Capital-labor substitution and misallocation

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Abstract

We explore the role of the elasticity of substitution between capital and labor (σ) in misallocation of resources. We show, both analytically and empirically using the cross-country firm level survey data, that the extent of misallocation is substantially large for low σ compared to the Cobb-Douglas value of one that the extant literature invariably assumes. When σ is low, dispersion in marginal products of capital across firms will be larger for given dispersion in capital-labor ratios because marginal product now declines more rapidly with increasing capital. The extent of misallocation is even larger when σ varies across firms. Given the overwhelming evidence that σ<1, our findings raise serious concerns about using the Cobb-Douglas production function in the misallocation literature. Careful accounting of the extent of misallocation is also important for policymaking.

Keywords: Misallocation, Elasticity of Substitution; CES Production Function; Total Factor Productivity
JEL Codes: D24; O11; O14; O47

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1. Introduction

There is a burgeoning literature in development macroeconomics that aims to evaluate the extent of misallocation of resources using micro-level data. This line of research is extremely important to understand the causes of vast differences in income per capita across countries. The differences in income per capita is explained, to a great extent, by the differences in the aggregate total factor productivity (TFP), which depends on TFP at the individual production unit level and allocation of resources across these production units. Misallocation occurs when, because of frictions, resources do not flow to the production units to take advantage of the highest marginal returns. Therefore, aggregate output is lower than the potential output that would have been if marginal returns were equalized across production units.

The extent of the misallocation crucially depends on the specification of the production function. The empirical literature, initiated by the seminal work of Hsieh and Klenow (2009),[1] invariably employs the Cobb-Douglas production function that is characterized by a unitary elasticity of substitution between capital and labor (σ).[2] Although this specification is very convenient for empirical exercise, it imposes an unrealistic restriction on σ, a key parameter of the production function. It is now well accepted that the value of σ differs from unity, more specifically it is less than 1 (see, Chirinko, 2008 for a survey; Knoblach, Roessler and Zwerschke, 2020 for a meta-analysis).

Elasticity of substitution refers to the ease with which capital can be substituted for labor when their relative price changes. To understand the role of σ in misallocation, first consider the case of the Cobb-Douglas production function. Under the restriction that $\sigma=1$, the ratio of marginal products of capital and labor can be expressed in terms of the capital-labor ratio. If resources are efficiently allocated, marginal products of both capital and labor would be the same for all production units and so are their capital-labor ratios. In the absence of efficient allocation, there will be dispersion in capital-labor ratios, which is generally referred to capital misallocation. The larger the dispersion in capital-labor ratios, the higher is the capital misallocation. However, if $\sigma\neq1$ (in the CES production function), capital misallocation also depends on σ. The higher is σ, the greater the similarity between capital and labor; thus the

[1] Restuccia and Richard (2013) and Hopenhayn (2014) provide nice reviews on the misallocation literature. See Restuccia and Rogerson (2017) for a lucid discussion.

[2] To the best of our knowledge, Whited and Zhao (2021) is the only study that uses the CES production function in calculating misallocation of financial assets in which the real benefit of finance is defined as a CES aggregate of debt and equity.

incremental capital is easily substituted for labor. Consequently, the capital-labor does not substantially increase from the technological point of view, which in turn resists the pull of diminishing returns to capital (Brown, 1968, p. 50). Therefore, heterogeneous production units differing only by their capital-labor ratios will generate smaller (larger) dispersion of marginal products when σ is high (low), and consequently misallocation will be lower (higher).

Extending the above argument, suppose that the two otherwise similar firms have differential access to credit (in terms of amount of loan) due to credit market imperfections. The firm with preferential access to credit will have lower marginal products of capital (MPK). However, MPK differential will be larger (lower) between the two firms when σ is low (high) because MPK will decline rapidly (slowly) for the firm receiving preferential credit. Aggregate output (efficiency) gain from reallocation of capital between the two firms will also be larger (lower) for the same reason. If these two firms also differ by their values of σ, then MPK differential will depend not only on their values of σ but also on which firm (with higher or lower σ) receives the preferential credit.

Given the importance of σ, this paper revisits the empirical literature on misallocation by introducing the CES production function to allow σ to depart from 1. To place our findings in the context of the extant literature, we evaluate the extent of misallocation and the aggregate output gain from reallocation of resources for different values of σ relative to the Cobb-Douglas value of 1. Specifically, we extend the canonical empirical framework of misallocation (e.g., Hsieh and Klenow, 2009) using the CES production function. In our baseline model, we assume the same σ for all firms (homogenous σ). The total factor productivity revenue (TFPR) is derived from the firm's optimization problem as a CES aggregate of output and capital wedges. The second-order log-linearized TFPR is a non-linear combination of σ, and output and capital wedges. We also derive expressions for efficient aggregate productivity and potential output gain from reallocation of resources.

To allow the possibility that capital-labor ratio of a firm also depends on its σ, we extend the model to vary σ across firms (heterogeneous σ_i). The misallocation now additionally depends on the dispersion of σ_is across firms, and its covariance with output and capital wedges in a highly non-linear way.

Our empirical exercise is based on the World Bank Enterprise Survey (WBES) data. This is a standardized firm level survey of formal businesses at the cross-country level in different years since 2005. The sample firms are in the manufacturing sector classified as ISIC2 codes 15-37 (see Section 3 for construction of our working data). We retain only those countries with at least 30 firms, giving a total of 153 countries. In the case of heterogeneous σ_i's, we vary σ across industry sub-categories (ISIC codes) but assume the same σ for all firms within an industry sub-category (σ_{ind}). We use industry σ_{ind}'s for the USA estimated by Chirinko and Mallick (2017) for all sample countries.

We find that both the TFPR misallocation and the aggregate output gain from reallocation, relative to the Cobb-Douglas value of σ=1, is large when σ is low and less than 1. The TFPR misallocation declines with σ but more rapidly when σ is low. It does not meaningfully depart from 1 for $\sigma \gtrapprox 0.8$. In contrast, the aggregate output gain is monotonically decreasing with σ, and is less than 1 for σ>1. For σ=0.34, which is the (unweighted) mean of heterogeneous σ_{ind}'s in our parameterization using the US values, TFPR misallocation and output gain are approximately 10 and 52 percent larger than the Cobb-Douglas value of 1, respectively. For heterogeneous σ_{ind}'s, TFPR misallocation is 9.5 larger, similar to the case of homogenous σ=0.34. Importantly, the capital misallocation for heterogeneous σ_{ind}'s is more than 300 percent larger than the Cobb-Douglas value of 1. As a counterfactual exercise, if the mean value of heterogeneous σ_{ind}'s is raised to 1 by rescaling σ_{ind}'s proportionately for all firms, capital misallocation is still approximately 46 percent larger. Our results suggest that capital misallocation is more responsive to the value of σ than TFPR misallocation.

However, we find that misallocation or aggregate output gain is not related to the economic development of a country. This finding is consistent with Inklaar et al. (2017), who observed similar lack of relationship using the Cobb-Douglas production function.

Our paper is built on Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) but our novel innovation is the introduction of the CES production function and exploring the role of σ in misallocation.[3] Although this approach abstracts from the origins of misallocation, we review the determinants of σ in the literature and relate them to the origins of misallocation in theoretical models.

Our paper is related to several strands of literature in economics. It identifies σ as an important link between misallocation and economic growth. There is a strand of literature (for example, de La Grandville, 1989; Klump and de La Grandville, 2000; de La Grandville and Solow, 2009) that emphasizes the importance of σ in economic growth. de La Grandville (1989) showed that at any stage of economic development, growth rate of per capita income is

[3] There are several studies that estimate misallocation in the manufacturing sector using the Cobb-Douglas production function that include, among others, Ezra (2013) in Chile, Kalemli-Ozcan and Sørensen (2014) and Cirera, Fattal Jaef and Maemir (2019) in Africa, Inklaar, Lashitew and Timmer (2017) at the cross-country (developing and transition countries) level and Chaudhry, Haseeb and Haroon (2017) in Pakistan. Studies that estimate misallocation in the agricultural sector include, among others, Adamopoulos and Restuccia (2020), Adamopoulos et al. (2022) and Chen et al. (2022). Banerjee and Duflo (2005) summarize microeconomic evidence of misallocation of capital.

increasing with the value of σ.[4] There is even a possibility of perpetual growth without technological progress if σ>1 (and above a critical value). Conversely, there is a gloomy possibility of perpetual slow-down if σ<1 (and below a critical value) (Mallick, 2010). The misallocation literature is based on the premise that larger misallocation leads to lower aggregate TFP, which is one of the reasons for economic underdevelopment. We show that low σ is related to larger misallocation thus undermining the prospect for economic growth.

Development accounting by Caselli (2005) shows the sensitivity of the cross-country income differences to the value of σ. When σ is close to 0.5, variation in productive factors accounts for almost all variations in per capita income across countries. The percentage variation is decreasing in σ and drops to 40 percent when σ equals 1. Our paper reinforces the role of σ in development accounting by documenting that misallocation of productive factors is large when σ is low.

Our paper also provides a potential link between business-cycles and misallocation. Oberfield (2013) and Sandleris and Wright (2014) document that misallocation increased markedly during prolonged recessions (crises) in Chile in the early 1980s and in Argentina in the early 2000s, respectively, resulting in declines in aggregate TFP. Propagation of business cycles also depend on σ. For example, Cantore et al. (2014) show that the business-cycle fluctuations in employment originating from (factor-augmenting) productivity shocks depend on σ although responses vary between RBC and NK models.

Our analytical and empirical results raise concerns about using the Cobb-Douglas production function in both empirical and theoretical misallocation literature. Similar concerns have also been raised in other branches of literature mentioned above. Given the overwhelming evidence of low σ, our results suggest that misallocation is substantially underestimated in the case of the Cobb-Douglas production. As σ is also influenced by institutional and policy frameworks of a country, our findings have important policy implications.

The rest of the paper proceeds as follows. In Section 2, we extend the canonical empirical framework using the CES production function. We describe the data in Section 3. The results are discussed in Section 4. The determinants of σ and their role in misallocation are reviewed in Section 5. Finally, Section 6 concludes.

[4] Mallick (2012) empirically tested this hypothesis and found empirical support.

2. Analytical Framework

We extend the canonical analytical framework by introducing the CES production function at the firm level. We first assume that all firms have the same elasticity of substitution between capital and labor (homogenous σ), and later allow σ to vary across firms (heterogeneous σ_i).

2.1 Homogenous σ

Total output in the manufacturing sector is a CES aggregate of firm (*i*) level output given by

$$Y = \left(\sum_i y_i^{\theta} \right)^{\frac{1}{\theta}}, \qquad \text{---(1)}$$

where $1/\theta$ is the mark-up over price and is given by $\theta = \frac{\gamma - 1}{\gamma}$; γ is the elasticity of substitution between goods produced by different firms. y_i denotes firm *i*'s real output. Cost minimization gives the firm's demand curve, $p_i y_i = \lambda \theta y_i^{\theta}$; here λ is the Lagrange multiplier.

Firm *i* maximizes the following profit function

$$\Pi_i = p_i y_i (1 - \tau_{yi}) - r k_i (1 + \tau_{ki}) - w l_i, \qquad \text{---(2)}$$

where τ_{yi} and τ_{ki} are output and capital wedges, respectively. Output wedge (τ_{yi}) is a tax (or subsidy) on final output affecting firm's output price idiosyncratically without altering the capital-labor composition. Capital wedge (τ_{ki}), for instance a lower than market interest rate paid by a firm due to political connections, impacts on the capital-labor ratio.

Output is produced using the CES production function given by

$$y_i = A_i \left[\alpha k_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) l_i^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \qquad \text{---(3)}$$

where, σ is the elasticity of substitution between capital and labor which is the same for all firms. When σ approaches 1, the CES production function becomes the Cobb-Douglas, $y_i = A_i k_i^{\alpha} l_i^{1-\alpha}$.[5] We assume neutral technological change to be consistent with the misallocation

[5] In the CES specification, $0 < \alpha < 1$ is the distribution parameter and does not have any direct interpretation. However, α becomes the share of capital in total output when σ = 1. In the normalized CES production function (normalized at some baseline values) α is the share of capital in total output at the point of normalization; both A and α become dependent on σ and the baseline values (Klump and de La Grandville, 2000). For simplicity, the baseline values can be set equal to 1 (see, Aquilina, Klump and Pietrobelli, 2006), and our approach can be considered accordingly, given that our empirical exercise is based on cross-section data.

literature that invariably uses a Cobb-Douglas production function in which neutral and factor-biased technological change cannot be differentiated.[6]

Profit maximization gives the following two first-order conditions with respect to capital and labor (combining with the demand function, $p_i y_i = \lambda\theta y_i^{\theta}$), in which marginal revenue products of capital and labor equal to rental rate and wage rate, respectively.

$$MRPK_i = \alpha A_i^{\frac{\sigma-1}{\sigma}}\theta p_i\left(\frac{y_i}{k_i}\right)^{\frac{1}{\sigma}} = \alpha\left(p_i A_i\right)^{\frac{\sigma-1}{\sigma}}\theta\left(\frac{p_i y_i}{k_i}\right)^{\frac{1}{\sigma}} = r\frac{(1+\tau_{ki})}{(1-\tau_{yi})} \qquad \text{---(4)}$$

$$MRPL_i = (1-\alpha)A_i^{\frac{\sigma-1}{\sigma}}\theta p_i\left(\frac{y_i}{l_i}\right)^{\frac{1}{\sigma}} = (1-\alpha)\left(p_i A_i\right)^{\frac{\sigma-1}{\sigma}}\theta\left(\frac{p_i y_i}{l_i}\right)^{\frac{1}{\sigma}} = \frac{w}{(1-\tau_{yi})} \qquad \text{---(5)}$$

When the production function is Cobb-Douglas, MPRK (MPRL) is expressed as the ratio of revenue to capital (labor). In the CES case, marginal revenue products cannot be separated from the TFPR; MPRK (MPRL) is a geometric average of the TFPR and the ratio of revenue to capital (labor) with weights being ρ=(σ-1)/σ and 1-ρ (=1/σ).

The marginal (physical) products of capital and labor are given, respectively, by

$$MPK_i = \alpha A_i^{\frac{\sigma-1}{\sigma}}\left(\frac{y_i}{k_i}\right)^{\frac{1}{\sigma}} \text{ and } MPL_i = (1-\alpha)A_i^{\frac{\sigma-1}{\sigma}}\left(\frac{y_i}{l_i}\right)^{\frac{1}{\sigma}}.$$

Combining equations (4) and (5), the ratio of the marginal products is expressed in terms of capital-labor ratio and σ that in turn can be expressed in terms the capital wedge.

$$\frac{MRPK_i}{MRPL_i} = \frac{\alpha}{1-\alpha}\left(\frac{k_i}{l_i}\right)^{-1/\sigma} = \frac{r}{w}(1+\tau_{ki}) \qquad \text{---(6)}$$

2.1.1 Capital Misallocation

Capital misallocation is defined as the dispersion (standard deviation) of logarithm the ratio of marginal revenue products across firms and is given by

$$sd\left(\ln\frac{MRPK_i}{MRPL_i}\right) = sd\left(1+\tau_{ki}\right) = \frac{1}{\sigma}sd\left(\ln\frac{k_i}{l_i}\right). \qquad \text{---(7)}$$

In the case of the Cobb-Douglas production (σ=1), the capital misallocation is entirely determined by the dispersion in capital-labor ratios across firms. The higher the dispersion in

[6] Without this assumption, the ratio of marginal products in equation (6) would also depend on factor-augmenting technological changes.

capital-labor ratios, the larger is the capital misallocation. However, for any given dispersion in capital-labor ratios, low σ leads to large capital misallocation. The reason is that with increasing capital, MPK declines rapidly when σ is low, thus firms with different capital-labor ratios will generate larger dispersion of marginal products.

Percentage change (gain) in output for firm *i* from decreasing capital wedge by one unit is derived as (see Appendix A)

$$d\ln y_i = -\gamma\left[\alpha + \frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)\right]d\ln(1+\tau_{ki}).$$

The extent of output gain of a firm from removing capital wedge depends both on its σ and capital-labor ratio. For σ<1 (>1), output gain decreases (increases) with capital-labor ratio.

2.1.2 TFPR Misallocation

To derive the *TFPR* in terms of output and capital wedges, we first write the marginal cost of firm *i* in the case of the CES production function as:

$$mc_i = \frac{1}{A_i}\left\{\alpha^\sigma r^{1-\sigma}\left[\frac{1+\tau_{ki}}{1-\tau_{yi}}\right]^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\left[\frac{1}{1-\tau_{yi}}\right]^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$$

$$= \frac{1}{A_i(1-\tau_{yi})}\left\{\alpha^\sigma r^{1-\sigma}(1+\tau_{ki})^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}$$

Given that price level (p_i) is a mark-up over marginal cost, the *TFPR* (p_iA_i) is written as

$$TFPR_i = \frac{1}{\theta(1-\tau_{yi})}\left\{\alpha^\sigma r^{1-\sigma}(1+\tau_{ki})^{1-\sigma} + (1-\alpha)^\sigma w^{1-\sigma}\right\}^{\frac{1}{1-\sigma}}.$$

This expression is highly non-linear; to get an intuitive sense of how σ impacts on the TFPR, we take a second-order Taylor approximation around σ=1:[7]

[7] See Kmenta (1967) for a second-order approximation of the CES production function.

$$
\begin{aligned}
\ln TFPR_i &\approx -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) - \ln\left(1-\tau_{yi}\right) + \alpha\left\{\ln\frac{r\left(1-\alpha\right)}{w\alpha}\left(1+\tau_{ki}\right)\right\} \\
&\quad + \frac{\alpha\left(1-\alpha\right)\left(1-\sigma\right)}{2}\left\{\ln\frac{r\left(1-\alpha\right)}{w\alpha}\left(1+\tau_{ki}\right)\right\}^2 \\
&= -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \left\{\alpha\beta + \frac{\alpha\left(1-\alpha\right)\left(1-\sigma\right)\beta^2}{2}\right\} - \ln\left(1-\tau_{yi}\right) \\
&\quad + \left\{\alpha + \alpha\left(1-\alpha\right)\left(1-\sigma\right)\beta\right\}\ln\left(1+\tau_{ki}\right) + \left\{\frac{\alpha\left(1-\alpha\right)\left(1-\sigma\right)}{2}\right\}\left[\ln\left(1+\tau_{ki}\right)\right]^2
\end{aligned}
\qquad \text{--(8)}
$$

where, $\beta = \ln\left[\frac{r\left(1-\alpha\right)}{w\alpha}\right]$.

The logarithm of TFPR is linear in output wedge but quadratic in capital wedge. Now take the total derivative as

$$d\ln TFPR_i = -d\ln\left(1-\tau_{yi}\right) + \left\{\alpha + \alpha\left(1-\alpha\right)\left(1-\sigma\right)\left[\beta + \ln\left(1+\tau_{ki}\right)\right]\right\} d\ln\left(1+\tau_{ki}\right),$$

and then from equation (6) substitute $\beta + \ln\left(1+\tau_{ki}\right) = -\frac{1}{\sigma}\ln\left(\frac{k_i}{l_i}\right)$ into the above equation to obtain

$$d\ln TFPR_i = -d\ln\left(1-\tau_{yi}\right) + \left\{\alpha + \alpha\left(1-\alpha\right)\frac{\sigma-1}{\sigma}\ln\left(\frac{k_i}{l_i}\right)\right\} d\ln\left(1+\tau_{ki}\right). \qquad \text{---(9)}$$

The effect of capital wedge on TFPR depends not only on whether σ is greater or smaller than 1 but also on the value of capital-labor ratio.[8] For k/l >1 (<1), it is increasing (decreasing) with σ. To see the quantitative implications, we can evaluate the term attached to the capital wedge in the bracket. We set α = 0.35 and consider two values of the capital-labor ratio—0.8 (smaller) and 1.2 (larger). For the lower value of capital-labor ratio, it is positive and decreases with σ approaching 0.299. In contrast, for the larger value of capital-labor ratio, it increases with σ approaching 1.1185 (it is negative for very low value of σ≤ 0.1).

The *TFPR* misallocation is defined as the standard deviation of ln($TFPR_i = p_iA_i$). The extent of misallocation for different values of σ, holding the dispersion in capital-labor ratios fixed, can be understood by examining equation (8). Substituting equation (6) into equation (8), the standard deviation will vary by the absolute value of (σ-1)/σ.[9] The magnitude of this

[8] More precisely, whether the capital-labor ratio is greater or smaller than 1. Since capital-labor ratio is an index number, its magnitude does not have any interpretation. Capital-labor ratio greater (smaller) than 1 can be thought of larger (lower) capital-labor ratio.

[9] To see, equation (6) to be substituted in equation (8).

absolute value is very large for low σ and decreases rapidly approaching 0 when σ=1; it slowly increases afterwards approaching 1, when σ approaches infinity.

2.1.3 Efficient Aggregate Output

Assuming that aggregate inputs are fixed so that firm level capital and labor sum to the respective aggregate levels, $K = \sum_i k_i$ and $L = \sum_i l_i$, at the efficient level of allocation ($\tau_{ki} = \tau_{yi} = 0$), capital-labor ratios will be the same for all firms and proportional to the aggregate capital-labor ratio.

$$\frac{k_i^e}{l_i^e} = \left(\frac{r}{w}\frac{1-\alpha}{\alpha}\right)^{-\sigma}$$
$$\Rightarrow \frac{k_i^e}{l_i^e} = \frac{k_j^e}{l_j^e} = \frac{K}{L} \qquad \text{---(10)}$$

The efficient level of output for firm *i* is derived as:

$$y_i^e = \frac{A_i^\gamma}{\sum_i A_i^{\gamma-1}}\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}. \qquad \text{---(11)}$$

Summing the efficient output of all firms, the efficient aggregate output is obtained as

$$Y^e = \left(\sum_i A_i^{\gamma-1}\right)^{\frac{1}{\gamma-1}}\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \qquad \text{---(12)}$$

where the efficient aggregate TFP is given by $\left(\sum_i A_i^{\gamma-1}\right)^{\frac{1}{\gamma-1}}$.

The aggregate output gain from reallocation is defined as Y^e/Y. It is decreasing with σ, which can be understood intuitively. When capital is reallocated to equalize marginal returns across firms, for aggregate output to increase the marginal contribution of the firm receiving additional unit of capital (lower capital-labor ratio) would be larger than the marginal decrease in output of the firm giving up capital (higher capital-labor ratio). The difference between the marginal contribution and marginal loss from reallocation of capital between the two firms will be larger when σ is low.

2.2 Heterogeneous σ_i

We now allow σ to vary by firms and the production function is rewritten as

$$y_i = A_i\left[\alpha k_i^{\frac{\sigma_i-1}{\sigma_i}} + (1-\alpha)l_i^{\frac{\sigma_i-1}{\sigma_i}}\right]^{\frac{\sigma_i}{\sigma_i-1}}.$$

The capital wedge is an extension of the case for homogeneous σ (the previous derivation goes through)

$$\frac{MRPK_i}{MRPL_i} = \frac{\alpha}{1-\alpha}\left(\frac{k_i}{l_i}\right)^{-1/\sigma_i} = \frac{r}{w}(1+\tau_{ki}).$$

However, the variance of the capital wedge now becomes complicated as it is determined not only by the variance of $\ln(k_i/l_i)$, but also by the variance of $(1/\sigma_i)$ and their covariance in a highly non-linear manner.[10]

$$sd\left(1+\tau_{ki}\right) = sd\left(\frac{1}{\sigma_i} * \ln\frac{k_i}{l_i}\right) \qquad \text{---(13)}$$

However, it can be understood intuitively why misallocation will be larger for heterogeneous σ_i. Suppose, two firms F1 and F2 have the same σ and initially they differed by their marginal products of capital due to their differences in capital-labor ratios. Also suppose that reallocating one unit of capital from F1 to F2 would equalize their marginal products. However, when F1 and F2 also vary by their σ_is, their marginal products of capital will change at a different rate than in the case of homogenous σ, and consequently will not equalize from reallocation of one unit of capital.

Capital-labor ratio of a firm might also depends, in addition to distortions, on how easily it can substitute capital for labor (σ_i). This dependence is accounted for in equation (13) as the variance consists of the covariance of σ_is and capital-labor ratios in a highly non-linear manner (see, footnote 10).

The equation for TFPR in terms of output and capital wedges is also an extension of the case for homogeneous σ (the previous derivation also goes through).

$$\begin{aligned}\ln TFPR_i \approx & -\ln\theta + \ln\left(\frac{w}{1-\alpha}\right) + \left\{\alpha\beta + \frac{\alpha(1-\alpha)(1-\sigma_i)\beta^2}{2}\right\} - \ln\left(1-\tau_{yi}\right) \\ & + \left\{\alpha + \alpha(1-\alpha)(1-\sigma_i)\beta\right\}\ln\left(1+\tau_{ki}\right) + \left\{\frac{\alpha(1-\alpha)(1-\sigma_i)}{2}\right\}\left[\ln\left(1+\tau_{ki}\right)\right]^2\end{aligned} \qquad \text{---(14)}$$

[10] Denoting $(1/\sigma_i) = x_i$ and $\ln(k_i/l_i) = z_i$, the variance in equation (14) can be written as $\mathrm{var}(x*z) = \mu_x^2\upsilon_z^2 + \mu_z^2\upsilon_x^2 + 2\mu_x\upsilon_{x,z^2} + 2\mu_z\upsilon_{x^2,z} + 2\mu_x\mu_z\upsilon_{x,z} + \upsilon_{x^2,z^2} - (\upsilon_{x,z})^2$ where μ is the sample mean of x or z, υ and υ^2 are the sample covariance and variance of x or z (and their squared terms), respectively.

The *TFPR* misallocation is the dispersion of this term. The variance is the sum of variances of each individual term in the RHS (except the first two constant terms) and their covariance. The last two variances in turn are determined by the formula in footnote (10). The covariance terms will also contribute to change in the dispersion in a non-linear manner. The joint determination of σ_i and $\ln(k_i/l_i)$ is accounted for in the variances and covariance in equation (14).

It is important to mention that the aggregate efficient output and therefore the output gain from reallocation cannot be derived for heterogeneous σ_i.

3. Data

3.1 Firm-level data

The World Bank Enterprise Survey (WBES)[11] is the source of our firm-level data. Since 2002, the WBES has been compiling cross-country standardized survey of formal business establishments employing five or more employees, following stratified random sampling method to ensure representativeness of the sample. The merit of this database is that the data are comparable across countries for a wide range of information at the firm level including, among others, financial and economic transactions, access to finance, obstacles to growth, corruption and competition. Although manufacturing and service sectors are the primary business sectors of interest, the firms in the manufacturing sector dominates in the sample. This corresponds to firms classified by ISIC2 codes 15-37 (Rev.3.1). Only formal (registered) companies are the target sample.

For our analyses, we need firm-level information about value added, capital stock and labor. Value added is calculated by subtracting total costs of intermediate inputs from total sales. Costs of intermediate inputs include costs of raw materials, and other expenditures, such as energy (fuel, electricity and water), communication services and transportation, incurred for production. Capital stock is the sum of book value of machinery, equipment and vehicles, and land and buildings. Number of employees is not adjusted for human capital and also data is missing for some firms. Total wage-bill is used instead (see Inklaar et. al., 2017).

[11] http://www.enterprisesurveys.org (accessed on November 3, 2019).

We implement the following steps to prepare the working data: i) only firms in the manufacturing sector are retained,[12] ii) firms having less than five employees are excluded, iii) firms with missing information of sales, intermediate input costs, capital stock, wage bills are excluded; and iv) firms with only positive value added are retained. Outliers are detected by inspecting the capital-output ratio and the capita-labor ratio, and two percent observations from each tail of the distribution of both ratios are deleted. Additionally, two percent observations from each tail of the distribution of the TFPR (for σ=1) are deleted. Finally, countries with at least 30 firms are retained, which gives a total of 153 countries (including some countries that were surveyed in multiple years). A detail list of countries and survey years is provided in Appendix C.

3.2 Elasticity of substitution between capital and labor (σ)

Many studies that attempted to estimate σ mostly focus on the aggregate value in the US context. Although there is no agreement on the precise value of σ, a mounting evidence is in favor of below-unity aggregate value of σ (see, Chirinko, 2008 for a survey; for recent studies see, León-Ledesma, McAdam and Willman, 2010; Chirinko and Mallick, 2017; Knoblach, Roessler and Zwerschke, 2020 for a meta-analysis).[13] For our parameterization, we need the value of σ for different manufacturing sub-categories (ISIC2 codes) at the cross-country level, which, to the best of our knowledge, are unavailable. We use the industry values of σ for the USA estimated by Chirinko and Mallick (2017; Table 5, p. 248). These authors, using the US KLEM data constructed by Dale Jorgenson, estimated the long-run (i.e., at low frequency) value of aggregate σ and also disaggregated values at the industry level that are comparable to ISIC2 codes in the WBES codes (see Appendix B for a complete list of the ISIC2 codes in the WBES and values of σ estimated by Chirinko and Mallick (2017)). These estimates are far below unity for the manufacturing sub-categories ranging between 0.078 (Food and Kindred Products) and 0.562 (Primary Metal Industries) with a (unweighted) mean of 0.34 and a standard deviation of 0.14. These numbers are broadly in line with Raval (2019) who, using manufacturing plant census, estimated σ for manufacturing sub-categories (two digit SIC codes) and found most estimates concentrated between 0.15 and 0.75.

[12] After following these steps, only two countries with more than 30 firms in the service sector would remain in the sample.
[13] Karabarbounis and Neiman (2014) is an exception, who estimated an aggregate value of σ greater than 1.

Recent studies that estimated aggregate σ for the manufacturing sector using micro data also obtained similar ranges. For example, estimates of σ by Raval (2019) and Oberfield and Raval (2021) for the aggregate manufacturing sector using the USA plant level microdata range between 0.3 and 0.7.[14]

Recognizing that the value of σ also varies across countries, we extend our empirical exercise using country level aggregate value of σ (that varies across countries but the same for all manufacturing sub-categories). To the best of our knowledge, Mallick (2012) is the only study that estimated aggregate values of σ for 90 countries using the PWT-6.1 data. The sample countries in Mallick (2012) match with 62 countries in our sample. Given the large variation in the values of σ across countries, we delete 5% extreme values from each tail of the distribution leaving 56 countries. A detail list is provided in Appendix C.

4. Results

We evaluate the extent of misallocation and aggregate output gain resulting from reallocation of resources across firms for different values of σ relative to the Cobb-Douglas value of σ = 1. We evaluate for a plausible range of σ from 0.1 to 2; however, given the overwhelming empirical evidence of low σ, we emphasize the results for σ<1. In the case of heterogeneous σ_is, we assume that σ varies across manufacturing sub-category (ISIC2 codes) but the same in all firms within a sub-category (σ_{ind}), which is not unrealistic.

In our benchmark evaluation, we set $\gamma = 3$,[15] and $\alpha = 0.35$, and retain only those countries with 30 firms. The benchmark results (graphs and tables) are presented as the ratio of the mean; the country average of misallocation (and other statistics) is calculated for different values of σ and it is then divided by the country average for σ=1.

4.1 Capital Misallocation

The extent of capital misallocation is calculated as the standard deviation of the capital wedges in equation (7). Relative to σ=1, it is solely determined by the term (1/σ). Misallocation decreases rapidly (slowly) with σ when σ is low (high).

[14] Herrendorf, Herrington and Valentinyi (2015) used aggregate data to estimate σ for the US manufacturing sector and obtained a higher value of σ (0.80) but it is still below unity.

[15] Although the mark-up over price varies across countries (Duarte and Rosa, 2015; Chirinko and Mallick, 2022), following the misallocation literature we use the same value of γ for all countries.

Figure-1 displays capital misallocation for different values of σ. For heterogeneous σ_{ind}'s, it is defined in equation (13), and is shown as the horizontal line in the Figure. The magnitude is 4.29 times larger than that for σ =1, which corresponds to the respective magnitude at homogenous σ≈0.233. In our data, the (unweighted) mean value of heterogeneous σ_{ind}'s is 0.34. For a stricter comparison, if this mean value is raised to the Cobb-Douglas value of 1 by proportionately increasing all σ_{ind}'s, the capital misallocation is 46 percent larger than that for σ=1. This finding suggests that variation in σ across firms enormously intensifies misallocation.

Analytical comparison of the extent of misallocation between heterogeneous σ_i and homogenous σ is difficult, but an intuitive explanation can be the following. For homogenous σ, marginal products across firms vary due to differences in capital-labor ratios. When firms also vary by their σ_is, marginal products will change at different rates across firms, thus widening the dispersion across firms, which leads to larger misallocation.

Insert Figure 1 here

Hsieh and Klenow (2009) compared misallocation (and efficiency gain) for India and China relative to the USA that has been followed in the subsequent literature as a popular comparison. Note that the direct India-China comparison does not require the USA as the benchmark (USA is also not in the sample). For homogenous σ=1, the capital misallocation is only two percent larger in India than in China, which is actually the dispersions in their capital-labor ratios, and would be the same for any homogenous value of σ. However, with heterogeneous σ_{ind}'s, the capital misallocation becomes 11 percent larger in India than in China.

Insert Table 1 here

The relative position of a country in terms of capital misallocation also differs between homogenous σ and heterogeneous σ_{ind}'s. The three countries with smallest capital misallocation in our sample are from Latin America—Columbia (2006 and 2017), Peru and Argentina. The countries with largest capital misallocation are African and ex-socialist (transition)[16] that include (in order) Angola, Guinea-Bissau, Bosnia and Herzegovina,

[16] Bartelsman, Haltiwanger and Scarpetta (2013) also observed larger distortions in transition economies of eastern and central Europe. Easterly and Fischer (1995) argued that σ is low in a socialist economy because these countries are characterized by a narrow range of capital goods. Some forms of the physical or human capital, such as market-oriented entrepreneurial skills, marketing and distribution skills, and information-sensitive physical and human

Lithuania, Hungary, Belarus, Mauritania, Cameroon and Cote d'Ivoire.[17] To see how the above country ranking would alter in the case of homogenous σ, consider the case of Bosnia and Herzegovina, which now ranks the second in terms of smallest capital misallocation preceded by Serbia and followed by Zimbabwe and Czech Republic.

4.2 TFPR Misallocation

The *TFPR* misallocation is calculated as the standard deviation of TFPR, which can alternatively be considered as the standard deviation of equation (8) for homogenous σ and as the standard derivation of equation (14) for heterogeneous σ_{ind}'s.

The TFPR misallocation is plotted in Figure 2 for different values of homogenous σ relative to σ=1. Table 1 also reports the results for some specific values of σ. The magnitude decreases with σ but rapidly when σ is low. For instance, when σ=0.2, it is 1.16 times larger than that for σ=1, which decreases to approximately 1.05 and 1.01 when σ equals 0.5 and 0.8, respectively. It then stabilizes at around 1 thereafter; indeed, misallocation decreases very slowly up to σ ≈1.2 and then increases but not meaningfully (for example, it is 1.006 at σ≈2). This pattern of change in TFPR misallocation can be understood by examining the standard deviation of equation (8) that we discussed in Section 3.1.2.

For heterogeneous σ_{ind}, the ratio is approximately 1.095 suggesting 9.5 percent larger TFPR misallocation relative to σ=1, which is shown by the horizontal line in the Figure. This extent of misallocation would occur at the value of homogenous σ close to the (unweighted) mean of σ_{ind}'s ≈ 0.34. For an alternative comparison, we re-calculate the overall misallocation by scaling up all σ_{ind}'s proportionately so that their mean value raises (from 0.34) to the Cobb-Douglas value of 1. The TFPR misallocation is only approximately 2 percent larger.

Insert Figure 2 here

Comparison across countries based on heterogeneous σ_{ind}'s and homogenous σ is also informative. For example, going back to the popular India-China comparison, the TFPR misallocation is 7.5 percent larger in India than in China for σ=1, which modestly increases to

capital were missing in a socialist economy. Mallick (2012) used socialist inclination of a country as an instrument for σ to investigate its role in economic growth. Our result is interesting since in our evaluation, σ varies only by industry but the same across all countries.

[17] This comparison is not strict as different countries were survey in different years.

9.6 percent for σ=0.5. In contrast, for heterogeneous σ_{ind}'s, the TFPR misallocation is only approximately 2 percent larger in India than in China.

The country rankings of TFPR misallocation for heterogeneous σ_{ind} and homogeneous σ are broadly similar but different from capital misallocation. For heterogeneous σ_{ind}'s, the five countries with the largest TFPR misallocation are in order: Thailand, Indonesia, Kyrgyz Republic, Nicaragua and Egypt (followed by Philippines, Vietnam, Georgia, Pakistan, and Afghanistan). For σ=1, these countries would now be (in order) Kyrgyz Republic, Indonesia, Thailand, Egypt and Philippines.

4.3 Aggregate Output Gain

The aggregate output gain, defined by ((Y^e/Y) -1) in equation (12) is calculated relative to σ=1, and is displayed in Figure 3 and also presented in Table 1 for some specific values of σ. The gain decreases monotonically with σ but the decrease slows when σ approaches 1. For example, when σ = 0.5, the gain is approximately 28 percent larger, which decreases to approximately 11 percent when σ = 0.7. The gain is greater than 1 for σ < 1, and less than 1 for σ > 1.

One important finding is that for σ<1, the aggregate output gain is larger than the TFPR misallocation and the difference is larger when σ is very low. This result is hard to explain because of extreme non-linearity of the two expressions, and requires further investigation.[18]

Insert Figure 3 here

4.4 σ Varying by Country (σ_c)

This exercise is similar to that for homogenous σ in that σ varies by country but not by industry sub-categories. It is intended to have a better idea about how misallocation would be underestimated, and country ranking would be misconstrued, if a uniform value of σ=1 is imposed on all countries. The sample countries now reduces to 56 countries. The (unweighted) mean of σ_c is 0.21 and ranges between 0.084 (Nicaragua) and 0.686 (Turkey).

[18] This finding resembles the results in a model by Asker, Collard-Wexler and Loecker (2014) in which higher adjustment costs lead to sluggish adjustments in inputs and as a result input variance is lower than revenue variance (Hopenhayn, 2014). If inputs are difficult to substitute, there might be less dispersions in inputs (misallocation) than the potential aggregate output gain. Note that adjustment costs changes the amount of capital stock, while capital wedge changes the price of investment goods, which in turn affects capital accumulation depending on the value of σ.

The country ranking now substantially alters (compared to the ranking in Section 4.1 for σ=1). The top five countries with lowest capital misallocation are Turkey, Nepal, China, Madagascar and Dominican Republic (followed by India), and all these countries have σ_c>0.5. On the other hand, countries with largest capital misallocation (in order) are Nicaragua, Cameron, Guatemala, Romania and Kenya, for all of which σ_c <0.1.

The country ranking in terms of TFPR misallocation is different from that in terms of capital misallocation. The top five countries with lowest TFPR misallocation are Bangladesh, Dominican Republic, El Salvador, Madagascar, and Cameroon. In contrast, countries with largest TFPR misallocation (in order) are Nicaragua, Thailand, Ghana, Nepal and Zimbabwe. Among these countries, Nepal has a high value σ=0.56 and Cameroon has a low value of σ=0.09.

These values of σ_cs should not be taken at the face value, and therefore the results should be treated with caution. Nonetheless, these results are informative and corroborates the previous results that the variation in σs than the variation in capital-labor ratios is more important in explaining capital misallocation, while TFPR misallocation differs due to differences in both σ and capital-labor ratio.

4.5 Misallocation and Output Gain by the Level of Economic Development

A startling result in Inklaar et al. (2017) based on the Cobb-Douglas production function is that misallocation does not vary with the stage of economic development. Here, we replicate their exercise using more recent data and also replicate for heterogeneous σ_{ind}'s. Although the sample countries include mostly developing and emerging economics, there is a large variation in per capita real GDP among them.

In Figure 4, capital misallocation for σ =1 is plotted against log of per capita real GDP in 2005 (Calculated from the PWT-9 data (www.ggdc.net/pwt)). The fit is slightly downward; the regression coefficient is -0.027 and statistically significant at the 5% level, suggesting a (weak) negative relationship.[19] However, for heterogeneous σ_{ind}'s, the negative relationship disappears (Figure 5); the fitted line is flat (the regression coefficient is -0.046 with a robust *t*-statistics of -0.39).

Insert Figures 4-10 here

[19] The result would be the same for any other value of homogenous σ; the regression coefficient would increase (decrease) proportionately for lower (higher) value of σ but the *t*-statistics would remain the same.

A similar pattern emerges in the relationship between TFPR misallocation and income level—a mild negative relationship for homogeneous σ =1 that disappears for homogeneous σ =0.5 and heterogeneous σ_{ind}'s (Figures 6-8). As an example, compare one of the richest and poorest countries in the sample, which are Trinidad and Tobago and Bangladesh. In 2005 per capita real GDP was approximately 14 times larger in Trinidad and Tobago than in Bangladesh. However, for σ =1, TFPR misallocation in Bangladesh (in 2007) is 0.5835 against 0.548 in Trinidad and Tobago (in 2010). For heterogeneous σ_{ind}, the misallocation in these two countries are 0.663 and 0.608, respectively.

The aggregate output gain is plotted for σ =1 and σ=0.5 in Figures 9-10, respectively. Once again, there is no relationship with the level of economic development (the fitted lines are mildly positive). These findings conform Inklaar et al. (2017), and require further investigation for an explanation.

4.6 Addressing Measurement Errors

Concerns about measurement errors in the data have legitimately been raised by several authors including, among others, Bils, Klenow and Ruane (2021). Since our objective is to evaluate misallocation for different values σ relative to σ =1, measurement errors are less of a concern. The statistics are calculated as the ratio of the means (ROM); for each value of σ, first the country average is calculated and then is divided by the country average for σ =1. One might doubt that this approach may not satisfactorily account for measurement errors. To check the robustness, we calculate the mean of the ratios (MOR); first calculate the ratio for different values of σ to σ=1 by country, and then take the country average of the ratios (statistics by country in Appendix C).[20] The results, presented in Appendix Table A1, are strongly robust.

4.7 Robustness

We check the robustness of our results in a variety of ways that are also standard in the empirical misallocation literature (see, Inklaar et al., 2017). In the first robustness check, we reset γ = 5 (Appendix Table A2). Note that this parameter enters the formula for only the aggregate output gain through the aggregate efficient TFP (capital and overall misallocation

[20] Both the ROM and the MOR are biased estimators (Rao, 2002). The ROM estimator is less dependent on the sample size and has a lower statistical uncertainty. Therefore, our benchmark estimation is based on the ROM. In contrast, the MOR estimator assigns an equal weight to each ratio and thus more dependent on the sample size.

are independent of γ). The second robustness check involves deleting 5% of TFPR from each tail of the distribution (based on σ =1), which give more conservative estimates (Appendix Table A3). Finally, we retain only the countries for which at least 100 firm observations are available (Appendix Table A4). A larger number of firms is useful for more precise estimation of the dispersion using σ_{ind}'s, as there will be more firms in each industry sub-category. The number of countries decreases to 64. The results are robust in all cases. The last exercise also suggests that our results are robust to the different combinations of the sample countries.

5. Factors Determining σ and Misallocation

The (indirect) approach that we follow quantifies misallocation and its impact but abstracts from the origins of misallocation. This approach also relies on the structure of the technology to identify misallocation (Hopenhayn, 2014). When Hicks (1932/1963) introduced σ, he realized it as a pure technological parameter. He pointed out the three possible ways in which substitution can take place—intra-sectoral substitution between known methods of production, inter-sectoral substitution of production, and substitution arising from new innovations. If σ is treated as a pure technology parameter, as Hicks did, then low σ can be interpreted as restricted choice of technology leading to misallocation of resources. When the restrictions on technology vary across firms, misallocation will be magnified.

However, σ is also treated as a general measure of the flexibility of the market system and therefore influenced by institutional settings (Klump and Preissler, 2000; Aquilina, Klump and Pietrobelli, 2006, de La Grandville and Solow, 2009; Solow, 2005). These include, among others, strength of labor unions (Maki and Meredith, 1987), customary and regulatory barriers to large changes in capital–labor ratios (de La Grandville and Solow, 2009), country's monetary and financial system (Klump and Preissler, 2000), openness to trade (Saam, 2008) and inclination to socialist system (Easterly and Fischer, 1995). Structural models are needed to understand the role of the specific institutional and policy aspects in misallocation and establish a causal link. Models that address some of the above features include, among others, the role of labor market regulation (Hopenhayn and Rogerson, 1993), credit market imperfections and financial frictions (Buera et al., 2011; Midrigan and Xu, 2014; Gopinath et al., 2017), trade barriers (Bai, Jin and Lu, 2021), and fluctuations in the interest rate leading to persistence in misallocation (Banerjee and Moll, 2010).

Although our paper does not explicitly attribute these specific institutional or policy aspects to the extent of misallocation, our results suggest that the CES production function incorporates these aspects more effectively than the convention approach using the Cobb-Douglas production function.

6. Discussions and Conclusions

Although the role of the elasticity of substitution between capital and labor (σ) has been emphasized in many areas in macroeconomics, it has been neglected in the misallocation literature. We investigate its role both analytically and empirically using the firm-level survey data at the cross-country level. We derive TFPR misallocation in terms of a non-linear combination of σ, and output and capital wedges. To position our contribution in the extant empirical literature, we compare misallocation and its impact for different values of σ relative to the Cobb-Douglas value of 1.

Using the WBES firm-level data at the cross-country level, we find that the TFPR misallocation is large for low σ but decreases then approaches and remains at around 1 for $\sigma \gtrapprox 0.8$. Capital misallocation is more sensitive to the value of σ than TFPR misallocation. The aggregate output gain also decreases monotonically with σ. Given the overwhelming evidence that σ is much smaller than 1, we conclude that the extent of misallocation is considerably underestimated in the empirical literature built on the Cobb-Douglas production function, and recommend using the CES production function.

The elasticity of substitution refers to the ease with which capital can be substituted for labor when their relative price changes. Therefore, low σ is interpreted as greater restrictions on the choice of technology in terms of capital-labor substitution that leads to larger misallocation of resources. The elasticity is also influenced by a country's institutional and policy features; therefore, our results have important policy implications.

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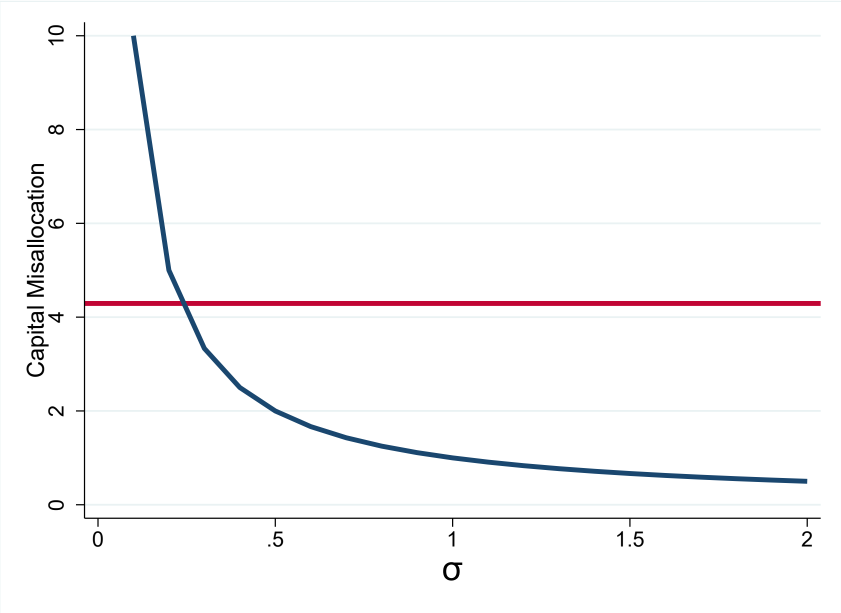
Tables and Graphs

Table 1: Misallocation and Aggregate Output Gain for different values of σ relative to σ = 1.

	Capital Misallocation	$TFPR_i$ Misallocation	Aggregate Output Gain	Aggregate Efficient TFPQ
$\sigma = 0.2$	5.000	1.137	1.733	1.615
$\sigma \approx 0.34$*	2.982	1.098	1.522	1.397
$\sigma = 0.5$	2.000	1.050	1.282	1.212
$\sigma = 1.5$	0.666	1.001	0.940	0.942
Heterogeneous σ_{ind}	4.317	1.095	-----	---

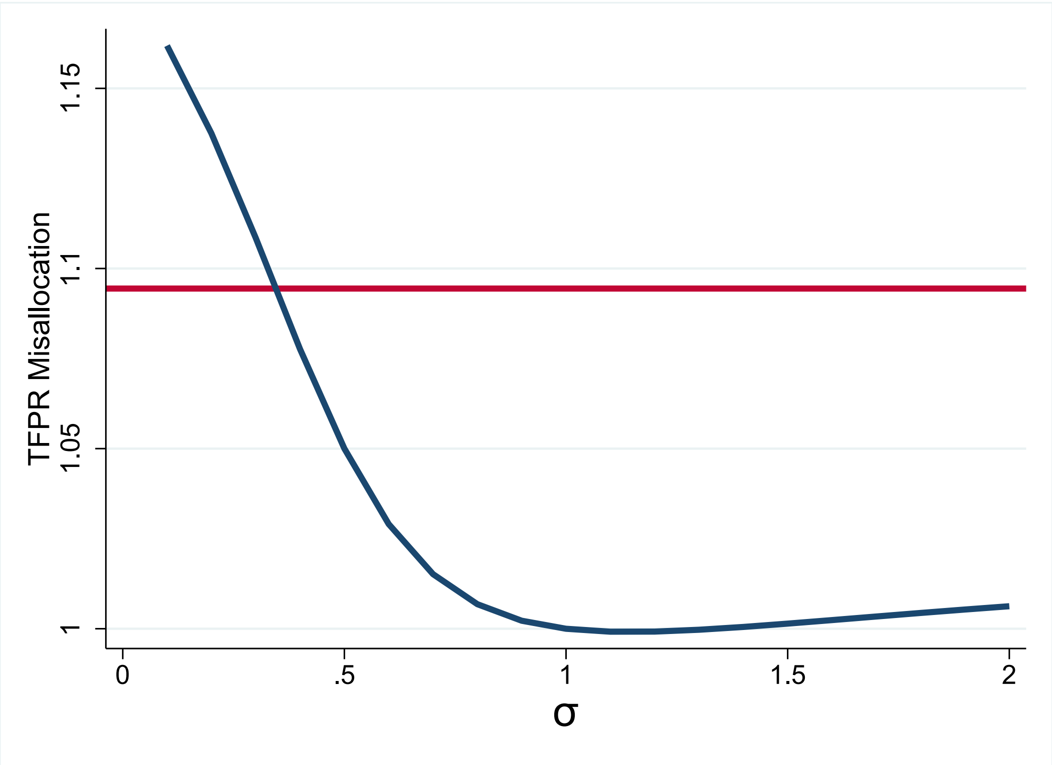
*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous $\sigma = 0.3353529$).

Figure 1: Capital Misallocation for Different Values of σ



Note: The horizontal line is the capital misallocation using heterogeneous σ_i relative to $\sigma = 1$ (=4.291), which is (approximately) equal to the capital misallocation for homogeneous $\sigma \approx 0.24$.

Figure 2: TFPR$_i$ Misallocation for Different Values of σ.



Note: The horizontal line is the TFPR misallocation using heterogeneous σ_i relative to $\sigma = 1$ (=1.094), which is (approximately) equal to the TFPR misallocation for homogeneous $\sigma \approx 0.34$.

Figure 3: Potential Output gain from reallocation for different values of σ.

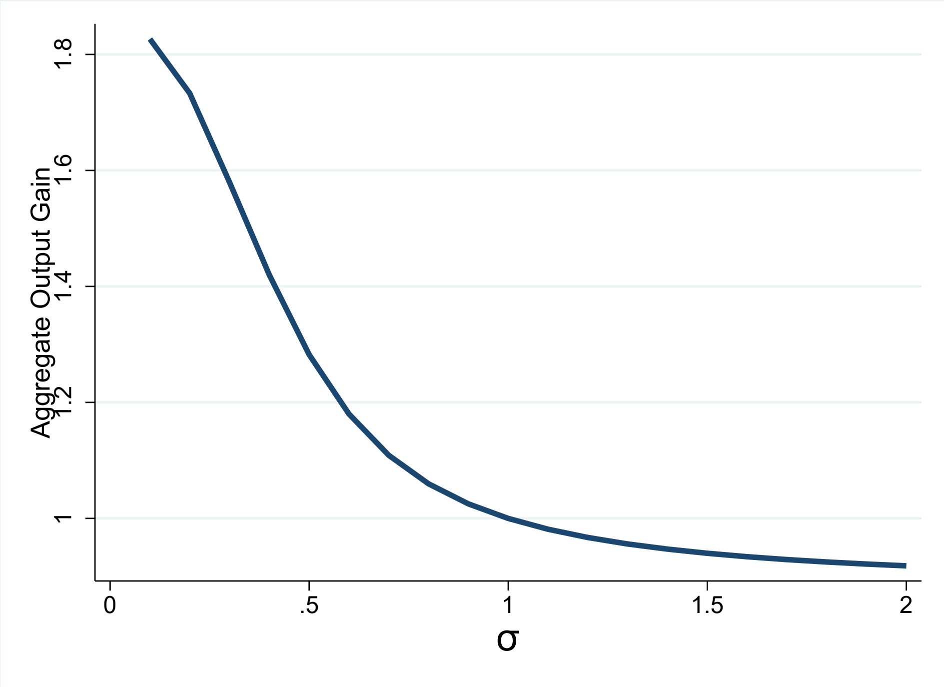
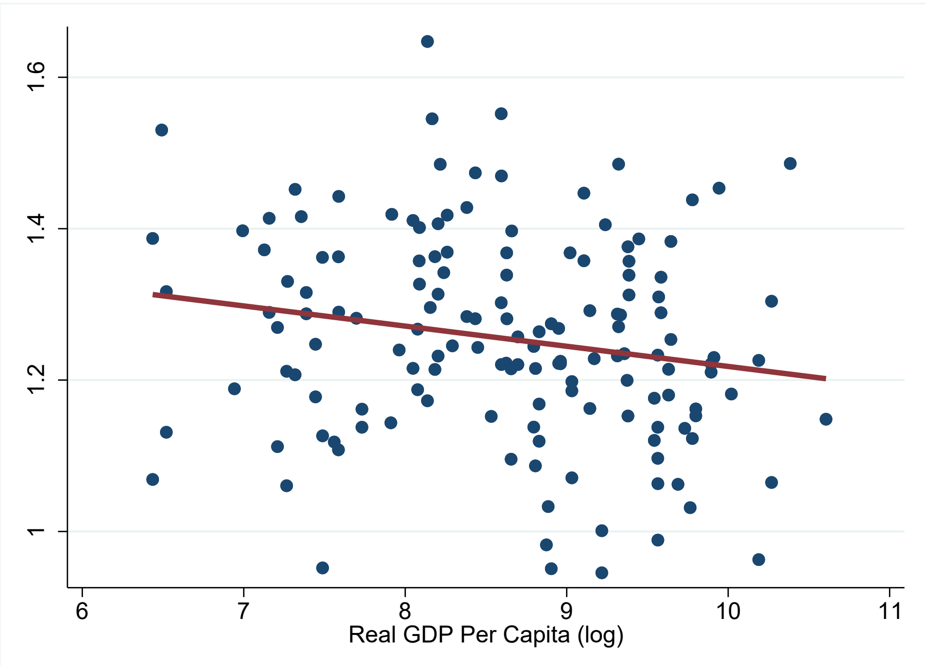
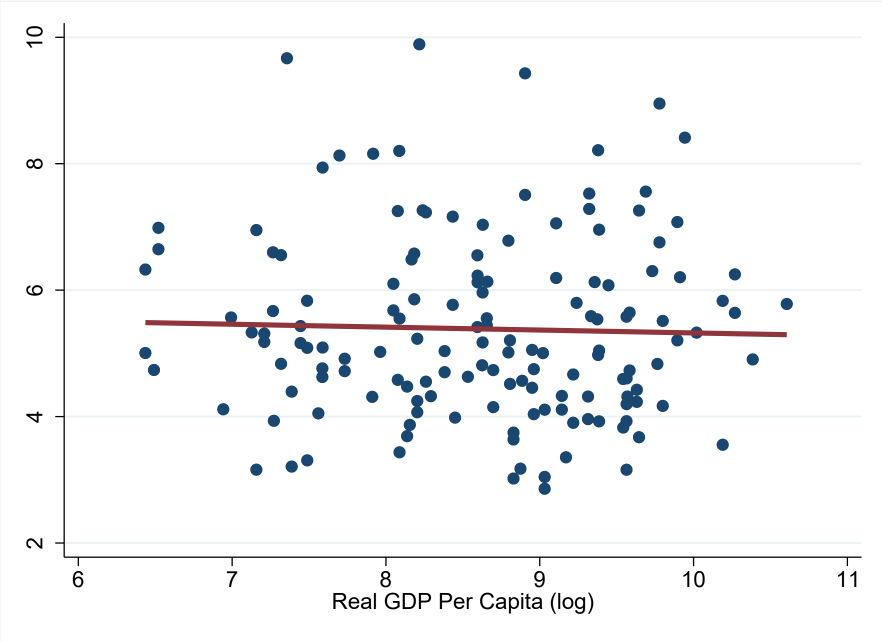


Figure 4: Capital Misallocation and Income Level (Homogeneous σ=1)



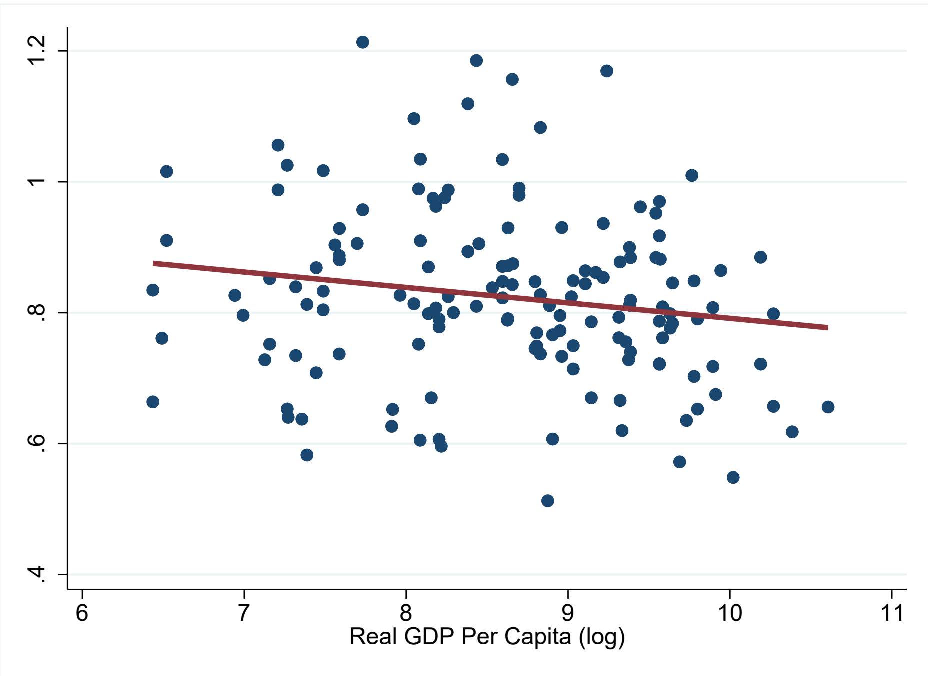
Note: Regression coefficient (robust t-statistics) is -0.027 (-2.18)

Figure 5: Capital Misallocation and Income Level (Heterogeneous σ_{ind})



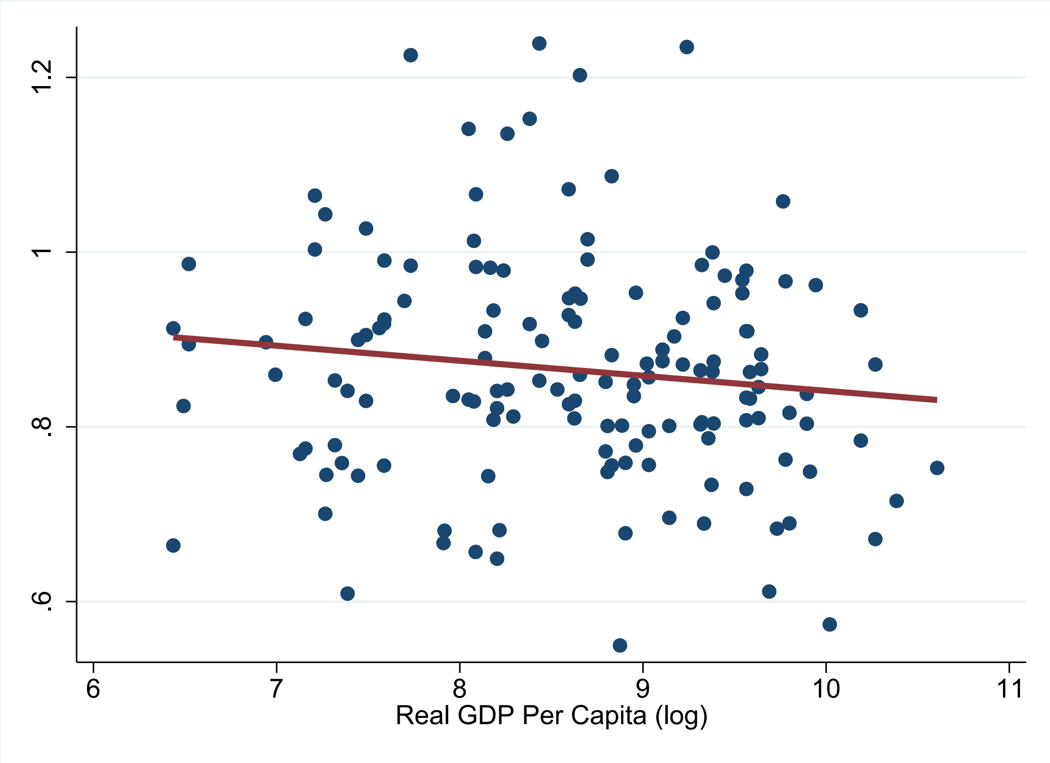
Note: Regression coefficient (robust t-statistics) is -0.046 (-0.39)

Figure 6: TFPR$_i$ Misallocation and Income Level (Homogeneous σ=1)



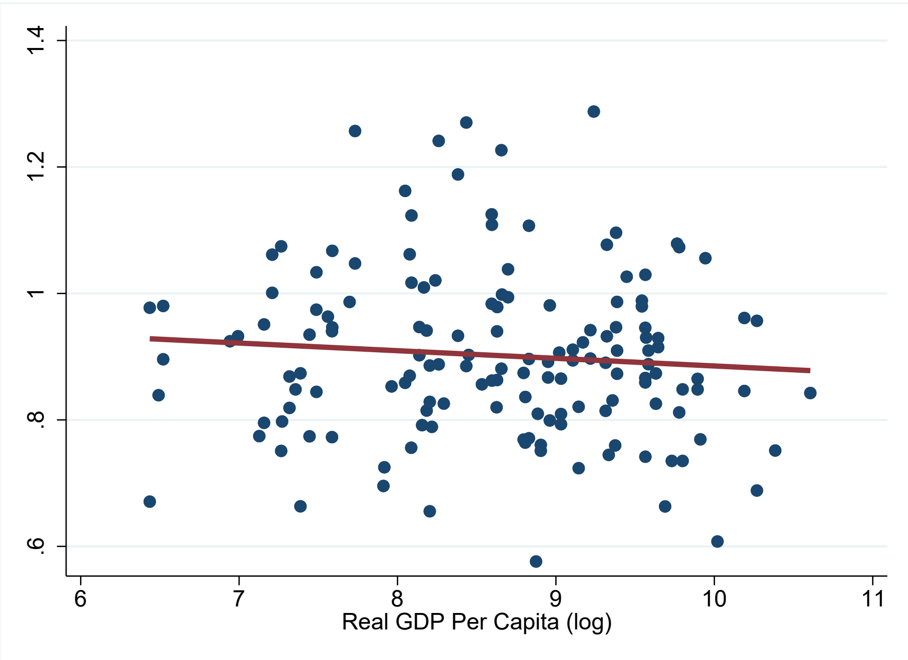
Note: Regression coefficient (robust t-statistics) is -0.024 (-2.11)

Figure 7: TFPRi Misallocation and Income Level (Homogeneous σ=0.5)



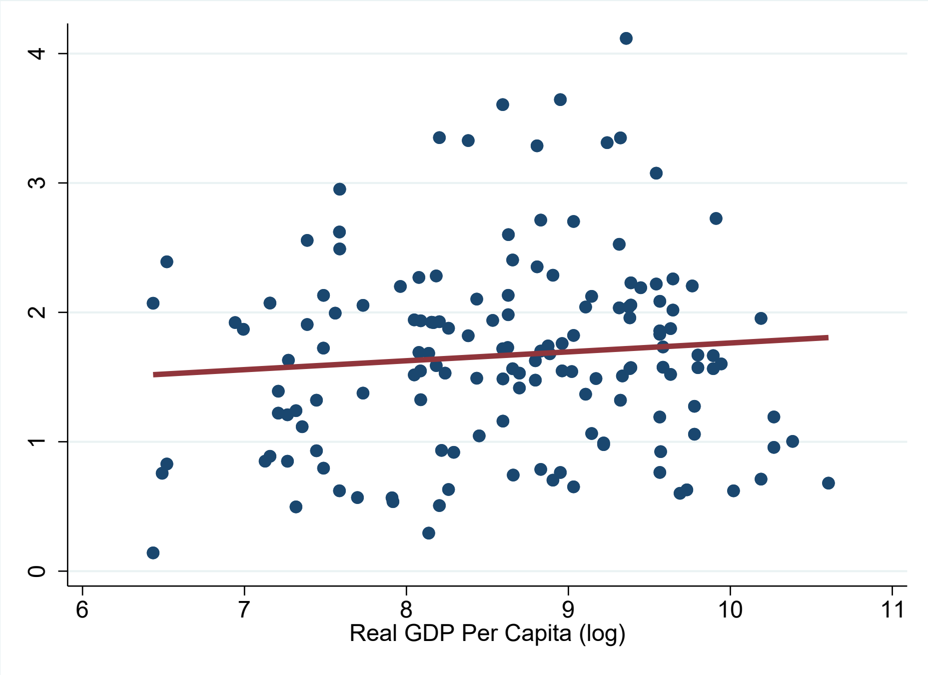
Note: Regression coefficient (robust t-statistics) is -0.017 (-1.63)

Figure 8: TFPRi Misallocation and Income Level (Heterogeneous σ_{ind})



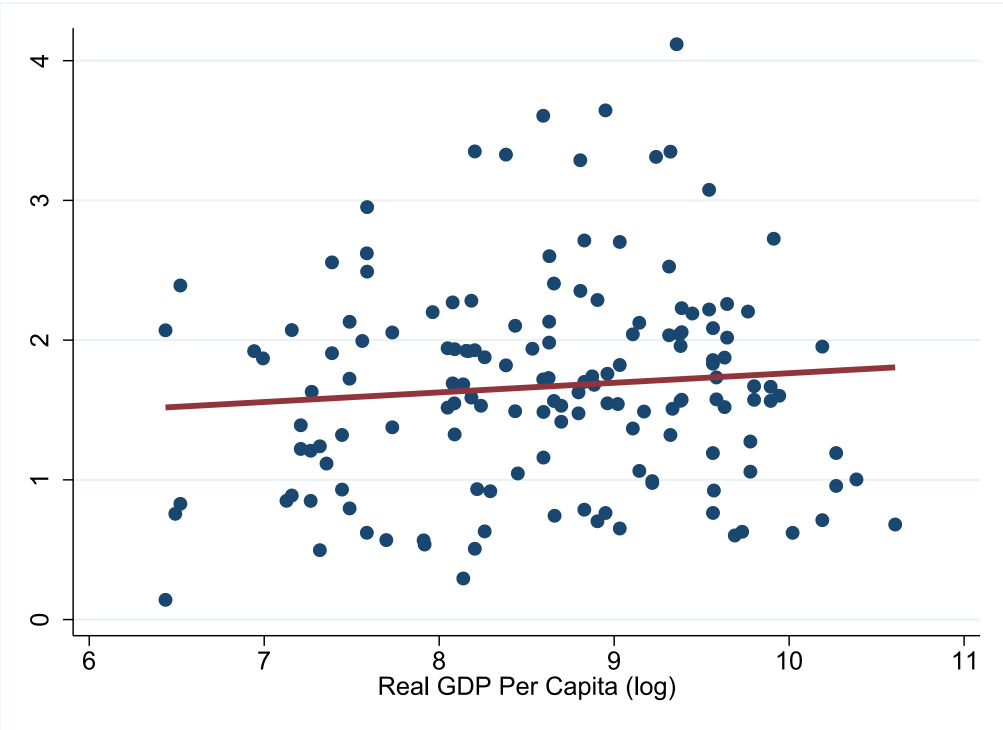
Note: Regression coefficient (robust t-statistics) is -0.012 (-1.12)

Figure 9: Aggregate Output Gain for σ = 1 by income level



Note: Regression coefficient (robust t-statistics) is 0.068 (1.07)

Figure 10: Aggregate Output Gain for σ = 0.5 by income level



Note: Regression coefficient (robust t-statistics) is 0.114 (1.25)

Appendix

Appendix A: Derivation of the output gain from removing capital wedge:

Firm-level output derived from cost minimization in equation (1) is given by

$$y_i = \left(\frac{p_i}{P}\right)^{-\gamma} Y$$

$$MRPL_i = (1-\alpha) A_i^{\rho} \theta p_i \left(\frac{y_i}{l_i}\right)^{1-\rho} = \frac{w}{(1-\tau_{yi})} \qquad \text{----(5)}$$

$$1+\tau_{ki} = \frac{\alpha}{1-\alpha}\frac{w}{r}\left(\frac{k_i}{l_i}\right)^{-1/\sigma} \qquad \text{----(6)}$$

Second-order Taylor approximation of the CES production function in equation (3):

$$\ln\left(\frac{y_i}{l_i}\right) = \alpha \ln\left(\frac{k_i}{l_i}\right) + \frac{1}{2}\frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)^2$$

Taking total derivative of the above equations:

$$d\ln y_i = -\gamma d \ln p_i$$

$$d\ln p_i = \frac{1}{\sigma}\left(d\ln y_i - d\ln l_i\right)$$

$$d\ln(1+\tau_{ki}) = -\frac{1}{\sigma} d\ln\left(\frac{k_i}{l_i}\right)$$

$$d\ln y_i - d\ln l_i = \left[\alpha + \frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)\right] d\ln\left(\frac{k_i}{l_i}\right)$$

Combining the above 4 equations, we obtain

$$d\ln y_i = -\gamma\left[\alpha + \frac{\sigma-1}{\sigma}\alpha(1-\alpha)\ln\left(\frac{k_i}{l_i}\right)\right] d\ln\left(1+\tau_{ki}\right)$$

Appendix B: List of Manufacturing Industries and Values of σ_{ind}.

ISIC2 Classification Codes	Industry Name	σ_i
15	Construction - General Contractors and Operative Builders	0.410
17	Construction - Special Trade Contractors	0.410
18*	Garments	0.408
19	Other Manufacturing	0.246
20	Food and Kindred Products	0.078
21	Tobacco Products	0.312
22	Textile Mill Products	0.204
24	Lumber and Wood Products, Except Furniture	0.484
25	Furniture and Fixtures	0.203
26	Paper and Allied Products	0.148
27	Printing, Publishing and Allied Industries	0.484
28	Chemicals and Allied Products	0.210
29	Petroleum Refining and Related Industries	0.294
33	Primary Metal Industries	0.562
34	Fabricated Metal Products	0.401
36	Electronic and Other Electrical Equipment and Components	0.486
37	Transportation Equipment	0.419

Note: σ_{ind}'s are from Chirinko and Mallick (2017; Table 5, p. 248). *This ISIC2 code does not exactly match with the industry code in Chirinko and Mallick (2017), so the aggregate value of σ is imputed for this industry.

Appendix C: Misallocation and Output Gain for different values of σ relative to σ = 1.

Country	Year	No. of firms	TFPR$_i$ Misallocation		Aggregate Output Gain			σ_c
			σ_{ind} /σ =1	σ_c /σ=1	σ =0.5/σ=1	σ =1.5/σ=1	σ_c /σ=1	
Afghanistan	2008	38	1.050		1.436	0.869		
Angola	2006	147	1.324		1.311	1.010		
Argentina	2006	338	1.101	1.182	1.105	0.995	1.445	0.112
Argentina	2010	511	1.190	1.325	1.275	0.918	1.756	0.112
Argentina	2017	45	1.031	1.038	1.125	0.974	1.659	0.112
Armenia	2009	41	1.141		1.286	0.902		
Azerbaijan	2009	68	1.022		1.051	0.988		
Bangladesh	2007	992	1.139	1.190	1.342	0.874	1.549	0.152
Bangladesh	2013	733	1.075	1.126	1.308	0.928	1.711	0.152
Belarus	2008	43	1.218		1.582	0.877		
Belarus	2013	52	1.167		1.198	0.957		
Bolivia	2006	159	1.032		1.115	0.984		
Bosnia and Herzegovina	2009	58	0.980		1.029	0.994		
Bosnia and Herzegovina	2013	44	1.253		1.454	0.814		
Botswana	2006	84	1.061		1.238	0.912		
Botswana	2010	38	1.028		1.134	0.967		
Brazil	2009	444	1.071	1.145	1.200	0.946	1.562	0.126
Bulgaria	2007	319	1.110		1.192	0.958		
Bulgaria	2009	58	1.116		1.238	0.943		
Bulgaria	2013	62	1.180		1.430	0.918		
Burundi	2006	66	1.103		1.019	1.016		
Cameroon	2009	63	1.112	1.184	1.252	0.911	0.761	0.089
Chile	2006	326	1.124	1.212	1.236	0.949	1.760	0.100
Chile	2010	534	1.166	1.251	1.236	0.926	1.429	0.100
China	2012	1195	1.087	1.031	1.174	0.949	1.144	0.548
Colombia	2006	473	1.080	1.196	1.288	0.934	2.153	0.147
Colombia	2010	510	1.111	1.188	1.093	0.971	1.184	0.147
Colombia	2017	100	1.019	1.060	1.007	1.010	1.061	0.147
Congo, DRC	2006	138	1.011		1.106	0.965		
Congo, DRC	2013	102	1.171		1.835	0.800		
Costa Rica	2010	163	1.100	1.173	0.936	1.034	0.885	0.114
Cote d'Ivoire	2009	67	1.089	1.127	0.963	1.019	0.929	0.144
Croatia	2007	209	1.073		1.212	0.947		
Croatia	2013	73	1.126		1.022	1.006		
Czech Republic	2009	38	1.172		1.550	0.808		
Czech Republic	2013	34	1.086		1.236	0.930		
Dominican Republic	2010	81	1.080	1.038	1.503	0.884	1.496	0.503
Ecuador	2006	204	1.090	1.178	1.892	0.824	3.660	0.126
Ecuador	2010	78	1.155	1.203	1.266	0.963	2.043	0.126

Egypt	2013	860	1.045		1.088	1.006		
Egypt	2016	153	1.061		1.011	1.005		
El Salvador	2006	242	1.138	1.205	1.091	0.985	1.346	0.191
El Salvador	2010	79	1.081	1.184	1.114	0.979	1.131	0.191
El Salvador	2016	60	1.049	1.139	2.116	0.800	4.181	0.191
Estonia	2009	58	1.205		1.181	0.966		
Estonia	2013	49	1.050		1.379	0.894		
Ethiopia	2011	45	0.984		1.131	0.954		
Ethiopia	2015	50	0.965		0.857	1.099		
Georgia	2008	52	1.130		1.413	0.856		
Georgia	2013	36	1.088		1.258	0.941		
Ghana	2007	237	1.157	1.278	0.983	1.028	1.044	0.141
Ghana	2013	85	1.074	1.084	1.302	0.938	1.920	0.141
Guatemala	2006	204	1.078	1.178	1.484	0.850	2.166	0.089
Guatemala	2010	158	1.091	1.177	1.584	0.934	2.934	0.089
Guinea	2006	89	1.211		1.256	0.958		
Guinea-Bissau	2006	32	1.331		1.628	0.866		
Honduras	2006	161	1.009	1.076	1.018	1.017	1.064	0.112
Honduras	2010	47	0.978	1.036	1.184	0.946	1.585	0.112
Hungary	2009	62	1.221		1.190	0.972		
India	2014	3734	1.032	1.009	1.214	0.937	1.203	0.515
Indonesia	2009	455	1.093		1.217	0.926		
Indonesia	2015	165	1.072		1.190	0.939		
Iraq	2011	358	1.020		1.234	0.910		
Israel	2013	98	1.216	1.417	1.765	0.932	3.562	0.136
Jamaica	2010	86	1.124		1.068	0.970		
Jordan	2013	144	1.040	1.060	1.025	1.028	1.101	0.331
Kazakhstan	2009	86	1.227		1.537	0.900		
Kazakhstan	2013	44	1.400		2.065	0.724		
Kenya	2007	318	1.049	1.091	0.947	1.051	1.049	0.094
Kenya	2013	122	1.060	1.113	0.951	1.036	0.974	0.094
Kosovo	2009	31	1.045		0.954	1.028		
Kyrgyz Republic	2009	34	1.036		1.091	0.961		
Kyrgyz Republic	2013	38	1.094		1.199	0.961		
Latvia	2009	56	1.159		1.167	0.951		
Lebanon	2013	83	1.043		1.147	0.979		
Lithuania	2009	57	1.155		1.197	0.938		
Lithuania	2013	37	1.265		1.532	0.846		
Macedonia, FYR	2009	62	1.079		1.374	0.903		
Macedonia, FYR	2013	70	1.035		1.275	0.959		
Madagascar	2009	99	1.116	1.062	2.525	0.796	2.103	0.565
Madagascar	2013	91	1.058	1.017	1.054	0.987	1.040	0.565
Malaysia	2015	143	1.068		1.336	0.861		
Mali	2007	240	1.246		1.265	0.945		
Mauritania	2006	67	1.249	1.290	0.769	1.145	0.670	0.098
Mauritius	2009	84	1.055		1.793	0.808		

Mexico	2006	646	1.063	1.134	1.305	0.915	1.669	0.087
Mexico	2010	881	1.094	1.190	1.279	0.925	1.920	0.087
Moldova	2009	77	1.037		1.099	0.982		
Moldova	2013	31	1.186		1.433	0.956		
Mongolia	2009	100	1.048		1.003	1.013		
Mongolia	2013	31	1.308		1.207	0.949		
Morocco	2013	46	0.997	1.064	0.946	1.043	0.958	0.102
Mozambique	2007	226	1.171		1.519	0.905		
Myanmar	2014	89	1.074		1.263	0.922		
Myanmar	2016	31	1.149		1.353	0.932		
Namibia	2006	77	0.999	0.996	0.969	1.019	0.945	0.200
Nepal	2009	66	1.014	1.008	1.373	0.909	1.288	0.563
Nepal	2013	158	1.005	1.004	1.057	0.987	1.044	0.563
Nicaragua	2006	164	1.077	1.119	1.178	0.959	1.499	0.084
Nicaragua	2010	41	1.257	1.343	2.343	0.751	5.701	0.084
Nigeria	2007	817	1.182	1.292	2.623	0.756	4.956	0.177
Pakistan	2007	76	1.118		1.347	0.960		
Pakistan	2013	89	1.086		1.188	0.923		
Panama	2006	87	1.067	1.062	1.230	0.940	1.505	0.265
Paraguay	2006	89	1.032		1.180	0.968		
Paraguay	2010	58	1.032		1.419	0.837		
Peru	2006	193	1.083		2.082	0.831		
Peru	2010	436	1.046		1.164	0.948		
Peru	2017	50	1.022		1.246	0.925		
Philippines	2009	305	1.044		1.091	0.979		
Philippines	2015	130	1.062		1.170	0.950		
Poland	2009	52	1.157		1.584	0.815		
Romania	2009	52	1.162	1.272	1.036	1.000	1.353	0.085
Romania	2013	82	1.127	1.191	1.329	0.938	2.144	0.085
Russian Federation	2009	235	1.082		1.161	0.961		
Russian Federation	2012	374	1.186		1.360	0.951		
Rwanda	2006	42	1.064		1.227	0.939		
Senegal	2007	204	1.110		1.281	0.933		
Serbia	2009	88	1.006		0.885	1.062		
Serbia	2013	53	1.051		1.004	1.005		
Sierra Leone	2017	42	1.118		1.436	0.892		
Slovak Republic	2009	37	1.139		1.332	0.922		
Slovenia	2009	71	1.048		0.961	1.020		
Slovenia	2013	51	1.198		1.671	0.890		
South Africa	2007	574	1.201		1.401	0.915		
South Sudan	2014	39	1.082		1.139	0.984		
Sri Lanka	2011	181	1.053	1.037	1.283	0.954	1.399	0.428
Swaziland	2006	57	1.100		1.305	0.952		
Sweden	2014	99	1.284		1.521	0.878		
Tajikistan	2008	37	1.066		1.216	0.940		
Tanzania	2006	168	1.035		0.987	1.017		

Tanzania	2013	98	1.115		1.538	0.847		
Thailand	2016	171	1.101	1.130	1.598	0.858	2.421	0.197
Trinidad and Tobago	2010	74	1.108		1.094	0.981		
Tunisia	2013	197	1.044	1.091	1.349	0.885	1.680	0.134
Turkey	2008	341	1.029	1.006	1.251	0.914	1.127	0.686
Turkey	2013	253	1.118	1.035	1.352	0.920	1.146	0.686
Uganda	2006	206	1.150		1.026	0.999		
Uganda	2013	39	1.048		0.951	1.047		
Ukraine	2008	146	1.055		1.026	0.994		
Ukraine	2013	188	1.090		1.348	0.930		
Uruguay	2006	136	1.069		1.124	0.977		
Uruguay	2010	137	1.123		1.618	0.879		
Uzbekistan	2008	69	1.048		1.369	0.886		
Uzbekistan	2013	71	1.015		1.053	0.988		
Vietnam	2009	515	1.055		1.319	0.924		
Vietnam	2015	99	1.060		1.212	0.942		
West Bank And Gaza	2013	50	1.046		1.062	0.986		
Yemen	2010	65	1.036		1.116	0.963		
Zambia	2007	239	1.093	1.178	1.254	0.936	1.634	0.133
Zambia	2013	89	1.076	1.116	1.015	1.007	1.085	0.133
Zimbabwe	2011	233	1.014	1.011	1.074	0.983	1.138	0.259
Zimbabwe	2016	46	1.016	1.036	1.472	0.865	1.815	0.259

Note: σ_{ind} = Heterogeneous σ that varies across ISIC2 classification industry codes but the same across all countries. σ_c = Homogenous σ that is the same across all industries but varies across countries.

Appendix Table A1: Misallocation and Aggregate Output Gain for different values of σ relative to σ = 1 (Mean of the Ratio).

	Capital Misallocation	$TFPR_i$ Misallocation	Aggregate Output Gain	Aggregate Efficient TFPQ
σ = 0.2	5.000	1.145	1.718	1.741
σ ≈ 0.34*	2.982	1.103	1.512	1.501
σ = 0.5	2.000	1.053	1.277	1.270
σ = 1.5	0.667	1.001	0.940	0.931
Heterogeneous σ_{ind}	4.328	1.099	---	-----

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ = 0.3353529).

Appendix Table A2: Misallocation and Aggregate Output Gain for different values of σ relative to σ = 1. (γ = 5)

	Aggregate Output Gain	Aggregate Efficient TFPQ
σ = 0.2	1.862	1.862
σ ≈ 0.34*	1.522	1.397
σ = 0.5	1.302	1.288
σ = 1.5	0.949	0.934

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ = 0.3353529).

Appendix Table A3: Misallocation and Aggregate Output Gain for different values of σ relative to σ = 1. (Gamma = 3; TFP-CD deleted 5%).

	$TFPR_i$ Misallocation	Aggregate Output Gain	Aggregate Efficient TFPQ
σ = 0.2	1.181	1.793	1.591
σ ≈ 0.34*	1.130	1.568	1.402
σ = 0.5	1.068	1.308	1.222
σ = 1.5	1.000	0.934	0.938
Heterogeneous σ_{ind}	1.127	---	-----

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ = 0.3353529).

Appendix Table A4: Misallocation and Aggregate Output Gain for different values of σ relative to σ = 1: Number of firms >=100.

	$TFPR_i$ Misallocation	Aggregate Output Gain	Aggregate Efficient TFPQ
σ = 0.2	1.135	1.699	1.565
σ ≈ 0.34*	1.096	1.507	1.382
σ = 0.5	1.049	1.277	1.211
σ = 1.5	1.002	0.942	0.939
Heterogeneous σ_{ind}	1.087	----	---

*Mean value of heterogeneous σ_{ind}'s is 0.3353529 (results compared with homogenous σ = 0.3353529). Number of country-year = 64.