Content Licensing with Endogenous Homing

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Abstract

This paper examines the licensing strategy of a monopoly content provider that supplies horizontally differentiated content through downstream distributors to consumers who can potentially purchase from both distributors. When consumers' additional gain from the second purchase is high, the mismatch cost is low, and the quality of the extra content is high, some consumers purchase from both firms, which is called multi-homing. Apart from that, all consumers purchase from either distributor. When some consumers multi-home, the content provider always licenses to only one distributor. When all consumers single-home, the content provider either licenses to one distributor or shares the licensing.

Keywords: Multi-homing, Licensing, Exclusive Dealing, Digital Content, Online Platform

JEL Codes: D43, L13, L42

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1 Introduction

With the rapid development of information technology and the growing popularity of the Internet, various platform-based Internet companies have seen global opportunities for growth. From e-commerce platforms, such as Taobao and Amazon, to Internet media platforms like Youtube, Internet platforms are involved in all aspects of our lives. According to "The 47th China Statistical Report on Internet Development" by CNNIC, until December 2020, the size of China's Internet users had reached 989 million, an increase of 85.4 million from March 2020, and the Internet penetration rate had reached 70.4%. Among them, the scale of Internet audio & video users had reached 927 million, up 76.33 million from March 2020, accounting for 93.7% of Internet users as a whole.[1]

The group of Internet users are huge in terms of number, penetration, and growth rate. With increasing Internet penetration, people's life styles have also experienced a great change. In the traditional scenario, people's shopping behavior usually takes place offline, and it may cost them a whole afternoon in the mall to buy just a pair of shoes. In contrast, consumers can browse through dozens of different brands of shoes on Internet platforms by simply opening a mobile application, and it only takes a few minutes to choose one.

Such a market structure of online platform competition has two important features. First, the distributors' sales approach differs from the traditional market structure. For example, in the video game industry, traditional game retailers only sell games individually and charge a price for each game cassette or disc. Some online platforms employ a new method of selling the right to play all the games on them as part of a membership package, such as the XGP of Xbox. Consumers can play all the games on the distributor by paying a monthly membership fee. Second, in the traditional sales model, goods sold by different shops are functionally identical, so consumers generally purchase goods from only one retailer. For example, in the audiovisual industry, a consumer's decision to buy a CD from a shop generally depends on which shop is nearest to him, and once he gets one, it would be unlikely for him to buy the same CD from another shop. However, in the online platform case, services offered by the distributors are differentiated in terms of content, interaction, data portability and so on. As a result of these two features, consumers have the incentive to purchase from two or more

[1]Data source: China Internet Network Information Center. The 47th China statistical report on internet development[J]. Office of the Central Leading Group for Cyberspace Affairs, 2021

distributors simultaneously, i.e., multi-homing. With the rapid development of information technology, the distributors that provide digital content to consumers become more and more differentiated, and consumers' cost of multi-homing reduces to almost zero. As a result, more and more consumers choose to multi-home to several distributors. In this paper, I focus on the difference between the single-homing only market and the multi-homing market, and analyze the licensing strategy of the upstream content provider.

In this paper, I investigate online platform markets with the above characteristics by constructing a model of vertical licensing competition. I focus primarily on the endogenization of consumers' homing choice, i.e., the freedom of consumers to decide to join one or two distributors. What determines consumers' choices of distributors and how should the optimal price of a distributor be set? The second half of the paper introduces a content provider who can produce additional content and license it to the distributors. In this revised case, this paper also interests how the content provider would set the licensee and the license price and whether the distributors would accept the license contract.

I consider the following two cases. First, I study the case where there is no content provider as a benchmark (Jeitschko et al., 2017).[2] There are two types of players in this case: two distributors that provide horizontally differentiated content and a group of consumers who have different preferences for the two distributors. The distributors gain profit by selling their memberships, and consumers who purchase the membership can view and use all content on the distributors they belong to. As a result, I derive the boundary area of the existence of multi-homing consumers. This setting lets us analyze the effect of multi-homing on consumers and distributors when the platforms can choose between the multi-homing and single-homing equilibria. Then, I extend this basic model by introducing a content provider. This content provider licenses the extra content it produces to a distributor under a fixed licensing fee. It can license the extra content exclusively to a distributor or both distributors. It also determines the contract term(s) at that moment. The distributor facing the contract term decides to accept it and provide this additional content to its members or reject it. Thus, the basic model is a special case of the extended model when both distributors have rejected the contract.

[2]Jeitschko et al. (2017) consider the same market structure in which the distributors are asymmetric. In this paper, for the simplicity of calculation, I assume that the distributors are symmetric. The main results hold without this assumption.

By analyzing the basic and extended models, I obtain the following conclusions. Firstly, when consumers' additional gain from the second purchase is intermediate, there are multiple equilibria. One is that both firms charge low prices, and then multi-homing consumers exist. The other kind of equilibria is that both firms charge high prices, and then all of the consumers single-home to either firm.[3] In such a situation, the firms are better off under single-homing equilibrium, while both the social welfare and the consumer surplus are getting worse. In the extended model, I analyze the content provider's licensing determination. It can be divided into three cases according to the value of parameters. When consumers' additional gain from the second purchase is large, some consumers purchase from both distributors no matter which licensing strategy is applied, which is called multi-homing. In this case, the content provider always licenses to only one distributor. When consumers can get a little additional gain from the second purchase, all consumers purchase from only one distributor no matter which licensing strategy is applied. In such a situation, content provider licenses to both distributors if the extra content is low rated compared with mismatch cost. On the other hand, if the extra content is high rated compared with mismatch cost, content provider licenses to only one distributor even though all consumers purchase from a single distributor. When the additional gain from the second purchase is intermediate, the equilibrium outcome depends on content provider's licensing determination. In this case, if the additional gain from the second purchase is large, content provider will license to only one distributor and some consumers multi-homing. If the additional gain from the second purchase is low, content provider will license to both distributors, and all consumers purchase from a single distributor.

This paper has a strong connection to the studies of multi-homing. Anderson et al. (2017) constructed a model that consumers can choose to be multi-homing and studied the relationship between price strategy and product quality. Jeitschko et al. (2017) derived the condition that all consumers choose single-homing or some of them multi-home in a Hotelling model, which is similar to my baseline model. However, both of them did not take the upstream provider into account. Many kinds of research showed that the incentive of agents to multi-home on one side depends on whether the other side multi-homes negatively, such as Gabszewicz and Wauthy (2004) and Bakó and Fátay(2019). However, in my model, the relationship between upstream licensing and downstream multi-homing may not be one-on-one.

[3]Jeitschko and Tremblay(2020) showed that this kind of mixed homing also occurs in a two-sided market.

There exist some cases in which the content provider licenses to only one distributor and all consumers also purchase from only one distributor.

Another aspect that this paper involves is licensing. According to Callister and Hall (2003), “a license is a contract, not necessarily in writing, in which one party (the licensor) transfers rights to use certain property to a user (the licensee) for some limited period or until some event.” A lot of studies about content licensing focus on the pay-TV industry. Armstrong (1999) considers such a vertical competition situation: one outside supplier of programming and two television retailers. As a result, he says that the seller should sell the rights exclusively to the firm with the initial competitive advantage in a fixed-fee case, which is socially unfavorable. Harbord and Ottaviani (2001) extend model of Armstrong that the downstream broadcasters are allowed to resale the extra contents, which relaxes the price competition of downstream broadcasters. Weeds (2016) considers that in a dynamic model, the content owner gains an advantageous position in the market through exclusivity and therefore has the incentive to opt for exclusivity. Chowdhury and Martin (2017) consider a two-sided newspaper market with an outside syndicate that offers comics to newspapers that are essential complementary goods to some consumers. As a result, an exclusive license will be achieved when the downstream varieties are substitutable. Also, when the downstream varieties are highly differentiated, the syndicate will share its licenses. Carroni et al. (2021) also construct a two-sided model, in which there is a ”superstar” seller who has all the bargaining power over her product. If the network externality exerted by the consumers is large enough, the superstar will choose non-exclusive licensing. Otherwise, she chooses exclusive licensing. In this article, I refer to the literature on outside licensing providers, which is the most common scenario in the digital content market like online music and video games, and construct a dynamic market with a fixed-fee licensing method.

Motivated by literature in these two aspects, Ishihara and Oki (2021) and Jiang et al. (2019) consider models that combine multi-homing and product licensing. They examine the vertical relationships between a monopoly licensing provider and two downstream distributors where consumers may access multiple distributors. Both of them find that when multi-homing consumers exist in the market, the content provider exclusively licenses the additional content to one of the distributors. Unlike these articles that assume the existence of multi-homing consumers, I consider a unified model in which both single-homing and multi-homing outcomes can be simultaneously attainable in some parameter sets. As the licensing strategies influence whether the subgame outcome is single-homing or multi-homing, this model allows us to analyze the interaction between licensing strategies and consumers' homing decisions, which has been out of scope in Ishihara and Oki (2021) and Jiang et al. (2019). In this paper, I show what would happen under different licensing strategies. As the endogenization of licensing choice affects consumers' purchasing behavior, there exists a parameter region in which some consumers purchase from both distributors if content provider exclusively licenses its content, and all consumers purchase only one product if content provider shares its content with both distributors. Consumers' homing choices in such an area in my model are different from the above two papers, in which consumers are assumed to be single-homing. As a result, consumers purchase from both distributors in a larger area than in the previous studies, and content provider is more willing to license exclusively.

The rest of the paper is organized as follows. In Section 2, I explain the basic model of digital products. In Section 3, I derive the equilibria of the basic model. I show that under certain conditions, there are multiple equilibria. In Section 4, I introduce a content provider into the basic model and analyze its licensing strategy.

2 Model

There are two types of agents in this model: two distributors S_1 and S_2, and a mass of consumers.[4]

- Consumers: Consumers uniformly distribute on the linear city [0,1]. The total number of consumers is normalized to 1. They can join the membership of each firm by paying the membership fee p_i $(i = 1, 2)$ to firm S_i, which are located at the two ends of the linear city.

  A consumer at $x \in [0, 1]$ can gain utility by joining the membership of S_1 according to the utility function $U = v_1 - tx - p_1$; gain utility by joining the membership of S_2 according to the utility function $U = v_2 - t(1 - x) - p_2$; gain benefits by joining the memberships of both S_1 and S_2 (multi-homing) according to the utility function $U = v_{12} - t - p_1 - p_2$ and gain 0 utility by joining no one. The parameter t represents the mismatch cost.

[4]Armstrong(1999) considered a similar situation in which he only involved the single-homing case.

v_i ($i = 1, 2, 12$) is the intrinsic benefit of belonging to each firm. If the consumer chooses to be multi-homing, the associating intrinsic benefit $v_{12} = V$, which is larger than both v_1 and v_2. As a result, the utility function of consumer $x \in [0, 1]$ is:

$$\begin{cases} U_1 = v_1 - tx - p_1 & \text{if the consumer chooses } S_1, \\ U_2 = v_2 - t(1-x) - p_2 & \text{if the consumer chooses } S_2, \\ U_{12} = V - t - p_1 - p_2 & \text{if the consumer multi-homes,} \end{cases} \tag{1}$$

For simplicity, three assumptions are given:

Assumption 1 $v_1 + v_2 \geq 3t$.

Assumption 2 $v_1 = v_2 = v$.

Assumption 3 $V > v$.

The first assumption ensures the full coverage of the Hotelling competition. Under this condition, all consumers on the linear city choose to belong to at least one of the firms in equilibrium. The second assumption shows that the two firms are symmetric. I define that $\Delta = V - v$ (> 0), which measures consumers' additional gain from the second purchase.[5]

- Distributors: Each distributor gains profits by selling the membership to consumers, which lets consumers access all of its content and services. I assume that the distributors can offer their content without any cost. Firm S_i decides p_i to maximize its profits $\pi_i = p_i d_i$, where $d_i \in [0, 1]$ denotes the number of consumers who join the membership of firm S_i.

In this game, firstly, distributors decide the membership fees p_i. Consumers decide on their purchasing behavior after observing the membership fees. I will study two kinds of equilibria: the equilibrium with multi-homing consumers and the one with only single-homing consumers.

[5] This assumption is not crucial for the main result. Even if the firms are asymmetric, similar equilibria exist, which has been shown by Jeitschko et al. (2017) In this case, I only consider the simplest setting.

3 Equilibrium with and without multi-homing consumers

In this section, I follow the calculation method in Jeitschko et al. (2017), and derive the pure strategy subgame perfect equilibria when multi-homing consumers exist. In my model, expanded from their model in which there always exist some single-homing consumers, both firms may offer their membership to all consumers. As a result, all consumers purchase from both distributors when Δ is large enough, and the membership prices are higher than what is shown in Jeitschko et al. (2017). Consumers at x single-home to S_1 if $U_1 > \max\{U_{12}, U_2\}$, single-home to S_2 if $U_2 > \max\{U_{12}, U_1\}$, and multi-home otherwise. Based on these three inequalities, I derive the following lemma.

Lemma 1 *When $\Sigma_k p_k < 2\Delta - t$, there exists multi-homing consumers in the market; when $\Sigma_k p_k \geq 2\Delta - t$, all of the consumers single-home to either firm.*

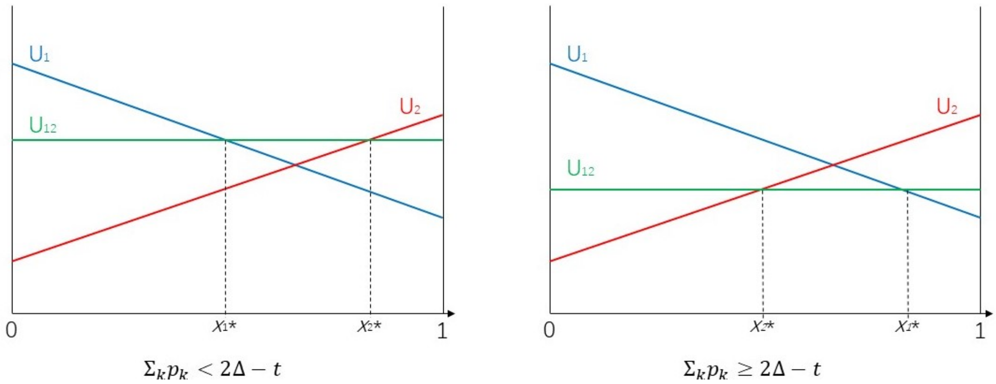


Figure 1: Multi-homing Consumer Existence

Proof: According to the utility function of consumers, the indifferent consumer between choosing S_1 and multi-homing locates at x_1^* such that $U_1(x_1^*) = U_{12}(x_1^*)$, which means that $v - tx_1^* - p_1 = V - t - p_1 - p_2$. By organizing this equation, I get that $x_1^* = 1 - (\Delta - p_2)/t$. The indifferent consumer between choosing S_2 and multi-homing locates at x_2^* such that $U_2(x_2^*) = U_{12}(x_2^*)$, so his location x_2^* is presented as $x_2^* = (\Delta - p_1)/t$. When there exist multi-homing consumers in the market, $x_1^* < x_2^*$. In order to fulfill this condition, $\Sigma_k p_k < 2\Delta - t$ must be satisfied. On the other hand, when all of the consumers single-home to either firm, $x_1^* \geq x_2^*$, which means $\Sigma_k p_k \geq 2\Delta - t$. Q.E.D.

3.1 Equilibrium when multi-homing consumers exist

In this section, suppose that $\Sigma_k p_k < 2\Delta - t$. The number of consumers who belong to firm S_i, $N_i, i = 1, 2$, is

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_i = \frac{\Delta - p_i}{t}. \tag{2}$$

Then, for the distributors, they choose their membership price p_i to maximize their profit $\pi_i = p_i N_i$. Taking the first order condition that $\partial\pi_i/\partial p_i = 0$, the optimal prices of firms are shown below. When multi-homing consumers exist,

$$p_i^{**} = \begin{cases} \dfrac{\Delta}{2} & if\ \Delta < 2t, \\ \Delta - t & if\ \Delta \geq 2t. \end{cases} \tag{3}$$

The corresponding number of members belonging to distributor S_i with the optimal price is

$$N_i^{**} = \begin{cases} \dfrac{\Delta}{2t} & \text{if } \Delta < 2t, \\ 1 & \text{if } \Delta \geq 2t, \end{cases} \tag{4}$$

and the maximizing profits of distributors are

$$\pi_i^{**} = \begin{cases} \dfrac{\Delta^2}{4t} & if\ \Delta < 2t, \\ \Delta - t & if\ \Delta \geq 2t. \end{cases} \tag{5}$$

By checking the incentive of distributors to deviate from this optimal price, the existence condition of multi-homing equilibria is shown below:

Lemma 2 *Pure strategy subgame perfect equilibria with multi-homing consumers exists when* $\Delta > 2t/(2\sqrt{2} - 1)$.

3.2 Equilibrium when multi-homing consumers do not exist

In this section, suppose that $\Sigma_k p_k \geq 2\Delta - t$. The number of consumers who belong to firm S_i, $N_i, i = 1, 2$, is

$$
\begin{aligned}
N_i &= \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where} \\
\hat{N}_i &= \frac{1}{2} - \frac{p_i - p_j}{2t}.
\end{aligned}
\tag{6}
$$

Then, for the distributors, they choose their membership price p_i to maximize their profit $\pi_i = p_i N_i$. Taking the first order condition that $\partial \pi_i / \partial p_i = 0$, the optimal prices of distributors are

$$p_i^* = t. \tag{7}$$

The corresponding number of members belonging to distributor S_i with the optimal price is

$$N_i^* = \frac{1}{2}. \tag{8}$$

and the maximizing profits of firms are

$$\pi_i^* = \frac{t}{2} \tag{9}$$

By checking the incentive of distributors to deviate from this optimal price, the existence condition of single-homing equilibria is shown below:

Lemma 3 *Pure strategy subgame perfect equilibria without multi-homing consumer exists when* $\Delta \leq \sqrt{2}t$.

3.3 Possibility of Multi Equilibria

In this section, I will show that under certain parameter values, there exist multiple equilibria with both multi-homing equilibrium and single-homing equilibrium. By Combining Lemma 2 and 3, there is a parameter range that both the multi-homing equilibrium and the single-homing equilibrium exist.

Proposition 1

- *When $\Delta > \sqrt{2}t$, at least some of the consumers are multi-homing.*

- *When $\Delta \leq 2t/(2\sqrt{2}-1)$, all of the consumers choose to single-home.*

- *When $2t/(2\sqrt{2}-1) < \Delta \leq \sqrt{2}t$, there are multiple equilibria. One is that both firms charge low prices, and then multi-homing consumers exist. The other kind of equilibrium is that both firms charge high prices, and then all of the consumers single-home to either firm.*

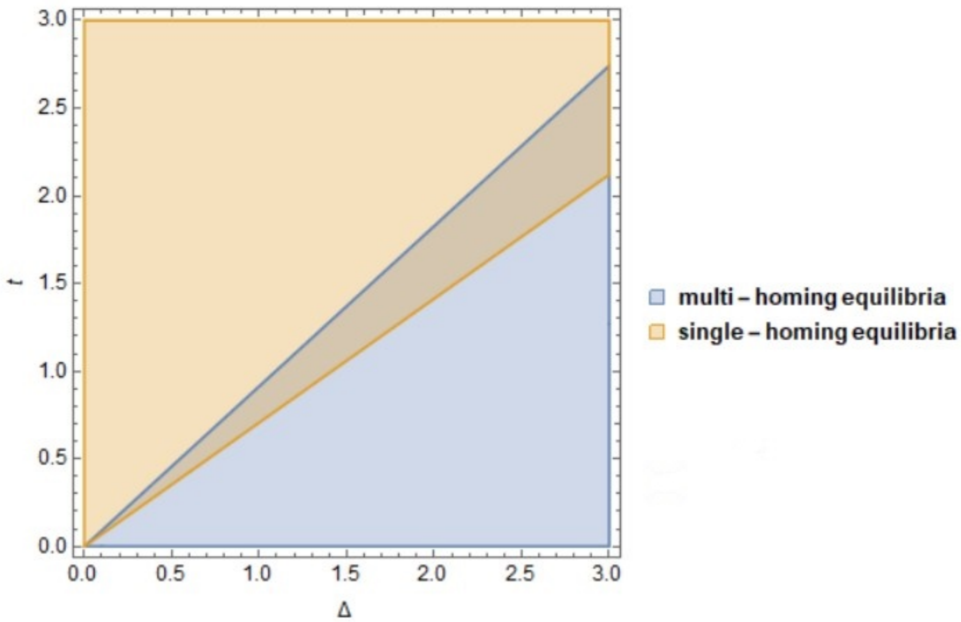


Figure 2: Parameter Range with Multi Equilibria

Although distributors can choose a multi-homing equilibrium when consumers' additional gain from the second purchase is moderate, they are both worse off in such a situation. There are two kinds of the opposite effect on distributors' profits when the equilibrium switches from the single-homing one to the multi-homing one. The first effect is the price effect. In the single-homing case, the mismatch cost t denotes the cost of consumers switching from one firm to the other. Due to this market power, the distributors can charge a higher price to their consumers. However, such an effect no longer exists in the multi-homing case because the consumers can purchase from both firms. Both distributors face a situation similar to the monopoly among their subscribers. As a result, the prices do not vary with t. The other effect of switching equilibrium on the profits is the demand effect. When all of the consumers single-home to either distributor, as long as the distributors keep their extent of differentiation, their demands will stay the same. On the other hand, a monopoly-like situation enables them to get higher demands in the multi-homing case than in the single-homing case. Note that because of the price effect, the profit of each distributor increases with the mismatch cost t, and the monopoly-like situation in the multi-homing case results in a decrease in profit with the mismatch cost t. As the price effect leads to a higher price in the single-homing equilibrium, while the demand effect leads to higher demand in the multi-homing equilibrium, the profit effect of equilibrium switching seems to be ambiguous. However, by comparing the profit of both firms in single- and multi-homing equilibrium that

$$\pi_{multi} = \begin{cases} \frac{\Delta^2}{4t} & , if\ \Delta < 2t, \\ \Delta - t & , if\ \Delta \geq 2t, \end{cases} \tag{10}$$

$$\pi_{single} = \frac{t}{2}, \tag{11}$$

the single-homing profit is always larger than multi-homing profit in the interval that $2t/(2\sqrt{2}-1) < \Delta \leq \sqrt{2}t$. This result shows that if multiple equilibria exist, consumers' additional gain from the second purchase is not high enough, so the price effect of the single-homing equilibrium exceeds the demand effect of the multi-homing equilibrium, which makes both distributors better off under the single-homing equilibrium. Such a result can be also seen in the asymmetric case.

Although both distributors prefer the single-homing equilibrium, it is not the same for the whole society. The total welfare $TW = \int_0^{1-N_2^*}(v - tx)dx + \int_{1-N_2^*}^{N_1^*}(V - t)dx + \int_{N_1^*}^{1}[v - t(1 - x)]dx$ is larger in the multi-homing case, where N_1^* and N_2^* respectively denotes the number of consumers who purchase the membership of S_1 and S_2. For the consumers, the consumer surplus $CS = TW - \pi_1 - \pi_2$, so the consumers are better off in the multi-homing equilibrium. The distributors prefer the single-homing equilibrium, while consumers prefer the multi-homing equilibrium. For the whole society, the multi-homing equilibrium is desirable.

Proposition 2 *Both distributors are better off in the single-homing equilibrium than the multi-homing equilibrium, while the social welfare and consumer surplus are worse off.*

4 Licensing from Content Provider

In this section, I introduce a content provider into the basic model. The content provider produces extra content but cannot sell it directly to consumers. It can only get profit through licensing to distributors. These distributors serve the same products as the distributors discussed above. There is no additional cost of producing a duplicate in the digital world, so its marginal cost is zero. It will provide extra content to the distributors as long as they have accepted the licensing contract. The provider can choose its licensing strategy and provide a content licensing contract to each distributor. It gains profits by charging a fixed fee to each distributor that accepts the contract. There are four kinds of sales strategies: licensing to only S_1; licensing to only S_2; licensing to both S_1 and S_2, and licensing to no one. The content provider makes its decision by maximizing its profit $\pi_M = f_1 + f_2$, where $f_i, i = 1, 2$ denotes the fixed fee charged to the ith distributor. The term of the licensing contract is common knowledge among all agents. To analyze the decision-making of each enterprise and consumers in this situation, I construct a four-step game.

1. The content provider decides the licensing strategy and the fixed licensing fee of extra content and offers a licensing contract to each of the chosen distributors.

2. Each of the chosen distributors decides whether to accept the licensing contract.

3. The distributors decide the membership prices.

4. Consumers choose the distributor(s) to access.

I use the backward induction to derive the pure strategy subgame perfect equilibrium in this game.

4.1 Membership prices and consumer decision

The situation that the content provider licenses to no one is the same as the baseline model, so proposition 1 applies to this case. Then, I will analyze the rest two licensing strategies: exclusive licensing and shared licensing.

4.1.1 Exclusive licensing

Without loss of generality, suppose that the content provider licenses extra content exclusively to S_1. The utility function of consumer $x \in [0,1]$ then becomes:

$$\begin{cases} U_1 = v + \delta - tx - p_1 & \text{if the consumer chooses } S_1, \\ U_2 = v - t(1-x) - p_2 & \text{if the consumer chooses } S_2, \\ U_{12} = V + \delta - t - p_1 - p_2 & \text{if the consumer multi-homes,} \end{cases} \tag{12}$$

where $\delta(>0)$ reflects the value of extra content. Following the same process in Section 3, the membership prices and consumer decisions can be derived as:

Lemma 4

- *When $\Delta > \bar{\Delta}$, at least some of the consumers are multi-homing. S_1 charges*

$$p_1 = \begin{cases} \frac{\Delta+\delta}{2} & \textit{if } \Delta < 2t - \delta, \\ \Delta + \delta - t & \textit{if } \Delta \geq 2t - \delta, \end{cases} \tag{13}$$

on its membership, and S_2 charges

$$p_2 = \begin{cases} \frac{\Delta}{2} & \textit{if } \Delta < 2t \\ \Delta - t & \textit{if } \Delta \geq 2t \end{cases} \tag{14}$$

on its membership. They respectively get the profit that

$$\pi_1 = \begin{cases} \dfrac{(\Delta+\delta)^2}{4t} - f_1 & if\ \Delta < 2t - \delta, \\ \Delta + \delta - t - f_1 & if\ \Delta \geq 2t - \delta, \end{cases} \quad \pi_2 = \begin{cases} \dfrac{\Delta^2}{4t} & if\ \Delta < 2t, \\ \Delta - t & if\ \Delta \geq 2t. \end{cases} \tag{15}$$

- *When $\Delta \leq \underline{\Delta}$, all of the consumers choose to single-home. S_1 charges*

$$p_1 = t + \frac{\delta}{3} \tag{16}$$

on its membership, and S_2 charges

$$p_2 = t - \frac{\delta}{3} \tag{17}$$

on its membership. They respectively get the profits that

$$\pi_1 = 2t(\frac{3t+\delta}{6t})^2 - f_1, \quad \pi_2 = 2t(\frac{3t-\delta}{6t})^2 \tag{18}$$

- *When $\underline{\Delta} < \Delta \leq \bar{\Delta}$, there are multiple equibria.*

Proof: See in the appendix A.

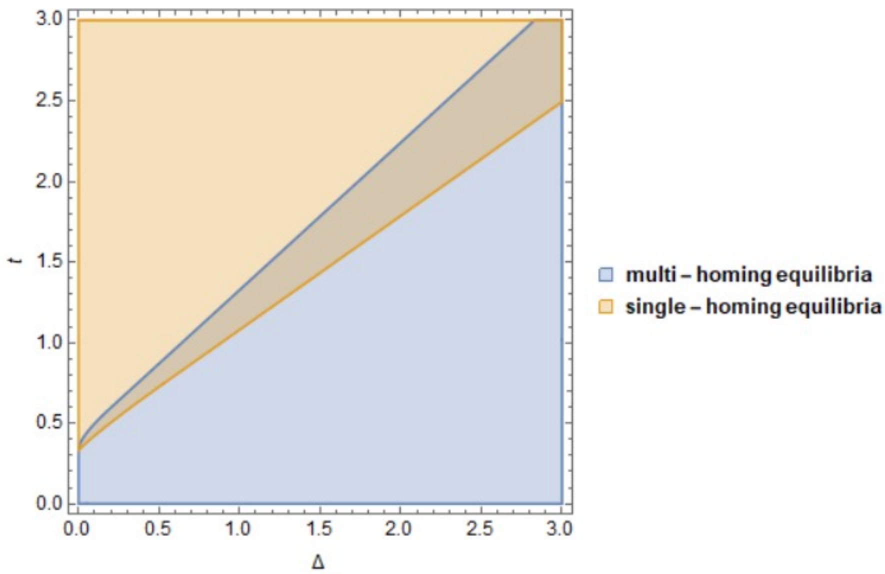


Figure 3: Multi Equilibria with Exclusive Licensing, $\delta = 1$

In this proposition, $\underline{\Delta}$ and $\bar{\Delta}$ are two variables that depend on t and δ. Both of them increase with t and decrease with δ. Therefore, if the content provider licenses the extra content exclusively, more consumers choose multi-homing than the case without licensing. By comparing the profits of both distributors under multi-homing and single-homing equilibria, I show that when $\Delta < \sqrt{2}t - (1 - \frac{\sqrt{2}}{3})\delta$, distributor S_1 gets higher profit under single-homing equilibrium than multi-homing equilibrium, and when $\Delta < \sqrt{2}t - \frac{\sqrt{2}}{3}\delta$, distributor S_2 gets higher profit under single-homing equilibrium. Both distributors prefer single-homing equilibrium when Δ and δ are low.

4.1.2 Shared licensing

In this case, the utility function of consumer $x \in [0,1]$ becomes:

$$\begin{cases} U_1 = v + \delta - tx - p_1 & \text{if the consumer chooses } S_1, \\ U_2 = v + \delta - t(1-x) - p_2 & \text{if the consumer chooses } S_2, \\ U_{12} = V + \delta - t - p_1 - p_2 & \text{if the consumer multi-homes.} \end{cases} \tag{19}$$

Note that for the multi-homing consumers, they can not get additional benefits from accessing to the extra content twice. Following the same process in Section 3, the membership prices and consumer decisions can be derived as:

Lemma 5

- *When $\Delta > \sqrt{2}t$, at least some of the consumers are multi-homing. Both S_1 and S_2 charge*

$$p_i = \begin{cases} \frac{\Delta}{2} & \textit{if } \Delta < 2t, \\ \Delta - t & \textit{if } \Delta \geq 2t \end{cases} \tag{20}$$

  on their membership. They get the profits that

$$\pi_i = \begin{cases} \dfrac{\Delta^2}{4t} - f_i & if\ \Delta < 2t, \\ \Delta - t - f_1 & if\ \Delta \geq 2t. \end{cases} \tag{21}$$

- *When $\Delta \leq 2t/(2\sqrt{2}-1)$, all of the consumers choose to single-home. Both S_1 and S_2 charge*

$$p_i = t \tag{22}$$

  on their membership. They get the profit that

$$\pi_i = \frac{t}{2} - f_i. \tag{23}$$

- *When* $2t/(2\sqrt{2}-1) < \Delta \leq \sqrt{2}t$, *there are multiple equibria.*[6]

4.2 Licensing contract

Denote $\pi_i(q_i, q_j)$ as the profit of distributor S_i when it chooses accepting strategy q_i and the other distributor chooses q_j. q_i indicates whether distributor i accepts the license ($q_i = 1$) or not ($q_i = 0$). If the content provider offers exclusive contract to distributor S_i, the distributor will accept it when $\pi_i(1,0) > \pi_i(0,0)$; and if the content provider offers shared contract to distributor S_i, distributor will accept it when $\pi_i(1,1) > \pi_i(0,1)$.

Since the single-homing profit is always larger than the multi-homing profit for each distributor when multiple equilibria exist with shared licensing, we suppose that the realized outcome is of the single-homing equilibrium under this condition. This is different from the assumption in Jiang et al. (2019), in which they assume that the multi-homing equilibrium would be achieved. Bringing the above prices into these conditions, I get the best response of distributor facing licensing contract.

Lemma 6 *If the content provider offers the exclusive contract to distributor S_i, the distributor will accept the licensing contract when*

$$f_i < \begin{cases} f^{ex}_{single} & if\ \Delta < \bar{\Delta}, \\ f^{ex}_{inter} & if\ \bar{\Delta} \leq \Delta < \sqrt{2}t, \\ f^{ex}_{multi} & if\ \Delta \geq \sqrt{2}t, \end{cases} \tag{24}$$

[6] Making $v' = v + \delta$, then the calculating steps for this case and the baseline model are identical except that both distributors should pay their licensing fee f_i.

where

$$f^{ex}_{single} = \frac{\delta}{3} + \frac{\delta^2}{18t}\ , \quad f^{ex}_{inter} = \begin{cases} \dfrac{(\Delta+\delta)^2}{4t} - \dfrac{t}{2} & if\ \Delta < 2t-\delta, \\ \Delta + \delta - \dfrac{3}{2}t & if\ \Delta \geq 2t-\delta, \end{cases}$$

$$f^{ex}_{multi} = \begin{cases} \dfrac{2\Delta\delta+\delta^2}{4t} & if\ \Delta < 2t-\delta, \\ \Delta+\delta-t-\dfrac{\Delta^2}{4t} & if\ 2t-\delta \leq \Delta < 2t, \\ \delta & if\ \Delta \geq 2t, \end{cases} \tag{25}$$

If the content provider offers the shared contract, both distributors will accept the licensing contract when

$$f_i < \begin{cases} f^{sh}_{single} & if\ \Delta < \bar{\Delta}, \\ f^{sh}_{inter} & if\ \bar{\Delta} \leq \Delta < \sqrt{2}t, \\ f^{sh}_{multi} & if\ \Delta \geq \sqrt{2}t, \end{cases} \tag{26}$$

where

$$f^{sh}_{single} = \frac{\delta}{3} - \frac{\delta^2}{18t}\ , \quad f^{sh}_{inter} = \begin{cases} \dfrac{t}{2} - \dfrac{\Delta^2}{4t} & if\ \Delta < 2t, \\ \dfrac{3}{2}t - \Delta & if\ \Delta \geq 2t, \end{cases} \quad f^{sh}_{multi} = 0. \tag{27}$$

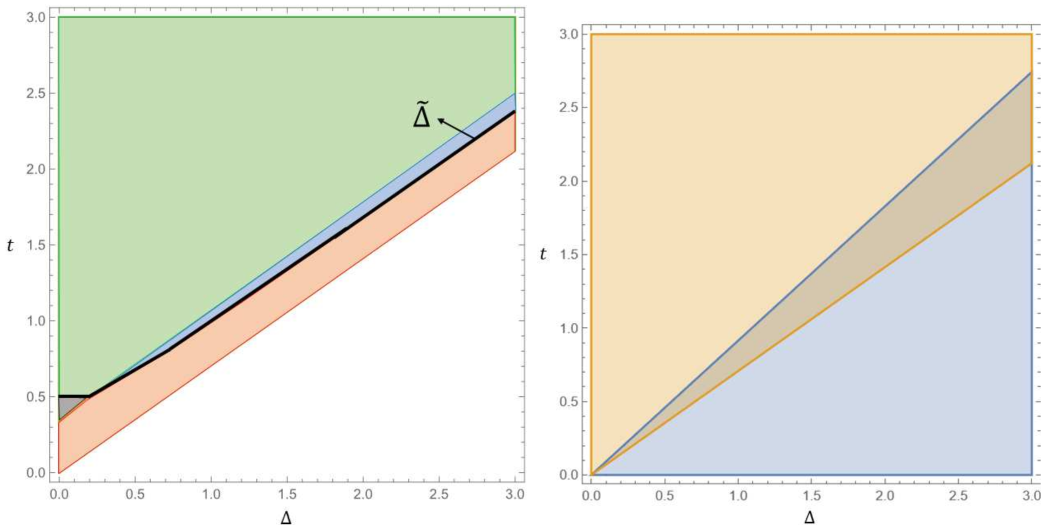
The content provider chooses the licensing fee f_i to maximize its profit $f_1 + f_2$, so it will charge

$$f^{\tau}_i = \begin{cases} f^{\tau}_{single} - \epsilon & if\ \Delta < \bar{\Delta}, \\ f^{\tau}_{inter} - \epsilon & if\ \bar{\Delta} \leq \Delta < \sqrt{2}t, \\ f^{\tau}_{multi} - \epsilon & if\ \Delta \geq \sqrt{2}t, \end{cases} \tag{28}$$

on the distributors, where $\tau \in \{ex,\ sh\}$, and ϵ is an arbitrarily small positive number. Since f_i is larger than 0, the content provider will always license to at least one distributor with a positive licensing fee.

Finally, the content provider decides whether it will license the extra content exclusively or share it. If it chooses exclusive licensing, its profit will be $\pi_M = f_i^{ex}$, and $\pi_M = 2f_i^{sh}$ if it chooses shared licensing.

Proposition 3 *The content provider shares hits license to both distributor when* $\Delta \leq \tilde{\Delta}$ *and* $t \geq \delta/2$*. Otherwise, it licenses the extra content exclusively.* [7]



(a) Licensing strategy (b) Equilibrium outcome without licensing

Figure 4: Cpntent Provider's Choice of Licensing Strategy when $\delta = 1$

Green and blue area in the left figure: the content provider chooses shared licensing;

Black, red and white area in the left figure: the content provider chooses exclusive licensing.

The right figure is the same as Figure 2.

In the left picture of Figure 4, we find five cases. In the green and black areas, all consumers single-home to one of the distributors no matter whether the content provider chooses shared licensing or exclusive licensing. The green area indicates the range of parameters in which the content provider chooses shared licensing and the black area indicates the range of parameters in which the content provider chooses exclusive licensing. This result shows that even for all consumers single-home, the content provider may license to only one distributor. In the blue and red areas, the existence of multi-homing consumers depends on the licensing strategy chosen by the content provider. If the content provider chooses shared licensing, all consumers will purchase from a single distributor; if it licenses to both distributors, some consumers will

[7]Detail of $\tilde{\Delta}$ is shown in Appendix B.

choose multi-homing. The blue area indicates the range of parameters in which the content provider chooses shared licensing and all consumers single-home. The red area indicates the range of parameters in which the content provider chooses exclusive licensing and some consumers multi-home. Finally, in the white area, there always exist some consumers who purchase from both distributors. In this case, the content provider always chooses exclusive licensing.

The condition $\tilde{\Delta}$ in Proposition 3 consists of two parts. First, t should be larger than 2δ. Second, Δ should be lower than a threshold $\tilde{\Delta}$. This result shows that when Δ is large and t and δ are low, the content provider chooses exclusive licensing, and some consumers multi-home. When Δ and δ are low and t is large, the content provider chooses shared licensing, and all consumers single-home. When t and Δ are low and δ is large, the content provider chooses exclusive licensing and all consumers single-home. The border line between the green and black areas under $\Delta = 0$ in Figure 4(a) ($t = \delta/2 = 1/2$) corresponds to the condition of Proposition 1 in Jiang et al. (2019) who assume consumers' single-homing behavior. This paper expands the parameter ranges by allowing endogenous choices of consumers' homing behavior and obtains the threshold value $\tilde{\Delta}$ that nest the scenario in Jiang et al. (2019).

In Jiang et al. (2019), content provider licenses to both distributor when $t \geq \delta/2$ if all consumers single-home. In our model, it licenses to both distributors when $\Delta \leq \tilde{\Delta}$ and $t \geq \delta/2$. This condition that depends on the value of Δ provides a larger area in which content provider exclusively licenses to one distributor and some consumers multi-home than Jiang et al. (2019) provides. This result comes, on the one hand, from the assumption that single-homing equilibrium will be achieved if there exists multiple equilibria. On the other hand, I take the case in which the existence of multi-homing consumers depends on the licensing strategy chosen by the content provider. This case has been ignored in Jiang et al. (2019) and Ishihara and Oki (2021). The variable $\tilde{\Delta}$ here takes a value between $\bar{\Delta}$ and $\sqrt{2}t$ when $t \geq \delta/2$.

When Δ is larger than $\sqrt{2}t$, there always exist some consumers who multi-home to both platforms no matter whether the content provider licenses the extra content exclusively or shared. In this case, the content provider cannot get additional profit by offering shared licensing because each distributor is not willing to accept the licensing contract with a positive licensing fee if the competitor accepts it. Therefore, it always exclusively licenses the extra content to any one of the distributors.

When Δ is less than $\bar{\Delta}$, all of the consumers single-home to any distributor no matter whether the content provider licenses the extra content exclusively or shared. Because $\partial f_i^{ex}/\partial\delta$ is larger than $\partial 2f_i^{sh}/\partial\delta$, the exclusive profit of content provider will be larger when δ is high, which is relevant to a small t. As a result, when $\Delta < \bar{\Delta}$, the content provider licenses exclusively when t is low and shares licensing when t is high.

When $\bar{\Delta} \leq \Delta < \sqrt{2}t$, all of the consumers will single-home to any distributor if the content provider shares its licensing, and part of the consumers will multi-home to both distributors if the content provider chooses exclusive licensing. In this case, the exclusive profit of the content provider increases with Δ, while the shared profit decreases with Δ. Therefore, the content provider licenses to one of the distributors when consumers' additional gain from the second purchase is high and shares licensing when it is low.

This proposition shows that when consumers' additional gain from the second purchase is high, or the mismatch cost for consumers is sufficiently low, the content provider licenses its extra content exclusively to a single distributor. On the other hand, when consumers' additional gain from the second purchase is low, the content provider shares its extra content. The quality of the extra content also plays an important role in the provider's decision. If the extra content is valuable enough to the consumers, the content provider always chooses exclusive licensing, no matter whether the distributors are differentiated or homogenous. In other words, if the content provider produces high-quality content, it can allocate the market and exclusively license its content to either distributor it wants.

4.3 Examples of content licensing with endogenous homing consumers

In this section, I will give two examples of content licensing decisions. According to Proposition 3, when consumers' additional gain from the second purchase is high and the mismatch cost is sufficiently low, there exist multi-homing consumers in the market, and the content provider exclusively licenses the extra content to one of the distributors. I show that this can happen in the real world.

4.3.1 Mobile game agency

With the popularity of mobile devices such as smartphones, the mobile game industry has grown at an incredible pace. According to the Korea Creative Content Agency, the estimated market size of the global mobile game was \$57.6 billion in 2017. Among them, China was the largest market in the world with 20.7% market share. To compete in this market, most game companies, especially mobile game companies, apply an entry method called "agency". By using this method, the game producers can license their games to a local agent in China instead of constructing a new company. One of the biggest mobile game agents in China, Bilibili, has obtained more than 5 billion CNY through its mobile game business, and most of them came from a game agency.

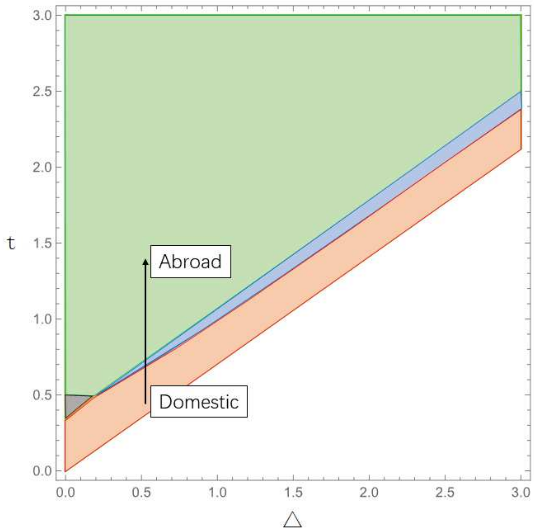


Figure 5: An Example of Mobile game agency

Now, consider a market with one game producer and two agents, which is the same as my model. Suppose that both of the agents are domestic companies. As it is easy for the gamers to attach to each agent, both the mismatch cost and consumers' additional gain from the second purchase can be considered to be low. As a result, this market falls in the bottom area in Figure 5. In this case, the game producer chooses exclusive licensing. [8] Then, let one of the agents be a foreign company. Because the game is a digital product, the transportation cost can be considered as 0, and the quality does not change. Meanwhile, I ignore the increase

[8]If consumers' additional gain from the second purchase is low enough that the parameters launch into the black area, all consumers single-home. On the other side, some consumers multi-home if the distributor difference is high.

in the consumer base. Compare with the original case, the mismatch cost for consumers increases. Since the two agents are located in different countries, it will be very costly for consumers to attach to both agents because of different languages or the usage of VPN. The variation of parameters are showed in Figure 5. As a result, the content provider tends to share its licensing to both distributors.

4.3.2 VoD market

The video-on-demand (VoD) market, in which people consume audiovisual content via online streaming services, has experienced rapid growth in recent years. A few typical examples include Netflix and Amazon Prime Video. According to Wayne (2018), more than 180 million consumers subscribe to these two platforms. One of the most important reasons for consumers to make a choice between the platforms is their exclusive content. Aguiar & Waldfogel (2018) show that there are 648 exclusive movies on Netflix, and 199 titles on Amazon Prime Video.

The issues about exclusivity in this market can be explained by this article. Suppose that there are two distributors in the market. If both distributors own large amounts of exclusive content, consumers will get high extra benefits from multi-homing to both distributors, so the content provider is more likely to license its product to only one of them, and consumers tend to multi-home. On the other hand, if the distributors own little exclusive content, consumers will not benefit from multi-homing to both distributors, so the content provider is more likely to share its product, and consumers tend to purchase from only one distributor. The quality of the extra content also plays an important role in this case. If the extra content is outstanding, which means δ is large, the content provider will choose to license it to only one distributor. This matches what happened with *House of Cards*, which is available exclusively on Netflix.

5 Conclusion

In this paper, I have studied the distributor competition of horizontally differentiated duopoly. The key departure of this paper from most of the existing literature is the endogenous homing choice. This paper simultaneously considers the single-homing and multi-homing outcomes in a unified model. As a result, both outcomes are simultaneously sustainable in some parameter sets. Concretely, when consumers' additional gain from multi-purchasing is high, there are multi-homing consumers in the market; when the additional gain from multi-purchasing is low, all consumers choose to single-home to either distributor. When the additional gain from multi-purchasing is moderate, the single-homing and multi-homing outcomes are sustainable. In this case, the distributor charges a low price when the competitor also charges a low price, and then multi-homing consumers exist; or it charges a high price when the competitor also charges a high price, and then all consumers single-home. Although the distributors can get higher profits in the single-homing equilibrium, both social welfare and consumer surplus are worse off. Then I introduce a content provider who produces extra content and can license it to both distributors or exclusively license it to either distributor. It charges a fixed fee from the licensee with zero cost. Then, if consumers' additional gain from multi-purchasing is high, the mismatch cost is low, and the value of the extra content is high enough, the content provider will license its product to only one distributor. On the other hand, if the additional gain from multi-purchasing is low, the mismatch cost is high, and the value of the extra content is low enough, the content provider will choose shared licensing. Note that, unlike previous research, the choices of the content provider and consumers are not one-to-one. If the additional gain from multi-purchasing is low, and the mismatch cost and the quality of extra content are moderate, the content provider will license to only one distributor, and all consumers will be single-homing. In other cases, either the content provider licenses to only one distributor and some consumers purchase from both distributors, or the content provider chooses shared licensing, and all consumers single-home.

There is still much space for improvement and further research in this paper. First, I only studied fixed-fee licensing in this paper. Wang (1998) and San Martín (2010) found that royalty is a better licensing method for the technology holder than fixed-fee. In the setting of this paper, the profit of the maker will become $\pi_M = r_1 x_1 + r_2 x_2$ if it uses the royalty method, and the profit of distributor i will be $\pi_i = (p_i - r_i)x_i$, where r_i is the royalty fee of licensing. The second concerned extension is that the maker and the distributors in this paper make their strategies depending on their profits. However, in reality, some large distributors, such as Amazon and Netflix, can bargain with the content provider. A more extreme example is vertical integration, where the distributor itself becomes the maker of the goods. As integration somewhat mitigates competition between distributors, it may be possible for the maker and consumers to choose to be multiple-homing simultaneously in the equilibrium. I will continue to explore these lines in the future, examining more possibilities for the digital content distributor market.

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Appendix

A Proof of Lemma 4

Suppose that the content provider licenses to S_1.

A.1 Equilibrium when multi-homing consumers exist

In this section, suppose that $\Sigma_k p_k < 2\Delta + \delta - t$. The number of consumers who belong to firm S_i, $N_i, i = 1, 2$, is

$$N_i = \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where } \hat{N}_1 = \frac{\Delta + \delta - p_1}{t}, \ \hat{N}_2 = \frac{\Delta - p_2}{t}. \tag{29}$$

Under the condition that $\Sigma_k p_k < 2\Delta + \delta - t$, $\Sigma_k N_k > 1$, which means that there exists multi-homing consumers in the market.

Then, for the firms, they choose their membership price p_i to maximize their profit $\pi_i = p_i N_i$. Taking the first order condition that $\frac{\partial \pi_i}{\partial p_i} = 0$, the optimal prices of firms are showed below. When multi-homing consumers exist,

$$p_1^{**} = \begin{cases} \frac{\Delta + \delta}{2} & if \ \Delta < 2t - \delta, \\ \Delta + \delta - t & if \ \Delta \geq 2t - \delta, \end{cases} \quad p_2^{**} = \begin{cases} \frac{\Delta}{2} & if \ \Delta < 2t, \\ \Delta - t & if \ \Delta \geq 2t. \end{cases} \tag{30}$$

The corresponding number of members belonging to firm S_i with the optimal price is

$$N_1^{**} = \begin{cases} \frac{\Delta + \delta}{2t} & \text{if } \Delta < 2t - \delta, \\ 1 & \text{if } \Delta \geq 2t - \delta, \end{cases} \quad N_2^{**} = \begin{cases} \frac{\Delta}{2t} & \text{if } \Delta < 2t, \\ 1 & \text{if } \Delta \geq 2t, \end{cases} \tag{31}$$

and the maximizing profits of firms are

$$\pi_1^{**} = \begin{cases} \frac{(\Delta + \delta)^2}{4t} - f_1 & if \ \Delta < 2t - \delta, \\ \Delta + \delta - t - f_1 & if \ \Delta \geq 2t - \delta, \end{cases} \quad \pi_2^{**} = \begin{cases} \frac{\Delta^2}{4t} & if \ \Delta < 2t, \\ \Delta - t & if \ \Delta \geq 2t. \end{cases} \tag{32}$$

Lemma A.1 *Pure strategy subgame perfect equilibria with multi-homing consumers exists when* $\Delta > \underline{\Delta}$.

Proof: To check whether the optimal price meets the condition of multi-homing existence, I put equation (30) into the multi-homing existing condition $\Sigma_k p_k < 2\Delta + \delta - t$. Then,it is derived that when $\Delta > t - \delta/2$, the existence condition of mult-homing condition is satisfied.

Then, I show that the firms have no incentive to change their membership prices. Note that since $p_i = p_i^*$ satisfies the first and second order condition of profit maximization under the condition that $\Sigma_k p_k < 2\Delta + \delta - t$, p_i^{**} is always the optimal price for firm S_i when multi-homing consumers exist. Therefore, I only check the situation that S_i raises its price to the level that $p_i + p_j^{**} \geq 2\Delta + \delta - t$, which means no consumer chooses to multi-home. As a result, the number of consumers accessing firm S_i under is

$$N_i = \max\{min\{\hat{N}_i, 1\}, 0\}, \text{ where}$$

$$\hat{N}_1 = \begin{cases} \dfrac{2t + 2\delta + \Delta - 2p_1}{4t} & if\ \Delta < 2t, \\ \dfrac{\Delta + \delta - p_1}{2t} & if\ \Delta \geq 2t, \end{cases} \quad \hat{N}_2 = \begin{cases} \dfrac{2t - \delta + \Delta - 2p_2}{4t} & if\ \Delta < 2t - \delta, \\ \dfrac{\Delta - p_2}{2t} & if\ \Delta \geq 2t - \delta. \end{cases} \tag{33}$$

Then, for the firm S_i, it chooses the membership price p_i to maximize its profit $\pi_i = p_i N_i$. Taking the first order condition that $\partial \pi_i / \partial p_i = 0$, the optimal price of S_i is

$$pd_1^{**} = \begin{cases} \dfrac{\Delta + 2\delta + 2t}{4} & if\ \Delta < 2t, \\ \dfrac{\Delta + \delta}{2} & if\ \Delta \geq 2t, \end{cases} \quad pd_2^{**} = \begin{cases} \dfrac{\Delta - \delta + 2t}{4} & if\ \Delta < 2t - \delta, \\ \dfrac{\Delta}{2} & if\ \Delta \geq 2t - \delta. \end{cases} \tag{34}$$

Put equation (34) into the single-homing condition $\Sigma_k p_k \geq 2\Delta + \delta - t$, then for S_1, the equilibrium takes inner point solution when $\Delta < 6t/5 - 2\delta/5$; for S_2, the equilibrium takes inner point solution when $\Delta < 6t/5 - 3\delta/5$. Otherwise, it takes the end point solution. As a result, firm S_i's optimal price when it escapes to the single-homing case is

$$pd_1^{**} = \begin{cases} \dfrac{\Delta + 2\delta + 2t}{4} & if\ \Delta < \frac{6}{5}t - \frac{2}{5}\delta, \\ \frac{3}{2}\Delta + \delta - t & if\ \frac{6}{5}t - \frac{2}{5}\delta \leq \Delta < 2t, \\ \Delta + \delta & if\ \Delta \geq 2t, \end{cases} pd_2^{**} = \begin{cases} \dfrac{\Delta - \delta + 2t}{4} & if\ \Delta < \frac{6}{5}t - \frac{3}{5}\delta, \\ \frac{3}{2}\Delta + \frac{1}{2}\delta - t & if\ \frac{6}{5}t - \frac{3}{5}\delta \leq \Delta < 2t - \delta, \\ \frac{\Delta}{2} & if\ \Delta \geq 2t - \delta. \end{cases} \tag{35}$$

The corresponding number of members belonging to firm S_i with this price is

$$Nd_1^{**} = \begin{cases} \dfrac{\Delta + 2\delta + 2t}{8t} & \text{if } \Delta < \frac{6}{5}t - \frac{2}{5}\delta, \\ 1 - \frac{\Delta}{2t} & \text{if } \frac{6}{5}t - \frac{2}{5}\delta \leq \Delta < 2t, \\ 0 & \text{if } \Delta \geq 2t, \end{cases} Nd_2^{**} = \begin{cases} \dfrac{\Delta - \delta + 2t}{8t} & \text{if } \Delta < \frac{6}{5}t - \frac{3}{5}\delta, \\ 1 - \frac{\Delta+\delta}{2t} & \text{if } \frac{6}{5}t - \frac{3}{5}\delta \leq \Delta < 2t - \delta, \\ 0 & \text{if } \Delta \geq 2t - \delta, \end{cases} \tag{36}$$

and the maximizing profit of firm S_i is

$$\pi d_1^{**} = \begin{cases} \dfrac{1}{2t}(\dfrac{\Delta + 2\delta + 2t}{4})^2 - f_1 & \text{if } \Delta < \frac{6}{5}t - \frac{2}{5}\delta, \\ (\frac{3}{2}\Delta + \delta - t)(1 - \frac{\Delta}{2t}) - f_1 & \text{if } \frac{6}{5}t - \frac{2}{5}\delta \leq \Delta < 2t, \\ -f_1 & \text{if } \Delta \geq 2t, \end{cases}$$

$$\pi d_2^{**} = \begin{cases} \dfrac{1}{2t}(\dfrac{\Delta - \delta + 2t}{4})^2 & \text{if } \Delta < \frac{6}{5}t - \frac{3}{5}\delta, \\ (\frac{3}{2}\Delta + \frac{1}{2}\delta - t)(1 - \frac{\Delta+\delta}{2t}) & \text{if } \frac{6}{5}t - \frac{3}{5}\delta \leq \Delta < 2t - \delta, \\ 0 & \text{if } \Delta \geq 2t - \delta. \end{cases} \tag{37}$$

Comparing this profit with the optimal profit under multi-homing existence condition, the firm S_i has no incentive to escape from the optimal price p_i^{**} when

$$
\begin{aligned}
&(3\delta < 2\sqrt{2}\sqrt{\Delta^2} \wedge 6t > 3\delta + 5\Delta \wedge 2\sqrt{2}\sqrt{(\delta+\Delta)^2} > 2\delta + \Delta + 2t) \\
&\vee (6t > 2\delta + 5\Delta \wedge (\left(2t < \delta + \Delta \wedge 2\Delta < \delta \wedge \delta \le 2\left(\sqrt{2}\sqrt{\Delta^2} + \Delta\right)\right) \\
&\vee (\delta > 2\left(\sqrt{2}\sqrt{\Delta^2} + \Delta\right) \wedge 2\sqrt{2}\sqrt{\Delta(6\delta + 5\Delta)} + 6\delta + 7\Delta > 18t))) \\
&\vee (2\delta + 5\Delta \ge 6t \wedge 2t > 3\Delta) \vee (2t > \Delta \wedge 2t \le 3\Delta \wedge \delta + \Delta > 2t))) \\
&\vee ((2t \ge \delta + \Delta \wedge (\left(5\sqrt{2}\sqrt{\delta^2} + 4\delta \ge 8t \wedge \sqrt{2}\sqrt{\delta^2} + \delta < 4t \wedge 2\left(\sqrt{2}\sqrt{(\delta+2t)^2} + t\right) < 6\delta + 7\Delta\right) \\
&\vee (4t > 3\delta \wedge 6t \le 2\delta + 5\Delta) \vee \left(8t > 5\sqrt{2}\sqrt{\delta^2} + 4\delta \wedge 6t \le 3\delta + 5\Delta\right))))).
\end{aligned}
\tag{38}
$$

Q.E.D.

A.2 Equilibrium when multi-homing consumers do not exist

In this section, suppose that $\Sigma_k p_k \ge 2\Delta + \delta - t$. The number of consumers who belong to firm S_i, $N_i, i = 1, 2$, is

$$
\begin{aligned}
N_i &= \max\{\min\{\hat{N}_i, 1\}, 0\}, \text{ where} \\
\hat{N}_1 &= \frac{1}{2} - \frac{p_i - p_j + \delta}{2t},\ \hat{N}_2 = \frac{1}{2} - \frac{p_i - p_j - \delta}{2t}.
\end{aligned}
\tag{39}
$$

Under the condition that $\Sigma_k p_k \ge 2\Delta + \delta - t$, $\Sigma_k N_k = 1$, which means that there is no multi-homing consumer in the market.

Then, for the firms, they choose their membership price p_i to maximize their profit $\pi_i = p_i N_i$. Taking the first order condition that $\partial\pi_i/\partial p_i = 0$, the optimal prices of firms are shown below. When multi-homing consumers exist,

$$p_1^* = t + \frac{1}{3}\delta,\ p_1^* = t - \frac{1}{3}\delta. \tag{40}$$

The corresponding number of members belonging to firm S_i with the optimal price is

$$N_1^* = \frac{1}{2} + \frac{1}{6t}\delta,\ N_2^* = \frac{1}{2} - \frac{1}{6t}\delta, \tag{41}$$

and the maximizing profits of firms are

$$\pi_1^* = 2t[\frac{1}{2} + \frac{\delta}{6t}]^2 - f_1,\ \pi_2^* = 2t[\frac{1}{2} - \frac{\delta}{6t}]^2. \tag{42}$$

Lemma A.2 *Pure strategy subgame perfect equilibria without multi-homing consumer exists when $\Delta \leq \bar{\Delta}$.*

Proof: To check the single-homing existence condition, I put equation(40) into the condition $\Sigma_k p_k \geq 2\Delta + \delta - t$. Then, it is derived that when $\Delta \leq 3t/2 - \delta/2$, the single-homing condition is satisfied.

Then, I show that the firms have no incentive to change their membership prices. Note that since $p_i = p_i^*$ satisfies the first and second order condition of profit maximization under the condition that $\Sigma_k p_k \geq 2\Delta + \delta - t$, p_i^* is always the optimal price for firm S_i when multi-homing consumers do not exist. Therefore, I only check the situation that S_i decreases its price to the level that $p_i + p_j^* < 2\Delta + \delta - t$, which means that there exist multi-homing consumers in the market. As a result, the number of consumers accessing firm S_i under is

$$N_i = \max\{min\{\hat{N}_i, 1\}, 0\}, \text{ where} \tag{43}$$

$$\hat{N}_1 = \frac{\Delta + \delta - p_1}{t},\ \hat{N}_2 = \frac{\Delta - p_2}{t} \tag{44}$$

Then, for the firm S_i, it chooses the membership price p_i to maximize its profit $\pi_i = p_i N_i$. Taking the first order condition that $\partial \pi_i / \partial p_i = 0$, the optimal price of S_i is

$$pd_1^* = \begin{cases} \dfrac{\Delta + \delta}{2} & , if\ \Delta < 2t - \delta, \\ \Delta + \delta - t & , if\ \Delta \geq 2t - \delta, \end{cases} \quad pd_2^* = \begin{cases} \dfrac{\Delta}{2} & , if\ \Delta < 2t, \\ \Delta - t & , if\ \Delta \geq 2t. \end{cases} \tag{45}$$

Put equation(45) into the multi-homing existing condition $\Sigma_k p_k < 2\Delta + \delta - t$, then for each distributor, the optimal prices can be divided into three cases.

$$pd_1^* = \begin{cases} 2\Delta + \frac{4}{3}\delta - 2t & , \, if \ \Delta < 2t - \delta \ and \ \Delta < \frac{4}{3}t - \frac{5}{9}\delta, \\ \dfrac{\Delta + \delta}{2} & , \, if \ \Delta < 2t - \delta \ and \ \Delta \geq \frac{4}{3}t - \frac{5}{9}\delta, \\ 2\Delta + \frac{4}{3}\delta - 2t & , \, if \ \Delta \geq 2t - \delta \ and \ \Delta < t - \frac{1}{3}\delta, \\ \Delta + \delta - t & , \, if \ \Delta \geq 2t - \delta \ and \ \Delta \geq t - \frac{1}{3}\delta, \end{cases}$$

$$pd_2^* = \begin{cases} 2\Delta + \frac{2}{3}\delta - 2t & , \, if \ \Delta < 2t \ and \ \Delta < \frac{4}{3}t - \frac{4}{9}\delta, \\ \dfrac{\Delta}{2} & , \, if \ \Delta < 2t \ and \ \Delta \geq \frac{4}{3}t - \frac{4}{9}\delta, \\ 2\Delta + \frac{2}{3}\delta - 2t & , \, if \ \Delta \geq 2t \ and \ \Delta < t - \frac{2}{3}\delta, \\ \Delta - t & , \, if \ \Delta \geq 2t \ and \ \Delta \geq t - \frac{2}{3}\delta. \end{cases} \tag{46}$$

The corresponding number of members belonging to firm S_i with this price is

$$Nd_1^* = \begin{cases} 2 - \frac{3\Delta + \delta}{3t} & , \, if \ \Delta < 2t - \delta \ and \ \Delta < \frac{4}{3}t - \frac{5}{9}\delta, \\ \dfrac{\Delta + \delta}{2t} & , \, if \ \Delta < 2t - \delta \ and \ \Delta \geq \frac{4}{3}t - \frac{5}{9}\delta, \\ 1 & , \, if \ \Delta \geq 2t - \delta, \end{cases}$$

$$Nd_2^* = \begin{cases} 2 - \frac{3\Delta + 2\delta}{3t} & , \, if \ \Delta < 2t \ and \ \Delta < \frac{4}{3}t - \frac{4}{9}\delta, \\ \dfrac{\Delta}{2t} & , \, if \ \Delta < 2t \ and \ \Delta \geq \frac{4}{3}t - \frac{4}{9}\delta, \\ 1 & , \, if \ \Delta \geq 2t, \end{cases} \tag{47}$$

and the maximizing profit of firm S_i is

$$\pi d_1^* = \begin{cases} (2\Delta + \frac{4}{3}\delta - 2t)(2 - \frac{3\Delta+\delta}{3t}) & , if\ \Delta < 2t - \delta\ and\ \Delta < \frac{4}{3}t - \frac{5}{9}\delta, \\ \dfrac{(\Delta+\delta)^2}{4t} & , if\ \Delta < 2t - \delta\ and\ \Delta \geq \frac{4}{3}t - \frac{5}{9}\delta, \\ 2\Delta + \frac{4}{3}\delta - 2t & , if\ \Delta \geq 2t - \delta\ and\ \Delta < t - \frac{1}{3}\delta, \\ \Delta + \delta - t & , if\ \Delta \geq 2t - \delta\ and\ \Delta \geq t - \frac{1}{3}\delta, \end{cases}$$

$$\pi d_2^* = \begin{cases} (2\Delta + \frac{2}{3}\delta - 2t)(2 - \frac{3\Delta+2\delta}{3t}) & , if\ \Delta < 2t\ and\ \Delta < \frac{4}{3}t - \frac{4}{9}\delta, \\ \dfrac{\Delta^2}{4t} & , if\ \Delta < 2t\ and\ \Delta \geq \frac{4}{3}t - \frac{4}{9}\delta, \\ 2\Delta + \frac{2}{3}\delta - 2t & , if\ \Delta \geq 2t\ and\ \Delta < t - \frac{2}{3}\delta, \\ \Delta - t & , if\ \Delta \geq 2t\ and\ \Delta \geq t - \frac{2}{3}\delta. \end{cases} \tag{48}$$

Comparing this profit with the optimal profit under single-homing condition, the firm S_i has no incentive to escape from the optimal price p_i^* when

$$\begin{aligned} &(2t > \delta \wedge 2t \geq \delta + \Delta \wedge 3t < 2\delta) \vee (12t > 4\delta + 9\Delta \wedge 3\sqrt{2}\sqrt{\Delta^2} \geq 4\delta \wedge 12t \leq 5\delta + 9\Delta) \\ &\vee (\delta + 3\Delta < 3t \wedge 2t \leq \delta) \vee (3t > 2\delta \wedge 12t > 5\delta + 9\Delta) \\ &\vee (12t \leq 5\delta + 9\Delta \wedge ((2t \geq \delta + \Delta \wedge \delta < 3\Delta \wedge 6\sqrt{2}\sqrt{\Delta^2} + 3\Delta < 7\delta) \\ &\vee (6\sqrt{2}\sqrt{\Delta^2} + 3\Delta \geq 7\delta \wedge 3\sqrt{2}\sqrt{\Delta^2} < 4\delta \wedge 3\sqrt{2}\sqrt{(\delta+\Delta)^2} < 2\delta + 6t))))) \\ &\vee ((((\frac{\delta^2}{t} + 27t > 12\delta + 18\Delta \\ &\wedge ((3t > 2\delta \wedge \sqrt{2}\sqrt{\delta^2} + \delta > 3t \wedge 2t < \delta + \Delta) \vee (3t > \delta \wedge 3t < 2\delta \wedge 3t \leq \delta + 3\Delta))) \\ &\vee (2t < \delta + \Delta \wedge 3t < 2\delta \wedge 2t > \delta \wedge 3t > \delta + 3\Delta) \\ &\vee (6t > 3\sqrt{2}\sqrt{\delta^2} + 2\delta \wedge 12t \leq 4\delta + 9\Delta \wedge \sqrt{2}\sqrt{(\delta+3t)^2} > 3(\delta + \Delta))). \end{aligned} \tag{49}$$

Q.E.D.

B Detail of $\tilde{\Delta}$

The parameters Δ, δ and t satisfy that

$$
\begin{aligned}
&(t \geq \frac{\delta}{3\sqrt{2}-3} \wedge 0 < \Delta < \frac{1}{3}(\sqrt{2}\delta - 3\delta + 3\sqrt{2}t))\\
&\vee (\frac{\delta}{2} < t < \frac{\delta}{3\sqrt{2}-3} \wedge 0 < \Delta < \frac{\delta^2 + 27t^2 - 12\delta t}{18t})\\
&\vee (\frac{\sqrt{6}\sqrt{650\Delta^2 - 457\sqrt{2}\Delta^2}}{193 - 132\sqrt{2}} + \frac{6\sqrt{2}\Delta - 11\Delta}{132\sqrt{2} - 193} < t < -\frac{\Delta}{3\sqrt{2}-5} \wedge 2t - \Delta < \delta \leq 3\sqrt{2}t - 3t)\\
&\vee (\Delta < t \leq \frac{\sqrt{6}\sqrt{650\Delta^2 - 457\sqrt{2}\Delta^2}}{193 - 132\sqrt{2}} + \frac{6\sqrt{2}\Delta - 11\Delta}{132\sqrt{2} - 193} \wedge 2t - \Delta < \delta < \frac{-\Delta^2 + 5t^2 - 2\Delta t}{2t}\\
&\vee (\delta \leq \Delta \wedge \frac{\sqrt{\delta^2 + 2\delta\Delta + 3\Delta^2}}{\sqrt{6}} < t \leq \frac{1}{6}(3\sqrt{2}\delta - 2\delta + 3\sqrt{2}\Delta))\\
&\vee (\Delta < \delta < \frac{3\Delta - 3\sqrt{2}\Delta}{3\sqrt{2}-5} \wedge \frac{\delta + \Delta}{2} \leq t \leq \frac{1}{6}(3\sqrt{2}\delta - 2\delta + 3\sqrt{2}\Delta))\\
&\vee (-\frac{\Delta}{3\sqrt{2}-5} < t < \mathrm{Root}[13\#1^4 - 44\#1^3\Delta + 18\#1^2\Delta^2 + 4\#1\Delta^3 + \Delta^4\&, 2]\\
&\wedge 6t - 3\sqrt{t^2 + 2\Delta t} \leq \delta < \frac{-\Delta^2 + 5t^2 - 2\Delta t}{2t})\\
&\vee (\frac{\sqrt{6}\sqrt{2\Delta^2 - \sqrt{2}\Delta^2}}{11 - 6\sqrt{2}} - \frac{\Delta}{6\sqrt{2} - 11} < t \leq -\frac{\Delta}{3\sqrt{2}-5} \wedge 3\sqrt{2}t - 3t < \delta < \frac{-\Delta^2 + 5t^2 - 2\Delta t}{2t}).
\end{aligned}
\tag{50}
$$