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Modeling and Forecasting the Electricity Price in Iran Using Wavelet-Based GARCH Model

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Abstract

The restructuring of Iranian electricity industry allowed electricity price to be determined through market forces in 2005. The main purpose of this paper is to present a method for modeling and forecasting the electricity prices based on complex features such as instability, nonlinear conditions, and high fluctuations in Iran during the spring 2013 and winter 2018. For this purpose, time-series data of the daily average electricity price was decomposed into one approximation series (low frequency) and four details series (high frequency) utilizing the wavelet transform technique. The approximation and details series are estimated and predicted by ARIMA and GARCH models, respectively. Then, the electricity price is predicted by reconstructing and composing the forecasted values of different frequencies as a proposed method (Wavelet-ARMA-GARCH). The results demonstrated that the proposed method has higher predictive power and can forecast volatility of electricity prices more accurately by taking into consideration different domains of the time-frequency; although, more errors are produced if the wavelet transform process is not used. The mean absolute percentage error values of the proposed method during spring 2017 to winter 2018 are significantly less than that of the alternative method, and the proposed method can better and more accurately capture the complex features of electricity prices.

Highlights

- This study investigates complex features of electricity price forecasting in Iran.
- This study proposed wavelet transform combined with ARMA-GARCH to predict the electricity price.
- The proposed model has increased predictive power and can capture the complex behavior of electricity prices.

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1. Introduction

The deregulation process and novel ways of conducting transactions in the electricity market have led to price uncertainty for electricity producers and consumers. The forecast of electricity prices is considered as a means of selecting a strategy by market participants. It allows electricity suppliers and consumers to adequately manage their investments, usage, plans, and risks to compensate for the effects of price fluctuations (Karakatsani & Bunn, 2004). As a commodity, electricity has spot-price forecasting features distinguishing it from other products because electricity is a non-reservable commodity in which production and consumption are carried out simultaneously. In addition, its prices are determined locally, the reserving capacity for this commodity is not available on networks, and there is no possibility for exchanges in the form of arbitrage. However, power generation companies could dedicate their capacity to cooperating auxiliary services, besides exchanging the produced steam instead of electrical energy.

Furthermore, electricity demand depends on unpredictable factors such as weather conditions, which exacerbate the impact of supply and demand shocks on its price. Electricity has a seasonal pattern as a commodity and responds to periodic fluctuations in demand (Bourbonnais & Meritet, 2007).

In Iran's electricity market, all companies are known as net sellers because there are no reserve markets similar to those in other countries. In other words, the Iranian market is considered as a wholesale market. However, some major net sellers, such as Khuzestan and Tehran, have motivations for managing market power. Accordingly, results of this study can be applied in the Iranian electricity market, and wholesale markets, as well as markets in which the net sellers tend to control the market. However, some researchers examined the Iranian electricity market using the same approach as other electricity markets. For example, Zarezadeh et al. (2008) studied the price forecasting model using artificial neural networks that operate electricity price data with temperature and load criteria to improve results. Shayeghi and Ghasemi (2013) proposed a hybrid model that combined the wavelet transform, least-square support vector machine (LSSVM) and the gravitational search algorithm and assessed applying electricity market price data from Iran, Spain and Ontario.

In this study, the Wavelet-ARMA-GARCH model is proposed to forecast electricity prices in the Iranian electricity market to achieve these aims. Despite the fact that the restructuring of Iran's electricity occurred in 2005, the need for more accurate forecasting models has increased.

The structure of this paper is as follows:

Section 2 presents the literature review, and section 3 deals with the methodology used in this study based on wavelet transforms and ARMA-GARCH models. Section 4 is the empirical results, and finally section 5 presents the conclusions.

2. Literature Review

While much research has been conducted and various methods proposed to address these issues, there are generally two approaches for predicting prices namely artificial intelligence-based methods such as artificial neural networks, learning machines, genetic and fuzzy logic algorithms, and time series analysis.

Hong et al. (2002) proposed to integrate a recurrent neural network with clustering through the fuzzy c-means (FCM) algorithm for predicting marginal prices in the PJM[1] electricity market. Yang and Lai (2005) proposed a global and local electricity price forecasting model based on a recurrent neural network in order to achieve a precise short-term forecast in the New England market using the Lyapunov's exponents. Contreras et al. (2003) proposed the ARIMA-based model for daily electricity price forecasts in Spain and California while González et al. (2005) described an Input-Output Hidden Markov Model for analyzing and forecasting electricity prices in Spain based on a series of dynamic models linked together by a Markov chain.

Conejo et al. (2005) presented a new method for daily electricity prices forecasting based on wavelet transform and ARIMA models. Based on 2002 electricity energy market data in Spain, the performance of the proposed method using the wavelet transform and ARMA model is superior to the direct use of ARIMA models.

Using GARCH method for the daily prediction of electricity price in Spain and California, Garcia et al. (2005) concluded that this model outperformed the ARIMA method when price fluctuations were present.

Catalão et al. (2007) introduced a three-layer feed-forward neural network approach developed by the Levenberg-Marquardt algorithm for the weekly prediction of electricity price in Spain and California.

Vahidinasab et al. (2008) used fuzzy c-means (FCM) algorithm for daily load pattern clustering and applied artificial neural networks with a modified Levenberg-Marquardt algorithm for price forecasting in PJM market.

A study by Bowden and Payne (2008) evaluating the time-series models of ARIMA, ARIMA-EGARCH, and ARIMA-EGARCH-M for hourly electricity prices at MISO hubs concluded that the predictive power of the ARIMA-EGARCH-M was superior to all other three models.

Tan et al. (2010) presented a wavelet transform-based price forecasting method with the combination of ARIMA and GARCH models for the Spanish and PJM electricity markets, and found the proposed method was significantly more accurate than other conventional forecasting methods.

Shrivastava et al. (2014) combined the Extreme Learning Machine method with wavelet transform to develop a combined model called WELM which is a wavelet transform-based technique. The results of experiments showed that this method was one of the best price forecasting techniques in Ontario, PJM, New York, and Italy.

[1] Pennsylvania, New Jersey, and Maryland.

Pany et al. (2015) used the local linear wavelet neural network to determine market clearing prices and the results revealed its good potential to accurately predict electricity prices.

Razak et al. (2016) presented a multi-stage optimization for a combination of the Least Square Support Vector Machine model and the genetic algorithm to achieve accurate electricity price forecasting in the market by optimized parameters and input characteristics.

Yang et al. (2017) used a combination of the wavelet transform, the Kernel extreme learning machine based on the self-adaptive particle swarm optimization, and an ARMA model for electricity price forecasting in the power markets of PJM, Australia, and Spain. After wavelet-transform decomposition, the stationary series were predicted as new input sets by ARMA model, whereas the non-stationary series were predicted using the SAPSO-KELM model.

Bento et al. (2018) introduced a combined method for electricity price forecasting in Spain and PJM using the wavelet transform before processing followed by a feed-forward neural network with Bat and Scaled Conjugate Gradient Algorithms to improve the neural network capabilities.

Researchers claimed that the wavelet transform could cause data decomposition and have a stable variance (Chang et al., 2019). They introduced a WT-Adam-LSTM model based on LSTM neural network, wavelet transform, and Adam optimization. Applied data from the French and New South of Australia reveals the high accuracy of the hybrid model.

Qiao and Yang (2020) applied the wavelet transform as a data preprocessing algorithm in their study. They planned a crossover experiment with schemes of wavelet transform parameter selection. This paper estimates each scheme with stacked auto-encoder and long short term memory, producing a new model WT-SAE-LSTM. They found that this method accurately predicted electricity prices in the US. Table 1 summarizes the research findings.

Table 1. A summary of background studies

Authors	Methodology	Key Findings
Hong et al. (2002)	Neural Network & Fuzzy c-mean	The RNNs were applied in the PJM market, and the recommended model was able to predict LMP values proficiently.
Contreras et al. (2003)	ARIMA	ARIMA methods were used to analyze California and Spain markets, and the errors were reasonable compared to ANN.
Yang and Lai (2005)	Recurrent Neural Network	The RNN-based electricity prices forecast model showed that the short-term forecast is accurate in New England.
González et al. (2005)	Input-Output Hidden Markov	The model applied using clearing prices in Spanish electricity market has provided dynamic information and accurate forecasts.
Conejo et al. (2005)	Wavelet Transform & ARIMA	An appropriate estimation period should be chosen. The forecasts of electricity prices in mainland Spain have been satisfactory.

Table 1 (Continued). A summary of background studies

Garcia et al. (2005)	GARCH	The proposed model has been tested on the Californian market and has shown that the GARCH model gives better results than the ARIMA model.
Catalão et al. (2007)	Neural Network & Levenberg-Marquardt	The result of the proposed neural network model revealed the accuracy of predictions in California and Spain.
Vahidinasab et al. (2008)	Artificial Neural Network & Fuzzy c-mean	The learning algorithm and ANN were used to forecast prices in the PJM market, and results were accurate in comparison with previous studies.
Bowden and Payne (2008)	ARIMA, ARIMA-EGARCH & ARIMA-EGARCH-M	The ARIMA-EGARCH-M model works better than ARIMA, ARIMAEGARCH. However, no method can clearly outperform others with respect to predictive performance.
Zarezadeh et al. (2008)	Artificial Neural Networks	The recommended ANN models can be considered as an appropriate technique to predict the average hourly price in the Iranian electricity market.
Tan et al. (2010)	Wavelet Transform, ARIMA & GARCH	The Wavelet-ARIMA-GARCH method has been tested for forecasting on the PJM and Spain markets.
Shayeghi and Ghasemi (2013)	Wavelet Transform, GSA & LSSVM	A reasonable accuracy of the hybrid Wavelet transform in LSSVM and the CGSA technique was evaluated utilizing the real price dataset in electricity markets of Spain, Ontario, and Iran markets.
Shrivastava et al. (2014)	Wavelet Transform & Extreme Learning Machine	The wavelet-ELM model techniques create less prediction errors than current approaches.
Pany et al. (2015)	Wavelet Transform & Neural Network	The Wavelet Neural Network model could prove the potential of the model topredict electricity price.
Razak et al. (2016)	Least Square Support Vector Machine & Genetic Algorithm	The LSSVM-GA model showed lower complexity with better prediction accuracy than current techniques.
Yang et al. (2017)	Wavelet transform, ARMA & kernel-based ELM	The SAPSO-KELM method provided more accurate predictions and feasibility than unique methods and other hybrid techniques.
Bento et al. (2018)	Wavelet Transform & Neural Network	The Neural Network & Wavelet approach can capture complex characteristics of price signals.
Chang et al. (2019)	Wavelet Transform, LSTM Neural Network & Adam Optimization	The WT-Adam-LSTM model could improve forecasting and capture appropriate behaviors accurately for electricity prices.

Table 1 (Continued). A summary of background studies

Qiao and Yang (2020)	Wavelet Transform, stacked autoencoder (SAE) and long short-term memory (LSTM)	The best performance of the WT-SAE-LSTM model for the US electricity price forecast was in the commercial, industrial and residential sectors, respectively.

Although many studies have highlighted challenges, the review of the relevant studies shows that despite the use of wavelet transform technique for forecasting electricity prices in other countries, no such study using that technique in combination methods and which can spot time-frequency domains has yet been conducted in Iran. The main contribution of this proposed method is to use a wavelet transform technique to decompose and restructure a price series, so that each time series can be predicted separately using a suitable ARMA and GARCH model in accordance with its characteristics. Finally, a prediction model can be reconstructed by adding together all the decomposed prediction models. This combined method could identify time-frequency domains, non-stationary and nonlinear characteristics, and high fluctuations in electricity prices.

3. Methodology

In this paper, a combination of wavelet transform with ARMA-GARCH model is used to predict the price of electricity as a proposed method, and then the predictive power of the proposed model is compared to ARMA-GARCH model without using wavelet transform. Thus, the distinction and the ability to use wavelet-transformation appear in the predictive model. The electricity price is a variable of the model and the data used in both models is based on the daily average price of electricity for 1,825 days from the spring of 2013 to the winter of 2018. The daily electricity price data is provided by Iran Grid Management Company (IGMC)[2].

3.1 Wavelet Transform

Wavelet transformation is used as a mathematical transformation to detect latent information in a signal or a time series. Some features of a time series are not visible in the time domain, viewing and investigating these characteristics is achieved by transferring the time series to other domains though. In fact, wavelet transform is a useful tool in which wave-shaped functions decompose a time series into a series of coefficients. This group of functions is created by moving a basis function called "mother wavelet or analyzing wavelet" (Mallat, 1999).

Each set derived from the wavelet coefficients represents a part of the time series on a different scale. The ability of wavelets to use time and their scale allows them to focus simultaneously on time and frequency domains.

[2] Available at: http://www.igmc.ir/

The length of the basic function of the wavelet transform is long-term in the time domain when it detects low-frequency phenomena and thus has a good frequency resolution. Conversely, when it detects high-frequency phenomena, the length of the basic function of the wavelet transform is short-term in the time domain and therefore produces good time resolution for such phenomena.

It is possible to detect and obtain all the information in a time series and link them to the specific time and place horizons by combining different kurtosis and time shifts of the mother wavelet transform.

In short, a wavelet function $\psi\ (t)$ is a function of the following time called the acceptance condition:

$$C_{\psi} = \int_{0}^{\infty} \frac{|\Psi(f)|}{f} df < \infty \quad (1)$$

Where, $\Psi(f)$ is a Fourier transform. This condition ensures that when $f \rightarrow 0$, $\Psi(f)$ will also approach zero immediately. In fact, to ensure that C_{ψ} < ∞ this condition should be imposed that $\Psi(0) = 0$, and it is equal to:

$$\int_{-\infty}^{+\infty} \psi\ (t)\ dt = 0 \quad (2)$$

The second condition for the wavelet function is that its energy should be unit:

$$\int_{-\infty}^{+\infty} |\psi\ (t)|^2\ dt = 1 \quad (3)$$

Continuous Wavelet Transform (CWT) is a function of two variables u and s, and is calculated from the multiplication of the desired function in the wavelet function and the integration of the product (Gençay et al., 2001). Assuming that the desired function x(t) is a function of time, the wavelet transform is as follows:

$$W(u,s) = \int_{-\infty}^{\infty} x(t)\ \psi_{u,s}(t)\ dt \quad (4)$$

Where, $\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s}\right)$ is the elementary wavelet function extended to s, and displaced with the value of u on the time axis. The resulting coefficients are actually a function of the two s (scale) and u (the amount of displacement in time) parameters, although the main function is only a function of the time parameter.

By applying different parent wavelets, which are displaced on the time axis and extended on a function, the complex structure of the function is subdivided into smaller components, a process defined "functional analysis or decomposition". If wavelet functions used in the analysis meet the wavelet acceptance condition, the main function can be obtained by using the following formula and with the inverse operations on the wavelet coefficients:

$$x(t) = \frac{1}{C_{\psi}} \int_{0}^{\infty} \int_{-\infty}^{\infty} W(u,s)\psi_{u,s}(t)\, du\, \frac{ds}{2} \tag{5}$$

It involves the synthesis or reconstruction of the function. The key feature of wavelet transforms is that they can completely reconstruct the second-order integrable functions.

The approximation of each discrete function or time series uses wavelet functions as follows:

$$f(t) = \sum_{k} s_{J,k}\phi_{J,k}(t) + \sum_{k} d_{J,k}\psi_{J,k}(t) + \sum_{k} d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \sum_{k} d_{1,k}\psi_{1,k}(t) \tag{6}$$

Where, J is the number of analysis or scales levels, and k is the amount of displacement per time at each level. $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are orthogonal wavelet functions which are expressed as follows:

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(\frac{t-2^{j}}{2^{j}}\right) \tag{7}$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^{j}}{2^{j}}\right) \tag{8}$$

Where, $\phi_{0,0}(t) = \phi(t)$ is termed the "father wavelet" and $\psi_{0,0}(t) = \psi(t)$ is termed the "mother wavelet". In general, $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are respectively called scaling functions and wavelet functions. The wavelet coefficients can be calculated using the following formula:

$$s_{J,k} \approx \int \phi_{J,k}(t) f(t)\, dt \tag{9}$$

$$d_{j,k} \approx \int \psi_{j,k}(t) f(t)\, dt \tag{10}$$

Where $S_{J,k}$ is called Smooth of the level j^{th} and $d_{j,k}$ is called Details of the level j^{th} (Crowley, 2007).

There are different wavelet functions with the most well-known being the Haar, Daubechies, Symmlet, and Coiflet wavelets. Haar wavelet is a special type of function known as the first wavelet, and is used for this study. Its advantage is its high processing speed which, accordingly, increases the process speed. The Haar wavelet's mother wavelet is defined according to Equation 11:

$$\Psi(t) = \begin{cases} 1. & 0 \le t < \frac{1}{2} \\ -1. & \frac{1}{2} \le t < 0 \\ 0. & \text{otherwise} \end{cases} \tag{11}$$

The scaling function is equals to:

$$\phi(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The Haar wavelet's mother wavelet is shown in Figure 1.

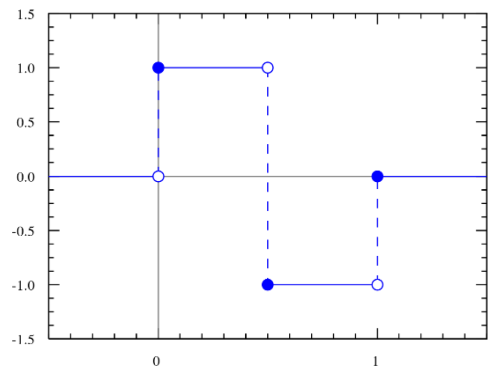

Figure 1. Haar wavelet
***Source**: Schleicher (2002)*

The integer j = 0,1, ..., J and $m = 2^j$ represents the wavelet's level. The J number represents the maximum level of experience (Lepik & Hein, 2014).

3.2 Mean Equation Modeling (ARIMA Models)

Time series models are related to Auto-Regressive Integrated Moving Average (ARIMA) models. For time series data such as Y_t, the ARIMA model is considered a tool for studying and possibly predicting future values of these series. In the ARIMA (p, d, q) process, the p, d, and q symbols represent the lag number of the autoregressive term, the differential order, and the lag number of moving average term, respectively. The optimal lag length of the models are determined using the Akaike Information Criterion (AIC) or the Schwarz Information Criterion (SIC) (Mills, 1991). The autoregressive moving average models are described as follows:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \cdots + \theta_q \phi_{t-q} \tag{13}$$

3.3 Volatility or Variance Modeling (GARCH Models)

Auto-Regressive Conditional Heteroscedasticity (ARCH) is used to model fluctuations or volatility. The ARCH model can explain the conditional variance process according to its past values and, finally, dynamic forecasting in time series models becomes possible based on their averages and variances (Engle, 1982).

Although it provides a good framework for the analysis of time series volatility, the ARCH model has some shortcomings; one of which is related to the determination of lag number of error terms. Another model known as the

Generalized Autoregressive Conditional Heteroscedasticity (GARCH) is being used to address these problems. In general, GARCH (p, q) is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \cdots + \alpha_q u_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-p}^2 \quad (14)$$

Based on the characteristics of the GARCH model, the conditional variance of Yt is estimated by an ARMA process (Bollerslev, 1986). To estimate uncertainty using the GARCH model, it is first necessary to determine the order of the ARIMA (p, d, q) model. This operation is similar to the ARMA models.

3.4 The Proposed Model (Wavelet-ARIMA-GARCH Model)

Following the selection of the Haar wavelet as the mother wavelet, the signal (original series) decomposes into an approximation and details signals. The approximate signal at each level of the decomposition can be decomposed again using the mother wavelet. Thus, achieving an approximate signal at a higher level, more detailed signals can be obtained. The approximate signal and the detail signal of order i are shown with the ai and di symbols, respectively. The sum of ai and $\sum_{i=0}^{n} d_i$ reconstructs the original signal. n determines the level of decomposition of the wavelet transform. This relationship is expressed in Equation 15.

$$s_t = a_n + d_1 + d_2 + \cdots + d_n \quad (15)$$

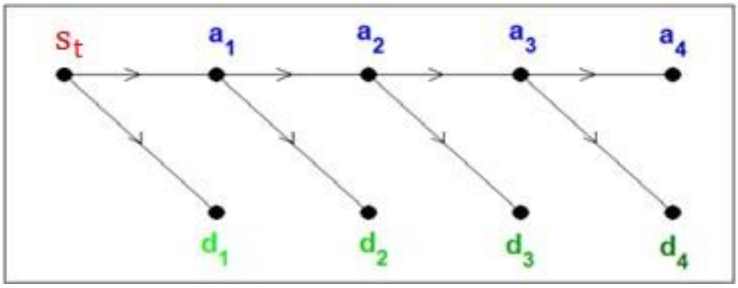

Figure 2. Decomposition levels

***Source**: Author's elaboration based on Schleicher (2002)*

Wavelet transform is used to analyze data for different time horizons prior to estimation in this study. As shown in Figure 2, the same time series formed by the average daily price of electricity has been decomposed to four levels using Haar wavelet transform. The results of this decomposition correspond to five coefficients categories; a4 is an approximate series in the fourth decomposition and d1, d2, d3, and d4 are details with different frequencies. Five new time series with specific characteristics are considered as outputs of this process. Thus, the relation of these five times series to the original signal can be defined by Equation 16.

$$s_t = a4 + d4 + d3 + d2 + d1 \quad (16)$$

By wavelet reconstruction, series of a4, d4, d3, d2 and d1 are called A4t, D4t, D3t, D2t and D1t. The relation among the approximate signals and the detail signals is expressed as follows with less loss:

$$P_t = A4_t + D4_t + D3_t + D2_t + D1_t \quad (17)$$

Each of these new time series can be modeled instead of estimating Pt, and an estimation model for the time series can be obtained by aggregating them. This is expressed in Equation 18.

$$\hat{P}_t = \hat{A4}_t + \hat{D1}_t + \hat{D2}_t + \hat{D3}_t + \hat{D4}_t \quad (18)$$

ARIMA models are used to estimate each new time series and are modeled using the Box-Jenkins method. Also, when examining the volatility of the models, GARCH models are used to estimate the variance equation for heteroscedasticity. Figure 3 is based on Tan et al. (2010) and shows a schematic view of the time-series estimation process in this study.

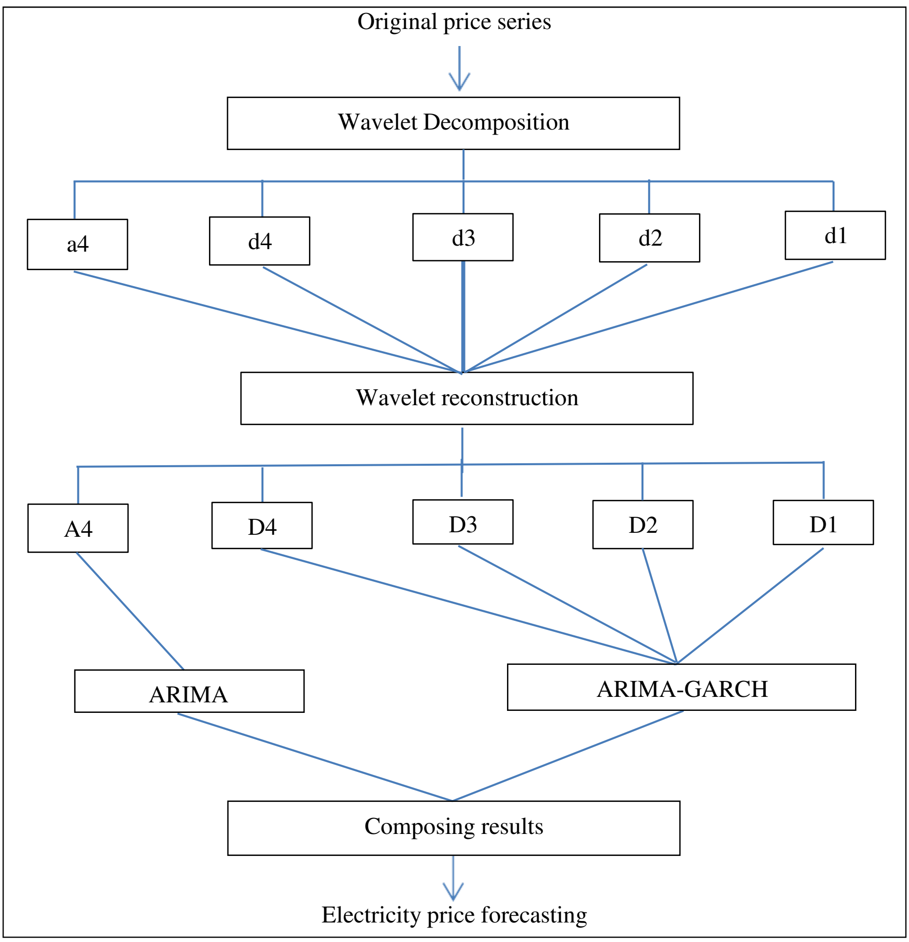

Figure 3. Procedure of the proposed method

Source*: Author's elaboration based on Tan et al. (2010)*

4. Empirical Results

4.1 Wavelet Transformation for Electricity Price

Figure 4 shows the results of the data analysis using Haar wavelet transform and its reconstruction in the dimension of time frequency. The decomposition order of the original signal by the Haar wavelet is selected at the fourth level. In Figure 4, st and a4 represent the behavior of the original signal and an approximate signal.

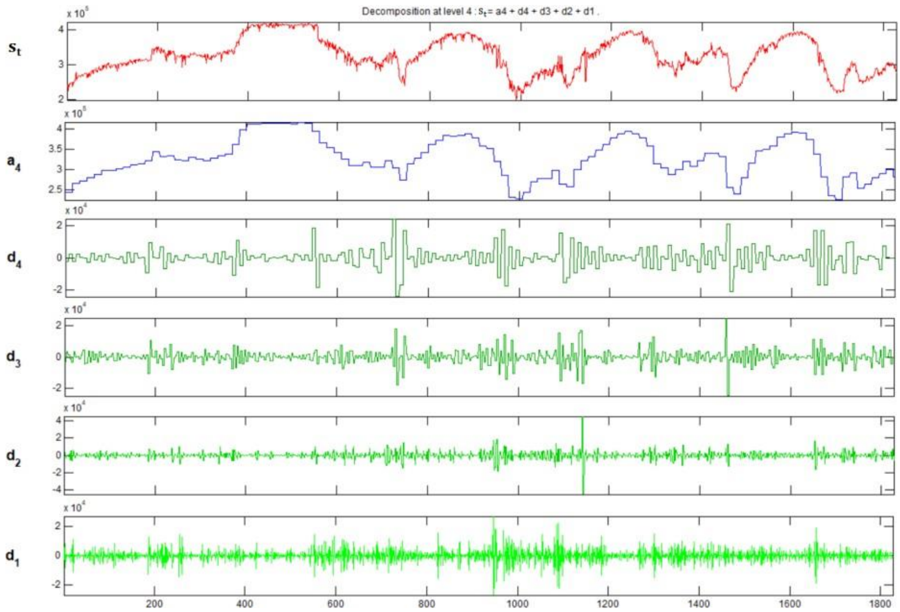

Figure 4. Time–Frequency analysis

***Source**: Research findings*

The series st represents the daily average price of electricity (original signal) based on the statistics published by Iran Grid Management Company. The sample size is 1,825 from 01.03.2013 to 28.02.2018. Data include the average price of electricity purchases in the wholesale market per MWh[3].

The series a4 is an approximate signal representing the daily average price of electricity de-trended at the fourth levels. It is observed that the fluctuations of this series replicate the original series of fluctuations of the daily average price of electricity. The series d4, d3, d2, and d1 show the series of fluctuations or noises (detail signals) at levels of four, three, two, and one, respectively.

di is the time interval of ith level, which is considered the interval (2i, 2i-1). Given that the original signal, the first (d1), second (d2), third (d3), and fourth

[3] Mega Watt per hour.

levels of decomposition (d4) represent the 1 to 2-day, 2 to 4-day, 4 to 8-day, and 8 to 16-day intervals, respectively.

4.2 Unit Root and Randomness Tests

It is necessary to ensure stationary and non-randomness before estimating and predicting a time series model [28]. Thus, the stationary test was performed for all time series generated by the decomposition and reconstruction of the original signal made by the wavelet transformation. In this study, the Augmented Dickey-Fuller (ADF) unit root test is used for the stationary test. The results of the unit root test are presented in Table 2. It is observed that $A4_t$,$D1_t$, $D2_t$, $D3_t$, and $D4_t$ are stationary at the 95% confidence interval. Since the daily average price of electricity (s_t) has a unit root and is non-stationary, the unit root was tested for in 1st difference (Δs_t), and it is stationary at the 95% confidence interval, that is $s_t \sim I(1)$.

The runs test is used for randomness test using a nonparametric approach. This test determines whether the series is set up randomly or systematically. If the variable is random, the series is indicated as unpredictable (Awiagah and Choi, 2018). The null hypothesis of the run test indicates the pattern of randomness of the variables. The result of the test shows that all variables are non-random, and the series are predictable.

Table 2. The result of unit root and randomness tests

Variables	Unit root test (ADF)			Randomness test (run test)			
	t-Statistic[a]	P-values	Result	mean	Z-value	P-values	Conclusion
s_t	-2.798	0.197	non-stationary	313513	-42.64	0.000	Non-randomness
Δs_t	-16.643	0.000	stationary	148	4.42	0.000	Non-randomness
$A4_t$	-3.731	0.020	stationary	313341	-46.15	0.000	Non-randomness
$D4_t$	-9.772	0.000	stationary	0	24.1	0.000	Non-randomness
$D3_t$	-11.449	0.000	stationary	0	-12.73	0.000	Non-randomness
$D2_t$	-14.758	0.000	stationary	0	-28.29	0.000	Non-randomness
$D1_t$	-22.006	0.000	stationary	0	-37.43	0.000	Non-randomness

a: Critical Value of ADF is -3.412 at 95% confidence interval.
Source: *Research findings*

4.3 Estimation of Combined Model: Wavelet- ARIMA-GARCH

4.3.1 Estimation of Approximation Signal A4t (Wavelet-ARIMA)

As shown in Table 2, the approximation signal ($A4_t$) is stationary and the ARMA (1,0) model was proposed for this signal using the lowest values of AIC, SIC, and the highest value of $\bar{R}^2$ (Table 3).

Table 3. Optimal lags of ARMA (p,q) for signal A4

ARMA(p,q)	$\bar{R}^2$	AIC	SIC
ARMA(1,0)	**0.988**	**19.950**	**19.959**
ARMA(1,1)	0.988	19.951	19.963
ARMA(0,1)	0.706	23.193	23.202
ARMA(1,2)	0.988	19.952	19.967

***Source**: Research findings*

The assumptions of no serial correlation between the error terms (LM test) and the homoscedasticity (ARCH test) were met according to the results reported in Table 4. Thus, the precision of the model is confirmed.

Table 4. ARCH test for $A4_t$

	Signal	$A4_t$: ARMA(1,0)
ARCH Test	F-statistic (Prob. F(3,1820))	0.273 (0.844)
	Chi-Square(N×R^2) (Prob. Chi-Square(3))	0.822 (0.844)

***Source**: Research findings*

As a result, the ARMA model (1,0) or AR (1) is selected and estimated as the final model. The results are shown in Table 5.

Table 5. The result of ARMA model for $A4_t$

	Signal	**$A4_t$: ARMA(1,0)**		
Mean equation	Variable	Coef.	t-stat	Prob.
	Constant	320970.6	14.859	0.000
	AR(1)	0.994	320.682	0.000
	SIGMASQ	26880122	163.647	0.000
Diagnostic Tests	$\bar{R}$	0.988		
	AIC	19.950		
	SIC	19.959		

***Source**: Research findings*

4.3.2 Estimation of Detail Signals (Wavelet-ARMA-GARCH)

The detail signals ($D1_t$, $D2_t$, $D3_t$, and $D4_t$) are stationary (Table 2), optimal lags of the ARMA (p,q) model for the detail signals are presented in Table 6.

Table 6. Optimal lags of ARMA(p,q) for Details signals

Signal	ARMA(p,q)	$\overline{R}^2$	AIC	SIC	Signal	ARMA(p,q)	$\overline{R}^2$	AIC	SIC
Signal $D1_t$	**ARMA(4,3)**	**0.5137**	**18.8814**	**18.908**	Signal $D2_t$	**ARMA(1,4)**	**0.7497**	**18.2662**	**18.2873**
	ARMA(3,2)	0.5037	18.8986	18.919		ARMA(1,5)	0.7495	18.2672	18.2914
	ARMA(2,1)	0.5041	18.8968	18.911		ARMA(2,4)	0.74968	18.2666	18.2907
	ARMA(1,0)	0.2018	19.3676	19.376		ARMA(2,3)	0.74908	18.2684	18.2895
	ARMA(0,1)	0.4995	18.9047	18.913		ARMA(3,3)	0.74892	18.2694	18.293
	ARMA(1,1)	0.4993	18.9058	18.917		ARMA(2,2)	0.6658	18.5458	18.5639
	ARMA(2,2)	0.5038	18.8979	18.916		ARMA(1,1)	0.4991	18.9456	18.9576
	ARMA(3,3)	0.5038	18.8991	18.923		ARMA(1,0)	0.0831	19.5456	19.5547
	ARMA(4,4)	0.5134	18.882	18.912		ARMA(0,1)	0.4994	18.9445	18.9535
Signal $D3_t$	**ARMA(2,6)**	**0.8282**	**17.973**	**18.00**	Signal $D4_t$	**ARMA(1,2)**	**0.6520**	**19.4388**	**19.4539**
	ARMA(1,6)	0.8096	18.072	18.099		ARMA(0,2)	0.5962	19.5870	19.5991
	ARMA(0,6)	0.7561	18.313	18.337		ARMA(1,1)	0.6494	19.4459	19.4579
	ARMA(2,5)	0.786	18.181	18.208		ARMA(1,0)	0.6467	19.4540	19.4631
	ARMA(3,5)	0.7901	18.167	18.289		ARMA(0,1)	0.4983	19.8071	19.816
	ARMA(3,4)	0.789	18.170	18.197					
	ARMA(3,3)	0.7477	18.34	18.370					
	ARMA(2,2)	0.6679	18.617	18.635					
	ARMA(1,0)	0.3674	19.252	19.261					
	ARMA(0,1)	0.4968	19.027	19.036					

Source: *Research findings*

Thus, the best estimate of the ARMA (p, q) for the time series $D1_t$, $D2_t$, $D3_t$, and $D4_t$ are respectively obtained as ARMA (4,3), ARMA (1,4), ARMA (2,6), and ARMA (1,2) models. The ARCH test for these models is shown in table 7.

Table 7. ARCH Test for Details Signals

Signal	F-statistic	Prob. F	Chi-Square($N \times R^2$)	Prob. Chi-Square	Result
$D1_t$	66.981	0.000	181.361	0.000	Heteroscedasticity
$D2_t$	45.458	0.000	165.754	0.000	Heteroscedasticity
$D3_t$	5.176	0.000	25.602	0.000	Heteroscedasticity
$D4_t$	84.704	0.000	495.496	0.000	Heteroscedasticity

Source: *Research findings*

Based on the results in Table 7, the constant variance hypothesis is rejected for all detail signals. Thus, it can be modeled as the variance equation. To do this, the coefficients of the variance equation must be positive and their sum, except for intercept, must be less than one. The optimal order of GARCH (p,q) process is determined by information criteria.

Table 8. Optimal lags of GARCH (p,q)for Details Signals

	GARCH (p,q)	AIC	SIC	Positive coefficients	Sum of coefficients < 1
Signal $D1_t$	**GARCH (1,1)**	**18.75086**	**18.78404**	**Yes**	**Yes**
	GARCH (0,1)	18.89725	18.92716	Yes	Yes
	GARCH (1,0)	18.75086	18.78404	No	Yes
	GARCH (2,2)	19.22127	19.26048	No	Yes
Signal $D2_t$	**GARCH (1,1)**	**18.19699**	**18.22414**	**Yes**	**Yes**
	GARCH (0,1)	18.26474	18.28887	No	Yes
	GARCH (1,0)	18.25357	18.2777	No	Yes
	GARCH (2,2)	18.80238	18.83555	No	Yes
Signal $D3_t$	**GARCH (1,1)**	**17.83664**	**17.87284**	**Yes**	**Yes**
	GARCH (0,1)	17.99391	18.02708	No	Yes
	GARCH (1,0)	18.14669	18.17987	Yes	No
	GARCH (2,2)	17.96192	18.00414	No	Yes
Signal $D4_t$	**GARCH (1,1)**	**18.93509**	**18.9562**	**Yes**	**Yes**
	GARCH (0,1)	19.40642	19.42451	Yes	Yes
	GARCH (1,0)	18.84628	18.86438	Yes	No
	GARCH (2,2)	18.85151	18.87865	No	Yes

***Source**: Research findings*

As shown in Table 8, for all $D1_t$, $D2_t$, $D3_t$, and $D4_t$ time series, the GARCH model (1,1) satisfies the necessary conditions. Thus, by combining the ARMA and GARCH model, ARMA(4,3)-GARCH(1,1), ARMA(1,4)-GARCH(1,1), ARMA(2,6)-GARCH(1,1), ARMA(1,2)-GARCH(1,1) are estimated for the signals of $D1_t$, $D2_t$, $D3_t$, and $D4_t$, respectively. Tables 9 and 10 show the results of the models estimation for detail signals. Using the GARCH (1,1) model, it can be observed that residuals of the models have a white noise and normal distribution.

Table 9. The result of ARMA-GARCH for $D1_t$ and $D2_t$

	Signal	**$D1_t$: ARMA(4,3)-GARCH(1,1)**			**$D2_t$: ARMA(1,4)-GARCH(1,1)**		
	Variable	Coef.	t-stat	Prob.	Coef.	t-stat	Prob.
Mean equation	Constant	0.187	1.030	0.302	-0.044	-0.079	0.936
	AR(1)	-1.500	-2.836	0.004	0.497	5.501	0.000
	AR(2)	-0.557	-0.925	0.354	-	-	-
	AR(3)	-0.040	-0.291	0.770	-	-	-
	AR(4)	-0.012	-0.342	0.731	-	-	-
	MA(1)	0.504	1.177	0.239	0.472	123.877	0.000
	MA(2)	-0.973	-37.86	0.000	-1.505	-134234.1	0.000
	MA(3)	-0.525	-1.258	0.208	-0.470	-124.388	0.000
	MA(4)	-	-	-	0.506	15163.05	0.000
Variance equation	Constant	51573.33	8.432	0.000	121393.3	11.285	0.000
	RESID(-1)^2	0.023	14.844	0.000	0.022	13.411	0.000
	GARCH(-1)	0.972	596.0125	0.000	0.953	284.986	0.000
Diagnostic Tests	$\bar{R}$	0.500			0.746		
	AIC	18.750			18.196		
	SIC	18.784			18.224		

***Source**: Research findings*

Table 10. The result of ARMA-GARCH for $D3_t$ and $D4_t$

	Signal	**$D3_t$: ARMA(2,6)-GARCH(1,1)**			**$D4_t$: ARMA(1,2)-GARCH(1,1)**		
	Variable	Coef.	t-stat	Prob.	Coef.	t-stat	Prob.
Mean equation	Constant	0.286	0.370	0.710	49.332	0.569	0.568
	AR(1)	0.805	24.863	0.000	0.389	3.126	0.001
	AR(2)	-0.386	-13.526	0.000	-	-	-
	MA(1)	-0.099	-50.713	0.000	0.313	1.520	0.128
	MA(2)	0.819	45.600	0.000	0.144	1.282	0.199
	MA(3)	-0.003	-4.137	0.000	-	-	-
	MA(4)	-0.995	-5627.664	0.000	-	-	-
	MA(5)	0.094	50.860	0.000	-	-	-
	MA(6)	-0.815	-45.406	0.000	-	-	-
Variance equation	Constant	76651.270	17.348	0.000	383336.1	24.703	0.000
	RESID(-1)^2	0.037	15.798	0.000	0.144	9.333	0.000
	GARCH(-1)	0.943	374.612	0.000	0.846	164.799	0.000
Diagnostic Tests	$\bar{R}$	0.828			0.612		
	AIC	17.836			18.935		
	SIC	17.872			18.956		

***Source**: Research findings*

Figures 5, 6, 7, and 8, respectively, show the matching of the estimated models of $\hat{D}1_t$, $\hat{D}2_t$, $\hat{D}3_t$ and $\hat{D}4_t$ against the actual detail signals of $D1_t$, $D2_t$, $D3_t$, and $D4_t$.

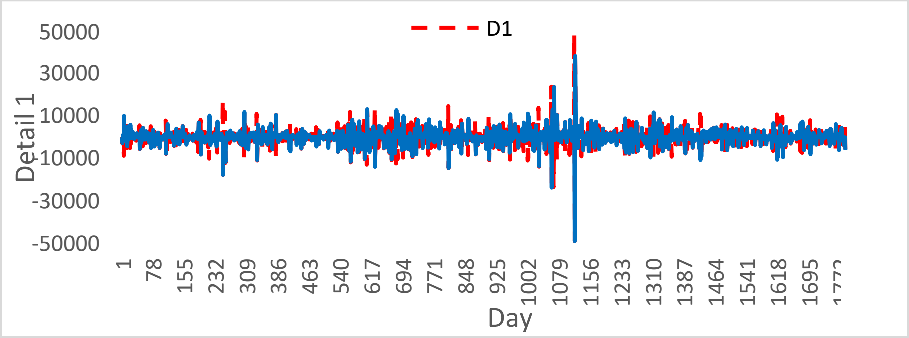

Figure 5. Actual and fitted values of D1 ($D1_t$&$\hat{D}1_t$)

***Source**: Research findings*

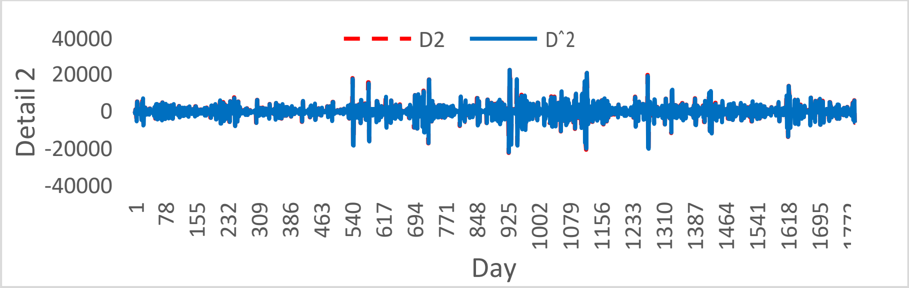

Figure 6. Actual and fitted values of D2 ($D2_t$&$\hat{D}2_t$)

***Source**: Research findings*

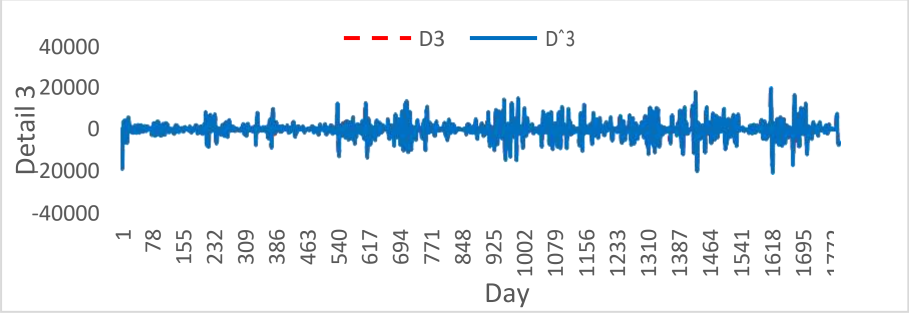

Figure 7. Actual and fitted values of D3 ($D3_t$&$\hat{D}3_t$)

***Source**: Research findings*

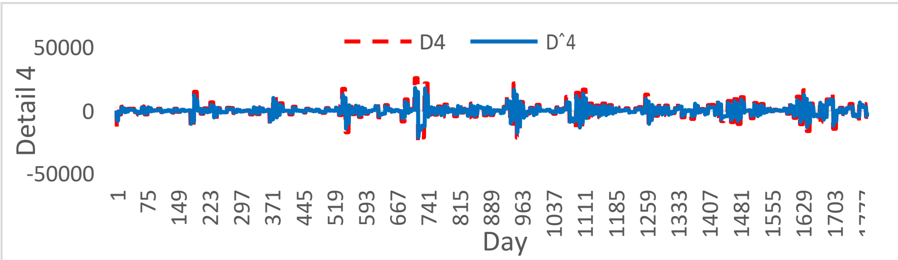

Figure 8. Actual and fitted values of D4 ($D4_t$&$\hat{D4}_t$)

***Source**: Research findings*

4.3.3 Estimation of the Proposed Model (Combined Wavelet-ARMA-GARCH)

The average daily price of electricity (s_t) as an original signal has been decomposed into four levels and five new time series by Haar wavelet. Equation 17 shows the relationship between these five times series and the original signal with less loss (P_t).

Since each time series has different characteristics, diagnostic tests need to be defined. The time series $A4_t$was estimated due to no autocorrelation and homoscedasticity in residuals term with the ARMA model. The time series $D1_t$, $D2_t$, $D3_t$, and $D4_t$were estimated due to no autocorrelation and the existence of the heteroscedasticity with the ARMA-GARCH model. Furthermore, the p and q orders are shown in Table 11.

Table 11. Model selection of proposed models

Signal / Condition	$A4_t$	$D1_t$	$D2_t$	$D3_t$	$D4_t$
Autocorrelation	No	No	No	No	No
ARMA(p,q)	ARMA(1,0)	ARMA(4,3)	ARMA(1,4)	ARMA(2,6)	ARMA(1,2)
Heteroscedasticity	No	Yes	Yes	Yes	Yes
GARCH (p,q)	-	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)	GARCH (1,1)
Proposed model	ARMA	ARMA-GARCH	ARMA-GARCH	ARMA-GARCH	ARMA-GARCH

***Source**: Research findings*

Thus, time series of $\hat{D1}_t$,$\hat{D2}_t$,$\hat{D3}_t$, and $\hat{D4}_t$have been estimated by ARMA-GARCH models. Then, the time series of the average daily price of electricity ($\hat{P}_t$) can be estimated with the aggregation of these time series (see Equation 18). Figure 9 shows matching the actual electricity price (s_t) and its estimated values ($\hat{P}_t$).

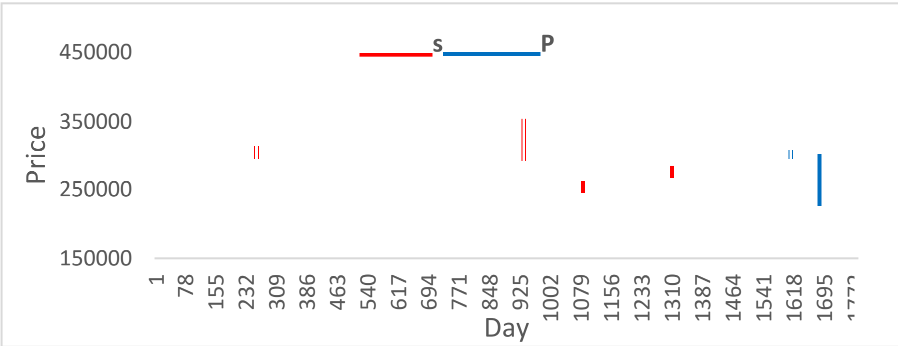

Figure 9. Actual and fitted values of Price (s_t&$\hat{P}$)
***Source**: Research findings*

4.4 Estimation of ARIMA-GARCH Model

To measure the efficiency of the wavelet transform technique, the alternative modeling on the electricity price (s_t) is the ARMA-GARCH model without wavelet transformation. As shown in table 2, electricity price is non-stationary, and Δs_t is used in the form of ARMA (4,5)-GARCH(1,1). Finally, we can compare the power prediction between the Wavelet-ARMA-GARCH and ARMA-GARCH models. The optimal lags of ARMA and GARCH models are presented in Tables 12 and 13.

Table 12. Optimal lags of ARMA(p,q) and ARCH test for selected Model (Δs_t)

	Signal Δs_t			
Info Criterion	ARMA(p,q)	$\bar{R}^2$	AIC	SIC
	ARMA(4,5)	0.122034	20.84402	20.8772
	ARMA(5,4)	0.102908	20.86286	20.89604
	ARMA(4,4)	0.105837	20.86024	20.89041
	ARMA(3,4)	0.099901	20.86519	20.89234
	ARMA(3,3)	0.080274	20.8855	20.90963
	ARMA(2,2)	0.080215	20.88447	20.90256
	ARMA(1,1)	0.069758	20.89461	20.90668
ARCH Test	F-statistic Prob. F(3,1820)		92.843 (0.000)	
	Chi-Square(N×R^2) Prob. Chi-Square(3)		242.093 (0.000)	

***Source**: Research findings*

Table 13. Optimal lags of GARCH(p,q)for Signal Δs_t

GARCH (p,q)	AIC	SIC	Positive coefficients	Sum of coefficients < 1
GARCH (1,1)	**20.56427**	**20.60355**	**Yes**	**Yes**
GARCH (0,1)	20.85929	20.89555	Yes	Yes
GARCH (1,0)	20.68233	20.71859	Yes	Yes
GARCH (2,2)	20.54442	20.58974	No	No

***Source**: Research findings*

The results of the ARMA-GARCH model concerning first difference of electricity price (Δs_t) are shown in Table 14.

Table 14. The results of the ARMA-GARCH Model (without Wavelet transform)

	Signal	**Δs_t : ARMA(4,5)-GARCH(1,1)**		
Mean equation	Variable	Coef.	t-stat	Prob.
	Constant	23.09666	0.235	0.814
	AR(1)	-0.556109	-617.377	0.000
	AR(2)	0.246969	271.753	0.000
	AR(3)	-0.554511	-693.830	0.000
	AR(4)	-1.000575	-1176.395	0.000
	MA(1)	0.280356	11.293	0.000
	MA(2)	-0.396895	-28.672	0.000
	MA(3)	0.619187	97.129	0.000
	MA(4)	0.841104	60.642	0.000
	MA(5)	-0.272441	-11.129	0.000
Variance equation	Constant	13311230	13.103	0.000
	$RESID(-1)^2$	0.401	19.812	0.000
	GARCH(-1)	0.435	16.041	0.000
Diagnostic Tests	$\bar{R}^2$	0.123617		
	AIC	20.56427		
	SIC	20.60355		

***Source**: Research findings*

In order to plot $\hat{\Delta s}_t$, first it converts into s'_t as Equation 19. The matching s'_t and the original price series (s) are plotted in figure 10.

$$\hat{\Delta s}_t = s'_t - s'_{t-1} \Rightarrow s'_t = s'_{t-1} + \hat{\Delta s}_t \qquad (19)$$

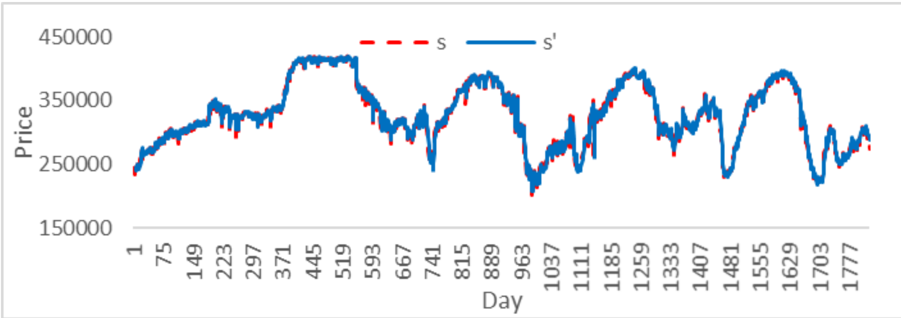

Figure 10. Actual and fitted values of Price(s_t&s_t')
***Source**: Research findings*

4.5 Evaluating Forecast Accuracy

The actual (s_t) and predicted ($\hat{P}_t$) values were compared using a predictive accuracy measure to express the predictive power between the proposed Wavelet-ARMA-GARCH-based model and the ARMA-GARCH-based model. This makes it possible to measure the prediction error value. The Mean Absolute Percentage Error (MAPE) is used to evaluate the predictive power that is not affected by units of measurement (De Myttenaere et al., 2016). By determining "m" as the length of the period, the time series estimation error of the electricity price for Wavelet-ARMA-GARCH-based model is calculated by Equation 20.

$$MAPE_{\hat{P}_t} = \frac{100 \sum_{t=T+1}^{T+m} \left| \frac{\hat{P}_t - s_t}{s_t} \right|}{m} \tag{20}$$

In order to calculate the estimation error of the electricity price with the ARMA-GARCH based model, $\hat{\Delta s}_t$ is first converted to s'_t according to Equation 19. Then, the MAPE is calculated using Equation 21.

$$MAPE_{s^F} = \frac{100 \sum_{t=T+1}^{T+m} \left| \frac{s_t^F - s_t}{s_t} \right|}{m} \tag{21}$$

The mean absolute percentage error for the Wavelet-ARMA-GARCH-based model in the 4-season, including Spring of 2017 to Winter of 2018, is 1.295%, 0.578%, 1.981%, and 1.120%, respectively. MAPE for four-season is equal to 1.244%. MAPE for the ARMA-GARCH-based model estimation during the above periods are respectively 1.653%, 0.814%, 1.988%, and 1.345%, and it is equal to 1.446% for total seasons. Consequently, it is necessary to use a statistical method to determine whether the difference in predictive performance of the two models of Wavelet-ARMA-GARCH and ARMA-GARCH is significant. This study uses the Diebold-Mariano test (DM), which provides a quantitative method to evaluate the prediction accuracy of electricity price forecasting models. The null hypothesis of DM test indicates that there are no significant differences in the performance of two prediction models, whereas the rejection of null hypothesis shows that one prediction model works better than the other[4].

The result of the DM test demonstrates that the absolute value of DM statistic is greater than 1.96 (except Autumn season) and the null hypothesis is rejected at the 5% level of significance. Thus, the predictive accuracy of the Wavelet-ARMA-GARCH model is better than the ARMA-GARCH model.

Table 15. MAPE (%) for the four season of Iran electricity market in spring 2017 to winter 2018

Interval time	Wavelet-ARMA-GARCH (WAG)	ARMA-GARCH (AG)	Diebold-Mariano test	conclusion
Spring	1.295%	1.653%	-2.132 (p-value = 0.0330)	WAG is the better forecast
Summer	0.578%	0.814%	-3.261 (p-value = 0.0011)	WAG is the better forecast
Autumn	1.981%	1.988%	-0.228 (p-value = 0.8190)	Forecast accuracy is equal
Winter	1.120%	1.354%	-2.029 (p-value = 0.0424)	WAG is the better forecast
Total (year)	**1.244%**	**1.446%**	**-2.409 (p-value = 0.0160)**	**WAG is the better forecast**

***Source**: Research findings*

As shown in the Table 15 and Figure 11, the predictive power of the Wavelet-ARMA-GARCH-based model is better than the ARMA-GARCH model. It can be concluded that the proposed model performs well on a seasonal (short-term) and annual (long-term) basis.

[4] The Diebold-Mariano test define theloss differential between the two forecasts by $d_t = L(e_{1.t}) - L(e_{2.t})$. The DM test statistic is $DM = \frac{\bar{d}}{\sqrt{var(\bar{d})}}$. Where $\bar{d}$ is the mean of the coefficient of d_t and var($\bar{d}$) is an estimate of the variance of $\bar{d}$. The null hypothesis of the DM test will be rejected at the 5% level if $|DM| > 1.96$ (Chen and etal., 2014).

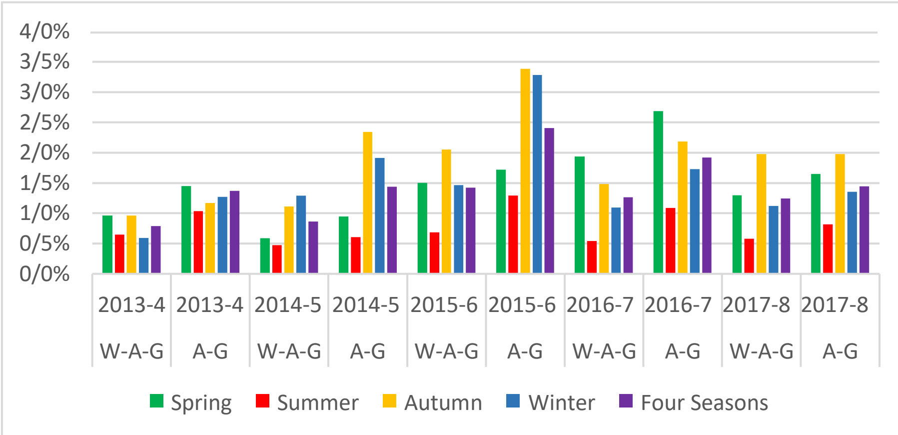

Figure 11. Mean Absolute Percentage Error:
Wavelet-ARMA-GARCH (W-A-G) & ARMA-GARCH (A-G)
***Source**: Research findings*

Figure 12 shows the actual values of the electricity price (s_t) and the fitted values obtained from the Wavelet-ARMA-GARCH Model ($\hat{P}_t$) and ARIMA-GARCH Model (s'_t) in winter 2018. It is mentioned that the mean absolute percentage error for the Wavelet-ARMA-GARCH-based model and ARIMA-GARCH model estimation are 1.120% and 1.354% in winter of 2018, respectively.

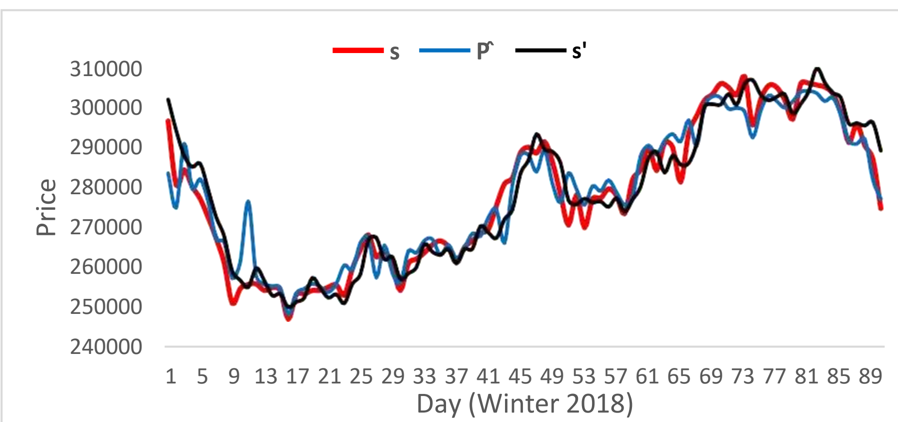

Figure 12. Actual (s_t) and fitted values of Price in
Wavelet-ARMA-GARCH ($\hat{P}_t$) and ARIMA-GARCH (s'_t)
***Source**: Research findings*

5. Conclusion

The electricity price has many complex features such as non-stationarity, nonlinearity, and high fluctuations. Finding the best model to capture these features of the electricity price is very important. This study examines the Iranian electricity market and attempts to present appropriate forecasting method. In this way, the combination of wavelet transformation with the ARMA-GARCH models is investigated as a proposed method. The wavelet-transformation technique has multiple resolution features; it can be used to study the time series with different resolutions. Thus, in order to account for the fluctuations in the structure of the forecasting model, the electricity price were decomposed into time-frequency dimensions using wavelet transformation. In this study, Haar wavelet was used to decompose the electricity price at four levels. Finally, the decomposed signals were estimated using ARMA and GARCH models separately, and the Wavelet-ARMA-GARCH model is called by reconstructing the decomposed fitted signals. Moreover, the ARMA-GARCH model was presented as an alternative forecasting method to compare with the Wavelet-ARMA-GARCH model.

The results show that the Wavelet-ARMA-GARCH-based model in four seasons from Spring of 2017 to Winter of 2018 (including MAPE 1.295%, 0.578%, 1.981%, and 1,120%) has better performance than that of the ARMA-GARCH model (including MAPE 1.653%, 0.814%, 1.988%, and 1.345%) . Accordingly, it can be concluded that the proposed Wavelet-ARMA-GARCH model performs well in forecasting the price of electricity in the Iranian electricity market.

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