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Abstract: This paper studies a non-cooperative bargaining problem with one buyer and many sellers, focussing on the tension between the complementarity intrinsic to such a setup and efficiency. We address this problem in a very general setup with a technology that allows for variable degrees of complementarity, a bargaining protocol that is symmetric and allows for both secret, as well as publicly observable offers, and strategies that allow for history dependence. We examine equilibria for all parameter values. Interestingly, and in contrast to most of the literature, we demonstrate that there is a large class of parameter values such that an asymptotically efficient equilibrium with a positive buyer payoff exists - thus demonstrating that strategic holdout is not a serious obstacle to the working of the Coase theorem. For robustness we examine alternative contractual forms, i.e. conditional and equity contracts, as well as variations that allow for multiple project implementation and asymmetric sellers.

JEL Classification Number - C78, D23, D62, L14.

Keywords - Multi-person bargaining, holdout, complementarity, efficiency, Coase theorem.

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1 Introduction

This paper studies a non-cooperative bargaining problem with one buyer and many sellers, focussing on the tension between the complementarity intrinsic to such a setup and efficiency. We address this problem in a very general setup with a technology that allows for variable degrees of complementarity, a bargaining protocol that is symmetric (in a sense described later) and allows for both secret, as well as publicly observable offers, and strategies that allow for history dependence.

Many economic activities involve a single buyer seeking to acquire and combine objects from several sellers, e.g. drug development often requires separate patents, land developers have to combine separate plots of land and firms often purchase assets of other firms. Further, firms often bargain with multiple unions, and, in case of financial distress, with multiple creditors. Coase's (1960) famous railroad example considers a situation where a railroad has to acquire plots of land from several farmers.[1]

Given the complementarity inherent in all such activities, received wisdom suggests that the outcome is likely to exhibit *holdout*, with sellers refusing to transact until others have already done so, when commencing production becomes more profitable, allowing the sellers who holdout to extract a greater share of the surplus. In such a scenario holdout is expected to cause inefficiencies, viz. delay, or the implementation of an inefficient project, even, in the presence of strong complementarities, a complete breakdown of negotiation.[2] The literature on one-buyer-many-seller bargaining problems with complementarity, viz. Cai (2000, 2003) and Menezes and Pitchford (2004), in fact demonstrates inefficiencies in the form of delays, thus seemingly corroborating this intuition.

The central motivation of this paper is to re-visit this conclusion, *inter alia* examining the applicability of the Coase theorem in this setup. While it is well known that informational problems can lead to inefficiencies, the literature on coalitional bargaining has identified strategic issues that may, even in the absence of informational issues, cause the Coase theorem to fail.[3] While one response to such strategic inefficiency has been to study random bargaining protocols, e.g. Okada (1996), another line of research examines bargaining protocols with renegotiation,

[1] Similarly, Cournot analyzed a problem where a brass manufacturer has to buy copper and zinc from two monopoly suppliers.

[2] In the context of land acquisition, many countries, including the USA, have promulgated eminent domain laws (that allow land acquisition for public purposes on payment of compensation), presumably to counter this holdout problem. One of our motivating examples comes from West Bengal, India, where the state government used the Land Acquisitions Act, 1894, to acquire land for building an automobile factory for Nano (the one lakh rupee car) in Singur. It has also been argued by some, e.g. Parisi (2002), that problems like excessive fragmentation can be be traced, at least partially, to such holdout problems. In the patents literature, Shapiro (2001), suggests that holdout can be serious obstacle to R&D.

[3] Chatterjee et. al. (1993), Bloch (1996) and Ray and Vohra (1997, 1997), among others, have pointed out the role of renegotiation in this context.

Seidmann and Winter (1998), Hyndman and Ray (2007), etc. In this paper, however, we examine a *deterministic* (though symmetric) bargaining protocol that does *not* allow for renegotiation. Remarkably enough, even then for a large class of parameter values we find that there is *some* equilibrium that is *asymptotically efficient*, i.e. the grand project is implemented and delay costs, if any, goes to zero as the discount factor approaches 1. Further, for a certain range of parameter values there is an efficient equilibrium that implements the grand project without any delay.

To this end we consider the interaction between one buyer and $n \geq 2$ sellers, all of whom have an object to sell. These objects can be combined to produce value. In particular, all sellers have identical objects, with a project using m objects having value $v(m)$. We assume that $v(m)$ is strictly super-additive, with $v(0) = 0$ and $v(n) = 1$.[4] This formulation allows for different degrees of complementarity, with the buyer being allowed to implement a 'partial' project that does not require the use of all objects.

The negotiation process that we consider is a natural extension of the Rubinstein (1982) bargaining model in which the agents, the buyer, as well as the sellers, make simultaneous offers to the other side of the market in alternate periods. Note that this protocol is symmetric in the sense that at no point during negotiation, is an active seller shut out of the bargaining process. As is standard in this literature, we introduce negotiation costs by assuming that agents discount future payoffs. Two scenarios are considered: one in which the offers made by the buyer to the sellers are publicly observable, so that the sellers get to observe the entire vector of offers made by the buyer and another in which the offers are 'secret' so that given a vector of offer, a seller can only observe his component of the offer vector.

The question of interest is the possibility of obtaining equilibria that are both asymptotically efficient, *and* in which the buyer obtains a strictly positive payoff.[5] The requirement that the buyer obtains a strictly positive payoff seems natural. This is because with a zero (or arbitrarily small) payoff for the buyer, implementing a project is problematic whenever, as seems realistic, there is some startup cost for the buyer. Unlike much of the literature where inefficiency takes the form of bargaining delays (see for example Cai (2000, 2003), and Menezes and Pitchford (2004)), in our set up, inefficiency can also result if the buyer fails to implement the grand project, the project that combines all the objects. We plan to study these properties when agents are sufficiently patient, i.e. in the limit, as discounting vanishes.

Our results indicate that the marginal contribution of the *last two* sellers, i.e. $1 - v(n-2)$ and $1 - v(n-1)$, are critical in determining whether the buyer can obtain a strictly positive payoff in an equilibrium. It turns out that if these contributions are very large, to be precise $v(n-2) = 0$ and $v(n-1) < \frac{1}{2}$, then it is impossible to support a strictly positive payoff for the

[4] Perfect complementarity thus arises as a special case where $v(m) = 0$, $\forall m < n$,.

[5] Given the folk-theorem like results in Chatterjee et. al. (1993) and Herrero (1985), the most that we can hope for here is the existence of at least one equilibrium that is efficient, at least asymptotically. See Hyndman and Ray (2007) on this issue though.

buyer in the limit as discounting vanishes. For the same set of parameter values, however, we establish the existence of an equilibrium outcome in which the grand project is implemented in the second period.

When $v(n-2) > 0$ or $v(n-1) > \frac{1}{2}$, then it is always possible to support strictly positive payoff for the buyer in *some* equilibrium. Moreover, when $v(n-1) > \frac{1}{2}$, there is an *efficient* equilibrium where the grand project is implemented in the first period and, moreover, the buyer obtains a payoff of 1. This result holds under both bargaining protocols, public, as well as secret offers. Next suppose that $v(n-1) < \frac{1}{2}$. We find that a buyer payoff close to $\frac{1}{2}$ can be sustained when the offers are publicly observable, while a buyer payoff close to $v(n-2)$ can be sustained for the secret offers case. Furthermore, in both these cases, the buyer implements the grand project, albeit at the end of period 2.

To summarize, these results imply that the holdout problem is 'resolved', in the sense of there being some 'efficient' equilibrium with a positive buyer payoff, provided either $v(n-2) > 0$, or $v(n-1) > \frac{1}{2}$. In fact, for $v(m)$ satisfying increasing returns,[6] the above two conditions reduce to a single one, namely $v(n-2) > 0$. For $v(m)$ satisfying increasing returns therefore, the holdout problem is 'resolved' whenever $v(n-2)$ is positive, no matter how small - results that clearly run counter to the literature. These results are even more surprising since, by insisting on equilibria that are not only 'efficient' but involves a positive buyer payoff, we bias our framework in favor of holdout. Further, these results imply that the Coase theorem does largely go through in this context.

While the proofs are rather involved (especially in view of the fact that we do not invoke stationarity), the intuition can be simply stated. First, assume that $v(n-2) = 0$ and $v(n-1) < 1/2$. Now consider a situation where the buyer has already reached an agreement with $n-2$ sellers. Since $v(n-2) = 0$, the buyer has an incentive to reach an agreement with at least one of the remaining two sellers. But given that $v(n-1) < \frac{1}{2}$, once the buyer has acquired $n-1$ objects, he will always continue negotiation with the n-th seller as well. Hence once the buyer reaches an agreement with $n-2$ sellers, *both* the remaining two sellers become critical. Thus *ex ante* all the sellers have a lot of bargaining power, so that the holdup problem is very severe and it is impossible to sustain a strictly positive payoff for the buyer in the limit when discounting vanishes.

We can now understand why the logic of holdout may not operate when $v(n-2) > 0$. First suppose $v(n-1) > \frac{1}{2}$. Note that once the buyer reaches an agreement with $n-1$ of the sellers, in any continuation game the buyer must obtain at least $v(n-1)$. In fact, given that $v(n-1) > \frac{1}{2}$, the buyer has a strong incentive to terminate the project immediately and implement $v(n-1)$. Since the seller then has a payoff of zero, this consideration considerably reduces the ability of the remaining seller, and *ex ante* of all the sellers, to holdout, allowing the buyer to extract

[6]Increasing returns is said to hold if $v(s) - v(s-1) \leq v(s+1) - v(s)$ for all s, with strict inequality at $s = n-1$.

all the surplus from trading. Next suppose that $v(n-1) < \frac{1}{2}$. Now even though in this case, the buyer will never implement a project of size $n-1$ so that the last seller has considerable bargaining power, when the buyer reaches agreement with $n-2$ sellers, the bargaining power of the remaining two sellers is not too high. This is because the buyer has now the option of exiting and obtaining $v(n-2)$. This extra option for the buyer reduces the *ex ante* bargaining powers of every seller, resulting in a strictly positive payoff for the buyer.

We next examine several extensions of the basic model. The first extension that we consider is where the buyer is allowed the choice of implementing multiple projects. Unlike earlier, we now assume that the buyer can collect $n-m$ objects, implement $v(n-m)$, and then obtain the remaining m objects and get $v(m)$. We find that our results are minimally affected in this case mainly because, typically, implementing multiple projects is an inefficient choice, and thus does not constitute a credible threat. Turning to the effect of asymmetry, we find that for $n = 2$, it is possible that with asymmetric sellers the buyer obtains a strictly positive payoff. Finally, consider parameter configurations such that the holdout problem is significant. In such cases one may ask whether it is possible to bypass the problems of 'holdout' if the buyer is not restricted to simple cash offer contracts. We show that this is indeed possible if the buyer could make 'conditional offers' that essentially takes the form of 'take-it-or-leave-it' offers to the set of active sellers. More interestingly, however, even without such contingent offers, preliminary analysis suggests that the holdout problem will be largely mitigated if the seller could offer equity contracts. Thus all the extensions seem to corroborate the basic message of this paper that the possibility of strategic holdout does not affect the applicability of the Coase theorem to any significant extent.

1.1 Relation to Existing Literature

This paper traces its ancestry to one of the most important recent research areas, the theory of coalitional bargaining. Following the seminal work of Rubinstein (1982), as well as the literature on core implementation, researchers have studied the non-cooperative foundations of various cooperative solution concepts, in particular the core and the Shapley value. While Gul (1989) and Hart and Mas-colell (1996) are concerned with the Shapley value, Chatterjee et. al. (1993), Serrano (1995) and Krishna and Serrano (1996) study implementing the core. Moreover, while Chatterjee et. al. (1993) consider exogenous but deterministic bargaining protocols, Okada (1996) examines a model with random proposers. There is also a relatively recent branch of this literature that tries to endogenize the process of coalition formation, e.g. Perry and Reny (1994), Bloch (1996), Ray and Vohra (1997, 1999) and Okada (2000), as well as allow for contractual renegotiation, e.g. Seidmann and Winter (1998), Hyndman and Ray (2007), etc.

Most of this literature, however, abstracts from the two-sided nature of many bargaining situations, leading naturally to issues of complementarity. This paper carries forward the research program initiated by the coalitional bargaining literature into this setup. Formal treatments of the holdout problem (using game theoretic arguments) were first provided in Eckart (1985) and Asami (1988). The theoretical literature was further developed in Cai (2000, 2003) and Menezes and Pitchford (2004). Like us, Cai (2000) analyzes a *cash-offer* model in which the seller is paid immediately after an agreement is arrived at. In contrast, Cai (2003) allows the buyer to offer a *contingent contract* that promises to pay the seller a given amount only when production is carried out. Cai (2000) shows that inefficiency in the form of delays must occur in an equilibrium. Furthermore, this delay cost persists even when the players are sufficiently patient.

Our results extend Cai (2000) by precisely identifying parameter configurations for which the holdout problem is serious. Moreover, given that Cai (2000, 2003) does not allow the buyer to negotiate with more than one seller at any given point of time (so that simultaneous offers by sellers are ruled out), it is not very clear whether the delay in Cai (2000, 2003) then follows because of the *holdout problem*, or from the modeling assumptions themselves. Our analysis demonstrates that Cai's results are robust to this criticism, since, with perfect complementarity, we establish the existence of bargaining breakdown even when the buyer can simultaneously negotiate with all active sellers. In sharp contrast to Cai (2000) however, we find that there is a large class of parameter values for which the holdout problem is largely mitigated. This difference can possibly be traced to three differences in formulation between the two papers. First, we allow for general production technologies (which however does allow for perfect complementarity as a special case), second, the bargaining protocol adopted by us is symmetric, and finally, we allow for strategies that are history dependent.

Menezes and Pitchford (2004) consider a *two*-seller framework that allows the buyer to negotiate with both these sellers at any given date. They also consider cash offer contracts. One significant difference is that once an agreement is reached with one of the sellers, the contribution of this seller is removed from the pie. The buyer and the remaining seller then bargain over the residual pie. In their model holdout takes the form of delay. The cost due to such delay, however, goes to zero when players become sufficiently patient.

The rest of the paper is organized as follows. Section 2 describes the framework, and, in 2.1, also establishes some preliminary results. Sections 3 and 4 identify conditions under which holdout is significant, whereas Section 5 demonstrates that holdout is 'resolved' for a large class of parameter values. Section 6 contains some extensions of the bargaining protocol, while some of the proofs are collected together in the Appendix.

2 The Framework

There are $n + 1$ agents, one buyer and $n \geq 2$ sellers. Every seller has an identical object each that can be combined to generate returns for the buyer. We write $v(s)$ to denote the return to the buyer when a project that combines s objects, $0 \leq s \leq n$, is implemented, where the grand project involves combining all the objects. $v(s)$ is assumed to be non-decreasing in s and we normalize units such that $v(0) = 0$ and $v(n) = 1$. The buyer is allowed to implement a project of size zero, which should be interpreted as the buyer exiting the game without acquiring any object. We assume that $v(s)$ is *strictly super-additive* in that $v(n) > v(s) + v(n - s)$ for any s, where $1 \leq s < n$. Consequently implementing any project other than the grand project, is inefficient. A somewhat stronger restriction that we sometimes invoke is that of *increasing returns*. Formally, increasing returns is said to hold if $v(s) - v(s - 1) \leq v(s + 1) - v(s)$ for all s, with strict inequality at $s = n - 1$.

The buyer and the sellers bargain over the price of the objects. The bargaining protocol that we use here is a simple variant of the standard Rubinstein (1982) procedure. Time is discrete and continues for ever, so that $t = 1, 2, \cdots$. At the start of any period t, there is a set of 'active' sellers who are yet to sell their objects. Each period t is divided into three sub-stages.

We begin by describing the first two stages. The first stage of t consists of one side of the market making offer(s) to the other side, followed, in stage two, by the acceptance/rejection decisions of the other side. We assume without loss of generality that the buyer makes his offers in odd numbered periods, whereas the sellers make their offers in even numbered ones. Thus at $t = 1, 3, \ldots$, the buyer offers a price vector to the set of sellers active at that point of time. These sellers then simultaneously decide whether to accept, or reject the offer made to each one. In even numbered periods, $t = 2, 4, \ldots$, on the other hand, the active sellers simultaneously make their offers. After observing all the offers, the buyer decides which of these offers to accept, if at all.

Continuing with our description of the bargaining protocol, we consider two alternative scenarios that differ as to what a seller knows regarding the offers received by the other sellers. In the first scenario, each seller can observe only her component of the buyer's offer and does not know what offers are received by other sellers. We call this the 'secret offer' case. (Chatterjee and Dutta (1998) refer to this as the telephone bargaining setup.) In the second scenario, on the other hand, each seller observes the entire vector of offers. We label this as the 'public offer' case. In either of this situation, once a price is agreed upon between the buyer and any seller, this seller immediately receives the agreed upon price and exits the game.

At the third stage of t, the buyer has the option of implementing a project of size k, where k denotes the number of objects that are acquired by the buyer till then. If the buyer implements a project of size k, the game is over.[7] When $k < n$ however, the buyer may continue bargaining when the game goes to the next period, with the number of active sellers being $n - k$. We assume, without loss of generality, that if $k = n$, then the project is immediately implemented. Let P_{it} denote a price for seller i in period t. If t is even, then P_{it} denotes an offer made by the i-th seller to the buyer, while if t is odd, then it denotes the offer made by the buyer to the i-th seller. Further, we assume that $P_{it} \in [-\beta, 1 + \beta]$ where $\beta > 0$. Thus, the assumption that

[7] In section 6.1, we discuss how our results generalize when the buyer is allowed to implement more than one projects.

the buyer has to make an offer to every seller, is with out loss of generality since the buyer can always make a negative offer to any seller which will surely be rejected by the seller. Moreover, a seller can always make an unacceptable offer by asking for a price strictly greater than 1. All agents are risk neutral and have a common discount factor δ, where $0 < \delta < 1$.

For any given play of the game, the history at the beginning of stage i of date t includes information on all the past offers, the acceptance/rejection decision of the players and the set of active sellers that are yet to sell their objects. For the public offer game, all such information is common knowledge among the players. For the secret offer game, the set of active sellers at the beginning of stage i of date t is common knowledge.

Our focus in this paper is to study subgame perfect equilibria in pure strategies. The central issue addressed here is if, for δ large, there exist equilibria that yield a positive payoff for the buyer, and if so, whether these equilibria are efficient.

2.1 Some Preliminary Results

In this sub-section we record some observations that will be used repeatedly in the rest of the paper. All these results hold irrespective of whether the buyer's offers are publicly observed or not. Consider a scenario where the buyer has already acquired $n-1$ of the objects and suppose that $v(n-1) < 1/2$. When δ is large, it is straightforward to extend Osborne and Rubinstein (1990, 3.12.2) to show that the buyer will prefer to reach an agreement with the remaining seller as well, and implement the grand project rather than the project of size $n-1$. From Rubinstein (1982), it also follows that in such a case, the continuation payoffs of the proposer and the responder are, respectively, $\frac{1}{1+\delta}$ and $\frac{\delta}{1+\delta}$.

Let δ^* satisfy

$$v(n-1) = \frac{\delta^{*2}}{1+\delta^*}. \tag{1}$$

When $v(n-1) < \frac{1}{2}$, δ^* as defined in (1) is strictly less than one and moreover, for $\delta > \delta^*$, $v(n-1) < \frac{\delta}{1+\delta}$.

Lemma 1 *Let $v(n-1) < \frac{1}{2}$ and $\delta > \delta^*$.*
(a) In any equilibrium, if the buyer implements a project of size k, then $k \neq n-1$.
(b) Consider any history that starts with exactly one seller. If t is odd, the buyer offers the price $\frac{\delta}{1+\delta}$ which is accepted by the seller, while if t is even, the seller asks for $\frac{1}{1+\delta}$, which is accepted by the buyer.

The next lemma puts a lower bound on seller payoffs when the buyer is making acceptable offers to all active sellers.

Lemma 2 *Suppose $v(n-1) < \frac{1}{2}$ and $\delta > \delta^*$. Consider any history with $m \geq 1$ active sellers at date t, where t is odd. If the equilibrium calls for the buyer to make an acceptable offer to all m sellers, then each seller must get at least $\frac{\delta}{1+\delta}$.*

Proof. Since at t, the buyer makes an acceptable offer to all existing sellers, if any of these sellers rejects, then at the end of period t, the buyer would have acquired exactly $n-1$ objects. Since $\delta > \delta^*$, from Lemma 1, the negotiation will continue in the next period when the deviating seller has a payoff of $\frac{1}{1+\delta}$. Thus by rejecting the offer, given that the rest of the sellers are accepting their offers, any seller can assure himself a payoff of $\frac{\delta}{1+\delta}$. ■

Given δ, let $Y_B(m,\delta)$ be the supremum of buyer payoffs in any continuation equilibria beginning from a history with m active sellers. Similarly, let $Y_i(m,\delta)$ denote the supremum of seller i's payoffs in any continuation equilibria starting from a history with m active sellers, with seller i being one of these active sellers.

Our next lemma provides an upper bound on these payoffs.

Lemma 3 *Let $v(n-1) < \frac{1}{2}$ and $\delta > \delta^*$. Fix an equilibrium and a history with $m \geq 1$ active sellers, so that seller i is one of the sellers active at the point. Then, $Y_B(m,\delta) \leq \frac{1}{1+\delta}$ and $Y_i(m,\delta) \leq \frac{1}{1+\delta}$.*

Proof. Please see Appendix A.

3 The Holdout Problem for $n = 2$: 'Negligible' Buyer payoff

For $n = 2$, the preceding lemmas allow us to completely characterize the set of equilibrium outcomes in this case. Our main result here is that in any equilibrium, agreement takes place in the first period itself and the grand project is implemented. The buyer's payoff in this equilibrium is however exactly $\frac{1-\delta}{1+\delta}$, which goes to zero as the players become sufficiently patient. This result holds irrespective of whether offers are secret, or not. Furthermore, the results hold even if one allows for mixed strategies. Thus, for the two seller case, the holdout problem turns out to be extremely severe.

Proposition 1 *Suppose $n = 2$ and let $\delta > \delta^*$. Then in any equilibrium of either the secret, or the public offers game, after any history starting with two active sellers, the following hold:*

(a) If t is odd, the buyer offers $\frac{\delta}{1+\delta}$ to each of these sellers.

(b) If t is even, seller i makes an offer of P_i and both offers are accepted by the buyer, then;

  (i) $1 - P_1 - P_2 = \frac{\delta(1-\delta)}{1+\delta}$, and

  (ii) $P_i \geq \frac{\delta^2}{1+\delta}$, for $i = 1, 2$.

(c) An equilibrium exists, where at $t = 1$, the buyer offers $\frac{\delta}{1+\delta}$ to both the sellers. These offers are accepted by the sellers and the buyer implements the grand project at the end of period 1.

At this point the following remark is useful.

Remark 1 *Proposition 1 thus implies that in any equilibrium, agreement is reached in the first period, resulting in a payoff of $\frac{1-\delta}{1+\delta}$ for the buyer. Clearly, as the agents become sufficiently patient, the buyer's payoff goes to zero in any equilibrium. We also note that although we have proved this result in the class of all pure strategy equilibria, the result holds even when players are allowed to randomize in their choice of strategies.*

The intuition for this proposition is straightforward when offers are secret. First, we observe that with $v(m)$ being strictly super-additive, we have $v(1) < \frac{1}{2}$. Since $\delta > \delta^*$, it follows from Lemma 1 that in every equilibrium that yields a positive payoff to the buyer, the grand project must be implemented. Thus, from Lemma 2, it follows that if the buyer has to make an acceptable offer to both the sellers, then each seller must be given at least $\frac{\delta}{1+\delta}$. Now when the offers are secret, there can not be an equilibrium, where the buyer first makes an offer to only one of the sellers (say seller 1) and then once it is accepted, negotiates with the the second seller. This is because the buyer can always offer the second seller $\frac{\delta}{1+\delta}$ along with the equilibrium offer to the first seller. Since offers are secret, such an action will not affect the acceptance decision of the first seller. The buyer must be better off from this deviation since the project is implemented one period earlier and thus saves on the discounting cost.

This argument, however does not hold when the buyer's offers are publicly observed. In this scenario, it is possible that along an equilibrium, the buyer first makes a very low offer to seller 1 and only when she accepts this offer, does the buyer start negotiating with the other seller. In such a case, if the buyer tries to make an acceptable offer to seller 2 in the same period, since offers are observable, the first seller will not accept her current offer. As it turns out however and as the proof below shows, it is the first seller who then has a profitable deviation. Following this, one can then establish that in any equilibrium that involves the buyer making acceptable offers, both sellers must accept and the logic of Lemma 2 thus applies.

Proof of Proposition 1. Because of strict super-additivity of $v(m)$, we have $v(1) < \frac{1}{2}$ and thus for $\delta > \delta^*$, Lemmas 1-3 hold. In particular, by Lemma 1, we have that in any equilibrium with a positive payoff for the buyer, the grand project must be implemented. Recall that $Y_B(2, \delta)$ denotes the supremum of the continuation payoff to the buyer in any equilibrium, starting from a history when both sellers are active. We will show that $Y_B(2, \delta) \leq \frac{1-\delta}{1+\delta}$ for all $\delta > \delta^*$. We first note that if t is odd, then the buyer can always make an offer of $\frac{\delta}{1+\delta}$ to each of the sellers. By Lemma 3, both sellers must accept and the buyer has a positive payoff. Thus, in any equilibrium, the buyer can never exit without acquiring any object as that will give him a zero payoff.

If the claim is false, then there exists an equilibrium in which the buyer's payoff is arbitrarily close to $Y_B(2, \delta)$, which in turn is strictly greater than $\frac{1-\delta}{1+\delta}$.

Clearly, in such an equilibrium, the payoff to at least one of the sellers (label him seller 1) must be strictly less than $\frac{\delta}{1+\delta}$. Let t be the date at which an agreement with seller 1 takes place. Clearly both sellers must be present at that date by Lemma 1. Now if at t, seller 1 himself was making an offer, he could have asked for a slightly higher price, the buyer could not have rejected since his payoff in this equilibrium was arbitrarily close to $Y_B(2,\delta)$. Therefore, this offer was made by the buyer. Furthermore, at that date, the buyer could not have made an acceptable offer to seller 2 as well. Since then by Lemma 2, seller 1 could have rejected his offer and obtained $\delta/(1+\delta)$ in the next period. The proof is now is complete in the secret offer case because if the buyer were to make an immediate offer of $\frac{\delta}{1+\delta}+\epsilon$ to the second seller along with the prescribed equilibrium offer to seller 1, seller 1 will continue to accept and, because of Lemma 3, seller 2 must accept it as well. Since the agreement now takes place in that period itself, the buyer will save on the discounting cost and thus for ϵ small, this must be a profitable deviation. This completes the argument for the secret offer case.

For the rest of the proof, we then assume that offers are public. Again suppose there exists an equilibrium in which the buyer's payoff is arbitrarily close to $Y_B(2,\delta)$, which in turn is strictly greater than $\frac{1-\delta}{1+\delta}$. As before let seller 1's payoff be strictly less than $\frac{\delta}{1+\delta}$, with the agreement with seller 1 being reached at date t. Clearly, by the preceding arguments, agreement with seller 2 takes place at $t+1$. If P is the price offer made to seller 1, the payoff to the buyer in this equilibrium, then is given by

$$K = -P + \frac{\delta^2}{1+\delta}.$$

This follows since at $t+1$, the seller makes the offer and by Lemma 1, the buyer obtains $\delta/(1+\delta)$. Since K is arbitrarily close to $Y_B(2,\delta) > 0$, we must have

$$P < \frac{\delta^2}{1+\delta}. \tag{2}$$

Suppose seller 1 rejects the buyer's offer. Since the buyer can not exit the game without acquiring the object, at $t+1$, let seller 1 make a counter offer of $P' = \frac{P}{\delta}+\epsilon$. Clearly, acceptance of this offer will be a profitable deviation for seller 1. We now argue that this offer will be accepted by the buyer for ϵ small. If the buyer were to accept seller 1's offer, his overall payoff (from period $t+1$ perspective) can not be less than

$$Y = \frac{-P}{\delta} - \epsilon + \frac{\delta}{1+\delta}.$$

This is because the buyer can always reject seller 2's offer and in $t+2$ offer $\delta/(1+\delta)$ to him which, by Lemma 1, will be accepted by seller 2. On the other hand, if he were to reject this offer, then the maximum that he can obtain is $\delta Y_B(2,\delta)$ which is arbitrarily close to δK. It is easy to check that $Y > \delta K$ if and only if $P < \frac{\delta^2}{1+\delta}$ which is true because of (2).

This proves that $Y_B(2,\delta) \leq \frac{1-\delta}{1+\delta}$.

Part (a) of the proposition now follows because when t is odd, the buyer can offer $\frac{\delta}{1+\delta}$ to each of the sellers which, by Lemma 3, must be accepted by the sellers. This results in a payoff of $\frac{1-\delta}{1+\delta}$ to the buyer.

To prove part (b) of the proposition, observe that for the buyer to accept both of the offers (P_1, P_2), $1 - P_1 - P_2$ must be at least $\frac{\delta(1-\delta)}{1+\delta}$. This inequality, however can not be strict since in that case any one of these sellers can increase her offer price a little. Furthermore, if any seller makes an unacceptable offer today, then because of Lemma 1 and part (a) of the Proposition, the agreement will be reached next period, with the buyer making an offer of $\frac{\delta}{1+\delta}$. This results in a payoff of $\frac{\delta^2}{1+\delta}$ to the seller. This proves that $P_i \geq \frac{\delta^2}{1+\delta}$.

Part (c) of the proposition, i.e. existence, is proved by construction. For the secret offers game, at every t odd, the buyer offers $\frac{\delta}{1+\delta}$ to all active sellers, with a seller accepting an offer if and only if she is offered at least $\frac{\delta}{1+\delta}$, whereas at t even, the sellers both ask for $\frac{\delta^2}{1+\delta}$, with the buyer accepting an offer vector if and only if he obtains at least $\frac{\delta(1-\delta)}{1+\delta}$ (a full specification is provided in footnote 15 later). At the end of any period t (odd or even), the buyer continues negotiation in the next period unless he can implement the grand project.

We then write down the strategies for the public offers case. It is sufficient to write down the accept/reject decisions for the sellers at t odd, the strategies at every other subgame mimic those for the secret offers case. Each seller accepts P_i if $P_i \geq \frac{\delta}{1+\delta}$, and rejects P_i if $P_i \leq \frac{\delta(1+\delta^2)}{2(1+\delta)}$. Otherwise, seller 1 accepts $P_1 \in (\frac{\delta(1+\delta^2)}{2(1+\delta)}, \frac{\delta}{1+\delta})$ if and only if $P_2 < \frac{\delta}{1+\delta}$ and seller 2 accepts $P_2 \in (\frac{\delta(1+\delta^2)}{2(1+\delta)}, \frac{\delta}{1+\delta})$ if and only if $P_1 \leq \frac{\delta(1+\delta^2)}{2(1+\delta)}$. ■

4 The Holdout Problem in the General Case

In this section we demonstrate that the negative results of the previous section survive with $n \geq 3$, but only if the conditions $v(n-2) = 0$ and $v(n-1) < 1/2$ *both* hold together. Under these conditions, Proposition 2 shows that for δ large, the buyer's payoff in any equilibrium is no more than $\frac{1-\delta}{1+\delta}$, being exactly zero when the offers are secret. Further, there is an equilibrium where a project of size zero is implemented. Thus Proposition 2 establishes the severity of the holdout problem in this case.

Proposition 2 *Suppose $n \geq 3$, $v(n-2) = 0$ and $v(n-1) < \frac{1}{2}$. Fix $\delta > max\{\delta^*, 1/2\}$. Then the following holds:*

(a) In any equilibrium of the secret offer game, the buyer's payoff is zero.

(b) In any equilibrium of the public offer game, the buyer's payoff is at most $\frac{1-\delta}{1+\delta}$.

(c) In both the secret offer and the public offer game:

  (i) There exists an equilibrium in which an agreement is reached at $t = 2$, with the buyer implementing the grand project.

  (ii) There exists an equilibrium in which the buyer exits at $t = 1$, resulting in a zero payoff to all players.

When the production technology satisfies *increasing returns*, we have that $v(n) - v(n-1) > v(n-1) - v(n-2)$. Consequently, $v(n-2) = 0$ implies that $v(n-1) < \frac{1}{2}$. This yields the following corollary:

Corollary 1 *Suppose $n \geq 3$ and $v(s)$ satisfies increasing returns. If $v(n-2) = 0$, then Proposition 2 holds for $\delta > \max\{\delta^*, \frac{1}{2}\}$.*

Before we turn to proving Proposition 2, a few remarks may be helpful.

Remark 2 *Observe that the case of perfect complementarity, viz. $v(m) = 0$, $\forall m < n$, is obviously covered by Proposition 2. From Proposition 2 then, whenever $n \geq 3$, the buyer's payoff is arbitrarily close to zero, being necessarily zero in the secret offer case. Interestingly, this result holds in the class of all strategies, and does not invoke stationarity. This is in contrast to Cai (2000, Theorem 2) who analyzes the case of perfect complementarity and proves a similar result that allows only stationary strategies.*

Remark 3 *It is of interest to note that unlike in the secret offer case, it is possible to support a strictly positive buyer payoff when the buyer's offers are publicly observed. To see how, let us consider the following action profile: in period $t = 1$, the buyer makes an offer of zero to all sellers. Further, all of the first $n-2$ sellers accept, while the last two sellers reject. In period $t = 2$, each of these two sellers asks for $P^* = \frac{1+\delta^2}{2(1+\delta)}$ and these offers are accepted by the buyer. To support such an outcome in equilibrium, the buyer needs to exit at the end of period 1 if any of the first $n-2$ sellers fails to accept the first period offer. Given such an exit threat, the first $n-2$ sellers can thus do no better than to accept the present offer. We note that such an action profile can not be supported when offers are secret as then the buyer can deviate at $t = 1$ and make an offer of $\frac{\delta}{1+\delta}$ to each of the last two sellers. This would constitute a profitable deviation for the buyer.*

Remark 4 *It is important to stress that in Proposition 2, the fact that buyer payoffs are arbitrarily close to zero, is critically dependant on our assumption that the players (especially the sellers) are not allowed to randomize in their accept/reject decisions. To see why this is so, consider the situation with n sellers and a technology that exhibits perfect complementarity. Assume that in period 1, the buyer makes an offer of $P' < 1/n$ to each seller: each seller accepts this offer with probability r^* strictly less than 1. Furthermore, the buyer continues negotiation with probability one only if no more than two sellers reject his offer and exits otherwise.[8] Given these strategies, if a seller rejects the current offer P', she now faces the risk of getting zero if at least two other sellers reject because in that event, the buyer exits the game. On the other hand, when*

[8] Since, for any history that starts with at least three sellers, there exists an equilibrium outcome (in pure strategies) in which every player receives zero (see part (ii) of Proposition 2c), this action profile of the buyer can be supported in an equilibrium.

no more than one other sellers reject her offer, this seller's payoff will be strictly higher than P'. It thus follows that there exists a value of r^* for which each seller is indifferent between accepting the offer P', or rejecting it. We, however, note that such outcomes are necessarily inefficient as with strictly positive probability, the buyer must exit the game without implementing any project. Consequently, when $v(n-2) = 0$, if one insists on efficiency and strictly positive payoff for the buyer, then these two are incompatible even when one allows for 'mixed strategies'.[9]

Proof of Proposition 2. Let $Y_B^g(m, \delta)$ denote the supremum of the buyer's payoff in any equilibrium for the game g, where $g = S$ if offers are secret, and $g = P$ if offers are public. We first prove that for any $m \geq 1$, these payoffs are bounded above by $\frac{1-\delta}{1+\delta}$ for all $\delta > \delta^*$.

Since $v(n-2) = 0$, and $v(n-1) < 1/2$, for $\delta > \delta^*$, in any equilibrium with a strictly positive payoff for the buyer, the buyer must implement the grand project. Thus, for $m = 2$, the result follows from the proof of Proposition 1. We now assume as an induction hypothesis that the result is true for all $m = 2, \ldots, n-1$. If the claim is false, then for some game g, $g \in \{S, P\}$, there is an equilibrium in which the grand project is implemented at some date t giving the buyer a payoff arbitrarily close to $Y_B^g(m, \delta)$, which in turn is strictly greater than $\frac{1-\delta}{1+\delta}$. Because of the induction hypothesis, the number of active sellers m can be either $m = n$, or $m = 1$.

First consider the case $m = n$. Since $n \geq 3$, and $\delta > 1/2$, it follows from Lemma 2 that the buyer could not have made an acceptable offer at that date. Thus, it is the sellers who are making the offers. But then any seller can ask for a slightly higher price which must be accepted by the buyer.

So it must be that $m = 1$. Label this seller 1. Now at $t-1$, all n sellers must have been present, otherwise, the induction hypothesis would have applied. Now if t is odd, then at $t-1$, seller 1 was making an unacceptable offer. If seller 1 at $t-1$ asked for $P \in (\frac{\delta^2}{1+\delta}, \frac{\delta}{1+\delta})$, the buyer would have accepted this offer. This would thus have been a profitable deviation for seller 1. Therefore t must be even. Thus seller 1 is asking for $P = \frac{1}{1+\delta}$ at date t. The buyer's payoff from $t-1$ onwards is thus $\frac{\delta^2}{1+\delta} - \sum_{i \neq 1} P_i$. Since $P_i \geq 0$ and the buyer's payoff is strictly greater than $\frac{1-\delta}{1+\delta}$, for every seller $i \neq 1$, we have

$$P_i < \frac{\delta^2}{1+\delta} - \frac{1-\delta}{1+\delta}. \tag{3}$$

If any of these sellers (say seller 2) rejected the buyer's offer at period $t-1$, given that the other sellers are accepting their respective offers, next period would begin with exactly two active sellers. In the continuation game, from Proposition 1(a), it follows that whenever the buyer has to make an offer with two sellers present, he will offer exactly $\delta/(1+\delta)$ to each of them. Thus, the worst payoff to seller 2 following this deviation is that the agreement takes place in $t+1$ with the buyer making an offer of $\delta/(1+\delta)$ to seller 2. Thus, from the perspective of period $t-1$,

[9] A formal statement and a proof of this assertion is available upon request.

seller 2 can assure himself a payoff of $\frac{\delta^3}{1+\delta}$. Since $\delta < 1$, it follows that $\frac{\delta^3}{1+\delta} > \frac{\delta^2}{1+\delta} - \frac{1-\delta}{1+\delta} > P_2$. The last inequality follows from (3). This will thus be a profitable deviation for player 2.

This proves part (b) of Proposition 2. We now prove that when the offers are secret $Y_B^S(m, \delta) = 0$ when $m \geq 3$. We prove this result when $m = 3$. Assume that the result is false and that $Y_B^S(3, \delta) > 0$. Then there is an equilibrium in which the buyer's payoff is arbitrarily close to $Y_B^S(3, \delta) > 0$. Moreover, since $\delta > 1/2$, there must be at least one seller (label him seller 1) who is getting strictly less than $\frac{\delta}{1+\delta}$ in this equilibrium. Further, let seller 1 be the last such seller. Let t be the period at which an agreement with this seller is being reached in this equilibrium. Let $\tilde{m}$ be the set of sellers that are active at this period. Clearly $m \geq 2$. Now we argue that t can not be odd. This is because a) either, at this date all active sellers are accepting in which case the buyer's payoff is negative if $\tilde{m} = 3$ (Lemma 2 and the fact that $\delta > 1/2$), or if $\tilde{m} = 2$, by Proposition 1 (part (a)), each seller is getting $\frac{\delta}{1+\delta}$, or b) agreement takes place with some sellers only at a later date. If the buyer were to offer $\frac{\delta}{1+\delta}$ to these sellers, then, these sellers must accept. Moreover since offers are secret, such a deviation will not affect the acceptance decision of any other seller. This will constitute a profitable deviation for the buyer. Therefore t is even. Now if seller 1 deviates and asks for a price slightly higher, but less than $\frac{\delta}{1+\delta}$, the buyer must accept. This follows since the payoff to the buyer in the original equilibrium was arbitrarily close to the supremum $Y_B^S(3, \delta)$. The rest of the proof then follows from a straightforward induction argument. This proves part (b) of the Proposition. For the proof of part (c), please see Appendix B. ■

5 Resolving the Holdout Problem: $v(n-2) > 0$

In this section, we demonstrate that the holdout problem is largely resolved whenever $v(n-2) > 0$ and the agents are patient (i.e. δ is sufficiently large). In this case, a positive buyer payoff can always be sustained, and moreover this can be sustained as an (asymptotically) efficient equilibrium. The exact results, however, depend on $v(n-1)$.

If $v(n-1) > 1/2$, then, an equilibrium exists where the grand project is implemented in period 1 itself, with the buyer extracting the entire surplus. The intuition for this result is the following. Given that $v(n-1) > 1/2$, in any continuation game with exactly one active seller, the buyer can credibly threaten to exit, resulting in a zero payoff for the remaining seller (see Lemma 4). Thus, no seller wants to hold out. This allows the buyer to come to an immediate agreement with all of the sellers. Interestingly, this result holds for all $v(n-1) > \frac{1}{2}$, and not just for $v(n-1)$ close to 1.

With $v(n-1) < 1/2$, however, the last remaining seller is in a very strong bargaining position since by Lemma 1, the buyer never implements a project of size $n-1$. Negotiations thus can only be concluded (and the grand project implemented) at the end of the second period. Although such equilibria exhibit delay, the delay cost associated with such equilibrium outcomes vanishes as δ goes to one. Further, given that the buyer can implement a project of size $n-2$ and get a positive payoff, he has some bargaining power which he can use to obtain a positive payoff in equilibrium.

We begin our analysis with the case where $v(n-1) < 1/2$.

5.1 $v(n-1) < \frac{1}{2}$

We begin with the secret offers case. For this case we show that the buyer's payoff is bounded above by $v(n-2)$. Moreover, for δ large, any payoff close to $v(n-2)$ can be sustained for the buyer.

Proposition 3 (Secret Offer Case) *Let $n \geq 3$, $v(n-2) > 0$, and $v(n-1) < \frac{1}{2}$.*

(a) If $\delta > max\{\delta^*, 1/2\}$, then in any equilibrium, the buyer's payoff is at most $v(n-2)$.

(b) Fix $\epsilon > 0$, then there exists $\delta(\epsilon) < 1$, such that for all $\delta > \delta(\epsilon)$, there exists an equilibrium in which the buyer implements the grand project at $t = 2$ and gets a payoff strictly greater than $v(n-2) - \epsilon$.

We omit a formal proof for part (a) of the proposition as it is a generalization of the proofs in Propositions 1 and 2. The argument is quite intuitive though. If the buyer were to get a payoff that is strictly greater than $v(n-2)$, it must be that the buyer implements the grand project in equilibrium. This is because with $v(n-1) < 1/2$ and $\delta > \delta^*$, Lemma 1 holds. However, when offers are secret, it is impossible for the buyer to make an acceptable offer to more than $n-2$ sellers at any period. That not all sellers can accept follows from Lemma 2, while if $n-1$ sellers accept, then for $\delta > \delta^*$, by Lemma 1, the buyer must negotiate with the remaining seller in the next period. Thus, project implementation will be delayed. This delay cost could have been saved if the buyer were to make a secret acceptable offer to the remaining seller. Such an offer will not change the acceptance strategies of the other sellers. Thus, in any equilibrium, at the final date of negotiation, the buyer must face at least two active sellers. A slight modification of the proof of Proposition 1 then allows us to show that the payoff to the buyer is no more than $max\{v(n-2), \frac{1-\delta}{1+\delta}\}$. Since $v(n-2) > 0$, the result then follows for δ large.

To prove part (b) of the proposition, consider the following action profiles for the players: in period 1, the buyer makes zero offers to all sellers, when each seller rejects this offer. In period 2, every seller asks for P^0 such that $1 - nP^0 = \delta v(n-2)$. These offers are accepted by the buyer and the grand project is implemented in period $t = 2$. In Appendix C, we show how this action profile can be sustained as a subgame perfect equilibrium.

Our next proposition deals with the public offer case. It is interesting to note that compared to the secret offer case, a higher buyer payoff can be sustained when offers are publicly observable.

Proposition 4 (Public Offer Case) *Let $n \geq 3$, $v(n-2) > 0$, and $v(n-1) < \frac{1}{2}$.*

(a) If $\delta > max\{\delta^*, 1/2\}$, then in any equilibrium, the buyer's payoff is at most $\frac{\delta}{1+\delta}$.

(b) *Fix $\epsilon > 0$, then there exists $\delta(\epsilon) < 1$, such that for all $\delta > \delta(\epsilon)$, there exists an equilibrium in which the buyer implements the grand project at $t = 2$ and gets a payoff strictly greater than $\frac{1}{2} - \epsilon$.*

The intuition as to why a higher buyer payoff can be sustained when offers are publicly observable, is as follows. When offers are publicly observed, it becomes possible for the buyer to make an acceptable offer to $n - 1$ sellers in a given period and this increases the bargaining power of the buyer and as Proposition 4 shows, allows him to capture approximately half of the total surplus.

Proof of Proposition 4. From Lemma 3, we know that the buyer's payoff in any equilibrium starting with any set of active sellers is at most $\frac{1}{1+\delta}$. Since $\delta > 1/2$, it also follows from Lemma 2 that at $t = 1$, if the buyer was to make an offer that will be acceptable to all sellers, the buyer's payoff will be negative. Thus, part (a) of the proposition follows.

To prove part (b) of the proposition, consider the following action profile for the players: at $t = 1$, the buyer offers zero to all sellers. All sellers but the first one accept. In period 2, the first seller asks for $\frac{1}{1+\delta}$ which is accepted by the buyer and the grand project is implemented at the end of period 2. In Appendix *D*, we show that for δ large enough, this action profile can be supported as part of a subgame perfect equilibrium. ■

Remark 5 *While the two preceding propositions focus on establishing the maximum payoff that a buyer can obtain in an equilibrium, they leave open the question as to whether, for $v(n-2) > 0$, it is possible to support (in equilibrium) any buyer payoff in $(0, v(n-2))$ for the secret offer case, and in $(0, \frac{1}{2})$ for the public offer case? Given the folk theorem like results in Chatterjee et. al. (1993) and Herrero (1985), these questions are of natural interest. It is possible to provide an affirmative answer to this when the technology satisfies an added restriction, namely that $\frac{v(s)}{s}$ is increasing in s.*[10] *It is easy to check that this assumption is weaker than the assumption of increasing returns. The following example shows why, in general, the result is not true. Let $n = 3$ and $v(1) = v(2) = 2/5$. Since in any equilibrium, there must be at least one seller whose payoff must be no more than $1/3$, the buyer can always implement a project of size 1 and get a payoff of $2/5 - 1/3 = 1/15$. It is thus possible to show that for this example, for δ close to 1, in any equilibrium where the grand project is implemented, the buyer's payoff must be at least $1/15$.*

The next sub-section contains our analysis for the case when $v(n - 1) > 1/2$.

[10] A buyer payoff of x at $t = 2$, where $x < v(n - 2)$, can be supported as follows. The equilibrium involves the buyer making unacceptable offers at $t = 1$, and the sellers all asking for a price of $(1 - x)/n$ at $t = 2$, which the buyer accepts. The assumption that $\frac{v(s)}{s}$ is increasing ensures, for example, that the buyer does not find it profitable to accept and implement a subset of these offers. A formal proof is available upon request.

5.2 $v(n-1) > \frac{1}{2}$

For these parameter values we show that there is an *efficient* equilibrium, that involves implementing the grand project without any delay, and moreover the buyer obtains the whole of the surplus. The argument critically relies on the buyer having a credible exit threat once he has acquired $n-1$ of the objects. The following lemma does exactly that.

Lemma 4 *Let $n \geq 3$, $v(n-1) > \frac{1}{2}$ and $\delta > v(n-1)$. Consider any history that starts at t with exactly one active seller. Then there is a continuation equilibrium such that at every t even, the buyer's payoff is $\frac{v(n-1)}{\delta}$.*

While the formal proof can be found in Appendix E, an informal discussion of the action profiles in this equilibrium may be useful. At every t even, the seller asks for $1 - \frac{v(n-1)}{\delta}$, which the buyer accepts, whereas at every t odd, the buyer offers the seller $\delta - v(n-1)$, which the seller accepts. If the seller asks for an amount higher than $1 - \frac{v(n-1)}{\delta}$, the buyer rejects and there is transition to a state where at every t odd the buyer asks for 1 which the seller accepts since otherwise the buyer exits from the game and implements the project of size $n-1$.

One can now use this lemma to show the existence of an equilibrium in which negotiation is concluded in period 1 with the buyer receiving the entire surplus from trade. The strength of this proposition arises from the fact that it holds for all $v(n-1) > \frac{1}{2}$, and not just for $v(n-1)$ close to 1.

Proposition 5 *Suppose $n \geq 3$, $v(n-1) > 1/2$ and $\delta > v(n-1)$. Then under both the secret offer, as well as the public offer case, there exists an equilibrium where the buyer implements the grand project at date $t = 1$ and receives a payoff of 1.*

While the proof can be found in Appendix E, here we outline the structure of the equilibrium profile. At every t even, the sellers all make unacceptable offers, whereas at every t odd, the buyer offers a payoff of zero to all sellers. At t odd, the sellers all accept. If the buyer faces exactly one active seller, he exits the game and implements a project of size $n-1$. Lemma 4 plays a critical role here in ensuring that in this case the buyer is actually indifferent between exiting the game, and continuing to bargain.

Remark 6 *Can any payoff in $(0, 1)$ be supported in an equilibrium when $v(n-1) > 1/2$? As in Remark 5 earlier, the answer is in the affirmative as long as $\frac{v(s)}{s}$ is increasing in s. When this condition fails however, say when $n = 3$, $v(1) = 2/5$ and $v(2) = 3/5$, a payoff of $1/15$ is impossible to support in an equilibrium where the grand project gets implemented.*

To summarize, our results thus show that the value of $v(n-2)$ and $v(n-1)$ are critical in determining whether the equilibria exhibit holdout. When $v(n-2) > 0$, holdout is not a problem as there always exist asymptotically efficient equilibria with strictly positive payoffs for the buyer. Moreover, for production technology satisfying increasing return, we also have the exact converse (see the corollary to Proposition 2), namely, that holdout occurs whenever $v(n-2)=0$. These results thus suggest that strategic holdout is not a serious obstacle to the working of the Coase theorem.

6 Extensions

In this section we examine some interesting variations of the bargaining protocol studied so far.

6.1 Multiple Projects

So far the buyer has been restricted to implement a single project. Does the possibility of implementing multiple projects improve the bargaining position of the buyer? From Propositions 3, 4 and 5 we know that when $n \geq 3$ and $v(n-2) > 0$, an efficient outcome can be obtained in equilibrium with the buyer receiving a positive surplus. Consequently, this question assumes greatest relevance when $v(n-2)=0$, which is the case we focus on.

If only a single project can be implemented then, because of Lemma 1, for $v(n-1) < \frac{1}{2}$ the buyer never implements a project of size $n-1$ and this allows individual sellers to hold out. However, when the buyer can implement more than one project, he can threaten to implement the project of size $n-k$, and then bargain with the remaining k sellers. For such a threat to be credible however, this strategy must give the buyer a strictly higher payoff compared to what he obtains if he instead continues negotiation (and implements the grand project). Indeed when $n \geq 3$, such a strategy is never optimal since $v(n-2)=0$ and thus Proposition 2 continues to hold even when the buyer can implement multiple projects. However for $n=2$, it is possible that $v(1)+\frac{v(1)}{2}$ is strictly greater than $\frac{1}{2}$. In such a case, for δ large, one can support an equilibrium in which the buyer offers exactly $\frac{\delta v(1)}{1+\delta}$ to both the sellers, and all such offers are accepted. Further, the buyer implements the grand project in period 1 and, for δ close to 1, the payoff to the buyer from this equilibrium is arbitrarily close to $1-v(1)$.

6.2 Asymmetric Sellers

How important is our assumption of seller symmetry for the results obtained so far? While a general treatment with n sellers would take us too far afield, we briefly discuss how, in a two-sellers context, asymmetry modifies our existing results.

Let $S=\{1,2\}$ be the seller set, with $v(\{1,2\})=1$ and $v(\{1\}) > v(\{2\})$. Strict superadditivity implies that $1 > v(\{1\})+v(\{2\})$. Since $v(\{2\}) < v(\{1\})$, it then follows that $v(\{2\}) < \frac{1}{2}$. If $v(\{1\}) < \frac{1}{2}$ as well, it is easy to check that for δ large, it is never optimal for the buyer to implement a project of size 1. Consequently, our earlier analysis holds and in equilibrium the grand project is necessarily implemented, with a result analogous to that of Proposition 1 going through.

However, when $v(\{1\}) > 1/2$, for δ is large, a result similar to Lemma 4 holds. It is then possible to show that when only seller 1 sells her object, the buyer can credibly threaten to implement a project of size 1 and obtain $v(\{1\})$. This results in a zero payoff for the second seller. Given this possibility, one can then prove the existence of an equilibrium in which the buyer offers $\frac{\delta}{1+\delta}$ to seller 1 and zero to seller 2, both sellers accept and the grand project is implemented in period 1.[11]

6.3 Alternative contractual forms

Our analysis in this paper shows that, even under very simple cash offer contracts, the holdout problem is largely mitigated whenever either $v(n-2) > 0$ and/or $v(n-1) > \frac{1}{2}$. For such parameter values, therefore, use of more complex contracts is unnecessary. However, for situations where the holdout problem does bite, it may be of interest to examine whether alternative contractual forms may restore efficiency.

6.3.1 Conditional Offers

We study the following game form: at t odd, with m sellers present, the buyer can make an offer to any subset of these active sellers. If all of these sellers accept, then the buyer is committed to accepting all these acceptances. If, however, any one of them rejects, no offers are accepted and the game moves to the next stage. At t even, the game form is the same as before. We call this the conditional offers case. Given that conditional offers increase the bargaining power of the buyer, it is of interest to know whether such offers allow the buyer to escape the holdout problem. The proposition below shows that for δ sufficiently large, a buyer payoff of 1 can always be sustained, even when $v(n-2)$ and $v(n-1)$ are *both* small.

Proposition 6 *Let $n \geq 2$, then there exists $\hat{\delta}$, such that for all $\delta > \hat{\delta}$, the outcome where the buyer implements the grand project at $t = 1$ and gets a payoff of 1, can be supported in an equilibrium.*

Proof. See Appendix F.

Note that for $v(n-1) < \frac{1}{2}$, this result is in sharp contrast to that under the unconditional offer case. If offers are unconditional and the buyer makes an offer that is accepted by all the active sellers, then a seller can always reject and ensure that she obtains the Rubinstein payoff. Under the conditional offers case, however, if a seller rejects her proposal, the subgame will begin with all of the other sellers. Furthermore, unlike in the situation of a bilateral bargaining, no seller can unilaterally reject an offer and ensure that her counteroffer will be accepted by the buyer.

[11] Since $v(\{2\}) < 1/2$, the buyer can never credibly exit with only the second seller's object and thus in any equilibrium, seller 1 must always extract $\frac{\delta}{1+\delta}$.

6.3.2 Equity contracts

Finally, we briefly consider equity contracts where the buyer sells a fraction α_i of the project value to seller i.[12] The contract α_i thus entitles seller i to a payoff of $\alpha_i v(k)$ in case a project of size k is implemented. While a full analysis is beyond the scope of this paper, preliminary analysis suggests that the holdout problem is largely mitigated in this case. In particular, for $n = 2$, we are able to show the existence of an equilibrium in which the buyer offers $\frac{\delta}{1+2\delta}$ to both sellers at $t = 1$, and these are accepted. Consequently the buyer's payoff goes to $1/3$ as δ goes to 1. The difference arises because, once an agreement has been reached with seller 1, say, then any subsequent bargaining will be over a smaller cake size, which reduces the bargaining power of seller 2.

7 Conclusion

This paper characterizes the conditions under which holdout may, or may not be significant, demonstrating that the problem is largely mitigated whenever there are at least three sellers and the marginal contribution of the last two sellers, either singly, or jointly, is not too large. The result is even more striking for production functions satisfying increasing returns: For any positive $v(n-2)$, there is an asymptotically efficient equilibrium with a positive payoff for the buyer. Thus the broad lesson of this paper would seem to be that, as long as the complementarity problem is not too extreme, the bargaining protocol does not shut active sellers out, and strategies can be history dependent, holdout is not a significant obstacle to the working of the Coase theorem. Interestingly, this view finds some support in the empirical literature on land acquisition. Benson (2005), for example, discusses examples where private railroads managed to collect the required plots without any government intervention.[13]

This insight is of relevance to the central debate in the coalitional bargaining literature: the tension between efficiency and strategic considerations. The results obtained in this paper are even more surprising given the very general setup adopted here, and the fact that, contrary to the coalitional bargaining literature, efficiency seems to require neither a random bargaining protocol, nor the ability to renegotiate.[14] What seem to be critical are strict super-additivity

[12] A closely related contract is the deferred payments one, where in contrast to equity contracts, the buyer promises to pay an absolute amount to the seller only when the buyer implements a project. Such contracts have been examined in Cai (2003).

[13] In the Indian context, for example, while the Nano project in Singur ran into problems, around the same time there were many instances of trouble free land acquisition by private agents, even in West Bengal.

[14] It is interesting that in the context of general coalitional games with binding offers, Hyndman and Ray (2007) find that in the presence of externality, efficiency does not obtain for four player games in Markovian equilibria even if one allows for renegotiation. It should be noted though that in contrast to Hyndman and Ray (2007), we have a two-sided bargaining structure, strict super-additivity and further

and a bargaining protocol that does not exclude any active seller.

These results are of potential interest to policy makers as well, especially in fields like land development. While policy prescription is beyond the scope of the present paper, our results suggest for example, that holdout cannot be used as a justification for eminent domain. Justification for such regulations, if any, must be found in other factors, e.g. incomplete information. This suggests one possible direction for future research, namely informational asymmetry. While the adverse efficiency implications of informational problems are well known, it would be of interest to see if, in the presence of holdout, informational problems are exacerbated.

8 Appendix

8.1 Appendix A: Lemma 3

Proof of Lemma 3. By Lemma 1, the result is clearly true for $m = 1$. So assume an induction hypothesis that the result is true for histories that begin with $m = 1, \ldots, n-1$ active sellers. Consider now an history that begins with $m = n$ active sellers.

(i) We first argue that $Y_i(n, \delta) \leq \frac{1}{1+\delta}$ for any seller i. Suppose not. Then there exists some seller i for whom $Y_i(n, \delta) > \frac{1}{1+\delta}$. This implies that there is an equilibrium outcome in which an offer P_i is agreed upon by the buyer and seller i at some date t (following this history) giving seller i a payoff that is arbitrarily close to $Y_i(n, \delta)$ which in turn is strictly greater than $\frac{1}{1+\delta}$. Now because of the induction hypothesis, all sellers must be active at this date.

Now if the buyer was making this offer, then by Lemma 2, each of the remaining seller j must be getting an offer $P_j \geq \frac{\delta}{1+\delta}$. If the buyer deviates by offering $P_i - \eta$ to seller i, seller i must accept for η small. Now if the remaining sellers were offered $P_j + \epsilon$ where $(n-1)\epsilon < \eta$, then all such sellers must accept it also. Clearly, this will constitute a profitable deviation for the buyer.

Therefore, it is seller i who is asking for P_i which is accepted by the seller. We first claim that the game must be over after this date with the buyer implementing a project. Otherwise, the game continues in the next period, when, by our induction hypothesis, the buyer's payoff in the continuation equilibrium is at most $\delta/(1+\delta)$. Since $P_i > \frac{1}{1+\delta}$, the buyer's payoff in this equilibrium is thus negative. Now we claim that all of the remaining sellers must be making an acceptable offer at this date as well. Otherwise, the buyer is implementing a partial project, and moreover, since $v(n-1) < 1/2$, the buyer's payoff then is negative in this equilibrium. Thus, at this date, all sellers make acceptable offers of P_j resulting in a payoff of $1 - \sum_j P_j$. Suppose the buyer deviates, rejects seller i's offer, but accepts the rest of the offers. Then in period $t+1$, the buyer will offer $\frac{\delta}{1+\delta}$ to seller i which, by Lemma 1, must be accepted by seller i. Such a deviation will yield a payoff of $\frac{\delta}{1+\delta} - \sum_{j \neq i} P_j$. Since $P_i > \frac{1}{1+\delta}$, the buyer will be better off from

allow for history dependent strategies.

this deviation. This establishes that the supremum of equilibrium seller payoff for any seller is $\frac{1}{1+\delta}$.

(ii) We now show that $Y_B(n, \delta) \leq \frac{1}{1+\delta}$. Otherwise, $Y_B(n, \delta) > \frac{1}{1+\delta}$ and thus there is an equilibrium outcome in which an agreement is reached in period t with the buyer getting a payoff arbitrarily close to $Y_B(n, \delta)$, which in turn is strictly greater than $\frac{1}{1+\delta}$. Since $v(n-1) < 1/2$ and the payoff to each seller must be non-negative, it must be that in this equilibrium, the buyer implements the grand project. Furthermore, at t, the number of active sellers must be n, otherwise, the induction hypothesis will apply. Now if t is odd, then it is the buyer who is making an acceptable offer to all of the n active sellers. Since $n \geq 2$, by Lemma 2, the buyer's payoff is no more than $\frac{1-\delta}{1+\delta}$, a contradiction. Therefore, at t, it is the sellers who are making these acceptable offers. Since the payoff to the buyer is more than $\frac{1}{1+\delta}$ and $n \geq 2$, at least one of the sellers is making an offer P_j that is strictly less than $\frac{\delta}{1+\delta}$. If this seller deviates and asks for a slightly higher price, the buyer must accept. This is because by accepting all the offers, his payoff will be arbitrary close to $Y_B(n, \delta)$ while if he rejects all offers, then his payoff is at most $\delta Y_B(n, \delta)$. Finally, if he accepts only a subset and continues, by induction hypothesis, his payoff from tomorrow is at most $\frac{\delta}{1+\delta}$. Thus, seller j has a profitable deviation. ■

For the proofs in Appendix B-F, we will number the sellers 1 through n and given any seller set S, the highest ranked seller is referred to as the first seller, while the lowest ranked seller is referred to as the last seller. Further, for expositional simplicity, in our specification of equilibrium strategies, we restrict attention only to those histories that may arise out of unilateral deviation by a given player (see, however, footnote 15).

8.2 Appendix B: Proposition 2(c)

Proof of Proposition 2(c)(i).

Secret Offer Case:

Consider any history h_t that starts with the active seller set S, with $|S| = m$. If $m = 1$, the strategies are as given by Lemma 1, while if $m = 2$, the strategies are given as in Proposition 1(c). Let $m \geq 3$.

At every t odd, the buyer offers zero to all active sellers. Seller i accept an offer P_i if and only if $P_i \geq \frac{1}{m}$.

At every t even, each seller asks for $P_i = 1/m$. The buyer accepts each of these offers. However, if any seller asks for more, the buyer rejects all the offers.[15]

[15] To define the strategies of the buyer for any arbitrary history, let $V(t, m)$ denote the continuation payoff of the players at the end of period t when m sellers remain and players play according to the strategies specified above. Consider now a history with $m \geq 3$ sellers and the offer vector $(P_1, \ldots, P_m)$. If the buyer accepts the offers made by the seller set S' with $|S'| = s$, then his continuation payoff is $Y(S') = V(t, m-s) - \sum_{i \in S'} P_i$. Let S^* maximize this payoff and let $s^* = |S^*|$. The buyer accepts the

Finally, consider any history at the end of a period t where the buyer has acquired k objects. The buyer implements if and only if $k = n$. He continues negotiation otherwise.

This completes the description of the strategies. Given these strategies, all sellers reject the first period offer of zero by the buyer. In the second period, each seller asks for $P = 1/n$. These offers are accepted by the buyer and the grand project is implemented in period 2.

Observe that in period 1 if seller i is offered less than $1/n$, she is supposed to reject. Is this an optimal response for seller i? Note that such a rejection is clearly optimal if all of the other sellers are accepting their offers. Since the offers are secret, seller $i's$ rejection is thus consistent with an offer of at least $1/n$ to the remaining sellers. Since such offers will be accepted by the other sellers, seller i will be better off rejecting an offer which is less than $1/n$.

It is straightforward to check that the strategy profile described above constitutes a subgame perfect equilibrium.

Public Offer Case:

Consider any history h_t that starts with the active seller set S, with $|S| = m$. If $m = 1$, the strategies are as given by Lemma 1, while if $m = 2$, the strategies are given as in Proposition 1(c).

Assume now that we have the strategies for players for all histories with m active players, where $m = 1, 2 \ldots, n-1$ and let $m = n$.

If t is odd, the buyer offers zero to all active sellers.

To define the acceptance/rejection decision of seller i, consider any arbitrary offer vector $(P_1, \ldots, P_n)$. By induction hypothesis, the response of the first $n-1$ sellers is well defined for the history that starts with $n-1$ sellers and the offer vector $(P_1, \ldots, P_{n-1})$. Let H be the set of sellers in $\{1, 2 \ldots, n-1\}$ that reject given the offer vector $(P_1, \ldots, P_{n-1})$. We now define the acceptance decision of the n-th player given P_n. If $|H| \geq 3$, then P_n is accepted if and only if $P_n \geq 0$. If $|H| = 2$, then P_n is accepted if and only if $P_n \geq \frac{\delta(1-\delta)}{1+\delta}$. However, if $|H| < 2$, P_n is accepted if and only if $P_n \geq \frac{\delta}{1+\delta}$.

If t is even, the first two sellers in S ask for $\frac{\delta}{1+\delta}$, while the third seller asks for $\frac{1-\delta}{1+\delta}$. The rest of the sellers ask for zero. The buyer accepts these offers. However, if any seller asks for more, the buyer rejects all offers.[16]

Finally, consider any history at the end of a period t where the buyer has acquired k objects. The buyer continues negotiation if and only if $k \neq n$.

Given these strategies, at $t = 1$, the buyer makes a zero offer to all sellers. The first three sellers reject, while the rest accept. At $t = 2$, the first two sellers ask for $\frac{\delta}{1+\delta}$ and the third seller asks for $\frac{1-\delta}{1+\delta}$. These offers are accepted by the buyer and the grand project is implemented at $t = 2$.

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offers in S^* if and only if $Y(S^*) \geq 0$. He rejects all offers otherwise.

[16] A construction similar to that of footnote 15 can be used to define the acceptance/rejection decision of a buyer for any arbitrary offer vector.

Proof of Proposition 2(c)(ii). Consider buyer strategies such that at $t = 1$, he makes unacceptable offers to all sellers, and exits at the end of the first period. From $t = 2$ onwards, the strategies follow those given in Proposition 2(c)(i).

8.3 Appendix C: Proposition 3(b)

Proof of Proposition 3(b). We will construct an equilibrium in which for δ large, the buyer's payoff is exactly $\delta v(n-2)$ and the grand project is implemented at $t = 2$.

Fix a history h_t that ends with the active seller set S and $|S| = m$. When $m = 1$, the strategies are specified in Lemma 1 and thus assume that $m \geq 2$.

If t is odd, the buyer offers zero to all sellers. The first two sellers accept an offer P_i if and only if $P_i \geq \frac{\delta}{1+\delta}$. All other sellers accept any nonnegative offer for all $t \geq 3$. For $t = 1$, however, these sellers accept an offer P_i if and only if $P_i \geq \frac{1-\delta}{1+\delta}$.

If t is even and $t \neq 2$, each seller asks for P such that $1 - mP = v(n-m)$. While at $t = 2$, each seller asks for P^0, where $1 - nP^0 = v(n-2)$. The buyer is supposed to accept these offers. If any seller asks for more, the buyer rejects all offers.

Finally, consider the implementation decision of the buyer at the end of any period t where the buyer has acquired k objects. The buyer implements the project of size k if and only if $v(k) > 0$. It continues negotiation otherwise.

We observe that given the specified strategies, all sellers in period 1 rejects the buyer's offer. In period 2, each seller offers P^0, and these offers are accepted by the buyer and the grand project is implemented in period $t = 2$.

We note that at $t = 2$, if any seller demands $P > P^0$, the buyer is better off rejecting such an offer. This is because by rejecting all offers at that period, the buyer hopes to get $v(n-2)$ next period since his offer of zero will be accepted by the last $n-2$ sellers. Thus, rejection yields a payoff of $\delta v(n-2)$ to the buyer. If the buyer were to accept the deviating seller's offer of $P > P^0$, his payoff is necessarily less than $\delta v(n-2)$. Further, following the buyer's rejection, the seller has a continuation payoff of zero since, in the next period, either she accepts an offer of zero, or there are $n-2$ sellers who do and the buyer exits. Hence no deviating seller has a profitable deviation at $t = 2$. It is also straightforward to check that given the acceptance strategies of the sellers, the buyer can not hope to make an acceptable offer to a group of sellers and get a higher payoff.

Finally, we note that for δ large, $v(n-2) > \frac{1-\delta}{1+\delta}$ and thus given the offer/acceptance decision of the sellers, for any $t \geq 3$, it is always optimal for the buyer to implement the project of size $n-2$, rather than continuing negotiation with the remaining two sellers. ■

8.4 Appendix D: Proposition 4(b)

Proof of Proposition 4(b). We begin with

Lemma 5. *For δ large, starting from any history h_t with exactly two active sellers, there is an equilibrium where the buyer obtains $\frac{v(n-2)}{\delta}$ at t even.*

Proof of Lemma 5. Choose δ such that $\frac{\delta^3}{1+\delta} > v(n-2)$.

For any history with $m = 1$ active seller, the strategy is given by Lemma 1. Consider now any history h_t that starts with $m = 2$ active sellers and at the start of $t-1$, the number of active sellers was greater than two.

Let $\tilde{P}$ satisfy $\frac{\delta}{1+\delta} - \tilde{P} = \frac{v(n-2)}{\delta}$. Since $v(n-2) \leq v(n-1) < 1/2$, for δ close to 1, $\tilde{P}$ is strictly positive. The strategies of the players in the continuation equilibrium will depend on the phase it is in, A or B. The first period always starts in phase A.

Strategies in Phase A.

If t is odd, the buyer offers zero to the first seller and $\delta\tilde{P}$ to seller 2. The first seller accept an offer P_1 if and only if $P_1 \geq \frac{\delta}{1+\delta}$. Seller 2 accepts an offer P_2 if and only if $P_2 \geq \delta\tilde{P}$ and $P_1 < \frac{\delta}{1+\delta}$.

If t is even, the first seller asks for $\frac{1}{1+\delta}$, while seller 2 asks for $\tilde{P}$. These offers are accepted by the buyer. If any seller asks for more, the buyer rejects both the offers.

In this phase, the buyer always continues negotiation if it has not acquired all objects and the period is odd.

If however, the period is even and the buyer still has $n-2$ objects, it will continue negotiation only if one of the sellers in that period have deviated and asked for a higher price. It will implement the project of size $n-2$ otherwise. Thus, if the sellers did not deviate from their offer strategies but the buyer rejected both of the offers, the buyer will implement the project of size $n-2$.

Transition.

If t is even, and one of the sellers deviate from the above strategies and the buyer rejects both of the offers, there will be a transition from phase A to phase B.[17] By construction, in phase B, it is the buyer who has to make an offer.

Strategies in Phase B.

The buyer makes an offer of zero to both players. Seller 2 accepts any non-negative offer if $P_1 < \frac{\delta}{1+\delta}$. While seller 1 accepts an offer if and only if $P \geq \frac{\delta}{1+\delta}$.

In stage B, at the end of the period, the buyer continues negotiation only if it has acquired $n-1$ objects. He will implement the project of size $n-2$ otherwise.

The state stays in phase B for precisely one period and will revert to phase A in the following period.

[17] It is important to note that the transition to phase B takes place only if one or both sellers deviate from their prescribed strategies.

Observe that in phase A, the payoff to the buyer is exactly $v(n-2)$ when t is odd, and it is $\frac{v(n-2)}{\delta}$, if t is even. Thus, at the end of an odd period, the buyer's discounted payoff by continuing is exactly $v(n-2)$. He also gets $v(n-2)$ by implementing the project, and thus it is optimal for him to continue in odd periods. If t is even, however and the sellers did not deviate from their equilibrium strategies, phase A will continue and thus if the buyer continues he will get exactly $\delta v(n-2)$ and by implementing he gets $v(n-2)$, thus, he is better off implementing the project. Finally, if t is even and the state is going to be in phase B, then by rejecting all offers today and continuing next period, the buyer expects to get $\frac{\delta^3}{1+\delta}$ which is strictly greater than $\frac{v(n-2)}{\delta}$ and thus the buyer will be better off continuing negotiation.

To check that the strategies of the sellers are optimal, it is sufficient to consider seller 2's offer when t is even. If she asks for any price greater than $\tilde{P}$, given that seller 1 is asking for $\frac{1}{1+\delta}$, phase A will transit to phase B. In phase B, the buyer will offer zero to her and the buyer will exit at the end of phase B if the seller rejects. Note that if at the end of phase B, the set of active seller continues to number two, the buyer will implement the project of size $n-2$. ■

Proof of Proposition 4(b) continued. Assume now that we have defined the strategies for the players for all histories h_t that start with m active sellers where $m = 1, 2, \ldots, n-1$ and let $m = n$.

If t is odd, the buyer offers zero to all active sellers.

To define the acceptance/rejection decision of a seller, consider any arbitrary offer vector $(P_1, \ldots, P_n)$. If $P_1 \geq \frac{\delta}{1+\delta}$, the first seller accepts P_1, while the acceptance decision of the sellers numbered 2 through n is the same as their decision for a history with seller set $\{2, ..., n\}$, for the offer vector $(P_2, , ..., P_n)$. On the other hand, if $P_1 < \frac{\delta}{1+\delta}$, the first seller rejects while the rest of the sellers accept any non negative offer.

If t is even, every seller asks for a price of 1. The buyer rejects all these offers. Furthermore, if there is an unilateral deviation by one seller who asks for $P > 0$, the buyer continues to reject all of the offers.[18]

Finally, if at the end of any period, the buyer has acquired $n-2$ objects, he implements a project of size $n-2$. It continues after any other histories.

Given these strategies, it is easy to check that at $t = 1$, the buyer offers zero to all sellers at $t = 1$. All sellers but the first one will accept. In period 2, the first seller asks for $\frac{1}{1+\delta}$ which is accepted by the buyer and the grand project is implemented at the end of period 2. ■

8.5 Appendix E: Lemma 4 and Proposition 5

Proof of Lemma 4. The proof involves constructing an equilibrium where at every t even, the seller asks for $\frac{v(n-1)}{\delta}$, which the buy accepts. The strategies are conditional on whether the game is in either of two phases A, or B.

[18] The acceptance/rejection decision of the buyer for any arbitrary offer vector can be constructed in the same fashion as that of footnote 15.

In *phase A*, at every t odd the buyer offers $\delta - v(n-1)$ to the seller, and the seller accepts if and only if he obtains at least $\delta - v(n-1)$. Whereas at every t even, the seller asks for $1 - \frac{v(n-1)}{\delta}$. The buyer accepts if and only if he obtains at least $\frac{v(n-1)}{\delta}$.

In this phase, the buyer always continues negotiation in case an offer is rejected.

Finally, there is *transition* to *phase B* if the seller asks for more than $1 - \frac{v(n-1)}{\delta}$.

In *phase B*, at every t odd the buyer offers $(1, 0)$, and the seller accepts if and only if he obtains at least 0. Whereas at every t even, the seller offers $(\delta, 1-\delta)$. The buyer accepts if and only if he obtains at least δ,

If t is even and an offer is rejected by the buyer, the buyer continues negotiation.

If however, t is odd and the seller rejects buyer's offer, the buyer implements the project of size $n-1$. In this case, if the buyer fails to implement the project and continues, phase B transits to phase A. ■

Proof of Proposition 5. The proof involves constructing equilibrium profiles such that at every t even, the sellers all make unacceptable offers, whereas at every t odd, the buyer offers a payoff of zero to all sellers. At t odd, the sellers all accept since otherwise the other sellers accept, and the buyer exits the game and implements a project of size $n-1$. We now formally describe the strategies.

For any history that starts with exactly one active seller, the strategies of the players are as specified in the proof of Lemma 4. For any history that starts with m sellers, $m > 1$, the strategies are as follows:

If t odd, the buyer offers zero to each of the active sellers. Each seller accepts any non-negative offer.

At the end of period t where t is odd, the buyer implements the project only if he has acquired at least $n-1$ objects. He continues otherwise.

If t even, each seller asks for $P = 1$. Given any offer vector $P = (P_1, P_2 \dots, P_m)$, let $Z = 1 - \sum_{i \in M} P_i$, where M is the set of active sellers. The buyer accepts every offer if $Z \geq \delta$. If $Z < \delta$, it rejects all offers.

At the end of period t, where t is even, the buyer implements the project only if he has acquired all the objects. He proceeds to the next period otherwise.

Consider a subgame with t odd, where $n-1$ of the offers have been accepted by the sellers. The buyer's payoff from implementing a project is $v(n-1)$, whereas if he continues to negotiate, then he obtains $\frac{v(n-1)}{\delta}$ in the next period (Lemma 4), so that exiting immediately is optimal. Further, note that these strategies work for the secret, as well as the public offer game. ■

8.6 Appendix F: Proposition 6

Proof of Proposition 6. For any history at date t with $m = 1$ active seller, the strategies of the players are given by Lemma 1 if $v(n-1) < 1/2$. If $v(n-1) > 1/2$, these strategies are given by Lemma 4. Next fix a history at date t with $m \geq 2$ active sellers:

If t is odd, the buyer offers zero to all sellers. This offer is conditional, i.e, it is binding on the buyer only if all sellers accept. For any offer vector $(P_1, \ldots, P_m)$, seller i accept P_i if and only if $P_i \geq 0$.

If t is even, each seller asks for $P = 1$. Given any vector of offers $\{P_i\}$, the buyer accepts all the offers if $1 - \sum_i P_i \geq \delta$. He rejects all the offers otherwise.

Finally, consider any history at the end of any period, with the buyer having acquired k objects. The buyer continues negotiation only if $k \neq n$.

It is easy to check that for δ large, this strategy profile constitutes an equilibrium. The buyer implements the grand project at date 1 and obtains 1. ■

9 Reference

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