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Abstract

In this study, we present a survey of (non-Walrasian) disequilibrium economics in which the gap between expressed demand and supply and between desired and realized transaction are allowed. We see a breif history of the disequilibrium theory and characteristics of it such as temorary equilibria with quantity adjustment and the discontinuity of dynamics due to regime switching. We redefine the disequilibrium economics by comparing with equilibrium economics, and find that the core of it is inconsistency of transaction that is emphasized as "dual-decision" by Robert Clower.

1 Introduction

Equilibrium has been a central concept in economic analysis, as Dixon (1990) points out. The Walrasian general equilibrium model, which was modernized in Debreu (1959) and Arrow and Hahn (1971), is not only a core of microeconomics in textbooks but also a base of current macroeconomic models, such as the Dynamic (Stochastic) General Equilibrium models. In model analyses the term "market-clearing" condition is often used, to simplify the descriptions of the economic system, efficiency problem, and consistency.

Disequilibrium economics is a term used these days by some macroeconomics researchers (see Mankiw and Weinzierl (2011), Michaillat and Saez (2015, 2019), Schoder (2017, 2020), van Aarle (2017), Dupor et al. (2019), and Eggertsson et al. (2019)). These studies address macroeconomic problems, such as involuntary unemployment and secular stagnation, and they sometimes utilize the rationing scheme constructed by disequilibrium economics. As Backhouse and Boianovsky (2012) argues, disequilibrium research has

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not been active for a long time. This implies that economic research that exploits disequilibrium economics tools may mislead readers about its essential structure, causing future research to suffer.

This study investigates disequilibrium economics to evaluate its potential. We discover that the spillover effect between markets is a core of disequilibrium economics and plays a significant role in economic problems. However, in the studies mentioned above, this issue is often overlooked and not properly addressed. Rationing, however, is frequently regarded as a feature of disequilibrium economics. Furthermore, rigid price variables such as wage and interest rates are commonly used to explain unemployment. One important spillover effect in a disequilibrium economy, which is emphasized in Patinkin (1956), is the effect of goods demand on labor demand. In short, a firm's intention to employ in disequilibrium is influenced by the perceived quantity constraint on sales. In disequilibrium economics, this is known as Keynesian unemployment, and it differs from other types of unemployment by rigidity. Therefore, it might be better to examine a brief history of disequilibrium economics and define the theoretical aspect of the disequilibrium situation. We find that the spillover effect of quantity signals when compared to wage rigidity and ZLB economics, can result in substantial and long-term unemployment.

The remainder of this paper is organized as follows. Section 2 gives a brief history of disequilibrium economics (also known as non-Walrasian or neo-Keynesian economics). The birth of the disequilibrium theory was influenced by Keynes's economic views, although the theme was wider. In Section 3, we discuss several properties of the disequilibrium theories. The analytical tools become interesting but challenging to employ as we deal with excess demand or supply situations. Why has disequilibrium economics been neglected for so long? We discuss the inherent difficulties and unsolved problems in Section 4. In Section 5, we redefine the disequilibrium model and compare it with the rigidity models. We summarize our research findings in Section 6.

2 The dawn of disequilibrium economics

This section provides a quick overview of disequilibrium economics from Keynes until the 1970s reevaluation of Keynes. See Cuddington et al. (1984, Chapter 3) and Backhouse and Boianovsky (2012) for additional information about the history.

2.1 After Keynes

Keynes's *General Theory* (Keynes, 1936) is a seminal work in disequilibrium economics. Using a unique consumption function, liquidity preference, and the effective demand principle, he demonstrated that involuntary unemployment can readily persist in the current economy. After publication, the book was interpreted using mathematical models. "Keynesian" economists, such as Hicks (1937) and Modigliani (1944), proposed the IS-LM model, in which the Keynesian situation could be interpreted as an extended classical model with sluggish price adjustment. This neoclassical synthesis has presented many tractable models, but it has been criticized by both (neo-)classical economists and (post-) Keynesian economists; for its lack of consistency and under evaluating the Keynesian situation.[1]

Patinkin (1956) was also able to merge Walrasian economics with Keynes's theory, and he discovered that Keynes's involuntary unemployment could be shown as the situation in which the employment and the labor supply detach; in other words, the labor market is in disequilibrium. Patinkin specified the spillover effect from goods to labor demand, which has been the core of disequilibrium economics.

In his seminal work (Clower, 1965), Robert Clower, who explored technical issues in dynamics, such as Clower and Bushaw (1954), also demonstrated that Keynes's model is illustrated by market disequilibrium. He presented a simple model in which transactions occur even if the set of prices does not reach an equilibrium level. Because the demand and supply quantities of markets could differ in terms of current prices, the realized transaction quantity is different on at least one side; therefore, demand or supply is rationed. Although the demand-supply gap model was not new, he introduced the dual-decision hypothesis, which is a key concept in disequilibrium economics. This hypothesis states that individuals who face rationing (the gap between the planned transaction and the realized transaction) will reconsider their other demands and supplies so that the individual revises their decision. The revised demand and supply are called *effective*, as opposed to *notional*, which is derived from a usual decision problem, such as utility maximization and profit maximization. Clower demonstrated that consumer demand is a function of the realized transaction (realized income) implying that the Keynesian consumption demand function is derived from a microeconomic optimization problem. Clower's novel idea immediately spread in the 1970s macroeconomics, with strong support from the book Leijonhufvud (1968). Leijonhufvud argued that Keynes' economics should be interpreted

[1] Leijonhufvud (1967) criticized "Keynesian" economics since it views Keynes's economics as a special case.

as an economy in which the Walrasian auctioneer is absent. In the absence of an auctioneer, a transaction must be executed even if the prices are not in equilibrium. He argued that Keynes assumed that the speed of quantity adjustment was higher than that of price adjustment.

2.2 Disequilibrium macroeconomics in the 1970s

Macroeconomists have developed several mathematical models that tackle quantity rationings after Clower and Leijonhufvud's basic works for disequilibrium interpretations of Keynes's *General Theory*. Most seem to rely on Leijonhufvud's quantity adjustment theory; prices are fixed in the short term or a static model, and they gradually change in dynamics.

The first model was Solow and Stiglitz (1968), which described labor market dynamics. Solow and Stiglitz introduced a simple rationing scheme, $Y = \min\{Y^d, Y^s\}$, where Y represents output, superscript d is the demand, and s is the supply. They demonstrated that under gradual real wage adjustment, which depends on excess demand for goods and labor, there is a possibility of persistent (involuntary) underemployment.

Barro and Grossman (1971) presented the "general" disequilibrium model, in which the spillover effects between the goods and labor markets are fully described. They synthesized Patinkin's labor market model and Clower's goods market model. Let us now describe the model's abstract. Let x_i denote the realized transactions of good i and let P be the price vector. The usual Walrasian (equilibrium) model explores the following situation:

$$x_i = x_i^{d*}(P^*) = x_i^{s*}(P^*), \ \forall i. \tag{1}$$

Demand and supply are functions of P (and other parameters), and transactions occur at the equilibrium price P^*. This is because the prices are adjusted to the equilibrium level before the actual trade occurs, and individuals are supposed to know it. The notional demand and supply x^{d*}, x^{s*} are derived from the usual optimization problems without any transaction rationing. Next, we considered a simple disequilibrium model. The transaction occurs under the short-side rule:

$$x_i = \min\{\tilde{x}_i^d(P, X), \tilde{x}_i^s(P, X)\}, \ \forall i, \tag{2}$$

where P is the prevailing price vector, $X = \{x_i\}_i$ is the set of realized transactions, and˜(tilde) indicates effective demand and supply. The dual-decision hypothesis emerges as X in each demand and supply; that is, the *realized* transactions affect demand and supply.

Barro and Grossman (1971) organized and compared the notional and the effective demand and supply by presenting two disequilibrium cases: excess supplies in labor and goods markets and excess demands in both. These disequilibrium regimes were sorted and labeled by Malinvaud (1977), and the names have been used in subsequent disequilibrium studies. For example:

Keynesian unemployment (KU) : Excess supplies exist in both labor and goods markets so that involuntary unemployment occurs due to the shortage of goods demand.

Classical unemployment (CU) : The labor market is in excess supply but the goods market is in excess demand, creating involuntary unemployment even though the demand of the goods is sufficient.

Repressed inflation (RI) : Excess demands exist in both labor and goods markets and therefore the price and the wage would rise during the adjustment process.

Underconsumption (UC) The labor market is in excess demand but the goods market is in excess supply, which means that the households restrict their consumption and labor supply so that the activity of the economy is restricted.

For the case in which there is no demand-supply gap, **equilibrium** or **Walrasian equilibrium (WE)** is labeled.

Bénassy (1975b) extended Barro-Grossman's interpretation of Patinkin-Clower's disequilibrium models, with the descriptions of the microeconomic view of transactions; the monetary exchange, and the rationing schemes.[2] He presented systematic tools to describe the dual-decision hypothesis and demonstrated the existence of the Keynesian equilibrium (K-equilibrium), which is defined by *effective* demands.

One characteristic of Bénassy's K-equilibrium is the specification of the perception of quantity constraint. Let's start with a certain set of expressed excess demand where the goods are indexed with i, $\{\tilde{z}_{ih}\}_{i,h}$, where h is the individual index. The rationing scheme is a set of correspondences $\{G_{ih}\}_{i,h}$. For simplicity, he assumes that the rationing scheme is described as a function: $\bar{\tilde{z}}_{ih} = G_{ih}(\{\tilde{z}_i\}_i)$ is the quantity constraint on the trade of goods i of individual h. Returning to the excess demand expressions, the effective demand of individual h is defined which is the set $\{\tilde{z}_{ih}\}_i$ which is an optimum under the

[2]His paper on monetary exchange was published in the same year (Bénassy, 1975a). He was interested in Barro-Grossman's general disequilibrium model as well as Clower's monetary theory (Clower, 1967); see Section 3.3 for money in disequilibrium.

budget constraint and affected by the perceived quantity constraints $\{\bar{\bar{z}}_{ih}\}_i$. To emphasize this dual decision, Bénassy's effective demand concept focuses only on the spillover effect. That is, the individual solves the optimization problem of every good repeatedly, in which the budget constraint for good h is not affected by $\bar{\bar{z}}_{ih}$.[3] Finally, the realized transactions are described as a set $\{\bar{z}_{ih}\}_{i,h}$, which is determined by the rationing scheme $\bar{z}_{ih} = F_{ih}(\{\tilde{z}_i\}_i)$, where F_{ih} is the rationing function.

Bénassy defined K-equilibrium as a set of three types of excess demand vectors $\{\tilde{z}_{ih}, \bar{z}_{ih}, \bar{\bar{z}}_{ih}\}_{ih}$, which satisfies (1) $\bar{\bar{z}}_{ih} = G_{ih}(\{\tilde{z}_i\}_i)$; (2) $\tilde{z}_{ih}$ is optimal under the quantity constraint $\{\bar{\bar{z}}_{ih}\}_i$; and (3) $\bar{z}_{ih} = F_{ih}(\{\tilde{z}_i\}_i)$ for every i, It is worth noting that the first two conditions say that K-equilibrium is an equilibrium of perception of quantity rationing and dual-decision under constraint; we should interpret the transaction as the equilibrium point of "quantity-tâtonnement." The realized quantity itself seems independent of the tâtonnement process, although the perceived quantity constraints are often equated with the realized rationing ($F_{ih} = G_{ih}$) in sequential works.

For macroeconomists, it is important to know how the disequilibrium regime (in particular, KU) originates and how persistent it is. They used comparative statics to formulate the relationships between price variables, stock variables, and regimes. Korliras (1975) extended Patinkin's employment model and explored how the two price variables (real wage and interest rate) affect involuntary unemployment in the short term. Barro and Grossman (1976) modified their model in 1971 by introducing dynamic optimization. Malinvaud (1977, 1980) developed simple models and emphasized the persistence of the KU regime. Muellbauer and Portes (1978) presented a graphical interpretation of the four disequilibrium regimes using offer curves which are derived from optimization problems of households and firms.[4]

[3]The Bénassy's effective excess demand for good i of household h is described as follows:

$$\max_{z_{ih}} U_h(\omega_h + z_h, M_h) \text{ subject to } p \cdot z_h + M_h \leq M_{h0},$$
$$\omega_h + z_h \geq 0, M_h \geq 0,$$
$$|z_{i'h}| \leq |\bar{\bar{z}}_{i'h}|, \text{ where } i' \neq i,$$

where U_h is h's utility function, M_h is the money holding, M_{h0} is the initial money holding, and ω_h is the initial allocation vector.

[4]Stoneman (1979) also provides a graphical interpretation of disequilibrium model.

2.3 Towards general disequilibrium model of microeconomics

Although Keynes's macroeconomics influenced disequilibrium economics, some researchers have explored the microeconomic features of disequilibrium as an extension of the general equilibrium model.

This representative work is called the fixed-price equilibrium model in Drèze (1975), which was written as a draft in 1971 (Backhouse and Boianovsky, 2012, p.111). He explored the world with rigid prices and quantity constraints and found the existence of trade or the Drèze equilibrium.[5] His work was extended by Grandmont and Laroque (1976) as work on Keynesian equilibrium, but Drèze himself initially treated his work as one in general equilibrium economics (Backhouse and Boianovsky, 2012, p.111).

Although the Drèze equilibrium is well "micro-founded" in that the descriptions about preferences and choices are consistent with the usual general equilibrium model, it lacked an important aspect of disequilibrium economics: the spill-over effect with quantity signals. Since the quantity constraints are fully perceived before the agents decide their demands and supplies, no one can notice the demand-supply gap in the realized trade. That is, no one is surprised by rationing. However, Bénassy's K-equilibrium concept lacks consistency. The agents repeatedly solve optimization problems for each good, and the expressed demands and supplies can violate their budget constraints.[6]

Some researchers introduced a stochastic rationing scheme for Keynesian macroeconomics, to overcome Bénassy's inconsistency and Drèze's intractability,(see Gale (1979), Green (1980), and Svensson (1980)). Although this concept overcomes the microeconomic consistency problem of rationing models, stochastic rationing is not used in the macroeconomic analysis; the exceptions are Ioannides (1983) and Honkapohja and Ito (1985).

[5]The constraints could be summarized as $(x, P) \in B$, where x is the set of excess demands, P is the price vector, and B is some constraint set. For instance, Drèze (1975) used an inequality constraint on prices, such as downward rigidity and quantity constraint for individual i, as the set of ranges: $x_i \in \prod_k [l_{ki}, L_{ki}]$, where k is a good index. Although Drèze allowed a slight change in prices, Younès (1975) reinterpreted it as a p-equilibrium, which is a fixed-price equilibrium.

[6]See Svensson (1980) for an example of this violation. For comparative studies on equilibria with quantity rationings, see Grandmont (1977, 1982) and Silvestre (1982, 1983).

3 Prosperity of disequilibrium economics: its characteristics

Based on disequilibrium theories in the early days, the disequilibrium model became a tool for economic analysis (Backhouse and Boianovsky, 2012, p.79) in the mid-1970s; for example, Dixit (1978) and Cuddington et al. (1984) used it for international trade. In this section, we discuss several characteristic issues in disequilibrium economics.

3.1 Dynamical characteristic of disequilibrium

As previously stated, the persistence of disequilibrium, particularly, the Keynesian unemployment regime, is one of the most important issues in disequilibrium macroeconomics. Although the early models such as Solow and Stiglitz (1968), Barro and Grossman (1976) and Muellbauer and Portes (1978) examined dynamic properties, it was Ito (1979) who presented the use of "discontinuous-righthand-side" differential equation technique that marked a turning point.[7][8]

The most characteristic dynamic property of the disequilibrium model is regime-switching. Depending on the demand-supply gaps in the goods and labor markets, the usual disequilibrium macroeconomic model has three or four regimes. The realized transaction is described using different equations, and discontinuity emerges in the dynamic equations. Take, for example, the dynamic equations in Malinvaud (1977), for which Honkapohja and Ito (1983, Section 4) provides the complete mathematical explanation.

There are three disequilibrium regimes (KU, CU, and RI), and WE are located at their center. The locations are determined by a combination of the price and wage (P, W). The adjustment of price and wage is determined by excess demand in the goods and labor markets, respectively. Therefore, the differential equations vary when the regime changes. Let R_x denote the

[7] For the basic concept of solution for this problem, see Filippov (1988).

[8] Of course, many works had analyzed the disequilibrium dynamics. Varian (1977) proved the existence of stable non-Walrasian and unstable Walrasian equilibrium, however, his analysis is limited because the regime-switching is not included. The complex price adjustment process was shown in Böhm (1978), although the paper was limited to within-regime dynamics and graphical explanations. Löfgren (1979) investigated the Barro-Grossman model's dynamics and concluded that there was not sufficient analysis on regime-switching. Malinvaud (1980) showed the stable KU steady state, but his analysis also lacked the regime-switching. Blad and Zeeman (1982) introduced the lagged adjustment to avoid the on-the-regime-boundary analysis.

region of the disequilibrium regime named x. Then,

$$(\dot{P}, \dot{W}) = \begin{cases} (f^1(P,W), g^1(P,W)) & \text{if } (P,W) \in \text{int}R_{KU}, \\ (f^2(P,W), g^2(P,W)) & \text{if } (P,W) \in \text{int}R_{CU}, \\ (f^3(P,W), g^3(P,W)) & \text{if } (P,W) \in \text{int}R_{RI}, \end{cases} \tag{3}$$

where $(f^i(P^*, W^*), g^i(P^*, W^*)) = (0, 0), i = 1, 2, 3$. Figure 1 illustrates the dynamics. It is worth noting that the dynamics on the RI-KU border are intractable using the usual techniques since the two systems meet at this boundary. Honkapohja and Ito (1983) used the Filippov solution, in which the dynamics are determined by the combination of vectors. See x_0 in this figure. We know that $(f^1(x_0), g^1(x_0))$ and $(f^3(x_0), g^3(x_0))$, and then we should calculate them and combine them. The newly produced vector goes along the line tangent to the boundary, so that the economy would go on a "sliding trajectory" and would converge to the WE. However, the economy can diverge along the sliding trajectory, an example of which is shown as x_1. Whether the sliding path converges or diverges depends on the shape of the boundary and the velocities of P and W (see Honkapohja and Ito (1983, Theorem 4.1)). If the vectors are balanced, the dynamics stop even though all f^i and g^i are not zero. This is called the *quasi-equilibrium* in Filippov (1988) and is shown as the QE in the figure.

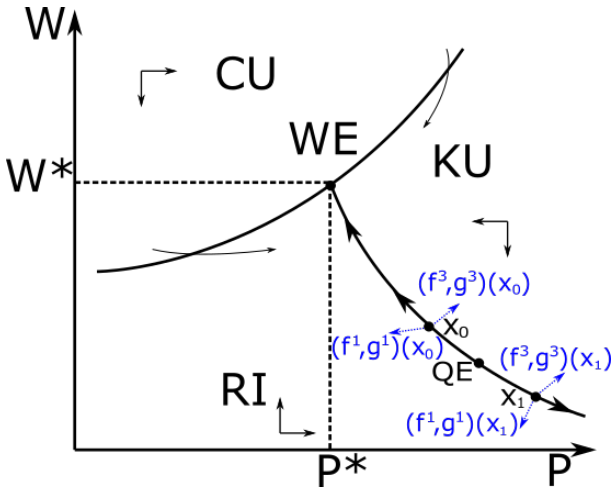

Figure 1: The phase diagram of Malinvaud's dynamics

The disequilibrium dynamics themselves were fascinating; therefore, they were applied to several economic issues. The shape of the Phillips curve depends on each regime as demonstrated by Picard (1983) and Chiarella et al. (2000, Chapter 5). Colombo and Weinrich (2003) also showed that the

Phillips curve emerges as an attractor of chaotic dynamics of a simple disequilibrium model. The dynamics of inventory in disequilibrium are a notable issue. As Blinder (1980) argued, the existence of inventory moderates the dynamics because the spillover effect is weakened unless a stockout occurs; for instance, Eckalbar (1985) discovered that the number of regimes drops to two.[9]

3.2 Disequilibrium Growth

As the dynamics are explored, the growth theory is often analyzed in disequilibrium economics. The most seminal is Ito's neoclassical growth model (Ito, 1978, 1980a). He introduced sluggish wage adjustments into Solow (1956). Although the regime changes during transitional dynamics, the economy converges into a neoclassical steady state in which the markets are in notional equilibria. This story seems to overlap Solow's view on long-run dynamics in Solow (1988, p.312); it seems "neoclassical synthesis" in disequilibrium economics since the balanced growth path or the steady-state in the long run is the same as that in equilibrium economics, but the short- and medium-run dynamics emphasize disequilibrium regimes such as Keynesian unemployment. This synthesized view is sometimes utilized: Ginsburgh et al. (1985) explored a Ramsey model, Sgro (1984) extended Ito (1980a) with monetary growth, and van Marrewijk and Verbeek (1994) used a two-sector approach.

Of course, many growth models do not depend on neoclassical synthesis. Nikaido (1980) and Hénin and Michel (1982) explored Harrodian instability in quantity constrained models. Böhm and Puhakka (1988) and Weddepohl and Yildirim (1993) built overlapping generation models although their main issues are not the growth path but the optimality and expectation respectively. Keynesian macroeconomists, who are not New Keynesian, focus on the KU regime and have developed numerous dynamic models; see Flaschel (1999), Chiarella and Flaschel (2000), Chiarella et al. (2000), Chiarella et al. (2005), Asada et al. (2011), and Murakami (2014).

3.3 The money on disequilibrium

As Clower treated this as the main issue in his 1967 paper (Clower, 1967), money is sometimes considered in disequilibrium economics (see Gale (1983)). In particular, money is thought to play an important role in transactions, the medium of exchange. The buyer handles the money and the seller receives it in pairwise trades (see Ostroy and Starr (1974)). Indeed, money

[9]For disequilibrium dynamics with inventories, see Honkapohja and Ito (1980), Green and Laffont (1981), Simonovits (1982), Duménil and Lévy (1987), and Hsu (1992, 1994).

promotes trade if sufficient information is shared, but hinders trade in effective demand failures.[10] In the Keynesian coordination failure situation (KU regime), the employee and the employer may improve their realized trade if they bartered the workforce and the commodity. However, as Bénassy (1975b) demonstrated, each trade requires the exchange of money. This payment constraint prevents the resolution of Keynesian unemployment.[11] Since money is always needed for payment, it is natural to address the cash-in-advance (CIA) constraint problem.[12] However, we should note that Clower himself denied that his idea was inherited in the CIA analyses (see Plassard (2017)). Furthermore, the dual-decision hypothesis seems to be the stricter version of the CIA constraint, since the expressions of demand and supply are restricted by the quantity constraint of the realized transaction, which adds flow constraints.

However, we should note that monetary constraints have not been treated *completely*, even in disequilibrium economics. As Rogers (1989, Chapter 3) argued, Clower and his followers could not describe monetary constraints on the transaction process in detail; the constraint works explicitly in *ex-post* transactions. This might be an issue in all economic theories rather than one particular to disequilibrium economics.

3.4 Econometrics for disequilibrium

Economists have also explored empirical studies to reinforce the progress of theoretical analyses in disequilibrium economics. For the basic assumption, Kawasaki et al. (1982, 1983) empirically showed that quantity adjustment is preferred to price changes in the German industry, using microdata. They also presented a simplified dynamic model with inventory dynamics in Kawasaki et al. (1983) and demonstrated that firms tend to choose quantity adjustment when the demand shock is transitory (or when they expect that the demand would recover soon). Although this empirical study succeeded in justifying the critical assumption that the firm prefers quantity adjustment, the theoretical analyses of this assumption within the disequilibrium framework have not succeeded.

[10] See Younès (1975) and Bénassy (1975a).

[11] His textbooks of disequilibrium (Bénassy, 1982, 1986) start with comparing barter economy and monetary economy using the diagrams in Clower (1967).

[12] Grandmont and Younès (1972) and Grandmont (1985) tried to build a general model for Clower's exchange with money payment, with Hicksian temporary equilibrium. In particular, Grandmont (1985) pointed out that the real balance effect with price adjustment cannot resolve the persistent excess supply situation (which could be interpreted as the KU regime).

Several econometric analyses of disequilibrium economics have focused on regimes, whether the economy is demand-constrained or supply-constrained. For the equilibrium framework, the observed economic activity, such as the prevailing price and traded quantity of goods, are supposed to be located on both demand and supply curves. However, for the disequilibrium framework without friction, the realized and observed transactions could differ from demand or supply. Distinguishing between the demanded and supplied quantity using the observed quantity was a central issue.

Fair and Jaffee (1972), Maddala and Nelson (1974), and Rosen and Quandt (1978) presented some methods to estimate whether markets are demand or supply-constrained using the observed price dynamics; see Ito (1980b) and Quandt (1988) for the summary. Although some studies have rejected the hypothesis of market equilibrium, their estimations depend heavily on the price adjustment formulation. Therefore, estimation remains a difficult and unresolved problem.

Recently, Juselius (2021) suggested a cointegrated-VAR (CVAR) approach, supported by Colander et al. (2008) and inspired by Guzman and Stiglitz (2020). Guzman and Stiglitz's theoretical work emphasizes inconsistency in trade, although the critical difference from the fluctuations in orthodox equilibrium models is not explicitly described in mathematical theory. Juselius argues that her CVAR model, which does not require consistency or stationarity in a base model, is consistent with the nonstationary time series in a disequilibrium economy.

3.5 Keynesian disequilibrium: Iwai model

Iwai (1974, 1981, 2018) also studied disequilibrium dynamics from a Keynesian and Wicksellian perspective, but the perspective is different from disequilibrium economics treated in this study.[13] He used quantity rationing on transactions but denied quantity adjustment in one period; instead, he described the streams of production and market transactions. Because production takes time, the firm should plan the quantity of goods supply and labor demand before the commodity market opens. Furthermore, the firm should also determine the goods' prices and wages before it goes to the market. Iwai supplied microeconomic theory for these uncertainties and showed that a flexible price change would bring instability.

From the perspective of the disequilibrium theory, one of the prominent characteristics of Iwai's model is that it distinguishes the planning time of

[13]Iwai says that he did not accept the Keynesian view by Clower and Leijonhufvud in his biography (Iwai, 2015).

quantity and the time when trades occur. This theory makes it difficult to reproduce the spillover effect of quantity adjustment (the dual-decision hypothesis) but sounds natural and realistic. Future disequilibrium research on quantity adjustment should integrate the advantages of this model.[14]

4 Impasse of analysis and frauds of disequilibrium models

When we look at the standard textbooks of macroeconomics today, we would rarely see descriptions of disequilibrium models. As argued, disequilibrium economics seems almost forgotten today. When we learn macroeconomics, it usually means that we learn (dynamic) general *equilibrium* economics.

After the prominent book (Barro and Grossman, 1976) was published, Barro immediately moved away from disequilibrium. He is now known as the economist of equilibrium rather than disequilibrium. As "New Classical" economics appeared, the concern about disequilibrium declined during the 1980s, in particular in the US.[15] Economists have pointed out that disequilibrium economics has shortcomings.

4.1 Shortcomings of disequilibrium theory

First, the reason why the price is "sticky" is not explained in the disequilibrium theory. However, this issue is unclear. The most explicit difference between equilibrium and disequilibrium economics is the existence of an auctioneer, as argued by Leijonhufvud (1968). From the equilibrium view, it is important to explain how the price stickiness occurs and why the agents quickly adjust their quantities in "micro-foundations," which usually means that the agents behave with some consistent choice. Although several researchers have explored this, their answers do not seem to be acceptable.[16] New Keynesian economics solved this problem after the disequilibrium economics almost disappeared by using frictions and imperfections

[14]For instance, Smolny (1993, 1998) consider the models with lagged sales in disequilibrium framework.

[15]Ito (1985) says that disequilibrium economics seemed accepted calmly in Europe and Japan and the equilibrium economics is preferred in the US. Mankiw (1990, p.1655) also mentioned this difference.

[16]For instance, Azariadis (1975) and Azariadis and Stiglitz (1983) presented the implicit contrast theory. In Keynesian economics, the perceived kinked demand curve might be a good tool for quantity-constrained micro-foundations (see Bénassy (1976) and Negishi (1979). However, a consistent model that synthesizes the kinked demand curve and disequilibrium regime model has not yet been developed.

about information; Mankiw argues that New Keynesian economics inherits disequilibrium properties, but it has proper micro-foundations (Backhouse and Boianovsky, 2012, p.7). However, we should be careful when accepting this idea. Most micro-founded price stickiness might not be suitable for the possible inconsistency in disequilibrium (see the definition of disequilibrium economics below). For instance, the Calvo pricing named after Calvo (1983) in standard macrodynamic models seems to assume that sales expectations are consistent so that it assumes that there is no disequilibrium in economy.

Second, transactions and rationing have not been fully explained (Howitt, 1979, p.60). Although Bénassy (1975b) presented a disequilibrium model in which the rationing scheme could take various forms, it is difficult to interpret each rationing as in decentralized markets. At this point, the micro foundation of the disequilibrium has not yet been completed.

Third, the equilibrium point of quantity adjustment is an ambiguous concept. Following Leijonhufvud (1968), almost all disequilibrium models describe the transaction as the equilibrium point of the quantity adjustment. This is the counterpart to the tâtonnement process in the general equilibrium model.[17] When we emphasize the spillover effect that arises from the quantity constraint, this is a little inconvenient; the agents make dual decisions because the transactions are different from the planned one, but these unexpected transactions continue to occur and the agents never change their policies. Although many works are related to this problem, there seems to be no complete or intuitive answer.[18] This problem might arise from a static framework: disequilibrium economics is the economics of dynamics.

4.2 Confusing terms

For disequilibrium economics, there is a serious problem that is not crucial for theoretical consistency but one that confuses researchers: various names and confusing technical terms.

Disequilibrium economics has been called *quantity constrained* (Negishi, 1979; Latham, 1980), *quantitative rationing* (Muellbauer and Portes, 1978;

[17]For the stability of quantity-tâtonnement, see Hayashi (1977). On this point, the theory of disequilibrium seems to be a compliment to the equilibrium model and not an alternative.

[18]For instance, the non-tâtonnement process such as Hahn and Negishi (1962) and Uzawa (1962) explored the dynamics of price adjustment in which the transaction by quantity rationing also occurs. However, the agents are consistent in that they are never surprised by the realized quantity and make no dual decision. Stochastic rationing, as previously mentioned, can blur this problem. In equilibrium with rational expectation, however, we again return to the problem of consistency; the agents are unaffected by quantity rationing, so we do not need the disequilibrium framework.

Sneessens, 1984; Fourgeaud et al., 1981; Movshovich, 1994), *fixed (ed) price equilibrium* (Eckalbar, 1981; Silvestre, 1982, 1983; Weddepohl, 1983; Michaillat and Saez, 2015), and *nonclearing market (NCM)* (Bénassy, 1993). This variation seems to originate from the difficult concept of disequilibrium.

The most serious problem is the word "disequilibrium." We can see many papers that include the word "disequilibrium" when we try to research disequilibrium economics on the Web. It is constructive to discuss the disequilibrium here.[19] Disequilibrium is the antonym of equilibrium; therefore, we compare the disequilibrium economics presented by Bénassy (1975b) with the general equilibrium framework.

Dixon (1990) supplied three properties of equilibrium economics. (P1) The behavior of agents is consistent, (P2) no agent has an incentive to change their behavior, and (P3) equilibrium is the outcome of some dynamic processes. Table 1 summarizes the properties of the general equilibrium and Bénassy's K-equilibrium. The properties of the general equilibrium model follow those of Dixon's original description. In this study, we consider the properties of the K-equilibrium. First, consistency (P1) is ambiguous in K-equilibrium. As the expressed demand and supply are determined by the optimization problem with quantity signals perceived, agents seem consistent with their choices. However, actual trade could easily betray their expressions (trade plans), since the realization and perception of constraints are independent of each other. The consistency property is regarded as violated when we focus on realized transactions. Second, P2 is ensured through payments in the monetary economy. In K-equilibrium, the agents do not change the expressions of demand and supply because they react optimally to the quantity and price signals. If they change the transaction, they should go through the monetary payment process (they cannot trade goods and labor directly). They cannot improve their transactions, even if they are ineffective (see Bénassy (1975b, Section 5)). Third, K-equilibrium is an equilibrium point of the quantity-perception tâtonnement process (see Bénassy (1975b, p. 509)). This property seems to come from Leijonhufvud's interpretation of Keynes's economics: quantity adjustment is completed before price adjustment finishes.

From this comparison, we find that the most prominent property of disequilibrium economics is inconsistency, which we also emphasize in Guzman and Stiglitz (2020). In other words, what the word disequilibrium says is that realized trades incorporate inconsistency. The main issue for disequilibrium economists is how to treat this inconsistency in disequilibrium economics. In

[19]The discussion here follows the concept in what is called Non-Walrasian and Neo-Keynesian disequilibrium economics.

the next section, we present how we treat inconsistency (the gap between the desired or planned trade and the realized trade) and the importance of the spillover effect.

Table 1: Equilibrium properties of equilibrium and disequilibrium models

	Equilibrium	K-equilibrium
P1	Demand equals supply	Unsatisfied?
P2	Actual trade equals desired trade	Payment constrant
P3	Tâtonnement	Adjustment of perception and expression

5 Definition of Non-Walrasian disequilibrium

Although the concern about disequilibrium had once declined, some researchers refer to disequilibrium models such as Barro and Grossman (1971) when they explore the unemployment and "Keynesian situation" in *secular stagnation* these days. Sometimes, the terms of disequilibrium economics are abused in analyses of involuntary unemployment, and several important issues are overlooked.[20] In this section, we examine the doctrinal and mathematical features of disequilibrium economics and define them. We should remember that disequilibrium economics is similar to the (general) "equilibrium" model; it explores the realized transactions in markets in which buyers and sellers meet.

1. The realized quantity could differ from the planned quantity for each individual. This condition implies a gap between the expressed demand and the expressed supply. In general equilibrium economics, demand and supply must be the same in transactions since the tâtonnement process finishes before the actual transaction occurs. However, in disequilibrium economics, individuals do not know whether their planned (desired) transactions are realized. This corresponds with rationing in many disequilibrium models, but it has a wider range. We allowed for possible instantaneous price adjustments.[21] This condition states that an economic individual never knows the realized transaction before making a decision.

For a model analysis, this definition is difficult to interpret. Notably, it simply states that $X \neq X^d$ or $X \neq X^s$ holds in the market of X. However,

[20]It does NOT mean that the analyses fail; our purpose is to redefine the disequilibrium economics and to find an unused analytical tool which is peculiar to it.

[21]On this point, our view is closer to the Keynesian economics in Negishi (1979) than that in Leijonhufvud (1968). However, our economics is not just Keynesian; it only emphasizes the possibility of disequilibrium.

we need to clarify the meaning of X^d or X^s. For instance, households somehow know how much rationing of their labor supply occurs; $L = (1-u)L^s$ always holds as $u \in (0,1)$. They optimize L^s, which means that the realized employment is also optimized because u is known to them (they are overbidding). In this example, they know that their labor supply is not realized but that the intended employment is realized; this is the same as an equilibrium model. To revise this problem for the disequilibrium model, we should set the realized value of u unknown when households make decisions. It should be noted that they do not know whether their planned quantity is realized or whether they perceive the possible quantity constraint. This unexpected constraint condition induces inefficient trade owing to insufficient information. As Bénassy (1975b) showed, agents fear the constraint and, as a result, reduce demand, which causes persistent inefficient trades.

2. The demand (and supply) quantities under the perception of quantity constraints and without it are different. When an individual perceives the possible quantity constraint on their transaction in at least one market, this means that their budget constraint, including the realized quantities, is different from the planned budget constraint without the perception of quantity constraint. This condition corresponds to Clower's *dual decision* hypothesis. This condition requires the individual to make a decision considering the (expected) realized quantity.

Demand and supply should be affected by the quantity signals so that the expected sales Y^e or employment L^e are included in the labor demand and goods demand functions; $L^d = L^d(Y^e)$ and $Y^d = Y^d(L^e)$ hold. In addition, the expectation (quantity signal) is affected by the realization or the realization itself at the equilibrium point of quantity adjustment; that is, $Y^e = Y^e(Y)$ and $L^e = L^e(L)$ hold. Finally, each demand (and supply) is affected by realized transactions in other markets. This is the spillover effect emphasized by Patinkin (1956) and Clower (1965).

In summary, individuals do not know the realizations of every market, but the realizations in markets are (indirectly) connected. This relationship should be maintained when we move from microstructure to macroeconomic analysis.

5.1 Rigidity models: rigid wage and ZLB

Here we conduct a brief comparison between a "non-Walrasian disequilibrium model" defined above and an equilibrium model which has a disequilibrium flavor.

First, we see the model with downward rigidity of wage.

The real wage is pegged at $\bar{w}$ so that the *nominal* labor demand can be

lower than the labor supply:

$$L = L^d(\bar{w}) < L^s = \bar{L} = \text{const.} \tag{4}$$

This is a classical unemployment regime (see figure 2). On this point, disequilibrium economics seems still alive; in fact, Dupor et al. (2019) refer to Barro and Grossman (1971) as a sticky wage model.

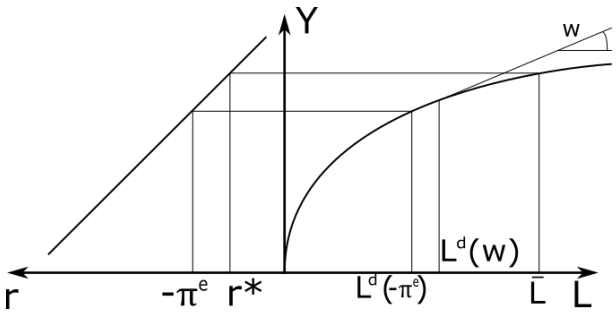

Figure 2: A simple example of ZLB model and rigid wage model

Second, we consider a zero-lower-bound (ZLB) economics model. The following example follows Blanchard (2016, Chapter 6). Production Y is determined by the real interest rate r. We define the natural interest rate r^* as the rate at which full employment $\bar{L}$ is ensured. Employment is determined by production $L = L(Y)$. Therefore, the system consists of the following equations.

$$Y = Y(r), \tag{5}$$
$$L = L(Y), \tag{6}$$
$$r = \max[r^*, -\pi^e], \tag{7}$$

where π^e denotes the expected inflation rate. When the ZLB constraint holds ($r > r^*$), involuntary unemployment ($L < \bar{L}$) continues (see figure. 2). Although unemployment in ZLB economics sometimes occurs through the goods demand term, this is classical unemployment because the rigidity of price in the funds market (real interest rate) is a resource of unemployment. Although these models are often complicated, the core structure seems to be the stickiness of the real (or nominal) interest rate.

Because employment is quantity adjusted, the employment condition in the above models can be rearranged as follows:

$$L = \min[L^d(r, w), \bar{L} = L^s]. \tag{8}$$

This rationing equation is the same as the ones in disequilibrium models; therefore, the ZLB works and secular stagnation analyses sometimes refer to disequilibrium economics.[22] However, the core of non-Walrasian disequilibrium economics is usually overlooked, that is, the spillover effect in disequilibrium or the dual-decision hypothesis.

Although involuntary and unexpected unemployment occurs, in the example above, households do not change their behavior. In other words, neither labor supply nor goods demand differs from that with full employment. This means that the disequilibrium analysis was incomplete. If the spillover effect is incorporated, the goods demand function depends on realized employment.

$$Y = Y(r, w, L) \tag{9}$$

If we proceed with the quantity adjustment process, the transaction is described as system $Y = Y(L)$ and $L = L(Y)$ under the given (r, w). We should notice that even if the price variables are equilibrium values (r^*, w^*), on which the full employment is ensured in the standard ZLB model, the full employment is not ensured in the quantity adjustment model.[23]

5.2 Simple non-Walrasian disequilibrium model

We now present a simple example of a disequilibrium model on the ZLB. Following Blanchard's textbook, we present a system that determines the number of goods produced Y and employment L:

$$Y = \min\{Y^d(r, w, x, L), Y^{s*}(w), \bar{Y}^s(\bar{L})\}, \tag{10}$$

$$L = \min\{\tilde{L}^d(Y, x), L^{d*}(w), \bar{L}\}, \tag{11}$$

$$w = \bar{w}, \tag{12}$$

$$r = \max\{r^*, -\pi^e\}, \tag{13}$$

where w is the real wage rate, r is the real interest rate, x is sales expectations, and π^e is the expected inflation rate. The notional supply Y^{s*} and notional demand L^{d*} are derived from the usual optimization problems without quantity constraints. The labor supply is inelastic $L^s = \bar{L}$, such that there is a physical constraint on production capacity $\bar{Y}^s$. If we suppose that the monetary authority has complete information about the real economy and there

[22]For instance, see Eggertsson et al. (2019, footnote 19). The simplified ZLB model in Palley (2019) does not specify the employment function in detail but seems to adopt this rationing scheme.

[23]Barro and Grossman (1971) showed the first example of unemployment under the equilibrium prices.

is no demand saturation, such that r^* always satisfies $\tilde{L}^d(Y^d(r^*), x) = \bar{L}$, then the resource of unemployment is price rigidity, and if r or w is too high, unemployment occurs. We should note that the unemployment mechanism itself does not change from the standard ZLB or rigid wage model, but that the quantity adjustment process is added to this model. The shortage of goods demand is enhanced through the spillover effect in labor demand $Y = Y^d(L(Y))$ (see figure 3). An unsolved problem is the spillover effect.

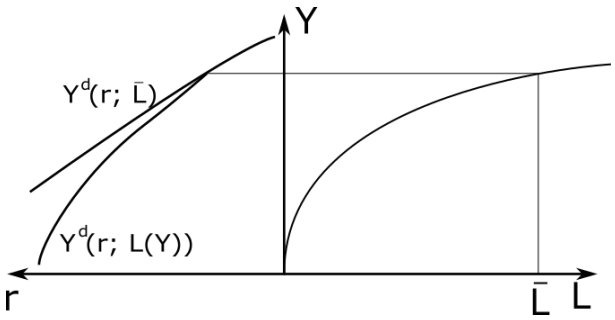

Figure 3: ZLB model with spill-over (dual-decision)

However, there is a possibility that another problem may cause involuntary unemployment. Suppose that sales expectation x is affected by realized production Y. If the firm becomes risk-avoiding and underestimates x, then there might not be a natural interest rate: $\tilde{L}^d(Y^d(r), x(Y^d(r))) < \bar{L}$ holds for any r. The pessimistic expectation of sales is maintained for the current shortage of goods demand, and the ZLB is no longer the main source of unemployment.[24] Figure 4 presents an example. The second quadrant depicts two goods demand curves. The higher one has a sufficient and fixed sales expectation $\bar{x}$, and the lower one has an underestimated $x(Y)$ function.

5.3 Problems on expectations

Subsequently, our interest moves to the formulation of sales expectation: why do firms underestimate it? If we set the perfect foresight model as the starting point, we must forcibly add some inconsistency or irrationality to the canonical model. This implies that the equilibrium model, in which the consistency of individuals and the model itself is embedded, cannot explain the persistent downturn in their natural assumptions, as Guzman and Stiglitz (2020) emphasized. Therefore, we cannot utilize the tools of mathematical

[24]Varian (1977) showed the persistent underestimated sales expectation and shortage of goods demand.

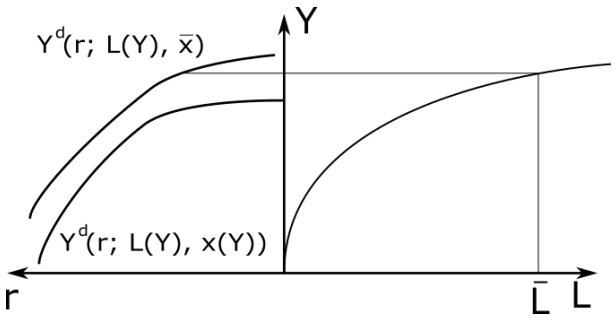

Figure 4: The case in which there is no natural interest rate

formulations of expectations that have grown in equilibrium without considering it. Instead, it is natural to construct formulations of micro-founded expectations within the disequilibrium framework.

Neary and Stiglitz (1983) found that the self-fulfilling prophecy works in a disequilibrium economy; the optimistic expectation for sales constraint in the next period promotes the current economic activities so that the current sales would be stimulated. Ogawa (2021a, Section 5) also points out that the recursiveness between the current constraint and expected constraint in the future so that the perfect foresight and rational expectation might be no sense in a disequilibrium economy. When we treat sound micro foundation in disequilibrium economics, the left issues are numerous.

6 Concluding remarks and future issues

In this study, we present a brief survey of disequilibrium economics. Disequilibrium economics was greatly affected by the changes in macroeconomic analysis tools. However, it has the potential to research various economic issues such as secular stagnation.

The most important characteristic in disequilibrium research is the dual-decision hypothesis, in which quantity signals have a spillover effect, and this issue has often been overlooked. A spillover effect exists on the relationship among the realized transactions, not the desired transactions. On this point, the dual-decision hypothesis comes from the most important feature, inconsistency. We should include this term in the disequilibrium models and estimate them in the future.

As the work has been forgotten for a long time, there are enormous issues in disequilibrium economics. Let us summarize them in what follows.

As we have seen in 4.1, some concepts of non-Walrasian disequilibrium are not well defined. How should we interpret and utilize the fixed point of

quantity-tâtonnement in real-world economy? This is one of the most important question for empirical analysis.[25] The primary question on basic assumption is still remained: why the sellers prefer quantity adjustment rather than price adjustment. Although Negishi (1979) and Palley (1997) refered to kinked demand curve theory, it is not synthesized with general disequilibrium theory. These days, some researchers, e.g., Dossche et al. (2010) and Ilut et al. (2020), tuckle the kinked demand curve so that constructive analysis on this issue could be expected. The central concept *rationing* is also ambiguous. In macroeconomic view, the short-side rule is so useful that it has usually been used without considering it. It is ignored how each individual in long side is rationed. If there are heterogenous individuals, the rationing on them never happens uniformly. This fact recursively affects macroeconomic activities so that the rationing rule matters.

Although the disequilibrium economics has developed its tools and derived many interesting conclusions, it is still a minor part of economics. How to promote discussion on disequilibrium? There are the two types of strategy. One is to revearl empirical merits. The current macroeconomics based on equilibrium theory emphasizes how the results of simulation accord with the data. The procedures such as calibration are valued. As the disequilibrium economics was replaced with equilibrium economics, the procedures of data-fitting are not adapted to disequilibrium so much. It is needed to identify the dual-decision effect in economy and to find what is peculiar to disequilibrium regime. In particular, KU regime is remarkable since the unemployment easily continues with low wage rate. The other is to reconstruct the model realistically. As we have refered, the quantity-tâtonnement is a counterpart of price-tâtonnement. This is not the direct description of a realistic transaction, but an approximation. If we treat disequilibrium directly, the trade should coexist with adjustment process. This problem, since Smale (1976), seems to argue that the model should be a stream of transactions rather than the fixed point. It is equal to abandon the property P3 in 4.2. The model of this type is presented by Iwai (1981) and recently Shiozawa et al. (2019) works on it. Of course, these strategies are not independent: more realistic model is expected more performance for data-fittling. We expect the this paper contributes to further analysis on disequilibrium.

[25] Ogawa (2021b) presents one example: entries and exits in labor market. The workers' entries and exits in matching economy could be interpreted as the quantity-adjustment process.

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