivity worker chooses education level e_H in the first stage, where:*
$e_H \in [\frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_L}, min\{\frac{(1-\gamma)(\theta_H - r_H)}{c_H}, \frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_H}\}]$
(iii) *the firm's belief is:*
(a) $\mu(0) = 0$ *and* $\mu(e_H) = 1$
(b) *for any $e \in (0, e_H)$:*
$\mu(e) \in [\frac{r_H - r_L - (1-\gamma)(\theta_L - r_L)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}, min\{1 - \frac{c_H(e_H - e)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}, \frac{c_L e}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}\}]$
or $\mu(e) = 0$
(d) *for $e > e_H$:* $\mu(e) = 0$ *or* $\mu(e) \in [\frac{r_H - r_L - (1-\gamma)(\theta_L - r_L)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}, 1]$
(iv) *the wage schedule is $w(e) = \mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)]$.*

Proof. The wage schedule of the firm follows directly from the manner in which γ reflects the firm's market power. In view of this, I will first show that, given the firm's wage schedule, the strategy of the worker maximises his utility; next, I prove that, given the worker's strategy, the firm's belief is derived from Bayes' rule whenever possible. However, prior to showing this, I explain that the if and only if part of the proposition that involves γ^* comes from the fact that the level of education e_H attained by a high-productivity worker is such that $e_H \in [\frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_L}, min\{\frac{(1-\gamma)(\theta_H - r_H)}{c_H}, \frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_H}\}]$. Now, since $c_H < c_L$ implies $\frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_L} < \frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_H}$, such an education level exists if and only if $\frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_L} \leq \frac{(1-\gamma)(\theta_H - r_H)}{c_H}$. Under the assumption $c_L(\theta_H - r_H) - c_H(\theta_H - \theta_L) > 0$, this is equivalent to $\gamma \leq \gamma^* \equiv \frac{c_L(\theta_H - r_H) - c_H(\theta_H - \theta_L)}{c_L(\theta_H - r_H) - c_H(\theta_H - \theta_L) + c_H(r_H - r_L)} \in (0,1)$.

Optimality of the worker's strategy. The optimality of the third period strategy of a high-productivity (low-productivity) worker accepting the firm's wage if it is at least equal to r_H (r_L) is obvious. Hence, I focus on the choice of education in the first stage.

The high-productivity worker. If he attains e_H level of education, then, according to the firm's wage schedule, he obtains the wage $w(e_H) = r_H + (1-\gamma)(\theta_H - r_H)$. Since this is at least as much as his self-employment income r_H, he accepts the wage and obtains a corresponding utility of $r_H + (1-\gamma)(\theta_H - r_H) - c_H e_H$. I will now show that, given the firm's wage schedule, he cannot obtain a utility higher than $r_H + (1-\gamma)(\theta_H - r_H) - c_H e_H$:

(a) Suppose he deviates by choosing an education level $e \in [0, e_H)$ such that $\mu(e) = 0$.

Then, the firm offers $w(e) = r_L + (1-\gamma)(\theta_L - r_L)$.

Firstly, if $w(e) < r_H$, then the high-productivity worker chooses self-employment and obtains a utility of $r_H - c_H e$. However, $e_H \leq \frac{(1-\gamma)(\theta_H - r_H)}{c_H} \Leftrightarrow r_H + (1-\gamma)(\theta_H - r_H) - c_H e_H \geq r_H \geq r_H - c_H e$. So, this is not a utility improving deviation.

Secondly, if $w(e) \geq r_H$, then he chooses the firm's wage, and obtains a utility of $r_L + (1-\gamma)(\theta_L - r_L) - c_H e$. Now, $e_H \leq \frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_H} \Leftrightarrow r_H + (1-\gamma)(\theta_H - r_H) - c_H e_H \geq r_L + (1-\gamma)(\theta_L - r_L) \geq r_L + (1-\gamma)(\theta_L - r_L) - c_H e$. Hence, this is not a utility improving deviation either.

It follows that if $e_H \leq min\{\frac{r_H - r_L + (1-\gamma)(\theta_H - r_H + r_L - \theta_L)}{c_H}, \frac{(1-\gamma)(\theta_H - r_H)}{c_H}\}$, then choosing education level $e < e_H$ such that $\mu(e) = 0$ is not a utility improving deviation.

(b) Suppose that the high-productivity worker deviates by choosing education $e \in (0, e_H)$ such that $\mu(e) > 0$. Part (a) above establishes that choosing zero education along with self-employment is not a utility improving deviation; so, choosing positive level of education along with self-employment cannot be a utility improving deviation. So, I only consider the case where the worker attains education $e \in (0, e_H)$ and accepts the corresponding wage $w(e) = \mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)]$. In this case, the utility is $\mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)] - c_H\, e$.

Now, when $e \in (0, e_H)$, the firm's belief is $\mu(e) \leq 1 - \frac{c_H(e_H - e)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}$. This inequality is equivalent to $\mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)] - c_H e \leq r_H + (1-\gamma)(\theta_H - r_H) - c_H e_H$. So, this is not a utility improving deviation.

(c) Suppose he deviates by choosing education $e > e_H$. Now, the highest wage that the worker can obtain is $r_H + (1-\gamma)(\theta_H - r_H)$. However, he obtains exactly this wage with a lower education level e_H. Hence, choosing $e > e_H$ is not a utility-improving deviation.

The low-productivity worker. If a low-productivity worker chooses $e_L = 0$, he obtains a utility of $r_L + (1-\gamma)(\theta_L - r_L)$ I will show that, given the firm's wage schedule, he cannot obtain a higher utility.

(d) If he chooses e_H, then he obtains a wage of $r_H + (1-\gamma)(\theta_H - r_H)$ and a utility of $r_H + (1-\gamma)(\theta_H - r_H) - c_L e_H$. Now, $e_H \geq \frac{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}{c_L}$ implies $r_H + (1-\gamma)(\theta_H - r_H) - c_L e_H \leq r_L + (1-\gamma)(\theta_L - r_L)$. Hence, choosing e_H is not a utility improving deviation for a low-productivity worker. This, in addition with the fact that $r_H + (1-\gamma)(\theta_H - r_H)$ is the highest wage offered by the firm, implies that attaining an even higher level of education $e > e_H$ is not a utility-improving deviation either.

(e) If the worker chooses $e \in (0, e_H)$ such that $\mu(e) = 0$, then the wage remains the same – so, this cannot represent a utility-improving deviation. Next, suppose he chooses

$e \in (0, e_H)$ such that $\mu(e) > 0$. Now, $w(e) = \mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)]$, with corresponding utility $\mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)] - c_L e$. In this case, the belief of the firm is $\mu(e) \leq \frac{c_L e}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}$, and this inequality is equivalent to $\mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)] - c_L e \leq r_L + (1-\gamma)(\theta_L - r_L)$. Hence, this is not a utility improving deviation either.

The firm's belief. Firstly, if a low productivity worker and a high productivity worker choose education level e_L and e_H, with $e_L \neq e_H$, then $\mu(e_L) = 0$ and $\mu(e_H) = 1$ is consistent with Bayes' rule.

Secondly, consider a level of education $e \in (0, e_H)$. The belief $\mu(e) = 0$ is consistent with Bayes' rule. At the same time, a belief $\mu(e) > 0$ is also consistent with Bayes' rule, and it may also be feasible for these levels of education, but only if the corresponding wage $w(e)$ is not lower than r_H. For if, on the contrary, $w(e) < r_H$ holds, then $w(e)$ will not be accepted by a high-productivity worker in the last stage of the game, in which case, for consistency with Bayes' rule, $\mu(e) > 0$ cannot hold. Thus, if $\mu(e) > 0$, then $w(e) \geq r_H \equiv \mu(e)[r_H + (1-\gamma)(\theta_H - r_H)] + (1-\mu(e))[r_L + (1-\gamma)(\theta_L - r_L)] \geq r_H$ must hold. That is, $\mu(e) \geq \frac{r_H - r_L - (1-\gamma)(\theta_L - r_L)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}$. In addition, I have shown in part (b) and part (e) above that when $e \in (0, e_H)$, then $\mu(e) \leq min\{(1 - \frac{c_H(e_H - e)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}), (\frac{c_L e}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)})\}]$ must hold. Putting these together gives the beliefs stated in the proposition.

Finally, consider an education level $e > e_H$. The belief $\mu(e) = 0$ is consistent with Bayes' rule. In addition, as explained above, for $\mu(e) > 0$ to be a feasible belief, the inequality $w(e) \geq r_H$ must also hold. This implies that, for education levels $e > e_H$, $\mu(e) = 0$ or $\mu(e) \geq \frac{r_H - r_L - (1-\gamma)(\theta_L - r_L)}{\gamma(r_H - r_L) + (1-\gamma)(\theta_H - \theta_L)}$ holds. ■

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