Cross Ownership, Loan Commitment, Managerial Delegation and the “Prisoner’s Dilemma”

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Abstract: This paper investigates the relationship between cross ownership, sales delegation and loan commitment. We find that under sales delegation, a higher degree of cross ownership decreases the optimal bank loan interest rate, which is beneficial to the firm profits. However, cross ownership reduces the firm output, leading a lower consumer surplus and social welfare. The policy implication is that antitrust authority and banking regulatory bureau should “coordinate” policies to mitigate the concerned stakeholders’ conflicts.

Keywords: Cross ownership; Sales delegation; Loan commitment

JEL Classifications: G32, G34, J53, L21

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Cross Ownership, Loan Commitment, Managerial Delegation and the “Prisoner’s Dilemma”

Abstract: This paper investigates the relationship between cross ownership, sales delegation and loan commitment. We find that under sales delegation, a higher degree of cross ownership decreases the optimal bank loan interest rate, which is beneficial to the firm profits. However, cross ownership reduces the firm output, leading a lower consumer surplus and social welfare. The policy implication is that antitrust authority and banking regulatory bureau should “coordinate” policies to mitigate the concerned stakeholders’ conflicts.

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1. Introduction

Loan commitments are firm commitments to provide credit under pre-specified terms and conditions. One major tool to finance corporations is via credit line commitments. In a written loan commitment, a lender charges fee for entering into an agreement under which it is obligated to fund or acquire a loan (or to satisfy an obligation of the other party under a specified condition). Loan commitment is one crucial financing instrument to acquire inputs or order equipment in advance for output-producing target. In microeconomic theory textbook, the price of capital in production process use the term rental rate without much discussion of the availability of internal and external funding.

Financial markets and product markets have important linkages (Brander and Lewis, 1986). Among them, loan commitment as an important financial instrument supporting firm investment and market competition (Maksimovic, 1990), ensures the borrower access to credit via signing loan commitment contract at a relatively low interest rate in the future, which plays an important role for firms planning make large purchases. Existing literature has discussed the issue of bank loan commitment from the perspective of strategic choice (Berkovitch and Greenbaum, 1991; Coscollá and Granero, 2003) and influencing factors (Graham et al. 2008; Mitani, 2014; Hollander and Verriest, 2016; Qian et al. 2019). In particular, Qian et al. (2020) argued that information asymmetry is a major obstacle in both formal and informal loan markets. Cross-ownership will enable lenders to collect more concrete and precise information about borrowers, and reduces the likelihood that the lender will require the borrower to provide collateral. Using a data set of 1091 intercorporate loans from China, they find strong support for the prediction that cross-ownership between lenders and borrowers lowers the collateral requirements by more than 50%. Quite differently, Lian et al. (2022) showed in a network market with managerial delegation that if the network externalities are positive and strong enough, sales delegation dominates the market share delegation; otherwise, the owner will choose the market share delegation. More importantly, they found that, if the network externalities are positive, the
optimal interest rate of a loan commitment decreases with an increase of network externalities, and firm’s delegation behavior will benefit the society, regardless of the delegation contract types. On the contrary, the optimal interest rate increases with an increase of network externalities, and firm’s delegation behavior will harm social welfare in presence of negative network externalities. Lian et al. (2022) did not taking the effect of cross ownership into consideration at determining optimal loan rate.

In contemporary economy, highly recognized that cross ownership and managerial delegation are the observed way of business operation for modern enterprise that attracts the extensive attention of scholars and the government. Cross ownership refers to a situation of holding non-controlling minority stakes in rivals, will reduce the incentive of firms to compete aggressively, therefore leading to higher product prices and lower industry output (Reynolds and Snapp, 1986), but social welfare may be improved even though market competition is hindered (Farrell and Shapiro, 1990). Note that on the one hand, in terms of cross ownership, some studies focus on the emergence of implicit collusion between managerial firms and examine the possibility of cross ownership resulting in collusion (Spagnolo, 2005; Gilo et al. 2006), and others investigate the welfare effect of cross ownership under different ownership structure (Chen et al. 2021; Hu et al. 2021; Lestage, 2021; Li and Shuai, 2022; Alipranti et al. 2022; Sun and Wang, 2022). On the other hand, the literature on strategic managerial delegation has grown significantly since the seminal contributions of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987), and Sklives (1987) (VFJS for shot), the outcome is that the sale delegation in Cournot market will lead to the “Prisoner’s Dilemma” situation. Management incentives can be divided into the following three types, that is, the objective function of a firm includes not only its own profit, but also output or revenue, market share and rival’s profit (Delbono and Lambertini, 2020). However, theoretical and empirical studies on cross ownership of firms, managerial delegation and loan commitment are still quite lacking. That motivates us to take cross ownership and managerial delegation into account of loan commitment decision in Cournot model, which will provide a new
supplement for existing research.

Our results show that in Cournot duopoly with loan commitment that the competitive banks have plenty of monies for the competing firms to sign up at lower interest rates\(^1\); when there exists cross ownership among firms, with the increase of degree of cross ownership, the optimal loan interest rate decreases and the profitability of firms increases. Meantime, firm output, consumer surplus and social welfare have declined. Moreover, in the case of sales delegation, the adverse effects of cross ownership on bank’s optimal loan interest rate, firm’s performance, and social welfare are mitigated.\(^2\) The “Prisoner’s Dilemma” situation is resolved. The results of this study have important implications for organization structure and financial policies.

The rest of this paper is organized as follows. Section 2 provides basic model setup. Section 3 considers sales delegation in the presence of loan commitment without cross ownership. Sections 4 studies cross ownership, sales delegation and loan commitment concurrently in Cournot product market. Section 5 compares the subgame perfect Nash equilibrium (SPNE) of the above cases in detail. Section 6 concludes the paper.

2. Basic Model

We consider Cournot competition between two firms competing in a homogeneous product market with quantity \(q_i\) \((i = 1, 2)\). The inverse demand function is given by \(p = a - Q\), where \(p\) is the price and \(Q = q_1 + q_2\) is the total output. For simplicity, we assume that the marginal production costs of both firms are constant and equal to 1.

Production is financed by loans from the competitive banking sector. We assume the market interest rate is \(r\). If firm \(i\) borrows at the interest rate of \(r\), the profits

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\(^1\) Our model setting was presented in finance literature by Maksimovic (1990) and used in Qian et al. (2019).

\(^2\) See La Pape and Wang (2018, 2020) from corporate governance structure to examine and delineate the intensity of conflicting interest among major stakeholders.
of firms are as follows

\[ \pi_1 = (a - 1 - q_1 - q_2 - r)q_1 \]  
\[ \pi_2 = (a - 1 - q_1 - q_2 - r)q_2 \]

Further, we consider the presence of loan commitments. Consistent with Qian et al. (2019), we assume that firm can obtain financing at interest rate \( r_i \) through loan commitment. Among them, \( r_i \in [0, r] \) and \( r < a - 1 < 3r \) to ensure a positive output and interest rate level. Written loan commitment fee \( F_i > 0 \) is determined by the break-even constraint of representative competing banks, which means the price of loan commitments is reasonable. Thus, the profit of firm \( i \) under the loan commitment \( r_i \) is

\[ \pi_i = (a - 1 - q_1 - q_2 - r_i)q_i - F_i \]  
\[ \pi_2 = (a - 1 - q_1 - q_2 - r_2)q_2 - F_2 \]

Consumer surplus and social welfare are respectively:

\[ CS = \frac{(q_1 + q_2)^2}{2} \]  
\[ SW = \sum_{i=1}^{2} \pi_i + \sum_{i=1}^{2} (F_i + r_i q_i) + CS \]

Next, based on a Cournot oligopoly model incorporating loan commitment, we study the following three cases. First, we consider the case of cross ownership. Second, consider managerial delegation represented by sales delegation. Third, consider both cross ownership and sales delegation.

We first consider the cross ownership among firms and examine how it affects the optimal bank loan interest rate, firm output and profits, consumer surplus and social welfare. In other words, firm 1 owns shares of firm 2, and firm 2 owns shares of firm 1. For simplicity, we assume that the two firms hold the same proportion of each other which is \( \alpha \) and \( \alpha < \frac{1}{2} \). Thus, either firm can share in its rival’s profits but not in
their output decisions. In this case, the profit of firm \( i \) is

\[
\Pi_i^{CO} = (1 - \alpha)\pi_1 + \alpha \pi_2 \tag{5a}
\]

\[
\Pi_i^{CO} = (1 - \alpha)\pi_2 + \alpha \pi_1 \tag{5b}
\]

where the superscript \( CO \) denotes the case of cross ownership and loan commitment.

The timing of this game is as follows. In the first stage, firm owners arrange financing simultaneously by acquiring loan commitments from competitive banks. In the second stage, the firms engage in quantities competition and decide their output independently. The SPNE is solved by backward induction.

In the second stage, the firms choose output \( q_i \) to maximize profit \( \Pi_i^{CO} \). From the first-order condition, we have

\[
\frac{\partial \Pi_i^{CO}}{\partial q_i} = a - q_i + 2q_i(\alpha - 1) + (1 + r_i)(\alpha - 1) - a\alpha = 0 \tag{6a}
\]

\[
\frac{\partial \Pi_i^{CO}}{\partial q_i} = a - q_i + 2q_i(\alpha - 1) + (1 + r_i)(\alpha - 1) - a\alpha = 0 \tag{6b}
\]

From (6a) and (6b), we obtain the following equilibrium output level

\[
q_i^{CO} = \frac{(1 - \alpha)(a - 1 + r_i + 2r_i(\alpha - 1) + 2\alpha(1 - a))}{3 + 4\alpha(\alpha - 2)} \tag{7a}
\]

\[
q_i^{CO} = \frac{(1 - \alpha)(a - 1 + r_i + 2r_i(\alpha - 1) + 2\alpha(1 - a))}{3 + 4\alpha(\alpha - 2)} \tag{7b}
\]

In the first stage, firm owners choose \( r_i \) to maximize \( \Pi_i^{CO} \). Substituting

\[ F_i = (r - r_i)q_i \] into \( \Pi_i^{CO} \), we have

\[
\frac{\partial \Pi_i^{CO}}{\partial r_1} = \frac{(\alpha - 1)(a - 1 - 6r + 4r_i + r_2 - r_2\alpha + r(7 - 2\alpha)\alpha + 2r_1\alpha(\alpha - 3))}{(3 - 2\alpha)^2} = 0 \tag{8a}
\]

\[
\frac{\partial \Pi_i^{CO}}{\partial r_2} = \frac{(\alpha - 1)(a - 1 - 6r + r_i + 4r_2 - r_2\alpha + r(7 - 2\alpha)\alpha + 2r_2\alpha(\alpha - 3))}{(3 - 2\alpha)^2} = 0 \tag{8b}
\]

From (8a) and (8b), we obtain the optimal rate of loan commitment

\[ 3 \]

In the scenario of competitive banking system, the interest rate is determined by the producers’ objective functions, which is also argued and used by Maksimovic (1990).
\[ r^{co} = r^1_{z}^{co} = r^2_{z}^{co} = \frac{(2\alpha^2 - 7\alpha + 6)r - a + 1}{(2\alpha - 5)(\alpha - 1)} > 0 \] (9)

By substituting (9) into (7a) and (7b), we obtain the SPNE outcomes. The output, profit, consumer surplus and social welfare are as follows

\[ q^{co} = q^1_{z}^{co} = q^2_{z}^{co} = \frac{(\alpha - 2)(a - r - 1)}{2\alpha - 5} > 0 \] (10a)

\[ \Pi^{co} = \Pi^1_{z}^{co} = \Pi^2_{z}^{co} = \frac{(1-a + r)^2(2-\alpha)}{(5 - 2\alpha)^2} > 0 \] (10b)

\[ CS^{co} = \frac{2(1-a + r)^2(\alpha - 2)^2}{(5 - 2\alpha)^2} > 0 \] (10c)

\[ SW^{co} = \frac{2(a-1-r)(2-\alpha)(3\alpha - 3 + 2r - (a-1+r)\alpha)}{(5 - 2\alpha)^2} > 0 \] (10d)

From (9) and (10), taking the derivative with respect to \( \alpha \) yields if \( \alpha \in (0, \frac{1}{2}) \)

\[ \frac{\partial r^{co}}{\partial \alpha} = \frac{(a-1-r)(4\alpha - 7)}{(2\alpha^2 - 7\alpha + 5)^2} < 0 \] (11a)

\[ \frac{\partial q^{co}}{\partial \alpha} = \frac{1 + r - a}{(5 - 2\alpha)^2} < 0 \] (11b)

\[ \frac{\partial \Pi^{co}}{\partial \alpha} = \frac{(1+r - a)^2(2\alpha - 3)}{(2\alpha - 5)^3} > 0 \] (11c)

\[ \frac{\partial CS^{co}}{\partial \alpha} = \frac{4(1+r - a)^2(2-\alpha)}{(2\alpha - 5)^3} < 0 \] (11d)

\[ \frac{\partial SW^{co}}{\partial \alpha} = \frac{2(a-1-r)(a-1+2r(2-\alpha))}{(2\alpha - 5)^3} < 0 \] (11e)

From (11), we have Proposition 1.

**Proposition 1:** A higher degree of cross ownership will reduce bank’s optimal loan interest rate, the firm’s output, consumer surplus and social welfare, while is beneficial to the firm’s profit.
Our result is consistent with conventional wisdom without managerial delegation that passive cross-holding reduces the incentive of firms to compete aggressively, therefore leading to higher product prices, lower industry output and social welfare (Reynolds and Snapp, 1986).

3. Managerial Delegation and Loan Commitment

In this section, we consider the case of managerial delegation and loan commitment. In particular, we assume that firm $i$ hires a manager with a sales delegation contract which the weight $\lambda_i$ is non-negative. The manager is motivated to choose $q_i$ to maximize

$$M_{i}^{SD} = \pi_i + \lambda_i q_i$$  \hspace{1cm} (12a)$$

$$M_{2}^{SD} = \pi_2 + \lambda_2 q_2$$ \hspace{1cm} (12b)

where the superscript $SD$ denotes the case of managerial delegation and loan commitment.

The timing of this game is as follows. In the first stage, firm owners arrange financing simultaneously by acquiring loan commitments from competitive banks. In the second stage, each firm’s owner commits to sales delegation contract to compensate to manager. In the third stage, the firms engage in quantities competition and decide their output independently. The SPNE is solved by backward induction.

In the third stage, the manager $i$ choose output $q_i$ to maximize profit $M_{i}^{SD}$.

From the first-order conditions, we have

$$\frac{\partial M_{i}^{SD}}{\partial q_i} = a - 1 - 2q_i - q_2 - r_1 + \lambda_i = 0$$ \hspace{1cm} (13a)$$

$$\frac{\partial M_{2}^{SD}}{\partial q_2} = a - 1 - q_1 - 2q_2 - r_2 + \lambda_2 = 0$$ \hspace{1cm} (13b)

From (13a) and (13b), we obtain the following equilibrium output level
In the second stage, the owner $i$ maximize $\Pi_{i}^{SD}$ by $\lambda_i$.

\[
\frac{d\Pi_{i}^{SD}}{d\lambda_i} = \frac{a - 1 - 2r_l + r_2 + 2\lambda_1 - \lambda_2}{9} = 0
\] (15a)

\[
\frac{d\Pi_{i}^{SD}}{d\lambda_2} = \frac{a - 1 - 2r_l - 2r_2 - \lambda_1 + 2\lambda_2}{9} = 0
\] (15b)

From (15a) and (15b), we have

\[
\lambda_{i}^{SD} = \frac{a - 1 - 3r_l + 2r_2}{5}
\] (16a)

\[
\lambda_{2}^{SD} = \frac{a - 1 + 2r_l - 3r_2}{5}
\] (16b)

In the first stage, firm owners choose $r_l$ to maximize $\Pi_{i}^{SD}$.

\[
\frac{d\Pi_{i}^{SD}}{dr_l} = \frac{2(1 + 15r - a - 12r_l - 2r_2)}{25} = 0
\] (17a)

\[
\frac{d\Pi_{i}^{SD}}{dr_2} = \frac{2(1 + 15r - a - 2r_l - 12r_2)}{25} = 0
\] (17b)

From (17a) and (17b), we obtain the optimal rate of loan commitment with sale delegation

\[
r_{i}^{SD} = r_{l}^{SD} = r_{2}^{SD} = \frac{1 - a + 15r}{14} > 0
\] (18)

By substituting (18) into (14) and (16), we obtain the SPNE outcomes. The weight of delegation contract, firm’s profit, consumer surplus and social welfare are as follows

\[
\lambda'_{i}^{SD} = \lambda_{i}^{SD} = \lambda_{2}^{SD} = \frac{3(a - 1 - r)}{14} > 0
\] (19a)
\[ q^{SD} = q_i^{SD} = q_z^{SD} = \frac{3(a-1-r)}{7} > 0 \]  
\[ \Pi^{SD} = \Pi_i^{SD} = \Pi_z^{SD} = \frac{3(1+r-a)^2}{49} > 0 \]  
\[ CS^{SD} = \frac{18(1+r-a)^2}{49} > 0 \]  
\[ SW^{SD} = \frac{6(a-1-r)(4a-4+3r)}{49} > 0 \]

4. Cross Ownership, Managerial Delegation and Loan Commitment

In this section, we further examine whether the equilibrium outcomes hold when the cross ownership and managerial delegation are incorporated in the loan commitment model concurrently. In the third stage, the manager \( i \) choose output \( q_i \) to maximize profit \( M_i^{CS} \). From the first-order conditions, we have

\[ \frac{\partial M_i^{CS}}{\partial q_i} = a - 1 - 2q_i - q_2 - r + \lambda_i = 0 \]  
\[ \frac{\partial M_i^{CS}}{\partial q_2} = a - 1 - q_i - 2q_2 - r + \lambda_2 = 0 \]

where the superscript \( CS \) denotes the case of cross ownership, managerial delegation and loan commitment.

From (20a) and (20b), we obtain the following equilibrium output level of each firm

\[ q_i^{CS} = \frac{a-1-2r_i + r_2 + 2\lambda_i - \lambda_2}{3} \]  
\[ q_z^{CS} = \frac{a-1+ r_i - 2r_2 - \lambda_i + 2\lambda_2}{3} \]

In the second stage, the owner \( i \) maximize \( \Pi_i^{CS} \) by \( \lambda_i \).
\[
\frac{\partial \Pi_{i}^{CS}}{\partial \lambda_{i}} = \frac{a - 1 - 2r_{1} + r_{2} + 3\alpha - 3a\alpha + 3r_{2}\alpha - 4\lambda_{i} + 6\alpha\lambda_{i} - \lambda_{2}}{9} = 0 \tag{22a}
\]
\[
\frac{\partial \Pi_{2}^{CS}}{\partial \lambda_{2}} = \frac{a - 1 + r_{1} - 2r_{2} + 3\alpha - 3a\alpha + 3r_{1}\alpha - 4\lambda_{2} + 6\alpha\lambda_{2} - \lambda_{1}}{9} = 0 \tag{22b}
\]

From (22a) and (22b), we have
\[
\lambda_{i}^{CS} = \frac{1}{4} \left(2(a - 1 - r_{2}) + \frac{3(2 - 2a + r_{1} + r_{2})}{5 - 6\alpha} + \frac{3(r_{1} - r_{2})}{2\alpha - 1}\right) \tag{23a}
\]
\[
\lambda_{2}^{CS} = \frac{1}{4} \left(2(a - 1 - r_{1}) + \frac{3(2 - 2a + r_{1} + r_{2})}{5 - 6\alpha} + \frac{3(r_{2} - r_{1})}{2\alpha - 1}\right) \tag{23b}
\]

In the first stage, firm owners choose \( r_{i} \) to maximize \( \Pi_{i}^{CS} \)
\[
\frac{\partial \Pi_{i}^{CS}}{\partial r_{1}} = \frac{1}{(5 - 6\alpha)^{2}} \left((18(r_{1} - r)\alpha^{3} + 3(2a - 2 + 25r - 24r_{1} - 3r_{2})\alpha^{2}ight.
\]
\[\left. - 2(a - 1 + 43r - 38r_{1} - 6r_{2})\alpha - 2(a - 1 - 15r + 12r_{1} + 2r_{2})\right) = 0 \tag{24a}\]
\[
\frac{\partial \Pi_{2}^{CS}}{\partial r_{2}} = \frac{1}{(5 - 6\alpha)^{2}} \left((18(r_{2} - r)\alpha^{3} + 3(2a - 2 + 25r - 3r_{1} - 24r_{2})\alpha^{2}ight.
\]
\[\left. - 2(a - 1 + 43r - 6r_{1} - 38r_{2})\alpha - 2(a - 1 - 15r + 2r_{1} + 12r_{2})\right) = 0 \tag{24b}\]

From (24a) and (24b), we obtain the optimal rate of loan commitment with sales delegation
\[
r_{i}^{CS} = r_{i}^{CS} = r + \frac{1 + r - a}{4 - 6\alpha} = \frac{(a - 1 - r)(6\alpha - 5)}{28 + 2\alpha(6\alpha - 23)} > 0 \tag{25}
\]

By substituting (25) into (21) and (23), we obtain the SPNE outcomes. The weight of delegation, output, profit, consumer surplus and social welfare are obtained as follows:
\[
\lambda^{CS} = \lambda_{i}^{CS} = \lambda_{2}^{CS} = \frac{(a - 1 - r)(3\alpha - 1)(6 + \alpha(3\alpha - 10))}{(3\alpha - 2)(14 + \alpha(6\alpha - 23))} > 0 \tag{26a}
\]
\[
q^{CS} = q_{i}^{CS} = q_{2}^{CS} = \frac{(a - 1 - r)(6 + \alpha(3\alpha - 10))}{14 + \alpha(6\alpha - 23)} > 0 \tag{26b}
\]
\[
\Pi^{CS} = \Pi_{i}^{CS} = \Pi_{2}^{CS} = \frac{(1 + r - a)^{2}(2 - 3\alpha)(6 + \alpha(3\alpha - 10))}{(14 + \alpha(6\alpha - 23))^{2}} > 0 \tag{26c}
\]
\[CS_{cs}^{CS} = \frac{2(1+r-a)^2(6+\alpha(3\alpha-10))^2}{(14+\alpha(6\alpha-23))^2} > 0 \]  
(26d)

\[SW^{CS} = \frac{2(a-1-r)\left(-8+8a+6r+(13-13a-10r)\alpha\right)+3(a-1+r)\alpha^2}{(14+\alpha(6\alpha-23))^2} > 0 \]  
(26e)

From (25) and (26), taking the derivative with respect to \(\alpha\) yields if \(\alpha \in (0, \frac{1}{3})\)

\[\frac{\partial r_{cs}^{CS}}{\partial \alpha} = \frac{2(a-1-r)(3\alpha(110+\alpha(6\alpha(3\alpha-2)-79)))-116)}{(2-3\alpha)^2(14+\alpha(6\alpha-23))^2} < 0 \]  
(27a)

\[\frac{\partial \lambda^{CS}}{\partial \alpha} = \frac{(1+r-a)(256+3\alpha(\alpha(3\alpha(15\alpha-64)+356)-292))}{(2-3\alpha)^2(14+\alpha(6\alpha-23))^2} < 0 \]  
(27b)

\[\frac{\partial q^{CS}}{\partial \alpha} = \frac{(1+r-a)(3\alpha(3\alpha-4)+2)}{(14+\alpha(6\alpha-23))^2} < 0 \]  
(27c)

\[\frac{\partial \Pi^{CS}}{\partial \alpha} = \frac{(1+r-a)^2(\alpha-2)(6\alpha-5)(3\alpha(3\alpha-4)+2)}{(14+\alpha(6\alpha-23))^3} > 0 \]  
(27d)

\[\frac{\partial CS^{CS}}{\partial \alpha} = \frac{4(1+r-a)^2(\alpha(10-3\alpha)-6)(3\alpha(3\alpha-4)+2)}{(14+\alpha(6\alpha-23))^3} < 0 \]  
(27e)

\[\frac{\partial SW^{CS}}{\partial \alpha} = \frac{2(a-1-r)(2-3\alpha(4-3\alpha))(2-3\alpha-a(2-3\alpha)-2r(6-\alpha(10-3\alpha)))}{(14+\alpha(6\alpha-23))^3} < 0 \]  
(27f)

From (27), we have Proposition 2.

**Proposition 2:** A higher degree of cross ownership matching sales delegation will decrease bank’s optimal loan interest rate, the firm’s output, consumer surplus and social welfare, while is beneficial to the firm’s profit.

No surprisingly, we find that the higher cross ownership will benefit the cross-owned firms at the expense of consumer surplus and social welfare.

5. Comparison of Equilibrium Outcomes

Now we compare the equilibrium outcomes of above cases if \(\alpha \in (0, \frac{1}{3})\), and
have

\[ r^{SD} > r^{CS} > r^{CO} \]  \hspace{1cm} (28a)

\[ q^{SD} > q^{CS} > q^{CO} \]  \hspace{1cm} (28b)

\[ \Pi^{SD} < \Pi^{CS} < \Pi^{CO} \]  \hspace{1cm} (28c)

\[ CS^{SD} > CS^{CS} > CS^{CO} \]  \hspace{1cm} (28d)

\[ SW^{SD} > SW^{CS} > SW^{CO} \]  \hspace{1cm} (28e)

From (28), we have Proposition 3.

**Proposition 3:** In Cournot duopoly with sales delegation and loan commitment contract, the bank’s optimal loan interest rate, the firm’s output, consumer surplus and social welfare are lower under cross ownership than non-cross ownership, while the equilibrium profit is higher by contrary.

Note that consume surplus and social welfare under cross ownership is the lowest, but the puzzling “Prisoner’s Dilemma” situation is resolved since delegated firm have incentive to produce more aggressively with loan commitment contract will produce more than the case in the presence of cross ownership. The resolving of puzzling result has important implications for organization structure, competition and financial policies.

6. Conclusions

In this paper, we develop a Cournot duopoly setting and investigate how cross ownership affects the optimal loan interest rate, firm output and profits, consumer surplus and social welfare. We find that in the absence of cross ownership, firms use sales delegation as a strategic commitment to gain a larger market share, which contributes to increasing consumer surplus and social welfare. When cross ownership exists between firms, the higher degree of cross ownership, the lower the optimal loan interest rate, will lead to the lower consumer surplus and social welfare, but the higher
the profits of firms. The results of this study have important implications for antitrust and financial policies. The antitrust authority should constantly monitor the variation of cross ownership among firms.
References


