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Abstract: This paper proposes a two-country AK model of growth with cross-country knowledge diffusion and endogenous migration to study the relationship between migration, income inequality and economic growth. In contrast with mainstream AK literature, we show that introducing knowledge diffusion from frontier to non-frontier countries makes AK models predict conditional convergence, with migration playing an important role in speeding up the catching-up process of non-frontier countries. When testing the robustness of the policy implications of the AK literature, we find that subsidizing capital accumulation in frontier countries stimulates migration and worldwide growth, but also that it increases cross-country inequalities in terms of both income and technology. On the contrary, subsidizing capital accumulation in non-frontier countries reduces migration and mitigates inequalities worldwide, but has no effects on the long-run pace of economic growth of the two countries.

JEL classification: E1, F1, O4

Keywords: Two-Country Model, Endogenous Growth, Labor Migration, Technology Transfer, Growth Policy

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1 Introduction

Modern macroeconomic literature has produced various dynamic macro-models to analyze the effect of migration on aggregate economic variables. Noticeable examples of these studies including two-country models are Lundborg and Segerstrom (2000, 2002), Bretscher (2001), Klein and Ventura (2007, 2009), Levine et al. (2010), Mandelman and Zlate (2012) and Ikhenaode and Parello (2020). This literature, which encompasses both models of exogenous and endogenous growth, usually does not model migration decisions, nor pay attention to the effects that spreading technical knowledge worldwide might have on both migration decisions and growth policies.

In this paper, we propose a two-country AK model of growth with cross-country knowledge diffusion and fully-endogenous migration to study the interplay between migration, per capita income inequality and economic growth. Through it, we try to answer the following questions: what is the effect of migration on the level of income per capita and the pace of economic growth of the sending and the receiving countries? Do these effects promote or prevent cross-country convergence in income per capita? Whereas migration resulted effective in generating a less unequal world economy, who is the winner and the loser of this process of convergence?

Testing for cross-country convergence in per capita income is one of the most debated issues in economics, especially during the 90s. Using growth regression techniques, Barro (1991) predicts a weak 'unconditional' divergence for the period 1960-1985, while Pritchett (1997) predicts strong income divergence between 1870 and 1990. However, when controlling for some specific correlates (e.g. investment/ratio, education, institutions, etc.), Barro and Sala-i-Martin (1991, 1992) and Mankiew et al. (1992) find evidence for conditional convergence, i.e. the tendency of poor countries to catch up with rich countries only if their economies share some structural parameters.

While the former two studies empirically support the diverging AK-like growth scheme *à la* Romer (1986) and Rebelo (1991), the latter three partially rehabilitate Neoclassical growth models with diminishing marginal returns to capital. However, recent contributions by Roy et al. (2016), Patel et al. (2021) and Kremer et al. (2021) have revisited the convergence debate of the 1990s by finding support for 'absolute' convergence since the 1980s on. In particular, Kremer et al. (2021) find evidence for a reduction in frontier-country growth and an increase in non-frontier country growth that is consistent with neoclassical growth models, and inconsistent with AK growth models.

In this paper, we will show that extending the AK framework to include cross-country learning-by-doing externalities and endogenous migration can re-establish the empirical validity of AK models. To do this, the paper develops a two-country world economy characterized by an asymmetric initial distribution in baseline technical knowledge, where one country behaves as the frontier country and the other country behaves as the non-frontier country. Moreover, to accommodate international mobility in labor and technology, paper assumes that (*i*) only workers from the non-frontier country can emigrate; (*ii*) the technology advances produced in the frontier country can spill over the non-frontier country in the form of cross-country productivity externality; (*iii*) emigration generates disutility.

In contrast with mainstream AK literature, this paper finds that introducing knowledge diffusion makes AK models predict 'conditional' convergence, thereby causing this class of endogenous growth models to be consistent with data. In our model, this finding occurs independently of the presence of migration and can be explained by the fact that knowledge diffusion lines up productivity levels across countries, thereby allowing the non-frontier country's income per capita to catch-up with that of the frontier country. However, when workers are allowed to move across countries, we find that migration contributes to bridge cross-country knowledge gaps and to speed up the catching-up process of non-frontier countries. This result is due to the positive 'scale' effect that migration is capable of generating in frontier countries, which causes non-frontier countries to grow temporarily faster than frontier countries.

Another related issue we would like to address in this paper is the robustness of the policy implications of the AK literature, when both technology and labor are no longer country-specific inputs. Indeed, whereas standard AK models predict that stimulating capital accumulation is helpful in increasing long-run growth, how can this result change if workers are allowed to move across countries?

As is well known, targeted government policies can produce permanent growth effects in AK models. For instance, in investment-based endogenous growth models à la Romer (1986) and Rebelo (1991), introducing capital accumulation subsidies results effective in raising economic growth and re-establishing first-best equilibria. In Barro's (1990) AK model with productive public spending, government can go even further by entering explicitly in the private production function of firms through the provision of nonrival and nonexcludable public services (e.g. property rights protection, defence, public infrastructure, etc.).

However, although all these policy results have been found to be robust by theoretical growth literature, can we establish that a pro-growth policy intervention undertaken in a frontier country has different effects than a policy undertaken in a non-frontier country when workers and technology can freely spread worldwide?

In an attempt to answer these questions, in the second part of the paper we calibrate the model to US and Mexican data and run three different simulation exercises consisting in: (*i*) introducing an investment subsidy in the frontier country; (*ii*) introducing an investment subsidy in the non-frontier country; (*iii*) introducing a pro-migration policy in the frontier country aiming at stretching the duration of the work permits of immigrants.

The main results of our simulations are the following. Introducing investment subsidies in frontier countries turns out to increase migration and stimulate growth worldwide, but also to increase cross-country inequalities both in terms of income and technology. On the contrary, introducing investment subsidies in non-frontier countries reduces migration and mitigates inequalities worldwide, but has no effects on the long-run pace of economic growth of the two countries. This result is due to the fact that, in our two-country AK model, what really matters for global growth is the pace with which frontier countries can generate new technical knowledge. And since only capital accumulation in frontier countries is able to improve productivity at a world level, subsidizing gross investment in non-frontier countries turns out to be ineffective to increase productivity, and then growth, on a global scale.

This paper contributes to several strands of economic literature. Firstly, it contributes to the literature on investment-based growth with learning-by-doing externalities, by providing a two-country extension of the Romer's (1986) and Rebelo's (1991) AK growth models to include migration. To the best of our knowledge, sofar only few papers have formally addressed the dynamic issues of immigration in an investment-based model of economic growth. In particular, the studies more closely related to this paper are Kemnitz (2001), which examines the growth and welfare implications of immigration through an investment-based endogenous growth model with external effects à la Romer (1986), and to Larramona and Sanso (2006), which presents a discrete-time two-country model to study how migration dynamics can affect economic growth and convergence in terms of both the capital/labor ratio and wages.

This paper differs from Kemnitz (2001) and Larramona and Sanso (2006) for the following features. First, whereas in Kemnitz (2001) the whole analytical structure is that of a closed-economy model with exogenous migration, our model focuses on a two-country framework with endogenous migration. Second, while Kemnitz (2001) and Larramona and Sanso (2006) set the focus of the paper on the linkages between migration and AK growth, our study extends the analysis to include cross-country knowledge diffusion and growth-enhancing policy. Third, differently from Larramona and Sanso (2006), who consider overlapping generations and focus on utility-driven migration decisions, this paper considers infinitely-lived agents who base their migration decision on the balance between cross-country wage gaps and the disutility generated by emigration.

Secondly, this paper is also related to the literature on migration and growth that plugs endogenous migration processes into two-countries endogenous growth models. Noticeable papers in this research stream are Urrutia (1998), Lundborg and Segerstrom (2000, 2002) and Mondal and Gupta (2008) and Ivus et al. (2022).

With respect to this branch of literature, our paper contributes as follows. First, whereas in this literature growth is mainly driven by R&D, in our study economic growth is the result of an unintended creation of technological knowledge due to learning by doing *à la* Arrow (1962). Second, while these models do not consider analyzing the effects of cross-country knowledge transmission, this paper distinguishes between the case in which technical knowledge can freely spread across countries from that in which it is strictly country specific and immobile. Such a distinction between mobile and immobile technical knowledge is particularly important in our study since it helps us to show to what extent migration is able to either reduce or expand income inequality across countries.

The outline of the paper is the following. Section 2 sets up the two-country growth framework and introduces the main economic relationships of the baseline model. It also characterizes the perfect-foresight general equilibrium of the model and determines the reduced-form dynamic system. Section 3 investigates about the role played by knowledge diffusion and migration to determine 'conditional' convergence. To this end, it compares the Balanced-Growth Path (BGP) equilibria of two different versions of the model: a reduced version of the benchmark model where only technical knowledge is allowed to move across countries; a full version of the benchmark model where both knowledge and workers are allowed to move across countries. Section 4 revisits the pro-growth policy predictions of the AK literature and analyzes the effects on convergence and growth of a pro-migration policy consisting in stretching the duration work permits in frontier countries. Finally, Section 5 concludes.

2 The model

2.1 Overview of the model

We consider an asymmetric world economy made up of two countries: a 'frontier' country (hereinafter "North"), denoted with the subscript "n", and a 'non-frontier' country (hereinafter "South"), denoted with the subscript "s". In each country, the total population consists of a continuum of dynastic households and production activities are carried out by perfectly competitive firms that assemble physical capital and labor services. Households supply labor inelastically and accumulate capital assets, whereas Governments levy taxes on labor income in order to finance subsidy policies to private investments. For simplicity, we abstract from money and other nominal assets, and assume that only individuals from the 'non-frontier' country find it convenient to migrate and work abroad.

2.2 Migration and households' consumption

In each country $i = \{n, s\}$, there is a fixed measure of identical households that provide labor services in exchange for wages. Each individual member of a household is infinitely lived and is endowed with one unit of labor, which is inelastically supplied to firms. The size of the representative household in North is equal to L_n, while that of the representative household in South is equal to L_s. Since only Southern workers are allowed to freely move across countries, at each time t there exists a stock M of Southern workers who currently resides and works abroad for Northern firms. This implies that the total supply of labor in South is given by $L_s - M$, while the total supply of labor in North is given by $L_n + M$.

The stock of immigrant labor is supposed to build gradually over time according to the following migration function

$$\dot{M} = L_e - \eta M, \ \ \eta \in [0, 1],$$

where L_e is the flow of Southern workers who decides to emigrates to North at time t and η is a given parameter capturing the probability that an exogenous return-inducing shock could induce immigrant workers to return to South. The latter parameter can be interpreted as the individual probability that a Southern worker could be forced to repatriate by Northern authorities based on legal reasons (Mandelman and Zlate, 2012).[1]

Denoting the immigrants-to-natives ratio of North (hereinafter, *immigration ratio*) by $m \equiv M/L_n$ and the ratio of new emigrates over the Southern workforce (hereinafter, *emigration ratio*) by letting $\mu \equiv L_e/L_s$, we can manipulate the previous differential equation to obtain

$$\dot{m} = \frac{\mu}{\ell_n} - \eta m, \tag{1}$$

where $\ell_n \equiv L_n/L_s$ is the relative size of the Northern workforce.

Each household in country i is modeled as a dynastic family that maximizes discounted lifetime utility. To simplify the model, we focus on stationary populations and suppose that in South emigrating generates disutility (see, e.g., Demurger, 2015; Ivlevs et al., 2019). In particular, we assume that the lifetime utility of an household residing in country i at time t is given by

$$\mathcal{U}_i = \int_0^{\infty} e^{-\rho t} \left(\log c_i - 1_{[i=s]} \frac{\chi \mu^{\gamma+1}}{\gamma+1} \right) \mathrm{d}t, \ \ \rho > 0, \ \ \chi > 0, \ \ \gamma \geq -1, \tag{2}$$

where c_i indicates the level of individual consumption expenditure of country i at time t, χ is a given parameter measuring how emigration generates disutility, γ is the inverse of the elasticity of migrant labor supply and $1_{[i=s]}$ is an indicator function attaining the value of 1 if the household resides in South and 0 otherwise.

The goal of the representative household in South is to maximize (2) subject to (1) and a country-specific budget constraint. In line with the literature on migration and remittances (see, e.g., Stark and Levhari, 1982; Stark, 1991; Yang and Choi, 2007; Mandelman and Zlate, 2012; Garip, 2014; Ikhenaode and Parello, 2020), we consider the presence of remittances as a form of a risk-sharing mechanism, such that all migrant workers care about the welfare of their own household and send home remittances in order to smooth consumption risks and support the welfare of household's members. Consequently, the flow budget constraint of the representative household of South can be described by the following dynamic equation

$$\dot{k}_s = r_s k_s + (1 - \tau_s)\, w_s \,(1 - m\ell_n) + (1 - \tau_n)\, w_m m \ell_n - c_s, \tag{3}$$

[1] In the remainder of the paper, we will treat η as the (exogenous) Poisson arrival rate of an event that might force an immigrant worker to quite her working position in North and repatriate to South. This, in turn, implies that the average duration of an employment position of immigrants equals $1/\eta$.

where r_s is the rate of return on Southern capital k_s, w_s and w_m are the wage rates received by each households' member employed in South, as native worker, and North, as immigrant worker, τ_i denotes the tax rate on labor income in country i and $\ell_n m = M/N_s$ is an alternate way to indicate the share of Southern workers currently employed in North at time t.

Similarly, in North the goal of the representative household is to maximize (2) subject to a country-specific budget constraint in the form

$$\dot{k}_n = r_n k_n + (1 - \tau_n) w_n - c_n, \tag{4}$$

where r_n is the rental rate of Northern capital k_n and w_n is the wage rate paid to northern workers.

Using Pontryagin's Maximum Principle, the solution to the two optimal control problems associated to each utility maximization problem is given by the following set of equations

$$\dot{c}_n = c_n (r_n - \rho) \tag{5}$$

$$\dot{c}_s = c_s (r_s - \rho) \tag{6}$$

$$\frac{\dot{\xi}}{\xi} + \frac{[(1 - \tau_n) w_m - (1 - \tau_s) w_s] \ell_n}{\xi c_s} = \rho + \eta \tag{7}$$

$$\mu = \left(\frac{\xi}{\chi \ell_n} \right)^{1/\gamma}, \tag{8}$$

where ξ is the shadow price of migration.

Static equation (8) establishes that the critical determinant of the optimal emigration ratio, μ, is the relative shadow price of migration, ξ, the path of which has to be determined through (7). Dynamic equations (5) and (6) are the well-known Euler conditions for consumption, according to which individual consumption expenditures c_n and c_s grow over time if and only if the market interest rates, r_n and r_s, exceed the subjective discount rate ρ. Finally, the differential equation (7) acts as a "pseudo" asset condition for emigration, according to which workers from South find it convenient to migrate if and only if the net wage rate paid to foreign workers in North, $(1 - \tau_n) w_m$, is large enough to compensate both the loss of the net Southern wage rate, $(1 - \tau_s) w_s$, and the disutility of emigrating.[2]

2.3 Technologies and production

In each country i, there is a continuum of identical competitive firms that produce a unique homogeneous commodity, Y_i, by combining physical capital and labor. We assume

[2]Notice that labor income taxation τ_i is indeed non completely non-distorsive, as it is able to alter the decision to emigrate to North.

that the shape of the production function is common across countries and that it takes the following Cobb-Douglas specification

$$Y_i = A_i K_i^{\alpha} N_i^{1-\alpha}, \;\; \alpha \in (0,1), \tag{9}$$

where A_i is an endogenous variable capturing the level of technology of country i at time t, and K_i and N_i are the capital stock and labor employment, the latter differing from the native populations because of migration. In particular, in South, N_s is given by only those native Southern workers who decide not to migrate to North, $L_s - M$, while in North, N_n consists of a mix of native and immigrant workers, where the contribution of each worker type to Northern production is captured by the CES aggregator $N_n \equiv \left[(1-\theta) L_n^{\lambda} + \theta M^{\lambda}\right]^{1/\lambda}$, where $\theta \in [0,1]$ is the share parameter and $\lambda < 1$ is the substitution parameter.

Using m and ℓ_n, each country-specific labor inputs can be written as

$$N_s \equiv (1 - m\ell_n) L_s \tag{10}$$

$$N_n \equiv \left(1 - \theta + \theta m^{1-1/\phi}\right)^{1/(1-1/\phi)} L_n, \tag{11}$$

where $\phi \equiv 1/(1-\lambda)$ is the elasticity of substitution between native and immigrant workers in North. Following Manacorda et al. (2012) and Ottaviano and Peri (2012), who find significant values for ϕ ranging from 7 to 20 (imperfect substitutability), in the remainder of the paper we will focus our analysis on the special case of $\phi > 1$, implying that there is some degree of substitutability in production between native and immigrant workers.

Since (9) is linearly homogeneous in capital and labor, we can use (10) and (11) to substitute for N_i in (9) to obtain

$$Y_s = A_s k_s^{\alpha} (1 - m\ell_n)^{1-\alpha} L_s \tag{12}$$

$$Y_n = A_n k_n^{\alpha} \left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)} L_n, \tag{13}$$

where $k_i \equiv K_i/L_i$ denotes the level of capital per native individual of country i at time t.

The goal of the representative firm in country i consists of maximizing profits subject to either (12) or (13). Let us suppose that the Government in each country i spurs private investments by introducing an investment subsidy $\sigma_i \in [0,1)$. Taking first-order conditions with respect to K_n and K_s yields the following pair of expressions for the rental rates of capital

$$r_s (1 - \sigma_s) = \alpha A_s k_s^{\alpha-1} (1 - m\ell_n)^{1-\alpha} \tag{14}$$

$$r_n (1 - \sigma_n) = \alpha A_n k_n^{\alpha-1} \left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}. \tag{15}$$

As is easy to see, changes in m have asymmetric impacts on r_s and r_n due to the different effect that migration has on the 'scale' of the workforce of each country. Indeed, an increase in m unambiguously increases the rental rate of North, while it decreases the rental rate of South. The opposite result instead holds for wages. To see this, it suffices to take the first-order conditions with respect to L_s, L_n and M to obtain

$$w_s = (1-\alpha) A_s k_s^{\alpha} (1 - m\ell_n)^{-\alpha} \tag{16}$$

$$w_n = (1-\alpha)(1-\theta) A_n k_n^{\alpha} \left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} \tag{17}$$

$$w_m = (1-\alpha)\theta A_n k_n^{\alpha} \left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi}. \tag{18}$$

For any given value of k_n and k_s, an increase in m increases the wage rates of native workers w_s and w_n, while it decreases that of immigrants, w_m.

2.4 The aggregate economy

In each country, technical knowledge A_i is assumed to depend upon capital accumulation. In particular, we postulate that each time that a firm increases its capital stock, the economy-wide stock of technical knowledge in North and South also increase, respectively according to

$$A_n = a_n k_n^{\psi_n}, \;\; a_n > 0 \tag{19}$$

$$A_s = a_s \varpi_s^{\psi_s}, \;\; a_s > 0, \;\; \varpi_s \equiv \left[(1-\omega) k_s^{1-1/\varphi} + \omega k_n^{1-1/\varphi}\right]^{\frac{1}{1-1/\varphi}}, \tag{20}$$

where a_i is a given parameter capturing the stock of baseline knowledge of the country i, ψ_i is a positive externality parameter and ϖ_s is an Hölder weighted power mean over stocks of technical knowledge, where $\omega \in [0,1)$ is a weight parameter capturing the contribution of the knowledge stock of North for the the formation of the productivity index A_s, and $\varphi \geq 0$ is an exogenous parameter that can used to generalize the productivity index to different types of means.[3]

In order to capture the different degree of development between countries, we throughout make the following assumptions. First, we suppose that both Southern and Northern capital are able to generate technological externalities in South according to (20), but Southern capital accumulation does not improve technological knowledge in North. Second, we assume that $a_n > a_s$, implying that the baseline knowledge of North is larger than that of South. Third, we follow Arrow (1962), Sheshinski (1967) and Romer (1986) in setting $\psi_i = 1-\alpha$, $(i = n, s)$. Substituting (19) into (13) and (20) into (12), it easy to show that aggregate (or social) technologies differ from private technologies and can be written as

$$y_n = a_n k_n \left(1 - \theta + \theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)} \tag{21}$$

[3]Generalized means, or Power mean or *Hölder* mean (after Otto Hölder), are a family of mean generating functions that can include all the most important means as special cases. For instance, for $\varphi = 0$, the *Hölder* mean gives the weighted geometric mean, while for $\varphi = 1$ it gives the weighted arithmetic mean.

$$y_s = a_s k_s \left[1-\omega+\omega\left(\frac{k_n}{k_s}\right)^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}} (1-m\ell_n)^{1-\alpha}, \tag{22}$$

where $y_n \equiv Y_n/L_n$ and $y_s \equiv Y_s/L_s$ are the level of output per native individual in North and South, respectively. Similarly, plugging (19) and (20) into (15) and (14), respectively, yields the following expressions for the rental rates on physical capital

$$r_n = \frac{\alpha a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}}{1-\sigma_n} \tag{23}$$

$$r_s = \frac{\alpha a_s \left[1-\omega+\omega\left(\frac{k_n}{k_s}\right)^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}} (1-m\ell_n)^{1-\alpha}}{1-\sigma_s}. \tag{24}$$

In contrast with standard AK models, in our two-country version with migration the aggregate level of the rental rates are no longer constant over time and turn out to depend on the 'scale' of each country's workforce. Indeed, from (23) and (24), it is easy to see that an increase in m *ceteris paribus* increases the rental rate on capital assets of North and decreases that of South through the two 'scale' components $1-\theta+\theta m^{1-1/\phi}$ and $1-m\ell_n$. This result echoes Parello (2022) and represents one of the main distinguishing features of this model with respect to standard AK models.

2.5 Wages and remittances

At each time t, the amount of remittances sent back by each migrant worker to South is equal to the difference between the wage rate that they earn in North, w_m, and what they consume, c_s. Hence, denoting the overall flow of remittances by R, it follows that the overall flow remittances received by the Southern economy at each t can be written as

$$R \equiv \left[(1-\tau_n)\, w_m - c_s\right] M. \tag{25}$$

To obtain the aggregate values of wages, we substitute (19) and (20) in (16)-(18) to obtain

$$w_s = (1-\alpha)\, a_s (1-m\ell_n)^{-\alpha} \left[1-\omega+\omega\left(\frac{k_n}{k_s}\right)^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}} k_s \tag{26}$$

$$w_n = (1-\alpha)(1-\theta)\, a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} k_n \tag{27}$$

$$w_m = (1-\alpha)\,\theta a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi} k_n. \tag{28}$$

Equations (26)-(28) establish that wages depend on the local stock of capital, k_s and k_n, and on the immigration ratio of North, m. Moreover, from (26)-(28) it is also possible to see that the sign of the 'scale' effect induced by an increase in m is positive for w_s and negative for w_m, while it turns out to have ambiguous effects for w_n.[4] Dividing (28) by (26), we can then obtain

$$\frac{w_m}{w_s} = \frac{\theta a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi} \left(\frac{k_n}{k_s}\right)}{a_s \left(1-m\ell_n\right)^{-\alpha} \left[1-\omega+\omega\left(\frac{k_n}{k_s}\right)^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}}}, \tag{29}$$

which establishes that, *ceteris paribus*, the level of the wage premium from migrating is increasing in the relative baseline knowledge of North, a_n/a_s, increasing in the relative capital stock of North, k_n/k_s, and ambiguous in the immigration ratio, m.

Finally, using (28) to substitute from w_m into (25) and then dividing the resulting expression by (22), it follows that the remittances-to-GDP ratio of South can be written as

$$\mathcal{R} = \frac{\left[(1-\tau_n)(1-\alpha)\theta a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi} \left(\frac{k_n}{k_s}\right) - \frac{c_s}{k_s}\right] m\ell_n}{a_s \left[1-\omega+\omega\left(\frac{k_n}{k_s}\right)^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}} \left(1-m\ell_n\right)^{1-\alpha}}, \tag{30}$$

where $\mathcal{R} \equiv R/Y_s$ denotes the overall value of remittances as a share of the Southern GDP.

Equations (27)-(30) concludes the description of our two-country AK growth model with migration. In the next section, we will characterize the dynamic equilibrium of the model and will study the dynamic properties of our two-country world economy.

2.6 Governments

Governments follow a balanced-budget rule. In North, equilibrium between revenues and outlays implies the following budget constraint

$$\tau_n \left(w_n + m w_m\right) = \sigma_n r_n k_n. \tag{31}$$

In South, equilibrium between revenues and expenses implies the following budget constraint

$$\tau_s \left(1-m\ell_n\right) w_s = \sigma_s r_s k_s. \tag{32}$$

For both equations (31) and (32), the left-hand side corresponds to the government revenues, whereas the right-hand side corresponds to the government expenditures. As governments conduct a zero profit policy, the income tax τ_i is assumed to endogenously adjust to balance the government budget, so that when a temporary deficit (surplus) takes place, the government reacts by raising (decreasing) τ_i.

[4] Indeed, differentiating (27) with respect to m, it is possible to establish that, everything equal, an increase in m either increases w_n if $\phi \in (1, 1/\alpha)$ or decreases w_n if $\phi > 1/\alpha$.

2.7 Characterization of the equilibrium

A dynamic equilibrium for the global economy consists of a set of allocations for the Northern households $\{c_n(t),\ k_n(t)\}_{t\in(0,\infty)}$, a set of allocations for the Southern households $\{c_s(t),\ k_s(t),\ m(t),\ \mu(t)\}_{t\in(0,\infty)}$, a triplet time paths for wage rates $\{w_n(t),\ w_m(t),$ $w_s(t)\}_{t\in(0,\infty)}$, and a pair time paths for rental rates $\{r_n(t),\ r_s(t)\}_{t\in(0,\infty)}$, such that: (i) households maximize discounted utility (2) subject to the migration function (1) and the two accumulation constraints (3) and (4); (ii) firms maximize profits subject to technology constraints (21) and (22); (iii) technical knowledge evolves over time according to (19) and (20); (iv) all markets clear.

Overall, the equilibrium system of the model consists of six differential equations governing the long-run dynamics of the aggregate economy and five static equations establishing rental and wage rates. The dynamic equations of the model are: the two Euler conditions for consumption (5) and (6); the two accumulation equations (3) and (4); the two laws of motion for the immigration and emigration ratios (1) and (7). The static equations of the model are: the two capital-market equilibrium conditions determining the equilibrium rental rates (23) and (24), the three labor-market conditions determining the equilibrium wage rates (26), (27) and (28), and the government budget constraints (31) and (32).

To solve the model, it is convenient to reduce the dynamic system of one dimension and focus on the following re-scaled variables: the consumption-to-capital ratio of North, $x_n \equiv c_n/k_n$, the consumption-to-capital ratio of South, $x_s \equiv c_s/k_s$, and the relative capital stock of North, $\kappa_n \equiv k_n/k_s$, the latter proxying for the technology distance separating the two countries involved in the process of migration.

Log-differentiating x_n, x_s, and κ_n with respect to time and then using (3)-(6), (23)-(24) and (26)-(28), we obtain

$$\frac{\dot{x}_n}{x_n} = x_n - \rho - (1-\tau_n)\,\hat{w}_n(m) \tag{33}$$

$$\frac{\dot{x}_s}{x_s} = x_s - \rho - (1-\tau_s)\,\hat{w}_s(m,\kappa_n)(1-m\ell_n) - (1-\tau_n)\,\hat{w}_m(m)\,\kappa_n m\ell_n \tag{34}$$

$$\begin{aligned}\frac{\dot{\kappa}_n}{\kappa_n} &= x_s - x_n + (1-\tau_n)\,\hat{w}_n(m) - (1-\tau_s)\,\hat{w}_s(m,\kappa_n)(1-m\ell_n) + \\ &\quad + \alpha\left[\frac{a_n\left(1-\theta+\theta m^{1-\frac{1}{\phi}}\right)^{\frac{1-\alpha}{1-1/\phi}}}{1-\sigma_n} - \frac{a_s\left(1-\omega+\omega\kappa_n^{1-\frac{1}{\varphi}}\right)^{\frac{1-\alpha}{1-1/\varphi}}(1-m\ell_n)^{1-\alpha}}{1-\sigma_s}\right] \\ &\quad - (1-\tau_n)\,\hat{w}_m(m)\,\kappa_n m\ell_n,\end{aligned} \tag{35}$$

where, based on (26)-(28), to simplify the system we set

$$\frac{w_s}{k_s} = (1-\alpha)\,a_s(1-m\ell_n)^{-\alpha}\left(1-\omega+\omega\kappa_n^{1-1/\varphi}\right)^{\frac{1-\alpha}{1-1/\varphi}} \equiv \hat{w}_s(m,\kappa_n) \tag{36}$$

$$\frac{w_n}{k_n} = (1-\alpha)(1-\theta)\, a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} \equiv \hat{w}_n(m) \tag{37}$$

$$\frac{w_m}{k_n} = (1-\alpha)\,\theta a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha\phi)/(\phi-1)} m^{-1/\phi} \equiv \hat{w}_m(m), \tag{38}$$

and using (31) and (32) we have that

$$\tau_n = \frac{\sigma_n \alpha a_n \left(1-\theta+\theta m^{1-1/\phi}\right)^{(1-\alpha)/(1-1/\phi)}}{(1-\sigma_n)\left[\hat{w}_m(m)\, m + \hat{w}_n(m)\right]} \tag{39}$$

$$\tau_s = \frac{\sigma_s \alpha a_s \left[1-\omega+\omega\kappa_n^{1-1/\varphi}\right]^{\frac{1-\alpha}{1-1/\varphi}} (1-m\ell_n)^{1-\alpha}}{(1-\sigma_s)(1-m\ell_n)\,\hat{w}_s(m,\kappa_n)}. \tag{40}$$

Next, rearranging (7) and then manipulating terms, we have that the shadow price of migration evolves over time according to

$$\frac{\dot{\xi}}{\xi} = \rho+\eta-\frac{\left[(1-\tau_n)\,\hat{w}_m(m)\,\kappa_n-(1-\tau_s)\,\hat{w}_s(m,\kappa_n)\right]\ell_n}{\xi x_s}. \tag{41}$$

Finally, Plugging (8) into (1), we can write the law of motion of the immigration ratio of North as

$$\frac{\dot{m}}{m} = \frac{1}{m\ell_n}\left(\frac{\xi}{\chi\ell_n}\right)^{1/\gamma} - \eta. \tag{42}$$

Starting from any pair of initial values $\kappa_n(0)$ and $m(0)$, the time evolution of all endogenous variables of the model can be determined from the dynamic system (33)-(42). In the system, variables x_n, x_s and ξ act as non-predetermined/jump variables, and κ_n and m act as predetermined/state variables. A rest point for system (33)-(42) occurs when $\dot{x}_n = \dot{x}_s = \dot{\kappa}_n = \dot{m} = \dot{\xi} = 0$ holds simultaneously. When this happens, the world economy is said to be on its BGP equilibrium, where immigration ratio of North, m^*, and shadow price of migration, ξ^*, are constant over time, while per capita consumptions, c_n and c_s, and capital stocks, k_n and k_s are time-varying variables growing exponentially over time according to

$$c_i(t) = c_i(0)e^{g_i^* t} \text{ and } k_i(t) = k_i(0)e^{g_i^* t} \text{ for } i = (s,n),$$

where g_i^* is the (constant) BGP growth rate of the economy i.

Equations (33)-(42) complete the construction of the reduced-form dynamic system of the model. Unfortunately, due to the mathematical complexity of the system, we are neither able to get a closed form solution for the long-run equilibrium, nor to establish the existence of a unique BGP equilibrium analytically. Consequently, in Section 3.2.1 we will calibrate the model to real data and will show that there exists a unique, asymptotically saddle-path stable, BGP equilibrium.

However, before we start our study of the dynamic property of the model, we need to answer the following questions: who is really responsible for the convergence in income per capita, either technology diffusion or migration? And also: what is the role played by migration in shaping the long-run dynamics of the model? To address these issues, in the next section we will present a reduced version of the benchmark model without migration and will perform some simulation exercises based on US and Mexico real data.

3 Migration, technology diffusion and growth

In this section, we analyze how knowledge diffusion and migration can effect long-run growth. To this end, we begin by presenting a reduced version of the two-country AK model of Section 2, where workers are not allowed to move across countries. Here our goal is to theoretically show how knowledge transmission can actually reduce income inequality and promote the emergence of a sustained process of cross-country convergence in income per capita. Then, we will calibrate both the reduced model without migration and the full model with migration to US and Mexico, and will investigate whether migration can play an active role in stimulating capital accumulation and reducing cross-country income inequality.

3.1 Technology diffusion as convergence device

Consider the benchmark model of Section 2 and assume that $\sigma_n = \sigma_s = \theta = m = 0$, such that governments do not provide any policy measure to support capital accumulation and Southern households have no incentive to send their members to North as immigrant workers.

Without migration, it can be shown that the 5×5 reduced-form equilibrium system (33)-(42) boils down to the simpler 3×3 dynamic system

$$\frac{\dot{x}_n}{x_n} = x_n - \rho - (1-\alpha)\, a_n \tag{43}$$

$$\frac{\dot{x}_s}{x_s} = x_s - \rho - (1-\alpha)\, a_s \left(1 - \omega + \omega \kappa_n^{1-1/\varphi}\right)^{\frac{1-\alpha}{1-1/\varphi}} \tag{44}$$

$$\frac{\dot{\kappa}_n}{\kappa_n} = x_s - x_n + a_n - a_s \left(1 - \omega + \omega \kappa_n^{1-1/\varphi}\right)^{\frac{1-\alpha}{1-1/\varphi}} . \tag{45}$$

In system (43)-(45), x_n x_s behave as control variables and κ_n behaves as state variable. The equilibrium properties of the long-run equilibrium of the model without migration are thus established by the following proposition.

Proposition 1 *If*

$$a_n > (1-\omega)^{(1-\alpha)/(1-1/\varphi)}\, a_s,$$

then the two-country AK model without migration predicts a unique BGP equilibrium where: (i) *cross-country income inequality is entirely determined by technology gap,* κ_n^**, according to*

$$\frac{y_n^*}{y_s^*} = \kappa_n^* = \left[\frac{\left(\frac{a_n}{a_s}\right)^{(1-1/\varphi)/(1-\alpha)} - (1-\omega)}{\omega} \right]^{1/(1-1/\varphi)} ; \tag{46}$$

(ii) *the whole world economy grows at the same constant rate*

$$g^* = \alpha a_n - \rho; \tag{47}$$

(iii) *the BGP equilibrium is asymptotically saddle-path stable.*

Proof. See Appendix A. ■

Two main results emerge from Proposition 1. The first result is that knowledge diffusion causes the 'unconditional' divergence characterizing standard AK models to disappear. In fact, the saddle-path stability of the BGP equilibrium shown in item (iii) of the proposition implies the tendency of the model to predict 'conditional' convergence in the same fashion of Neoclassical growth models; i.e., the tendency of all countries in the world economy to converge towards the same growth rate, but not towards the BGP level of income per capita.

To better grasp this point, consider the special case of $\omega = 0$, which contemplates the case of a two-country dynamic model consisting of two independent AK economies without migration and cross-country spillovers. In such a scenario, it is easy to check that the BGP growth rates of North and South are $g_n^* = \alpha a_n - \rho$ and $g_s^* = \alpha a_s - \rho$ respectively, from which it follows that $g_n^* > g_s^*$ since $a_n > a_s$. From this and (46), it follows that the per capita income ratio y_n/y_s perpetually diverges over time, due to the fact that the BGP value of the technology distance variable κ_n tends to explode as time passes.

Thus, in line with standard AK models *à la* Romer (1986), the world economy characterized by an asymmetric distribution of baseline technological knowledge, a_i, and immobile labor predicts cross-country 'unconditional' divergence in income per capita, and then a progressive (and perpetual) increase in cross-country income inequality, in a fashion similar to that documented by Barro and Sala-i-Martin (1992) and Mankiew et al. (1992). Yet, as equation (47) in Proposition 1 clearly shows, when technological knowledge is allowed to move internationally, κ_n acts as a converging device since it stops the tendency of productivity growth in South of lagging behind that of North.

The second result emerging from Proposition 1 is that what really matters for long-run economic growth is the baseline knowledge of North. Indeed, for 'conditional' convergence to replace 'unconditional' divergence, it must be that technology transmission is strong enough to guarantee labor productivity to equalize at a global level, thereby allowing the per capita income ratio, y_n^*/y_s^*, to stabilize around an equilibrium level.

However, assuming that both North and South are in their own BGP equilibria, what are the effects of lifting labor mobility barriers on their economies? Is it possible to establish that migration can generate asymmetric effects on their economies, and thus to produce winners and losers from a macroeconomic point of view?

To address these questions, in the rest of this section we will calibrate the model for the case of US and Mexican economies and study this issue numerically.

3.2 Migration, growth and income inequality

3.2.1 Calibration

This section provides a quantitative analysis of the effects of migration on the two-country world economy. Table 1 shows the benchmark values for the parameters used in the simulations.

Parameter	Description	Value
a_n	Baseline knowledge stock of North	0.301
a_s	Baseline knowledge stock of South	0.147
α	Capital share	0.33
ϕ	Substitution elasticity for workers	20
θ	Immigrant labor share	0.409
φ	Substitution elasticity for capital	20
ω	Southern weight on productivity	0.5
ρ	Subjective discount rate	0.05
γ	Elasticity of immigrant workers supply	0.38
η	Return migration rate	0.153
ℓ_n	Relative size of Northern population	2.578
χ	Emigration disutility parameter	338,226

Table 1: Benchmark calibration of the model.

As for the baseline knowledge parameters a_n and a_s, the ratio a_n/a_s is set to 2.055 to match the average ratio of Mexican immigrants over the total US population of roughly 0.033 for the year 2020,[5] whereas a_n is set equal to 0.301 to match the U.S. average

[5]Source: United Nations, Department of Economic and Social Affairs, data on International Migrant Stock 2020.

annual GDP per capita growth of roughly 2% over the period 1960 – 2019.[6] Further, we choose the capital share parameter $\alpha = 0.33$ to match the empirical evidence of Gollin (2002). To parametrize ϕ we follow Ottaviano and Peri (2012), who find an elasticity of substitution between US natives and immigrants of 20, whereas the CES share parameter θ is set to 0.409 to match the average wage ratio between migrant and native workers of 1.22 over the decade 2010 – 2020.[7]

Regarding the parameters related to households preferences and population characteristics, we follow the findings of Gourinchas and Parker (2002) and set the subjective discount rate ρ to 0.05. In order to match observed data on average annual flow of immigrants from Mexico towards the U.S. during the period 2010 – 2019 of approximately 0.13% over the total U.S. population, we choose $\eta = 0.153$. This, in turn, implies an average duration for a working position of immigrants of $1/\eta \approx 6.5$ years.

The population size ratio ℓ_n is set to match the ratio between the U.S. total population and the aggregate population in Mexico of roughly 2.578.[8] The elasticity γ is instead set to 0.38 (i.e. $\gamma = 2.632$) to match the estimates on the Frisch Elasticity of Heathcote et al. (2014).[9] Moreover, we set the disutility parameter χ equal to 338,226 so that the remittances-to-GDP ratio of Mexico is equal to roughly 4%. Finally, as there are is no empirical evidence on how to calibrate the cross-country spillovers parameters, the elasticity of substitution between Northern and Southern capital is set equal to the elasticity of substitution between Northern and immigrant workers, i.e. $\varphi = \phi = 20$, whereas the share parameter $\omega =$ is set to 0.5 for simplicity.

3.2.2 Quantitative analysis

Based on the calibrated parameters shown in Table 1, we can run two distinct simulation exercises: one to obtain the BGP equilibrium of the reduced model without migration and one to obtain the BGP equilibrium of the complete model with migration.[10] The results of the simulations are reported in Table 2.

According to our simulations, migration dramatically slows down long-run economic growth at a world level by, on average, 2.95 pp. per year. However, such a reduction in g^* is not equally spread across countries. Indeed, as Table 2 clearly states, allowing for international labor mobility reduces cross-country income and wage inequality ($y_n^*/y_s^* \downarrow$

[6]Source: World Bank, Constant GDP per capita for the United States, retrieved from FRED, Federal Reserve Bank of St. Louis.

[7]Source: U.S. Bureau of Labor Statistics, Employed full time: Earnings of foreign born as percent of native born 16 years and over, retrieved from FRED, Federal Reserve Bank of St. Louis.

[8]Source: Penn World Table 10 database.

[9]Note that the chosen parametrization of γ is also within the estimates ranges reported by Pencavel (1986) and Whalen and Reichling (2017) of 0 – 0.45 and 0.27 – 0.53, respectively.

[10]As far as the analytical details of the derivation of BGP equilibrium of the baseline model with migration is concerned, interested readers are referred to the online Appendix B, available on request.

Variable	Without migration	With migration	Change
g^*	0.0495	0.0200	-2.95 p.p.
κ_n^*	4.9326	2.8123	-43.0%
m^*	n.a.	0.0330	...
ξ^*	n.a.	9.4898	...
y_n^*/y_s^*	4.9326	2.8123	-42.4%
$\mathcal{R}^*$	n.a.	0.0400	...
w_n^*/w_m^*	n.a.	1.2200	...
w_m^*/w_s^*	n.a.	2.0534	...
w_n^*/w_s^*	4.93262	2.5052	-49.2%

Table 2: BGP effects of labor migration.

and $w_n^*/w_s^* \downarrow$), and stimulates a process of catching up in technical knowledge ($\kappa_n^* \downarrow$), thereby signaling that migration is particularly beneficial for South.

The mechanics explaining such results are not difficult to grasp and rotate around the positive 'scale' effect that migration generates in North, which stimulate Northern firms to increase gross investment to accumulate more capital. This, in turn, enlarges the stock of technical knowledge of North and determines an increase in the flow of cross-country knowledge externality that spills over to South. Moreover, increased productivity due to cross-country externality and the emergence of remittances as a new source of funding for Southern capital accumulation cause the South's level of income per capita, y_s, and wages, w_s, to grow temporarily faster than North, thereby inducing the per capita income ratio, y_n/y_s, to shrink over time.

In conclusion, according to our analysis, migration can accelerate convergence in productivity and income per capita. However, a natural question raising from this result is whether the policy implications of standard AK models may change if technical knowledge and labor are allowed to freely flow internationally. We will dedicate the remainder of the paper to tackling this question.

4 Growth-enhancing policies

In this section, we propose three comparative statics exercises to assess how pro-migration and pro-growth policies can affect long-run growth. We will begin by analyzing how a pro-growth policy consisting in the introduction of a subsidy to capital accumulation can affect the long-run pace of global growth. In doing so, we will distinguish the case in which it is the frontier country that introduces the policy, from that in which it is the non-frontier country that introduces the subsidy. Then, we will restrict our attention to analyzing the effects on growth of introducing a pro-immigration policy that increases the average duration of work permits in North.

4.1 Investment subsidy in North

We begin by simulating a benchmark scenario where there are no subsidies to capital accumulation, i.e. $\sigma_n = \sigma_s = 0$, and no taxes levied by the governments, $\tau_n = \tau_s = 0$. Then, we suppose that, at a certain moment of time, the Northern government decides to introduce a 5% ($\sigma_n = 0.05$) investment subsidy to stimulate capital accumulation and growth in North. Table 3 shows the comparative statics results.

Variable	Benchmark	After-policy	Change
g^*	0.0200	0.0237	0.37 p.p.
κ_n^*	2.8123	3.1311	11.3%
m^*	0.0330	0.0337	0.07 p.p.
ξ^*	9.4898	10.052	5.9%
y_n^*/y_s^*	2.8123	2.9745	5.8%
$\mathcal{R}^*$	0.0400	0.0487	0.87 p.p.
w_n^*/w_m^*	1.2200	1.2213	0.1%
w_m^*/w_s^*	2.0534	2.1639	5.4%

Table 3: BGP effects of introducing a 5% investment subsidy in North.

As shown by the table, introducing a 5% investment subsidy in North stimulates Northern firms to rent more capital and hire more workers. This pushes upwardly both the immigration ratio, m^* (+0.07 pp.), and the relative capital stock of North, κ_n^* (+11.3%), but has a negative impact on cross-country income inequality, as it causes the ratio y_n^*/y_s^* to increase in the new BGP equilibrium by 5.8%.

As far as economic growth is concerned, because g^* permanently increases in the post-policy BGP equilibrium by almost 0.4 pp., it follows that the long-run effects that the subsidy exerts on the BGP equilibrium growth rate are positive overall. Such a rise in g^* can be explained by the growth-pushing effect induced by the subsidy σ_n, which generates a positive incentive to accumulate capital in North and, in turn, stimulates growth not only North, but also in South through cross-country knowledge spillovers.

Interestingly, even though the native-immigrant wage ratio w_n^*/w_m^* is barely affected by the introduction of the subsidy, the emigration wage premium w_m^*/w_s^* stretches in the new BGP equilibrium (+5.4%). This is because the introduction of the subsidy to Northern investment makes it profitable for Northern firms to increase workers' demand in North. Further, the relative increase in the immigrant wage, along with the surge in immigrant workers, strengthen the overall flow of remittances to South, so that $\mathcal{R}^*$ rises by 0.87 pp. in the new BGP equilibrium.

4.2 Investment subsidy in South

We now turn our attention to the case where it is the Southern government that decides to introduce a 5% subsidy to private investment, whereas in North no policy is undertaken. Table 4 shows the results of our comparative statics analysis.

Variable	Benchmark	After-policy	Change
g^*	0.0200	0.0200	0
κ_n^*	2.8123	2.5162	-10.5%
m^*	0.0330	0.0321	-0.09 p.p.
ξ^*	9.4898	8.8348	-6.9%
y_n^*/y_s^*	2.8123	2.6486	-5.8%
$\mathcal{R}^*$	0.0400	0.03318	-0.7 p.p.
w_n^*/w_m^*	1.2200	1.2183	-0.1%
w_m^*/w_s^*	2.0534	1.9427	-5.4%

Table 4: BGP effects of introducing a 5% investment subsidy in South.

Overall, introducing the investment subsidy in South causes a 5.8% drop in per capita income ratio, y_n^*/y_s^*, implying that promoting capital accumulation policies in developing countries can effectively reduce income inequality worldwide. The emergence of this catching-up process in income per capita can be explained by the fact that increasing σ_s leads to an increase in capital accumulation and employment in South, and to a decrease in the immigration, ratio m^* (-0.09 pp.), and technology distance, κ_n (-10.5%). Moreover, in line with the increase in workers' demand in South, we have that the emigration wage premium w_m^*/w_s^* decreases in the new equilibrium by 5.4%, whereas the native-immigrant wage ratio w_n^*/w_m^* remains mostly unaffected by the presence of the subsidy, as it falls slightly by only 0.1%.

As far as the remittances-to-income ratio is concerned, the impact of the subsidy on the level of $\mathcal{R}^*$ is clearly negative (-0.037 pp.). This is the result of the combined effect of a decrease in the nominal flow of remittances received by the Southern economy, R^*, and an increase in Southern income, Y_s^*.

Interestingly, the BGP equilibrium growth rate g^* is virtually unaffected by the introduction of the subsidy. This is a relevant result for our analysis as it emphasize the fact that only Northern pro-growth policies are able to permanently rise the global growth (see Section 4.1), whereas the introduction of the same policy in South fails to stimulate long-run growth at a world level.

4.3 Extending visas' duration in North

In this subsection, we investigate the effects of extending visas' duration in North. Visa regulations can indeed affect the migration flows between North and South by easing the issuance of long-period work permits, and therefore affect the overall performance of the entire world economy.

Consider the parametrization without investment subsidy of Table 1 and suppose that each economy is in its own BGP equilibrium. Assume that, at $t = 0$, the repatriation rate η permanently decreases by 50% in response to the introduction of a new migration policy that aims at stretching the time validity of visas.

Variable	Benchmark	After-policy	Change
g^*	0.0200	0.0216	1.6 p.p
κ_n^*	2.8123	3.5342	25.7%
m^*	0.0330	0.0782	4.52 p.p.
ξ^*	9.4898	14.843	56.4%
y_n^*/y_s^*	2.8123	3.5342	25.7%
R^*	0.0400	0.1004	6.04 p.p.
w_n^*/w_m^*	1.2200	1.2738	4.4%
w_m^*/w_s^*	2.0534	2.0867	1.7%

Table 5: BGP effects of a 50% permanent fall in the repatriation rate, η.

Table 5 shows the results of the comparative statics analysis. Because of the decrease in η, the average duration of the working positions of immigrants doubles from 6.54 to 13 years. This has an important positive impact on the shadow price of emigration, ξ^*, which increases in the after-policy BGP equilibrium by +56.4%.

The increase in ξ^* causes the BGP immigration rate, m^*, to rise by 4.52 pp. and per capita income inequality, y_n^*/y_s^*, to increase by 25.7%. The increase in y_n^*/y_s^* can be explained by the positive 'scale' effect that the rise in m^* generates on the Northern economy, and thus by the increase in Northern labor supply that makes it profitable for Northern firms to rise gross investment in physical capital to expand production. Indeed, the rise in m^* causes the relative size of the capital stock of North, κ_n^*, to increase in the after-policy BGP equilibrium, but such an increase in Northern investment comes along with only a slight increase in the native-immigrant wage ratio, w_n^*/w_m^*, which increases by only 4.4%.

As far as the emigration wage premium is concerned, our numerical analysis reveals that the ultimate impact that a fall in η has on the wage ratio, w_m/w_s, is positive (1.7%), owing to the tendency of capital accumulation and productivity to increase relatively more in North than in South. This, in turn, implies that the net economic benefit that Southern households receive from increasing emigration mainly comes from the increased flow of remittances, $\mathcal{R}^*$, that makes their disposable income increase by 0.6 GDP points in the after-shock BGP equilibrium.

Summing-up, what emerges from our results is that a pro-migration policy in North is very effective in reallocating labor across countries, but also that it dramatically enlarges per capita income inequality worldwide. Moreover, as Table 5 clearly shows, easing enlarging work permits in frontier countries has negligible effects on the equilibrium growth rate of the world economy, as it rises g^* by less than 0.2 percentage points.

5 Conclusions

In this paper, we have analyzed the effects of international migration on economic growth and income inequality through a two-country AK model with cross-country knowledge diffusion and endogenous labor migration. In the model, we assumed that: (*i*) baseline technical knowledge is asymmetrically distributed across countries, so that one country can be referred to as a frontier country, whereas the other country is regarded as a non-frontier country; (*ii*) progress in technical knowledge in the frontier country can spill over the non-frontier country in the form of productivity externality; (*iii*) workers from the non-frontier country are able to emigrate, whereas workers from the frontier country never find it convenient to move and work abroad. These features allow us to pursue the double objective of formally addressing the dynamic issues of immigration in an investment-based model of endogenous growth, and of reconciling the AK literature with the recent empirical evidence for cross-country 'unconditional' convergence.

The findings of this paper are as follows. First, we find that the introduction of knowledge diffusion makes AK models predict conditional convergence in the same fashion that Neoclassical growth models do. This result holds true regardless of the presence of international migration, as knowledge spillovers allow the non-frontier country's income per capita to catch-up with that of the frontier country by bridging productivity levels across countries. Second, we show that migration is able to speed up the catching-up process of non-frontier countries by generating a positive scale effect in the frontier countries. Third, we find that investment subsidy generates different effects on the two-country economy depending on which country introduces the subsidy policy. Indeed, since only capital accumulation in frontier countries is able to improve productivity at a world level, subsidizing gross investment in non-frontier countries turns out to be ineffective to increase productivity, though able to mitigate worldwide income inequalities. On the contrary, subsidizing investments in frontier countries stimulates migration and worldwide growth, but also increases cross-country inequalities in terms of both income and technology.

These results are in line with the recent empirical trend to convergence, which has been driven by a faster catch-up growth for non-frontier countries and a slower growth of the frontier for the last couple decades (Kremer et al., 2021). Nonetheless, our analysis builds on a simple two-country AK model and can be further extended to address other issues for future research. One significant issue to be pursued in future work would be to extend our model to also allow for trade and financial integration across countries. The AK literature already shows that international trade can act as a converging tool that makes all countries grow at the same long-run rate (see, e.g., Acemoglu and Ventura, 2002) and empirical research suggests that migration may spur bilateral trade through a number of channels (see, e.g., Ortega and Peri, 2012, 2014; Genc, 2014; Ottaviano et al., 2018), thereby affecting the interplay between migration, income inequality and growth dynamics.

Another interesting avenue of research it would be worth address is to extend the presented model to include human capital. As Greiner and Semmler (1996) and Greiner (2003) show, poverty traps as well as both local and global indeterminacy may take place when technical change is disembodied and the contribution of new gross investment to the formation of human capital is assumed to be declining in time. Hence, allowing investments to differently affect the building up of physical and human capital and assuming that physical and human capital are characterized by different depreciation rates, as in Greiner and Semmler (1996) and Greiner (2003), may lead to different growth dynamics for the world economy.

Appendix

A. Proof of Proposition 1.

This Appendix provides the formal demonstration of Proposition 1. First, using the system (43)-(45) and setting $\dot{x}_n = \dot{x}_s = \dot{\kappa}_n = 0$, we can obtain the following (unique) rest point

$$x_n^* = x_s^* = \rho + (1-\alpha)\, a_n \tag{A.1}$$

$$\kappa_n^* = \left[\frac{\left(\frac{a_n}{a_s}\right)^{(1-1/\varphi)/(1-\alpha)} - (1-\omega)}{\omega} \right]^{1/(1-1/\varphi)} . \tag{A.2}$$

Recalling equations (21) and (22) for Northern and Southern per capita income, respectively, it is easy to check that $y_n^*/y_s^* = \kappa_n^*$ for $m^* = 0$. This proves part (i) of the proposition.

We now move to part (ii) of Proposition 1. Let us consider the Euler equations (5) and (6). Recall that in this version of the model $\sigma_n = \sigma_s = \theta = m = 0$. Consequently, from (5) and (6), we can obtain the following steady-state growth rate for Northern and Southern consumption

$$\frac{\dot{c}_n}{c_n} = \frac{\dot{c}_s}{c_s} = \alpha a_n - \rho. \tag{A.3}$$

Because in the BGP equilibrium k_n, c_n, k_s, and c_s must necessarily grow at the same rate, we can conclude that the following relations $\dot{c}_n/c_n = \dot{k}_n/k_n = \dot{c}_s/c_s = \dot{k}_s/k_s = g^* = \alpha a_n - \rho$ hold in the long run. This demonstrates part (ii) of Proposition 1.

To prove the last part of Proposition 1, we take the first-order Taylor expansion of system (A.1)-(A.2) around the rest point, $\langle \hat{x}_n, \hat{x}_s, \hat{\kappa}_n \rangle$, to obtain

$$\begin{pmatrix} \dot{x}_n \\ \dot{x}_s \\ \dot{\kappa}_n \end{pmatrix} = \hat{J} \begin{pmatrix} x_n - \hat{x}_n \\ x_s - \hat{x}_s \\ \kappa_n - \hat{\kappa}_n \end{pmatrix},$$

where

$$\hat{J} = \begin{pmatrix} \rho + (1-\alpha)\, a_n & 0 & 0 \\ 0 & \rho + (1-\alpha)\, a_n & -\Upsilon\, (\alpha - 1)\, \omega \left[\rho + (1-\alpha)\, a_n\right] \\ -\left[\frac{\left(\frac{a_n}{a_s}\right)^{\frac{(1-1/\varphi)}{(1-\alpha)}} - (1-\omega)}{\omega}\right]^{\frac{1}{1-\frac{1}{\varphi}}} & \left[\frac{\left(\frac{a_n}{a_s}\right)^{\frac{(1-1/\varphi)}{(1-\alpha)}} - (1-\omega)}{\omega}\right]^{\frac{1}{1-\frac{1}{\varphi}}} & -\Upsilon \end{pmatrix},$$

is the Jacobian matrix of the linearized system and

$$\Upsilon \equiv (\alpha - 1)\, a_s \left(\frac{a_n}{a_s}\right)^{\frac{1/\varphi - \alpha}{1-\alpha}} \left[1 - \omega - \left(\frac{a_n}{a_s}\right)^{\frac{1-1/\varphi}{1-\alpha}}\right]$$

is a collection of given parameters, with $\Upsilon > 0$ if and only if

$$a_n > (1-\omega)^{(1-\alpha)/(1-1/\varphi)}\, a_s. \tag{A.4}$$

In the system, x_n and x_s act as non-predetermined/jump variables and κ_n acts as predetermined/state variable. As a result, if two eigenvalues of the matrix of coefficients are positive and the other one is negative, then the rest point, $\langle \hat{x}_n, \hat{x}_s, \hat{\kappa}_n \rangle$, is saddle-path stable, implying that the BGP equilibrium of the reduced model without migration is asymptotically stable.

Let us denote each element of $\hat{J}$ placed at the ith row and jth column by $\hat{\jmath}_{ij}$. The determinant of the matrix $\hat{J}$ is given by

$$\det(\hat{J}) = \hat{\jmath}_{11} \left(\hat{\jmath}_{22}\hat{\jmath}_{32} - \hat{\jmath}_{23}\hat{\jmath}_{32}\right) = -\Upsilon\alpha \left[\rho + (1-\alpha)\, a_n\right]^2,$$

which is necessarily negative under condition (A.4). The trace of $\hat{J}$ is instead given by

$$\mathrm{Tr}(\hat{J}) = \hat{\jmath}_{11} + \hat{\jmath}_{22} + \hat{\jmath}_{33} = 2\rho + (1-\alpha)\, a_n \left[1 + (1-\omega)\left(\frac{a_n}{a_s}\right)^{\frac{1/\psi - 1}{1-\alpha}}\right] > 0.$$

From linear algebra, we know that for a $N \times N$ matrix with N distinct eigenvalues, the following relations holds:

$$\det(\hat{J}) = \prod_{n=1}^{N} \lambda_n, \quad \mathrm{Tr}(\hat{J}) = \sum_{n=1}^{N} \lambda_n,$$

where λ_n is the nth eigenvalue of the $N \times N$ matrix. From this, it follows that since $\det(\hat{J}) < 0$, then either all the eigenvalues of $\hat{J}$ are negative, or two eigenvalues are positive and one negative. However, because $\mathrm{Tr}(\hat{J}) > 0$, we can rule out the latter hypothesis and conclude that only two eigenvalues out of three are positive, while the remaining one is negative.

This proves that the BGP equilibrium is asymptotically saddle-path stable under condition (A.4) and concludes the proof of Proposition 1.

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