Role of worker flows in the relationship between job offers and employment

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Abstract

This study investigates the mechanism determining changes in job offers and analyzes the role of worker flows in the relationship between job offers and employment. It applies a queueing system to dynamic general equilibrium models and analyzes economic fluctuations. The queueing system helps in considering the relationship between job offers and employment and changes in job offers in response to shocks. The numerical simulations indicate that worker flows influence changes in job offers in response to a productivity shock. Although employment fluctuations remain constant, changes in job offers amplify when fewer workers join and/or more workers leave the firm, whereas they decrease when more workers join and/or fewer workers leave. The shock responses of other variables in each model are in line with the reactions of the standard dynamic general equilibrium models. These findings provide important insights into labor market dynamics.

Keywords: Queueing system, Job offer, Employment fluctuation, Economic fluctuation, Efficiency wage

Classification codes: E24, E32, J20, J33

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1. Introduction

At the macroeconomic level, job offers are one of the critical indicators when discussing economic policies because they reflect labor market conditions. At the organizational level, job offers play a crucial role in ensuring the necessary amounts of employment. However, job vacancies may not always be filled, for reasons such as disagreement regarding employment contracts and leaving. The resulting unfulfilled job vacancies lead to a gap between changes in job offers and fluctuations in employment. Cabo and Martín-Román (2019), Goux et al. (2001), and Hamermesh and Pfann (1996) discuss the fluctuations in new hiring and employment using the dynamic models of labor demand. Studies such as Chiarini and Piselli (2005), Lindé (2009), and Mitra et al. (2019) analyze employment fluctuations using dynamic general equilibrium (DGE) models considering the demand and supply sides of labor. The fluctuations analyzed in these frameworks do not correspond to changes in job offers because job vacancies are not always filled. This gap highlights the need for a framework analyzing changes in job offers.

In the given context, this study investigates the mechanism of the changes in determining job offers and expounds on the relationship between job offers and worker flows through a theoretical analysis. We use two DGE models with and without unemployment, respectively. The model with unemployment in this study is based on the efficiency wage model by Collard and de la Croix (2000), which is extended to investigate the change in job offers. The other model extends the standard DGE model. The change in job offers can be analyzed by applying queueing theory, which deals with issues related to waiting. The job offers correspond to the finite capacity of the queues, and employment corresponds to the workers in the queue. Given this, the relationship between job offers and employment is derived and introduced to the firms' optimization problem.

In efficiency wage models, even if there is excess supply in the labor market, firms do not reduce wages because labor productivity depends positively on wages. The literature considers four types of models—the shirking, adverse selection, labor turnover, and gift exchange models. In the shirking model, firms pay higher wages to prevent workers from slacking because of the greater cost of lost income (e.g., Gomme, 1999; Martin and Wang, 2020; Shapiro and Stiglitz, 1984). In the adverse selection model, firms hire the best workers by offering higher wages (Weiss, 1980). In the labor turnover model, firms pay higher wages to reduce labor turnover and save on training and hiring costs (e.g., Campbell III, 1994; Salop, 1979; Stiglitz, 1974). In the gift exchange model, employees work harder in return for a higher pay from firms (e.g., Akerlof, 1982; Collard and de la Croix, 2000; Danthine and Kurmann, 2004). This study extends the gift exchange model to simulate the behavior of job offers in response to a productivity shock.

Some studies investigate the labor market by applying a queueing system unlike the one used in this study. Deutsch and David (2020) assume that workers and jobs arrive at a system independently, and the job is assigned to a worker or discarded within a limited time. This job assignment yields a higher (lower) gain if there is a match (mismatch) between the worker and the job offer. Deutsch and David (2020) analyze the optimal choices of workers in the system. Meanwhile, Feigin and Landsberger (1981) construct a model using an unemployment queue and discuss the stationary distribution of unemployment. Sattinger (2010) supposes that workers search and join queues at firms to acquire a job and considers the workers' decision regarding which queues to enter.

This study uses the queueing theory to reveal the relationship between job offers and employment and shows that a large number of job offers leads to a significant increase in employment. It also shows that, at the same level of job offers, employment increases when more workers join the firm and/or fewer workers leave the firm. In the matching literature, many studies assume the Cobb–Douglas function in the matching function, such as Leduc and Liu (2016), Wesselbaum (2011), and Zanetti (2019). The matches depend on unemployed workers and job vacancies. The relationship between job offers and employment in this study shares a commonality with the matching function in that employment increases as labor demand increases. Based on numerical model simulations, this study also shows that substantial changes in job offers owing to productivity shocks do not necessarily increase employment variations. This is because the worker inflows and outflows influence only the reaction of job offers to the shock. The reactions are amplified when fewer workers join the firm and/or more workers leave the firm, though the employment fluctuations remain constant. In each model, the responses of the other variables to the shock are consistent with the reaction of a standard DGE model and an efficiency wage model, respectively.

This study makes the following contributions. First, it introduces the queueing system to construct macroeconomic frameworks that can analyze changes in job offers, showing how workers' flows influence changes in job offers resulting from the shock. This provides important perspectives on labor market dynamics. Second, the numerical simulations predict how the balance between workers' inflows and outflows influences the change in job offers. This indicates the need to examine both the change in job offers and workers' flows when discussing economic policies. Suppose that workers' flows change to fewer workers joining the firm and/or more workers leaving the firm. Then, employment may not increase significantly with the shock because of changes in workers' flows. If we focus only on the changes in job offers without paying attention to the changes in workers' flows, the resulting employment policy may be inadequate.

The remainder of this paper is organized as follows. Section 2 describes the relationship between job offers and employment according to the queueing theory. Section 3 analyzes the dynamics of the standard DGE model with a queueing system. Section 4 investigates the dynamics of the efficiency wage model with a queueing system. Finally, Section 5 concludes the study.

2. Queueing system in the labor market

In this section, we analyze the relationship between job offers and employment according to the queueing theory. Using Kendall's notation, the system is classified as M/M/1 with finite capacity. We consider the situation that the firm offers jobs, and workers join the firm and leave after working for some time.

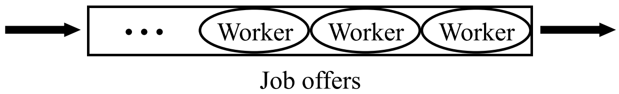

Figure 1. Job offers and employment in the model

Figure 1 illustrates the relationship between job offers and employment. The firm makes J_t job offers. We assume that the number of employees the firm hires cannot exceed that of J_t job offers. Then, the maximum number of employees is the number of job offers, indicated by the size of the square in the figure. The number of workers in the square represents employees. The arrows indicate increases or decreases in the number of employees. The average employment L_t is expressed as follows:

$$L_t = \sum_{n=0}^{J_t} nP_n, \tag{1}$$

where n is the number of employees, and P_n is the probability of n workers in the steady state.

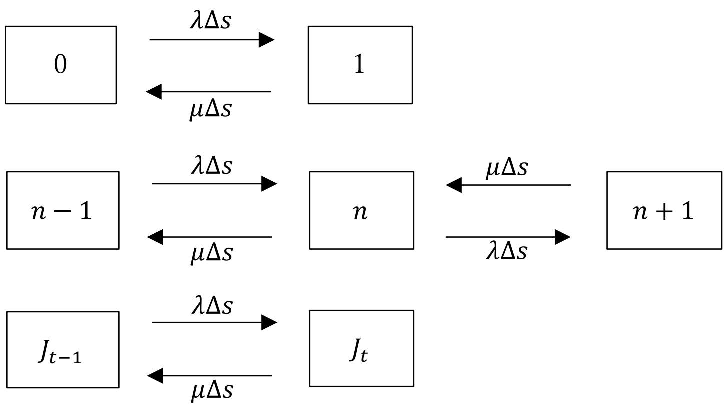

Figure 2. Transition diagram of the system

Using the queueing theory, P_n can be derived. We assume that workers join the firm in a Poisson process at a constant $\lambda > 0$—the average number of workers joining the firm per unit of time. We also assume that workers leave the firm in a Poisson process at a constant $\mu > 0$—the average number of workers leaving the firm per unit of time. In this case, the first worker to join the firm is not necessarily the first to leave. Suppose that a unit of time is T; then, λT

employment contracts are signed within T. Furthermore, T is divided into N; then, the probability of joining the firm is $\lambda T/N$. Replacing T/N with a time interval Δs, the probability of increasing the number of employees by one is expressed as $\lambda\Delta s$. We assume that the interval is sufficiently small and that employment cannot be increased by more than one worker. Similarly, suppose that a unit of time T is divided into N, and μT employment contracts are cancelled within T. Then, the probability of leaving the firm is $\mu T/N$. Thus, the probability of decreasing the number of workers by one is denoted by $\mu\Delta s$.

Figure 2 shows the change patterns in the number of employees. The number of employees is indicated in squares, and the arrows indicate increases or decreases in the number of employees. When the system is in a steady state, we derive P_n by considering the following cases: (i) $n = 0$, (ii) $0 < n < J_t$, and (iii) $n = J_t$. The upper transition in Figure 2 corresponds to case (i). We obtain $\lambda\Delta sP_0 = \mu\Delta sP_1$ if $n = 0$, where $\lambda\Delta sP_0$ is the probability of increasing the number of employees from $n = 0$ and $\mu\Delta sP_1$ is the probability of decreasing the number of employees from $n = 1$. The equation is transformed as follows:

$$P_1 = \theta P_0, \tag{2}$$

where $\theta = \lambda/\mu$, which is called traffic intensity in the queueing theory. Traffic intensity measures the relative imbalance between the inflow and outflow of workers. The middle transition in Figure 2 corresponds to case (ii). We obtain $\lambda\Delta sP_{n-1} + \mu\Delta sP_{n+1} = (\lambda\Delta s + \mu\Delta s)P_n$ if $0 < n < J_t$. In this expression, $\lambda\Delta sP_{n-1}$ is the probability of increasing from $n-1$, $\mu\Delta sP_{n+1}$ is the probability of decreasing from $n+1$, and $(\lambda\Delta s + \mu\Delta s)P_n$ is the probability of increasing and decreasing from n. The equation is transformed as follows:

$$P_{n+1} = (1+\theta)P_n - \theta P_{n-1}. \tag{3}$$

The lower transition in Figure 2 corresponds to case (iii).We obtain $\mu\Delta sP_{J_t} = \lambda\Delta sP_{J_t-1}$ if $n = J_t$, where $\mu\Delta sP_{J_t}$ is the probability of decreasing from J_t and $\lambda\Delta sP_{J_t-1}$ is the probability of increasing from $J_t - 1$. The equation is transformed as follows:

$$P_{J_t} = \theta P_{J_t-1}. \tag{4}$$

We derive the probability of n by using Eqs. (2)–(4). First, P_n can be expressed as a function of P_0. By substituting Eq. (2) into Eq. (3) with $n = 1$ to eliminate P_1, we obtain $P_2 = \theta^2 P_0$. By substituting $P_2 = \theta^2 P_0$ into Eq. (3) with $n = 2$ to eliminate P_2, we obtain $P_3 = \theta^3 P_0$. Similarly, we express P_n as follows:

$$P_n = \theta^n P_0. \tag{5}$$

In Eq. (5), $P_{J_t-1} = \theta^{J_t-1}P_0$ when $n = J_t - 1$. Using $P_{J_t-1} = \theta^{J_t-1}P_0$ to eliminate P_{J_t-1} from Eq. (4), we obtain $P_{J_t} = \theta^{J_t}P_0$. Therefore, Eq. (5) holds in $0 < n \le J_t$.

Second, P_0 can be derived as a function of J_t. It holds that the probabilities sum to one $\sum_{n=0}^{J_t} P_n = 1$. By substituting Eq. (5) into $\sum_{n=0}^{J_t} P_n = 1$, we obtain $P_0 \sum_{n=0}^{J_t} \theta^n = 1$. From $\sum_{n=0}^{J_t} \theta^n = (1-\theta^{J_t+1})/(1-\theta)$, $P_0 \sum_{n=0}^{J_t} \theta^n = 1$ is transformed as follows:

$$P_0 = \frac{1-\theta}{1-\theta^{J_t+1}}. \tag{6}$$

Finally, P_n can be expressed as a function of J_t. Using Eq. (6) to eliminate P_0 from Eq. (5), we obtain $P_n = \theta^n(1-\theta)/(1-\theta^{J_t+1})$. As θ approaches 1, both $\theta^n(1-\theta)$ and $1-\theta^{J_t+1}$ approach 0. By L'Hôpital's rule, we obtain the following equations:

$$\lim_{\theta \to 1} \frac{\theta^n(1-\theta)}{1-\theta^{J_t+1}} = \frac{1}{J_t+1}.$$

We obtain the probability of n as follows:

$$P_n = \begin{cases} \frac{\theta^n(1-\theta)}{1-\theta^{J_t+1}} & \text{for} \quad \theta \neq 1, \\ \frac{1}{J_t+1} & \text{for} \quad \theta = 1, \end{cases} \tag{7}$$

where $0 \leq n \leq J_t$.

By substituting Eq. (7) into Eq. (1), we obtain the average employment as follows:

$$L_t = \begin{cases} \frac{\theta\left[1-(J_t+1)\theta^{J_t}+J_t\theta^{J_t+1}\right]}{(1-\theta)(1-\theta^{J_t+1})} & \text{for} \quad \theta \neq 1, \\ J_t/2 & \text{for} \quad \theta = 1. \end{cases} \tag{8}$$

Eq. (8) expresses the relationship between job offers and average employment, as shown in Figure 3. This indicates that a large number of job offers leads to increased employment. The θ increases when more workers join the firm and/or fewer workers leave the firm, which is the case with larger λ and/or smaller μ. At the same level of job offers, the larger θ brings about larger employment. In sections 3 and 4, we introduce the aforementioned relationship into the models and analyze the behavior of job offers in response to a productivity shock.

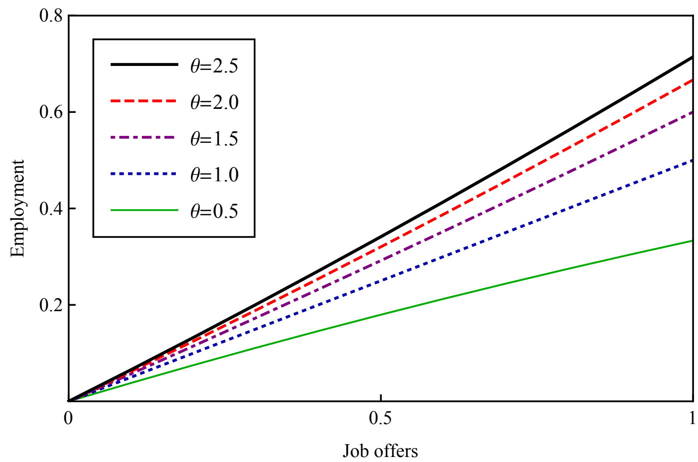

Figure 3. Relationship between job offers and average employment

In matching models, the Cobb–Douglas matching function is assumed, such as in Leduc and

Liu (2016), Wesselbaum (2011), and Zanetti (2019). The number of matches is a function of unemployed workers and job vacancies. The relationship between job offers and average employment in this study is similar to the matching function in that employment increases as labor demand increases.

3. A DGE model without unemployment

In this section, we build a DGE model with a queueing system in the labor market. The economy consists of a representative household and firm. We also conduct a numerical analysis to investigate economic fluctuations and changes in job offers resulting from the productivity shock.

3.1 Model

The representative household maximizes the following utility function, which is the same as that assumed in Blanchard and Galí (2010):

$$\sum_{t=0}^{\infty} \beta^t \left[logC_t - \chi \frac{L_t^{1+\nu}}{1+\nu} \right],$$

where $0 < \beta < 1$ is the discount factor, χ is the disutility from working, $1/\nu$ is the Frisch elasticity of labor supply, C_t is consumption, and L_t is labor supply.

The household's budget constraint is as follows:

$$C_t + I_t = R_t K_t + w_t L_t, \tag{9}$$

where I_t is investment, R_t is the rental rate of capital, K_t is capital, and w_t is the wage rate. The law of motion for capital stock is as follows:

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{10}$$

where $0 < \delta < 1$ denotes the depreciation rate of capital. From Eqs. (9) and (10), the constraint can be expressed as follows:

$$K_{t+1} = (R_t + 1 - \delta)K_t + w_t L_t - C_t. \tag{11}$$

The household maximizes its utility subject to Eq. (11). At the beginning of the first period, K_0 is given. The first-order conditions with respect to C_t, L_t, and K_{t+1} are as follows:

$$\frac{1}{C_t} = \Lambda_t, \tag{12}$$

$$\chi L_t^{\nu} = \Lambda_t w_t, \tag{13}$$

$$1 = \beta \frac{\Lambda_{t+1}}{\Lambda_t}(R_{t+1} + 1 - \delta), \tag{14}$$

where Λ_t is the Lagrange multiplier. We impose the following transversality condition:

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1} = 0.$$

From Eqs. (12) and (14), we obtain the following Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta(R_{t+1} - \delta + 1). \tag{15}$$

Using Eqs. (12) and (13), we obtain the following labor supply equation:

$$\frac{w_t}{C_t} = \chi L_t^{\nu}. \tag{16}$$

The representative firm produces Y_t according to the following Cobb–Douglas production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{17}$$

where $0 < \alpha < 1$ is the capital share in production, and A_t is productivity. We assume that productivity follows a first-order autoregressive process:

$$logA_t = \rho logA_{t-1} + \varepsilon_t, \tag{18}$$

where $-1 < \rho < 1$ is the autoregressive parameter, and ε_t is the shock to productivity, which is set to 0 in the steady state. The business cycle literature assumes the same exogenous law of motion of productivity. As discussed in Eq. (8), we assume that L_t is a function of the job offers.

The firm chooses K_t and J_t to maximize the following profit:

$$\pi_t = A_t K_t^{\alpha}[L(J_t)]^{1-\alpha} - R_t K_t - w_t L(J_t).$$

The first-order conditions for profit maximization are as follows:

$$R_t = \alpha \frac{Y_t}{K_t}, \tag{19}$$

$$w_t = (1-\alpha)\frac{Y_t}{L(J_t)}. \tag{20}$$

The equilibrium in the goods market is as follows:

$$C_t + I_t = Y_t. \tag{21}$$

Eqs. (8), (10), and (15)–(21) describe the behavior of the variables Y_t, C_t, I_t, K_t, L_t, J_t, R_t, w_t, and A_t.

3.2 Numerical analysis in the model

Table 1 lists the parameters. The discount factor β is set to 0.99, which is commonly used in the literature. Following Blanchard and Galí (2010), the inverse Frisch elasticity of labor supply ν is set to 1.0. We suppose that the disutility of working χ is 1.0, as in Furlanetto (2011). The depreciation rate of capital δ is set to 0.025, which is in line with Chiarini and Piselli (2005), Lindé (2009), and Zanetti (2019). As in Collard and de la Croix (2000), the capital share in production α is set to 0.36. The persistence of the shock in productivity ρ is set to 0.95, following Lendvai et al. (2013). Suppose that $\varepsilon_t = 0$ and $A_t = A_{t-1} = A$ in the steady state. Then, we obtain the steady-state productivity $A = 1$ from Eq. (18). We assume that a positive temporary shock to productivity occurs: Productivity increases by 1% in period 0, that is, ε_0 is set to 0.01, and the ε_t in the other periods are 0. Suppose that θ is set to 1.0 in the baseline simulation. In the sensitivity analysis, we also examine the cases where θ is 0.6, 0.8, and 1.2. Appendix A presents the steady-state values.

Table 1. Parameters in the standard dynamic general equilibrium model

Parameters		Value
α	Capital share in production	0.36
β	Discount factor	0.99
δ	Depreciation rate of capital	0.025
θ	Ratio between the inflow and outflow of workers	1.0
ρ	Autoregressive parameter	0.95
ν	Inverse Frisch elasticity of labor supply	1.0
χ	Disutility from working	1.0

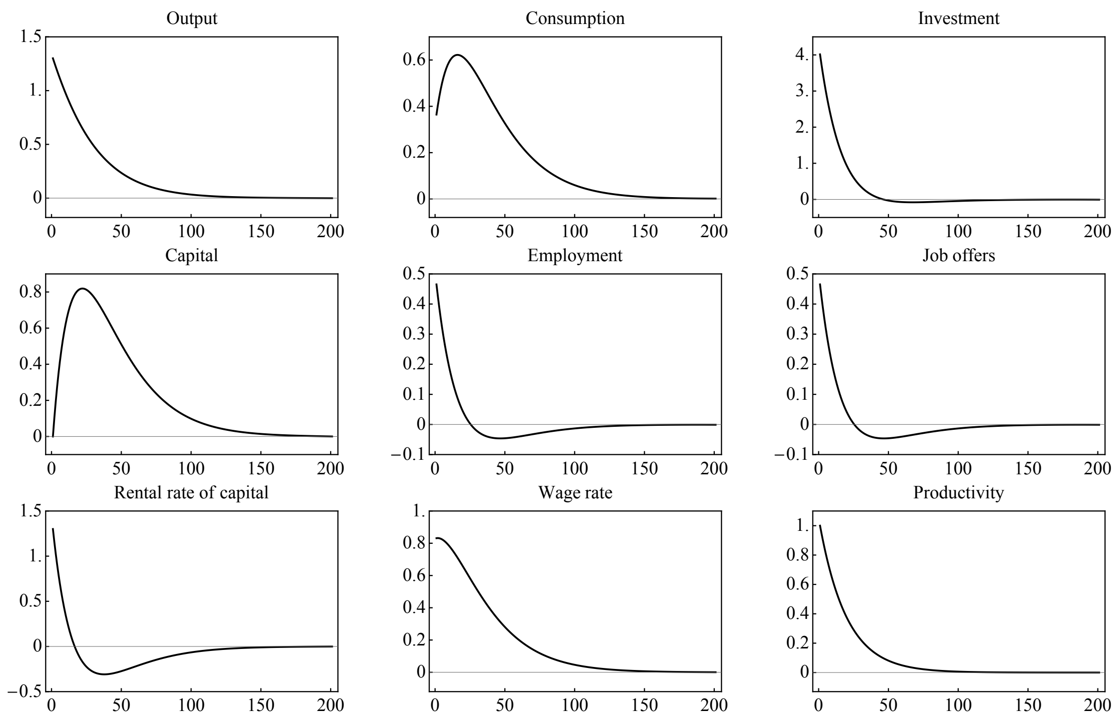

Figure 4. Response to the productivity shock in the standard dynamic general equilibrium model
Note: The horizontal and vertical axes represent time and percentage, respectively. The figure shows the percentage deviation of the variables from their steady-state values when the shock occurs.

Figure 4 shows the responses to the productivity shock. Productivity increases in period 0 and gradually returns to the steady state according to the first-order autoregressive process. The results of the numerical simulations are in line with the results of a standard DGE model. Specifically, the results show that a positive temporary shock to productivity increases the output, consumption, investment, capital, employment, rental rate of capital, and wage rate.

The positive productivity shock increases the marginal products of capital and labor, which increases capital and employment and thereby, the output. To increase the labor input, a firm increases its job offers. The increase in demand for these inputs leads to an increase in the rental rate of capital and wage rate. Additionally, an increase in income leads to an increase in consumption and investment. When $\theta = 1$, the same changes in job offers and employment are observed.

Figure 5 shows the response of job offers to the productivity shock. The numerical simulations show that a smaller θ amplifies the change in job offers in response to the productivity shock. Specifically, the response of job offers in period 0 is approximately 0.72%, 0.51%, 0.47%, and 0.45% if θ is 0.6, 0.8, 1.0, and 1.2, respectively. When the ratio is small, fewer workers join and/or more workers leave the firm. In this situation, firms should increase their job offers to hire the necessary number of workers in response to the productivity shock. In this setting, the reactions of the other variables are not affected by the change in θ. Therefore, substantial changes in job offers in response to the shock do not necessarily increase employment fluctuations. Firms increase job offers not only to employ a large number of workers but also to deal with the situation of a small θ. Moreover, changes in job offers are larger than those in employment when $\theta < 1$, whereas changes in job offers are smaller when $\theta > 1$.

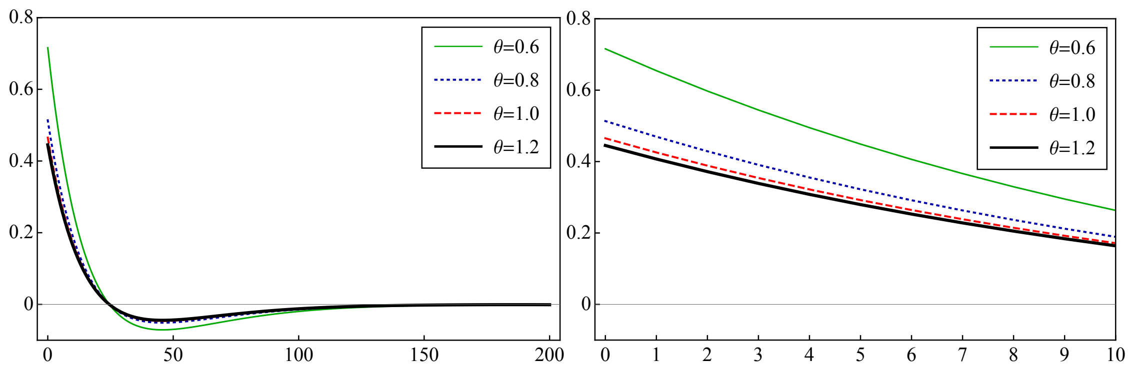

Figure 5. Response of job offers in the standard dynamic general equilibrium model

Note: The horizontal and vertical axes represent time and percentage, respectively. The figure shows the percentage deviation of job offers from the steady-state value when the shock occurs. The left- and right-hand sides of the figure show the periods from 0 to 200 and 0 to 10, respectively.

4. A DGE model with unemployment

In this section, we analyze the efficiency wage model with the queueing system in the labor market. This framework is based on the gift exchange model by Collard and de la Croix (2000), which is extended to investigate the change in job offers. As in section 3, the economy consists of a representative household and firm. We explore the reaction of the DGE model to a positive temporary productivity shock.

4.1 Efficiency wage model

The representative household maximizes the following utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[logC_t - d_t \left(e_t - \phi - \gamma log\left(\frac{w_t}{w_t^a}\right) - \psi log\left(\frac{w_t}{w_{t-1}}\right) \right)^2 \right],$$

where $0 < \beta < 1$ is the discount factor, C_t is consumption, e_t is effort; w_t is the wage rate, and w_t^a is an alternative wage rate; the constant parameter ϕ denotes the effort level that the household is willing to provide, the constant parameter γ expresses the sensitivity of the effort to the ratio of the wage to the alternative wage, and the constant parameter ψ is the sensitivity of the effort to the ratio of the wage to the previous wage. We assume that d_t is a dummy variable: $d_t = 1$ if the worker is employed, and $d_t = 0$ otherwise. If we assume $\gamma = 0$, then the employment in the steady state cannot be expressed parametrically. If we assume $\psi = 0$, then the employment will remain constant over the business cycle. Tripier (2006) also assumes the same type of effort function.

The household's budget constraint is as follows:

$$C_t + I_t = R_t K_t + w_t L_t, \tag{22}$$

where I_t is investment, R_t is the rental rate of capital, K_t is capital, and L_t is labor supply. The law of motion for capital stock is as follows:

$$K_{t+1} = (1 - \delta) K_t + I_t, \tag{23}$$

where $0 < \delta < 1$ denotes the depreciation rate of capital. From Eqs. (22) and (23), we express the constraint as follows:

$$K_{t+1} = (R_t + 1 - \delta) K_t + w_t L_t - C_t. \tag{24}$$

The household maximizes its utility subject to Eq. (24). It is assumed that K_0 is given. The first-order conditions with respect to C_t, e_t, and K_{t+1} are as follows:

$$\frac{1}{C_t} = \Lambda_t, \tag{25}$$

$$e_t = \phi + \gamma log\left(\frac{w_t}{w_t^a}\right) + \psi log\left(\frac{w_t}{w_{t-1}}\right), \tag{26}$$

$$1 = \beta \frac{\Lambda_{t+1}}{\Lambda_t} (R_{t+1} + 1 - \delta), \tag{27}$$

where Λ_t is the Lagrange multiplier. We impose the following transversality condition:

$$\lim_{t\to\infty} \beta^t \Lambda_t K_{t+1} = 0.$$

From Eqs. (25) and (27), we obtain the following Euler equation:

$$\frac{C_{t+1}}{C_t} = \beta(R_{t+1} + 1 - \delta). \tag{28}$$

The representative firm produces Y_t, according to the following Cobb–Douglas production function:

$$Y_t = A_t K_t^{\alpha} (e_t L_t)^{1-\alpha}, \tag{29}$$

where $0 < \alpha < 1$ is the capital share in production, and A_t is productivity. We suppose that productivity follows a first-order autoregressive process:

$$logA_t = \rho logA_{t-1} + \varepsilon_t, \tag{30}$$

where $-1 < \rho < 1$ is the autoregressive parameter, and ε_t is the shock to productivity. As discussed in Eq. (8), we assume that L_t is a function of J_t.

The firm's profit is expressed as follows:

$$\pi_t = A_t K_t^{\alpha} [e_t L(J_t)]^{1-\alpha} - R_t K_t - w_t L(J_t).$$

The firm chooses K_t, w_t, and J_t to maximize profit, subject to Eq. (26):

$$R_t = \alpha \frac{Y_t}{K_t}, \tag{31}$$

$$L(J_t) = (1-\alpha)\frac{Y_t}{e_t}\left(\frac{\gamma+\psi}{w_t}\right), \tag{32}$$

$$w_t = (1-\alpha)\frac{Y_t}{L(J_t)}. \tag{33}$$

Using Eq. (32) to eliminate $(1-\alpha)\, Y_t/[w_t L(J_t)]$ from Eq. (33), we obtain the effort level as follows:

$$e_t = \gamma + \psi. \tag{34}$$

The effort level depends only on the constant parameters. From Eqs. (26) and (34), the firm controls the wage rate such that the effort is constant over time. Eq. (34) corresponds to a transformation of the Solow condition in which the elasticity of effort with respect to the wage rate equals 1 as $e(w_t) = e'(w_t)w_t$.

We assume that the unemployment rate is $1 - L_t$. The alternative wage is assumed as follows:

$$w_t^a = w_t L_t, \tag{35}$$

which is the average labor income. We assume that the unemployment compensation is zero. de la Croix et al. (2009) and Kaufman (2002) also discuss the alternative wage.

The equilibrium in the goods market is as follows:

$$C_t + I_t = Y_t. \tag{36}$$

The model consists of the variables Y_t, C_t, I_t, K_t, L_t, J_t, R_t, w_t, w_t^a, e_t, and A_t. The system of equations defining the model consists of Eqs. (8), (23), (26), (28)–(31), and (33)–(36).

4.2 Numerical analysis in the efficiency wage model

According to Collard and de la Croix (2000), we set the parameters, as displayed in Table 2. The parameter ϕ is set such that the steady-state value of employment L is 0.9. From Eqs. (26), (34), and (35) in the steady state, we obtain $\phi = \gamma(logL + 1) + \psi$. Suppose that θ is set to 1.0 in the baseline simulation. In the sensitivity analysis, we also examine the cases where θ is 0.6, 0.8, and 1.2. Suppose that $\varepsilon_t = 0$ and $A_t = A_{t-1} = A$ in the steady state. Then, the steady-state productivity $A = 1$ is obtained from Eq. (30). We assume that a positive temporary shock to productivity occurs: Productivity increases by 1% in period 0, that is, ε_0 is set to 0.01, and the ε_t in the other periods are 0. Appendix B presents the steady-state values.

Table 2. Parameters in the efficiency wage model

Parameter		Value
α	Capital share in production	0.36
β	Discount factor	0.99
δ	Depreciation rate of capital	0.025
θ	Ratio between the inflow and outflow of workers	1.0
ρ	Autoregressive parameter	0.95
ϕ	Effort level that the household is willing to provide	3.60518
γ	Sensitivity of the effort to the ratio of the wage to the alternative wage	0.9
ψ	Sensitivity of the effort to the ratio of the wage to the previous wage	2.8

Figure 6 shows the responses to the productivity shock. The direction of changes in variables in response to the shock is similar to the reaction of the standard DGE model discussed in section 3. The productivity increases in period 0 and gradually returns to the steady state, according to Eq. (30). The marginal products of capital and labor increase as a result of the positive productivity shock. The increase in the demand for capital and labor increases capital and employment, which increases the output. To increase the labor input, the firm increases its job offers. When $\theta = 1,$ the same changes in job offers and employment are observed. The increase in demand for capital leads to an increase in the rental rate of capital. Furthermore, the wage rate increases because the firm controls the wage rate such that the effort remains constant. When the firm does not increase the wage rate, the effort decreases by an increase in the alternative wage, as shown in Eqs. (26) and (35). An increase in income increases consumption and investments.

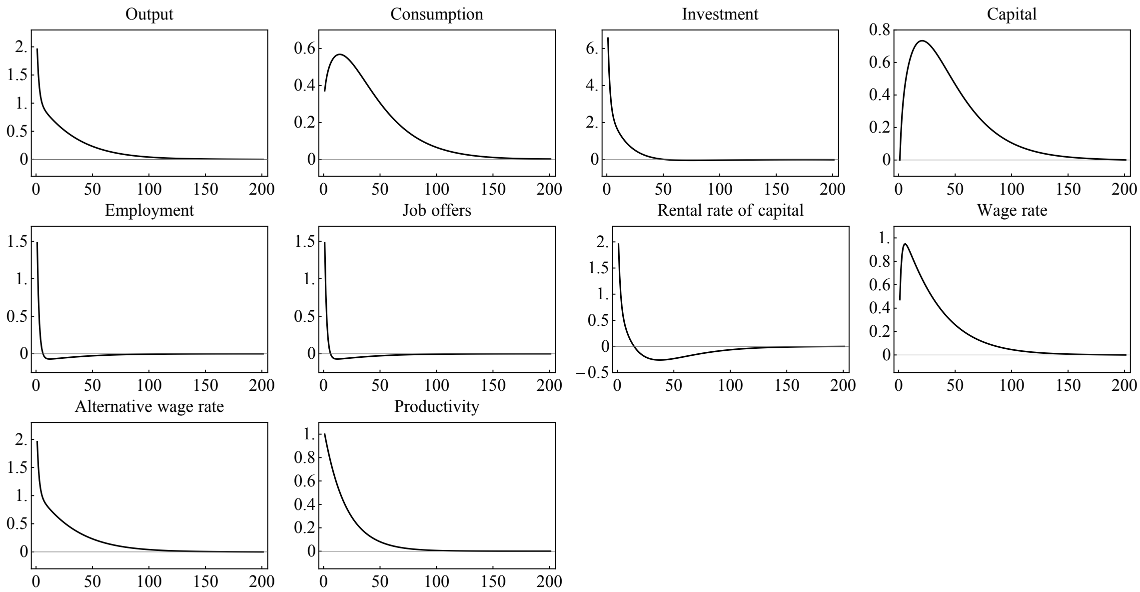

Figure 6. Response to the productivity shock in the efficiency wage model

Note: The horizontal and vertical axes represent time and percentage, respectively. The solid lines represent the percentage deviations of the variables from their steady-state values when the shock occurs.

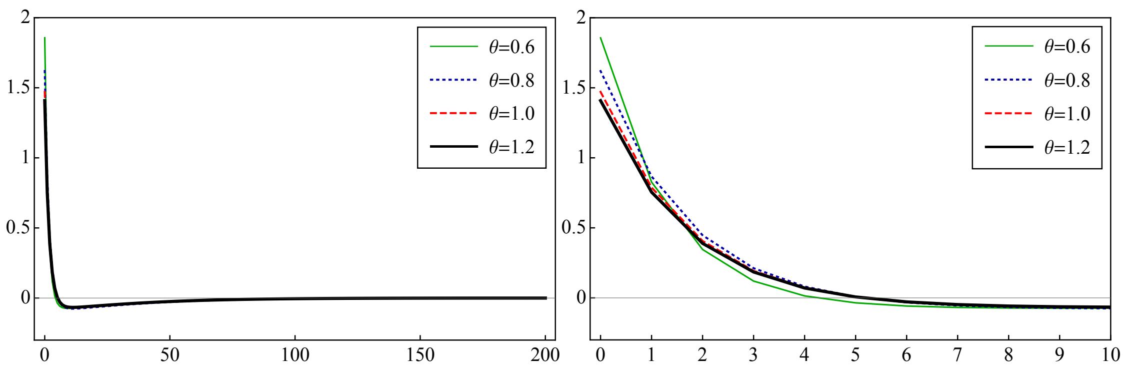

Figure 7. Response of job offers in the efficiency wage model

Note: The horizontal and vertical axes represent time and percentage change, respectively. The figure shows the percentage deviation of job offers from the steady-state value when the shock occurs. The left- and right-hand sides of the figure show the periods from 0 to 200 and 0 to 10, respectively.

Figure 7 shows the reaction of job offers to the productivity shock. The numerical simulation indicates that a smaller θ increases the change in job offers to the productivity shock: The response of job offers in period 0 is approximately 1.86%, 1.62%, 1.47%, and 1.41% if θ is 0.6, 0.8, 1.0, and 1.2, respectively. Similar to the analysis in section 3, in this setting, the change in θ

has no effect on the other variables. The small θ is caused by fewer workers joining the firm and/or more workers leaving the firm, which is the case with a smaller λ and/or a larger μ. The firms should increase their job offers to employ the necessary number of workers in response to the productivity shock in this situation. The simulation shows that a substantial change in job offers in response to the shock does not necessarily lead to significant employment fluctuations. The results also show that changes in job offers are larger than those in employment when $\theta < 1$, whereas changes in job offers are smaller when $\theta > 1$.

The simulations indicate that we should examine both the change in job offers and the worker flows when discussing economic policies. If we observe only the change in job offers, even though θ is small, then we would anticipate that a significant change in job offers leads to significant employment fluctuations. If so, the economic policy may be inadequate in terms of reducing the unemployment rate.

Although the increase in productivity brings about an increase in employment in this study, Mandelman and Zanetti (2014) demonstrate the negative response of labor input to a positive productivity shock because of an increase in hiring costs through an increase in productivity. Moreover, Mumtaz and Zanetti (2016) indicate that a positive productivity shock makes the production of greater output possible with fewer labor inputs, which reduces employment. These studies' findings are in line with the estimation results in Galí (1999). The models in sections 3 and 4 should be extended to demonstrate the employment responses.

5. Conclusions

The introduction of the relationship between job offers and employment, according to the queueing system, into the macroeconomic models allows for an analysis of changes in job offers. The relationship indicates that job offers and worker flows influence employment. The models predict that changes in job offers in response to the productivity shock depend on the ratio between the inflow and outflow of workers. A small ratio between the inflow and outflow of workers gives rise to significant changes in job offers in response to the productivity shock. Moreover, the ratio of the inflow and outflow of workers influences only the reaction of job offers in response to the shock. Therefore, significant changes in job offers do not necessarily cause significant employment fluctuations.

The simulations indicate that both the change in job offers and workers' flows need to be examined when discussing economic policies. If we focus only on the significant changes in job offers, notwithstanding the small ratio between the inflow and outflow of workers, we may anticipate a significant change in job offers to lead to significant employment fluctuations. The economic policy may be inadequate in terms of employment policy in such a situation.

Some issues remain to be addressed. The ratio determining the worker flows λ and μ is assumed to be given. Such flows are influenced by household preferences, economic policies, and labor market institutions. Hence, it is crucial to examine their underlying decision mechanism. Although a positive productivity shock increases employment in this study, some studies demonstrate a negative response of labor input to a positive productivity shock. Given this, it is important to study the negative response of employment to a positive productivity shock.

Future studies can extend these models to study heterogeneous households and firms to demonstrate various changes in job offers and employment. By extending these models, the effects of change in labor market institutions (e.g., unemployment benefits and labor adjustment costs) on employment fluctuations can be analyzed. Future studies can also examine the impact of economic policies on job offers.

Appendix A: Steady state in the standard dynamic general equilibrium model

In the steady state, from Eqs. (8), (10), and (15)–(21), we obtain the following:

$$L = \begin{cases} \frac{\theta[1-(J+1)\theta^{J}+J\theta^{J+1}]}{(1-\theta)(1-\theta^{J+1})} & \text{for} \quad \theta \neq 1, \\ J/2 & \text{for} \quad \theta = 1, \end{cases} \tag{A1}$$

$$I = \delta K, \tag{A2}$$

$$1 = \beta(R - \delta + 1), \tag{A3}$$

$$\frac{w}{C} = \chi L^{\nu}, \tag{A4}$$

$$Y = AK^{\alpha}L^{1-\alpha}, \tag{A5}$$

$$logA = \rho logA, \tag{A6}$$

$$R = \alpha\frac{Y}{K}, \tag{A7}$$

$$w = (1-\alpha)\frac{Y}{L}, \tag{A8}$$

$$C + I = Y. \tag{A9}$$

Eqs. (A1)–(A9) show the steady-state values of the variables without subscripts. From Eq. (A6), we obtain $A = 1$. From Eq. (A3), we obtain R as follows:

$$R = \beta^{-1} + \delta - 1. \tag{A10}$$

Using Eqs. (A5), (A10), and $A = 1$, we transform Eq. (A7) as follows:

$$\frac{K}{L} = \left(\frac{\beta^{-1}+\delta-1}{\alpha}\right)^{\frac{-1}{1-\alpha}}. \tag{A11}$$

Using Eq. (A5) and $A = 1$, we transform Eq. (A8) as follows:

$$w = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}. \tag{A12}$$

By substituting Eq. (A11) into Eq. (A12) to eliminate K/L, we obtain w. Using $A = 1$, Eq. (A5) can be transformed as follows:

$$\frac{Y}{L} = \left(\frac{K}{L}\right)^{\alpha}. \tag{A13}$$

Using Eq. (A2) to eliminate I from Eq. (A9) and multiplying $1/L$, we obtain the following:

$$\frac{C}{L} = \frac{Y}{L} - \delta\frac{K}{L}. \tag{A14}$$

Using Eqs. (A11) and (A13) to eliminate K/L and Y/L from Eq. (A14), we obtain the following:

$$\frac{C}{L} = \left(\frac{\beta^{-1}+\delta-1}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}} - \delta\left(\frac{\beta^{-1}+\delta-1}{\alpha}\right)^{\frac{-1}{1-\alpha}}. \tag{A15}$$

Multiplying both sides of Eq. (A4) by L, we obtain the following equation:

$$L = \left(\frac{w}{\chi}\right)^{\frac{1}{1+\nu}}\left(\frac{C}{L}\right)^{\frac{-1}{1+\nu}}. \tag{A16}$$

From Eqs. (A11), (A12), (A15), and (A16), we obtain L. By substituting L into Eq. (A1), we obtain J. By substituting L into Eqs. (A11) and (A15), we obtain K and C, respectively. By substituting K into Eq. (A2), we obtain I. By substituting K and L into Eq. (A5), we obtain Y.

Appendix B: Steady state in the efficiency wage model

In the steady state, from Eqs. (8), (23), (26), (28)–(31), and (33)–(36), we obtain the following:

$$L = \begin{cases} \frac{\theta[1-(J+1)\theta^{J}+J\theta^{J+1}]}{(1-\theta)(1-\theta^{J+1})} & \text{for} \quad \theta \neq 1, \\ J/2 & \text{for} \quad \theta = 1, \end{cases} \tag{B1}$$

$$I = \delta K, \tag{B2}$$

$$e = \phi + \gamma log\left(\frac{w}{w^{a}}\right), \tag{B3}$$

$$1 = \beta(R + 1 - \delta), \tag{B4}$$

$$Y = AK^{\alpha}(eL)^{1-\alpha}, \tag{B5}$$

$$logA = \rho logA, \tag{B6}$$

$$R = \alpha\frac{Y}{K}, \tag{B7}$$

$$w = (1-\alpha)\frac{Y}{L}, \tag{B8}$$

$$e = \gamma + \psi, \tag{B9}$$

$$w^{a} = wL, \tag{B10}$$

$$C + I = Y. \tag{B11}$$

Eqs. (B1)–(B11) show the steady-state values of the variables without subscripts. From Eq. (B6), we obtain $A = 1$. By substituting Eqs. (B9) and (B10) into Eq. (B3) to eliminate e and w^a, we obtain $logL = (\phi - \psi)/\gamma - 1$. From $exp(logL) = L$, we transform $logL = (\phi - \psi)/\gamma - 1$ as follows:

$$L = exp\left(\frac{\phi-\psi}{\gamma} - 1\right). \tag{B12}$$

Using Eq. (B12) to eliminate L from Eq. (B1), we obtain J. From Eq. (B4), we obtain R as follows:

$$R = \beta^{-1} + \delta - 1. \tag{B13}$$

Using Eqs. (B7), (B9), and $A = 1$ to eliminate Y/K, e, and A from Eq. (B5), we obtain the following:

$$\frac{K}{L} = \left[\frac{R}{\alpha(\gamma+\psi)^{1-\alpha}}\right]^{\frac{-1}{1-\alpha}}. \tag{B14}$$

By substituting Eq. (B13) into Eq. (B14), we obtain the following:

$$\frac{K}{L} = \left[\frac{\beta^{-1}+\delta-1}{\alpha(\gamma+\psi)^{1-\alpha}}\right]^{\frac{-1}{1-\alpha}}. \tag{B15}$$

Using Eq. (B12) to eliminate L from Eq. (B15), we obtain K. By substituting K into Eq. (B2), we obtain I. By substituting Eq. (B9), Eq. (B12), $A = 1$, and K into Eq. (B5), we obtain Y. Using Eq. (B9) and $A = 1$ to eliminate e and A from Eq. (B5), we obtain the following:

$$\frac{Y}{L} = (\gamma + \psi)^{1-\alpha}\left(\frac{K}{L}\right)^{\alpha}. \tag{B16}$$

From Eqs. (B8), (B15), and (B16), we obtain w. By substituting Eq. (B12) and w into Eq. (B10), we obtain w^a. Using Eq. (B11) to eliminate I from Eq. (B2) and multiplying $1/L$, we obtain the following:

$$\frac{C}{L} = \frac{Y}{L} + \delta\frac{K}{L}. \tag{B17}$$

From Eqs. (B12) and (B15)–(B17), we obtain C.

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