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# Welfare effects of common ownership in an international duopoly\*

Yi Liu<sup>†</sup> and Toshihiro Matsumura<sup>‡</sup>

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## Abstract

We formulate an international oligopoly model in the presence of global common ownership. We theoretically investigate how common ownership affects the volume of international trade in an oligopoly market and global welfare. We find that welfare decreases (increases) with the degree of common ownership when the international transport costs are low (high).

**JEL classification codes:** L13, F12, K21

**Keywords:** overlapping ownership, transport cost, welfare-improving production substitution

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# 1 Introduction

A distinct feature of financial markets in recent years is the high concentration in the investment industry. Several big institutional investors such as BlackRock, Vanguard, and State Street began to own non-negligible shares in most major listed firms globally (Nikkei Market News, 2018/10/24). Moreover, Moreno and Petrakis (2022) show in their dynamic model that all stationary equilibria involve large common investors holding symmetric portfolios regardless of whether firms face price or quantity competition. Therefore, we expect common ownership to prevail in the long run. Such common ownership reduces firms' incentive to compete (Azar et al., 2018; Moreno and Petrakis, 2022), and has become a central issue in recent antitrust debates (Elhauge, 2016; Backus et al., 2021).

Although the literature emphasizes the anti-competitive and welfare-reducing effects of common or overlapping ownership (Reynolds and Snapp, 1986; Farrell and Shapiro, 1990; Gilo et al., 2006; Azar et al., 2018), several studies also point out the possible welfare-improving effects of common ownership. While common ownership lessens competition in product markets and raises prices, partial ownership by common owners in the same industry may lead firms to internalize industry-wide externalities and improve welfare. López and Vives (2019) assert that common ownership internalizes the positive externality of R&D, showing that this welfare-improving effect may dominate the competition-reducing effects when the degree of common ownership is relatively low. In other words, they suggest a possible inverted U-shaped relationship between the degrees of common ownership and welfare. Sato and Matsumura (2020) investigate a free-entry market and find that common ownership internalizes the business-stealing effect, thus moderate common ownership may improve welfare.<sup>1</sup> They also show that significant common ownership always reduces welfare. Again, an inverted U-shaped relationship is seen between the degree of common ownership and welfare. Chen et al. (2021) investigate vertically related markets. They demonstrate

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<sup>1</sup>For a discussion on the business-stealing effect in free-entry markets, see Mankiw and Whinston (1986).

that common ownership mitigates the problem of double marginalization, and that this welfare-improving effect dominates the competition-reducing effects on downstream markets, if the competition among downstream firms is weak. Hirose and Matsumura (2022) investigate the relationship between common ownership and firms' commitments to environmental corporate social responsibility. They show that common ownership may improve welfare, but it weakens firms' incentive for effective emission-reducing commitments. However, no study has analyzed how common ownership affects trading volumes and global welfare in the presence of international oligopolies.

This study considers the welfare effect of common ownership in an international duopoly. We formulate a symmetric two-country model with one firm in each country. Each duopolist chooses the quantities for the home-market and exports. The export incurs additional costs for international transport. We find that global welfare decreases (increases) with common ownership when the transport cost is low (high). Our results suggest that even a high level of common ownership may improve welfare. Moreover, we demonstrate a possible U-shaped relationship between the degree of common ownership and welfare. In other words, we show that a low (high) level of common ownership may harm (improve) welfare. We incorporate international transport costs into the model, an important component of international trade, and explain how it affects the relationship between common ownership and global welfare. This constitutes the contribution of our study.

The remainder of this paper is organized as follows. Section 2 describes the model formulation. Section 3 (4) presents equilibrium outcomes (welfare implications) for linear demand. Section 5 extends our analysis to the case of general demand. Section 6 presents the conclusion.

## 2 The Model

We formulate a symmetric two-country two-firm model. There are two countries, A and B, and two firms, 1 and 2. Firm 1 (2) is the home (foreign) firm in country A, and firm 2 (1) is the home (foreign) firm in country B. We assume that each firm's marginal cost for its home (foreign) market is  $c$  ( $c + t$ ), where  $c$  is the marginal cost of production,  $t$  is the international transport cost, and  $c$  and  $t$  are non-negative constants. Let  $q_i^A$  ( $q_i^B$ ) be the output of firm  $i$  supplied for market A (B), where  $i = 1, 2$ . The inverse demand function in country A (B) is  $p^A = a - Q^A$  ( $p^B = a - Q^B$ ), where  $Q^A := q_1^A + q_2^A$  ( $Q^B := q_2^B + q_1^B$ ). We adopted Dixit's (1984) segmented market setup. In other words, consumer and independent trader arbitrage is assumed to be prohibitively costly.<sup>2</sup>

The profits of firm 1 ( $\pi_1$ ) and firm 2 ( $\pi_2$ ) are respectively

$$\pi_1 = (p^A - c)q_1^A + (p^B - c - t)q_1^B, \quad (1)$$

$$\pi_2 = (p^A - c - t)q_2^A + (p^B - c)q_2^B. \quad (2)$$

Following recent theoretical literature on common ownership (López and Vives, 2019), we assume that each firm  $i$  has the following objective function:

$$\psi_i = \pi_i + \lambda\pi_j,$$

where  $\pi_i$  is firm  $i$ 's profit,  $\pi_j$  is its rival's profit, and  $\lambda$  is the degree of common ownership.<sup>3</sup> We restrict our attention to the case in which the solution is interior. In other words, we assume that both firms are active in both markets. The solution is interior if and only if  $\lambda < \bar{\lambda} := (a - c - 2t)/(a - c)$ . Therefore, our analysis does not cover the case of high  $\lambda$ .

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<sup>2</sup>This assumption is not essential. Unless transport costs for consumers or independent traders are strictly lower than those of firms, arbitrage plays no role in our model.

<sup>3</sup>Prior studies have also investigated this type of payoff interdependence using a coefficient-of-cooperation model (Cyert and Degroot, 1973; Escrihuela-Villar, 2015) and a relative profit maximization model (Escrihuela-Villar and Gutiérrez-Hita, 2019; Hamamura, 2021; Matsumura and Matsushima, 2012; Matsumura et al., 2013).

Global welfare  $W$  is the sum of the two firms' profits and consumer surplus of the two countries. It is given by

$$W = \pi_1 + \pi_2 + \frac{(Q^A)^2}{2} + \frac{(Q^B)^2}{2}. \quad (3)$$

### 3 Equilibrium

Because of the symmetry between the two countries, we focus on the competition in market A. The first-order conditions of firms 1 and 2 are

$$p^{A'} q_1^A + (p^A - c) + \lambda p^{A'} q_2^A = 0, \quad (4)$$

$$p^{A'} q_2^A + (p^A - c - t) + \lambda p^{A'} q_1^A = 0. \quad (5)$$

Substituting  $p = a - q_1^A - q_2^A$ , we obtain the following reaction functions:

$$R_1^A(q_2^A) = \frac{a - c - (1 + \lambda)q_2^A}{2}, \quad (6)$$

$$R_2^A(q_1^A) = \frac{a - c - t - (1 + \lambda)q_1^A}{2}. \quad (7)$$

From these reaction functions, we obtain the following equilibrium outputs:

$$q_1^{A*} = \frac{(a - c)(1 - \lambda) + t(1 + \lambda)}{(3 + \lambda)(1 - \lambda)}, \quad (8)$$

$$q_2^{A*} = \frac{(a - c)(1 - \lambda) - 2t}{(3 + \lambda)(1 - \lambda)}, \quad (9)$$

$$Q^{A*} = \frac{2(a - c) - t}{3 + \lambda}. \quad (10)$$

Superscript \* denotes the equilibrium outcome.

From these equations, we obtain the following lemma:

**Lemma 1** (i)  $q_1^{A*}$  increases with  $t$ . (ii)  $q_2^{A*}$  and  $Q^{A*}$  decrease with  $t$ . (iii)  $q_1^{A*}$  increases with  $\lambda$  if and only if

$$\frac{t}{a - c} > \frac{(1 - \lambda)^2}{5 + 2\lambda + \lambda^2}.$$

(iv)  $q_2^{A*}$  and  $Q^{A*}$  decreases with  $\lambda$ . (v)  $\partial q_1^{A*}/\partial\lambda \geq \partial q_2^{A*}/\partial\lambda$ , where equality holds if and only if  $t = 0$ .

**Proof** See the Appendix

An increase in  $t$  raises firm 2's marginal cost for market A, which reduces  $q_2^{A*}$  (direct cost effect). As the strategies are strategic substitutes, it increases  $q_1^{A*}$  through strategic interactions between the two firms (indirect strategic effect). As the direct cost effect dominates the indirect strategic effect under the stability condition, an increase in  $t$  decreases  $Q^{A*}$ .

When  $\lambda$  is larger, each firm is more concerned with its rival's profit. Thus, an increase in  $\lambda$  always reduces each firm's output to increase its rival's profit when firms have the same marginal cost (i.e.,  $t = 0$ ). However, under cost heterogeneity (i.e.,  $t > 0$ ), an increase in  $\lambda$  may stimulate the home firm's production, which seems to be counter-intuitive. This is because the home firm's production is more efficient than that of the foreign firm from the viewpoint of joint-profit-maximization. When  $\lambda$  is larger, the equilibrium combination of outputs is close to the cooperative (joint-profit-maximizing) one. Thus, the foreign firm has a stronger incentive than the home firm to reduce its output. Because the strategies in the second stage are strategic substitutes, a reduction in the foreign firm's output naturally increases the home firm's output. This effect can be significant, especially when  $t$  is high, and may dominate the standard output-reducing effect owing to common ownership. Consequently, the home firm's output may increase with  $\lambda$ .

Even when  $q_1^{A*}$  decreases with  $\lambda$ , the output-reducing effect of common ownership is greater for the foreign firm than the home firm. This leads to Lemma 1(v).

By symmetry, we obtain  $q_1^{A*} = q_2^{B*}$ ,  $q_2^{A*} = q_1^{B*}$ , and  $Q^{A*} = Q^{B*}$ . Substituting these into

the inverse demand function, the profits of each firm, and global welfare yields

$$p^* = \frac{a(1 + \lambda) + 2c + t}{3 + \lambda}, \quad (11)$$

$$\begin{aligned} \pi_1^* = \pi_2^* := \pi^* &= \frac{[(a - c)(1 + \lambda) + t][(a - c)(1 - \lambda) + t(1 + \lambda)]}{(3 + \lambda)^2(1 - \lambda)} \\ &+ \frac{[(a - c)(1 + \lambda) - t(2 + \lambda)][(a - c)(1 - \lambda) - 2t]}{(3 + \lambda)^2(1 - \lambda)}, \end{aligned} \quad (12)$$

$$W^* = \left( \frac{2(a - c) - t}{3 + \lambda} \right)^2 + 2\pi^*. \quad (13)$$

## 4 Results

We now discuss how the degree of common ownership affects welfare.

**Proposition 1** (i) The common equilibrium price in the two countries,  $p^*$ , increases with  $\lambda$ . (ii) The equilibrium profit of each firm,  $\pi^*$ , increases with  $\lambda$ . (iii)  $\partial W^*/\lambda < (=, >)0$  if  $t < (=, >)\hat{t}$ , where

$$\hat{t} := \frac{(1 - \lambda)(a - c)[(3 + \lambda)\sqrt{(3 + \lambda)(1 + \lambda)} + \lambda^2 - 1]}{5\lambda^2 + 14\lambda + 13} > 0.$$

(iv)  $\hat{t}$  decreases with  $\lambda$ .

Common ownership harms consumer surplus (Proposition 1(i)), and increases firms' profits (Proposition 1(ii)). These standard results are intuitive. Proposition 1(iii) states that common ownership improves (harms) welfare if transport cost is high (low).<sup>4</sup> We explain the intuition behind this result. Each firm's marginal cost for the home market is lower than that for the foreign market because of international transport costs. In other words, each firm's supply for the home market is more profitable than that for the foreign market. In the presence of common ownership, each firm is concerned with the rival's profit. Thus, each firm reduces its supply to the foreign market more significantly than it does to the home market. This reduces the (weighted) average of the two firms' costs

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<sup>4</sup>We can show that the solution is interior when  $t = \hat{t}$ . Thus, there exists  $t$  such that an increase in  $\lambda$  improves (harms) welfare.



and increases their joint profits. This welfare-improving effect is more pronounced when  $t$  is higher, dominating the welfare-reducing effect owing to smaller total output (smaller consumer surplus). Therefore, common ownership improves welfare if  $t$  is high.<sup>5</sup>

Moreover, Proposition 1(iii,iv) suggests a possible non-monotone relationship between the degree of common ownership and welfare. Because  $\hat{t}$  is decreasing in  $\lambda$ , it is possible that  $t < \hat{t}$  ( $t > \hat{t}$ ) holds when  $\lambda$  is small (large). Thus, the relationship between  $W$  and  $\lambda$  can be U-shaped. Several studies on common ownership show a possible non-monotone relationship (López and Vives, 2019; Sato and Matsumura, 2020). However, they suggest that a moderate degree of common ownership improves welfare but a significant degree does not. By contrast, our result suggests that a significant degree of common ownership can improve welfare even if a moderate degree of common ownership harms welfare. This is because an increase in  $\lambda$  more effectively induces welfare-improving production substitution when  $\lambda$  is larger.<sup>6</sup>

In summary, there are three (two monotone and one non-monotone) patterns in the relationship between  $W$  and  $\lambda$ . If  $t$  is sufficiently low (high),  $W$  always decreases (increases) with  $\lambda$  (i.e., monotone relationship appears). If  $t$  is moderate,  $W$  decreases (increases) with  $\lambda$  when  $\lambda$  is small (large). However, in our analysis, an inverted U-shape relationship does not appear.

## 5 Extension: general demand

In the previous section, we use a linear-demand function. In this section, we extend our analysis to a more general demand case. The common demand function for each country market is  $p^i = p(Q^i)$  where  $i = A, B$ . Because of the symmetry of the two countries, we focus on market A only and drop the superscript A until we discuss equilibrium outcomes. We assume that  $p' < 0$  and  $p'' \leq 0$  as long as both  $p$  and  $Q$  are positive. This guarantees

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<sup>5</sup>See Lahiri and Ono (1988) for discussions of welfare-improving production substitution.

<sup>6</sup>See the proof of Lemma 1(v), which states that  $\partial q_1^{A*}/\partial \lambda - \partial q_2^{A*}/\partial \lambda$  increases with  $\lambda$ .

that the second-order conditions are satisfied (because  $2p' + p''(q_i + \lambda q_j) < 0$  holds), the strategies are strategic substitute (because  $(1 + \lambda)p' + p''(q_i + \lambda q_j) < 0$  holds), and the stability condition is satisfied (because  $|2p' + p''(q_i + \lambda q_j)| > |(1 + \lambda)p' + p''(q_i + \lambda q_j)|$  holds), where  $i, j = 1, 2$ , and  $i \neq j$ .<sup>7</sup>

We restrict our attention to the case where the solution is interior. In other words, we assume that both firms are active in both markets. Therefore, our analysis does not cover cases with high  $t$  and  $\lambda$ . Let  $\bar{t}(\lambda)$  be the upper bound of  $t$  that yields the interior solution. From (5), we obtain  $\bar{t} = p - c + \lambda p' q^M$  where  $p^M$  ( $q^M$ ) is the price (output) when the home firm is the monopolist. As the monopoly price is independent of  $\lambda$  and  $t$ ,  $\bar{t}$  decreases with  $\lambda$ .<sup>8</sup>

(4) and (5) yield the equilibrium output. By totally differentiating (4) and (5), we obtain

$$\frac{\partial q_1}{\partial t} = -\frac{p'(1 + \lambda) + p''(q_1 + \lambda q_2)}{p'\Omega_1}, \quad \frac{\partial q_2}{\partial t} = \frac{2p' + p''(q_1 + \lambda q_2)}{p'\Omega_1}, \quad \frac{\partial Q}{\partial t} = \frac{1 - \lambda}{\Omega_1}, \quad (14)$$

$$\frac{\partial q_1}{\partial \lambda} = \frac{\Omega_2}{\Omega_1}, \quad \frac{\partial q_2}{\partial \lambda} = -\frac{[2p' + p''(q_1 + \lambda q_2)]\Omega_2 + p'q_2\Omega_1}{\Omega_1\Omega_3}, \quad \frac{\partial Q}{\partial \lambda} = \frac{-p'[(1 - \lambda)\Omega_2 + q_2\Omega_1]}{\Omega_1\Omega_3}, \quad (15)$$

where

$$\Omega_1 = (1 - \lambda)[(3 + \lambda)p' + p''(1 + \lambda)Q], \quad (16)$$

$$\Omega_2 = [p'(1 + \lambda) + p''q_1]q_1 - (2p' + p''q_2)q_2, \quad (17)$$

$$\Omega_3 = p'(1 + \lambda) + p''(q_1 + \lambda q_2). \quad (18)$$

Lemma 2 shows the properties of  $\Omega_1$ ,  $\Omega_2$ , and  $\Omega_3$ , which are critical for determining the relationship between  $\lambda$  and the equilibrium outcomes.

**Lemma 2** (i)  $\Omega_1 < 0$ . (ii)  $\Omega_2 > 0$  if  $t$  is sufficiently low (i.e., sufficiently close to 0). (iii)  $\Omega_2 < 0$  if  $t$  is sufficiently high (i.e., sufficiently close to  $\bar{t}$ ). (iv)  $\Omega_3 < 0$ .

**Proof** See the Appendix.

<sup>7</sup>We can replace the assumption  $p'' \leq 0$  with  $p' + p''Q < 0$  (the industry's marginal revenue is decreasing).

<sup>8</sup>If  $t \geq \bar{t}$ , each firm becomes the monopolist in its home market; thus, a further increase in  $\lambda$  does not affect the equilibrium outcomes.

Again, we use superscript A\* (B\*) to denote equilibrium outcomes in market A (B). By symmetry of the two countries,  $q_1^{A*} = q_2^{B*}$ ,  $q_2^{A*} = q_1^{B*}$ , and  $Q^{A*} = Q^{B*}$ . From (14),(15), and Lemma 2, we obtain the following lemma.

**Lemma 3** (i)  $q_1^{A*}$  increases with  $t$ . (ii)  $q_2^{A*}$  and  $Q^{A*}$  decrease with  $t$ . (iii)  $q_1^{A*}$  increases (decreases) with  $\lambda$  if  $t$  is sufficiently high (low). (iv)  $q_2^{A*}$  and  $Q^{A*}$  decrease with  $\lambda$ . (v)  $\partial q_1^{A*}/\partial \lambda \geq \partial q_2^{A*}/\partial \lambda$ , where the equality holds if and only if  $t = 0$ .

**Proof** See the Appendix.

Because of the symmetry between the two countries, global welfare  $W$  is given by:

$$W^* = 2 \left[ \int_0^{Q^{A*}} p(q) dq - c q_1^{A*} - (c+t) q_2^{A*} \right]. \quad (19)$$

Then we obtain

$$\frac{\partial W^*}{\partial \lambda} = 2 \left[ p(Q^{A*}) \frac{\partial Q^{A*}}{\partial \lambda} - c \frac{\partial q_1^{A*}}{\partial \lambda} - (c+t) \frac{\partial q_2^{A*}}{\partial \lambda} \right]. \quad (20)$$

We now present the result.

**Proposition 2** (i)  $\partial W/\partial \lambda < 0$  if  $t$  is sufficiently low. (ii)  $\partial W/\partial \lambda > 0$  if  $t$  is sufficiently high.

**Proof** See the Appendix.

Proposition 2 provides limit results. Proposition 2(i) (Proposition 2(ii)) characterizes the welfare consequence of common ownership when  $t$  is low (high). However, the three effects of common ownership discussed in the previous section do not depend on the linearity of the demand (Lemma 3(iii-v)). Common ownership reduces total output, and thus, harms consumer surplus. It increases the home firm's market share, which improves production efficiency, because it reduces international transport costs. The latter effect dominates the former only when  $t$  is high. Therefore, our main result that common ownership may or may not improve welfare, which depends on  $t$ , holds true. Based on these facts, we conjecture that common ownership is more likely to improve welfare when  $t$  is higher.

## 6 Concluding remarks

In this study, we investigate how common ownership affects the volume of international trade in an oligopoly market and global welfare. We find that common ownership reduces international trade. Welfare decreases (increases) with the degree of common ownership when the international transport cost is low (high). Therefore, common ownership can improve welfare, especially when the trade costs are high.

However, we do not consider trade policies in this study. Common ownership affects trade volume and tariff revenue when import tariffs are imposed. In this case, common ownership has another welfare effect in the presence of import tariff. Thus, common ownership may affect government incentives to formulate trade policies. Future research can extend our study in this direction.

In this study, we consider common ownership that induces non-profit-maximizing behavior. Other aspects such as corporate social responsibility also affect firms' objective functions. Another interesting extension of this study could incorporate a different type of non-profit maximizing behavior into our analysis and investigate how the interaction between different types of non-profit-maximizing objectives affects global welfare.<sup>9</sup>

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<sup>9</sup>See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Bárcena-Ruiz and Garzón (2005a, b) for a discussion on the relationship between the welfare-maximizing objective of a public firm and international trade. See Bárcena-Ruiz and Sagasta (2022) for a discussion on the relationship between corporate social responsibility and international trade.

## Appendix

### Proof of Lemma 1

From (8), (9), and (10), we obtain

$$\frac{\partial q_1^{A*}}{\partial t} = \frac{1 + \lambda}{(3 + \lambda)(1 - \lambda)} > 0, \quad \frac{\partial q_2^{A*}}{\partial t} = -\frac{2}{(3 + \lambda)(1 - \lambda)} < 0, \quad \frac{\partial Q^{A*}}{\partial t} = -\frac{1}{3 + \lambda} < 0,$$

which implies Lemma 1(i,ii).

Again, from (8), (9), and (10), we obtain

$$\begin{aligned} \frac{\partial q_1^{A*}}{\partial \lambda} &= \frac{-(1 - \lambda)^2(a - c) + (5 + 2\lambda + \lambda^2)t}{[(3 + \lambda)(1 - \lambda)]^2}, \\ \frac{\partial q_2^{A*}}{\partial \lambda} &= -\frac{(1 - \lambda)^2(a - c) + 4(1 + \lambda)t}{[(3 + \lambda)(1 - \lambda)]^2} < 0, \\ \frac{\partial Q^{A*}}{\partial \lambda} &= \frac{-2(a - c) + t}{(\lambda + 3)^2} < 0, \end{aligned}$$

which implies Lemma 1(iii,iv).

$\partial q_1^{A*}/\partial \lambda - \partial q_2^{A*}/\partial \lambda = t/(1 - \lambda)^2$ , which implies Lemma 1(v). Q.E.D.

### Proof of Proposition 1

From (11), we obtain  $\partial p^*/\lambda = (2a - 2c - t)/(3 + \lambda)^2 > 0$ , which implies Proposition 1(i).

From (12), we obtain

$$\frac{\partial \pi^*}{\partial \lambda} = \frac{4[(a - c)(a - c - t)(1 - \lambda)^3 + t^2(6\lambda + 3\lambda^2 + 7)]}{(\lambda - 1)^2(\lambda + 3)^3} > 0.$$

This implies Proposition 1(ii).

From (12) and (13), we obtain

$$\frac{\partial W}{\partial \lambda} = \frac{-4(a - c)(a - c - t)(1 - \lambda)^2(1 + \lambda) + 2t^2(5\lambda^2 + 14\lambda + 13)}{(1 - \lambda)^2(\lambda + 3)^3}. \quad (21)$$

The denominator of (21) is positive. As  $\lambda \geq 0$ , the numerator of (21) is positive (zero, negative) if  $t > (=, <) \hat{t}$ , where

$$\hat{t} = \frac{(1 - \lambda)(a - c)[(\lambda + 3)\sqrt{(\lambda + 3)(\lambda + 1)} + \lambda^2 - 1]}{5\lambda^2 + 14\lambda + 13} > 0.$$

This implies Proposition 1(iii).<sup>10</sup>

We have

$$\frac{d\hat{t}}{d\lambda} = \frac{-(5\lambda + 3)(a - c)(\lambda + 3)^2[13\lambda + 4\lambda^2 + \lambda^3 + 14 - (1 - \lambda)(3 + \lambda)\sqrt{(\lambda + 1)(\lambda + 3)}]}{(5\lambda^2 + 14\lambda + 13)^2(\lambda + 3)\sqrt{(\lambda + 1)(\lambda + 3)}}.$$

As  $13\lambda + 4\lambda^2 + \lambda^3 + 14 - (1 - \lambda)(3 + \lambda)(3 + \lambda) = 16\lambda + 9\lambda^2 + 3\lambda^3 + 5 > 0$  and  $\lambda + 3 > \sqrt{(\lambda + 1)(\lambda + 3)}$ , we obtain  $13\lambda + 4\lambda^2 + \lambda^3 + 14 - (1 - \lambda)(3 + \lambda)\sqrt{(\lambda + 1)(\lambda + 3)} > 0$ . This implies Proposition 1(iv). Q.E.D.

### Proof of Lemma 2

$\Omega_1$  can be rewritten as follows:

$$\Omega_1 = (1 - \lambda) \left[ \overbrace{(p' + p''q_1) + (p' + \lambda p''q_2)}^{(-)} + \overbrace{(p' + p''q_2) + \lambda(p' + p''q_1)}^{(-)} \right] < 0.$$

This implies Lemma 2(i).

The sign of  $\Omega_2$  depends on  $q_1$  and  $q_2$ .

$$\begin{aligned} \Omega_2 &= [p'(1 + \lambda) + p''q_1]q_1 - (2p' + p''q_2)q_2 \\ &= [p'(1 + \lambda) + p''q_1 + \lambda p''q_2]q_1 - (2p' + p''q_2 + \lambda p''q_1)q_2 \\ &= \overbrace{[(1 + \lambda)p' + p''(q_1 + \lambda q_2)]}^{(-)} q_1 - \overbrace{[2p' + p''(q_2 + \lambda q_1)]}^{(-)} q_2. \end{aligned}$$

Therefore,  $\Omega_2$  is more likely to be positive when  $q_2^{A^*}/q_1^{A^*}$  is smaller.

If  $t$  is sufficiently low (i.e., sufficiently close to zero), then  $q_2 \rightarrow q_1$ . Hence,  $\Omega_2|_{t \rightarrow 0} = -(1 - \lambda)p'q > 0$ . This implies Lemma 2(ii). If  $t$  is sufficiently high (i.e., sufficiently close to  $\bar{t}$ ),  $q_2 \rightarrow 0$ . Hence,  $\Omega_2|_{t \rightarrow \bar{t}} = [(1 + \lambda)p' + p''q_1]q_1 < 0$ . This implies Lemma 2(iii).

$\Omega_3 < 0$  is obtained because  $p' < 0$  and  $p'' < 0$ . Q.E.D.

### Proof of Lemma 3

<sup>10</sup>Because  $20\lambda^5 + 11\lambda^4 - 164\lambda^3 - 414\lambda^2 - 360\lambda - 117 < 0$ , we obtain  $0 < \hat{t} < (a - c)/2$ ; thus global welfare can increase with  $\lambda$  without violating the assumption of the interior solution.

From (14) and (16), we obtain Lemma 3(i,ii). Note that  $(p' + p''q_1) + \lambda(p' + p''q_2) < 0$  and  $(p' + p''q_2) + \lambda(p' + p''q_1) < 0$ .

Lemma 3(iii) is derived from (15) and Lemma 2(i,ii,iii).

From (15), we have

$$\begin{aligned} \frac{\partial q_2^{A*}}{\partial \lambda} &= - \frac{[2p' + p''(q_1 + \lambda q_2)] \{ [p'(1 + \lambda) + p''q_1]q_1 - [p'(1 + \lambda) + p''q_2]q_2 \}}{\Omega_1 \Omega_3} \\ &\quad - \frac{p'q_2(1 - \lambda)[(p' + p''q_2) + \lambda(p' + p''q_1)]}{\Omega_1 \Omega_3} \\ &= - \frac{[2p' + p''(q_1 + \lambda q_2)](q_1 - q_2)[p'(1 + \lambda) + p''(q_1 + q_2)]}{\Omega_1 \Omega_3} \\ &\quad - \frac{p'q_2(1 - \lambda)[(p' + p''q_2) + \lambda(p' + p''q_1)]}{\Omega_1 \Omega_3} < 0, \end{aligned}$$

where we use  $q_1 \geq q_2$ .

Similarly, we have

$$\frac{\partial Q^{A*}}{\partial \lambda} = \frac{-p'(1 - \lambda)Q[(p' + p''q_1) + \lambda(p' + p''q_2)]}{\Omega_1 \Omega_3} < 0.$$

Thus, Lemma 3(iv) is obtained.

$\Omega_2$  can be re-written as  $\Omega_2 = (q_1 - q_2)(p' + p''Q) + p'(\lambda q_1 - q_2)$ . Thus, from (15), we present  $\partial q_1^{A*}/\partial \lambda - \partial q_2^{A*}/\partial \lambda$  as follows:

$$\begin{aligned} &\frac{\partial q_1^{A*}}{\partial \lambda} - \frac{\partial q_2^{A*}}{\partial \lambda} \\ &= \frac{\Omega_2[p'(3 + \lambda) + 2p''(q_1 + \lambda q_2)] + p'q_1\Omega_1}{\Omega_1 \Omega_3} \\ &= \frac{p'(3 + \lambda)(q_1 - q_2)[p'(1 + \lambda) + p''Q] + p''[2(q_1 + \lambda q_2)\Omega_2 + p'q_2(1 - \lambda^2)Q]}{\Omega_1 \Omega_3} \\ &= \frac{q_1 - q_2}{\Omega_1 \Omega_3} \Delta, \end{aligned}$$

where

$$\Delta = p'(3 + \lambda)[p'(1 + \lambda) + p''Q] + p'' \{ 2(q_1 + \lambda q_2)p''Q + p'[2(1 + \lambda)q_1 - q_2 + \lambda(4 + \lambda)q_2] \}.$$

Because we assume  $p' < 0$  and  $p'' \leq 0$ , we obtain  $\Delta > 0$ . Since  $q_1 \geq q_2$  and equality holds only if  $t = 0$ , we obtain Lemma 3(v). Q.E.D.

## Proof of Proposition 2

From (15) and (20), we obtain

$$\frac{\partial W^*}{\partial \lambda} \Big|_{t \rightarrow 0} = \frac{2\Omega_2(1+\lambda)}{\Omega_1\Omega_3} (p' + p''q_1) [2(p-c) - t] < 0,$$

where we use Lemma 2 and the fact that  $q_2 \rightarrow q_1$  as  $t \rightarrow 0$ . This implies Proposition 2(i).

Similarly, we obtain

$$\begin{aligned} \frac{\partial W^*}{\partial \lambda} \Big|_{t \rightarrow \bar{t}} &= \frac{2\Omega_2}{\Omega_1\Omega_3} \{ -(p-c)p'(1-\lambda) + [(p-c) + \lambda p'q^M](2p' + p''q^M) \} \\ &= \frac{2\Omega_2}{\Omega_1\Omega_3} \{ (p-c)[(1+\lambda)p' + p''q^M] + \lambda p'q^M(2p' + p''q^M) \}, \\ &= \frac{2\Omega_2}{\Omega_1\Omega_3} (1-\lambda)(p-c)(p' + p''q^M) > 0, \end{aligned}$$

where we use Lemma 2 and the facts  $\bar{t} = p - c + \lambda p'q^M$  and  $p - c + p'q^M = 0$ . Note that if  $t \rightarrow \bar{t}$ , then  $q_1^{A*} \rightarrow q^M$ . This implies Proposition 2(ii). Q.E.D.



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